Studies on Nanofluids: Convection Flows and Diverse Applications

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By

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DECLARATION

I hereby declare that the work presented in the thesis entitled “Studies on Nanofluids: Convection Flows and Diverse Applicaions” is based on the original work done by me under the guidance of Dr. Julie Andrews and has not been included in any other thesis submitted previously for the award of any degree. The contents of the thesis are undergone plagiarism check using iThenticate software at C.H.M.K. Library, University of Calicut, and the similarity index found within the permissible limit. I also declare that the thesis is free from AI generated contents.

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This is to certify that the thesis titled ***"Studies on Nanofluids: Convection Flows and Diverse Applications"*** submitted by **Rose Paul** to the **University of Calicut** in partial fulfillment of the requirements for the award of the Degree of ***Doctor of Philosophy in Mathematics*** is a record of original research work carried out by her under my supervision. The content of this thesis, in full or in parts, has not been submitted by any other candidate to any other University for the award of any degree or diploma.

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ABSTRACT

A substance that has the ability to flow is referred to as a fluid. Fluids are broadly classified into two types: liquids and gases. The branch of physics that deals with the behavior of fluids at rest is known as fluid statics. The combined study of fluid statics and the behavior of fluids in motion, known as fluid dynamics, is termed fluid mechanics.

Nano fluid is a colloidal mixture in which a base fluid (water, oil, ethylene glycol, etc.) is mixed with nanometre-sized particles (metals, carbides, oxides or carbon nanotubes). Nano fluids tend to upgrade and stabilize the thermal properties of the fluid which marked a revolution in the field of fluid dynamics.

The description of a system using mathematical concepts and language is known as mathematical model and the process of developing a mathematical model is known as mathematical modelling. Mathematical models and its use in natural sciences, engineering disciplines and social sciences. A mathematical model helps to explain a system and to study the effects of different components and also to make predictions about its behaviour.

The thesis entitled *Studies on nanofluids: convection flows and diverse applications* has been arranged into 7 chapters. Chapter 1 introduces the basic concepts, preliminaries and definitions to the reader. Chapter 2 explains The Impact of Volume Fraction and Nanoparticles in the Flow of Nanofluids Along the Boundary Layer Over a Flat Plate. Chapter 3 explains Impact of Volume Fraction on Boundary Layer Flow of Nanofluids Over a Moving Plate. Chapter 4 explains Effect of Mixed Convection, Plate Inclination, and Nanoparticle Volume Fraction on Nanofluid Boundary Layer Flow. Chapter 5 contains Application of Nanofluids. Chapter 6 and 7 present the concluding remarks of the thesis and proposals for future work.

പഠനസംഗ്രഹം

ഒഴുകുന്ന പദാർത്ഥങ്ങളെ ദ്രവങ്ങൾ എന്നു പറയുന്നു. ദ്രാവകങ്ങൾ, വാതകങ്ങൾ എന്നിങ്ങനെ രണ്ട് തരം ദ്രവങ്ങൾ ഉണ്ട്. ചലനാവസ്ഥയിലുള്ള ദ്രവങ്ങളെകുറിച്ചുള്ള പഠനമാണ് ഫ്ളൂയിഡ് ഡയനാമിക്സ്.

അടിസ്ഥാന ദ്രവങ്ങളായ ജലം, എണ്ണ, എത്തിലീൻ ഗ്ലൈക്കോൾ എന്നിവയിൽ നാനോ പദാർത്ഥങ്ങളായ ലോഹങ്ങൾ, ലോഹ ഓക്സൈഡുകൾ, കാർബൈഡുകൾ, കാർബൺ നാനോ ട്യൂബുകൾ മുതലായവ സംയോജിപ്പിക്കുമ്പോൾ ഉണ്ടാകുന്ന കോളോയിഡൽ മിശ്രിതങ്ങളാണ് നാനോ ദ്രവങ്ങൾ. നാനോപദാർത്ഥങ്ങൾ ചേരുമ്പോൾ അടിസ്ഥാനദ്രവങ്ങളുടെ താപചാലകശേഷി വർദ്ധിക്കുന്നു. ഈ കണ്ടെത്തലിനു ഫ്ളൂയിഡ് ഡയനാമിക്സ് പഠനശാഖയിൽ വിപ്ലവകരമായ മാറ്റങ്ങൾ കൊണ്ടുവരാൻ സാധിച്ചിട്ടുണ്ട്.

ഗണിതശാസ്ത്ര ആശയങ്ങളും ഭാഷയും ഉപയോഗിച്ചുകൊണ്ടുള്ള ഒരു വ്യൂഹത്തെ ഗണിതശാസ്ത്രമാതൃകയെന്നും അത്തരം മാതൃകകൾ വികസിപ്പിച്ചെടുക്കുന്ന പ്രക്രിയയെ ഗണിതശാസ്ത്ര മോഡലിംഗ് എന്നും പറയുന്നു. ജീവശാസ്ത്രം, സാമൂഹ്യശാസ്ത്രം, എഞ്ചിനീയറിങ് തുടങ്ങി വിവിധ പഠനമേഖലകളിൽ ഗണിതശാസ്ത്ര മാതൃകകളുടെ ഉപയോഗങ്ങളുണ്ട്. ചലനാവസ്ഥയിലുള്ള ദ്രവങ്ങളുടെ പ്രത്യേകതകൾ ഗണിതശാസ്ത്രാശയങ്ങൾ ഉപയോഗിച്ച് വികസിപ്പിക്കുന്നതാണ്.

"സ്റ്റഡീസ് ഓൺ നാനോഫ്ളൂയിഡ്സ്: കൺവെക്ഷൻ ഫ്ളോസ് ആൻഡ് ഡൈവേഴ്സ് അപ്ലിക്കേഷൻസ് " എന്ന പ്രബന്ധത്തിൽ 7 അദ്ധ്യായങ്ങളാണ് ഉൾപ്പെടുത്തിയിട്ടുള്ളത്. ഫ്ളൂയിഡ് ഡയനാമിക്സിന്റെ അടിസ്ഥാന നിർവചനങ്ങളും ആശയങ്ങളും അതിനെക്കുറിച്ചുള്ള പഠനങ്ങളുടെ അവലോകനവുമാണ് ഒന്നാമത്തെ അദ്ധ്യായത്തിൽ ഉൾപ്പെടുത്തിയിരിക്കുന്നത്. പ്രയോഗ സാധ്യതകളെ മുൻനിർത്തികൊണ്ട് നാനോദ്രവങ്ങളുടെ വിവിധ സാഹചര്യങ്ങളിലുള്ള ഒഴുക്കിനെ പ്രതിപാദിക്കുന്ന പഠനങ്ങളും അതിന്റെ അവലോകനവുമാണ് 2 മുതൽ 4 വരെയുള്ള അദ്ധ്യായത്തിലുള്ളത്. നാനോഫ്ളൂയിഡുകളുടെ പ്രയോഗം, ഈ പഠനങ്ങളിൽനിന്നുള്ള നിഗമനങ്ങൾ ഭാവിയിലേക്കുള്ള സാധ്യതകൾ, ഉപസംഹാരം എന്നിവയാണ് 5 മുതൽ 7 വരെയുള്ള അദ്ധ്യായങ്ങളിൽ ഉൾപ്പെടുത്തിയിരിക്കുന്നത്.

I dedicate this work

to my parents who paved the way for me during their lifetime,

to my husband and children,

to my friends

for their unconditional love and support.

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Rose Paul

CONTENTS

CHAPTER 1

INTRODUCTION

1.1 Fluids

Etymologically, the word fluid derives its root word from a Latin word fluere, which means to flow. The unique feature of a fluid is the capability to flow and not to have a definite shape. Fluids are mainly two types; which are liquids and gases, are substances capable of flowing. Fluid dynamics also known as hydrodynamics, focused on the study of the static and mobile properties of fluids or substances. The study of fluid behaviour at static phase is termed as fluid statics and the study of fluid behaviour at mobile phase, in absence of pressure force termed as fluid kinematics. Hydrodynamics as an intersectional field of mathematics and physics critically studies the role of force in impacting the fluid dynamics.

1.2 Nanofluids

(here after nanofluids called as nfs)

1.2.1 History of Nanofluids

Nfs is a term coined by Choi (Choi & Eastman, 1995) to describe a new class of nanotechnology-based heat transfer fluids with thermal properties superior to their host fluids or conventional particle fluid suspensions. These fluids are crucial for ultra-high-performance cooling, which is indispensable and requisite for a wide range of industry-related innovations in the field of automation in a worldwide developmental leap of devices, machines and scientific acumen. Nfs are composed of nano-sized particles, typically less than $100nm$, that are suspended in liquids (Das et al., 2006). A nanometre, a measure derived from the morpheme nano, represents a scale of one part per billion of a meter. The prime objective of nfs is their potential to achieve the zenith of thermal properties (Al-Douri, 2022) (Mamdouh, 2023). This is accomplished by ensuring uniform dispersion on one hand and stable suspension of nanoparticles on other hand, inside host fluids at the lowest concentrations. Nfs, with their ability to act as coolants, can significantly enhance and catalyse the thermal properties of the host fluids, resulting in more effectual, operative, and uniform heat removal from systems that require precise temperature control at high heat variations (Ramesh & Prabhu, 2011). Compared to pure liquid nfs offers new possibilities to enhance performance of heat transfer hence they are considered to be the next generation heat transfer fluids and having different properties related to transfer of heat as compared to other formal fluids (Wang & Mujumdar, 2007).

1.2.2 Advancement of Nfs

Nanofluids are the colloidal suspension that contain nanoparticles in a base fluid, showing substantial advancement due to their properties like thermal conductivity, viscosity, etc. and imaginable usage across various fields (Choi & Eastman, 1995) (Fuskele & Sarviya, 2017).

1.2.3 Incompressible fluid and compressible fluid

A fluid has two key properties; incompressible and zero viscosity. The former is density remains constant under external forces and the latter is no internal friction between its particles. A fluid which has incompressible property. Such fluids are said to be incompressible fluids. Generally, liquids are considered as incompressible fluids. A fluid whose density alters when a force is applied on it is called as compressible fluids. Gases are generally regarded as compressible fluids.

1.2.4 Real fluids and Ideal fluids

The fluids with viscosity are known as real fluids. Hypothetical fluid which are incompressible and without viscosity is known to be ideal fluids.

1.2.5 Newtonian fluids and non-Newtonian fluids

The fluids that obey Newton's law of viscosity are called Newtonian fluids (Franco & Partal, 2010). Newton' s law of viscosity states that the shear stress within a fluid is directionally proportional to the velocity gradient (Newman, 2008). Newtonian fluids have constant viscosity regardless of the shear rate - rate of deformation (Whorton et al., 2025). Some examples of such fluids are water, gases and alcohol. Non-Newtonian fluids which do not obey Newton's law of viscosity (Chhabra, 2010). Examples for such fluids are blood, paint and toothpaste.

1.2.6 Single-phase and Multi-phase fluids

A fluid having only one state of matter like either liquid or gas is termed as the single-phase fluids and when a fluid shows a mixture of states like liquid droplets in a gas or bubbles in a liquid is termed as multi-phase fluid

1.3 Types of fluid flow:

1.3.1 Steady and Unsteady flow (Nabavi, 2009):

Steady fluid flow is the type of fluid flow in which the properties of fluids like pressure, velocity, and density remain unchanged at a particular point of time. It can be mathematically represented as partial derivative of any fluid property with respect to time is zero. Whereas the unsteady fluid flow is the case in which the properties at a point alter with time, represents that the partial derivative of at least one fluid property is not zero with respect to time

1.3.2 Compressible and Incompressible fluid:

Compressibility means the ability to change the volume of a fluid with mass. Gas, vapour, steam are considered as compressible fluids, as they don’t have definite volume. Compressible fluid flow accounts for changes in density due to variation in temperature and pressure and it requires the set of equations of conservation- mass, momentum and energy. The reason is that density is no longer constant. In compressible flow analysis the phenomenon like expansion of fan, shock waves and changes in acoustic speed are important. Compressible fluid flow is having applications such as rocket expulsion, high speed gas pipelines and supersonic aircraft. In case of incompressible fluids, they are considered to possess a definite volume. Incompressible flows are simpler to analyze and it is used in situation involving low-speed gas flow. As compared with incompressible fluid, compressible flow is more complex due to variable density and important for understanding high-speed gas dynamics.

1.3.3 Laminar and Turbulent flow:

The type of fluid flow in which the individual streamline moves parallelly in a regular well-ordered manner is called laminar fluid flow. In this case the mixing of fluid layers is observed. The flow of blood through blood capillaries and flow of oil through circular pipe are example for laminar flow. There are no cross-currents perpendicular to the direction of flow, nor eddoes or swirls of fluids. Turbulent flow of fluid indicates the irregular path in which the streamlines are mixed with each other, so that the mixing of fluid layers is observed. While smoking the movement of smoke closer to the cigarette considered as the example of laminar flow and the movement of smoke after certain distance can be considered to be the example of turbulent flow.

1.3.4 Rotational and Irrotational flow:

Vorticity (Ω) measures the rotation of fluid and is defined as the curl of the velocity. Rotational flow is the flow in which the fluid particles rotate about their own axis (Childs, 2010) (when $\Omega \neq 0$). Water swirling in a draining sink is an example of rotational flow. Irrotational flow is the flow in which fluid particles do not rotate about their own axis (Taylor, 1917) (when $\Omega = 0$). Flow around an airplane wing in potential flow theory is an example of irrotational flow.

1.3.5 1, 2 and 3-Dimensional flow:

Based on the number of coordinates in space, fluid flow can be classified as one, two, three dimensional. In one dimensional form the velocity, temperature and concentration of flow profiles are explained with space and time coordinates. Two dimensional and three-dimensional flow are described

with more than one space coordinate. One-dimensional flow varies in one direction ($x-$ direction), two-dimensional flow varies in two directions (both x and $y-$ directions), whereas three- dimensional flow varies with three directions (x, y and $z-$ directions)

1.3.6 Uniform and non-uniform flow:

Uniform flow refers to the condition where the velocity of fluid flow remains constant in both magnitude and direction at every point in the flowing fluid, at any specific time (Chaudhry, 2008). Non-uniform flow refers to the condition where the velocity of fluid flow changes at every point in the flowing fluid at any specific time.

1.4 Fundamental Equations of Fluid Flow

1.4.1 Equation of Continuity

The equation of continuity is a mathematical statement of the law of conservation of mass specifically applied to fluid flow (Al Balushi, 2015). According to the law, the rate of increase in fluid's mass within a given region equals the rate of flow through its boundary. It essentially states that for the steady flow of an incompressible fluid, the mass flow rates (i.e., the mass of fluid passing a point per unit time) remains constant throughout the flow.

In vector form, it can be expressed as (Bansal, 1977):

$$\frac{\partial \rho}{\partial t} + \nabla . \left(\rho \vec{V} \right) = 0$$

where ρ is the fluid density and $\vec{V}$ is the velocity vector.

This law also expressed as in follows:

$$m = \rho \mathcal{A} \vec{V}$$

where ρ is the fluid density; $\mathcal{A}$ is the area of cross-section; $\vec{V}$ is the velocity vector (as mentioned above).

For steady compressible flow, the equation of continuity simplifies to:

$$\nabla.\left(\rho\vec{V}\right) = 0$$

For incompressible flow, the equation of continuity simplifies to:

$$\nabla.\vec{V} = 0$$

1.4.2 Equation of Momentum (Navier-Stokes' Equation)

The equation of momentum is derived from Newton's second law of motion which states that the total force amounts to the rate of change of linear momentum. In vector notation it is written in the form (Bansal, 1977):

$$\rho\frac{D\vec{V}}{Dt} = \rho\vec{F} + \mu\nabla^2\vec{V} - \nabla P$$

Here ρ is the fluid density, μ is the dynamic viscosity, $\vec{V}$ is the velocity vector, P is the pressure, $\vec{F}$ is the force, and $\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \left(\vec{V}.\nabla\right)$ is expressed as the material derivative (Kumar et al., n.d.).

1.4.3 Equation of Energy

The law of conservation of energy states that the total energy in a closed system is conserved (Crowell, 2001). This forms the basis for deriving the equation of energy. It is written as (Bansal, 1977):

$$\rho\frac{DE_t}{Dt} = \rho\left(v_i\frac{Dv_i}{Dt} + \frac{DI}{Dt} + v_j\frac{\partial K}{\partial x_j}\right)$$

where E_t is the total energy, ρ is the density, K is the potential energy, I is the internal energy, and $\left(v_i, v_j\right)$ are the velocity components.

1.5 Reynolds Number

In fluid dynamics, Reynolds number refers to the ratio of inertial force to the viscous force determines the flow pattern (Xue et al., 2012). If the value of Reynolds number is high, it implies that an inertial force that corresponds to a turbulent flow pattern. If value of Reynolds number is low, it implies that an increased viscous force that corresponds to a laminar flow pattern.

1.6 Dimensional Analysis

Dimensional analysis is conducted based on its units and dimensions; it is the study of correspondence of quantities in a physical problem. Dimensional analysis is used to analyze the relationships between physical quantities and to check the consistency of equations.

1.7 Non-Dimensional Parameters

Non-Dimensional parameters are also called as dimensionless parameters, are ratios of one force to another. It yields a dimensionless number.

1.7.1 Prandtl Number

Prandtl number is the quotient of time momentum diffusivity to thermal diffusivity, measures the relative importance of heat conduction and fluid viscosity (Animasaun et al., 2022). It is denoted by the symbol Pr. The formula of Prandtl number is given by,

$Pr = \frac{\nu}{\alpha}$ or $Pr = \frac{\mu C_p}{K}$

Where, Pr is the Prandtl number, α is the thermal diffusivity, υ is the kinematic viscosity, C_p is the specific heat capacity of the fluid, μ is the dynamic viscosity, and K is the thermal conductivity (Bohne et al., 1984).

1.7.2 Skin Friction Coefficient

It is a dimensionless parameter of shearing stress exerted by fluid motion on the surface of a body. It is often denoted as C_f.

