MODELLING AND FORECASTING OF MARINE FISH LANDINGS IN KERALA USING MULTIPLE TIME SERIES MODELS

SATHIANANDAN.T.V.

A THESIS SUBMITTED TO

UNIVERSITY OF CALICUT
KERALA

IN PARTIAL FULFILMENT OF REQUIREMENTS
FOR THE AWARD OF DEGREE OF

DOCTOR OF PHILOSOPHY

NOVEMBER 2000

DEDICATED TO MY BELOVED PARENTS

AND

MY CHILDREN ADERSH AND AVINASH

CERTIFICATE

This is to certify that the Thesis entitled **Modelling and Forecasting of Marine Fish Landings in Kerala using Multiple Time Series Models** submitted to the University of Calicut for the award of **Degree of Doctor of Philosophy in Statistics** is a record of original research work done by Sathianandan.T.V. during the period of his study in the Department of Statistics, University of Calicut, Kerala, under my supervision and guidance and the Thesis has not formed the basis for award of any Degree/Diploma/Associateship/Fellowship or similar title to any candidate of any University or Institute.

SIGNATURE OF THEGUIDE

DECLARATION

I hereby declare that the matter embodied in this Thesis is the result of investigations carried out by me in the Department of Statistics, University of Calicut, under the supervision and guidance of Dr. Kumarankutty, Professor & Head (Retired), Department of Statistics, University of Calicut and it has not been submitted for award of any Degree/Diploma/Associateship/Fellowship of any other University or Institute.

SUPERVISOR

CANDIDATE

MODELLING AND FORECASTING OF MARINE FISH LANDINGS IN KERALA USING MULTIPLE TIME SERIES MODELS

SATHIANANDAN.T.V.

A THESIS SUBMITTED TO

UNIVERSITY OF CALICUT
KERALA

IN PARTIAL FULFILMENT OF REQUIREMENTS
FOR THE AWARD OF DEGREE OF

DOCTOR OF PHILOSOPHY

NOVEMBER 2000

ACKNOWLEDGEMENTS

I sincerely wish to place on record my gratitude to Dr.K.Kumarankutty, M.Sc., M.S., Ph.D., Professor & Head (Retired), Department of Statistics, University of Calicut, for his supervision, guidance and constant encouragement during the period of study and in the preparation of this Thesis.

I am grateful to Dr.M.Deveraj, former Director, Central Marine Fisheries Research Institute, Cochin for granting me study leave. I wish to express my gratitude to Dr. Mohan Joseph, Director, Central Marine Fisheries Research Institute, Cochin for extending the facilities to carryout this research work. I sincerely thank The Indian Council of Agricultural Research, New Delhi for the award of Senior Research Fellowship to me for taking up the Ph.D. programme. I thank the Additional Director General of Meteorology (Research), India Meteorological Department, Pune for providing necessary environmental data used in this study. I wish to thank Shri. K. Balan, Senior Scientist & former Head, Fishery Resources Assessment Division, Central Marine Fisheries Research Institute, Cochin for his help in the collection of data and constant encouragement through out the programme. I thank Shri. K. Narayana Kurup, Head, Fishery Resources Assessment Division, Central Marine Fisheries Research Institute, Cochin for the help and encouragement shown towards me for this work. I thank Dr. M. Srinath, Senior Scientist and Officer-in-charge of the Computer Centre, Central Marine

Fisheries Research Institute, Cochin for going through the Thesis and offering valuable modifications and suggestions.

I am grateful to Dr. N. Raju, Sr. Lecturer, Department of Statistics, University of Calicut for all the help he has shown towards me during the period of this study and for critically going through the thesis. I wish to place on record my gratitude to Dr. M. Manoharan, Head, Department of Statistics, University of Calicut for his encouragement and help during the period of this study. I thank Shri. Jensen, P.O., Research Scholar, University of Calicut for his help during the preparation of this thesis.

My sincere thanks are due to Shri.P.P. Pavithran, Technical Assistant, Fishery Resources Assessment Division, Central Marine Fisheries Research Institute, Cochin for his help in the preparation of this manuscript. I thank Dr. K. Sunilkumar Mohamed, Scientist (Sr. Scale), Central Marine Fisheries Research Institute and Dr. K. K. Joshi, Scientist, Central Marine Fisheries Research Institute for their constant encouragement and helps in the preparation of my thesis.

I am thankful to Shri. C. Mohanan, Miss. Sindhu.T.V., Shri. Arun.A.C. and Shri. K. Bava for the help they have shown towards me in the final stage of preparation of my thesis. My wife Mrs. Sathi Sathian has been constantly encouraging me and providing all possible help through out the period of this programme. I am indebted to her.

CONTENTS

		Pages
Introduction		1-6
Chapter-1	Univariate ARIMA modelling of marine fish landings in Kerala.	7-79
Appendix-I	Tables	i – xiii
	Charts	xiv – xxxii
Chapter-2	Vector Autoregressive modelling of marine fish landings in Kerala.	80-135
Appendix-II	Tables	i – xxv
Chapter-3	Modelling marine fish landings using VARMA models	136-159
Appendix-III	Tables	i – ii
Chapter-4	Modelling marine fish landings with environmental variables	160-184
Appendix-IV	Tables	i – iv
Chapter-5	Time series relations through canonical path analysis and time series of moving sums.	185-204
Appendix-V	Tables	i – iii
Appendix-VI	List of Computer Software Developed	i - ii
Summary and Conclusions		205 - 210
References		211-223

INTRODUCTION

Marine fisheries is an important sector in India and contributes 1.2 percent to the nation's Gross Domestic Production. It earns a foreign exchange worth 41,500 million rupees. The current marine fish production in the country is 2.4 million tonnes which is about 4 times the production in the early fifties. This accounts for a contribution of 3 percent of the world marine fish production and India is one among the top 10 fish producing countries. India has a coastline of 8,129 Kilometers. About 5 million people living in the coastal areas are engaged in fishing and other related activities for their livelihood. Fishing in marine fisheries sector is carried out by about 1.8 lakh non-motorised traditional fishing crafts, 32,000 motorized traditional crafts and 47,000 mechanized vessels.

Among the maritime states, Kerala has a prominent place with regard to marine fish production in the country which contributes to almost 25% of the total marine fish production though the total coast line covered by the state is only 590 Kms which is about one-tenth of the Indian coast line. The state has two major fishing harbours at Cochin and Sakthikulangara and about 220 landing centers distributed over 304 fishing villages. In the year 1996, total marine fish production from the state was 5.72 lakh tonnes which accounts for 23.62 % of the total marine fish production in the country. During the period 1987-96 Kerala accounted for an average landings of 5.45 lakh tonnes which is about 25.29 % of the average production in the country during this period. The marine fish production in the country, during this period, varied between 1.65 million tonnes (in 1987) and 2.42 million tonnes (in 1996) with an average of 2.16 million tonnes. The contribution from Kerala during this period ranged between 3.03 lakh tonnes (in 1987) and 5.72 lakh tonnes (in 1996). In this period the percentage contribution by the state towards marine fish production in the country was maximum in the year 1990 which is 30.94% and the minimum observed was 18.39% in the year 1987.

Among different marine fishery resources that contribute towards total marine fish landings in Kerala, oil sardine (*Sardinella longiceps*) and Indian mackerel (*Rastreliger kanagurta*) are the two prominent species. In the year 1996, the contribution by mackerel is about 1.28 lakh tonnes which is 22.45% of the total landings in the state and that by oil sardine was about 30,000 tonnes which is 5.35% of the state total. During the period 1987-96 these two species on an average accounted for about 25.29% of the landings in the state. Average landings of oil sardine in this period was 72,493 tonnes which is 13.29% of the average annual landings in the state and the average landings of mackerel in this period was 68,747 tonnes which is 12.61% of the average annual landings in Kerala. Other major contributors, based on their average landings during this period are Penaeid prawns (9.90%), Perches (9.60%), Carangids (9.00%), Anchovies (6.89%), Cephalopodes (3.33%), Lesser sardines (3.32%), Soles (3.05%), Tunas (3.01%), Sciaenids (2.24%), Saurida and Saurus (2.07%), Ribbon fish (1.81%) and Seer fish (1.19%).

Forecasting of marine fish production is very much essential for proper planning. Fluctuation in marine fish production affect the processing industry, export earnings, employment and income to fishermen community, marketing and cost of marine fish products. Advance information about future production will help in proper planning, storage, distribution and to take necessary measures according to the situation. Since many of the marine fish species depend each other due to factors like prey-predator relation, competing for a common food resource and influenced by a common environmental condition, it is always desirable to know about the inter-relations that exist between landings of different species. Marine fish landings observed over a period of time can be treated as a time series process generated by a mechanism and can be studied in this context to see the trend in landings, inter-relations among landings of different species, periodicity in the landings etc., using available time series techniques. Apart form this, by exploiting these factors time series models can be used to fit landings data and can further be used to forecast future values with more precision.

Time series analysis is a branch of statistics which can help in resolving the above problems by treating marine fish production as a time series process generated over time and examining them for secular trend, cyclical or periodic behaviour and other kinds of fluctuations using the tools available in time series analysis. Once these are identified, a time series model can be formulated which can be used for forecasting. Marine fish production statistics that are generated over time depend on past values and is influenced by seasonal and irregular kind of fluctuations. By analysing it as a time series this dependency can be exploited to develop a suitable time series model. Since it is composed of unknown irregular fluctuations, which are not deterministic in nature, (a deterministic model is not suitable for modelling marine fish landings time series) a stochastic time series model is most appropriate for modelling marine fish landings time series.

A model using which we are able to calculate the probability that a future value of the series is lying between two specified limits is termed as a stochastic model. In this contest, a time series is considered as a stochastic process and a sample or consecutive values of the time series is considered as a realization of the stochastic process that generated the time series. An important class of stochastic models that are widely used to represent time series is the stationary models which assume that the stochastic process that generate the time series remains in equilibrium about a constant mean level. A non-stationary process does not have any natural mean. A stochastic process is said to be strictly stationary if its properties are unaffected by change of time origin. A sequence of random variables $\{a_t\}$ is called a white noise series if these are random drawings from a fixed distribution with zero mean and constant variance. The kind of stochastic models considered for time series analysis are based on the concept that a process in which successive values are highly dependent can be regarded as generated from a series of independent shocks, a_t, which are white noise. If a white noise process $\{a_t\}$ is transformed into a stochastic process $\{z_t\}$ by a weighted sum of past values so that $z_t = \mu + a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \cdots$, then it is called a linear filtering. Using the back shift operator B notation, where B operates on the time series $\{a_t\}$ such that $B^k a_t = a_{t-k}$, for values of $k = 1,2,\cdots$ we can write the above equation as

$$z_t = \mu + a_t + \psi_1 B a_t + \psi_2 B^2 a_t + \cdots$$

$$= \mu + (1 + \psi_1 B + \psi_2 B^2 + \cdots) a_t$$

$$= \mu + \psi(B)a_t$$

where $\psi(B) = 1 + \psi_1 B + \psi_2 B^2 + \cdots$ which is a polynomial in the back shift operator B and is known as the transfer function of the filter. If the sequence of weights in the transfer function are finite or infinite and convergent then the filter is said to be stable and the generated stochastic process $\{z_t\}$is said to be stationary. When the sequence of weights $\{\psi_i\}$ is infinite and not converging then the process $\{z_t\}$ is non-stationary. For a stationary process the parameter μ will be the mean about which the process varies and for a non-stationary process it is only a reference point for the level of the process.

The most popular class of stationary type of stochastic model used for time series modelling is the Autoregressive Integrated Moving Average (ARIMA) model introduced by Box and Jenkins (1976). This class includes, autoregressive models, moving average models, random walk models, autoregressive moving average models, integrated models and seasonal models. In the first chapter univariate seasonal ARIMA models are used to fit quarterly landings of selected marine fish species/groups in Kerala. The inter-relations between landings are examined in the second chapter through cross correlation analysis and by modelling selected groups together using Vector Autoregressive models which is the multivariate version of the univariate autoregressive models. In the third chapter vector autoregressive moving average models are used as an alternative to higher order vector autoregressive models. In the fourth chapter the relationship between marine fish landings and environmental variables is examined by using data on environmental variables recorded at Cochin and monthly landings of marine fishes at Cochin Fisheries Harbour. This was carried out through cross correlation analysis and by fitting vector autoregressive models with environmental variables as exogenous vector series and marine fish landings as output vector series. In the fifth chapter a method of finding relation between time series was developed using canonical analysis and path coefficients. Also the properties of moving sums of different types of univariate time series are examined in this chapter.

In all chapters, except for the first, where ever computations were involved necessary computer softwares were developed in C language. An exhaustive list of the softwares developed for the study is given at the end. Tables and charts are given in appendix of each chapter.

CHAPTER-1

Univariate ARIMA modelling of marine fish landings in Kerala.

Introduction:

In univariate autoregressive model, the current value of a process is expressed as a linear aggregate of past values of the series along with a random shock. If by $\{z_t\}$, we mean a stationary process to represent a time series sequence and $\{\tilde{z}_t\}$ is the process which is the deviation from a central value μ, the mean of the process, then an autoregressive model of order p, denoted by AR(p), takes the form $\tilde{z}_t = \phi_1\tilde{z}_{t-1} + \phi_2\tilde{z}_{t-2} + \cdots + \phi_p\tilde{z}_{t-p} + \varepsilon_t$. Here $\{\varepsilon_t\}$ is a sequence of random shocks which are assumed to be independently and identically distributed with expectation zero and constant variance σ^2. Parameters of this model are $(\mu, \phi_1, \phi_2, \cdots, \phi_p, \sigma^2)$. Using the back shift operator notation we can write the AR(p) model as

$$\begin{aligned}\tilde{z}_t &= \phi_1 B\tilde{z}_t + \phi_2 B^2\tilde{z}_t + \cdots + \phi_p B^p\tilde{z}_t + \varepsilon_t \\ &= (\phi_1 B + \phi_2 B^2 + \cdots + \phi_p B^p)\tilde{z}_t + \varepsilon_t\end{aligned}$$

That is $\phi(B)\tilde{z}_t = \varepsilon_t$ where $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$ is a polynomial of degree p in the back shift operator B. By considering the linear filter representation of the above AR(p) model the condition for stationarity of the process can be derived in terms of the roots of the characteristic equation $\phi(B) = 0$. This condition is that all the roots of the p^{th} degree polynomial equation $\phi(B) = 0$ are more than one in absolute terms.

Another popular representation of a time series is using moving average models. In this model the deviations of $\tilde{z}_t$ are represented by a linear combination of a finite number of previous random shocks. The expression for a moving average model of order q, MA(q) model, is

$$\begin{aligned}\tilde{z}_t &= a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \cdots - \theta_q a_{t-q} \\ &= \theta\ (B) a_t\end{aligned}$$

where $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q$ is a q^{th} degree polynomial in the backshift operator B. Since the linear filter format of this model is the same and it contain only finite number of terms, a moving average model is always stationary. Using the inverse function $\pi(B) = \theta^{-1}(B)$, the linear process $\tilde{z}_t = \theta\ (B) a_t$ can have the infinite autoregressive representation $\pi(B)\tilde{z}_t = a_t$ where $\pi(B) = \sum_{j=0}^{\infty} \pi_j\ B^j$. This process is invertible when the sequence of weights π_j converge. The required condition for this is that the roots of the characteristic polynomial equation $\theta(B) = 0$ lie out side the unit circle and it is the condition for invertibility of a MA(q) model.

The model that combines the above two kinds of models is the mixed autoregressive moving average (ARMA) model. The autoregressive moving average model with p autoregressive terms and q moving average terms is represented by ARMA(p,q) and its mathematical expression is,

$$y_t - \phi_1 y_{t-1} - \phi_2 y_{t-2} - \cdots - \phi_p y_{t-p} = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \cdots - \theta_q a_{t-q}$$

This can be written as

$$\phi(B) y_t = \theta(B) a_t$$

where $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$ is a polynomial in the back shift operator B of degree p and $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots \theta_q B^q$ is a polynomial in B of degree q. The condition for stationarity of the model is the same as that for an AR(p) model and the condition for invertibility of the model is the same of an MA(q) model. Hence the conditions are that the roots of polynomials $\phi(B)$ and $\theta(B)$ lie out side the unit circle. In most of the practical situations, the observed time series show non-stationary behaviour and do not vary about a constant mean. In such situations it may be possible to represent the series by an ARMA model with a generalized AR operator polynomial, say $\varphi(B)$ for which one or more roots are unity so that it can be factorized as $\varphi(B) = \phi(B)(1 - B)^d$ where $\phi(B)$ is a polynomial in B with all its roots out side the unit circle. Box and Jenkins (1976) introduced a class of non-stationary models by integrating ARMA models with provision for the representation for unit root non-stationarity. This model permits violation of the stationarity condition by allowing some roots to lie on the unit circle. These models are termed as autoregressive integrated moving average (ARIMA) models. If there are d roots that fall on the unit circle, then the model representation is

$$\phi(B)(1 - B)^d y_t = \theta(B) a_t$$

This is equivalent to transforming the original series into another series by successively differencing it d times and then representing the ultimate differenced series by an ARMA(p,q) model. The parameter d is termed as the order of differencing and p and q are respectively the order of autoregressive and moving average terms in the model. Cressie (1988) had shown that differencing a series d times will remove a polynomial trend of degree d existing in the time series which may be the cause for non-stationarity in the series.

The general seasonal ARIMA model with AR order p, MA order q, order of differencing d, seasonality s, seasonal AR order P, seasonal MA order Q and order of seasonal differencing D is represented by ARIMA(p,d,q)(P,D,Q)s and its expression is

$$\varphi(B^s)\phi(B)\nabla^d\nabla_s^D y_t = \gamma + \Theta(B^s)\theta(B)a_t$$

where $\nabla = (1-B)$, $\nabla_s = (1-B^s)$, $\phi(B) = 1-\phi_1 B - \cdots - \phi_p B^p$

$\varphi(B^s) = 1-\varphi_1 B^s - \cdots - \varphi_P B^{sP}$, $\theta(B) = 1-\theta_1 B - \cdots - \theta_q B^q$

and $\Theta(B^s) = 1-\Theta_1 B^s - \cdots - \Theta_Q B^{sQ}$ which are polynomials in B. The conditions for stationarity and invertibility of the differenced series $\nabla^d\nabla_s^D y_t$ is that the roots of all these polynomials lie out side the unit circle.

Review of literature

Antony Raja (1973) studied oil sardine fishery and related it to rainfall, atrosia and availability of juveniles in July-September. Jensen (1976) examined the autocorrelation structure of Atlantic menhaden catch and used a second order autoregressive model for prediction of menhaden catch. Pierce (1979) developed an R-square that measures the explanatory power of a time series regression model relative to that of past values of dependent variable. He considered the innovation variance of the dependent variable as the variance to be explained by the regression model. Bhattacharyya (1979) modified the Box-Jenkins univariate time series model to incorporate intervention effects for a case study on effectiveness of seat belt legislation on the Queens land road toll through intervention analysis. In a second approach he considered a casual model with a proxy explanatory variable and ARMA error. Using this model he could quantify the long run legislative effect as a specific level of the explanatory variable to be a reduction of 46% in road deaths. Van Winkle *et. al.*(1979) examined the stripped bass commercial fisheries data of Atlantic coast for periodicity in the appearance of dominant year-classes through autocorrelation and spectral analysis.

Godolphin (1980) proposed a method for testing the order of an ARMA model based on serial correlations of residuals which has advantage in terms of its sensitivity in discriminating between alternative models. Mendelssohn (1980) used Box-Jenkins models to forecast fishery dynamics. Newbold (1980) established that the Lagrange multiplier test with an ARMA(p+k,q) model and a test based on k residual autocorrelations are equivalent for testing the adequacy of a fitted ARMA model. Noble (1980) analyzed mackerel landings in India for three decades and found a set pattern of recurring ups in and around the confluence of two decades and downs in the middle of once decade. He suspected the presence of a ten-year cycle in the mackerel fishery. Pearlman (1980) obtained the likelihood function of an ARMA process using Kalman filtering and have shown that this method is more efficient compared to other methods when the order of the moving average part is more than 5. Poskitt and Tremayne (1980) developed a procedure for the diagnostic checking of ARMA models as an extension of score multiplier test for testing ARMA(p,q) process against ARMA(p+r,q+s) model.

Saila *et. al.* (1980) analyzed monthly average of per day catch of rock lobster from New Zealand using monthly averages, harmonic regression and ARIMA models and found ARIMA models most suitable for short term forecasts.

Kedem and Slud (1981) proposed a test statistic, as an application of higher order crossing to test the goodness of fit of an ARMA model and compared it with the Box-Pierce portmanteau statistic. Milhoj (1981) proposed a goodness of fit statistic for time series models as a frequency domain analog of the Box-Pierce portmanteau statistic and its asymptotic properties were compared with the portmanteau test. Stocker and Hilborn (1981) considered stock production models and time series models for short term forecasting of marine fish stocks. Woodward and Gray (1981) introduced the concept of generalized partial autocorrelation function and discussed its usefulness in ARMA model identification. They compared the S-array method and Box-Jenkins procedure of model identification. Campbell (1982) proposed a recursion for M-estimates for the parameters of a finite order autoregressive process through stochastic approximation methods. Harvey and Mckenzie (1982) presented an algorithm for finite sample prediction from ARIMA process based on Kalman filter. Hannan and Rissanan (1982) suggested a recursive estimation procedure for ARMA order. This method involved estimation of the innovations by fitting a long autoregression to the data and then a series of regressions of the observations on the estimates of innovations. They established asymptotic properties of the estimates under very general conditions. Godolphin and Gooijer (1982) developed an iterative procedure for solving the likelihood equations for estimation of parameters of a Gaussian moving average process by expressing the estimator of each parameter as a linear combination of a suitably large set of sample serial correlations. Anderson (1982) made an empirical examination of Box-Jenkins procedure of forecasting which consists of data transformation, model identification, parameter estimation and diagnostic checking. Tiao and Tsay (1983) investigated the consistency properties of least square estimates of AR parameters in ARMA models and obtained consistent estimates of orders of autoregression for a given model.

Potsher (1983) developed a stepwise testing procedure using Lagrangian multiplier test to determine the order of an ARMA process. Taniguchi (1983) have shown that an appropriate modification of the ML and quasi ML estimators of Gaussian ARMA process are second order asymptotically efficient. He used the degree of concentration of sampling distribution, up to second order, as a measure of efficiency. McLeod and Sales (1983) developed an algorithm for calculation of approximate likelihood of ARMA and multiplicative seasonal ARMA model that is more efficient for the regular non-seasonal ARMA model. Clarke (1983) presented an algorithm that enables diagnostic checking for the adequacy of an invertible ARMA model for a given sample time series. Godolphin and Unwin (1983) presented a simple procedure for deriving the covariance matrix for the ML estimators of a Gaussian ARMA process.

Melard (1984) developed a fast algorithm to compute the exact likelihood function of a stationary ARMA process as an improved version of Pearlman's (1980) method with the quick recursion switching suggested by Gardner *et. al.* (1980). This program is efficient both in terms of computing time and amount of storage. McLeod (1984) presented a duality theorem and its properties with application to multiplicative seasonal ARMA models. These applications include a method for calculating covariance matrix of estimated parameters, formula for variances of residual autocorrelations and distribution of inverse partial autocorrelations. Hannan and Kavalieris (1984) developed an estimation method for an ARMA system including the estimation of orders as a modification of the method proposed by Hannan and Rissanen (1982). Kohn and Ansely (1984) used Kalman filtering for a pure seasonal moving average model and have shown that considerable computational savings can be achieved by using a result of Ansley (1979). Godolphin (1984) described a simple procedure for obtaining the estimator of parameters of an ARMA process that was derived from ML procedure assuming Gaussian residuals and two simplifying assumptions. Abraham and Ladolter(1984) investigated the usefulness of inverse autocorrelations as a model identification tool for time series models. Through simulation they established that the inverse autocorrelations are less powerful than the partial autocorrelations for an autoregressive process. Said (1984) proposed a test for unit roots in ARMA models of unknown order based on an approximation of ARMA by an autoregressive process. Solo (1984) derived a Lagrangian multiplier test for testing the order of differencing in ARIMA models and provided a numerical illustration. Tsay (1984) proposed a unified approach for the tentative specification of order of stationary and non-stationary ARMA models and presented an iterative regression procedure to produce consistent estimates of autoregressive parameters. Based on these estimates he defined an extended sample autocorrelation function and used it for order determination that eliminates the need to determine the order of differencing to produce stationarity in modeling a time series. Davies and Petruccell (1984) have shown that the general partial autocorrelation that has been used as a tool for order identification in ARMA models has unstable behavior when applied to time series of moderate length and its use in determining moving average order is very limited. Dickey *et. al.* (1984) have computed percentiles of distributions for time series that have a unit root at seasonal lag by Monte Carlo integration for finite samples and by analytic techniques and Monte Carlo integration for the limit case.