1.7.3 Nusselt Number

The effectiveness of heat convection at a surface can be measured using Nusselt number (Suresh et al., 2012). Nusselt number is the ratio of convective thermal convection to total thermal conduction, and can be denoted by Nu.

1.7.4 Grashof Number

Grashof number can be expressed as the ratio of buoyancy to viscous forces (Mahony, 1957). It is used in heat transfer, and is represented by Gr (Devi & Andrews, 2011).

$$Gr = \frac{\text{Buoyancy force}}{\text{Viscous force}}$$

The formula for Gr is given as: $Gr_{nf} = \frac{g\beta_{nf}L^3(T_w - T_\infty)}{(\nu_{nf})^2}$,

where: Gr_{nf} - Grashof number

g - acceleration due to gravity (m/s^2)

β - volumetric coefficient of thermal expansion $(1/K)$

L - characteristic length (m)

$\Delta T = T_w - T_\infty$ - difference in temperature between the surface and the surrounding fluid (T surface – T ambient)

V_{nf} is the kinematic viscosity of the fluid (m^2/s).

1.8 Nanofluid characteristics and their validation

Nfs is the type of fluid that containing nanoparticles of nanometre size. The nanoparticles used in nanofluids are typically made up of metals, oxides or carbon nanotube (Ahmed, 2019). Nfs are the colloidal suspension of nanoparticles in a base fluid in which are engineered (Silvestrov & Rančić, 2016). The properly prepared nanofluids are expected to give the benefits like more stability, higher heat conduction, microchannel cooling without clogging, reduced chances of erosion and reduction in pumping power (Das et al., 2006). Water, ethyl alcohol and oil are the commonly used base fluids. Nanofluids show enhanced thermal properties especially thermal conductivity than other base fluids and also with improves heat transfer, this property utilized in cooling system, electronics, etc. Nanoparticles in nanofluids have the ability to remain uniformly dispersed without sedimentation. Its stability can be improved using surfactants. The specific heat produced by nfs is usually lower than the base fluids and it is based on the concentration of nanoparticles. The particles with nanometre size with superior physical properties are used as the nano phase materials. Mechanical, optical, thermal, electrical and magnetic features of nanoparticles are better than ordinary materials. The nanoparticle volume fraction influences the density, viscosity, specific capacity and thermal conductivity of nfs

Two different varieties of nfs are considered throughout this work:

i) Titanium dioxide - Water nanofluid

ii) Silver - Water nanofluid

1.8.1 Density of the nanofluid

Density of nfs is one of the crucial properties that influence the various features like heat transfer, flow behaviour, etc. it mainly depends on the nanoparticle and base fluid, as well as the volume fraction of nanoparticle (Hussain, 2016). For both cases of titanium dioxide -water nfs and silver-water nfs, the effective density of the nfs is given by

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s$$

According to Santra et al. (2009) and (Xuan & Roetzel, 2000) ρ_f and $\rho_s$are densities of the base fluid and solid particle respectively

1.8.2 Viscosity of the nanofluid

Viscosity measures the fluid resistance to flow. Viscosity is an important property of nfs which directly influences the flow of fluids, pressure drop, thermal performance and pumping power. (Pak & Cho, 1998) measured viscosities of the dispersed fluids with Al_2O_3 and TiO_2 particles at a 10% volume concentration and were approximately 200 and 3 times greater than that of water. Usually, the viscosity of nanofluid is higher than their base fluid at higher nanoparticle concentration. In both situations of titanium dioxide -water nfs and silver-water nfs, the dynamic viscosity of the nfs as given by Brinkman (1952) is as follows:

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$$

where μ_f is the dynamic viscosity of the base fluid and ϕ is the solid volume fraction.

1.8.3 Specific heat capacity of the nanofluid

Specific heat capacity of nfs is a crucial thermophysical property that explain how much thermal energy the fluid can store per unit mass per degree of temperature rise (Zhou et al., 2010). It depends upon the mass fraction or volume fraction of nanoparticles, specific heat of nanoparticle as well as the base fluid and density of nanoparticles. The specific heat of nanofluid decreases with an increase in the volume concentration and increases with temperature (Nelson et al., 2009). It essentially having application in cooling, thermal storage and heat exchange.

The specific heat capacity of the two nanofluids titanium dioxide -water nanofluid and silver-water nanofluid, is given by

$$(c_p)_{nf} = (1-\phi)(c_p)_f + \phi(c_p)_s$$

as given by (Gosselin & da Silva, 2004), where $(c_p)_f$ and $(c_p)_s$ are specific heat capacities of the base fluid and solid particle respectively.

1.8.4 Thermal Conductivity of the nanofluid

Thermal conductivity is an important property of nfs in which it directly influences the efficiency of heat transfer, making nfs valuable in heat exchangers, solar thermal conductors and cooling systems (Ahmadi et al., 2018). Thermal conductivity is the ability of material or fluids to conduct heat (Sarviya & Fuskele, 2017). Choi coined the term nfs and proposed that by adding low concentration of nanoparticles of materials having higher thermal conductivity than the base fluid then the thermal conductivity of the base fluid can be increased (Choi & Eastman, 1995). It is expected that the ultrafine solid particle is able to increase the thermal conductivity and heat transfer performance, since the thermal conductivity of solid metals is higher than that of base fluids (Dinarvand et al., 2015). Nfs have higher thermal conductivity than the base fluids. The thermal conductivity for two types of nanofluids considered in this study can be determined as follows (Devi & Andrews, 2011):

$k_{nf} = k_f \left(\frac{\left({k_s}/{k_f}\right)+(n-1)-(n-1)\phi\left(1-\left({k_s}/{k_f}\right)\right)}{\left({k_s}/{k_f}\right)+(n-1)+\phi\left(1-\left({k_s}/{k_f}\right)\right)} \right)$, where $n = \frac{3}{\Psi}, \Psi$ is sphericity, $\Psi = 1$ for sphere

k_{nf} is thermal conductivity of nanofluid

k_f is thermal conductivity of base fluid

ϕ is the volume fraction of nanoparticle.

1.9 Introduction to heat transfer

Heat transfer is the movement of energy from one point to another point by virtue of a temperature difference (Isachenko et al., 1980). It is considered to be a fundamental concept in physics & engineering. The heat transfer mechanism determines the rate at which energy is exchanged with a system, as a result of temperature difference (Nellis & Klein, 2008). It also helps predict the temperature distribution within the regions of matter. Thermodynamics analysis focuses on the amount of heat transfer that occurs as a system undergoes a process from one equilibrium state to another (Gyftopoulos & Beretta, 2012). Nowadays in modern technology as well as in various field of earth science such as geophysics, meteorology, this field of study is considered to be important.

1.9.1 The different modes of heat transfer

The different types of heat transfer mechanisms can also be referred as modes of heat transfer. The basic modes of heat transfer are given below (Aliu et al., 2020),

1. Conduction
2. Radiation
3. Convection

1 Conduction

Conduction can be defined as the heat transfer from one part of body at a higher temperature to another part of the same body at lower temperature (Hahn & Özisik, 2012). It involves the transfer of energy from a molecule with higher energy to a molecule with lower energy, hence the conduction process said to be works on molecular level (Hahn & Özisik, 2012). The conduction of heat can be observable within gases, where the average kinetic energy of molecules in the higher temperature region is greater than those in the lower-temperature region (Aliu et al., 2020). There is a continuous transport of energy from high temperature region to those of lower temperature (Aliu et al., 2020). While considering the liquid, the molecules are more closely placed than that of Gases (Aliu et al., 2020).

2 Radiation

Radiation otherwise termed as thermal radiation, refers to the electromagnetic waves emitted by a body due to its temperature, and at the expense of its internal energy. It involves the emission of electromagnetic waves. Radiation can transfer energy through an empty space, unlike the other modes of heat transfer (conduction and convection). In simplest form, radiation can occur in a vacuum or through matter. According to wavelength the electromagnetic spectrum classifies radiation into different types like gamma rays, X-rays, UV rays, infrared, microwaves and radio waves (Aliu et al., 2020). More energetic and heat producing radiations have shorter wavelength. Temperature, surface properties, distance, angle of incidence are considered to be the factor that affect radiation. Sunlight warming the earth, infrared heaters, microwave oven, heat lamp in food warmers are some of the examples where the phenomenon of radiation is applied.

Radiation is considered to be one of the crucial heat transfer mechanisms in thermodynamics.

3 Convection

Convection is the process where can transfer of heat through the movement of fluids including both liquid and gases. The movement of fluid can be natural or forced. It is the transfer of heat between two bodies by current of moving fluid convection mode of heat transfer & not solely heat transfer process it requires movement of fluids to transport energy

Convection heat transfer can be classified into two types (Jaluria, 1980):

a) Natural or Free convection

b) Forced convection

a) Natural or free Convection

Natural or Free convection is the condition in which the fluid flow is caused by the buoyancy forces that are induced by the density difference due to variation of the fluids (Aliu et al., 2020). An example of free convection heat transfer is the cooling of hot components on a vertical array of circuit boards in air. Its technological application involves the cooling of nuclear reactors and the use of heat sinks in turbine blades (Han et al., 2012).

(b) Forced Convection.

Forced convection is the conditions in which the fluid is being driven or forced along some mechanisms other than thermal gradient at surface. External equipment’s like pumps, compressors or fans are used to create motion of fluids, enabling them to travel among the surface. An every day example of forced convection is fanning ourself to keep us cool, the other examples include cooling system of an automobile engine, cooling fan in computer, etc.

1.9.2 Boundary conditions

Boundary conditions are very crucial in every physical problem. The boundary conditions of velocity vector are given as follows: in the case of a viscous fluid, both the normal and tangential components of velocity must be continuous along the boundary. But in the case of an inviscid fluid, the normal component of velocity must be continuous, while the tangential component of velocity may be discontinuous.

Let [] represent the jump in the contained numbers when crossing the interface from medium 1 to media 2, and let $\hat{n}$ represent the unit vector oriented perpendicular to the interface between media 1 and 2. The boundary conditions for the velocity are given by

$$\hat{n}.[\vec{q}] = 0$$

$$\hat{n} \times [\vec{q}] = 0$$

1.10 Nanofluid flow transport model

Nfs transport model explain about the movement heat transfer characteristics of a fluid containing nanoparticles. For predicting and understanding of nfs, the nfs flow transport model is important in various fields including microelectronics, electronic cooling and heat exchangers.

1.10.1 Tiwari Das model

The fluid flow and heat transfer of nfs is well explained through Tiwari Das model. It is an attractive mathematical model for modelling Newtonian fluids, having the capacity to replicate a wide range of Newtonian nfs. he model incorporates the improvement in thermal conductivity due to the presence of nanoparticles (Tiwari & Das, 2007).

CHAPTER 2

THE IMPACT OF VOLUME FRACTION AND NANOPARTICLES IN THE FLOW OF NANOFLUIDS ALONG THE BOUNDARY LAYER OVER A FLAT PLATE

2.1 Introduction

In today's modern world, ultra-high-performance cooling is one of the most vital needs of many industrial technologies. Indeed, the concept of dispersing solid particles in liquids to enhance their thermal conductivity has been known for a while. This is because, solid particles generally have higher thermal conductivities than liquids. One of the significant challenges with dispersing solid particles in liquids to enhance thermal conductivity is sedimentation. In order to overcome this limitation, Choi and Eastman introduced the nano-suspensions (Choi & Eastman, 1995). Nanofluids were prepared by the stable suspension of nanometre sized particles (usually size less than 100nm) in a base fluid. This type of fluid has enhancive thermal properties than the conventional fluids.

In this work, forced convective, viscous, nonsequential boundary layer flow of a two-dimensional problem over a flat surface with heat transfer nanofluids is explored. Titanium dioxide (TiO_2) and Silver (Ag) are the nanoparticles used to prepare nanofluids and water is considered as the base fluid. The Pr value of water is taken as 7.02 with base temperature $293K$. The flow is modeled using PDEs which are nonlinear. By employing the similarity transformations, these heading equations are converted into ODEs and are then solved using the MATLAB bvp5c solution procedure.

2.2 Objectives

In this work, forced convective, viscous, nonsequential boundary layer flow of a two-dimensional problem over a flat surface with heat transfer nanofluids is explored. Titanium dioxide (TiO_2) and Silver (Ag) are the nanoparticles used to prepare nanofluids and water is considered as the base fluid. To investigate the effect different nanoparticles and their volume fraction in the flow of nanofluids along the boundary layer over a flat plate. Also, analyse the changes in the viscosity, density, specific heat capacity, thermal conductivity, Prandtl number, skin friction coefficient and Nusselt number.

2.3 Methods

The infinite flat plate oriented parallel to the uniform velocity U_∞ of the liquid flow. The temperature and velocity which are away from the plate are represented as T_∞ and U_∞, respectively. Consider the origin of the coordinates be at the fore edge of the plate, the x- and y- axes are taken aligned with the uniform flow and perpendicular to the plate respectively. A pair of nanofluids, namely Titana-Water nanofluid and Silver-Water nanofluid, are considered in this study. Both the nanoparticles are off the spherical shape with diameter $100nm$. The measurements of viscosity, thermal conductivity specific heat capacity, Prandtl number and density for different volume fraction of different nanofluids with nanoparticle are numerically found out at the first stage. Then changes occurred in velocity field, Nusselt number and skin friction coefficient are analyzed.

The effective thermo-physical properties of nanofluids are given by Table 2.1 (Ali & Salam, 2020), (Chein & Huang, 2005), (Loganathan & Vimala, 2014) and the given physical properties of the base liquid(water) and nanoparticles are exhibited in Table 2.2 (Bachok et al., 2012a), (Akter et al., 2022).

The governing equations of the flow are (Nandeppanavar & Shakunthala, 2016):

Continuity equation:
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.3.1}$$

Momentum equation:
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon_{nf}\frac{\partial^2 u}{\partial y^2} \tag{2.3.2}$$

Energy equation: $$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{nf}}{\rho_{nf}(C_p)_{nf}}\left(\frac{\partial^2 T}{\partial y^2}\right) \qquad (2.3.3)$$

where u and v are components of velocity in the x and y directions, T- temperature. The contribution of viscous dissipation is disregarded form the energy equation. The boundary constraints are given by (Nandeppanavar & Shakunthala, 2016):

$$At\ y\ =0, u=v=0,\ T=T_w$$

$$As\ y\ \rightarrow\infty, u=U_\infty, T\ =T_\infty\ . \qquad (2.3.4)$$

To solve the problem, the following dimensionless variables are implemented (Motsumi & Makinde, 2012):

$$\Psi(x,y)=\left(U_\infty v_f x\right)^{\frac{1}{2}}f(\eta), \qquad \eta=y\left(\frac{U_\infty}{v_f x}\right)^{\frac{1}{2}}, \qquad \theta=\frac{T-T_\infty}{T_w-T_\infty}, \qquad (2.3.5)$$

here $\Psi(x,y)$ is the stream function which satisfies the equation for continuity with $u=\frac{\partial\Psi}{\partial y}$ and $v=-\frac{\partial\Psi}{\partial x}$; θ is the dimensionless temperature.

By employing these variables, the components of velocity can be described in terms of the newly defined variables

$$u=U_\infty f'(\eta) \qquad (2.3.6)$$

$$v=\frac{1}{2}\left(\frac{U_\infty v_f}{x}\right)^{\frac{1}{2}}(\eta f'(\eta)-f(\eta)) \qquad (2.3.7)$$

Through the application of these transformations, the energy and momentum equations can be reduced to the ordinary differential equations which are nonlinear as:

$$f'''+\frac{1}{2}\left((1-\phi)^{2.5}\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\right)ff''=0 \qquad (2.3.8)$$

$$\theta''+\frac{1}{2}\frac{\left(\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\left[(1-\phi)+\phi\frac{(C_p)_s}{(C_p)_f}\right]\right)(Pr)_f f\theta'}{\left(k_{nf}/k_f\right)}=0 \qquad (2.3.9)$$

where $(Pr)_f = \frac{\upsilon_f}{\alpha_f}$, $\upsilon_f = \frac{\mu_f}{\rho_f}$ and $\alpha_f = \frac{k_f}{\rho_f (C_p)_f}$

The boundary conditions are then reduced to

$$\eta = 0, f = 0, f' = 0, \theta = 1$$

$$\eta \to \infty, f' = 1, \theta = 0 \tag{2.3.10}$$

The dimensionless quantities like Nusselt number Nu and skin friction coefficient C_f are considered in this study. These quantities are defined as (Devi & Andrews, 2011)

$$C_f = \frac{\tau_w}{\rho_f {U_\infty}^2}, \quad \text{where } \tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{2.3.11}$$

$$Nu = \frac{x q_w}{k_f (T_w - T_\infty)}, \qquad \text{where } q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{2.3.12}$$

By equation (2.3.5), we get

$$C_f (Re_x)^{\frac{1}{2}} = \frac{1}{(1-\phi)^{2.5}} f''(0) \tag{2.3.13}$$

$$Nu (Re_x)^{-\frac{1}{2}} = -\frac{k_{nf}}{k_f}\theta'(0) \tag{2.3.14}$$

where $Re_x = \frac{U_\infty x}{\upsilon_f}$, is the local Reynolds number.

The values that obtained for specific heat capacity, density, viscosity, thermal conductivity by the numerical method of both the nanofluids are found using MATLAB. The set of differential equations which are nonlinear (2.3.8), (2.3.9), could be derived through similarity transformations, subject to the boundary conditions (2.3.10) are solved using MATLAB bvp5c solution procedure.

2.4 Results and discussions

Analysing the forced convective boundary layer flow of various nanofluids across a flat plate in the two-dimensional problem is analysed. The numerical values of thermo-physical characteristics of different nanofluids for different volume fraction of nanoparticle are noted. Also, the variations observed in the velocity field are examined through the heat transfer coefficient and skin friction coefficient.

An alternative expression for the impact of volume fraction on the density of TiO_2-H_2O and Ag-H_2O nanofluids are exhibited in Figure 2.1. The figure illustrates a clear trend: as ϕ, so the density of the nanofluid. Also, the increase is more in the case of Ag--water nanofluid. This is because, Ag particles are denser than TiO_2 nanoparticles. Figure 2.2 shows that the viscosity is the same for both the nanofluid for a fixed volume fraction. The viscosity of the nanofluid is solely depended on the viscosity of base liquid. Further, the nanofluid's viscosity is directly proportional to volume fraction.