Hallin *et. al.* (1985) proposed a class of linear serial rank statistics for testing white noise against alternatives of ARMA serial dependency and provided efficiency properties of the asymptotically most efficient score generating functions. Jensen (1985) analyzed the catch and catch per unit effort data for Atlantic menhaden and Gulf menhaden through autocorrelation analysis to test for time lags and to develop forecasting models. Srinath and Datta (1985) used ARIMA models for forecasting marine products export from India and found that the forecasts made using these models were close to the actual values. Tsay and Tiao (1985) derived a canonical analysis for time series modelling using the second order moment structure of time series models. They proposed a canonical correlation approach for order determination in ARMA models that can handle both stationary and non-stationary processes. Franke (1985) proposed an algorithm for recursive calculation of parameters of an ARMA process generalizing the recursions of Levinson (1946) and Durbin (1960). Poskitt (1986) presented an identification criterion for model selection in the context of general class of parametric time series models that is asymptotically equivalent to a Bayes decision rule and considered the special case of ARMA order determination. Ljung (1986) developed a portmanteau statistic for testing adequacy of an estimated ARMA model based on autocorrelations of residuals. Hannan *et. al.*(1986) considered the estimation of parameters for a scalar linear system with rational transfer function and proposed a computational scheme using Givens transfer function which economize the three-stage recursive algorithm of Hannan and Rissanen. Bustos and Yohai (1986) proposed two classes of robust estimates for ARMA models, one based on residual autocovariance estimates and the other based on truncated residual autocovariances. These estimates were compared with least square, M and GM estimates.

Box *et. al.*(1987) suggested a method for estimating the trend of a multiplicative seasonal ARIMA model as a component of the models forecast function which is a linear combination of roots of the AR operator that are associated with trend. Holden (1987) developed a general model that leads to ARMA models for Poisson distributed variates and applied it to data on aircraft hijacking attempts. Kreiss (1987) proved locally asymptotic normality for ARMA processes and constructed locally asymptotic minimax estimators for the parameters of the model that will achieve the smallest possible covariance matrix asymptotically. Masarotto (1987) provided estimates of ARMA parameters that are consistent at the nominal Gaussian model and are insensitive to outliers. Misra and Uthe (1987) applied time trends analysis to contaminant levels in Canadian Atlantic cod and illustrated the use of MANOCOVA for time series trends investigation. Poskitt and Tremayne (1987) applied selection criteria for order determination in linear time series models via posterior odds ratios and by applying the concept of grades of evidence proposed by Jeffreys (1961). Poskitt (1987) again proposed a modification to the order selection strategy for ARMA models proposed by Hannan and Rissanen that eliminate the bias and involves the use of alternate model selection criterion. Wei (1987) considered the asymptotic performance of least square predictors of regression models and applied these results to non-stationary autoregressive time series. He constructed a statistic, to show how many times a non-stationary time series should be differenced in order to obtain a stationary time series. Hannan and Poskitt (1988) discussed about regression procedures for the estimation of parameters in ARMA models to provide initial estimates for iterative maximization of Gaussian likelihood. Brockett *et. al.*(1988) developed a test procedure based on estimated bispectrum values to test whether a sample time series is generated by a linear process. Cressie (1988) have shown that the amount of differencing d in an ARIMA(p,d,q) model can be read from a sequence of graphs based on the concept of generalized covariances known as variograms. Shaman and Stine (1988) presented a simple expression for the bias of autoregressive coefficients in least square and Yule-Walker estimators with unknown finite order.

Crafts *et. al.* (1989) constructed a new index of industrial production for Britain for the years 1700-1913 and using a structural time series model estimated by Kalman filter the index was decomposed into trend and cyclic components. Hall (1989) proposed a new test procedure for testing unit roots in a time series with moving average innovations that is based on an instrumental variable estimator. Hemerly and Davis (1989) discussed about the problem of order determination for AR models and have shown that order selection based on Rissanen's predictive least square principle is strongly consistent when the series is generated by an AR process with given upper bound to the order of the process. Knight (1989) have seen that the estimate of order of an AR model chosen to minimize Akaike's information criterion is weakly consistent. Hurvich and Tsai (1989) derived a bias correction to the Akaike's information criterion AIC for regression and AR time series models that are of particular use when sample size is small. This method is asymptotically efficient and was found better than other asymptotic methods and its application to ARMA models and nonstationary models were discussed. Potscher (1989) developed a sufficient condition for strong consistency of estimators of order of a general nonstationary autoregressive model based on AIC minimization criterion. Stergiou (1989) analyzed monthly catches of pilchard from Greek waters using autoregressive integrated moving average models and identified two models for describing the dynamics of the fishery and forecasting up to 12 months ahead.

Hurvich *et. al.*(1990) proposed a new estimator AICI of Kullback-Leibler information for Gaussian autoregressive model selection in small samples. Kreiss (1990) studied the problem of testing linear hypothesis about the parameter vector of an autoregressive model with finite order and developed asymptotically optimal statistical tests. Pukkila *et. al.* (1990) proposed a powerful method of determining order of a ARMA(p,q) model based on an AR order determination criterion and on linear estimation methods. Fearn and Maris (1991) used Box-Jenkins approach to design and control algorithm for feed back loop controlling the addition of dried gluten to bread making flour in a flour mill. Guilkey and Schmidt (1991) considered the tests of null hypothesis of a unit root in time series against the alternative that the series is level stationary. Hurvich and Tsai (1991) studied the bias in AIC information criterion and its bias corrected version AICC for selection of regression and autoregressive models. Kavalieris (1991) introduced an information theoretic order estimation procedure for estimation of parameters and model orders in an ARMA system and pointed out that the Hannan and Rissanen's (1982) method may over estimate the model orders. Li (1991b) considered the problem to decide whether both regular and seasonal differencing or just one of them would suffice to transform a series into stationary. He provided a Lagrange multiplier test and obtained the large sample representation of the test statistic in terms of integrals of Wiener process. Noble and Sathianandan (1991) used autoregressive integrated moving average models to study the trend in all India mackerel catches. Mills and Mills (1992) examined the seasonal patterns and components of quarterly economic time series and found that seasonality is much smaller in prices and interest rates than outputs and its components.

Ahn (1993) considered a nonstationary time series that can accommodate deterministic and stochastic trends and developed a test statistic for testing stochastic trend based on Lagrange multiplier principle. Galbraith and Walsh (1994) examined a non-maximum likelihood estimator for parameters of an MA model derived directly from the coefficient of an approximating AR model. Monti (1994) proposed a test of goodness of fit for time series models based on squared residual partial autocorrelations that is asymptotically chisquare. Ng and Perron (1995) analyzed the choice of the truncation lag for the Said-Dickey test for the presence of unit root in general ARMA model and have shown that the deterministic relationship between the truncation lag and sample size is dominated by data dependent rules that take sample information into account.

Materials and Methods

Quarterwise total marine fish landings, quarterwise landings of selected species and species groups in Kerala during the period 1960-96 was used in this study to find suitable univariate seasonal time series models belonging to the popular class of ARIMA models. The selection of species and species groups were made based on their commercial importance and their contribution towards total landings in the state. The necessary data for the study were collected from the "National Marine Living Resources Data Center " of the Central Marine Fisheries Research Institute at Cochin, Kerala. Other than total marine fish landings the species and species groups selected for the study were oil sardine (*Sardinella longiceps*), Indian mackerel (Restralliegur kanaguta), anchovies, lesser sardines, penaeid prawns, thressocles, ribbon fishes and tuna.

For estimation of parameters of the model the "trends" module in SPSS software was used. The algorithm used in this module for ARIMA estimation is the one given by Melard (1984). This is a fast algorithm for calculating exact likelihood of a stationary ARMA model under the assumption that the innovations are independently and identically distributed as $N(o,\sigma^2)$ and it uses a modified Kalman filter recursion based on a state-space representation of the model. For the iteration in the algorithm the parameter accuracy was fixed as 0.001, minimum reduction in residual sum of squares allowed was 0.001 percent, maximum value for the Marquardat constant was chosen as 10×10^9 and the maximum number of iterations was fixed as 50. Testing of the suitability of an estimated model was carried out by a χ^2 statistic, Q provided in Ljung and Box (1978) based on the autocorrelations of residuals given by

$$Q = T(T+2)\sum_{k=1}^{m}[r_k^2/(T-k)]$$

where r_k is the autocorrlation of lag k of the residuals and this will have (*m-p-q*) degrees of freedom where *p* and *q* are the orders of the model fitted.

According to the method of Box-Jenkins the autocorrelation function (*acf*) approaches zero linearly for a stationary time series. But in practice when we examine the *acf* such a clear picture will not be seen in the plot of autocorrelation functions of the original and differenced series. Cressie (1988) developed a graphical procedure based on variograms for estimating *d*, the order of differencing to be applied on a nonseasonal time series to make the series stationary. His method is based on variograms which are functions characterising the second order dependence properties of a time series. He presented three quantities semivariogram, linvariogram and quadvariogram, which are respectively based on I_1, I_2 and I_3 processes. An I_d process is an intrinsic random function of order *(d-1)*. Suppose $Z=(Z_{t_1},\cdots,Z_{t_n})'$ are observations at time points

$\{t_i : i = 1,\cdots,n\}$, X an $n \times d$ matrix with i^{th} row $(1, t_i, t_i^2, \cdots, t_i^{d-1})$ and λ a vector of size n called a generalized increment vector of order d such that $X'\lambda = 0$ then a process $\{Z_t : t \geq 0\}$ is an I_d process when $\sum_{i=1}^{n} \lambda_i Z_{t_i+u}$, $u \geq 0$ is second order stationary for any $\{t_i : i = 1,\cdots,n\}$ and any generalized increment vector λ. Semivariogram of a non-seasonal time series $\{Z_t\}$ denoted by $\gamma(h)$ is defined as $E(Z_{t+h} - Z_t) = 2\gamma(h)$. The relation between semivariogram and autocovariance of the time series is $\gamma(h) = C(0) - C(h)$ so that for a time series with negligible dependence between data far apart the asymptote of the semivariogram will be the variance of the time series. A time series for which $E(Z_t) = \mu$ and $E(Z_{t+h} - Z_t) = 2\gamma(h)$ is called intrinsically stationary time series and for such a time series the unbiased estimate of semivariogram is

$$\hat{\gamma}(h) = \tfrac{1}{2}\sum_{t=1}^{n-h}(Z_{t+h} - Z_t)^2 / (n-h), \text{ for } h=1,2,...$$

where n is the sample size and h is the lag. The linvariogram for lag h, denoted by $\gamma_1(h)$, of a time series is defined as

$$\gamma_1(h) = \left\{\sum_{j=1}^{h-1} \frac{\alpha(j)}{2j(j+1)}\right\} \Big/ h \quad \text{for } h=2,3,....$$

where $\alpha(h) = Var(hZ_i - (h+1)Z_{i+1} + Z_{i+h+1})$ and $\gamma_1(h)$ is estimated by using the formula

$$\hat{\gamma}_1(h) = \left\{\sum_{R(h)}^{h-1} \frac{\hat{\alpha}(j)}{2j(j+1)}\right\} \Big/ h \quad \text{for } h=2,3,...$$

$$\hat{\alpha}(h) = \sum_{R(h)} (hZ_{t_i} - (h+1)Z_{t_j})^2 \Big/ M(h),$$

$$R(h) = \{(i,j,l) : (t_i, t_j, t_l) = (i, i+1, i+h+1) \text{ or } (i, i-1, i-h-1)\}$$

and *M(h)* is the number of distinct triplets in *R(h)*. The quadvariogram for lag *h*, denoted by $\gamma_2(h)$, is defined as

$$\gamma_2(h) = 6\left[\sum_{j=1}^{h-2}\{(h-j-1)\beta(j)\} \div \{4j(j+1)(j+2)\}\right] \Big/ h^3$$

where $\beta(h) = Var(-h(h+1)Z_i + 2h(h+2)Z_{i+1} - (h+2)(h+1)Z_{i+2} + 2Z_{i+h+2})$.

The estimate of quadvariogram is given by

$$\hat{\gamma}_2(h) = 6\left[\sum_{j=1}^{h-2}\left\{(h-j-1)\hat{\beta}(j)\right\} \div \{4j(j+1)(j+2)\}\right] \Big/ h^3 \quad \text{for } h=3,4,\ldots \text{ where}$$

$$\hat{\beta}(h) = \sum_{R(h)}(-h(h+1)Z_{t_i} + 2h(h+2)Z_{t_j} - (h+1)(h+2)Z_{t_l} + 2Z_{t_m})^2 \Big/ \sum_{R(h)} 1 \quad \text{and}$$

$$R(h) = \{(i,j,l,m) : (t_i, t_j, t_l, t_m) = (i, i+1, i+2, i+h+2) \text{ or } (i, i-1, i-2, i-h-2)\}.$$

The degree of differencing *d* required to achieve stationarity of a non-seasonal time series can be found by looking at the plots of the scaled versions of semivariogram, linvariogram and quadvariogram, the scaling quantities being the estimated variances of the processes $\{Z_t\}, \{\nabla Z_t\}$and$\{\nabla^2 Z_t\}$ respectively. The leveling out of the semivariogram is an indication that *d=0* for the time series. A comparison of the leveling out of the estimated semivariogram can be made with its expected values. If the initial leveling out occurs for the scaled semivariogram then its expected value is given by *e(h)=1* which is independent of the lag and its standard error is approximately equal to $(1.35)/\sqrt{n}$ where *n* is the sample size. The graph of the scaled linvariogram levels out or not is an indication of whether *d=1* or *d=2*. When the initial leveling out occurs for the scaled linvariogram then $d=1$ with expected value for the scaled linvariogram for lag h is given by $e(h) = \frac{1}{2} - \frac{1}{2h}$ and its standard error is approximately equal to $(0.15)/\sqrt{n}$. Similarly, if the initial leveling out occurs for the scaled quadvariogram then it is an indication that $d=2$ or $d=3$. When this occurs the expected value of the scaled quadvariogram for lag h is given by $e(h) = \frac{1}{4} - \frac{1}{2h} - \frac{1}{4h^2} + \frac{1}{2h^3}$. The standard error of the quadvariogram is approximately equal to $(0.09)/\sqrt{n}$. The standard errors of the three variograms are independent of the lag h for $\frac{n}{3} \leq h \leq \frac{n}{2}$ .To examine the leveling out of the variograms, the observed variograms can be checked to see whether they fall in the confidence band, $e(h) \pm 2SE$, for the above range of values of lag h.

The method proposed by Box and Jenkins for identification of orders of a time series model is based on the properties of autocorrelation funciton *(acf)* and partial autocorrelation function *(pacf)* of the time series. For a stationary time series $\{Z_t\}$ with mean μ the autocorrelation at lag k denoted by ρ_k is defined as $\rho_k = E[(Z_t - \mu)(Z_{t-k} - \mu)]$ and its estimate is given by $\hat{\rho}_k = \sum_{t=1}^{T-k}(Z_t - \bar{Z})(Z_{t+k} - \bar{Z})$. The *acf*, $\{\rho_k; k = 0,1,2,\cdots\}$ of a Moving average process of order q (MA(q) process) will have the property that $\rho_k = 0 \quad for\ k = q+1, q+2, \cdots$ which is known as the cutoff property at lag q. Hence if the sample *acf* of a time series has a cutoff behavior at a finite lag q then the generating process can be considered as an MA(q) process. The *acf* of an AR process or a mixed ARMA process gradually tails off to zero. For a time series if the sample *acf* $\{\hat{\rho}_k; k = 0,1,2,\cdots\}$ is such that we can find a positive integer q so that

$$\hat{\rho}_i \in [-z_{\alpha/2}\hat{\sigma}(\hat{\rho}_i),\ z_{\alpha/2}\hat{\sigma}(\hat{\rho}_i)] \text{ for } i = q+1, q+2, \cdots \text{ where } \hat{\sigma}(\hat{\rho}_i) = \sqrt{\frac{1}{T}(1+2\sum_{j=1}^{q}\hat{\rho}_j^2)}$$

and z is the critical value of the standard normal variate then we can regard the process as a moving average process of order q. The conditional correlation between Z_t and Z_{t+k} given the intermediate values $Z_{t+1},\cdots,Z_{t+k-1}$ is called partial autocorrelation at lag k for the series $\{Z_t\}$ and it is denoted by ϕ_{kk}. The sequence of values $\{\phi_{kk}; k = 0,1,2,\cdots\}$ is known as the *pacf* of the time series. The *pacf* of a pure autoregressive process of order p (AR(p) process) will have the property that $\phi_{kk} = 0$ *for* $k = p+1, p+2, \cdots$. That is the *pacf* of an AR(p) process will cutoff after lag p. If the *pacf* of a sample time series is such that we can find a positive integer p so that $\hat{\phi}_{kk} \in [-z_{\alpha/2}\frac{1}{T},\ z_{\alpha/2}\frac{1}{T}]$ for $k = p+1, p+2, \cdots$ then we consider the sample as a realization for an AR(p) process. This method of identification is not useful for a mixed ARMA model when neither p nor q is zero. For such models the *acf* will consist of a mixture of damped exponential and/or damped sine waves after the *q-p* lags and the *pacf* will consist of a mixture of damped exponential and/or damped sine waves after the *p-q* lags. In general the graphs of *acf* and *pacf* would not yield any unique values of p and q for a mixed ARMA model.

An alternative method of identification of orders of an ARMA model is to use criterion selection where the orders p and q by minimizing a quantity that is a function of the estimate of white noise variance and the orders. Among this methods the popular criteria are one proposed by Akaike (1972) known as AIC criterion and the other is Bayesian Information criterion proposed by Schewatz, known as SBC. The AIC criterion for a mixed ARMA process is defined as $AIC(p,q) = \ln(\tilde{\sigma}^2_{p,q}) + 2(p+q)/T$ where $\tilde{\sigma}^2_{p,q}$ is the maximum likelihood estimate of the innovation variance when the orders of the model are p and q respectively. The SBC criterion is given as $BIC(p,q) = \ln(\tilde{\sigma}^2_{p,q}) + (p+q)\ln(T)/T$.

Results

1.1. Total Marine Fish landings

Quarterwise total marine fish landings in Kerala during 1960-96 period was used to identify and estimate a suitable univariate autoregressive moving average model. The maximum annual landings observed during this period was 6,62,890 tonnes (t) in the year 1990 and the minimum observed was 1,92,470 t in 1962. The average of total marine fish landings during this period is 3,395,770 t with a coefficient of variation (CV) of 29.49 %. The average landings during the period 1960-80 (first phase) was 3,38,181 t (with CV 19.8 %) and that during the period 1981-96 (second phase) was 4,71,356 t (with CV 26.5%). This shows an increase in landings of about 39.4% in the second phase compared to the first.

The average of total landings in the first quarter for the period 1960-96 was 76,828*t* (CV=35.9%), that for the second quarter was 64,420*t* (CV=47.9%), corresponding value for the third quarter was 1,23,335*t* (CV=43.5%) and for the fourth quarter it was 1,31,187*t* (with 28.6% CV). The minimum and maximum values of landings in this period for the first quarter were 33,755*t* in 1960 and 1,59,079*t* in 1971. The corresponding values for the second quarter were 12,738*t* in 1962 and 1,31,577*t* in 1990, for the third quarter the values were 34,480*t* in 1962 and 2,23,474*t* in 1994 and for the fourth quarter these values were 59,551*t* in 1980 and 2,20,551*t* in 1989.

As an initial step towards model building for the quarterwise total marine fish landings in the state, to study the patterns existing in the autocorrelation function (*acf*) and partial autocorrelation function (*pacf*), the values of these functions were computed up to lag 36 for the original data, the data generated by first order regular difference, seasonally differenced data and for the data generated by applying both regular and seasonal differences of order one and it is presented in table.1.1. Autocorrelations of the original series showed highly significant values at lags that are multiple of four and the strength of autocorrelations was found decreasing as the lag increases. The maximum autocorrelation observed for this series was 0.702 at lag 4. The data being quarterwise landings this is an indication of seasonality of period 4 present in the data. Partial autocorrelations were significant and high up to lag 5 for the original data. For the seasonally differenced data the *acf* was highly significant at lags 1 and 4 and the behavior of the *pacf* was also similar. Interestingly, when the original data were subjected to simple first order differencing its *acf* showed very highly significant (maximum -0.728 for lag 2) pattern at lags that are multiples of 2 with the sign of autocorrelation changing alternatively. Partial autocorrelations of this series were significant only at lags 2,3,4,7 and 14. When the series was subjected to both seasonal and simple differencing each of order one, the *acf* and *pacf* of the resultant series were found significant at lags 1 and 4. But this analysis does not show any pattern helpful for identification of orders of a suitable ARIMA model apart from showing that the data is seasonal and a suitable seasonal model may give better approximation for the generating process.

To examine whether the seasonally differenced series is stationary or not variograms were computed up to lag 70 using the seasonally differenced data and the scaled variograms are shown in fig.1.1.1. From the plots of the scaled versions of semivariogram (γ), linvariogram ($\gamma(1)$) and quadvariogram ($\gamma(2)$) up to lag 70, it can be seen that $\gamma(1)$ leveled at around 0.5 and $\gamma(2)$ leveled out around 0.2 both after lag 6. The scaled semivariogram fluctuated between 0.584 and 1.330, scaled linvariogram ranged between 0.306 and 0.515 and the scaled quadvariogram ranged between 0.115 and 0.265. The expected value and standard error (SE) for the scaled semivariogram were 1.0 and 0.1125 and many values fall out side the 1.0 ± 2.0 SE level. Standard errors for the scaled lin and quad variograms were 0.0125 and 0.0075 respectively. This indicates that first order differencing will be required to make the seasonally differenced series stationary. Hence in the general univariate seasonal ARIMA$(p,d,q)(P,D,Q)s$ type model the values of d and D were both taken as 1. Since the *acf* and *pacf* does not give any clear idea about the orders of the model the alternative approach using the AIC and SBC criterion were used for model identification.

The order parameters of the model ARIMA(*p*,*d*,*q*)(*P*,*D*,*Q*)*s* are *p*, *d*, *q*, *P*, *D* and *Q* in which *d* and *D* were fixed as 1 based on the earlier analysis. For estimating the remaining order parameters AIC and SBC values were calculated along with log likelihood and standard error, after fitting each model from a set of 318 different models corresponding to different values of the order parameters taking values *p*,*q* = 0,1,...,5 and *P*,*Q* = 0,1,2. The minimum AIC value observed was 3332.89 for the model ARIMA(0,1,2)(0,1,1)4 with log likelihood value -1663.44 and standard error 27019.49, the maximum observed value of AIC was 3407.58 for the model ARIMA(1,1,0)(0,1,0)4 with log likelihood -1702.79 and standard error 36026.12. The minimum value of SBC observed was 3332.89 and it also was corresponding to the model ARIMA(0,1,2)(0,1,1)4. The maximum value observed for SBC was 3405.14 corresponding to the model ARIMA(3,1,0)(0,1,0)4 with log likelihood -1699.57 and standard error 35459.97. Since both these criterion suggested the same model ARIMA(0,1,2)(0,1,1) as the suitable representation for the total marine fish landings time series it was selected as the required model.