The variation specific heat capacity $\left(C_p\right)_{nf}$ of nanofluid against the volume fraction is depicted in Figure 2.3. It is evident that as the volume fraction increases, $\left(C_p\right)_{nf}$ decreases. Also, the decrease is slightly more for Ag-water nanofluid. Figure 2.4 gives the variation in the thermal conductivity of nanofluids with respect to ϕ. The data suggests that Ag-water nanofluid exhibits sufficiently greater thermal conductivity compared to TiO_2-water nanofluid. Further, k_{nf} increases with the increase in ϕ. The variation in the Prandtl number of different nanofluids against ϕ is depicted in Figure 2.5. It is clear that Pr decreases with the increase in ϕ. Ag-water nanofluid shows much more decrease in Pr with increasing ϕ, as compared to TiO_2-water nanofluid.

The change in the velocity profile for both the nanofluids at fixed volume fraction values $(\phi = 0.04 \; and \; \phi = 0.07)$ are displayed in Figure 2.6(a) and Figure 2.6(b), respectively. Figures show that, the velocity profile is slightly more for the Ag-water nanofluid when compared with TiO_2-water nanofluid. The change in temperature profiles for the two nanofluids for different volume fractions $(\phi = 0.04 \; and \; \phi = 0.07)$ are displayed in Figure 2.7(a) and Figure 2.7(b). From the figures it is clear that the temperature is slightly more for titana-water nanofluid as compared with the other. The table 2.3 offers another perspective on how the non-dimensional skin friction coefficient changes for both nanofluids as the volume fraction varies. The trend shows that with higher volume fraction ϕ, there is a noticeable increase in the skin friction coefficient. Additionally, the skin friction coefficient is higher for Ag-water nanofluid as compared to TiO_2-water nanofluid.

The variation in the Nusselt number for the two nanofluids is displayed in Table 2.4. There is a noted increase in the Nusselt number as ϕ increases. The Nusselt number is more for Ag-water nanofluid for a fixed volume fraction of nanoparticle.

2.5 Conclusion

The effect of TiO_2-water and Ag-water nanofluids is investigated. This investigation focuses on a two-dimensional forced convective, laminar, nonlinear boundary layer flow over a flat plate. Observations reveal alterations in both the thermal and velocity fields. Additionally, variations are observed in the skin friction coefficient and Nusselt number across different nanoparticles. It's evident from the research that silver water nanofluid surpasses titanium dioxide water nanofluid in terms of thermal conductivity and heat transfer rate. Moreover, the physical attributes like viscosity, density, and thermal conductivity of nanofluids escalate proportionally with the volume fraction of nanoparticles. Conversely, specific heat capacity and Prandtl number exhibit an inverse relationship with the volume fraction of nanoparticles, decreasing as it increases.

Table 2.1: The effective thermo-physical properties of nanofluids

Effective density	$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$
Specific heat capacity	$(C_p)_{nf} = (1-\phi)(C_p)_f + \phi(C_p)_s$
Dynamic viscosity	$\frac{\mu_{nf}}{\mu_f} = (1-\emptyset)^{-2.5}$, where $\mu_f = 0.001002$
Thermal conductivity	$\frac{k_{nf}}{k_f} = \left(\frac{\left({k_s}/{k_f}\right)-(n-1)\phi\left(1-\left({k_s}/{k_f}\right)\right)+(n-1)}{\left({k_s}/{k_f}\right)+(n-1)+\phi\left(1-\left({k_s}/{k_f}\right)\right)}\right)$, where $n = \frac{3}{\Psi}, \Psi$ is sphericity, $\Psi = 1$ for sphere
Prandtl number	$Pr_{nf} = \frac{\mu_{nf}(C_p)_{nf}}{k_{nf}}$

Table 2.2: Physical properties of water, TiO_2, and Ag

	ρ (Kg/m^3)	C_p $(J/Kg.K)$	k $(W/m.K)$
Water	997.1	4179	0.613
TiO_2	4250	686.2	8.9538
Ag	10500	235	429

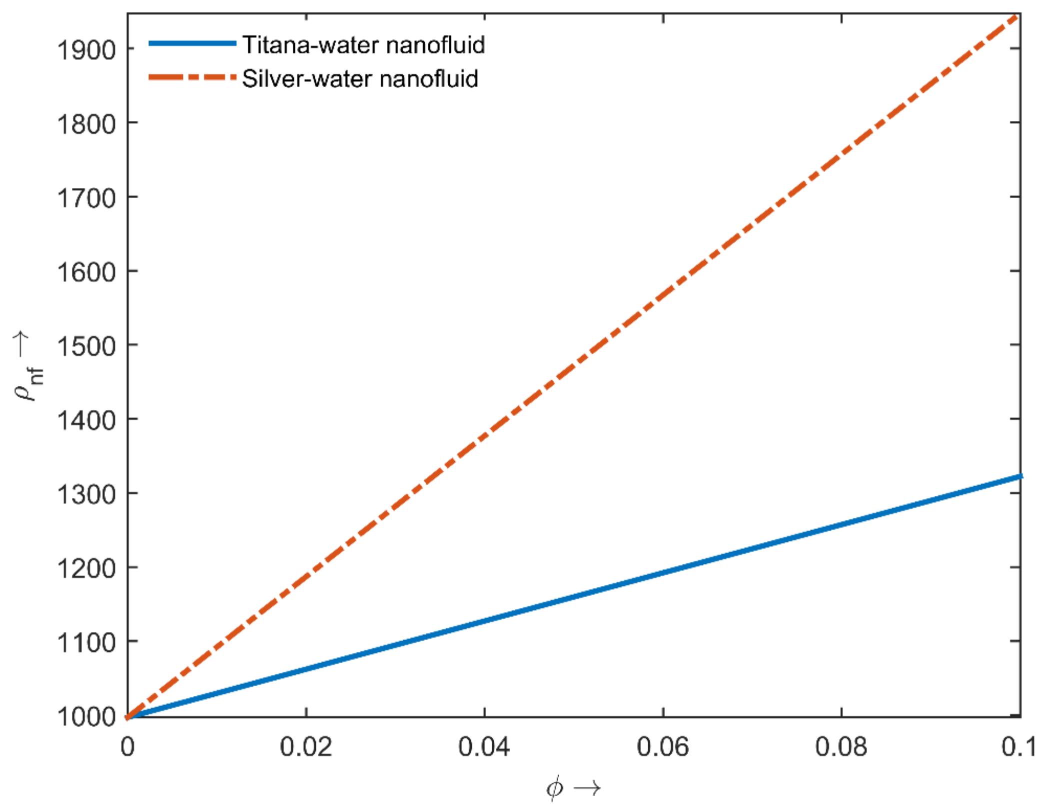

Figure 2.1: Density of $TiO_2 - H_2O$ and $Ag - H_2O$ nanofluids vs ϕ

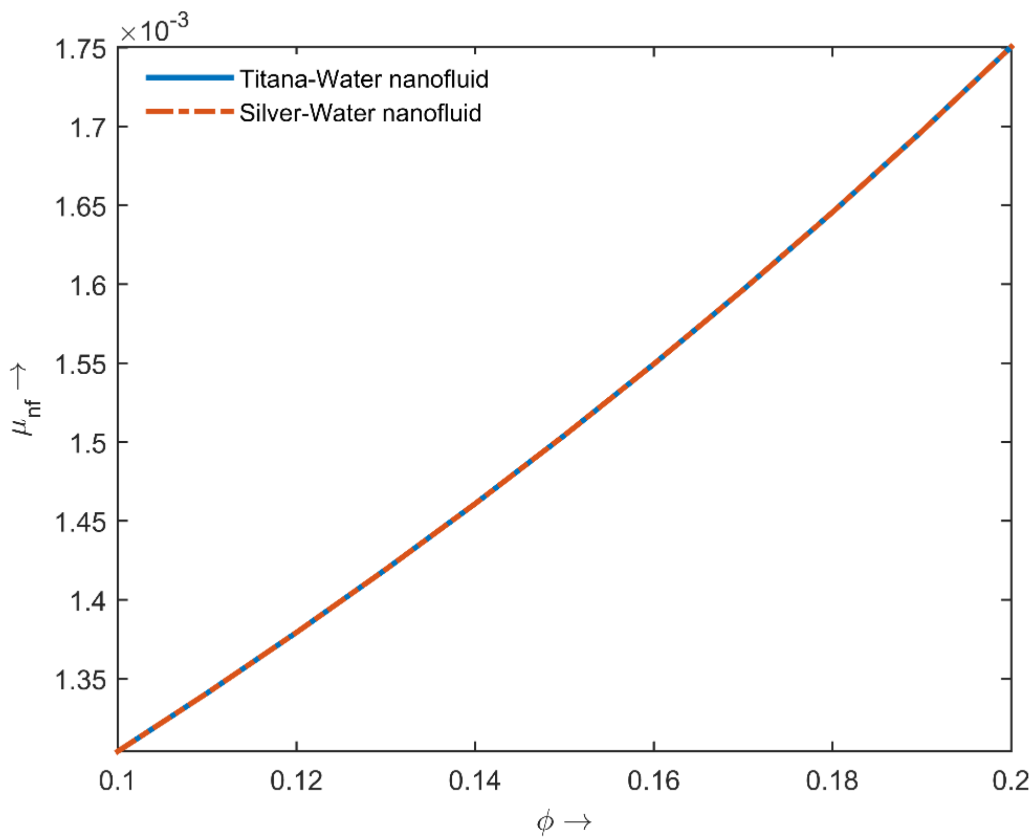

Figure 2.2: Viscosity of $TiO_2 - H_2O$ and $Ag - H_2O$ nanofluids vs ϕ

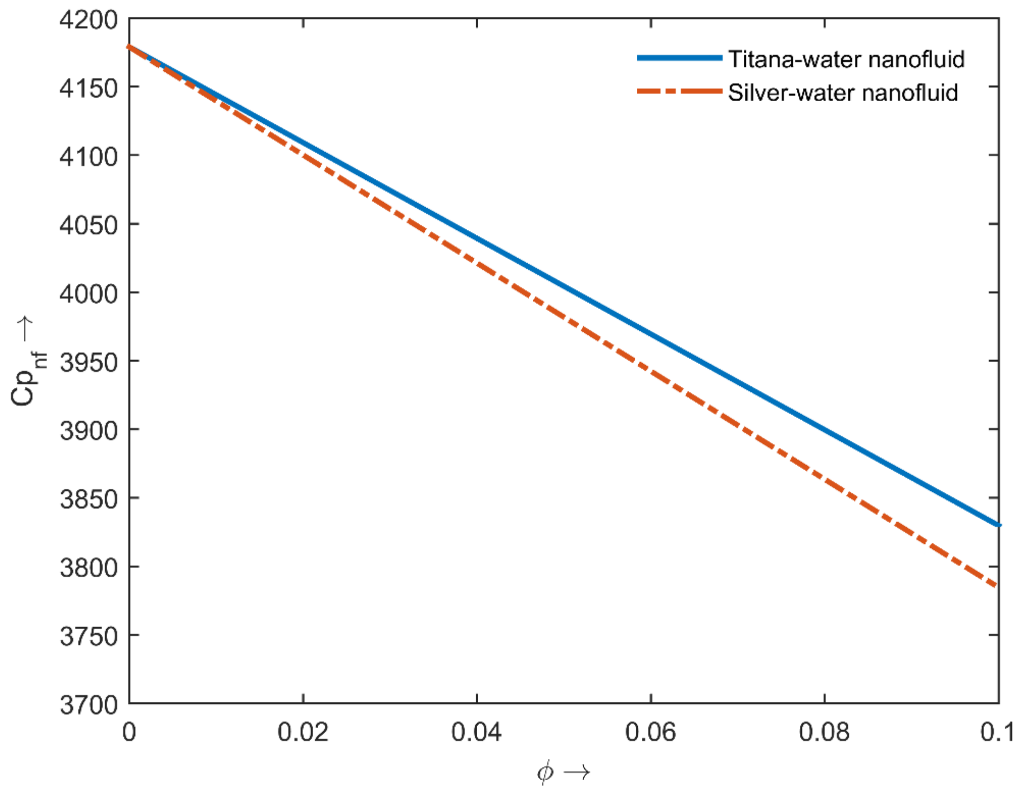

Figure 2.3: $\left(C_p\right)_{nf}$ **of** $TiO_2 - H_2O$ **and** $Ag - H_2O$ **nanofluids vs** ϕ

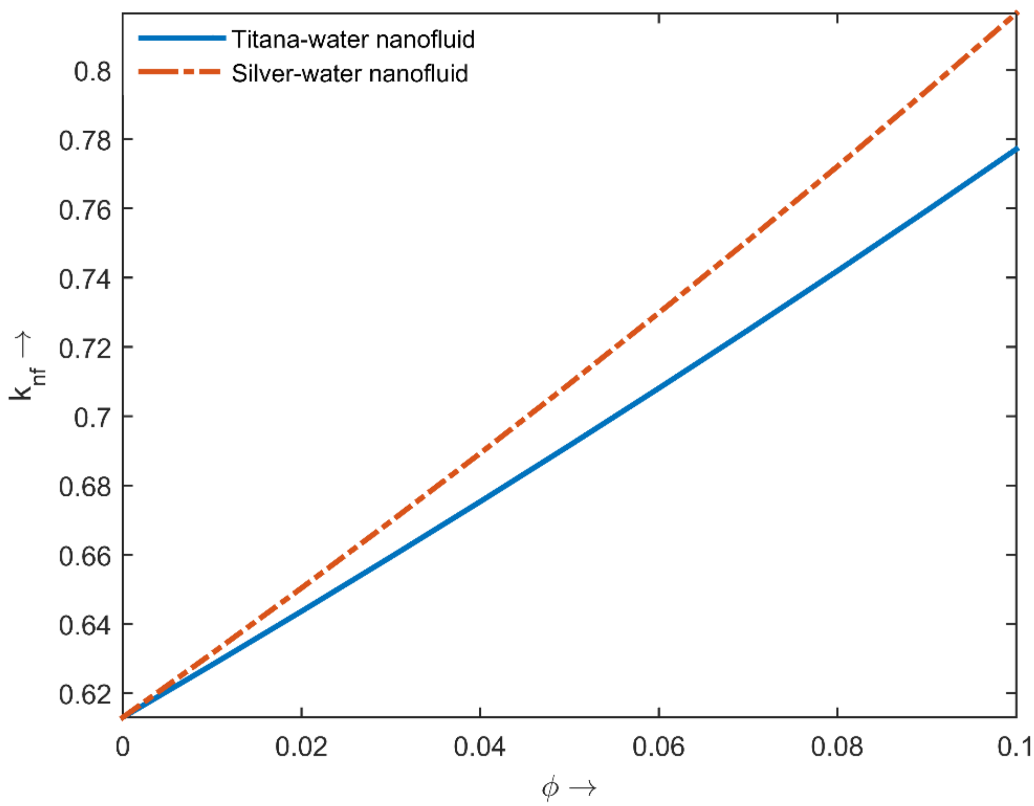

Figure 2.4: Thermal conductivity of $TiO_2 - H_2O$ and $Ag - H_2O$ nanofluids vs ϕ