The iteration process in the estimation of this process was terminated after 9 iterations when the change in all the parameter estimates were less than 0.001. The final estimates of parameters of the model are $\theta_1 = 0.434490$ (standard error 0.0763, p < 0.001), $\theta_2 = 0.436495$ (standard error 0.0796, p < 0.001) and $\Theta_1 = 0.715598$ (standard error 0.0706, p < 0.001). Autocorrelations up to lag 36 were calculated for the residual series generated by using the estimated model and was examined for their significance. Almost all the autocorrelations were non-significant except that at lag 12 with the value -0.127 which is not high. To test the suitability of the estimated model, Box-Ljung χ^2 statistic was computed using 36 autocorrelations of the residuals and its value is 26.376 which is not significant (p=0.88). The estimate of residual variance is 730053532. Hence the fitted model is suitable to represent the series. Since there are no autoregressive terms in the model the model is stationary. The characteristic polynomial for the moving average part is $(1-0.434490B-0.436495B^2)$ and its roots are 1.0972 and -2.0998. Since these roots fall out side the unit circle it satisfies the condition for invertibility of the model. The characteristic polynomial corresponding to the seasonal moving average part in the model is $(1-0.715598B^4)$ and the absolute value of the roots of this polynomial is 1.3974 which again lies out side the unit circle. Hence the model estimated is invertible. The original series and the fitted values using the estimated model are shown in fig.1.1.2. The algebraic form of the final estimated model is

$$(1-B)(1-B^4)\widetilde{Z}_t = (1-0.715598\,B^4)(1-0.434490\,B-0.436495\,B^2)\varepsilon_t$$

1.2. Intervention model for total marine fish landings

By the introduction of crafts fitted with outboard engines in late eighties in Kerala a prominent change in the landings was expected. To examine whether this has really caused any significant change in total marine fish landings an intervention analysis was carried out. As a first step towards this, using the data prior to this period (1960-87) a suitable ARIMA model was fitted. Autocorrelations and partial autocorrelations of the original series, differenced series, seasonally differenced series and the series with both the kinds of differencing are given in table.1.2. The patterns of *acf* and *pacf* for this data also showed seasonality but failed to give any clear indication regarding orders of a suitable model. Hence, criterion selection based on AIC and SBC was used to select orders of a seasonal ARIMA model. Among the 315 different models attempted for values of regular orders p,q = 0,1,...,5; seasonal orders P,Q = 0,1,2 the model ARIMA(0,1,2)(0,1,1)4 gave minimum values for both AIC (2488.33) and SBC (2496.35). Maximum values of AIC and SBC criterion were 2558.41 for ARIMA(1,1,0)(0,1,1)4 and 2564.62 for ARIMA(3,1,0)(0,1,0)4. The model ARIMA(0,1,2)(0,1,1)4 was then selected and the parameters were estimated as before. The estimation algorithm terminated after 7 iterations when the change in the parameter estimates were less than 0.001%. The estimates of the parameters were $\theta_1 = 0.440870$ (standard error 0.0909, p < 0.001), $\theta_2 = 0.525322$ (standard error 0.0907, p < 0.001) and $\Theta_1 = 0.742818$ (standard error 0.0820, p < 0.001). The log likelihood for these estimates was -1241.16 and the estimate of residual variance was 662719054. The characteristic polynomial for moving average term is $(1 - 0.440870B - 0.525322B^2)$ and it has 1.0225 and -1.8617 as its roots which fall out side the unit circle limit. Roots of the seasonal moving average polynomial $(1 - 0.742818B^4)$ have absolute value equal to 1.3462, which also lies out side the unit circle limit indicating that the estimated model is invertible. To examine the suitability of the fitted model, residuals were computed using the fitted model and *acf* of the residual series up to lag 36 were calculated. The maximum value for the residual *acf* was -0.124 which is not significant. The Box-Ljung χ^2 value computed using residual *acf* up to lag 36 is 21.635 which is not significant (p=0.972).

Hence the fitted model is a good approximation for the process generating this time series.

This fitted model was then used for intervention analysis. For this an auxiliary variate was generated with value 0 for the periods before intervention (from 1960 to 1987) and the variable was assigned the value 1 for the intervention period. This new variable was then included as a regression variable in the already estimated model, which is similar to inclusion of a step function in the model, and the model parameters were then re-estimated. The new estimates of model parameters are $\theta_1 = 0.50440$ (standard error 0.14397, $p \leq 0.001$), $\theta_2 = 0.48883$ (standard error 0.10474, $p < 0.001$), $\Theta_1 = 0.70057$ (standard error 0.06919, $p < 0.001$) and the estimate of the coefficient for the auxiliary variable was b=53770.4339 (standard error 14163.89, $p < 0.001$). The estimate of standard error was 25592.47, log likelihood corresponding to these estimates was -1656.84 and the estimate of residual variance was 654974673. This indicates that by inclusion of an intervention term from 1988 onwards there is a reduction of about 10.28% in the residual variance. The observed landings and the fitted values according to the intervention model are plotted in fig.1.2.1. From the estimate of the coefficient of the auxiliary variable we can infer that on an average there is an increase of about 53,770*t* in the total marine fish landings by the introduction of the new crafts fitted with outboard engines into the fishery. The algebraic form of the estimated intervention model with X representing the auxiliary variable is

$$(1-B)(1-B^4)\widetilde{Z}_t = 53770.43385X + (1-0.50440B-0.48883B^2)(1-0.70057B^4)\varepsilon_t$$

1.3. Oil sardine landings

Oil sardine (*Sardinella longiceps*) is one among the important marine fish species that contributes maximum towards the marine fish landings in Kerala. During the period 1960-96 the contribution by oil sardine towards total marine fish landings in the state reached 71.6% in the year 1968. Fluctuations in the landings of this species over years is very high so that the minimum contribution by this species came to even less than 1% in the year 1994. It is believed that there is a cyclical behaviour in the landings of this species with a periodicity of 11 years (Sathianandan and Alagaraja, 1998). The maximum observed annual landings of oil sardine during this period was 2,47,048 tonnes in 1968 and the minimum observed was 1,554 tonnes in the year 1994.

Quarterwise landings during 1960-96 was used to develop a suitable time series model for the landings of oil sardine. Average catch and coefficient of variability for different quarters in this period are 32,868*t* (CV=67.7%) for the first quarter, 12,594*t* (CV=74.4%) for the second quarter, 21,750*t* (CV=80.8%) for the third quarter and 55,358*t* (CV = 69.3%) for the fourth quarter. During this period the maximum observed catch in the first quarter was 92,424*t* in 1965, that in the second quarter was 37,462*t* in 1991, for the third quarter this value was 74,671*t* in 1989 and for the last quarter the maximum was 1,47,821*t* in 1960. An initial analysis was carried out by computing *acf* and *pacf* for the actual series, for the series with regular difference, seasonally differenced series and for the series by applying both regular and seasonal difference and these are given in table.1.3.

The *acf* of original data have shown highly significant values at almost all lags that are multiples of two. The maximum observed *acf* was 0.567 at lag 4. Partial autocorrelations of this series was found significant up to lag 8 with a maximum of 0.434 at lag 4. The *acf* of the series generated by a regular first order difference of the original series also showed highly significant values for all lags which are multiples of 2, the maximum being 0.619 at lag 4. Partial autocorrelations of the differenced series have significant values up to lag 7 with geometric decay as lag increases and the maximum *pacf* value observed was -0.608 at lag 2. The behavior of the original and differenced series were not similar to that of a stationary series and the series being quarterly data having higher autocorrelations for lags which are multiples of 4 is seasonal. A seasonal differencing of first order may be required to reduce the series into a stationary series and a seasonal ARIMA model may be a better model to represent the series. The seasonally differenced series have significant *acf* at lags 1, 4, 11, 15, 19, 20 and 23 and the maximum is 0.379 at lag 1. The *pacf* of this series were significant at lags 1, 4, 11, 19 and 24 with a maximum of 0.379 at lag 1. Though the *acf* and *pacf* of the seasonally differenced series behaves like that of a stationary series these functions did not give any conclusion regarding the orders of the model to be fitted. The *acf* of the series generated by applying both regular and seasonal differencing on the original series have significant values at lags 1, 4, 15, 18 19 and 23 with maximum value -0.434 at lag 4 and its *pacf* were significant at lags 1, 2, 4, 5, 8, 12 and 24 with maximum value –0.443 at lag 4.

To ascertain the need for a regular differencing of the seasonally differenced data variograms were computed using the seasonally differenced data and the standardized quantities of the semi, lin and quad variograms are given in fig.1.3.1. The scaled semivariogram computed for this data was found to fluctuate between 0.606 and 1.343 and many of the values fall out side the expected range of 1.0 ± 2 SE for a stationary series where SE = 0.1125. The scaled linvariogram ranged between 0.307 and 0.503 and it was found to level out after lag 8 at around 0.50. The behavior of the quad variogram was also similar and it fluctuated between 0.115 and 0.259 and was found to level out around 0.20. The standard errors for scaled lin and quad variograms were 0.0125 and 0.0075 respectively. This suggests that a first order differencing of the seasonally differenced data will ensure stationarity. Accordingly the seasonal model, ARIMA$(p,d,q)(P,D,Q)s$, was attempted to fit by fixing the values of d and D as 1 with seasonality $s = 4$. Since the analysis using *acf* and *pacf* of the original and transformed series does not give any valuable conclusion regarding the orders of a suitable model, criterion selection based on AIC and SBC were used to estimate the model orders.

Seasonal ARIMA$(p,d,q)(P,D,Q)s$ for different values of p,q=0,1,...,5; P,Q=0,1,2; d=0,1 and D=1 were initially fitted and the AIC and SBC criterion were computed. With d=0, among 315 different models attempted, the minimum AIC value observed was 3262.43 for the model ARIMA(1,0,0)(0,1,1)4 with log likelihood -1629.22 and the maximum AIC value was 3307.32 for the model ARIMA(2,0,3)(0,1,0)4 with log likelihood -1648.66. The model ARIMA(1,0,0)(0,1,1)4 also gave the minimum SBC value 3268.37. Maximum SBC value was 3323.63 corresponding to the model ARIMA(5,0,5)(2,1,2)4 with log likelihood value -1627.03. For d=1 the model corresponding to minimum AIC value of 3248.81 among the 314 different models attempted was ARIMA(1,1,1)(1,1,1) with log likelihood -1620.41. The minimum SBC value observed was 3259.45 for the model ARIMA(0,1,2)(0,1,1)4 and log likelihood of the model was -1622.29. The maximum AIC value observed was 3325.42 for ARIMA(1,1,0)(0,1,0)4 with log likelihood -1661.71.

The model ARIMA(1,1,1)(1,1,1)4 was estimated and the estimation algorithm was terminated after 11 iterations when the change in the parameter estimates were less than 0.001. Final estimates of parameters of the model are $\hat{\phi}_1 = 0.455945$ (standard error 0.0839, $p < 0.001$), $\hat{\theta}_1 = 0.999856$ (standard error 4.1409, $p > 0.80$), $\hat{\Phi}_1 = 0.139765$ (standard error 0.1158, $p > 0.20$) and $\hat{\Theta}_1 = 0.795725$ (standard error 0.0808, $p < 0.001$). The maximum correlation found between the parameter estimates was 0.6407 between $\hat{\theta}_1$ and $\hat{\Phi}_1$. Log likelihood value corresponding to this estimates of the parameters was -1620.41, AIC value was 3248.81 and SBC value was 3260.67. The estimate of residual standard error was 19779.71. The characteristic polynomial corresponding to the regular AR term in the model is $(1-0.455945x)$ with root 2.1932, and that corresponding to the seasonal AR term is $(1-0.139765x^4)$ with absolute value of the roots equal to 7.1548. Since all these roots fall out side the unit circle limit the estimated model is stationary. The characteristic polynomial corresponding to the regular MA term is $(1-0.999856x)$ with root 1.0001 and that for the seasonal MA term is $(1-0.795725x)$ with absolute value of the roots equal to 1.2567. Though the roots of the seasonal MA polynomial lie out side the unit circle, the root of the regular MA polynomial is very close to the unit circle boundary for invertibility of the model. Hence the estimated model is not strictly invertible. Residual series generated by using the estimated model were used to examine the suitability of the fitted model. Autocorrelations were computed up to lag 36 for the residual series and the maximum value observed was –0.160 at lag 19 and it is the only significant autocorrelation. The value of Box-Ljung χ^2 calculated using 36 autocorrelations of the residuals series is 28.510 which is not significant (p=0.809).

For the model ARIMA(0,1,2)(0,1,1)4 which corresponds to the minimum SBC value the iteration process in the estimation concluded after 30 iterations when the percentage reduction in residual sum of squares was less than 0.001. Final estimates of the model parameters are $\hat{\theta}_1 = 0.548735$ (standard error 0.1010, $p < 0.001$), $\hat{\theta}_2 = 0.439810$ (standard error 0.0899, $p < 0.001$) and $\hat{\Theta}_1 = 0.729457$ (standard error 0.0720, $p < 0.001$). Estimate of residual standard error was 20015.50, log likelihood was -1622.28, AIC value was 3250.56 and SBC value was 3259.45. Characteristic polynomial corresponding to the regular MA term is $(1 - 0.548735x - 0.439810x^2)$ having roots 1.010 and -2.251 out of which one root is close to the unit circle boundary. The characteristic polynomial for seasonal MA term is $(1 - 0.729457x^4)$ and it has 1.371 as the absolute value of the roots and hence all its roots fall in the region out side the unit circle. Since one of the roots of the regular MA polynomial is close to the unit circle region the estimated model is not strictly invertible. Since there is no AR term in the model the model is stationary. Analysis of the residuals generated using the estimated model revealed significant values for the residual *acf* at lags 4 and 19 maximum being

0.174 at lag 4. The Box-Ljung χ^2 calculated using *acf* of the residuals up to lag 36 was 42.005 which is not significant (p = 0.227).

Since these two models are not strictly invertible the model ARIMA(1,0,0)(0,1,1)4 which yielded minimum values for both AIC and SBC when $d = 0$ was also estimated and examined. For this model the estimation process was terminated after 5 iterations when the reduction in the residual sum of squares was found less than 0.001 percent. Final estimates of the parameters are $\hat{\phi}_1 = 0.466848$ (standard error 0.0757, p<0.001) and $\hat{\Theta}_1 = 0.732283$ (standard error 0.0621, p < 0.001). The characteristic polynomial for the AR term is $(1 - 0.466848x)$ with root 2.142 which is outside the unit circle limit so that the estimated model is stationary. The characteristic polynomial corresponding to the seasonal MA part is $(1 - 0.732283x^4)$ and the absolute value of its roots is 1.36 which again is outside the unit circle and hence the model is invertible also. Estimate of residual standard error corresponding to this model was 19746.96 and log likelihood value was -1629.22. The *acf* of residual series generated using the estimated model have significant values at lags 11, 19 and 20 with maximum –0.172 at lag 11. The Box-Ljung χ^2 calculated using 36 autocorrelations of the residuals series is 36.46 which is not significant (p=0.447). There was no significant correlation between the estimates of the parameters. Compared to the earlier models this model is strictly stationary and invertible and the residual standard error is comparatively less for this model. Hence this model was chosen as the final model for this time series and plot of observed catch and its expected vales as per the model is given in fig.1.3.2. The estimated model can be expressed as

$$(1-B^4)(1-0.466848B)\widetilde{Z}_t = (1-0.732283B^4)\varepsilon_t$$

1.4. Indian mackerel landings

During the period 1960-96 the average annual landings of mackerel in Kerala was 35,244 tonnes with a coefficient of variation of 88.21%. This on an average is about 8.91 % of the total marine fish landings in the state during this period. The maximum and minimum landings of mackerel during this period were 1,28,411 tonnes in 1996 and 3599 tonnes in 1969 respectively. The average landings during the period 1960-80 was 25,011 tonnes (with CV 81.1%) and that during 1981-96 period was 48,675 tonnes (with CV 76.24%). The maximum percentage contribution by this species towards total landings in Kerala was 24.07% in the year 1963 and the minimum was 1.04% in 1968. The average landings of this species in the first quarter during 1960-96 was 6,285 t (CV=137.1%), that for the second quarter was 5,905 t (CV=108.65%), for the third quarter the value was 10,559 t (CV=144.14%) and in the fourth quarter this value was 12,496 t (CV=98.9%). The maximum and minimum landings in the first quarter during this period were 53,501 t in 1971 and 333 t in 1969 respectively. For the second quarter these values are 29,967 t in 1971 and 14 t in 1968. During this period in the third quarter the minimum observed catch was 199 t in 1975 and the maximum was 67,887 t in 1994. For the fourth quarter these values were 49,828 t in 1989 and 1,346 t in 1979. The time series data on quarterwise landings of this species show high variation at different periods. During the last part of the series variation is very high and the series show an increasing trend with high fluctuations. Hence to reduce the variability in the data it was transformed by using natural logarithms and the transformed series was used to find a suitable time series model.

The *acf* and *pacf* up to lag 36 were computed for the transformed series, its first difference, seasonal difference and for the series generated by applying both regular and seasonal difference which is given in table.1.4. For the transformed series the maximum *acf* observed was 0.499 at lag 4 and the maximum observed *pacf* was 0.443 at lag 1. For this series *acf* were found significant at lags 1, 3, 4, 5, 7, 8, 9, 12, 20, 24 and 28 and *pacf* were found significant at lags 1, 3, and 4. For the series generated by a regular difference of the log transformed data the *acf* were significant at all lags which are multiples of two with a maximum of -0.473 at lag 2. The *pacf* of this series were significant at lags 1, 2, 3 and 10 with maximum at lag 2 which is -0.544. For the series generated by a seasonal difference of order 1, with seasonality 4, the *acf* were significant at lags 1, 4 and 8 with a maximum of 0.286 at lag 1 and its *pacf* were significant at lags 1, 4, 8 and 12 with -0.315 as the maximum at lag 4. The a maximum of *acf* of the series obtained by applying both seasonal and regular difference on the transformed data have significant values at lags 1, 2, 3, 4, 6, 21 and 33 with maximum value -0.327 at lag 1. The maximum observed *pacf* of this series was -0.388 at lag 4 and it has significant values at lags 1, 2, 3, 4, 5, 8, 20 and 22. The analysis of the series by using *acf* and *pacf* does not give any clue regarding the order of the process generating this series but gives some indications regarding seasonality in the data as shown by the *acf* and *pacf* of the seasonally differenced data.

Hence a seasonal ARIMA model may suit the series for which the orders are to be determined by other methods like minimum AIC and SBC criterion.

The log transformed series on quarterwise landings of mackerel was seasonally differenced and used to compute the variograms up to lag 70 to examine the requirement of a regular differencing. The scaled variograms and their confidence limit under the hypothesis of stationarity are shown in fig.1.4.1. The scaled semivariogram ranged between 0.718 and 1.245, scaled linvariogram ranged between 0.329 and 0.492 and the scaled quadvariogram was found to range between 0.111 and 0.246. The standard errors for scaled semi, lin and quad variograms under stationarity assumption were 0.1125, 0.0125 and 0.0075 respectively. For the scaled semivariogram only 2 values out of 70 at lags 1 and 4 were found to fall out side the confidence region and the scaled lin and quad variograms leveled out after few initial lags and all points fall within the confidence limits there after. Since most of the points are within the confidence limit for the semi variogram it is an indication that no regular differencing is required to make the series stationary. Based on this analysis the order of regular differencing was taken as zero and that for the seasonal differencing was taken as 1 in the seasonal type model ARIMA(p,d,q)(P,D,Q)s for which d=0 and D=1.

To determine the suitable orders of the seasonal ARIMA model, for values of p,q=0,1,...,5; P,Q=0,1,2 ; d=0, D=1 the AIC and SBC criterion were computed by estimating the model using log transformed data. Out of the 315 different models estimated the model ARIMA(3,0,0)(2,1,2)4 was found to have minimum AIC value which was 443.23 and the model corresponding to the minimum SBC value 455.20 was ARIMA(1,0,0)(0,1,1)4. The maximum AIC value obtained was 489.76 for the model ARIMA(3,0,0)(0,1,0)4 and the maximum SBC value obtained was 503.87 for the model ARIMA(3,0,2)(1,1,0)4. The estimate of standard error and log likelihood for the model corresponding to minimum AIC was 1.0782 and -213.61 respectively and for the model that gave maximum AIC value these quantities were respectively 1.3065 and -240.88. For the model with minimum SBC value these estimates were 1.1145 and -220.14 and for the model with maximum SBC value the estimates were 1.2624 and -234.54.

Using the log transformed time series of quarterwise landings of mackerel the model selected based on minimum AIC criterion ARIMA(3,0,0)(2,1,2)4 was estimated. The estimation algorithm terminated after 9 iterations when the reduction in residual sum of squares was less than 0.001 percent. Final estimates of these parameters are $\hat{\phi}_1 = 0.431122$ (standard error 0.0841, p<0.001), $\hat{\phi}_2 = -0.084324$ (standard error 0.0945, p>0.37), $\hat{\phi}_3 = 0.254399$ (standard error 0.0917, p<0.01), $\hat{\Phi}_1 = 0.901061$ (standard error 0.1559, p<0.001), $\hat{\Phi}_2 = -0.288687$ (standard error 0.1058, p<0.01), $\hat{\Theta}_1 = 1.639510$ (standard error 0.1497, p<0.001) and $\hat{\Theta}_2 = -0.767463$ (standard error 0.1238, p<0.001). The estimate of Φ_1 was found to have high correlation with the estimates of Θ_1 (0.796) and Θ_2 (0.790) and also the estimates Θ_1 *and* Θ_2 were highly correlated (r=-0.970). The characteristic polynomial corresponding to the regular AR term is (1- 0.431122 x + 0.084324 x^2 - 0.254399 x^3) and its roots are 1.3153, -0.4919+1.6573i and -0.4919-1.6573i (absolute value 1.7288). For the seasonal AR term the characteristic polynomial is (1-

0.901061 x^4 + 0.288687 x^8) and the absolute value of its roots is 1.1680. Since all the roots of both the regular and seasonal AR characteristic polynomials fall out side the unit circle the estimated model is stationary. The characteristic polynomial corresponding to the seasonal MA term is (1- 1.639510 x^4 + 0.767463 x^8) and the absolute value of its roots is 1.0336. Though these roots are outside the unit circle boundary region it is very close. Using the estimated model the residual series was generated and *acf* up to lag 36 were computed for the residual series. None of the *acf* values were significant and the maximum observed *acf* for residuals was -0.143 at lag 20. Box-Ljung χ^2 calculated using 36 *acf* of the residuals series up to lag 36 was 19.941 which is not significant (p=0.986).

For the model ARIMA(1,0,0)(0,1,1)4 NC estimated for the log transformed quarterwise landings of mackerel, which correspond to the minimum SBC value, the estimation algorithm terminated after 4 iterations when the change in residual sum of squares was found less than 0.001 percent. Final estimates of parameters of the model are $\hat{\phi}_1 = 0.415719$ (standard error 0.0766, p<0.001) and $\hat{\Theta}_1 = 0.726612$ (standard error 0.0630, p<0.001). The characteristic polynomial for the AR term is (1- 0.415719 x) with root 2.4055 which falls out side the unit circle region and hence the estimated model is stationary. For the seasonal MA term the characteristic polynomial is (1- 0.726612 x^4) with 1.0831 as the absolute value for the roots which also falls outside the unit circle so that the estimated model is invertible also. Residual series was generated with this model and the *acf* up to lag 36 were computed for the residuals. All the *acf* values were found non significant maximum being -0.141 at lag 14. The Box-Ljung χ^2 calculated using *acf* up to lag 36 of the residuals series was 28.29 which was not significant (p=0.817). This model is parsimonious compared to the first and it is stationary and invertible. Hence this model was chosen as the final model for this time series and plot of observed and expected catch is given in fig.1.4.2. The algebraic expression for the estimated model is

$$(1-0.415719B)(1-B^4)\widetilde{Z}_t = (1-0.726612B^4)\varepsilon_t$$

1.5. Anchovies landings

The average annual landings of anchovies during 1960-96 was 20,697 tonnes with a coefficient of variation of 74.88%. During this period, on an average this group contributed about 5.2% towards total landings in the state. The maximum landings by anchovies was 55,042*t* in the year 1983 and the minimum was 2,718*t* in 1965. During 1960-80 period the average landings of anchovies was 10,107*t* (CV=41.87%) and the average during the period 1981-96 was 34,596*t* (CV=40.0%). The percentage contribution by this group was maximum in the year 1983 (14.27%) and it was minimum in the year 1965 (0.80%). The average landings of anchovies in different quarters during 1960-96 period were 1,652*t* in first quarter (CV=87.4%), 4,154*t* in second quarter (CV=116.67%), 7,264*t* in the third quarter (CV=98.9%) and 7,627*t* in the fourth quarter (CV=80.4%). The percentage landings in the four quarters during the period under study on an average were 7.98%, 20.07%, 35.10% and 36.85% respectively. In the time series of quarterwise landings of anchovies the variability in data is very high during the last few years. Hence a logarithmic transformation was applied to stabilize the variability in data.

The *acf* and *pacf* up to lag 36 were calculated for the log transformed quarterwise landings, its regular difference, its seasonal difference with seasonality 4 and for the series generated by applying both regular and seasonal difference to the log transformed series which is presented in table 1.5. The *acf* of the log transforms series have significant and high values at lags which are multiples of 4 showing the behaviour of seasonal time series with seasonality 4. The *pacf* of this series has high and significant values at lags 1, 3, 4, 8, 20 and 28. The highest observed *acf* vlue was 0.624 at lag 4 and the highest observed *pacf* value was 0.520 at lag 4. The *acf* of the series generated by a regular difference of the log transformed series were significant and high at all lags that are multiples of 2 with a maximum of 0.474 at lag 4 and the *pacf* of this series were significant at lags 1, 2, 3, 7, 19 and 27 with maximum of -0.549 at lag 3. For the series obtained by a seasonal difference of the log transformed series the *acf* were significant at lags 4, 13 and 17 with 0.370 as the maximum at lag 4. The *pacf* were significant at lags 4, 8, 16 and 24 with -0.386 as the maximum at lag 4. This is an indication that even the seasonally differenced series is not fully free from seasonality present in the series and a seasonal model will suit better for this series. When both regular and seasonal differences were applied to the log transformed series both the *acf* and *pacf* of the resulting series had significant values at many lags and it does not show any identifiable pattern. The maximum observed *acf* for this series was -0.468 at lag 1 and the highest *pacf* observed was also -0.468 at lag 1. The above analysis failed to give any conclusion regarding the orders of a suitable model, and hence criterion selection based on maximum values of AIC and SBC was adopted for selection of suitable model. To examine further about the requirement of regular difference variogram analysis was carried out using the series generated by seasonal difference of the log transformed series and fig.1.5.1. show plots of different scaled variograms along with expected values and confidence limits. The scaled semivariogram fluctuated between 0.723 and 1.380 but only 4 values out of 70 were found to fall outside the confidence limits. Scaled lin and quad variograms leveled out after certain lags and are very close to the expected values there after. Scaled linvariogram fluctuated between 0.368 and 0.523 with a standard error of 0.0125 and the scaled quad variogram ranged between 0.123 and 0.267 with a standard error of 0.0075. This suggests that regular differencing of the series is not necessary and the series can be considered as stationary.

Based on these results AIC and SBC criterion were used to identify a suitable seasonal model of the type ARIMA (p,d,q) $(P,D,Q)s$ for the log transformed quarterwise landings of anchovies with $d = 0$ and $D = 1$. For values of $p,q = 0, 1,...,5$; $P,Q=0, 1, 2$; estimation was carried out and the AIC and SBC criterion were computed for 315 different models. The model corresponding to the minimum AIC value 366.44 was ARIMA(4,0,0)(1,1,2)4 and that corresponding to the minimum SBC value 379.54 was ARIMA(1,0,1)(0,1,1)4 with corresponding log likelihood values -176.22 and -182.32 respectively. The maximum observed values for AIC and SBC criterion were 418.78 for ARIMA(3,0,0)(0,1,0)4 and 435.32 for the model ARIMA(4,0,5)(1,1,2)4 respectively. These two models corresponding to minimum AIC and SBC values were estimated and compared for their suitability.

For the model ARIMA (4,0,0) (1,1,2) 4 the estimation algorithm terminated after 13 iterations when the decrease in residual sum of squares was less than 0.001 percent. Final estimates of the parameters in the model are $\hat{\phi}_1 = 0.188095$ (standard error 0.0732, p<0.05), $\hat{\phi}_2 = -0.073838$ (standard error 0.0775, p>0.3); $\hat{\phi}_3 = 0.251373$ (standard error 0.0726, p<0.001), $\hat{\phi}_4 = 0.570424$ (standard error 0.1431, p<0.001), $\hat{\Phi}_1 = 0.395996$ (standard error 0.1692, p<0.05), $\hat{\theta}_1 = 1.762684$ (standard error 0.1292, p<0.001) and $\hat{\theta}_2 = -0.795038$ (standard error 0.1257, p<0.001). The SBC value was 387.23 and the estimate of residual standard error was 0.8226. The polynomial corresponding to the regular AR terms in the model is (1-0.188095 x + 0.073838 x^2 - 0.251373 x^3- 0.570424 x^4) and its roots are 1.0202, -1.3781, -0.0414+1.1159*i* and -0.0414-1.1159*i*. All these roots fall outside the unit circle region which is the required condition for stationarity of the model. Roots of the seasonal AR polynomial (1- 0.395996 x^4) have 1.2606 as the absolute value for its roots, so that these roots also fall out side the non-stationarity region. Hence the estimated model is stationary. The seasonal MA polynomial (1-1.762684 x^4 + 0.795038 x^8) has 1.0291 as the absolute value for its roots so that these roots also fall outside the unit circle region for non-invertibility and hence the model estimated is invertible. When the correlation between the estimates were examined it was found maximum between $\hat{\theta}_1$ and $\hat{\theta}_2$ and there is high correlation between $\hat{\phi}_1$ and $\hat{\phi}_4$, $\hat{\phi}_3$ and $\hat{\phi}_4$, $\hat{\Phi}_1$ and $\hat{\theta}_2$ and also between $\hat{\Phi}_1$ and $\hat{\phi}_4$. To examine the suitability of the estimated model residuals were calculated based on this model and *acf* up to lag 36 were computed. The maximum observed residual *acf* value was -0.156 which is not significant. Box-

Ljung χ^2 value calculated using residual *acf* up to lag 36 was 30.73 which is not significant (p=0.717). Hence the estimated model is a suitable approximation to the series.

For the model based on SBC criterion, ARIMA(1,0,1)(0,1,1)4 for log transformed quarterly anchovies landings, the estimation algorithm terminated after 9 iteration when the reduction in residual sum of squares was found less than 0.001%. Final estimate of parameters of the above model are $\hat{\phi}_1$ 0.991353 (standard error 0.0270, p<0.001), $\hat{\theta}_1 = 0.856914$ (standard error 0.0553, p<0.001) and $\hat{\Theta}_1 = 0.959118$ (standard error 0.0836, p<0.001). The maximum likelihood corresponding to this model is -182.32 and the estimate of residual standard error was 0.8439. The characteristic polynomial corresponding to the regular AR term in the model is (1- 0.991353 x) and its root is 1.0087 which is close to the unit circle boundary region. The characteristic polynomial corresponding to the MA term is (1- 0.856914 x) with root 1.1670 and is well outside the unit circle region for non-invertibility. But for the characteristic polynomial for the seasonal MA term (1- 0.959118 x^4), the absolute value of its roots is 1.0105 which again is close to the boundary region for non-invertibility. There is high correlation between the estimates of ϕ_1 and θ_1 which is 0.764 and $\hat{\phi}_1$ and $\hat{\theta}_1$ are also significantly correlated. Residual analysis was carried out by computing *acf* up to lag 36 for the residual series generated using the estimated model. The maximum value observed for the residual *acf* was –0.172 at lag 9 which is significant. The Box-Ljung χ^2 value based on 36 autocorrelations of the residual was 49.02 which is not significant (p=0.078). When the two models were compared, the one based on minimum AIC criterion, ARIMA(4,0,0)(1,12)4 was found to fit better than the one based on SBC for the reasons that it has resulted in smaller residual variance, none of the residual autocorrelations were significant and the Box-Ljung χ^2 value is comparatively small and not at all significant. Plot of the observed catches of anchovies and expected catch computed based on the estimated model is shown in fig.1.5.2. The algebraic form of the final estimated model is

$$(1-0.395996B^4)(1-0.188095B+0.073838B^2-0.251373B^3-0.570424B^4)\widetilde{Z}_t$$
$$=(1-1.762684B^4+0.795038B^8)\varepsilon_t$$

1.6. Lesser sardine landings

During 1960-96 period Lesser sardines on an average accounted for about 3.83 percent of the total marine fish landings in Kerala. The maximum contribution by lesser sardines in this period was 13.92 % in the year 1973 and minimum was 0.76 % in 1985. The average landings by this group during this period was 15,263*t* with a coefficient of variation of 80.55%. The minimum observed annual landings of lesser sardines was 2,473*t* in 1985 and the maximum was 62,421*t* in 1973. During 1960-80 period the average landings by this group was 16,479*t* (CV=82.03%) which is about 4.87% of the total landings and during 1981-96 period the average landings was 13,667*t* (CV=75.06%) which is about 2.9% of the total landings in this period. The average landings by this group during 1960-96 in different quarters were 2,818*t* in the first quarter (CV=95.74%), 1,985*t* in the second quarter (CV=89.87%), 3,113*t* in the third quarter (CV=141.37%) and 7,347*t* in the fourth quarter (CV=112.45%). The average quarterwise percentage distribution of landings of lesser sardines in this period were 18.46%, 13.00%, 20.40% and 48.14% respectively for first, second, third and fourth quarter. In the time series of quarterwise landings of lesser sardines there is very high fluctuation in the middle and later stages of the period under study. Hence a transformation of the series was made by natural logarithm before analysing the series.

The *acf* and *pacf* up to lag 36 were computed for the log transformed quarterwise landings of lesser sardines, its regular difference, its seasonal difference and for the series obtained by applying both regular and seasonal differences, which is presented in table.1.6. For the log transformed series the *acf* were significant at lags 1, 3, 4, 8, 14, 16, 21 and 29 with a maximum of 0.488 at lag 4. The *pacf* of this series were significant at lags 1, 4 and 8, the maximum being 0.452 at lag 4. This is an indication that the time series is seasonal with seasonality 4. For the series obtained as regular difference of the log transformed series *acf* were significant at many lags with out any clear pattern and the maximum observed was 0.453 at lag 8. The *pacf* of this series were found significant at lags 1, 2, 3, and 7 with –0.535 as the maximum at lag 4. For the seasonally differenced series the *acf* was found significant only at lag 4 at which the autocorrelation is -0.424. The *pacf* of this series were significant at lags 4, 12 and 19 the maximum being -0.458 at lag 4. For the series obtained by applying both regular and seasonal differences on the log transformed series *acf* were significant at lags 1, 3 and 4 with a maximum of -0.506 at lag 4. The *pacf* of this series have significant values at lags 1, 2, 4 and 5 with maximum -0.431 at lag 1. Even though there is no clear pattern visible in the *acf* and *pacf* of these series for model identification, they indicated that a seasonal model will fit better for the series because the seasonally differenced series is also not free from significant *acf* and *pacf* values at lag 4 and its multiples.

Scaled variograms were computed using the seasonally differenced log transformed quarterwise landings of lesser sardines and these are plotted along with expected values and $\pm 2SE$ limits in fig.1.6.1. The standard errors for the scaled variograms were 0.1125, 0.0125 and 0.0075 respectively for semi, lin and quad variograms. The scaled variogram was found to fluctuate between 0.734 and 1.409, with some points falling outside the confidence limit even at higher lags. The scaled linvariogram was found to level out around the expected values after a few initial lags so also the scaled quadvariogram. The scaled linvariogram was found to range between 0.357 and 0.517 and the scaled quad variogram was found to vary between 0.121 and 0.263. The leveling of the scaled lin and quad variograms and the fluctuation of semi variogram even at higher lags indicate the need for a regular differencing of the deseasonalized series. Hence, for the estimation of a seasonal model for the series, the regular difference order parameter was taken as 1.

Among different models for values of $p,q = 0, 1, ...,5$; $P, Q = 0, 1, 2$; d, D = 1 the model that yielded the minimum AIC value 382.25was ARIMA(5,1,0)(0,1,2)4, and that corresponding to the minimum SBC value 393.48 was ARIMA(0,1,1)(0,1,1)4. These models were estimated and checked for their suitability. For ARIMA (5,1,0) (0,1,2)4 model the estimation algorithm stopped after 12 iterations when the reduction in residual sum of squares was less than 0.001%. Final estimates of the parameters in the model are $\hat{\phi}_1 = -0.727988$ (standard error 0.0831, p<0.001), $\hat{\phi}_2 = -0.455917$ (standard error 0.0982, p<0.001), $\hat{\phi}_3 = -0.302575$ (standard error 0.1117, p<0.008), $\hat{\phi}_4 = 0.608445$ (standard error 0.1068, p<0.008), $\hat{\phi}_5 = 0.237267$ (standard error 0.8956, p<0.01), $\hat{\Theta}_1 = 1.764207$ (standard error 0.0807, p<0.001) and $\hat{\Theta}_2 = -0.784631$ (standard error 0.0769, p<0.001). Some of these estimates were highly correlated and the maximum correlation was -0.979 between $\hat{\Theta}_1$ and $\hat{\Theta}_2$, next being 0.929 between $\hat{\phi}_2$ and $\hat{\phi}_4$. The log likelihood corresponding to these estimates was -184.13, SBC value was 402.99 and the estimate of residual standard error was 0.8622. The characteristic polynomial for the regular AR terms is $(1+0.727988x+0.455917x^2+0.302575x^3-0.608445x^4-0.237267x^5)$ and its roots are 1.4169, -1.0268, -2.8442, (-0.0551+1.0077*i*) and (-0.0551-1.0077*i*). Characteristic polynomial corresponding to the seasonal MA term in the model is $(1-0.764207x^4+0.7846311x^8)$ and 1.0308 is the absolute value of its roots which is out side the unit circle boundary so that the model estimated is invertible. The *acf* up to lag 36 were computed for the residual series generated using the estimated model and the maximum observed *acf* value was 0.183 at lag 6 which is the only significant value. The value of Box-Ljung χ^2 statistic computed using *acf* of residuals up to lag 36 was 31.875 which is not significant (p= 0.665).

For the second model ARIMA(0,1,1)(0,1,1)4 which yield the minimum SBC value, the estimation process was concluded after 5 iterations when the reduction in residual sum of squares was lass than 0.001%. Final estimate of parameters in the model are $\hat{\theta}_1 = 0.750384$ (standard error 0.6177, p<0.001) and $\hat{\Theta}_1 = 0.790039$ (standard error

0.5953, p<0.001). There do not exist any significant correlation between $\hat{\theta}_1$ and $\hat{\Theta}_1$. Log likelihood corresponding to this estimated model was -191.55. Characteristic polynomial for the regular MA term is $(1-0.750384x)$ and its root is 1.3327 and it lies out side the unit circle. The characteristic polynomial for the seasonal MA term is $(1-0.790039x^4)$ and the absolute value of its roots is 1.2658. These roots also lie outside the unit circle boundary for non-invertibility and hence this model is invertible. The maximum value of *acf* observed for the residual series generated using the estimated model was -0.162 at lag 29 and it was not significant. The Box-Ljung χ^2 calculated based on *acf* of residuals up to lag 36 was 40.50 which is not significant (p=0.278). This model have the desired properties of stationarity and invertibility compared the other three models and is parsimonious. Thus this model was chosen as the suitable model to represent the time series on quarterwise landings of lesser sardines in Kerala. The observed catch and its expected value according to the estimated model are plotted fig.1.6.2. The algebraic form of the estimated model is

$$(1-B^4)(1-B)\widetilde{Z}_t = (1-0.790039B^4)(1-0.750384B)\varepsilon_t$$

1.7. Penaeid Prawns landings

During the period 1960-96 the average percentage contribution by penaeid prawns towards total marine fish landings in Kerala was 10.39% with a maximum of 18.91% in 1973 and a minimum of 3.69% in 1960. Average annual landings of penaeid prawns was 40,418*t* with a coefficient of variation of about 41.81%. The maximum observed annual catch of this group during this period was 84,770*t* in 1973 and the minimum observed was 12,798*t* in 1960. The average annual landings during 1960-80 period was 37,027*t* (CV=48.58%) and that during 1981-96 was 44,869*t* (CV=31.59%). Quarterly average of penaeid prawn landings during 1960-96 period were 6,673*t* in the first quarter (CV=57.55%), 10,672*t* in the second quarter (CV=59.69%), 17,090*t* in the third quarter (CV=65.66%) and 5,982*t* in the fourth quarter (CV=60.15%). On an average the percentage of landings by penaeid prawns in different quarters were 16.51% in first quarter, 26.41% in the second quarter, 42.28% in the third quarter and 14.80% in the fourth quarter. Since there is high fluctuation to some extent in landings during the middle of the period log transformation was applied to reduce variability and the transformed series was used for analysis.

The *acf* and *pacf* up to lag 36 were computed for the log transformed series, its regular difference, seasonal difference and for the series obtained by applying both regular and seasonal differences and it is shown in table.1.7. For the series without any differencing, the *acf* were significant at all lags that are multiples of 4 and also at lags 1, 3, and 30 with a maximum of 0.546 at lag 4. Its *pacf* were significant at lags 1, 3, 4 and 8 with a maximum of 0.499 also at lag 4. This clearly is an indication that the series is seasonal with seasonality 4. The differenced series have more number of significant values for the *acf* especially at lags that are multiples of 4, the maximum being 0.516 at lag 4. The magnitude of autocorrelation was found to increase for the series obtained by applying a regular difference. The *pacf* of the differenced series have significant values at lags 1, 2, 3 and 7 with -0.624 as the maximum at lag 3. For the series obtained as a seasonal difference of the log transformed series, the only significant *acf* value was -

0.368 at lag 4 and the only significant *pacf* was 0.379 also at lag 4. This indicates that mere a first order seasonal differencing is enough to simplify the *acf* structure of the series. Since there still exist significant *acf* and *pacf* at lag 4, a seasonal model will suit better for the series. When both regular and seasonal differences were applied to the series, the *acf* and *pacf* were found significant at more number of lags. The *acf* were significant at lags 1, 3, 4, 6 and 25 with a maximum of –0.421 at lag 1 and significant *pacf* were at lags 1, 2, 4, 5 and 8 maximum being –0.421 at lag 1. The analysis using *acf* and *pacf* suggested that a seasonal ARIMA model with seasonality 4 would be a better model to represent the log transformed quarterwise landings of Penaeid prawns.

Variograms were computed using the seasonally differenced log transformed series and the scaled semi variogram was found to vary between 0.621 and 1.363. Only very few points fall outside the confidence limits for semi variogram and the lin and quad variograms level out after a few initial lags. The scaled linvarogram was found to vary in the range 0.356 to 0.493 and the scaled quadvariogram fluctuated between 0.119 and 0.246. Plots of the scaled variograms are given in the fig.1.7.1. This suggests that there is no necessity for a regular differencing for the series. Based on this analysis seasonal ARIMA models were attempted for the series with out any regular differencing and with a seasonal differencing of seasonality 4. For estimaion of the orders criterion selection based on AIC and SBC were used. Seasonal ARIMA model of the form ARIMA(p,d,q) $(P,D,Q)s$ were initially fitted for different values of p,q=0, 1,..., 5; P,Q= 0, 1, 2; d=0 and D=1 using the log transformed quarterwise landings of penaeid prawns. The values of AIC and SBC were obtained for 315 such models and the minimum AIC value observed was 272.41 for ARIMA(0,0,4)(2,1,2)4 and the minimum SBC value was 279.83 for ARIMA(0,0,1)(0,1,1)4. The maximum AIC value observed was 312.45 for ARIMA(3,0,0)(0,1,0)4 and the maximum SBC value observed was 329.45 for ARIMA(5,0,5)(2,1,2)4. The two models corresponding to the minimum values of AIC and SBC were estimated and compared.

For the model ARIMA(0,0,4)(2,1,2)4, the estimation algorithm terminated after 22 iterations when the change in the estimate of parameters were found to be less than 0.001. Final estimate of parameters in the model are $\hat{\theta}_1 = -0.177829$ (standard error 19.6694, p>0.99) $\hat{\theta}_2 = -0.080378$ (standard error 16.1927, p>0.99), $\hat{\theta}_3 = -0.098269$ (standard error 17.7724, p>0.99), $\hat{\theta}_4 = 0.804199$ (standard error 15.8694, p>0.95), $\hat{\Phi}_1 = 1.917048$ (standard error 0.0421, p≃0.0), $\hat{\Phi}_2 = -0.997742$ (standard error 0.0408, p≃0.0) $\hat{\Theta}_1 = -1.891108$ (standard error 0.3041, p≃0.0) and $\hat{\Theta}_2 = -0.968287$ (standard error 0.3017, p<0.002). The estimates of all the moving average parameters were found to be non-significant due to high estimates of their standard errors and these estimates were found to be highly correlated to each other. High correlation was also seen between the estimates of moving average parameters and seasonal AR and seasonal MA parameters. To see whether the estimated model posses stationary and invertibility properties, roots of characteristic polynomials were computed. The characteristic polynomial corresponding to the regular MA term is $(1 + 0.177829x + 0.0803780x^2 + 0.098269x^3 - 0.804199x^4)$ and its roots are 1.1627, -

1.00002, (-0.0203+1.0339*i*) and (-0.0203-1.0339*i*) (absolute value 1.0341). Hence some of the roots are close to the unit circle boundary for non-invertibility, and hence the model can not be considered as strictly non-invertible. Roots of the seasonal AR polynomial $(1-1.917048x^4+0.997742x^8)$ have 1.0003 as its absolute value and hence these roots are close to the unit circle boundary for stationarity of the model. Hence the model can not be considered strictly as stationary. The characteristic polynomial for the seasonal MA terms in the model is $(1+1.891108x^4+0.968287x^8)$ and the absolute value of its roots 1.0040 is close to the unit circle boundary. The estimate of residual standard error corresponding to this model was 0.5726, log likelihood was -128.20 and SBC value was 296.17. Residual analysis was carried out by computing residual *acf* up to lag 36 using the residual series generated based on the estimated model. None of these residual *acf* were significant and the maximum was 0.117 at lag 6. Box-Ljung χ^2 value based on *acf* up to lag 36 of the residuals was 17.168 which is not significant ($p>0.99$). Though the residual analysis showed acceptance of the estimated model it is not preferable since it does not posses properties of stationarity and invertibility, and also due to high standard error for some of the estimates.