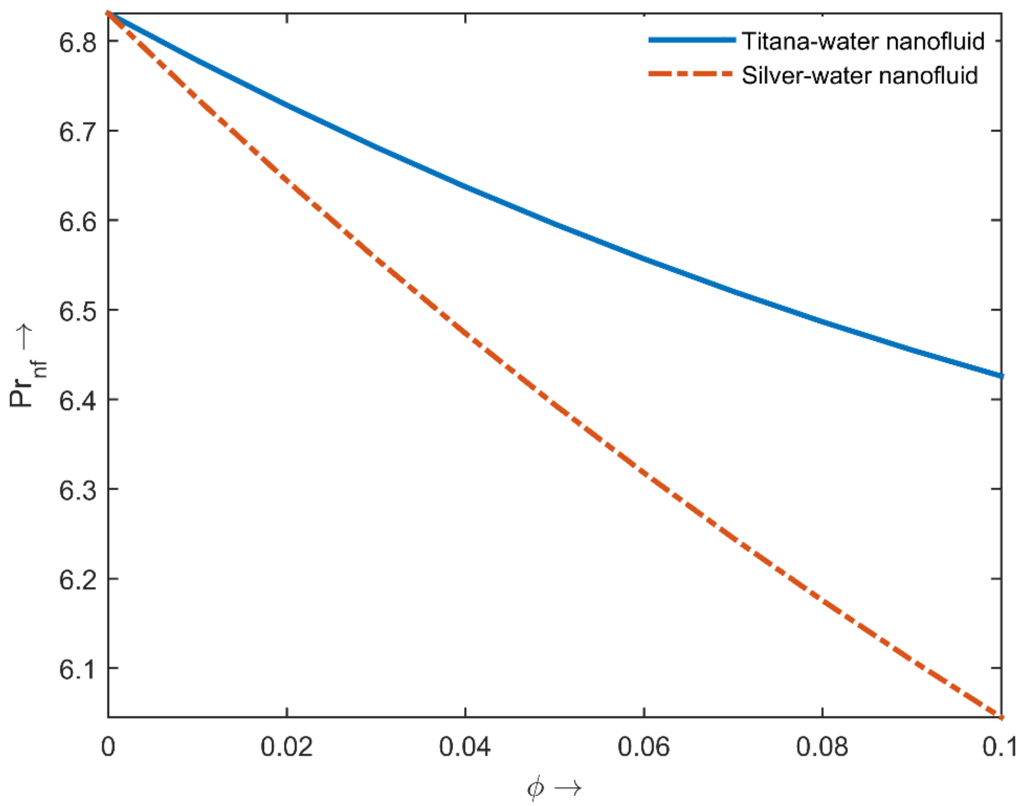

Figure 2.5: Prandtl number of $TiO_2 - H_2O$ and Ag $- H_2O$ nanofluids vs ϕ

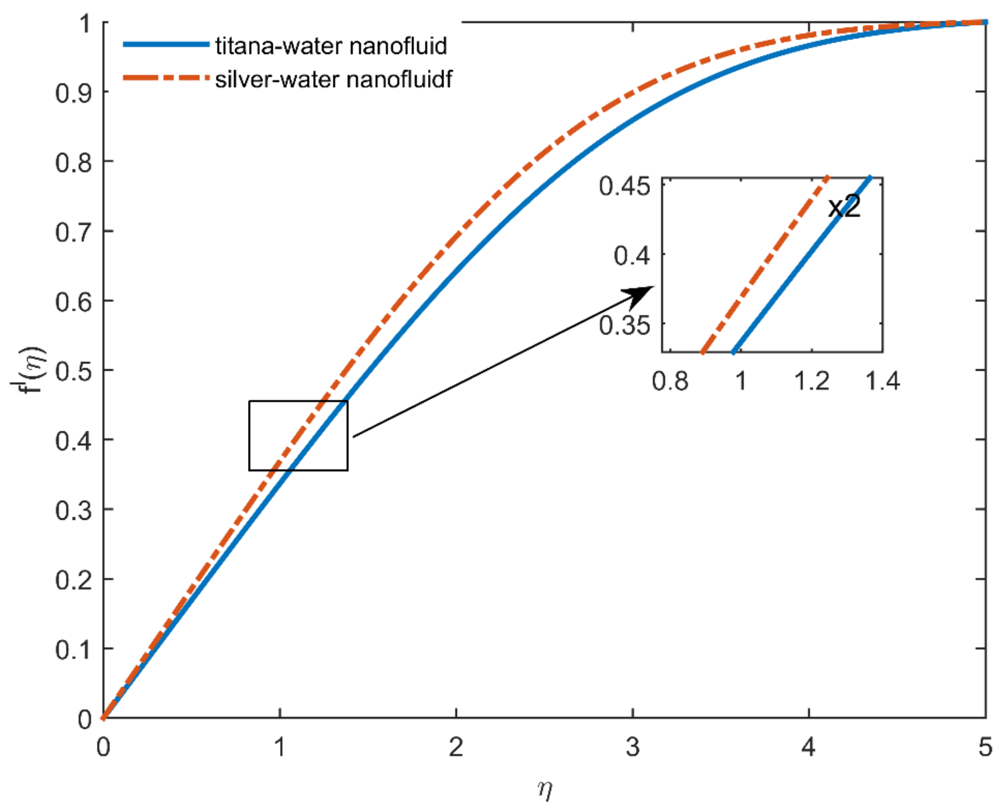

Figure 2.6(a): Change in velocity profile of two nanofluids for $\phi = 0.04$

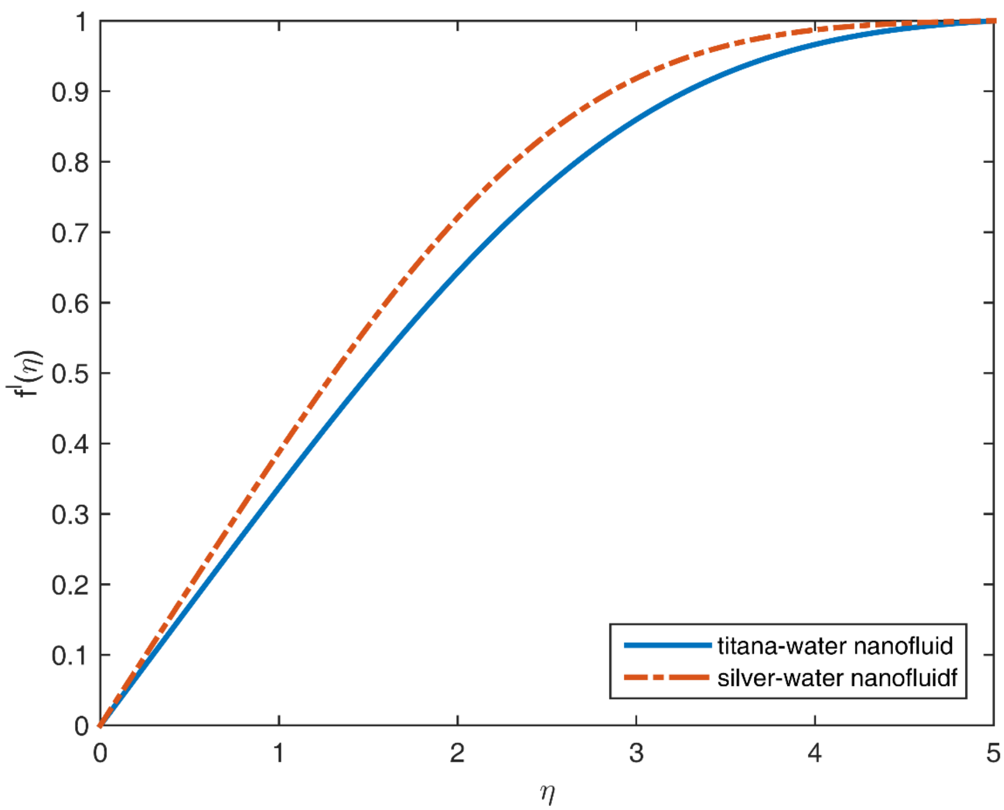

Figure 2.6(b): Change in velocity profile of two nanofluids for $\boldsymbol{\phi = 0.07}$

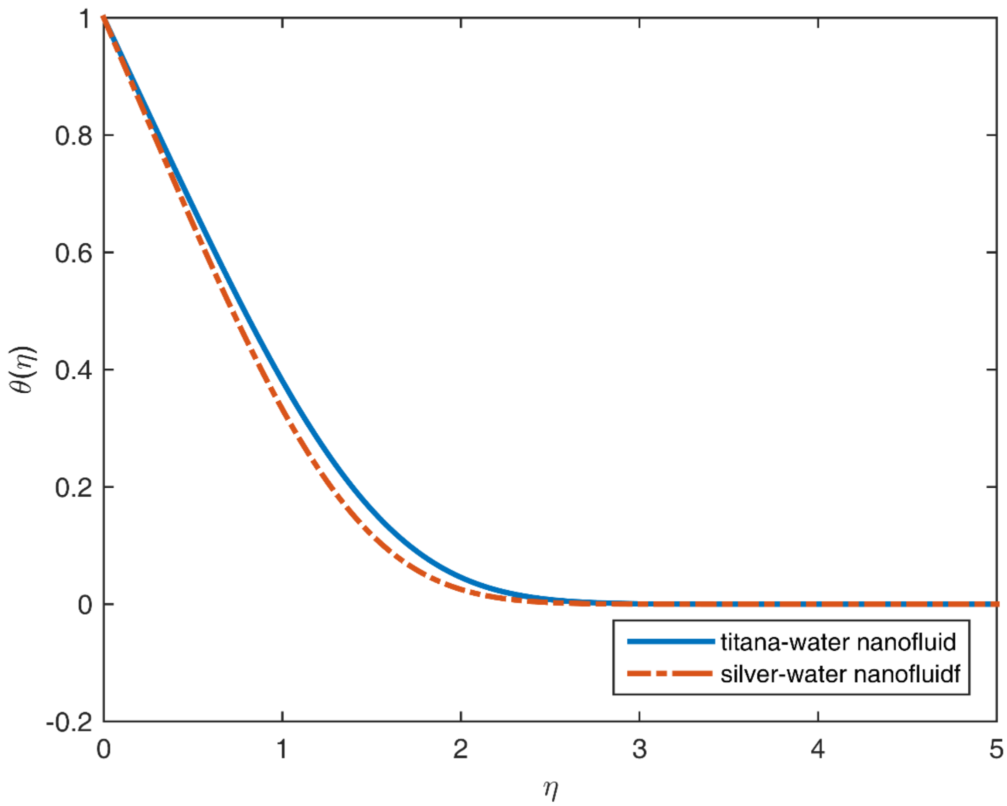

Figure 2.7(a): Change in temperature profile of two nanofluids for $\phi = 0.04$

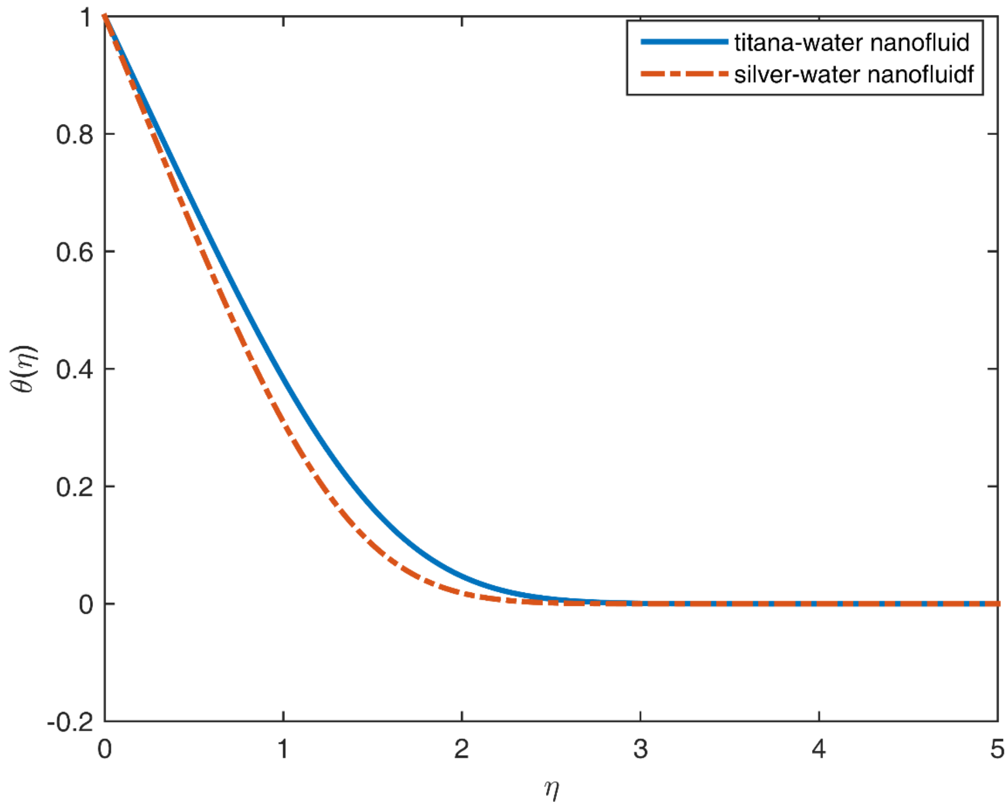

Figure 2.7(b): Change in temperature profile of two nanofluids for $\boldsymbol{\phi = 0.07}$

Table 2.3: Skin friction coefficient

ϕ	TiO_2-water nanofluid	Ag-water nanofluid
0.01	0.34578	0.35508
0.02	0.35557	0.37407
0.03	0.36554	0.39313
0.04	0.3757	0.41227
0.05	0.38606	0.43151

Table 2.4: Local Nusselt number

ϕ	TiO_2-water nanofluid	Ag-water nanofluid
0.01	0.66578	0.68716
0.02	0.68234	0.72464
0.03	0.69887	0.7617
0.04	0.71536	0.79839
0.05	0.73182	0.83474

CHAPTER 3

IMPACT OF VOLUME FRACTION ON BOUNDARY LAYER FLOW OF NANOFLUIDS OVER A MOVING PLATE

3.1 Introduction

The primary focus of the investigation is the nonlinear boundary layer flow of nanofluids which is two-dimensional, laminar, forced convective and viscous, placed upon a continuously moving flat plate, which directs attention to the heat transfer characteristics. The nanofluids are developed by disseminating Titanium dioxide $(TiO_2)$and Silver (Ag) nanoparticles in water, functioning as the principal fluid medium. 7.02 forms the Prandtl number for water and 293 K stands the temperature of the base fluid. The leading equations demonstrating the flow mean to be nonlinear partial differential equations (PDEs), which has been simplified into ordinary differential equations (ODEs) through similarity transformations. The ODEs are subsequently solved numerically using MATLAB's boundary value problem solver, bvp5c.

3.2 Model Formulation

This analysis tries to observe the nonlinear and steady flow of laminar boundary layer, which is of an incompressible and viscous nanofluid under the condition of forced convection. The study also focuses on the motion of a moving flat plate adapted to the direction of a uniform free stream velocity U_∞. The physical model is of a flat plate of infinite length which is moving with a velocity mentioned by u_w. It is also noted that the free stream settings are explained with a uniform velocity U_∞ and temperature T_∞ , which is found to be far from the surface of the plate. The system of Cartesian Coordination is accepted, in which the origin is considered to be placed at the edge of the plate. The x-axis seems to be in the direction of external flow, whereas the y- axis is found to be perpendicular to the plate surface. Therefore, it could be understood that the flow assumes to be two-dimensional, steady, and also systematised by the classical Prandtl boundary layer equation.

This observation employs two nanofluid models: Titania-Water and Silver-Water nanofluids, each of which contains spherical nanoparticles which has a uniform diameter of 10 nm. Primarily, the effective thermophysical properties- namely thermal conductivity, dynamic viscosity, density, specific heat capacity, and Prandtl number - are first evaluated numerically across a range of nanoparticle volume fractions (Safiei et al., 2020). Further, the impact of the parameters on the velocity profile, Nusselt number, and skin friction coefficient is examined to evaluate the flow dynamics and thermal behaviour of the nanofluids.

Table 3.1 summarises the effective thermophysical properties of the nanofluids according to (Ali & Salam, 2020), (Chein & Huang, 2005). Table 3.2 distinctly lists the physical properties of the base fluid(water) and the nanoparticles. (Bachok et al., 2012b), (Akter et al., 2022).

The performance of the flow is overseen by the subsequent set of reckonings (Nandeppanavar & Shakunthala, 2016):

Continuity equation: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (3.2.1)$$

Momentum equation: $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon_{nf}\frac{\partial^2 u}{\partial y^2} \qquad (3.2.2)$$

Energy equation: $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{nf}}{\rho_{nf}(C_p)_{nf}}\left(\frac{\partial^2 T}{\partial y^2}\right) \qquad (3.2.3)$$

In this context, u and v denotes the velocity components along with x- and y directions, as T specifies temperature. The effect of viscous indulgence is neglected in the energy equation. The system is subject to the subsequent boundary conditions (Bachok et al., 2012b):

$At\ y = 0,\ \ T = T_w, u = u_w$ and $v = 0$

$$u = U_\infty \text{ and } T = T_\infty \text{ when } y \to \infty \tag{3.2.4}$$

3.3 Method of solution

In order to facilitate the solution, the following non-dimensional variables are defined (Motsumi & Makinde, 2012):

$$\Psi(x,y) = \sqrt{U_\infty \upsilon_f x} f(\eta), \qquad \eta = y\sqrt{\frac{U_\infty}{\upsilon_f x}}, \qquad \theta = \frac{T_\infty - T}{T_\infty - T_w}, \tag{3.3.1}$$

where $\Psi(x,y)$ denotes the stream function, confirming compliance with the continuity equation under $u = \frac{\partial \Psi}{\partial y}$ and $v = -\frac{\partial \Psi}{\partial x}$; θ agrees to the scaled (non-dimensional) temperature.

The velocity components are reformulated using the variable of newly introduced dimensional parameters as

$$u = f'(\eta) U_\infty \tag{3.3.2}$$

$$v = 0.5\sqrt{\frac{U_\infty \upsilon_f}{x}}\left(\eta f'(\eta) - f(\eta)\right) \tag{3.3.3}$$

These transformations can be applied to the equations of energy and momentum, to

simplify them to a set of nonlinear ordinary differential equations of the form:

$$f''' = -0.5\left(\left[(1-\phi) + \phi\frac{\rho_s}{\rho_f}\right](1-\phi)^{2.5}\right) f f'' \tag{3.3.4}$$

$$\theta'' + \frac{1}{2}\frac{\left(\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\left[(1-\phi)+\phi\frac{(C_p)_s}{(C_p)_f}\right]\right)(Pr)_f f\theta'}{\left({k_{nf}}/{k_f}\right)} = 0 \tag{3.3.5}$$

where $(Pr)_f = \frac{\upsilon_f}{\alpha_f}, \upsilon_f = \frac{\mu_f}{\rho_f}, \alpha_f = \frac{k_f}{(C_p)_f \rho_f}$

Therefore, the conditions of the boundary take a simplified form (Rose Paul, 2024):

$f = 0, f' = U, \theta - 1 = 0$ when $\eta = 0$

$f' - 1 = 0$ and $\theta = 0$ as $\eta \to \infty$ (3.3.6)

Here $U = \frac{u_w}{U_\infty}$.

The dimensionless parameters of the study take into account the Nusselt number (Nu) and the skin friction coefficient (C_f). The quantities are defined through the following expressions (Devi & Andrews, 2011):

$$C_f = \frac{\tau_w}{\rho_f {U_\infty}^2}, \text{ where } \tau_w = \mu_{nf} \left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{3.3.7}$$

$$Nu = \frac{x q_w}{k_f (T_w - T_\infty)}, \text{ where } q_w = -k_{nf} \left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{3.3.8}$$

Applying equation (3.3.1) yields

$$C_f \sqrt{Re_x} = \frac{f''(0)}{(1-\phi)^{2.5}} \tag{3.3.9}$$

$$Nu (Re_x)^{-\frac{1}{2}} = -\frac{k_{nf}}{k_f} \theta'(0) \tag{3.3.10}$$

where $Re_x = \frac{U_\infty x}{\upsilon_f}$, is considered as the local Reynolds number.

The MATLAB numerically compute the thermophysical properties of both nanofluids including specific heat capacity, viscosity, density and the thermal conductivity. The similarity transformations formulate the nonlinear differential equations (3.3.4) and (3.3.5) and they are also accompanied by the boundary conditions specified in equation (3.3.6). The subsequent equations can be solved using the bvp5c boundary value problem solver of MATLAB

3.4 Discussion and Finding

The current study observes the nonlinear forced convection flow of a nanofluid, which is flowing under laminar flow conditions over a continuously moving flat plate. To improve the understanding of the physical behaviour, numerical simulations which are conducted for a range of key governing parameters, with the results that are illustrated graphically. Computations are conducted for the selected values of the nanoparticle volume fraction and pate velocity, with the $U = 0.1, 1, 3, 5, 7$ to investigate its influence on the flow and heat transfer characteristics.