For the second model ARIMA(0,0,1)(0,1,1)4 the estimation algorithm concluded after 4 iterations when the percentage reduction in residual sum of squares was found less than 0.001. The final estimate of parameters of the model are $\hat{\theta}_1=-0.184908$ (standard error 0.0830, $p<0.03$) and $\hat{\Theta}_1=0.618036$ (standard error 0.0704, $p<0.001$). The polynomial corresponding to the regular MA terms of the model is $(1+0.184908x)$ and its root is -5.4081 which falls outside the unit circle boundary for non-invertibility. Roots of the seasonal MA polynomial $(1-0.618036x^4)$ have 1.1278 as its absolute value so that these roots also fall outside the unit circle. Hence the model is invertible and stationary since there are no autoregressive terms in the model. The *acf* up to lag 36 were computed for the residual series generated based on the estimated model and all were found non-significant. The maximum observed for the residual *acf* was 0.137 at lag 6 and it is not significant. Using the *acf* up to lag 36 of the residuals the Box-Ljung χ^2 computed was 20.240 which is not significant (p=0.984) and hence the model can be used as an approximation for the generating series. Using this estimated model expected landings were computed for the original series and it is shown in fig.1.7.2. along with observed landings. The algebraic form of the final estimated model is

$$(1-B^4)\tilde{Z}_t = (1-0.618036B^4)(1+0.184908B)\varepsilon_t$$

1.8. Tuna landings

During the period 1960-96, the average annual landings of Tuna in Kerala was 8,336*t* with a coefficient of variation of 82.7%. This on an average is about 1.96% of the total landings in the state. During this period the maximum tuna landings observed was 32,615*t* in the year 1990 (4.92 % of total landings) and the minimum landings of Tuna observed was 723*t* (0.38% of total landings) in the year 1962. The minimum percentage contribution by Tuna towards the total landings in the state was 0.31% in 1969. During 1960-80 period, the average landings of Tuna was 4,513*t* (CV=88.1%) and the average landings in 1981-96 period was 13,353*t* (CV=50.12%). This showed a fourfold increase in the landings of Tuna during recent past. For time series modelling of Tuna landings, quarterwise landings during 1960-96 period was used. During this period the average landings of Tuna in different quarters were 1,337*t* in the first quarter (CV=85.94%), 2,762*t* in the second quarter (CV=90.62%), 1,630*t* in the third quarter (CV=113.56%) and 2,607*t* in the fourth quarter (CV=125.51%). The average percentage contribution by different quarters towards the annual landings of Tuna were 16.04%, 33.13%, 19.56% and 31.27% respectively for the four quarters. When the time series of quarterwise landings of Tuna were examined by plotting the variability in the series was found to be high in the later periods and a logarithmic transformation was carried out to reduce this variability before it is subjected to analysis for modelling.

Autocorrelations and partial autocorrelations calculated for the log transformed series on quarterwise landings of Tuna, its seasonal difference, its regular difference and both the kind of differences are given in table.1.8. Among the *acf* calculated up to lag 36 for the log transformed series almost all values were significant except those at lags 23, 27, 33, 34 and 35 and the maximum was 0.737 at lag 4. The *pacf* of this series was found significant at lags 1, 2, 3, 4, 8, 9, 12 and 19 with 0.579 as the maximum at lag 4. At lags that are multiples of 4, *acf* were high compared to other lags which is an indication that the series is seasonal with seasonality 4. The non-dying characteristic of *acf* is an indicator of non-stationarity in the series and a regular differencing of the series may be required to make the series stationary. When the log transformed series was subjected to regular differencing, the *acf* of the resulting series was also found to have highly significant values at most of the lags especially at lags which are multiples of two. The highest value of *acf* for this series was 0.559 at lag 12. The *pacf* of this series was found significant at lags 1, 2, 3, 7, 10, 11 and 18 with 0.621 as the maximum at lag 3. For the series obtained by a seasonal differencing of log transformed data the only significant *acf* was -0.440 at lag 4 and its *pacf* were significant at lags 4, 7, 8, 15 and 20. Maximum value of *pacf* observed for this series was -0.450 at lag 4. This clearly indicate that there is some amount of seasonality left even after the application of a seasonal differencing and hence a seasonal ARIMA model will be a better choice for modelling the series. When both regular and seasonal differencing were applied to the log transformed series, the *acf* of the resulting series was found significant at lags 1, 3 and 4 and its *pacf* was found significant at lags 1, 2, 4, 5, 6, 8, 10 and 14. The strength of the *pacf* was found to decrease for higher lags and the maximum values observed for both *acf* and *pacf* of this series was -0.482 at lag 1.

Using the series generated by a seasonal difference of the log transformed series on quarterwise landings of Tuna, variograms were computed up to lag 70 and plots of these variograms is given in fig.1.8.1. The scaled lin and quad variograms leveled out after a few initial lags and the scaled semi variogram was found to fluctuate between 0.720 and 1.454. A few points of the semivariogram were found to fall outside the 2 standard error limits. The scaled lin variogram ranged between 0.373 and 0.497 and all points after leveling out were within the $\pm 2SE$ limits. The scaled quadvariogram ranged between 0.123 and 0.245 and after leveling out all points were found to fall within the limits. This suggests that a regular differencing is required to make the series stationary and accordingly D was fixed as 1.

Selection of the suitable seasonal ARIMA model was based on AIC and SBC criterion. For this purpose 315 different models for values of p,q=0, 1, 2, ...,5; P,Q = 0, 1, 2; d = 1 and D=1 were initially estimated and these criterion were computed. The model corresponding to the minimum AIC value of 336.49 was ARIMA(2,1,4)(0,1,1)4 and that corresponding to the minimum SBC value 344.66 was ARIMA(0,1,1)(0,1,1)4. The maximum AIC value observed was 439.88 for ARIMA(1,1,0)(0,1,1)4 and the maximum SBC value was 442.85 for the model ARIMA(1,1,0)(0,1,0)4. The two models that correspond to the minimum values of AIC and SBC were finally estimated and compared for their properties.

For the minimum AIC model ARIMA(2,1,4)(0,1,1)4, the estimation algorithm concluded after 14 iterations when the change in parameter estimates were found less than 0.001. Final estimate of parameters of this model are $\hat{\phi}_1 = 0.111320$ (standard error 0.2722, p<0.001), $\hat{\phi}_2 = -0.980845$ (standard error 0.0206, p<0.001), $\hat{\theta}_1 = 0.928317$ (standard error 2.2337, p>0.678), $\hat{\theta}_2 = -1.097043$ (standard error 24.4780, p>0.96), $\hat{\theta}_3 = 0.724822$ (standard error 16.6682, p>0.96), $\hat{\theta}_4 = 0.045721$ (standard error 1.05521, p>0.96), and $\hat{\Theta}_1 = 0.874686$ (standard error 0.0646, p < 0.001). To examine whether this model satisfy the conditions for stationarity and invertibility, the roots of the characteristic polynomials were computed. Roots of the characteristic AR polynomial $(1 - 0.111320x + 0.980845x^2)$ are (0.0567+1.0081i) and (0.0567-1.0081i) and their absolute value is 1.0097 which is close to the unit circle boundary for stationarity. Hence the model is not strictly stationary. The characteristic polynomial corresponding to the regular MA terms is $(1-0.928317x+1.097043x^2-0.724822x^3-0.045721x^4)$ with roots 1.2634, -17.3113, (0.0973+0.9953*i*) and (0.0973-0.9953*i*) (absolute value is 1.000006). Two roots of this polynomial fall almost on the unit circle and hence the model does not possess invertibility property. Roots of the characteristic polynomial $(1-0.874686x^4)$ for the seasonal MA term has absolute value 1.0340 and hence these roots are also close to the boundary region for invertibility. All together this model can not be accepted because the model is neither stationary nor invertible. Also most of the estimated coefficients are highly correlated especially the moving average coefficients, and the estimates of moving average coefficients have high standard errors as a result of which these coefficients are non-significant.

For the model ARIMA(0,1,1,)(0,1,1,)4 the estimation algorithm concluded after 5 iterations. Final estimates of parameters of the model are $\hat{\theta}_1=0.825934$ (standard error 0.0531, p<0.001) and $\hat{\Theta}_1=0.744906$ (standard error 0.0622, p<0.001). Characteristic polynomial for the regular MA term is $(1-0.825934x)$ with root 1.2108. The characteristic polynomial for the seasonal MA term is $(1-0.744906x^4)$ and the absolute value of its roots is 1.0764. Since both these roots fall outside the unit circle boundary for non-invertibility, the estimated model is invertible. The model is stationary because there is no AR term in the model. Residual standard error corresponding to this model was 0.7711, log likelihood was -167.37 and AIC value was 338.73. Residual series were generated for this series using the estimated model and *acf* up to lag 36 were computed for the residuals. The maximum observed residual *acf* was -0.161 at lag 19 and it is not significant. Box-Ljung χ^2 calculated based on residual *acf* up to lag 36 is 32.731 which is not significant (p=0.625). This suggests acceptance of the estimated model ARIMA (0,1,1)(0,1,1)4 as a good approximation to the generating series. The observed catch and catch predicted using the estimated model are shown in fig.1.8.2. The algebraic form of the estimated model is

$$\left(1 - B^4\right)\left(1 - B\right)\tilde{Z}_t = \left(1 - 0.825934B\right)\left(1 - 0.744906B^4\right)\varepsilon_t$$

1.9. Thrissocles landings

The average contribution by this species group towards total landings in the state during 1960-96 was 0.62% with a maximum of 1.37% in the year 1992 and the minimum was 0.23% in 1961. The average annual landings by Thrissocles during this period was 2,587*t* (CV=69.89%) with a maximum of 7,676*t* in 1972 and a minimum of 630*t* in 1961. During the initial 1960-80 period the average landings was 1,843*t* (CV=48.4%) and during the last 1981-96 period the average landings was 3,564*t* (CV=61.67%). The average quarterwise landings during 1960-96 period was 383*t* in first quarter (CV=121.67%), 537*t* in the second quarter (CV=108.19%), 1060*t* in the third quarter (CV=72.17%) and 608*t* in the last quarter (CV=121.55%). The quarterwise percentage distribution of thrissocles landings on an average were 14.80%, 20.75%, 40.96% and 23.49% respectively for the four quarters. There is a slight upward trend in the landings of Thrissocles and the variability in the last few years are high. Hence a transformation of the series by natural logarithms was carried out before estimating a suitable model.

An initial analysis was carried out by computing *acf* and *pacf* for the original log transformed series, its regular difference, seasonal difference and both regular and seasonal difference and these are given in table.1.9. For the log transformed series the *acf* were significant at many lags including the lags which are multiples of 4 and the maximum *acf* value was 0.507 at lag 4. The *pacf* of this series have significant values at lags 1, 3, 4, 8 and 16 with maximum 0.370 at lag 4. This clearly indicates that there exists seasonality in the data and seasonal differencing of the log transformed series is necessary to remove seasonality in the data. When the log transformed quarterwise landings series was subjected to regular differencing the resulting series had significant *acf* at all lags that are multiples of 2, and the maximum value observed was 0.434 at lag 16. The *pacf* of this series was found significant at lags 1, 2, 3, 7 and 15 with -0.508 as the maximum at lag 2. For the series generated by a seasonal difference of log transformed series the *acf* was significant at lags 4 and 16 with a maximum of -0.443 at lag 4 and its *pacf* was significant at lags 4, 8 and 12 with a maximum of -0.460 at lag 4. Since there exists significant *acf* at lags that are multiples of 4 for the seasonally differenced series a seasonal model will suit better for this series. The *acf* of the series obtained by applying both regular and seasonal differencing to the log transformed series have significant values at lags 1, 3, 4, 5, 12 and 16 with a maximum value of -0.533 at lag 4 and its *pacf* had significant values at lags 1, 2, 4, 8 and 16 with a maximum of –0.479 at lag 4. This analysis failed to give any conclusion regarding orders of the seasonal model and hence criterion selection was used to identify a suitable seasonal ARIMA model.

Since there is a linear upward trend seen in the series, it may be required to apply a regular difference along with seasonal difference. To ascertain this variograms were calculated using the series generated by a seasonal difference of log transformed series and plot of the scaled values of semivariogram, linvariogram and quadvariogram are given in fig.1.9.1. along with respective expected values and confidence limits. The scaled semivariogram was found to fluctuate between 0.651 and 1.433 and some of the values are found to lie outside the confidence limits including some at higher lag. The scaled linvarigram was found to vary between 0.359 and 0.529 and the scaled quadvariogram ranged between 0.122 and 0.267. The scaled linvariogram and scaled quadvariogram were found to level out at around 0.4 and 0.2 respectively. Hence it suggests that a regular differencing will make the series stationary.

To estimate orders of the general seasonal type model ARIMA$(p,d,q)(P,D,Q)s$ with different values of p,q=0,1,..., 5; P,Q=0,1,2; D=1; d=1 and s=4 leading to 315 different models, were fitted and the AIC and SBC criterion were computed. Among these models the one which gave the minimum AIC value 402.14 was ARIMA(1,1,1)(1,1,2)4 and minimum SBC value 412.62 was obtained for the model ARIMA(0,1,1)(0,1,1)4. The maximum AIC and SBC values obtained were respectively 507.41 and 510.38 both the values correspond to the same model ARIMA(1,1,0)(0,1,0)4. For the model ARIMA(1,1,1)(1,1,2)4 the iteration of the estimation algorithm stopped after 5 iterations when the change in the parameters estimates were found less than 0.001. Final estimates of parameters in the model are $\hat{\phi}_1 = 0.209237$ (standard error 0.0981,

$p<0.05$), $\hat{\theta}_1 = 0.912306$ (standard error 0.0490, $p<0.001$), $\hat{\Phi}_1 = -0.998765$ (standard error 0.0221, $p<0.001$), $\hat{\Theta}_1 = -0.161557$ (standard error 0.1391, $p>0.20$) and $\hat{\Theta}_2 = 0.811702$ (standard error 0.1283, $p<0.001$). The log likelihood corresponding to these estimates of the model was -196.07 and SBC value was 416.96. The characteristic polynomial for the AR term is $(1-0.209237x)$ and its root is 4.7793. The polynomial corresponding to the seasonal AR term is $(1+0.998765x^4)$ and the absolute value of its roots is 1.0003. Since these roots are close to the boundary of non-stationary region, the model is not strictly stationary. The polynomial corresponding to the MA term is $(1-0.912306x)$ with 1.0961 as its root. The seasonal MA polynomial for the model is $(1+0.161557x^4-0.811702x^8)$ and its roots have 1.0497 and 1.0037 as absolute value. So the model is not strictly invertible. The estimated standard error for the residuals is 0.9313. Using the estimated model residuals and the *acf* of residuals up to lag 36 were computed. The maximum observed residual *acf* was 0.173 at lag 17 that is not significant. The Box-Ljung χ^2 calculated using residual *acf* up to lag 36 was 29.406 which is not significant ($p=0.773$).

For the model ARIMA(0,1,1)(0,1,1,)4 the estimation algorithm stopped after 8 iterations when the change in the estimates of parameters were less than 0.001. Final estimates of the parameters of the model are $\hat{\theta}_1 = 0.834907$ (standard error 0.0504, $p<0.001$) and $\hat{\Theta}_1 = 0.860635$ (standard error 0.0534, $p<0.001$). The characteristic polynomial for the MA term is $(1-0.834907x)$ and its root is 1.1977. For the seasonal

MA term the characteristic polynomial is $(1-0.860635x^4)$ and its roots have absolute value 1.0382. Log likelihood corresponding to this estimate of parameters was -201.35, SBC value was 412.69 and standard error of the residuals was 0.9694. Among the residual *acf* computed up to lag 36, only one at lag 16 with value 0.264 was found significant. The Box-Ljung χ^2 value was 42.563 when calculated using residual *acf* up to lag 36 which is not significant (p=0.209).

Among these two models the model ARIMA(1,1,1)(1,1,2)4 was preferred for the reasons that the residual acf were non-significant, loglikelihood is comparatively high and the residual variance was low. The observed values of the time series on thrissocle landings and the expected catch according to this fitted model are shown in fig.1.9.2. The mathematical expression for the fitted models is

$$(1-B^4)(1-B)(1+0.998765B^4)(1-0.209237B)\widetilde{Z}_t = (1+0.161557B^4-0.811702B^8)(1-0.912306B)\varepsilon_t$$

1.10. Ribbon Fish landings

The average annual landings of Ribbon fish in the state during 1960-96 period was about 10,769*t* (CV=75.23%). This is about 2.82% of the total landings in the state during this period. The minimum annual landings by this species was 169*t* in 1964 and the maximum landings observed was 30,192 *t* in 1974. During the period 1960-80 the average annual landings was 11,285*t* (CV=81.75%) and during 1981-96 period the average was 10,091*t* (CV=63.35%) which are respectively about 3.34% and 2.14% of the total marine fish landings in the state during these two phases. Quarterwise average annual landings of this species during 1960-96 period for the four quarters are 355*t* (3.30%), 1610*t* (14.95%), 6274*t* (58.26%) and 2529*t* (23.49%) respectively.

Since the quarterwise landings of ribbon fishes showed high fluctuations it was transformed by natural logarithms before analysis. An initial autocorrelation analysis was carried out on the transformed series. The *acf* and *pacf* up to lag 36 were computed for the log transformed series, its regular difference, seasonal difference and for the series obtained by applying both regular and seasonal difference and is given in the table.1.10. The *acf* at all even lags were significant for the non-differenced series with a maximum of 0.579 at lag 4. The *pacf* of this series were significant at lags 1, 2, 3, 4 and 7 with a maximum of 0.427 at lag 4. For the regular differenced series also the *acf* at all even lags were highly significant with more intensity and the maximum value was –0.583 at lag 2. For the seasonally differenced series the *acf* were significant at lags 1, 4, 11 and 12 with –0.333 as the maximum at lag 4 and significant *pacf* found for this series were at lags 1, 4, 5, 8 and 12 with –0.314 as the maximum at lag 4. For the series obtained by applying both regular and seasonal difference to the log transformed series the significant *acf* were at lags 1, 4, 5 and 7 and *pacf* were significant at lags 1, 4, 8, 12 and 13. The maximum *acf* observed for this series was –0.452 at lag 4 and the maximum *pacf* observed was –0.503 also at lag 4. These results indicate that the quarterly landings of ribbonfishes show seasonal behavior with seasonality 4. Hence a seasonal ARIMA model with seasonal differencing parameter $D=1$ will suit better for this series.

To examine the necessity of a regular differencing the scaled variograms up to lag 70 were computed using the seasonally differenced log transformed series and plots of scaled values of semi, lin and quad varigrams with respective confidence limits are shown in figure.1.10.1. The scaled semivariogram fluctuated between 0.700 and 1.351, scaled linvariogram varied between 0.341 and 0.514 and the scaled quadvariogram varied between 0.122 and 0.264. Both the scaled linvariogram and scaled quadvariogram leveled out after few initial lags and there after all the points were found very close to the expected values and they fall within the $\pm 2SE$ limits. For the scaled semivariogram very few points in the initial stage were found to fall out side the $\pm 2SE$ limits. Hence it is not necessary to apply a regular differencing on this series to make it stationary.

To select a suitable order for the seasonal ARIMA model for this series the AIC and SBC order selection criterion were computed for different values of the order parameters $p.q = 0,1,\ldots,5$; $P,Q = 0,1,2$; d=0 and D=1 by fitting models for different combinations of order parameters. The minimum AIC value observed was 533.33 for the model ARIMA(1,0,1)(0,1,1)4 and the maximum observed AIC value was 587.02 for ARIMA(1,0,1)(0,1,0)4. Minimum SBC value observed was 539.97 for ARIMA(1,0,0)(0,1,1)4 and maximum observed SBC value was 595.31 for ARIMA(3,0,0)(0,1,0)4. These two models corresponding to minimum values of AIC and SBC were estimated and compared. For the model ARIMA(1,0,1)(0,1,1)4 the estimation algorithm concluded after 6 iterations when the reduction in residual sum of squares was less than 0.001%. The final estimates of parameters of the model are $\hat{\phi}_1 = 0.731783$

(standard error 0.1205, $p < 0.001$), $\hat{\theta}_1 = 0.359239$ (standard error 0.0249, $p < 0.05$) and $\hat{\Theta}_1 = 0.781891$ (standard error 0.0606, $p < 0.001$). The characteristic polynomial corresponding to the AR term is $(1 - 0.731783\,x)$ and its root is 1.3665 that lies outside the unit circle boundary for non-stationarity. The characteristic MA polynomial $(1 - 0.359239\,x)$ has 2.7837 as its root that falls outside the unit circle boundary for non-invertibility. The characteristic polynomial corresponding to the seasonal MA term of the model is $(1 - 0.781891\,x^4)$ and the absolute value of its root is 1.0634 which again is outside the unit circle. Hence the estimated model is both stationary and invertible. Among the parameter estimates maximum correlation was found between $\hat{\phi}_1$ and $\hat{\theta}_1$ which is 0.872. Residual analysis was carried out for the model by computing the residual sequence, residual variance and residual autocorrelations. The estimate of residual standard error was 1.5070 and the log likelihood was –263.66. Among the *acf* calculated up to lag 36 for the residual series the maximum was 0.132 at lag 7 that is not significant. Using the residual *acf* up to lag 36 the calculated value of the Box-Ljung χ^2 statistic was 24.846 which is not significant (*p*=0.919). Hence the estimated model fits well for the series.

For the second model ARIMA(1,0,0)(0,1,1)4 selected based on minimum SBC criterion, the estimation algorithm stopped after 4 iterations when the reduction in residual sum of squares was found less than 0.001%. Final estimates of parameters in the model are $\hat{\phi}_1 = 0.455929$ (standard error 0.0749, *p*<0.001) and $\hat{\Theta}_1 = 0.762475$ (standard error 0.0588, *p*<0.001). The characteristic polynomial corresponding to the AR term in the model is $(1-0.455929\,x)$ and its root 2.1933 falls outside the unit circle region for non-stationarity. The characteristic polynomial for the seasonal MA term in the model is $(1-0.762475\,x^4)$ and its roots have 1.3115 as the absolute value, so that these roots fall outside the unit circle region for non-invertibility. Hence the estimated model is both stationary and invertible. No significant correlation was found between the estimates of these parameters. Residual *acf* up to lag 36 computed for the residual series generated based on the estimated model have 0.152 as the highest value at lag 7 and it is not significant. The estimate of residual standard error was 1.5155 and log likelihood corresponding to these estimates was –265.02. The Box-Ljung χ^2 calculated based on residual *acf* up to lag 36 was 26.903 is not significant ($p = 0.864$). Hence this model can also be used as an approximation to the generating series. Anyhow, the first model has lesser value for the Box-Ljung χ^2 and the highest autocorrelation of residuals is the least for this model. Hence the first model was taken as the suitable model for this time series. The observed landings and expected landings according to the estimated model are plotted in fig.1.10.2. The algebraic form of the estimated model is

$$(1-B^4)(1-0.731783\,B)\,\widetilde{Z}_t = (1-0.781891\,B^4)(1-0.359239\,B)\varepsilon_t$$

Discussion

For the time series on quarterwise total marine fish landings, the seasonal ARIMA model fitted can be expanded to get the expression

$$\begin{aligned}(\widetilde{z}_t - \widetilde{z}_{t-4}) = (\widetilde{z}_{t-1} - \widetilde{z}_{t-5}) + \varepsilon_t - 0.434490\,\varepsilon_{t-1} - 0.436495\,\varepsilon_{t-2} \\ - 0.715598\,\varepsilon_{t-4} + 0.310920\,\varepsilon_{t-5} + 0.312355\,\varepsilon_{t-6}\end{aligned}$$

In this model the difference in landings for two successive years of a quarter is obtained as the similar difference for the immediate preceding quarter with additional terms for error correction using residuals up to lag 6. In the time series data used for estimating the above model there were some values which exceeded the 2σ limits. These values correspond to the first quarter of 1971, fourth quarter of 1989, second quarter of 1990 and first quarter of 1991. These data points could be outliers and may affect the parameter estimates of the model and forecasts. It was observed that when the effect of these suspected outliers were nullified by replacing them with respective quarterly averages, there was a reduction of about 5% in the total variation. Forecasts of landings and their standard errors were calculated using the estimated model for different quarters in 1997 and 1998. These are given in table.1.11. The maximum difference between forecasted and observed landings was for the third quarter of 1997. This difference was 2.63 times the standard error of the forecast and about 61.85 % of the observed landings.