It is found that Figure 3.1 expresses the velocity of the base fluid when the plate is stationary (i.e., when $U = 0$ and volume fraction of nanoparticle $\phi = 0$). As $\phi = 0$, there is no influence of nanoparticle in the fluid. It is also understood that the nanofluids velocity matches the velocity of the base fluid, water. Figure 3.2 exposes the velocity of nanofluid when the plate seems to be at rest and the volume fraction is $\phi = 0.1$. The Ag – water nanofluid shows somewhat greater velocity when compared to the TiO_2 water nanofluid.

Figure 3.3(a) and Figure 3.3(b) depicts the velocity profile for $\phi = 0.1$ and different velocities of the plate $U = 0.1, 1, 3, 5, 7$ of titana-water and silver-water nanofluids respectively. It is observed that the velocity increases for both nanofluids when the plate's velocity is raised.

The Figure 3.4 displays the temperature profile of base fluid water (i.e., when the volume fraction ϕ of nanoparticle is zero in the mixture) and when the velocity of plate is zero (i.e., $U = 0$). Figure 3.5 expresses the temperature profiles for titana – water and silver- water nanofluids at $\phi = 0.1$ and $U = 0$. It is clear from the above observations that, for the volume fraction, the titania-water nanofluid shows a little greater temperature than the silver-water nanofluid.

The change in the temperature profile of titana- water nanofluid and silver-water nanofluid at a fixed velocity of plate ($U = 1$) and of different volume fractions of nanoparticles ($\phi = 0.01, 0.03, 0.05, 0.07, 0.09, 0.1$) is depicted in Figure 3.6 (a) and Figure 3.6 (b) respectively. The temperature of volume fraction of nanoparticles increases with increase in temperature of the titana-water nanofluid. This increase creates a decrease in the temperature of the silver-water nanofluid.

Table 3.3 gives an added insight into the variation of the skin friction coefficient for both the nanofluids as per the changes in plate velocity and in nanoparticle volume fraction. The table also shows that the quantity $-\frac{1}{(1-\phi)^{2.5}} f''(0)$ of the two nanofluids are directly proportional to the volume fraction of nanoparticles. It is also understood that $-\frac{1}{(1-\phi)^{2.5}}f''(0)$ increases with the increase in $U = \frac{u_w}{U_\infty}$ for a fixed volume fraction.

Table 3.4 show how heat transfer coefficient depends on ϕ and plate velocity in the case of both nanofluids. It also indicates that the local Nusselt number increases when the nanoparticle volume fraction is increased. It is also seen that the Nusselt number rises as the plate velocity increases as per a fixed volume fraction. It also proves that the Nusselt number is a little greater for silver-water nanofluid when related to that of the titana-water nanofluid.

3.5 Conclusion

The study investigates the velocity of the plate, and the effect of volume of nanoparticles of TiO_2-water and Ag-water nanofluids. It is identified that the flow of nanofluids over a moving plate is under forced convection conditions and is a two-dimensional, nonlinear and laminar boundary flow. Through the observation, a notable variation is identified in both the thermal and velocity fields. Subsequently, it is evident across the various types of nanoparticles that there are differences in the skin friction coefficient and Nusselt number. The velocity of Silver-water nanofluid is slightly more whereas, the temperature is more for Titana-water nanofluid. The quantities $-\frac{1}{(1-\phi)^{2.5}}f''(0)$ and $-\frac{k_{nf}}{k_f}\theta'(0)$ show an upward nature with increasing nanoparticles volume fraction in both nanofluids. Moreover, the two quantities ascend with the velocity of the plate for a fixed volume fraction.

Table 3.1: The effective thermophysical properties of the nanofluids

Effective density	$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$
Specific heat capacity	$(C_p)_{nf} = (1-\phi)(C_p)_f + \phi(C_p)_s$
Dynamic viscosity	$\mu_{nf} = \mu_f(1-\emptyset)^{-2.5}$, where $\mu_f = \frac{1002}{1000000}$
Thermal conductivity	$k_{nf} = k_f\left(\frac{\left(k_s/k_f\right)+(n-1)-(n-1)\phi\left(1-\left(k_s/k_f\right)\right)}{\left(k_s/k_f\right)+(n-1)+\phi\left(1-\left(k_s/k_f\right)\right)}\right)$, where $n = \frac{3}{\Psi}$, Ψ represents the sphericity, for sphere its value is 1
Prandtl number	$Pr_{nf} = \frac{\mu_{nf}(C_p)_{nf}}{k_{nf}}$

Table 3.2: Physical properties of water, TiO_2, and Ag

	ρ (Kg/m^3)	C_p $(J/Kg.K)$	k $(W/m.K)$
TiO_2	4250	6862×10^{-1}	89538×10^{-4}
Ag	10500	235	429
Water	9971×10^{-1}	4179	613×10^{-3}

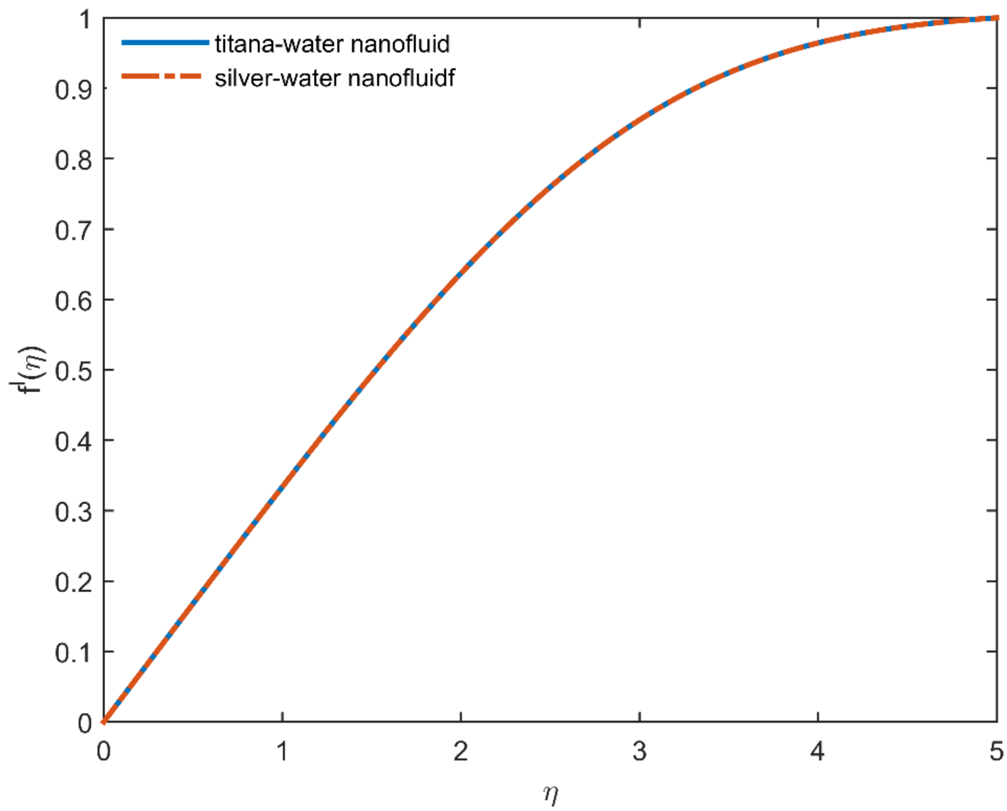

Figure 3.1: Velocity of base fluid when $U = 0$ and $\phi = 0$

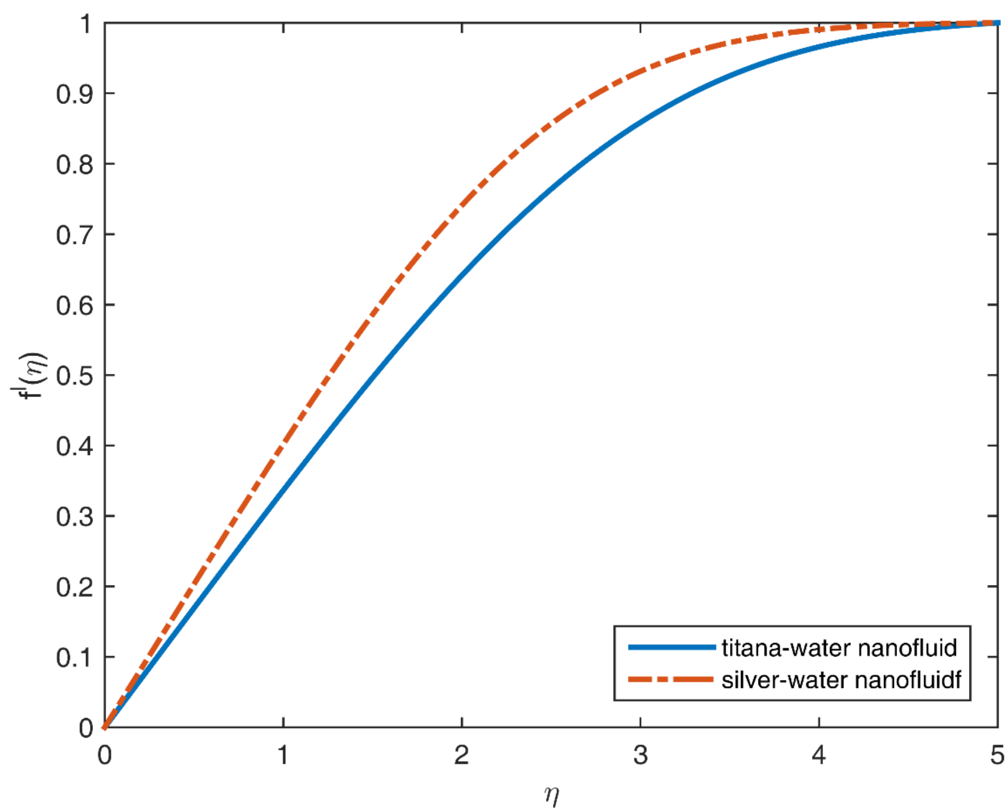

Figure 3.2: Velocity of base fluid when $U = 0$ and $\phi = 0.1$

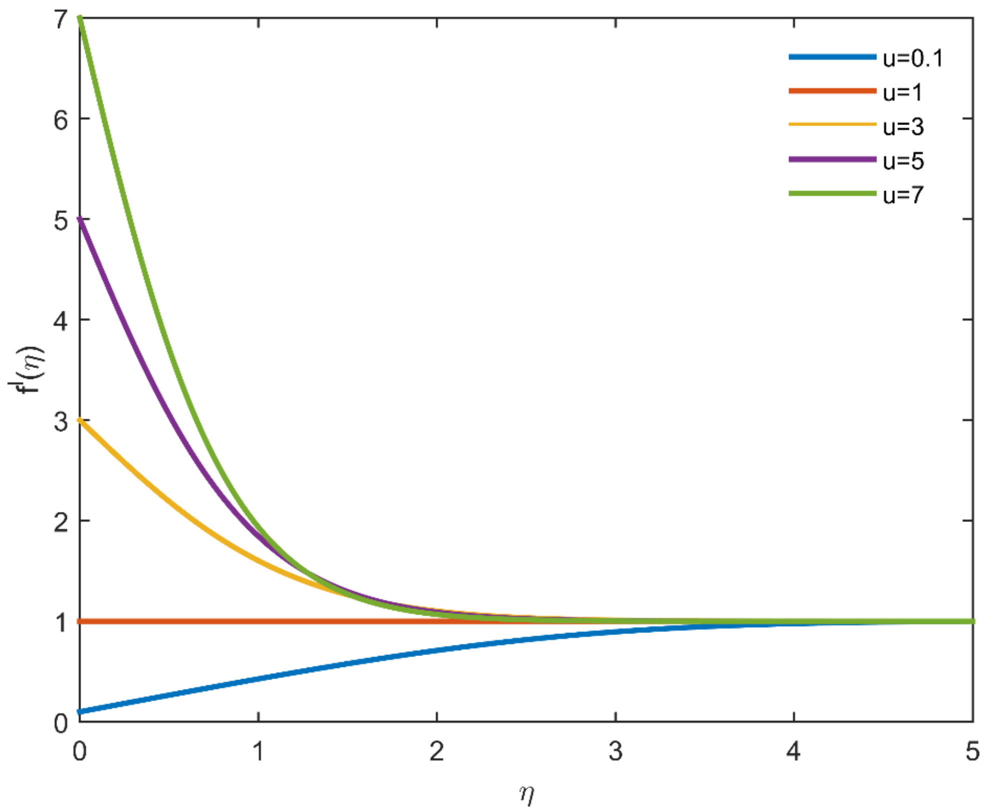

Figure 3.3(a): Velocity of $TiO_2 - H_2O$ nanofluid for $\phi = 0.1$ and different U values

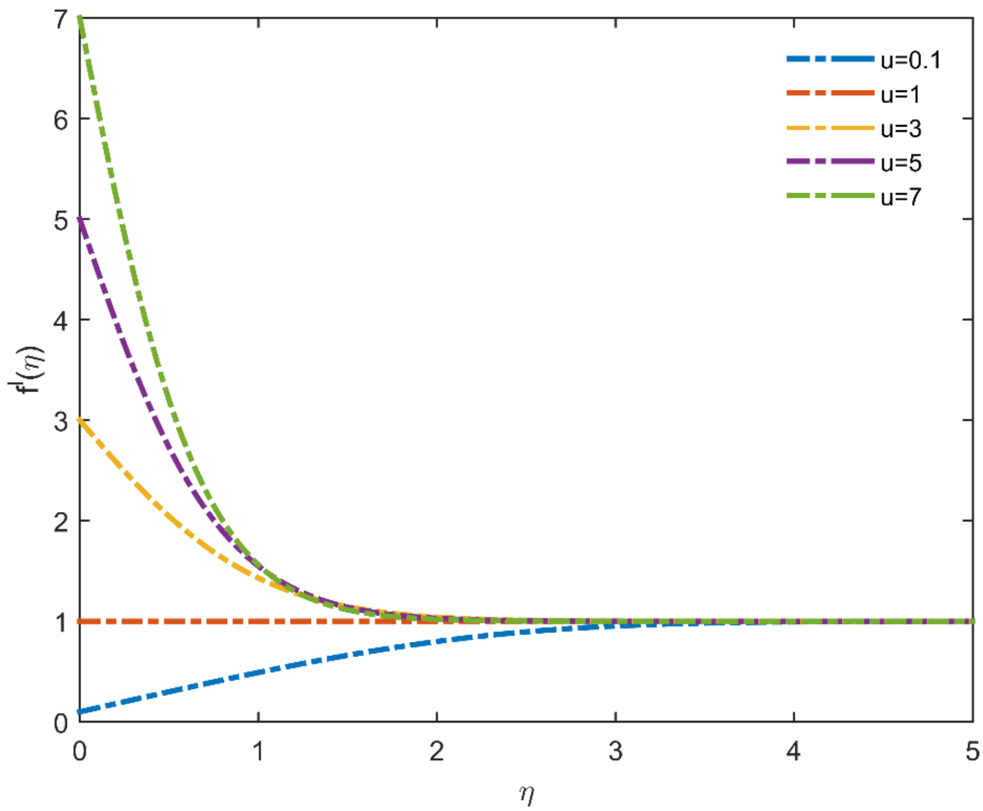

Figure 3.3(b): Velocity of $Ag - H_2O$ nanofluid for $\phi = 0.1$ and different U values