For the time series on quarterwise landings of oil sardine the seasonal model estimated can be written as

$$(\tilde{z}_t - \tilde{z}_{t-4}) = 0.466848\,(\tilde{z}_{t-1} - \tilde{z}_{t-5}) + \varepsilon_t - 0.732283\,\varepsilon_{t-4}$$

This model indicate that the difference in landings of a quarter in two successive years is proportional to a similar difference for the preceding quarter with additional error correction terms using residuals up to lag 4. In the data used for fitting the model there were few values that could be outliers as they exceeded the 2σ limit. These values were for the fourth quarter of 1960, fourth quarter of 1964, first quarter of 1965, third quarter of 1966, third and fourth quarter of 1968, first quarter of 1971, third quarter of 1989, third quarter of 1990 and third quarter of 1991. When the effect of these suspected outliers were nullified by replacing them with the respective quarterly averages, a reduction in the variance of the sample series up to 38% was observed. Forecasts of quarterly landings for the years 1997 and 1998 were made using this model and these are given in table.1.11. along with standard errors of forecasts. The maximum difference between the observed and forecasted values was 21,350 tonnes for the first quarter of 1997. This was about 1.08 times the standard error of the forecast and it comes to about 73% of the observed catch.

The model estimated for the time series on quarterwise landings of mackerel can be written in the form

$$(\tilde{z}_t - \tilde{z}_{t-4}) = 0.415719\,(\tilde{z}_{t-1} - \tilde{z}_{t-5}) + \varepsilon_t - 0.726612\,\varepsilon_{t-4}$$

This model also indicate that the difference in landings between the same quarter of two successive years is proportional to a similar difference with the preceding quarters. The model also had a term for error correction using residual at lag 4. In the sample series on mackerel landings used for estimating the model there were few data points that are to be suspected as outliers as they fall out side the 2σ limits. These values were for first and second quarters of 1971, fourth quarter of 1989, fourth quarter of 1990, third quarter of 1994 and second and third quarters of 1996. A reduction in the variance of the sample data up to 60% was obtained when these values were replaced with respective quarterly averages. Using the estimated model, quarterwise landings were predicted for 1997 and

1998 and these are given in table.1.11. along with standard errors of forecasts. The maximum difference between forecasted and observed landings was about 14,800 tonnes in the first quarter of 1997. This was about 1.71 times the standard error of the forecast and was about 65.67% of the observed landings.

The model fitted for anchovies landings can be expanded to get the expression

$$\begin{aligned}\tilde{z}_t &= 0.188095\,\tilde{z}_{t-1} - 0.073838\,\tilde{z}_{t-2} + 0.251373\,\tilde{z}_{t-3} + 1.900232\,\tilde{z}_{t-4} - 0.262580\,\tilde{z}_{t-5} \\ &+ 0.103078\,\tilde{z}_{t-6} - 0.350916\,\tilde{z}_{t-7} - 1.099970\,\tilde{z}_{t-8} + 0.074485\,\tilde{z}_{t-9} - 0.029239\,\tilde{z}_{t-10} \\ &+ 0.099452\,\tilde{z}_{t-11} + 0.199675\,\tilde{z}_{t-12} + \varepsilon_t - 0.762684\,\varepsilon_{t-4}\end{aligned}$$

According to this expression the landings in a particular quarter depend on landings in earlier quarters even up to three years back. In the sample data on quarterwise landings of anchovies also there were few values that exceeded the 2σ limit of the series which could be outliers. These values were corresponding to the landings in third and fourth quarter of 1983, third quarter of 1984, third quarter of 1988, second quarter of 1989, fourth quarter of 1991, third quarter of 1992 and second quarter of 1993. It was found that when the effects of these points were nullified by replacing these values with respective quarterly averages the total variation in the sample series reduced by about 55%. Forecasts were made for different quarters of 1997 and 1998 using the estimated model and the maximum difference between the observed and forecasted values was about 8,400 tonnes. This difference comes to about 0.60 times the standard error of the forecast and it is about 107% of the observed landings. The forecasts and standard errors are given in table.1.11.

The estimated model for lesser sardine can be brought to the form,

$$(\tilde{z}_t - \tilde{z}_{t-4}) = (\tilde{z}_{t-1} - \tilde{z}_{t-5}) + \varepsilon_t - 0.750384\,\varepsilon_{t-1} - 0.790039\,\varepsilon_{t-4} + 0.592833\,\varepsilon_{t-5}$$

According to this model the difference in landings in a specified quarter of two successive years is obtained as a similar difference for the preceding quarter with terms for error correction involving residuals up to lag 5. In the sample series on quarterwise landings of lesser sardines used for estimating this model few values were above the 2σ limit. These values could be outliers and they correspond to the landings in third and fourth quarter of 1973, third quarter of 1974, second quarter of 1975, third quarter of 1976 and first quarter of 1991. The reduction in the variance of the series was about 70% when these values were replaced with respective quarterly averages. Predicted values of quarterwise landings for the years 1997 and 1998, computed using this model are given in table.1.11. along with standard errors of forecasts. The maximum difference between forecasted and observed values was 8,600 tonnes for the fourth quarter of 1998. This was about 0.98 times the standard error of the forecast and comes to about 53.63 % of the observed landings.

For the time series on quarterwise landings of penaeid prawns the estimated model takes the form

$$(\tilde{z}_t - \tilde{z}_{t-4}) = \varepsilon_t - 0.184908\,\varepsilon_{t-1} - 0.618036\,\varepsilon_{t-4} - 0.114280\,\varepsilon_{t-5}$$

The relation explained by this model is that the difference in landings between the same quarter in successive years can be obtained using residuals of past values up to lag 5. In the time series data used to estimate this model there were few observations exceeding the 2σ limits and these observations could be outliers. These values correspond to third quarter of 1973, third quarter of 1975, fourth quarter of 1988, second quarter of 1989, first quarter of 1990, fourth quarter of 1991, first quarter of 1992 and second quarter of 1994. The effects of these observations were examined by replacing them with respective quarterly averages and this caused a reduction of about 47% in the variance of the sample series. Forecasts of landings were made using the estimated model for different quarters in 1997 and 1998 and these are given in table.1.11. along with standard errors. The difference between observed and forecasted values was maximum for the first quarter of 1997 which is about 7,900 tonnes. This difference was 1.42 times the standard error of the forecast and it amounts to 46.69% of the observed landings.

The seasonal ARIMA model fitted to the time series on quarterwise landings of Tuna can be written as

$$(\tilde{z}_t - \tilde{z}_{t-4}) = (\tilde{z}_{t-1} - \tilde{z}_{t-5}) + \varepsilon_t - 0.825934\,\varepsilon_{t-1} - 0.744906\,\varepsilon_{t-4} + 0.615243\,\varepsilon_{t-5}$$

This model indicate that the difference between landings of the same quarter in two successive years is obtained as a similar difference for the previous quarter with terms for error correction using residuals up to lag 5. It was found that in the sample series used for estimating this model few values exceed the 2σ limits and these values could be outliers. These values were corresponding to the first quarter of 1976, first quarter of 1979, second quarter of 1980, third and fourth quarter of 1990, third quarter of 1994 and first quarter of 1996. When these suspected outliers were replaced with the respective quarterly averages, the variance of the sample series was reduced by 62%. Using the estimated model forecasts of Tuna landings were made for different quarters in 1997 and 1998. These forecasts and their standard errors are given in table.1.11. The difference between forecasts and observed values was maximum for the fourth quarter of 1998, which was about 1,400 tonnes. This difference was 0.33 times the standard error of the forecast and 15.55% of the observed landings.

For the time series on quarterwise landings of Thrissocles the model estimated can be brought to the form

$$\begin{aligned}\tilde{z}_t &= 1.209237\,\tilde{z}_{t-1} - 0.209237\,\tilde{z}_{t-2} + 0.001235\,\tilde{z}_{t-4} - 0.001493\,\tilde{z}_{t-5} + 0.00258\,\tilde{z}_{t-6} \\ &+ 0.998765\,\tilde{z}_{t-8} - 1.207744\,\tilde{z}_{t-9} + 0.208979\,\tilde{z}_{t-10} + \varepsilon_t - 0.912306\,\varepsilon_{t-1} \\ &+ 0.161557\,\varepsilon_{t-4} - 0.147389\,\varepsilon_{t-5} - 0.811703\,\varepsilon_{t-8} + 0.740522\,\varepsilon_{t-9}\end{aligned}$$

From this expression it can be seen that the values of this series at a time point depend on past values even up to lag 10. The dependence was more on its own past values and residuals at lags 1, 8 and 9. In the sample data used for estimation of this model some values were found to fall out side the 2σ limits which could be outliers. These values were for the third quarter of 1976, fourth quarter of 1988, third quarter of 1992, first quarter of 1993, first and second quarter of 1994, first and second quarter of 1995 and fourth quarter of 1996. When these values were replaced with respective quarterly averages the variance of the series reduced by 48%. Forecasts of quarterwise landings were made for the years 1997 and 1998 using the estimated model and these are given in table.1.11. along with standard errors of forecasts. The difference between observed and forecasted values was maximum for the first quarter of 1998 and this was about 1360 tonnes. The difference in forecast was 1.05 times the standard error of the forecast and was about 52% of the observed value.

The expression for estimated model for the time series on Ribbonfish landings is

$$(\tilde{z}_t - \tilde{z}_{t-4}) = 0.731783\,(\tilde{z}_{t-1} - \tilde{z}_{t-5}) + \varepsilon_t - 0.359239\,\varepsilon_{t-1} - 0.781891\,\varepsilon_{t-4}$$

This indicate that the difference in landings of a certain quarter in two successive years is proportional to a similar difference for the previous quarter and the model has error correction terms using residuals up to lag 4. Here also the sample data had few values exceeding the 2σ limits that could be outliers. These values were for second quarter of 1965, second quarter of 1966, first quarter of 1967, fourth quarter of 1971, third quarter of 1973, third quarter of 1974, third quarter of 1979, first quarter of 1994 and fourth quarter of 1996. When the effect of these suspected outliers were nullified by replacing them with respective quarterly averages, the total variation in the sample series reduced by about 47%. The estimated model was used to forecast landings in different quarters of 1997 and 1998 and these are given in table.1.11. along with standard errors of forecasts. The maximum difference found between observed and forecasted values was 8,485 tonnes for the third quarter of 1998. This difference was about 1.27 times the standard error of forecast and about 70% of the observed landings.

It is evident from these studies that in each of these univariate time series models the influence of outliers is of great concern as they badly affect the estimate of parameters and forecasts based on the estimated model. It is necessary to detect outliers present in the sample data before analysis and use appropriate techniques to deal with the problem of outliers while modelling such time series data. The outlier problem itself is a major topic in time series analysis and is beyond the scope of the present study.

Appendix-I (Tables and Charts)

Table.1.1. Autocorrelations and partial autocorrelations computed for the time series on Total Marine Fish landings in Kerala (1960-96) – (i) original series (ii) first order difference (iii) seasonal difference and (iv) both regular and seasonal difference.

Autocorrelation					Partial Autocorrelation				
Lag	(i)	(ii)	(iii)	(iv)	Lag	(i)	(ii)	(iii)	(iv)
1	0.384	-0.092	0.412	-0.221	1	0.384	-0.092	0.412	-0.221
2	-0.133	-0.728	0.091	-0.137	2	-0.329	-0.742	-0.095	-0.195
3	0.263	-0.016	-0.050	0.146	3	0.601	-0.452	-0.063	0.073
4	0.702	0.710	-0.343	-0.450	4	0.412	0.192	-0.353	-0.464
5	0.249	-0.056	-0.133	0.105	5	-0.232	-0.053	0.201	-0.087
6	-0.157	-0.623	-0.048	0.082	6	0.043	-0.151	-0.083	-0.120
7	0.215	-0.012	-0.061	-0.053	7	0.117	-0.232	-0.036	0.003
8	0.613	0.665	-0.022	0.049	8	0.165	0.112	-0.150	-0.231
9	0.187	-0.074	-0.038	-0.001	9	-0.152	-0.100	0.066	-0.034
10	-0.175	-0.576	-0.056	0.075	10	0.050	-0.080	-0.087	0.055
11	0.181	0.033	-0.161	-0.091	11	0.033	-0.075	-0.203	-0.088
12	0.517	0.593	-0.149	-0.060	12	0.007	0.017	-0.076	-0.185
13	0.118	-0.095	-0.071	0.018	13	-0.070	-0.080	0.021	-0.096
14	-0.185	-0.569	-0.018	-0.117	14	0.044	-0.219	-0.018	-0.148
15	0.218	0.105	0.156	0.165	15	0.176	-0.044	0.054	-0.001
16	0.506	0.564	0.127	-0.035	16	-0.022	-0.045	-0.073	-0.234
17	0.093	-0.113	0.139	0.070	17	-0.010	-0.005	0.150	0.080
18	-0.196	-0.492	0.089	0.160	18	-0.034	0.081	-0.090	0.087
19	0.147	0.079	-0.133	-0.215	19	-0.097	-0.025	-0.152	-0.054
20	0.403	0.534	-0.089	0.065	20	-0.007	0.145	0.001	-0.056
21	0.001	-0.167	-0.119	-0.064	21	-0.128	-0.106	-0.032	-0.012
22	-0.223	-0.511	-0.100	-0.138	22	0.086	-0.135	-0.044	-0.044
23	0.188	0.167	0.069	0.203	23	0.115	-0.041	-0.018	0.019
24	0.397	0.501	0.002	-0.097	24	-0.032	-0.063	-0.069	-0.145
25	-0.018	-0.166	0.048	0.019	25	0.005	0.031	0.082	-0.013
26	-0.238	-0.465	0.086	0.016	26	-0.057	-0.015	0.014	-0.166
27	0.131	0.135	0.113	0.012	27	-0.045	-0.062	0.176	0.105
28	0.343	0.500	0.103	0.139	28	0.044	0.037	-0.064	0.067
29	-0.076	-0.183	-0.052	-0.142	29	-0.112	-0.016	-0.070	0.011
30	-0.280	-0.413	-0.045	0.059	30	-0.022	0.175	-0.009	0.032
31	0.036	0.118	-0.115	-0.084	31	-0.242	-0.035	-0.066	0.007
32	0.232	0.418	-0.083	-0.105	32	0.020	-0.035	-0.028	-0.036
33	-0.111	-0.137	0.069	0.175	33	-0.001	-0.028	0.021	-0.028
34	-0.291	-0.375	0.018	-0.039	34	0.009	-0.029	0.026	0.014
35	0.010	0.127	0.022	0.039	35	0.028	0.001	-0.017	0.041
36	0.175	0.396	-0.012	0.059	36	-0.013	0.028	-0.036	0.022

Table.1.2. Autocorrelations and partial autocorrelations computed for the time series on Total Marine Fish landings in Kerala (1960-87) – (i) original series (ii) first order difference (iii) seasonal difference and (iv) both regular and seasonal difference.

Autocorrelation					Partial Autocorrelation				
Lag	(i)	(ii)	(iii)	(iv)	Lag	(i)	(ii)	(iii)	(iv)
1	0.192	-0.079	0.379	-0.170	1	0.192	-0.079	0.379	-0.170
2	-0.488	-0.737	0.003	-0.177	2	-0.544	-0.748	-0.164	-0.212
3	0.029	0.017	-0.126	0.140	3	0.409	-0.330	-0.079	0.073
4	0.546	0.637	-0.414	-0.452	4	0.227	0.082	-0.398	-0.485
5	0.021	-0.045	-0.159	0.100	5	-0.184	-0.047	0.191	-0.022
6	-0.447	-0.593	-0.036	0.076	6	-0.050	-0.222	-0.145	-0.180
7	0.028	0.003	-0.023	-0.041	7	0.103	-0.263	0.005	0.047
8	0.515	0.621	0.027	0.085	8	0.196	0.095	-0.190	-0.226
9	-0.005	-0.068	-0.030	-0.070	9	-0.177	-0.125	0.044	-0.037
10	-0.425	-0.538	-0.002	0.130	10	0.049	-0.056	-0.053	0.073
11	0.017	0.029	-0.131	-0.081	11	-0.026	-0.156	-0.227	-0.083
12	0.435	0.540	-0.152	-0.103	12	0.085	0.011	-0.096	-0.124
13	-0.024	-0.055	-0.060	0.049	13	-0.084	-0.056	-0.074	-0.127
14	-0.402	-0.552	-0.003	-0.124	14	-0.007	-0.239	0.036	-0.081
15	0.104	0.082	0.175	0.135	15	0.199	-0.079	-0.001	-0.003
16	0.494	0.530	0.167	0.004	16	0.045	-0.113	-0.040	-0.174
17	0.026	-0.045	0.179	0.091	17	0.100	0.076	0.157	0.165
18	-0.381	-0.488	0.082	0.118	18	-0.097	0.000	-0.089	0.039
19	0.010	0.038	-0.140	-0.202	19	-0.010	0.025	-0.076	0.008
20	0.370	0.519	-0.085	0.069	20	-0.036	0.195	-0.004	-0.016
21	-0.116	-0.136	-0.135	-0.094	21	-0.225	-0.130	-0.043	-0.002
22	-0.407	-0.494	-0.082	-0.084	22	0.107	-0.008	0.003	0.033
23	0.098	0.146	0.057	0.200	23	-0.006	-0.036	-0.082	0.044
24	0.379	0.469	-0.035	-0.121	24	0.014	0.038	-0.064	-0.069
25	-0.100	-0.120	0.029	0.015	25	-0.033	0.057	0.020	-0.035
26	-0.394	-0.456	0.056	0.010	26	-0.047	-0.006	0.043	-0.096
27	0.058	0.089	0.079	0.024	27	0.019	-0.036	0.118	0.169
28	0.374	0.506	0.051	0.111	28	0.032	0.094	-0.109	0.001
29	-0.132	-0.159	-0.096	-0.141	29	-0.093	-0.001	-0.031	0.010
30	-0.395	-0.421	-0.071	0.068	30	-0.015	0.155	-0.043	0.034
31	0.024	0.099	-0.128	-0.103	31	-0.180	-0.019	-0.107	-0.036
32	0.311	0.416	-0.065	-0.122	32	0.008	0.004	-0.058	-0.106
33	-0.093	-0.118	0.137	0.204	33	-0.036	-0.083	0.057	-0.051
34	-0.320	-0.381	0.089	0.004	34	0.087	-0.026	0.016	0.096
35	0.078	0.135	0.064	0.018	35	0.028	0.025	-0.071	-0.029
36	0.284	0.381	0.025	0.066	36	-0.010	-0.023	0.012	0.031

Table.1.3. Autocorrelations and partial autocorrelations computed for the time series on Oil sardine landings in Kerala (1960-96) – (i) original series (ii) first order difference (iii) seasonal difference and (iv) both regular and seasonal difference.

Autocorrelation					Partial Autocorrelation				
Lag	(i)	(ii)	(iii)	(iv)	Lag	(i)	(ii)	(iii)	(iv)
1	0.310	-0.156	0.379	-0.227	1	0.310	-0.156	0.379	-0.227
2	-0.168	-0.569	0.058	-0.132	2	-0.292	-0.608	-0.100	-0.193
3	0.139	-0.072	-0.082	0.147	3	0.359	-0.502	-0.080	0.074
4	0.567	0.619	-0.360	-0.434	4	0.434	0.181	-0.347	-0.443
5	0.131	-0.076	-0.146	0.027	5	-0.245	-0.072	0.151	-0.191
6	-0.210	-0.455	0.024	0.156	6	0.005	-0.133	0.033	-0.060
7	0.072	-0.082	0.003	-0.034	7	0.065	-0.270	-0.065	0.022
8	0.477	0.580	0.035	0.012	8	0.205	0.107	-0.095	-0.202
9	0.080	-0.028	0.042	0.115	9	-0.162	0.030	0.074	0.020
10	-0.284	-0.460	-0.096	-0.026	10	-0.091	-0.081	-0.114	0.067
11	-0.017	-0.085	-0.201	-0.123	11	0.021	-0.136	-0.200	-0.076
12	0.375	0.555	-0.145	-0.010	12	0.062	0.073	-0.039	-0.178
13	0.004	-0.058	-0.088	-0.019	13	-0.137	-0.094	0.027	-0.032
14	-0.291	-0.458	-0.010	-0.117	14	0.040	-0.185	-0.058	-0.161
15	0.042	-0.021	0.186	0.203	15	0.146	-0.114	0.085	0.019
16	0.406	0.561	0.084	0.044	16	0.068	0.080	-0.099	-0.051
17	0.001	-0.085	-0.045	-0.070	17	-0.114	-0.058	-0.076	-0.006
18	-0.291	-0.370	-0.056	0.191	18	0.022	0.126	-0.065	0.116
19	-0.070	-0.071	-0.283	-0.240	19	-0.163	-0.022	-0.242	-0.131
20	0.269	0.456	-0.182	-0.045	20	-0.001	-0.017	-0.017	-0.036
21	-0.028	-0.074	-0.030	0.019	21	-0.012	-0.128	-0.082	-0.137
22	-0.236	-0.368	0.067	-0.045	22	0.105	-0.161	0.056	0.015
23	0.068	0.023	0.197	0.218	23	0.167	0.008	-0.049	0.105
24	0.349	0.428	0.067	-0.128	24	-0.008	-0.067	-0.174	-0.216
25	0.037	-0.049	0.082	0.080	25	0.083	0.052	0.145	0.016
26	-0.212	-0.358	0.025	0.006	26	-0.035	0.015	-0.015	0.021
27	0.035	-0.051	-0.023	-0.096	27	-0.015	-0.125	-0.024	0.078
28	0.360	0.484	0.005	0.057	28	0.133	0.145	-0.109	-0.101
29	0.019	-0.099	-0.001	-0.040	29	-0.145	-0.046	0.052	0.032
30	-0.193	-0.342	0.030	-0.010	30	0.061	0.062	-0.054	0.063
31	0.059	-0.008	0.050	0.042	31	-0.061	-0.018	-0.075	-0.004
32	0.342	0.436	0.031	0.021	32	0.040	0.013	-0.021	-0.042
33	0.013	-0.088	0.003	0.011	33	-0.008	-0.009	0.018	0.004
34	-0.206	-0.302	-0.046	-0.021	34	0.016	0.005	0.001	0.049
35	-0.003	-0.034	-0.041	0.015	35	0.005	0.015	-0.040	0.077
36	0.261	0.372	-0.048	-0.023	36	-0.028	-0.040	-0.087	-0.038

Table.1.4. Autocorrelations and partial autocorrelations computed for the time series on Mackerel landings in Kerala (1960-96 log transformed) – (i) original series (ii) first order difference (iii) seasonal difference and (iv) both regular and seasonal difference.