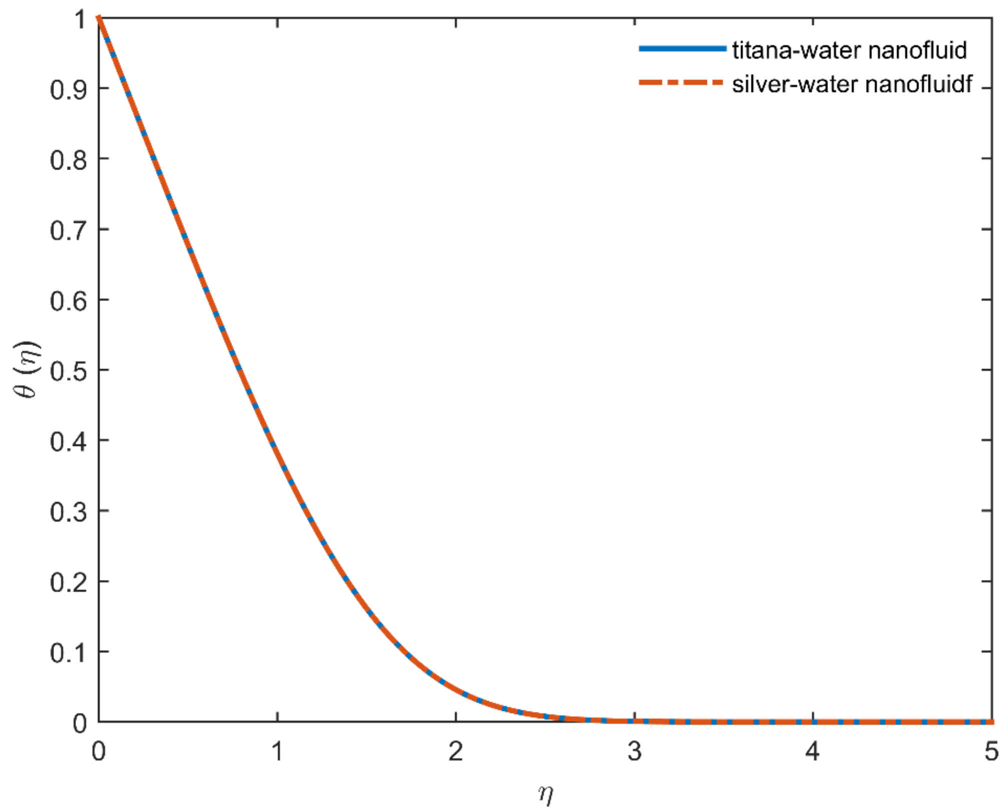

Figure 3.4: Temperature profile of base fluid when $U = 0$

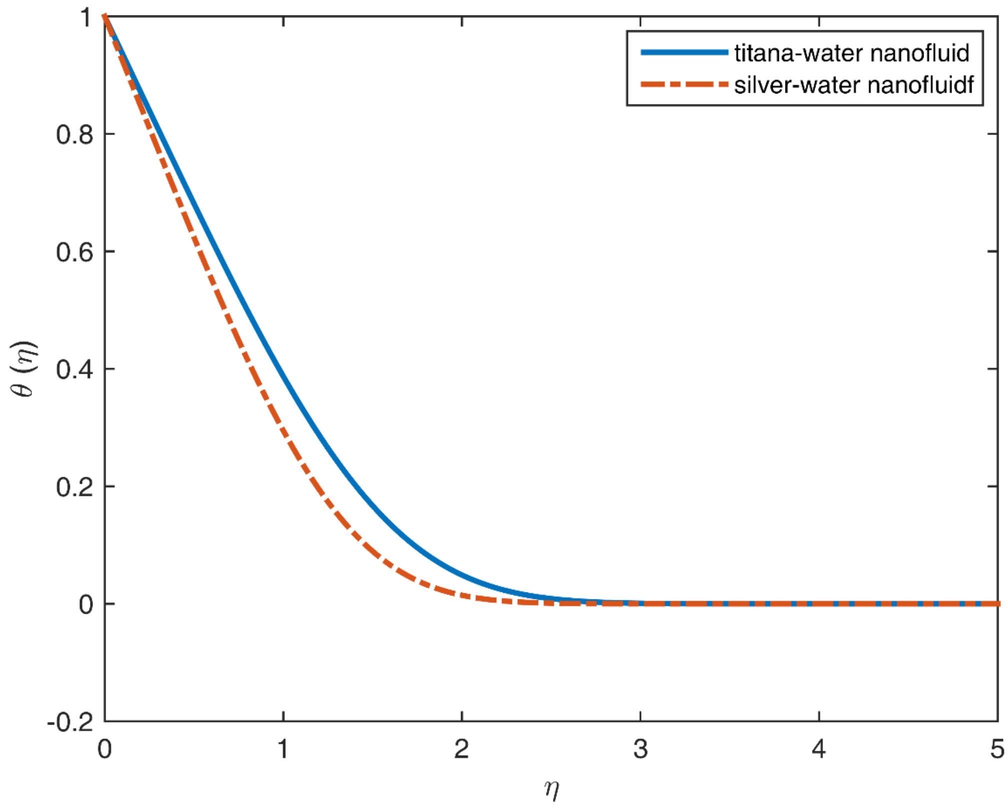

Figure 3.5: Temperature profile of $TiO_2 - H_2O$ and Ag $- H_2O$ nanofluids when $\phi = 0.1$ and $U = 0$

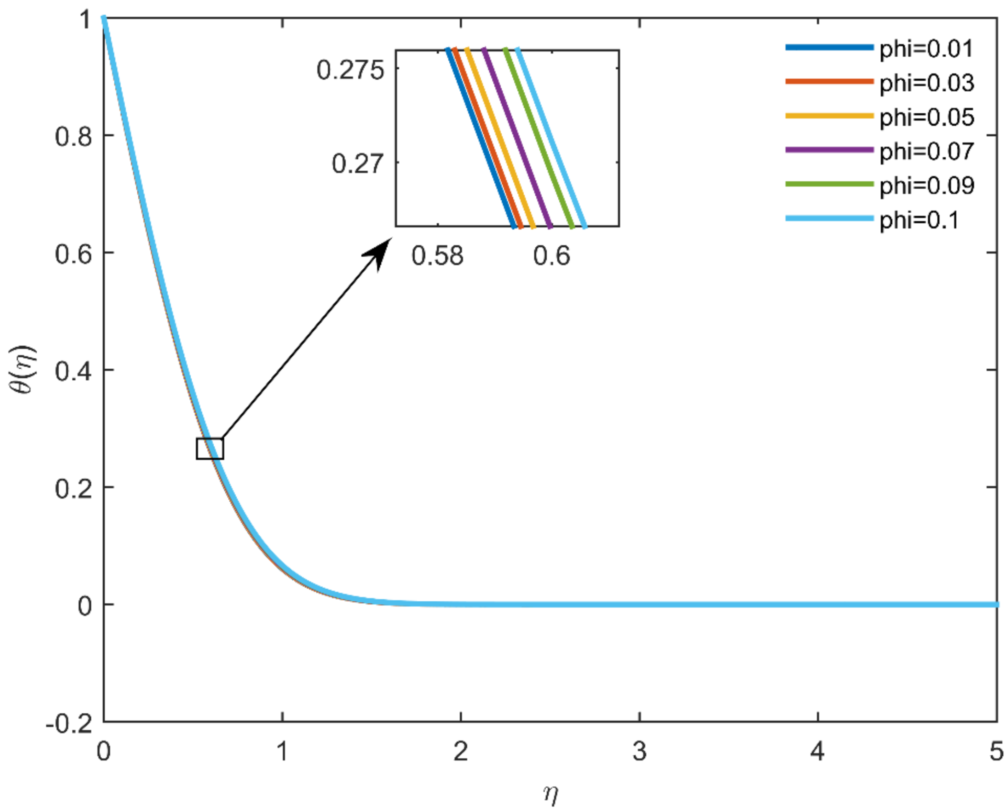

Figure 3.6(a): Temperature profile titana-water nanofluid for $U = 1$ and different ϕ values

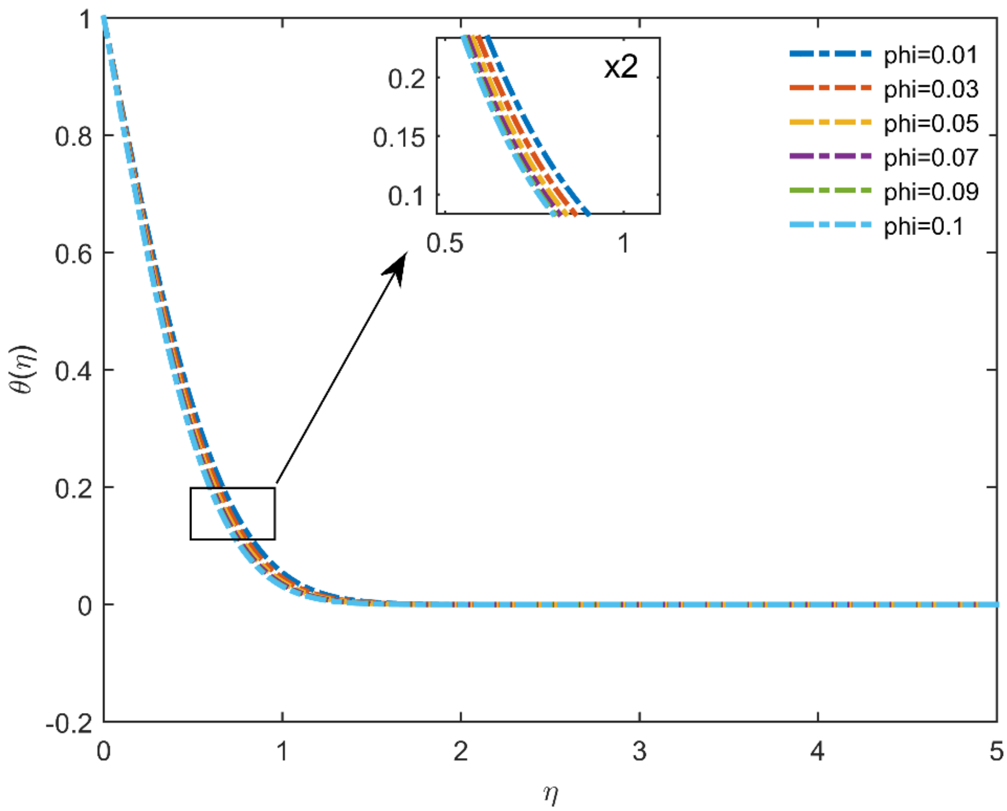

Figure 3.6(b): Temperature profile silver-water nanofluid for $U = 1$ and different ϕ values

Table 3.3: Skin friction coefficient

ϕ	$U = \frac{u_w}{U_\infty}$	$-\frac{1}{(1-\phi)^{2.5}} f''(0)$	
		TiO_2-water	Ag-water
0.01	3.0	1.74438626074681	1.79654106479544
	5.0	4.34577811812284	4.47573030418044
	7.0	7.58672196728113	7.81359145947041
0.03	3.0	1.84512648082344	1.99688571208917
	5.0	4.59675569541862	4.97487021817254
	7.0	8.02487137893067	8.68497906725936
0.05	3.0	1.94927157716633	2.19622632486472
	5.0	4.85621418541833	5.47149769718031
	7.0	8.47782621658340	9.55197745839068
0.07	3.0	2.05719274163873	2.39658440699277
	5.0	5.12507841432166	5.97065631205471
	7.0	8.94720103732131	10.4233926868005
0.1	3.0	2.22701119697695	2.70249269083692
	5.0	5.54814424945923	6.73277318916245
	7.0	9.68577569544769	11.7538736033104

Table 3.4: Local Nusselt number

$\boldsymbol{\phi}$	$\boldsymbol{U = \frac{u_w}{U_\infty}}$	$-\frac{k_{nf}}{k_f}\theta'(0)$	
		$\boldsymbol{TiO_2}$**-water**	**Ag-water**
0.01	3.0	2.51995	2.600251
	5.0	3.222519	3.325081
	7.0	3.798116	3.918932
0.03	3.0	2.638408	2.871331
	5.0	3.373711	3.671126
	7.0	3.976166	4.32648
0.05	3.0	2.756293	3.133612
	5.0	3.524173	4.005841
	7.0	4.153357	4.720632
0.07	3.0	2.873681	3.389211
	5.0	3.674007	4.331941
	7.0	4.329811	5.104596
0.1	3.0	3.048975	3.763218
	5.0	3.897769	4.808974
	7.0	4.593336	5.666203

CHAPTER 4

EFFECT OF MIXED CONVECTION, PLATE INCLINATION, AND NANOPARTICLE VOLUME FRACTION ON NANOFLUID BOUNDARY LAYER FLOW

4.1 Introduction

This study investigates the laminar and viscous boundary layer flow of nanofluids in a two-dimensional domain over an inclined plane, with a focus on non-linear behaviour and incorporating the effects of mixed convection. The nanofluids are prepared by dispersing Titanium dioxide (TiO_2) and Silver (Ag) nanoparticles in water, which serves as the base fluid. For the analysis, the Prandtl number (Pr) of water is assumed to be 7.02, and the reference temperature is $293\ K$. The governing flow behavior is described by a set of nonlinear partial differential equations (PDEs). The concept of similarity transformations can be applied to convert these equations into nonlinear ordinary differential equations (ODEs). They can then be solved using MATLAB's bvp5c solver.

4.2 Formulation of the problem

This study examines the nonlinear, steady, laminar boundary layer flow of an incompressible, viscous nanofluid past an inclined flat plate under the influence of mixed (free and forced) convection. The plate, assumed to be infinitely long, is inclined at an angle α to the horizontal. By applying the Cartesian coordinate system, the x-axis is taken in the direction parallel to the plate, and the y-axis is considered normal to it. The free-stream flow, directed parallel to the plate, has a uniform velocity U_∞, while the plate is maintained at a constant temperature T_w, which exceeds the ambient temperature T_∞. Gravitational acceleration acts vertically downward. The problem is modelled as a two-dimensional flow governed by the classical Prandtl boundary layer equations. The two types of nanofluids used here are Titania–water and Silver–water, each containing spherical nanoparticles with a diameter of $100nm$. The analysis focuses on velocity profile and temperature profiles. Additionally, it also examines the skin friction coefficient and the heat transfer coefficient.

Table 4.1 presents the effective thermophysical properties of the nanofluids (Ali & Salam, 2020) (Chein & Huang, 2005) and the relevant physical characteristics of the base liquid (water) and nanoparticles are shown in Table 4.2 (Bachok et al., 2012a) (Akter et al., 2022).

The fundamental equations governing the flow are as follows (Nandeppanavar & Shakunthala, 2016):

Continuity equation:
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4.2.1}$$

Momentum equation:
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon_{nf}\frac{\partial^2 u}{\partial y^2} + g\beta_{nf}(T - T_\infty)\sin\alpha \tag{4.2.2}$$

Energy equation:
$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{nf}}{\rho_{nf}(C_p)_{nf}}\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{4.2.3}$$

Here, u and v represent the velocity components in the x- and y-directions, respectively; T denotes the temperature; υ_{nf} is the kinematic viscosity of the nanofluid; and α is the inclination angle of the plate with respect to the horizontal. The effect of viscous dissipation is neglected in the energy equation.

The applicable boundary conditions are expressed as (Bachok et al., 2012a):

$$At\ y = 0, u = u_w, v = 0,\ T = T_w$$

$$As\ y \to \infty, u = U_\infty, T = T_\infty\ . \tag{4.2.4}$$

4.3 Method of solution

To simplify the analysis, the problem is reformulated using the following dimensionless variables (Motsumi & Makinde, 2012):

$$\Psi(x,y) = \left(U_\infty \upsilon_f x\right)^{\frac{1}{2}} f(\eta), \quad \eta = y\left(\frac{U_\infty}{\upsilon_f x}\right)^{\frac{1}{2}}, \qquad \theta = \frac{T-T_\infty}{T_w-T_\infty} \tag{4.3.1}$$

here, $\Psi(x,y)$ denotes the stream function that satisfies the continuity equation, with the velocity components defined as $u = \frac{\partial\Psi}{\partial y}$ and $v = -\frac{\partial\Psi}{\partial x}$. The variable θ represents the dimensionless temperature. Using these variables, the velocity components can be represented as (Rose Paul, 2024):

$$u = U_\infty f'(\eta) \tag{4.3.2}$$

$$v = \frac{0.5}{\sqrt{x}}\left(U_\infty \upsilon_f\right)^{\frac{1}{2}}\left(\eta f'(\eta) - f(\eta)\right) \tag{4.3.3}$$

These transformations can be applied to simplify the equations of momentum and energy to nonlinear ordinary differential equations of the form:

$$f''' + \frac{1}{2}\left((1-\phi)^{2.5}\left[(1-\phi) + \phi\frac{\rho_s}{\rho_f}\right]\right) f f''$$

$$+ \frac{1}{2}\left((1-\phi)^{2.5}\left[(1-\phi) + \phi\frac{\rho_s}{\rho_f}\right]\right)\lambda\theta\sin\alpha = 0 \tag{4.3.4}$$

$$\frac{1}{(Pr)_{nf}}\theta'' + \frac{1}{2}\left((1-\phi)^{2.5}\left[(1-\phi) + \phi\frac{\rho_s}{\rho_f}\right]\right) f\theta' = 0 \tag{4.3.5}$$

where $(Pr)_{nf} = \frac{\upsilon_{nf}}{\alpha_{nf}}$, $\lambda = \frac{(Gr_x)_{nf}}{\left(Re_x{}^2\right)_{nf}}$, $(Gr_x)_{nf} = \frac{g\beta_{nf}x^3(T_w-T_\infty)}{\upsilon_{nf}{}^2}$ and $(Re_x)_{nf} = \frac{U_\infty x}{\upsilon_{nf}}$

Accordingly, the boundary conditions can be simplified as:

$$\eta = 0, f = 0, f' = 0, \theta - 1 = 0$$

$$\eta \to \infty, f' - 1 = 0, \theta = 0 \tag{4.3.6}$$

The Nusselt number (Nu) and the skin friction coefficient (C_f) are taken into consideration here. Their definitions are given as follows (Devi & Andrews, 2011):

$$C_f = \frac{\tau_w}{\rho_f {U_\infty}^2}, \quad \text{where} \;\; \tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{4.3.7}$$

$$Nu = \frac{x q_w}{k_f (T_w - T_\infty)}, \qquad \text{where} \;\; q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{4.3.8}$$

By equation (4.3.1), we get

$$C_f (Re_x)^{\frac{1}{2}} = \frac{1}{(1-\phi)^{2.5}} f''(0) \tag{4.3.9}$$

$$Nu (Re_x)^{-\frac{1}{2}} = -\frac{k_{nf}}{k_f} \theta'(0) \tag{4.3.10}$$

where $Re_x = \frac{U_\infty x}{v_f}$, is the local Reynolds number.

The nonlinear differential equations (4.3.4) and (4.3.5), derived via similarity transformations and governed by the boundary conditions (4.3.6), are solved using MATLAB's bvp5c method.

4.4 Results

Convective and nonlinear flow of a nanofluid over an inclined plate under laminar conditions has been analysed in this work. To better understand the physics of the problem, we perform numerical calculations using different values for the physical parameters, such as α, λ and ϕ, and are shown graphically. The values of angle of inclination α considered here are $\alpha = 0°, 15°, 30°, 45°, 60°$. Also, $\lambda = 0.1, 0.5, 1, 1.5$ are the chosen values of mixed convection parameter α. Obtained results are represented by means of graphs and tables.

Figure 4.1 depicts velocity profiles of the two nanofluids for different values of α, the angle of inclination of the plate $\left(\alpha = 0, \frac{\pi}{12}, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}\right)$ and for fixed λ and ϕ values $(\lambda = 1, \phi = 0.01)$. From the figure it is clear that, for both the nanofluids the velocity ascends with the α. The increase is slightly more for silver-water nanofluid for any fixed value of α.

The changes occurred in the velocity profile of the two nanofluids for different values of λ, the mixed convection parameter $(\lambda = 0.1, 0.5, 1.0, 1.5)$ and for fixed α and ϕ values $\left(\alpha = \frac{\pi}{4}, \phi = 0.01\right)$ are displayed in figure 4.2. Figure shows that, velocity of both the nanofluids is directly proportional to the values of λ. Also, as in the previous case the increase is slightly more for silver-water nanofluid for any fixed λ value.

Figure 4.3 displays the changes in the velocity field of the two nanofluids for different values of volume fraction of nanoparticles $(\phi = 0.1, 0.05, 0.01.0)$, and for a fixed α and λ $\left(\alpha = \frac{\pi}{4}, \lambda = 1\right)$. The velocity of the both the nanofluids are increasing with the increase in volume fraction of nanoparticles. Silver-water nanofluids shows more deviations with the volume fraction as compared to Titana-water nanofluid.

The temperature distribution of the two nanofluids for different values of λ, the mixed convection parameter $(\lambda = 0.1, 0.5, 1.0, 1.5)$ and for fixed α and ϕ values $\left(\alpha = \frac{\pi}{4}, \phi = 0.01\right)$ are depicted in figure 4.4. As λ increases, the temperature θ decreases for both the two nanofluids. The mixed convection parameter λ reduces the thickness of the thermal boundary layer. The temperature decreases more for Silver-water nanofluid as compared to Titana-water nanofluid.

Figure 4.5 displays the changes in the thermal field the two nanofluids for different values of α, the angle of inclination of the plate $\left(\alpha = 0, \frac{\pi}{12}, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}\right)$ and for fixed λ and ϕ values $(\lambda = 1, \phi = 0.01)$. As in the previous case, for both the two nanofluids, θ is inversely proportional to the angle of inclination α and the decrease in temperature is more for Silver-water nanofluids as compared to the other.

Table 4.3 shows the changes in the value of non-dimensional quantity $\frac{1}{(1-\phi)^{2.5}} f''(0)$ with respect to ϕ and the angle of inclination α for a fixed value of mixed convection parameter λ $(\lambda = 1)$. It can be seen that in case of both the nanofluids, the increase in volume fraction of nanoparticles increases the quantity $\frac{1}{(1-\phi)^{2.5}} f''(0)$. Also, for a fixed volume fraction the quantity $\frac{1}{(1-\phi)^{2.5}} f''(0)$ increases with the increase in α.

Increase is slightly more for Silver-water nanofluids as compared to Titana-water nanofluids.

The changes in the value of non-dimensional quantity $\frac{1}{(1-\phi)^{2.5}} f''(0)$ with respect to the angle of inclination α of the plate and the mixed convection parameter λ for a fixed value of volume fraction of the nanoparticles $(\phi = 0.01)$ is displayed in Table 4.4. Table shows that, for both the two nanofluids the quantity $\frac{1}{(1-\phi)^{2.5}} f''(0)$ increases with the increase in the value of λ. Also, for a fixed λ, $\frac{1}{(1-\phi)^{2.5}} f''(0)$ increases with the angle of inclination of the plate with the horizontal. As in the previous case, this increase is slightly more for Silver-water nanofluid as comparing with the Titana-water nanofluid.

Table 4.5 gives the changes in the value of non-dimensional quantity $-\frac{k_{nf}}{k_f}\theta'(0)$ with respect to the nanoparticle's volume fraction and the angle of inclination α for a fixed value of mixed convection parameter λ $(\lambda = 1)$. Both the two nanofluids shows the same trend. The quantity $-\frac{k_{nf}}{k_f}\theta'(0)$ increases with the increase in volume fraction of nanoparticles. Also, for a fixed volume fraction the quantity $-\frac{k_{nf}}{k_f}\theta'(0)$ ascends with α. Increase is slightly more for Silver-water nanofluids as compared to Titana-water nanofluids.

The changes in the value of non-dimensional quantity $-\frac{k_{nf}}{k_f}\theta'(0))$ with respect to the angle of inclination α of the plate and the mixed convection parameter λ for a fixed value of volume fraction of the nanoparticles $(\phi = 0.01)$ is depicted in Table 4.6. Table shows that, for both the two nanofluids the quantity $-\frac{k_{nf}}{k_f}\theta'(0)$ increases with the increase in the value of λ. Also, for a fixed λ, $-\frac{k_{nf}}{k_f}\theta'(0)$ increases with the angle of inclination of the plate with the horizontal. As in the previous case, this increase is slightly more for Silver-water nanofluid as comparing with the Titana-water nanofluid.

4.5 Conclusion

The combined free-forced convective, viscous, steady, and nonlinear boundary layer flow over an inclined plate with heat transfer nanofluids, in a two-dimensional setup, is analysed. The effect of mixed convection parameter, the angle of inclination of plate and volume fraction of nanoparticle in the flow are studied. The variations in thermal and velocity fields are noted along with the skin friction coefficient and heat transfer coefficient. The velocity of both the nanofluids is directly proportional to λ, α and the volume fraction ϕ. Whereas the temperature of the two nanofluids is inversely proportional to λ and α. Of the two nanofluids the velocity and temperature of Silver-water nanofluid is slightly more as compared with Titana-water nanofluid in all cases. The non-dimensional quantities $\frac{1}{(1-\phi)^{2.5}} f''(0)$ and $-\frac{k_{nf}}{k_f}\theta'(0)$ ascends with $, \alpha$ and the volume fraction ϕ. And the increase is slightly more for Silver-water nanofluid is slightly more as compared with Titana-water nanofluid in all cases.

This study investigates the laminar and viscous boundary layer flow of nanofluids in a two-dimensional domain over an inclined plane, with a focus on non-linear behaviour and incorporating the effects of mixed convection. The angle of inclination (α), volume fraction ϕ of nanoparticle, and effects of the mixed convection parameter (λ) on the flow and thermal behaviour have been explored. Significant variations are observed in the velocity profile and temperature profiles, skin friction coefficient and heat transfer coefficient. The velocity of both nanofluids increases with rising λ, α, and ϕ, whereas the temperature decreases with increasing λ and α. Among the nanofluids considered, the Silver–water nanofluid consistently exhibits slightly higher velocity and temperature values than the Titania–water nanofluid. Furthermore, the dimensionless quantities $\frac{1}{(1-\phi)^{2.5}} f''(0)$ and $-\frac{k_{nf}}{k_f}\theta'(0)$ increase with λ, α, and ϕ, with the Silver–water nanofluid showing a slightly greater rate of increase compared to the Titania–water nanofluid.

Table 4.1: The effective thermophysical properties of the nanofluids

Effective density	$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$
Specific heat capacity	$(C_p)_{nf} = (1-\phi)(C_p)_f + \phi(C_p)_s$
Dynamic viscosity	$\mu_{nf} = \mu_f(1-\emptyset)^{-2.5}$, where $\mu_f = 100210^{-6}$
Thermal conductivity	$k_{nf} = k_f\left(\frac{\left({k_s}/{k_f}\right)+(n-1)-(n-1)\phi\left(1-\left({k_s}/{k_f}\right)\right)}{\left({k_s}/{k_f}\right)+\phi\left(1-\left({k_s}/{k_f}\right)\right)+(n-1)}\right)$, where $n = \frac{3}{\Psi}$, , for sphere $\Psi = 1$ (Ψ is sphericity)
Prandtl number	$Pr_{nf} = \frac{\mu_{nf}(C_p)_{nf}}{k_{nf}}$

Table 4.2: The physical characteristics of the base liquid and nanoparticles

	ρ (Kg/m^3)	C_p $(J/Kg.K)$	k $(W/m.K)$
TiO_2	4250	6862×10^{-1}	89538×10^{-4}
Ag	10500	235	429
Water	9971×10^{-1}	4179	613×10^{-3}

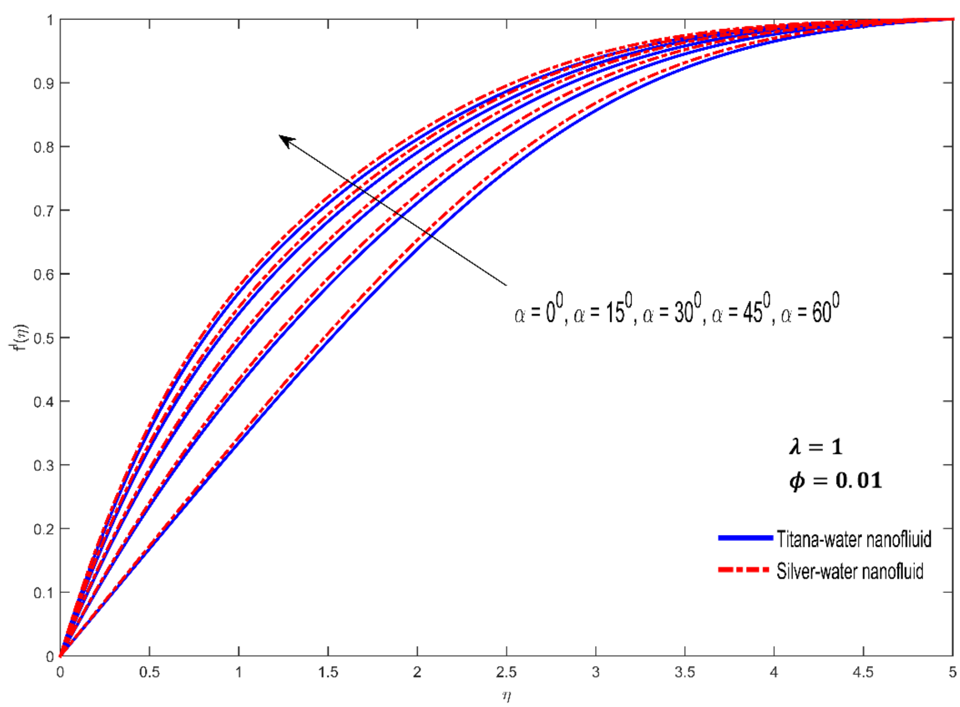

Figure 4.1: Dimensionless velocity profiles of Titana-water and Silver-water nanofluids for $\lambda = 1, \phi = 0.01$ and different α values

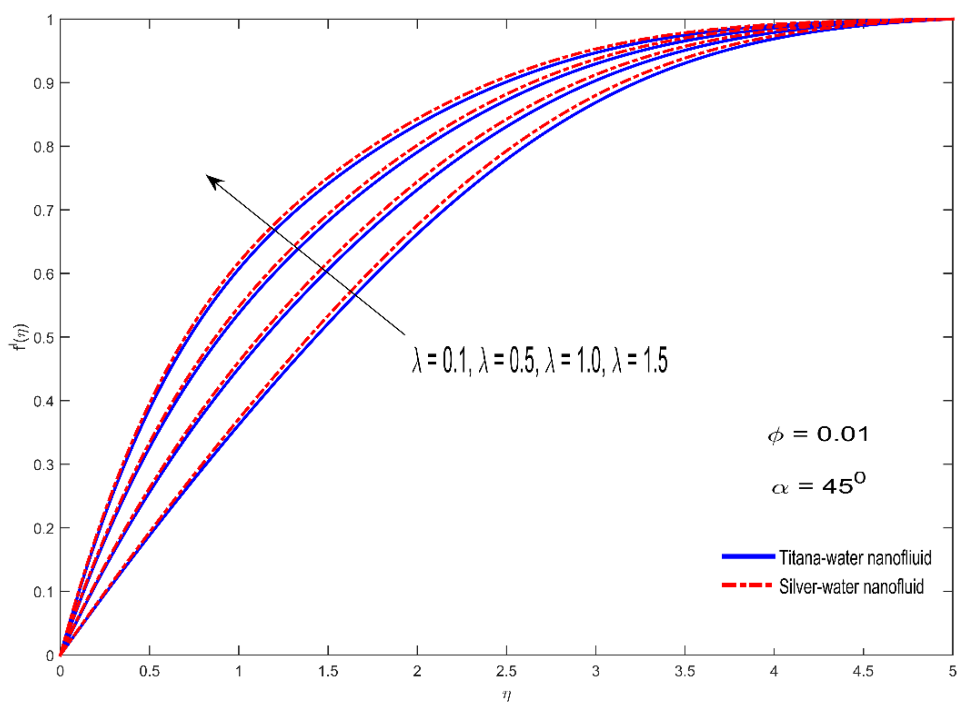

Figure 4.2: Dimensionless velocity profiles of Titana-water and Silver-water nanofluids for $\alpha = 45^{\circ}, \phi = 0.01$ and different λ values

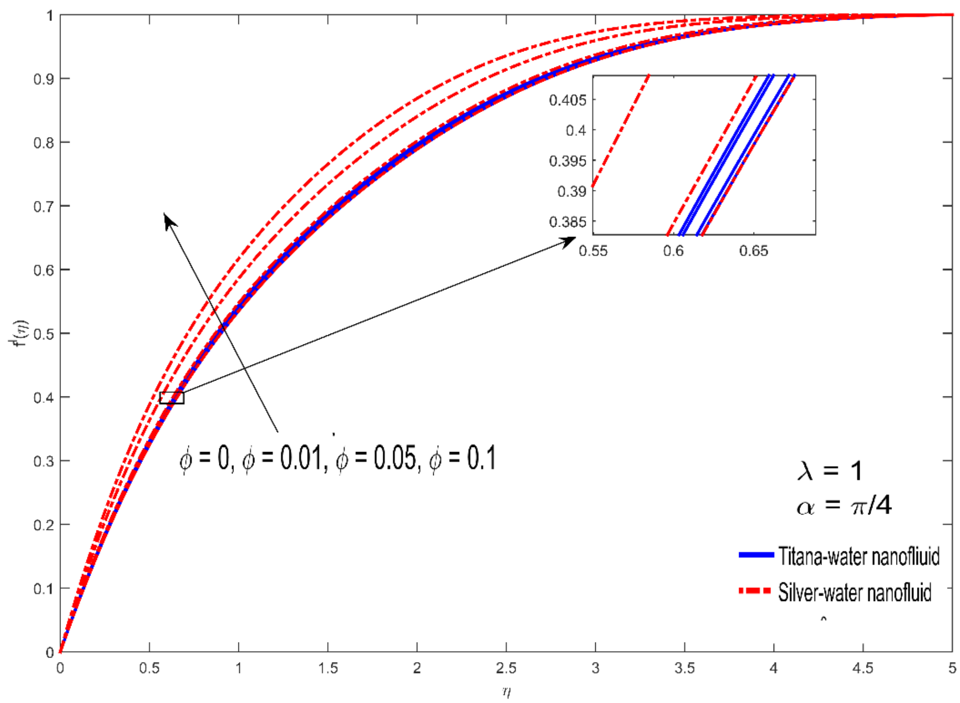

Figure 4.3: Dimensionless velocity profiles of Titana-water and Silver-water nanofluids for $\lambda = 1, \alpha = 45^{\circ}$ and different ϕ values

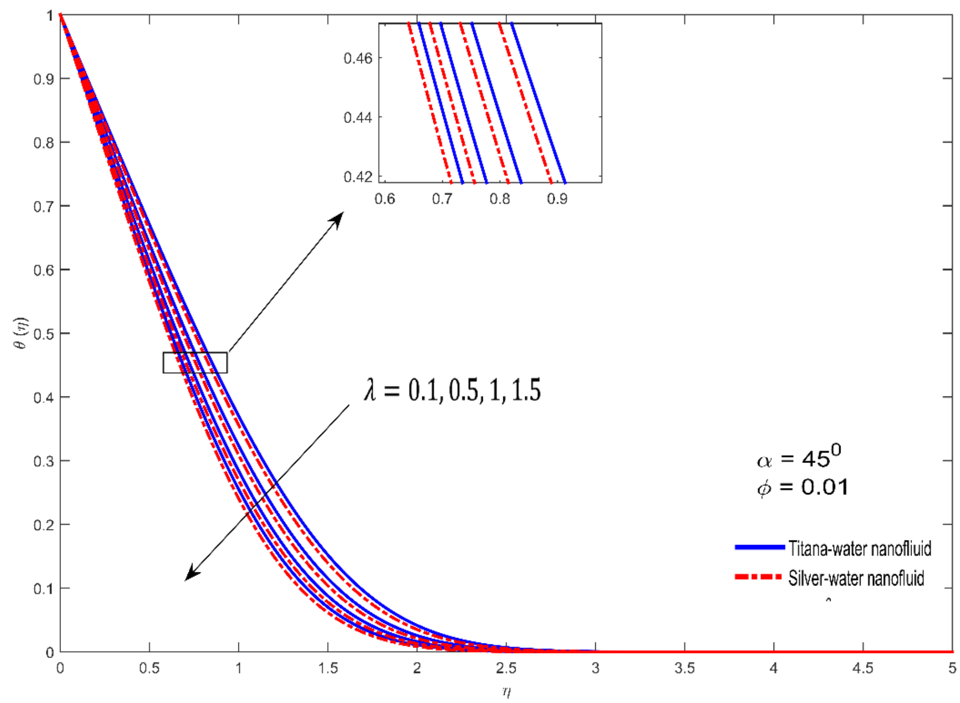

Figure 4.4: Dimensionless temperature profiles of Titana-water and Silver-water nanofluids for $\alpha = 1, \phi = 0.01$ and different λ values