Autocorrelation					Partial Autocorrelation				
Lag	(i)	(ii)	(iii)	(iv)	Lag	(i)	(ii)	(iii)	(iv)
1	0.443	-0.215	0.286	-0.327	1	0.443	-0.215	0.286	-0.327
2	0.125	-0.473	0.032	-0.186	2	-0.088	-0.544	-0.055	-0.328
3	0.327	0.032	0.052	0.232	3	0.383	-0.361	0.063	0.055
4	0.499	0.350	-0.255	-0.366	4	0.286	-0.043	-0.315	-0.388
5	0.277	-0.023	-0.040	0.065	5	-0.028	-0.003	0.162	-0.191
6	0.085	-0.252	0.083	0.250	6	-0.058	-0.054	0.028	0.019
7	0.175	-0.034	-0.150	-0.134	7	-0.002	-0.120	-0.170	0.033
8	0.309	0.246	-0.199	-0.157	8	0.080	0.014	-0.229	-0.323
9	0.163	0.046	-0.023	0.118	9	-0.055	0.072	0.150	-0.173
10	-0.039	-0.331	-0.016	-0.035	10	-0.124	-0.197	0.040	-0.064
11	0.133	0.057	0.036	0.071	11	0.151	-0.057	-0.049	0.080
12	0.244	0.306	-0.008	0.071	12	0.011	0.093	-0.208	-0.116
13	0.015	-0.076	-0.149	-0.108	13	-0.135	0.033	-0.023	-0.103
14	-0.135	-0.310	-0.139	-0.075	14	-0.079	-0.106	-0.034	-0.121
15	0.057	0.048	-0.016	0.062	15	0.055	-0.140	0.008	-0.010
16	0.206	0.252	0.017	0.064	16	0.105	-0.103	-0.104	-0.022
17	0.076	0.030	-0.048	-0.033	17	0.071	0.016	-0.102	-0.084
18	-0.093	-0.241	-0.062	0.069	18	-0.041	-0.004	-0.049	0.019
19	0.008	-0.067	-0.172	-0.096	19	-0.018	-0.076	-0.152	-0.006
20	0.183	0.208	-0.149	-0.149	20	0.047	-0.108	-0.149	-0.217
21	0.124	0.080	0.091	0.190	21	0.082	-0.033	0.076	-0.078
22	-0.025	-0.271	0.059	-0.068	22	0.006	-0.171	-0.045	-0.170
23	0.136	0.015	0.117	0.040	23	0.158	-0.087	0.072	-0.010
24	0.270	0.294	0.123	0.102	24	0.057	0.090	-0.080	-0.061
25	0.075	-0.099	-0.017	-0.116	25	-0.119	-0.031	-0.025	-0.002
26	-0.004	-0.156	0.012	-0.018	26	0.015	0.104	-0.084	-0.094
27	0.092	-0.023	0.065	0.059	27	-0.119	-0.071	0.009	-0.072
28	0.214	0.216	0.036	-0.004	28	0.056	-0.015	-0.005	-0.053
29	0.086	-0.038	0.011	-0.043	29	-0.012	-0.114	-0.011	-0.067
30	0.009	-0.055	0.047	0.105	30	0.100	0.153	0.010	0.039
31	-0.007	-0.113	-0.062	-0.113	31	-0.172	0.028	-0.098	-0.035
32	0.105	0.116	-0.016	-0.065	32	-0.041	-0.046	-0.029	-0.142
33	0.082	0.095	0.124	0.176	33	0.039	-0.007	0.096	0.019
34	-0.035	-0.109	0.006	-0.076	34	-0.001	0.012	-0.065	-0.023
35	-0.037	-0.034	-0.004	-0.003	35	-0.026	0.046	-0.021	-0.013
36	0.005	0.092	-0.005	0.099	36	-0.038	0.065	-0.041	0.027

Table.1.5. Autocorrelations and partial autocorrelations computed for the time series on Anchovies landings in Kerala (1960-96 log transformed) – (i) original series (ii) first order difference (iii) seasonal difference and (iv) both regular and seasonal difference.

Autocorrelation					Partial Autocorrelation				
Lag	(i)	(ii)	(iii)	(iv)	Lag	(i)	(ii)	(iii)	(iv)
1	0.334	-0.331	0.061	-0.468	1	0.334	-0.331	0.061	-0.468
2	0.122	-0.326	0.015	-0.058	2	0.012	-0.490	0.011	-0.355
3	0.350	-0.048	0.096	0.280	3	0.345	-0.549	0.094	0.112
4	0.624	0.474	-0.370	-0.428	4	0.520	0.016	-0.386	-0.335
5	0.275	-0.139	-0.024	0.195	5	0.009	0.028	0.039	-0.166
6	0.106	-0.271	-0.035	-0.029	6	-0.051	-0.064	-0.047	-0.238
7	0.284	-0.043	-0.018	0.014	7	-0.010	-0.247	0.084	0.051
8	0.533	0.425	-0.021	0.085	8	0.217	0.027	-0.208	-0.029
9	0.215	-0.161	-0.172	-0.161	9	-0.047	-0.055	-0.163	-0.142
10	0.092	-0.200	-0.038	0.063	10	-0.003	-0.026	-0.058	-0.204
11	0.250	-0.025	-0.020	0.042	11	-0.019	-0.126	0.049	0.005
12	0.446	0.280	-0.087	-0.196	12	0.089	-0.158	-0.163	-0.259
13	0.250	-0.041	0.213	0.247	13	0.111	-0.051	0.140	-0.033
14	0.114	-0.173	0.058	-0.084	14	0.027	-0.006	-0.042	-0.157
15	0.219	-0.053	0.060	0.076	15	-0.025	-0.073	0.121	0.237
16	0.404	0.329	-0.061	0.031	16	0.043	0.144	-0.305	0.005
17	0.133	-0.173	-0.230	-0.239	17	-0.175	-0.066	-0.101	-0.109
18	0.088	-0.060	0.033	0.243	18	0.018	0.076	0.009	-0.013
19	0.150	-0.157	-0.146	-0.206	19	-0.103	-0.246	-0.046	-0.003
20	0.405	0.333	0.047	0.020	20	0.226	-0.089	-0.085	-0.110
21	0.183	-0.062	0.164	0.177	21	0.047	-0.044	0.019	-0.100
22	0.061	-0.168	-0.041	-0.193	22	0.001	-0.106	0.025	-0.128
23	0.171	-0.053	0.147	0.188	23	0.012	-0.062	0.149	0.171
24	0.357	0.314	-0.037	-0.065	24	-0.010	0.039	-0.189	0.022
25	0.128	-0.100	-0.095	-0.077	25	-0.054	0.074	-0.065	0.042
26	0.038	-0.114	0.006	0.112	26	-0.067	0.127	-0.129	-0.049
27	0.076	-0.185	-0.123	-0.170	27	-0.164	-0.212	0.005	-0.002
28	0.383	0.397	0.069	0.096	28	0.207	-0.051	-0.083	-0.135
29	0.158	-0.046	0.103	0.089	29	0.045	-0.051	0.110	-0.029
30	-0.007	-0.206	-0.040	-0.105	30	0.053	-0.009	-0.017	0.021
31	0.086	-0.111	0.013	0.056	31	-0.028	-0.091	-0.009	0.003
32	0.356	0.380	-0.031	-0.031	32	0.066	0.083	-0.039	0.003
33	0.095	-0.114	-0.032	-0.027	33	-0.148	-0.004	0.004	0.055
34	-0.015	-0.157	0.034	0.016	34	-0.058	-0.008	-0.078	-0.009
35	0.088	-0.097	0.050	-0.031	35	-0.018	-0.003	0.035	0.009
36	0.333	0.388	0.123	0.115	36	0.022	0.026	-0.024	-0.048

Table.1.6. Autocorrelations and partial autocorrelations computed for the time series on Lesser sardine landings in Kerala (1960-96 log transformed) – (i) original series (ii) first order difference (iii) seasonal difference and (iv) both regular and seasonal difference.

Autocorrelation					Partial Autocorrelation				
Lag	(i)	(ii)	(iii)	(iv)	Lag	(i)	(ii)	(iii)	(iv)
1	0.190	-0.425	0.156	-0.431	1	0.190	-0.425	0.156	-0.431
2	0.075	-0.145	0.060	-0.060	2	0.040	-0.397	0.037	-0.302
3	0.187	-0.115	0.071	0.293	3	0.172	-0.535	0.058	0.175
4	0.488	0.450	-0.424	-0.506	4	0.452	0.055	-0.458	-0.410
5	0.057	-0.240	-0.054	0.144	5	-0.114	-0.032	0.105	-0.287
6	0.019	-0.077	0.078	0.157	6	-0.037	-0.041	0.139	-0.064
7	0.115	-0.125	-0.057	-0.163	7	-0.012	-0.279	-0.021	0.051
8	0.417	0.453	0.093	0.127	8	0.263	0.118	-0.149	-0.131
9	-0.023	-0.213	0.019	0.034	9	-0.122	0.144	0.025	-0.061
10	-0.107	-0.191	-0.114	-0.140	10	-0.161	-0.093	-0.006	-0.032
11	0.115	0.045	-0.013	0.125	11	0.068	-0.045	-0.025	0.095
12	0.253	0.312	-0.137	-0.067	12	0.010	0.032	-0.193	-0.058
13	-0.117	-0.225	-0.120	-0.089	13	-0.079	-0.019	-0.049	-0.135
14	-0.114	-0.140	0.003	0.064	14	-0.036	-0.125	0.015	-0.161
15	0.112	0.064	0.055	0.040	15	0.069	-0.142	0.128	0.098
16	0.226	0.275	0.041	-0.036	16	0.086	-0.028	-0.123	-0.002
17	-0.103	-0.188	0.089	0.101	17	-0.018	-0.010	-0.006	-0.003
18	-0.120	-0.118	-0.005	-0.103	18	-0.033	0.048	-0.012	-0.176
19	0.048	0.037	0.032	0.003	19	-0.099	-0.070	0.193	0.064
20	0.152	0.269	0.034	0.087	20	0.034	0.034	-0.063	0.148
21	-0.183	-0.222	-0.088	-0.065	21	-0.071	0.008	-0.160	0.123
22	-0.148	-0.042	-0.080	0.009	22	-0.050	0.116	-0.153	-0.141
23	-0.052	-0.005	-0.097	0.038	23	-0.173	-0.049	0.085	0.025
24	0.050	0.201	-0.139	-0.176	24	-0.017	-0.136	-0.101	-0.101
25	-0.165	-0.175	0.071	0.124	25	0.087	-0.118	0.033	0.073
26	-0.091	0.021	0.070	0.013	26	0.073	0.032	-0.088	-0.138
27	-0.057	-0.067	0.040	-0.021	27	-0.071	-0.101	0.118	0.059
28	0.082	0.277	0.053	0.119	28	0.061	0.054	-0.085	0.029
29	-0.220	-0.261	-0.116	-0.108	29	-0.080	-0.054	-0.056	0.130
30	-0.098	0.069	-0.092	-0.005	30	0.026	0.095	-0.135	-0.066
31	-0.092	-0.099	-0.092	-0.044	31	-0.113	-0.071	0.003	-0.124
32	0.069	0.244	-0.008	0.037	32	0.051	-0.024	0.050	0.021
33	-0.162	-0.221	0.031	-0.058	33	-0.006	-0.093	-0.050	0.026
34	-0.028	0.089	0.127	0.052	34	0.068	-0.030	-0.024	-0.110
35	-0.034	-0.068	0.172	0.083	35	0.017	0.044	0.146	-0.007
36	0.059	0.182	0.054	-0.036	36	-0.065	-0.026	0.010	0.076

Table.1.7. Autocorrelations and partial autocorrelations computed for the time series on Penaeid prawn landings in Kerala (1960-96 log transformed) – (i) original series (ii) first order difference (iii) seasonal difference and (iv) both regular and seasonal difference.

Autocorrelation					Partial Autocorrelation				
Lag	(i)	(ii)	(iii)	(iv)	Lag	(i)	(ii)	(iii)	(iv)
1	0.207	-0.342	0.097	-0.421	1	0.207	-0.342	0.097	-0.421
2	-0.038	-0.281	-0.046	-0.108	2	-0.084	-0.450	-0.056	-0.347
3	0.183	-0.146	0.015	0.241	3	0.219	-0.624	0.025	0.046
4	0.546	0.516	-0.368	-0.399	4	0.499	-0.054	-0.379	-0.390
5	0.146	-0.137	-0.011	0.119	5	-0.020	-0.094	0.086	-0.275
6	0.004	-0.140	0.124	0.190	6	0.035	0.042	0.079	-0.059
7	0.074	-0.218	-0.084	-0.159	7	-0.098	-0.231	-0.091	-0.030
8	0.461	0.463	-0.015	-0.002	8	0.264	0.104	-0.148	-0.244
9	0.105	-0.107	0.053	0.071	9	-0.092	0.041	0.098	-0.172
10	-0.056	-0.176	-0.002	-0.084	10	-0.038	-0.063	0.077	-0.089
11	0.087	-0.112	0.090	0.110	11	0.043	-0.055	0.020	0.043
12	0.394	0.412	-0.013	-0.047	12	0.104	0.150	-0.124	-0.148
13	0.043	-0.143	-0.031	-0.002	13	-0.070	0.093	0.074	-0.037
14	-0.079	-0.158	-0.047	0.023	14	-0.053	0.023	-0.022	0.028
15	0.038	-0.131	-0.108	-0.029	15	-0.019	-0.126	-0.099	0.082
16	0.374	0.441	-0.097	-0.042	16	0.139	0.093	-0.171	-0.128
17	0.015	-0.161	-0.014	0.031	17	-0.087	-0.038	0.048	-0.073
18	-0.086	-0.182	0.008	-0.047	18	0.027	-0.109	0.013	-0.095
19	0.118	-0.077	0.121	0.039	19	0.128	-0.115	0.062	-0.045
20	0.390	0.430	0.152	0.122	20	0.117	0.061	0.034	0.033
21	-0.003	-0.177	-0.047	-0.157	21	-0.062	0.007	-0.043	-0.100
22	-0.114	-0.153	0.030	0.103	22	-0.080	0.013	0.072	0.029
23	0.031	-0.070	-0.062	-0.053	23	-0.075	0.050	-0.036	-0.022
24	0.238	0.308	-0.055	-0.098	24	-0.131	-0.087	0.020	-0.098
25	0.006	-0.034	0.132	0.204	25	0.049	0.083	0.104	0.045
26	-0.156	-0.205	-0.046	-0.143	26	-0.088	-0.029	-0.044	-0.057
27	-0.007	-0.057	0.031	0.044	27	0.027	0.048	0.072	0.059
28	0.211	0.324	0.019	0.021	28	-0.030	0.083	-0.042	-0.066
29	-0.075	-0.111	-0.019	-0.043	29	-0.067	-0.002	0.085	0.082
30	-0.188	-0.154	0.032	0.070	30	-0.039	0.015	-0.059	0.111
31	-0.030	-0.029	-0.063	-0.044	31	-0.052	-0.042	-0.104	0.025
32	0.148	0.309	-0.074	0.002	32	0.006	0.087	-0.040	0.051
33	-0.148	-0.146	-0.097	-0.015	33	-0.110	-0.026	-0.080	0.032
34	-0.232	-0.176	-0.100	-0.057	34	-0.022	-0.113	-0.084	-0.035
35	-0.015	0.031	0.019	0.032	35	0.067	-0.017	-0.007	-0.104
36	0.130	0.276	0.070	0.051	36	-0.013	-0.083	0.062	-0.030

Table.1.8. Autocorrelations and partial autocorrelations computed for the time series on Tuna landings in Kerala (1960-96 log transformed) – (i) original series (ii) first order difference (iii) seasonal difference and (iv) both regular and seasonal difference.

Autocorrelation					Partial Autocorrelation				
Lag	(i)	(ii)	(iii)	(iv)	Lag	(i)	(ii)	(iii)	(iv)
1	0.481	-0.550	0.056	-0.482	1	0.481	-0.550	0.056	-0.482
2	0.532	0.099	0.021	-0.041	2	0.391	-0.292	0.018	-0.356
3	0.476	-0.305	0.060	0.288	3	0.203	-0.621	0.058	0.134
4	0.737	0.543	-0.440	-0.438	4	0.579	0.005	-0.450	-0.319
5	0.440	-0.357	-0.110	0.085	5	-0.045	-0.087	-0.067	-0.365
6	0.511	0.137	0.061	0.075	6	0.054	-0.008	0.110	-0.340
7	0.440	-0.327	0.090	0.061	7	-0.012	-0.340	0.193	0.126
8	0.713	0.503	0.004	-0.111	8	0.334	-0.083	-0.265	-0.186
9	0.465	-0.235	0.119	0.158	9	0.077	0.118	0.027	-0.112
10	0.452	0.090	-0.064	-0.071	10	-0.136	0.218	-0.016	-0.180
11	0.352	-0.398	-0.111	-0.105	11	-0.219	-0.194	0.076	-0.003
12	0.664	0.559	0.039	0.109	12	0.195	0.033	-0.091	-0.146
13	0.398	-0.293	-0.009	-0.066	13	-0.037	-0.031	0.059	-0.107
14	0.435	0.144	0.061	-0.025	14	0.031	0.118	0.037	-0.252
15	0.319	-0.375	0.179	0.163	15	-0.137	-0.048	0.211	0.103
16	0.595	0.500	-0.011	-0.114	16	0.029	0.024	-0.137	-0.116
17	0.349	-0.243	0.011	0.021	17	-0.064	0.057	0.077	-0.017
18	0.355	0.159	-0.003	0.084	18	-0.089	0.192	-0.014	-0.045
19	0.205	-0.450	-0.174	-0.144	19	-0.183	-0.068	0.017	0.148
20	0.513	0.518	-0.075	0.025	20	0.062	0.042	-0.189	-0.093
21	0.287	-0.232	-0.020	0.039	21	-0.057	-0.011	0.046	0.014
22	0.301	0.135	-0.039	-0.042	22	-0.009	-0.010	-0.059	-0.068
23	0.175	-0.403	0.016	-0.019	23	-0.019	-0.111	0.021	0.030
24	0.462	0.501	0.108	0.150	24	0.083	-0.024	-0.078	0.009
25	0.225	-0.241	-0.083	-0.102	25	-0.040	-0.039	-0.059	0.070
26	0.243	0.148	-0.084	-0.062	26	-0.024	-0.048	-0.136	-0.147
27	0.105	-0.369	0.032	0.076	27	-0.015	-0.022	0.083	-0.143
28	0.352	0.382	0.006	-0.050	28	-0.034	-0.122	0.094	-0.064
29	0.199	-0.155	0.071	-0.010	29	0.069	-0.069	0.029	-0.014
30	0.208	0.185	0.161	0.156	30	0.017	0.035	-0.020	0.053
31	0.029	-0.386	-0.045	-0.067	31	-0.078	0.156	-0.087	-0.014
32	0.254	0.320	-0.127	-0.120	32	-0.163	-0.051	-0.033	-0.167
33	0.142	-0.089	0.019	0.155	33	0.070	0.031	0.134	-0.018
34	0.121	0.120	-0.127	-0.161	34	-0.010	-0.062	-0.006	-0.024
35	-0.020	-0.358	0.030	0.073	35	0.070	0.049	0.004	0.092
36	0.211	0.342	0.052	0.077	36	-0.050	0.043	-0.123	-0.019

Table.1.9. Autocorrelations and partial autocorrelations computed for the time series on Thrissocles landings in Kerala (1960-96 log transformed) – (i) original series (ii) first order difference (iii) seasonal difference and (iv) both regular and seasonal difference.

Autocorrelation					Partial Autocorrelation				
Lag	(i)	(ii)	(iii)	(iv)	Lag	(i)	(ii)	(iii)	(iv)
1	0.351	-0.275	0.163	-0.428	1	0.351	-0.275	0.163	-0.428
2	0.054	-0.394	0.046	-0.037	2	-0.079	-0.508	0.020	-0.270
3	0.283	0.007	0.017	0.245	3	0.333	-0.431	0.007	0.144
4	0.507	0.354	-0.443	-0.533	4	0.370	-0.054	-0.460	-0.479
5	0.254	-0.028	-0.004	0.292	5	0.012	0.022	0.180	-0.139
6	0.033	-0.302	-0.035	0.016	6	-0.070	-0.134	-0.049	-0.076
7	0.208	-0.055	-0.106	-0.104	7	0.072	-0.280	-0.074	0.041
8	0.441	0.386	0.007	0.129	8	0.206	0.017	-0.231	-0.221
9	0.159	-0.067	-0.087	-0.104	9	-0.113	-0.043	0.056	-0.080
10	-0.009	-0.269	-0.025	-0.006	10	-0.026	-0.101	-0.039	-0.086
11	0.171	0.015	0.028	0.169	11	0.025	-0.113	-0.031	0.207
12	0.334	0.290	-0.179	-0.190	12	0.062	0.004	-0.366	-0.191
13	0.122	-0.047	-0.067	-0.004	13	-0.030	-0.008	0.015	-0.198
14	-0.028	-0.253	0.052	0.049	14	-0.022	-0.077	0.086	-0.175
15	0.154	-0.081	0.084	-0.126	15	0.051	-0.291	0.135	0.006
16	0.428	0.434	0.300	0.216	16	0.259	0.030	-0.021	-0.035
17	0.149	-0.035	0.163	0.033	17	-0.044	0.086	0.064	0.072
18	-0.096	-0.299	-0.024	-0.049	18	-0.131	0.070	-0.041	0.086
19	0.054	-0.036	-0.123	-0.024	19	-0.113	-0.017	-0.103	0.001
20	0.247	0.334	-0.177	-0.133	20	-0.047	0.074	-0.069	-0.142
21	0.022	-0.067	-0.034	0.088	21	-0.115	-0.031	0.105	0.057
22	-0.138	-0.273	-0.023	-0.004	22	-0.027	-0.111	-0.058	0.012
23	0.076	0.035	0.016	0.005	23	0.100	-0.029	-0.025	-0.016
24	0.235	0.344	0.048	0.137	24	0.029	0.126	-0.011	0.100
25	-0.052	-0.129	-0.137	-0.182	25	-0.103	0.014	-0.131	-0.012
26	-0.146	-0.251	-0.038	0.028	26	0.040	-0.031	-0.049	-0.053
27	0.066	0.079	0.006	0.083	27	0.041	0.005	0.007	-0.021
28	0.182	0.219	-0.096	-0.141	28	0.011	-0.097	-0.026	-0.011
29	0.012	-0.025	0.064	0.113	29	0.096	0.022	-0.001	0.001
30	-0.120	-0.251	0.022	-0.039	30	-0.022	-0.006	-0.046	0.016
31	0.072	0.005	0.035	-0.065	31	0.006	-0.028	-0 039	-0.034
32	0.234	0.298	0.161	0.123	32	0.024	-0.010	0.001	-0.032
33	0.013	-0.050	0.071	-0.013	33	-0.025	-0.015	0.050	0.053
34	-0.128	-0.227	0.019	-0.036	34	-0.026	-0.031	-0.051	-0.079
35	0.018	0.030	0.016	0.084	35	-0.007	0.013	0.095	0.046
36	0.118	0.226	-0.124	-0.161	36	-0.048	0.035	-0.050	-0.022

Table.1.10. Autocorrelations and partial autocorrelations computed for the time series on Ribbonfish landings in Kerala (1960-96 log transformed) – (i) original series (ii) first order difference (iii) seasonal difference and (iv) both regular and seasonal difference.

Autocorrelation					Partial Autocorrelation				
Lag	(i)	(ii)	(iii)	(iv)	Lag	(i)	(ii)	(iii)	(iv)
1	0.348	-0.160	0.297	-0.364	1	0.348	-0.160	0.297	-0.364
2	-0.094	-0.583	0.119	0.014	2	-0.245	-0.624	0.034	-0.136
3	0.222	-0.048	-0.074	0.048	3	0.422	-0.502	-0.130	0.004
4	0.597	0.578	-0.333	-0.452	4	0.427	0.070	-0.314	-0.503
5	0.214	0.028	0.031	0.265	5	-0.135	0.107	0.272	-0.143
6	-0.184	-0.571	0.005	-0.083	6	-0.157	-0.242	-0.014	-0.148
7	0.159	0.021	0.105	0.198	7	0.192	-0.085	0.030	0.159
8	0.460	0.516	-0.058	-0.067	8	0.050	0.103	-0.275	-0.224
9	0.078	-0.001	-0.129	-0.063	9	-0.158	0.053	0.073	-0.070
10	-0.293	-0.500	-0.107	0.106	10	-0.141	-0.051	-0.061	0.036
11	-0.005	-0.063	-0.238	-0.136	11	-0.041	-0.187	-0.165	0.127
12	0.351	0.530	-0.196	-0.021	12	0.110	-0.002	-0.310	-0.291
13	0.022	0.020	-0.127	0.002	13	-0.051	-0.006	0.067	-0.161
14	-0.323	-0.525	-0.057	-0.072	14	-0.053	-0.203	-0.011	-0.198
15	0.016	-0.045	0.107	0.120	15	0.148	-0.219	0.075	0.058
16	0.390	0.548	0.111	0.092	16	0.170	0.013	-0.151	-0.071
17	0.052	0.041	0.009	0.001	17	-0.053	0.015	-0.035	0.009
18	-0.327	-0.477	-0.082	-0.028	18	-0.081	0.079	-0.104	-0.092
19	-0.075	-0.122	-0.134	-0.122	19	-0.133	-0.096	-0.001	0.026
20	0.322	0.490	-0.035	0.014	20	0.053	-0.092	-0.111	-0.045
21	0.088	0.076	0.039	-0.006	21	0.080	-0.080	-0.035	-0.001
22	-0.225	-0.408	0.126	0.065	22	0.079	0.020	-0.044	-0.090
23	-0.014	-0.101	0.113	0.077	23	-0.038	-0.028	0.014	0.041
24	0.320	0.404	0.031	-0.085	24	0.030	-0.078	-0.078	-0.083
25	0.118	0.104	0.058	0.036	25	0.076	-0.064	0.053	0.026
26	-0.202	-0.365	0.000	-0.029	26	0.050	0.074	-0.046	-0.042
27	-0.036	-0.148	-0.009	-0.045	27	-0.082	-0.029	-0.011	0.013
28	0.317	0.405	0.030	0.022	28	0.026	0.011	-0.021	-0.088
29	0.133	0.105	0.038	-0.047	29	-0.021	-0.021	0.072	-0.017
30	-0.178	-0.348	0.116	0.096	30	0.026	0.025	0.037	0.064
31	-0.041	-0.143	0.069	-0.015	31	-0.050	0.024	-0.055	0.035
32	0.292	0.386	0.047	0.032	32	0.003	0.074	0.002	-0.041
33	0.097	0.136	-0.028	-0.002	33	-0.072	0.049	0.039	-0.028
34	-0.259	-0.403	-0.090	-0.122	34	-0.027	-0.079	0.049	-0.010
35	-0.104	-0.126	0.028	0.079	35	0.045	-0.005	0.075	0.081
36	0.217	0.366	-0.003	-0.011	36	-0.034	0.041	-0.018	0.053

Table.1.11. Forecasts and standard errors for the different seasonal ARIMA models fitted to marine fish landings in Kerala.