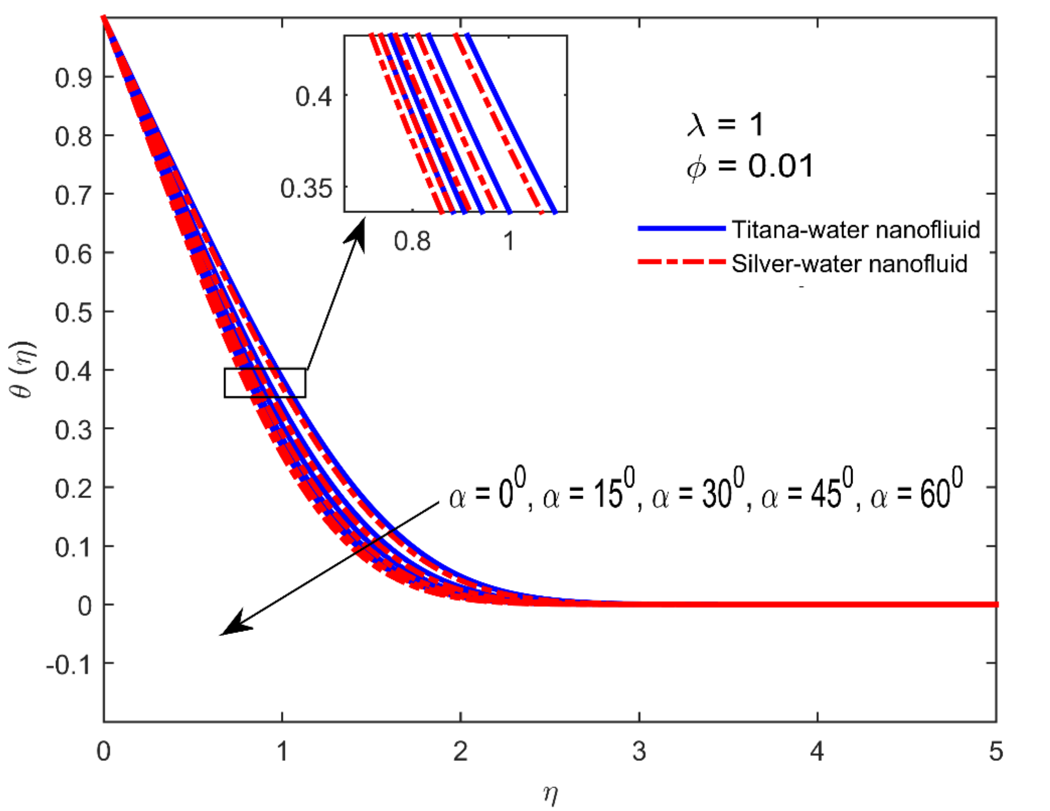

Figure 4.5: Dimensionless temperature profiles of Titana-water and Silver-water nanofluids for $\lambda = 1, \phi = 0.01$ and different α values

Table 4.3: Variation in $\frac{1}{(1-\phi)^{2.5}}f''(0)$ for Titana-water nanofluid and Silver-water nanofluid for different ϕ and α when $\lambda = 1.0$

ϕ	α	$\frac{1}{(1-\phi)^{2.5}}f''(0)$	
		TiO_2-water	Ag-water
0	0	0.336152343901079	0.336152356285676
	$\frac{\pi}{12}$	0.525986291580013	0.525986296594510
	$\frac{\pi}{6}$	0.681591017275126	0.681591249991089
	$\frac{\pi}{4}$	0.805079710567183	0.805079739126025
	$\frac{\pi}{3}$	0.895155004556717	0.895155027433198
0.01	0	0.345776055350397	0.355078854262693
	$\frac{\pi}{12}$	0.541588029633645	0.557554622462070
	$\frac{\pi}{6}$	0.702042308088541	0.723259891835723
	$\frac{\pi}{4}$	0.829363819510326	0.854685143862003
	$\frac{\pi}{3}$	0.922229283328594	0.950522491424374
0.05	0	0.386061795509434	0.431509717947878
	$\frac{\pi}{12}$	0.606633793155207	0.683753652376946
	$\frac{\pi}{6}$	0.787210553054405	0.889427629479209
	$\frac{\pi}{4}$	0.930447154229002	1.05233145125168
	$\frac{\pi}{3}$	1.03490227865265	1.17104788992209

0.1	0	0.441159732507585	0.529982788678272
	$\frac{\pi}{12}$	0.694892509065945	0.845609535925431
	$\frac{\pi}{6}$	0.902514530885302	1.10239520920713
	$\frac{\pi}{4}$	1.06717057950278	1.30561839850071
	$\frac{\pi}{3}$	1.18723374794063	1.45366245684930

Table 4.4: Variation in $\frac{1}{(1-\phi)^{2.5}} f''(0)$ for Titana-water nanofluid and Silver-water nanofluid for different λ and α when $\phi = 0.01$

λ	α	$\frac{1}{(1-\phi)^{2.5}} f''(0)$	
		TiO_2-water	Ag-water
0.1	0	0.345776055350397	0.355078854262693
	$\frac{\pi}{12}$	0.366991689947773	0.377040079966478
	$\frac{\pi}{6}$	0.386358155774012	0.397080541216608
	$\frac{\pi}{4}$	0.402703252413952	0.413990122030943
	$\frac{\pi}{3}$	0.415078479525657	0.426790326283318
0.5	0	0.345776055350397	0.355078854262693
	$\frac{\pi}{12}$	0.447750461056246	0.460575467218748
	$\frac{\pi}{6}$	0.535396982213767	0.551158351906948
	$\frac{\pi}{4}$	0.606512306393574	0.624617639910153
	$\frac{\pi}{3}$	0.658997068452929	0.678816493039919
1.0	0	0.345776055350397	0.355078854262693
	$\frac{\pi}{12}$	0.541588029633645	0.557554622462070
	$\frac{\pi}{6}$	0.702042308088541	0.723259891835723
	$\frac{\pi}{4}$	0.829363819510326	0.854685143862003
	$\frac{\pi}{3}$	0.922229283328594	0.950522491424374

1.5	0	0.345776055350397	0.355078854262693
	$\frac{\pi}{12}$	0.629619105399222	0.648480512897358
	$\frac{\pi}{6}$	0.854798710607021	0.880935489207298
	$\frac{\pi}{4}$	1.03140571424900	1.06317687597904
	$\frac{\pi}{3}$	1.15951710136083	1.19535366293883

Table 4.5: Variation in $-\frac{k_{nf}}{k_f}\theta'(0)$ for Titana-water nanofluid and Silver-water nanofluid for different ϕ and α when $\lambda = 1.0$

ϕ	α	$-\frac{k_{nf}}{k_f}\theta'(0)$	
		TiO_2-water	Ag-water
0	0	0.643253	0.643253
	$\frac{\pi}{12}$	0.707245	0.707245
	$\frac{\pi}{6}$	0.75197	0.75197
	$\frac{\pi}{4}$	0.783912	0.783912
	$\frac{\pi}{3}$	0.805622	0.805622
0.01	0	0.659691	0.680875
	$\frac{\pi}{12}$	0.725479	0.749273
	$\frac{\pi}{6}$	0.771427	0.796918
	$\frac{\pi}{4}$	0.804233	0.830899
	$\frac{\pi}{3}$	0.826526	0.853978
0.05	0	0.725131	0.827106
	$\frac{\pi}{12}$	0.797995	0.912179
	$\frac{\pi}{6}$	0.848777	0.970968
	$\frac{\pi}{4}$	0.885001	1.012763
	$\frac{\pi}{3}$	0.909606	1.041106

0.1	0	0.806202	1.003425
	$\frac{\pi}{12}$	0.887629	1.108305
	$\frac{\pi}{6}$	0.944312	1.180421
	$\frac{\pi}{4}$	0.984722	1.231588
	$\frac{\pi}{3}$	1.012161	1.266253

Table 4.6: Variation in $-\frac{k_{nf}}{k_f}\theta'(0)$ Titana-water nanofluid and Silver-water nanofluid for different λ and α when $\phi = 0.01$

λ	α	$-\frac{k_{nf}}{k_f}\theta'(0)$	
		TiO_2-water	Ag-water
0.1	0	0.659691	0.680875
	$\frac{\pi}{12}$	0.667484	0.688993
	$\frac{\pi}{6}$	0.674437	0.696233
	$\frac{\pi}{4}$	0.680192	0.702222
	$\frac{\pi}{3}$	0.684484	0.706686
0.5	0	0.659691	0.680875
	$\frac{\pi}{12}$	0.695554	0.718196
	$\frac{\pi}{6}$	0.723582	0.747305
	$\frac{\pi}{4}$	0.744779	0.769294
	$\frac{\pi}{3}$	0.759658	0.784721
1.0	0	0.659691	0.680875
	$\frac{\pi}{12}$	0.725479	0.749273
	$\frac{\pi}{6}$	0.771427	0.796918
	$\frac{\pi}{4}$	0.804233	0.830899
	$\frac{\pi}{3}$	0.826526	0.853978

1.5	0	0.659691	0.680875
	$\frac{\pi}{12}$	0.751404	0.776165
	$\frac{\pi}{6}$	0.810465	0.837351
	$\frac{\pi}{4}$	0.851244	0.87956
	$\frac{\pi}{3}$	0.878484	0.907744

CHAPTER 5

APPLICATIONS OF NANOFLUIDS

Nanofluids have a wide range of applications. Nanofluids have expensive applications in tribology as well as in medicine. Recent studies have shown that nanofluids have the ability to enhance the performance of real-world devices and systems, such as automatic transmissions (Choi, 2009).

5.1 Cooling applications

Nanofluids are present in various cooling applications such as vehicle cooling, transformer cooling, and electronic cooling.

5.2 Crystal cooling

The primary application of nfs is developing an advanced cooling technology to cool crystal silicon mirror used in high intensity X-ray sources (Lee & Choi, 1996). Nfs reduce the thermal resistance and increases the power densities, this property is highly evident in nfs cooled silicon microchannel heat exchanger rather than other microchannel like water and nitrogen.

Nano electronics is the use of nanotechnology in electronic components, particularly transistors (B. Yu & Meyyappan, 2006). Nanotechnology is defined as the utilization of technology which are less than 100 nm in size. Electronics is a field focused on developing electronic components and systems at the nanoscale. Nanofluids are used as coolants in a of electronic applications. Some of the applications are the following

- Nano-fluid heat pipes
- Microchannel heat exchangers
- Micro Channel Heat Sink (MCHS)

- Jet cooling
- Transformer cooling
- Spare and Nuclear System Cooling

5.2.1 Nanofluid Heat Pipes

Gold nfs used for working fluid in a circular heat pipe (Isai et al. (2004)), it helps to reduce significant reduction in thermal resistance compared to Deionized (DI) water at the Same charge volume. This development is effective in cooling devices for ultra-high-heat-flur electronic systems. A nanofluid Oscillating Heat Pipe (OHP) using water-based nanofluids with Aluminum Oxide (Al_2O_3) nanoparticles can remove heat in excess of 1000 W/cm^2 (Ma et. al (2006))

5.2.2 MCHS effectiveness

Experiments based on Silicon Micro Channel Heat Sink (MCHS) found that, water-based nanofluids containing cuprous oxide (CuO) nanoparticles absorb more beat than water, resulting in lower MCHS wall temperatures (Chein & Chuang, 2007). Studies conducted in steady laminar flow of nanofluids in microchannels concluded that copper nanoparticles at low volume fractions in high Prandtl number fluids enhance the heat transfer performance of MCHS (Koo & Kleinstreuer, 2005). Other valuable inference on MCHS performance is that nanofluids have the potential to improve MCHS performance (Chein & Huang, 2005).

5.2.3 Jet Cooling Systems

Heat transfer enhancement capability of nfs inside typical radial flow impingement jet cooling systems show that nfs can increase the average wall heat transfer coefficient and decrease the wall shear stress (Palm et al., 2006).

5.2.4 Vehicle Cooling

Nanoparticles is applied, not only in coolants, but also in transmission fluids, engine oils, gear oils and other fluids and lubricants (Lockwood et al., 2005). Nfs are good in thermal management and better lubrication. Real rotary blade coupling of a power transmission system of a real-time four-wheel drive vehicle show that cuprous oxide (CuO) nanoparticles have the lowest temperature distribution at both high and low rotating speed and implies better beat transfer effect (Zeng et al., 2005).

5.2.5 Transformer Cooling

Transformer cooling application of nfs helps to reduce size and weight. Nanoparticles can be used to increase the capability of transformers to transfer heat. Heat transfer properties of transfer oils can be improved using nanoparticle additive like nfs -based transformer oil (Xuan & Li, 2000) and (W. Yu et al., 2007).

5.2.6 Space and Nuclear systems cooling

Nanofluid, have the ability to double or triple the Critical Heat Flux (CHF) in pool boiling (You et al., 2003) and (Vassallo et al., 2004). Nanoparticle deposition causes high surface wettability, which helps to explain the thermal properties of nfs. The use of nfs with significantly higher CHF values could enable a much safer operation of commercial or military nuclear reactors.

5.3 Cooling Defence Applications

Nfs provide advanced cooling technology for submarines, military vehicles and high-power laser systems (Saidur et al., 2011). In defence applications, they not only serve as coolants, but also as multifunctional fluids with added thermal energy storage or energy harvesting through chemical reactions. This technology is applied in high-powered military electronics, military vehicle component, radars and lasers. Directed energy weapon cooling and power electronics are some potential military applications of nfs.

5.4 Tribological Applications of Nfs

Que et al. (1997) and (Liu et al., 2004), found that nanoparticles like Iridium Oxides (IrO_2) and Zirconium oxide (ZrO_3) decrease friction on surfaces. The nfs have the ability to reduce friction and wear, this property widely utilized in various oil and gas drilling application. Load-carrying capacity, of anti-wear and friction reduction are the enhanced tribological properties of nanoparticles. These properties are beneficial for surface-modified nanoparticles stably dispersed in mineral oils, making them particularly effective in reducing wear and tear.

5.5 Biomedical Applications

Nfs and nanotechnology are effective in biomedical applications. Cancer therapy using nfs is an innovative area in medical field (Chen et al., 2019). In cancer therapy, iron-based nanoparticles serve as carriers for drugs or radiation, and they are guided to tumours with magnets (Mekheimer et al., 2018). Nfs could be used to produce higher temperatures around tumours to kill cancerous cells without affecting healthy tissue (Jordan et.al (1999)). It could improve surgical safety by cooling the surgical region to improve patient survival and lower the risk of organ damage.

CHAPTER 6

CONCLUSION

The central goal of this research is to perform a detailed theoretical and computational analysis of nanofluid flow across flat, moving, and inclined plate configurations. The study of nanofluid flow holds significant importance across multiple applications such as cooling of crystal silicon mirrors, nanoelectronics, and electronic devices, as well as in heat pipes, jet cooling systems, engine and transformer cooling, spacecraft and nuclear reactor thermal management, military technologies, biomedical applications, and more. Flow characteristics like velocity and temperature, along with physical parameters such as drag coefficients and heat transfer rates, are analyzed and discussed. Fluid motion is described using the Tiwari-Das nanofluid model. The study focuses on water-based nanofluids formulated with titana (TiO_2) and silver (Ag) nanoparticles.

- A higher volume fraction of nanoparticles results in an elevated temperature field.
- Among water-based nanofluids containing TiO_2 and Ag nanoparticles, silver (Ag) plays a prominent role in enhancing fluid velocity.
- As the fluid temperature rises, the heat transfer rate, indicated by the Nusselt number, decreases, since it quantifies the heat exchanged from the surface to the fluid.

CHAPTER 7

FUTURE RECOMMENDATIONS

In the future, the works presented in this study can be extended In the following directions:

Here are some future research recommendations for convective flows of nanofluids in sheet geometry:

1. Sheet geometry modifications: Study the effects of varying sheet thickness, waviness, or perforations on convective flows and heat transfer
2. Turbulent flow regimes: Extend research to turbulent flow regimes, which are more representative of industrial applications
3. Multi-phase flows: Study convective flows of nanofluids in multi-phase systems, such as gas-liquid or liquid-liquid flows

LIST OF PUBLICATIONS

1. Rose Paul, Julie Andrews, “Nanofluids and Applications”, International Journal of Engineering Technology Science and Research, Volume 5, Issue 3, ISSN 2394– 3386, 633-638, (2018)

2. Rose Paul, Julie Andrews, Naveen V V, P. Suriyakumar, “The Impact of Volume Fraction and Nanoparticles in the Flow of Nanofluids along the Boundary Layer Over a Flat Plate”, International Journal of Communications on Applied Nonlinear Analysis, Vol 31, No. 2, ISSN: 1074-133X, 259-269, (2024), https://doi.org/10.52783/cana.v31.541.

LIST OF PRESENTATIONS

1. Rose Paul, “Nanofluids and Applications” in The International Conference on Innovative Research in Engineering, Science, Management and Humanities (ICIRESMH-2018) organized by Conference Info in association with Academic Science at The Institution of Engineers India, Visvesvaraya Bhavan, Hyderabad, Telangana, India on 18 March 2018.

2. Rose Paul, “Nanofluids: Coolants for the New Era” in The National Seminar on Materials, Methods, Sensors for Electromagnetic Applications organized by the Post Graduate and Research Department of Physics, Christ College (Autonomous), Irinjalakuda, Kerala, India during 24-25 January 2019.

3. Rose Paul, “Nanofluids: Synthesis and Diverse Applications” in The National Seminar on Advanced Mathematics and Statistics organized by the Post Graduate Department of Mathematics and Department of Statistics, Little Flower College, Guruvayur, Kerala, India during 8-9 August 2019.

4. Rose Paul, “Comparative Study of Thermal and Physical Properties of Al_2O_3-Water and AgO_2-Water Nanofluids in View of Different Volume Fractions” in the DST-PURSE, Govt. of India Supported Satellite Sympisium at Bharathiar University, Coimbatore during 13-15 September 2022.

5. Rose Paul, “A Study on the Thermal and Physical Properties of Titana-Water and Alumina Water Nanofluids” in The International Conference on Mathematics of Intelligent Computing and Data Science (ICMICDS 2022) organized by the Department of Mathematics, Rajagiri School of Engineering and Technology and Centre for Topology and Applications (CETA) on 16 September 2022

LIST OF SYMBOLS

Ω	Vorticity
ρ	Fluid density
$\vec{V}$	Velocity vector.
$\mathcal{A}$	Area of cross-section
M	Dynamic viscosity
P	Pressure
F	Force
Et	Total energy
K	Potential energy
I	Internal energy
(vi, vj)	Velocity components.
Pr	Prandtl number,
α	Thermal diffusivity,
υ	Kinematic viscosity,
Nu	Nusselt number
Gr_{nf}	Grashof number
g	Acceleration due to gravity
β	Volumetric coefficient of thermal expansion
L	Characteristic length
V_{nf}	Kinematic viscosity of the fluid
$\left(c_p\right)_f$	Specific heat capacity of the base fluid
$\left(c_p\right)_s$	Specific heat capacities of the solid particle
k_{nf}	Thermal conductivity of nanofluid
k_f	Thermal conductivity of base fluid
ϕ	Volume fraction of nanoparticle.
$\Psi(x, y)$	Stream function

θ	Dimensionless temperature.
C_f	Skin friction coefficient
Re_x	Local reynolds number.
$(C_p)_{nf}$	Specific heat capacity of nanofluid
U_∞	Uniform free stream velocity

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