Sl. No.	Species/ Group	Model	Year	Quarter	Forecast	Standard error
1	Total landings	ARIMA(0,1,2)(0,1,1)4	1997	I	100879	27020
				II	112168	31041
				III	214978	31236
				IV	164947	31430
			1998	I	100224	33356
				II	114601	34263
				III	217412	34554
				IV	167381	34843
2	Total landings	ARIMA(0,1,2)(0,1,1)4 (with intervension)	1997	I	97351	25656
				II	107720	28686
				III	211433	28686
				IV	160565	28687
			1998	I	93681	29798
				II	108202	30090
				III	211915	30091
				IV	161048	30091
3	Oil sardine	ARIMA(1,0,0)(0,1,1)4	1997	I	8836	19747
				II	5666	21793
				III	12325	22214
				IV	14884	22304
			1998	I	10436	23157
				II	6413	23338
				III	12674	23378
				IV	15047	23386
4	Mackerel	ARIMA(1,0,0)(0,1,1)4 (log transformed)	1997	I	7740	8663
				II	14192	17203
				III	33794	41480
				IV	22439	27602
			1998	I	6298	8037
				II	13027	16725
				III	32611	41911
				IV	22109	28419

Table.1.11. continued

Sl. No.	Species/ Group	Model	Year	Quarter	Forecast	Standard error
5	Anchovies	ARIMA(4,0,0)(1,1,2)4 (log transformed)	1997	I	2771	2279
				II	6770	5666
				III	11499	9632
				IV	16343	14036
			1998	I	2757	2462
				II	8188	7369
				III	14407	12974
				IV	16439	15058
6	Lesser sardines	ARIMA(0,1,1)(0,1,1)4 (log transformed)	1997	I	1237	1131
				II	883	832
				III	955	927
				IV	7706	7677
			1998	I	1197	1294
				II	855	954
				III	924	1063
				IV	7456	8814
7	Penaeid prawns	ARIMA(0,0,1)(0,1,1)4 (log transformed)	1997	I	9072	5604
				II	13813	8678
				III	18658	11722
				IV	6629	4164
			1998	I	8968	6018
				II	13813	9289
				III	18658	12548
				IV	6629	4458
8	Tuna	ARIMA(0,1,1)(0,1,1)4 (log transformed)	1997	I	2218	1710
				II	4489	3513
				III	3683	2924
				IV	4572	3682
			1998	I	2276	1982
				II	4607	4086
				III	3780	3412
				IV	4693	4309

Table.1.11. continued

Sl. No.	Species/ Group	Model	Year	Quarter	Forecast	Standard error
9	Thrissocles	ARIMA(1,1,1)(1,1,2)4 (log transformed)	1997	I	1696	1587
				II	1090	1063
				III	2124	2095
				IV	972	965
			1998	I	1269	1298
				II	1803	1864
				III	2308	2402
				IV	1972	2067
10	Ribbon fishes	ARIMA(1,0,1)(0,1,1)4 (log transformed)	1997	I	504	759
				II	961	1546
				III	4419	7334
				IV	5068	8548
			1998	I	358	635
				II	749	1345
				III	3680	6655
				IV	4433	8044

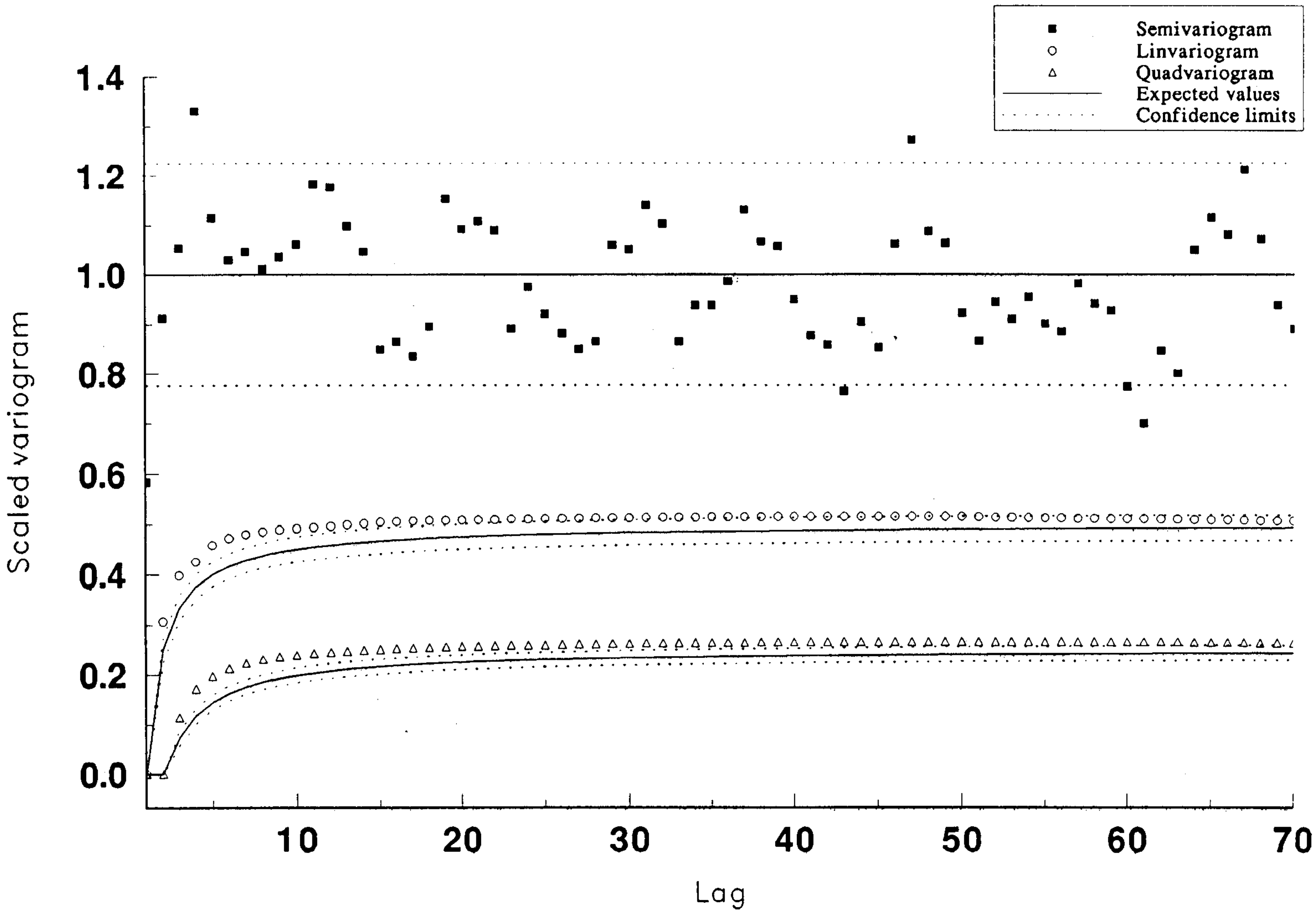

Fig.1.1.1. Plot of scaled variograms for the seasonally differenced total marine fish landings with 2 standard error limits.

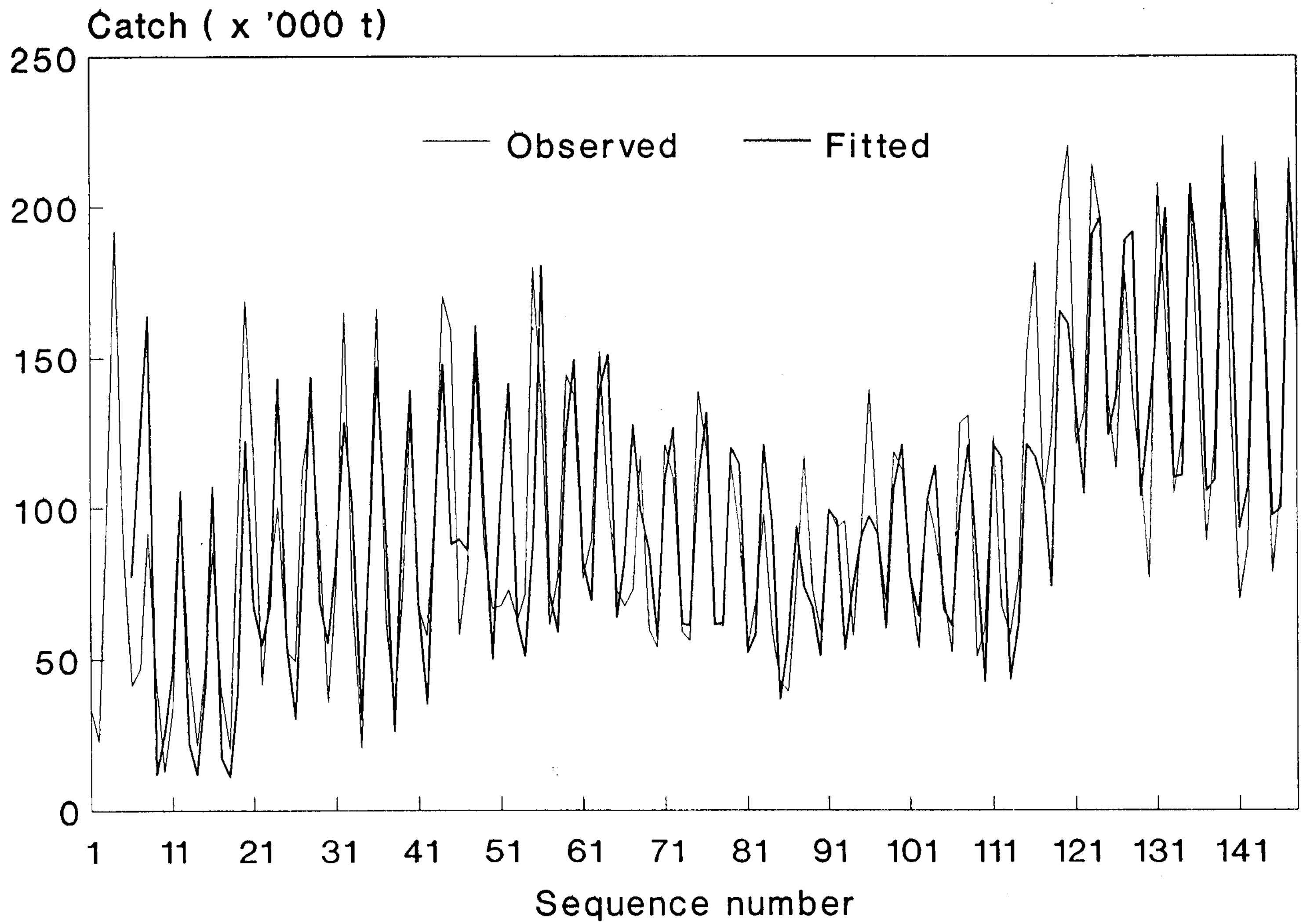

Fig.1.1.2. Plot of observed and fitted values of quarterwise total marine fish landings in Kerala during 1960-96 using the model ARIMA(0,1,2)(0,1,1)4.

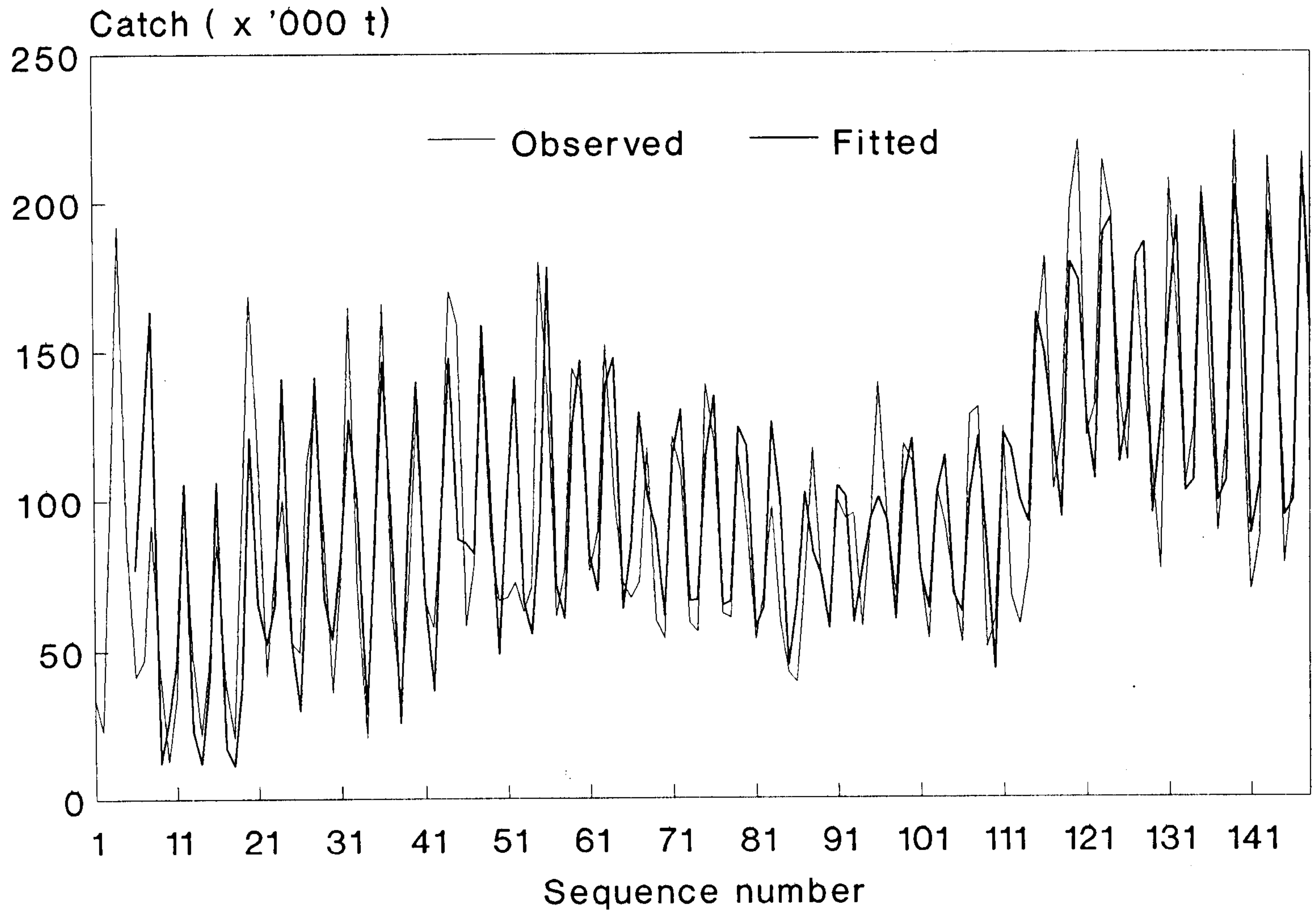

Fig.1.2.1. Plot of observed and fitted values of quarterwise total marine fish landings in Kerala during 1960-96 using the model ARIMA(0,1,2)(0,1,1)4 with intervesion.

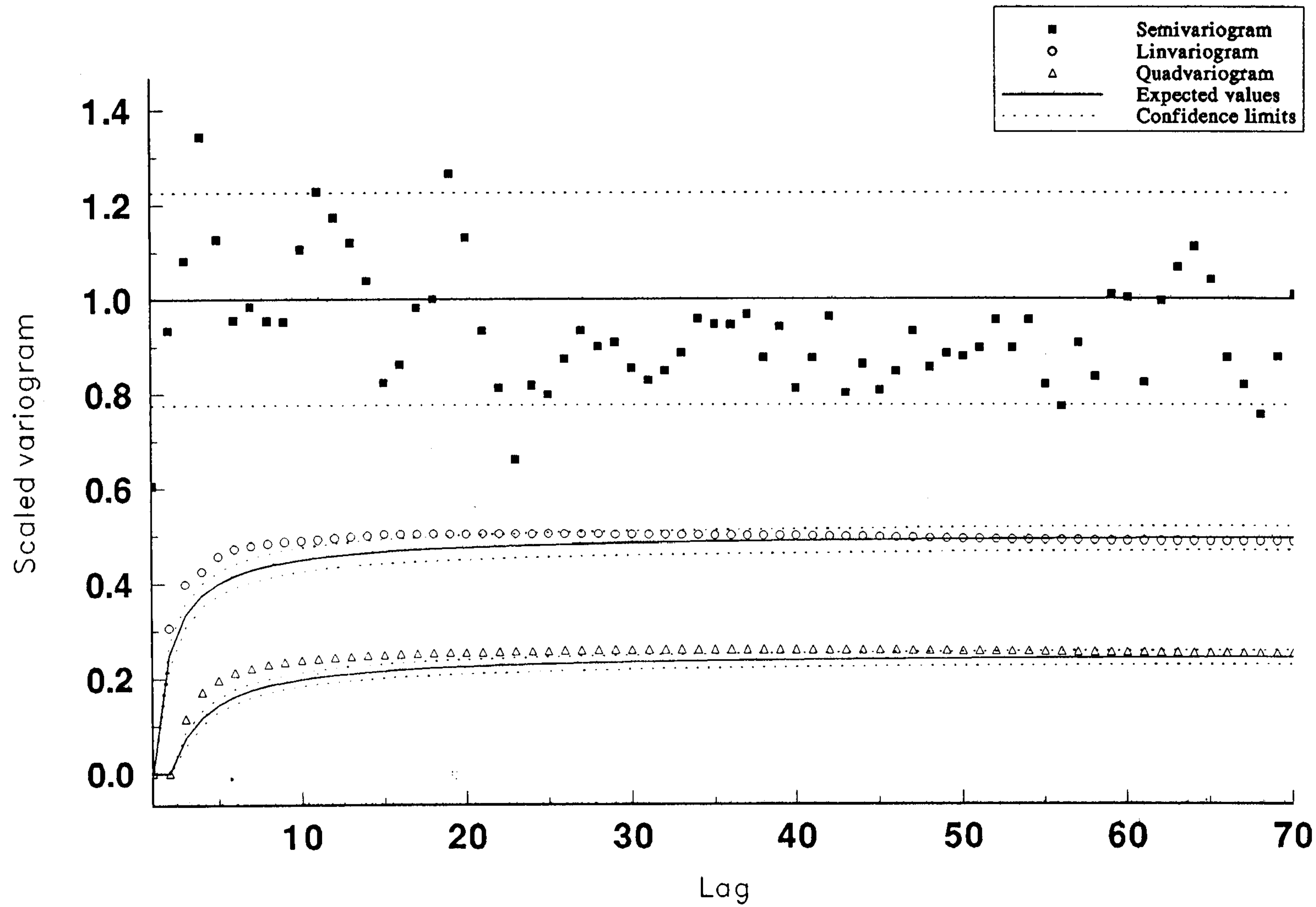

Fig.1.3.1. Plot of scaled variograms for the seasonally differenced oil sardine landings with 2 standard error limits.

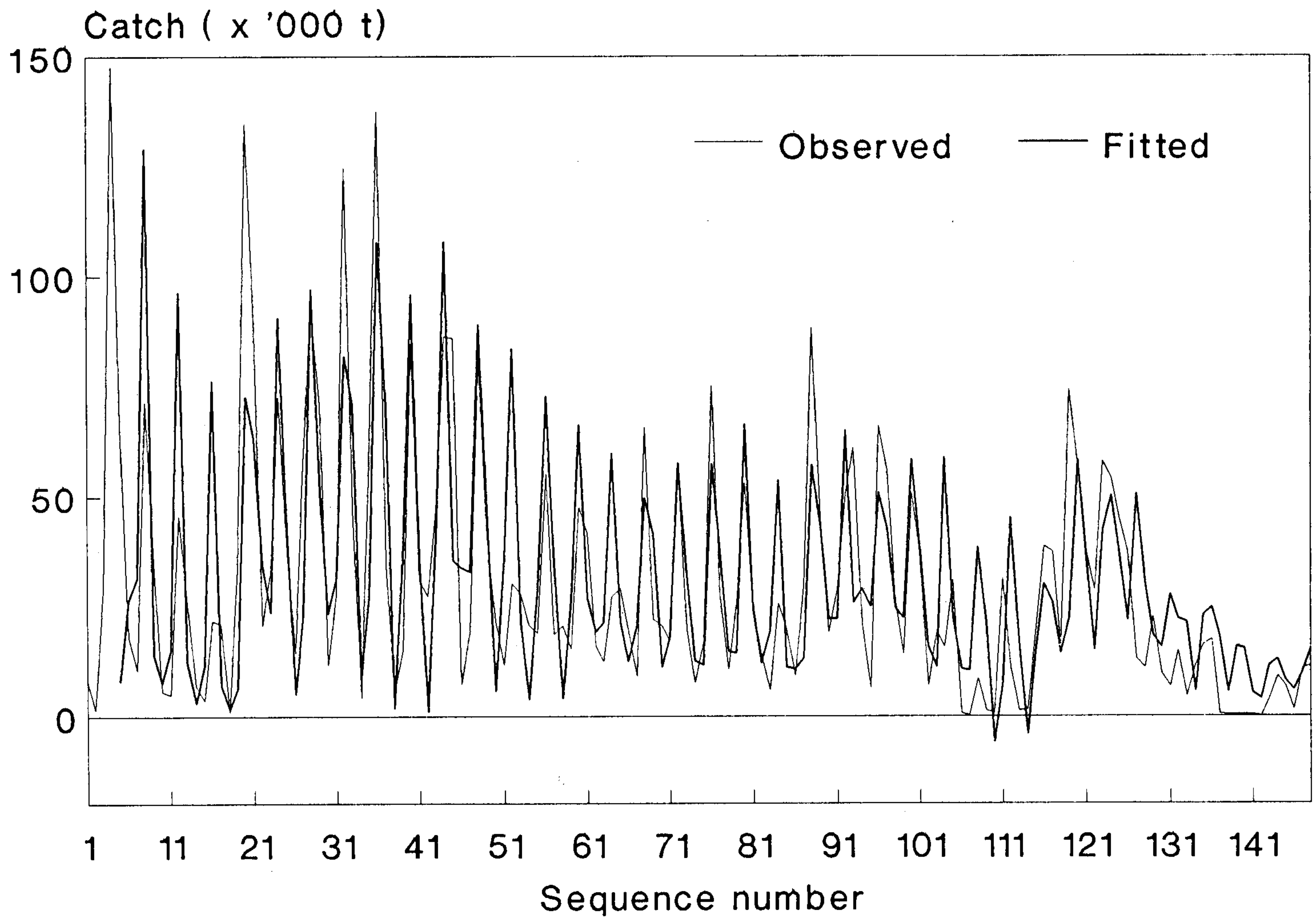

Fig.1.3.2. Plot of observed and fitted values of quarterwise landings of oil sardine in Kerala during 1960-96 using the model ARIMA(1,0,0)(0,1,1)4.

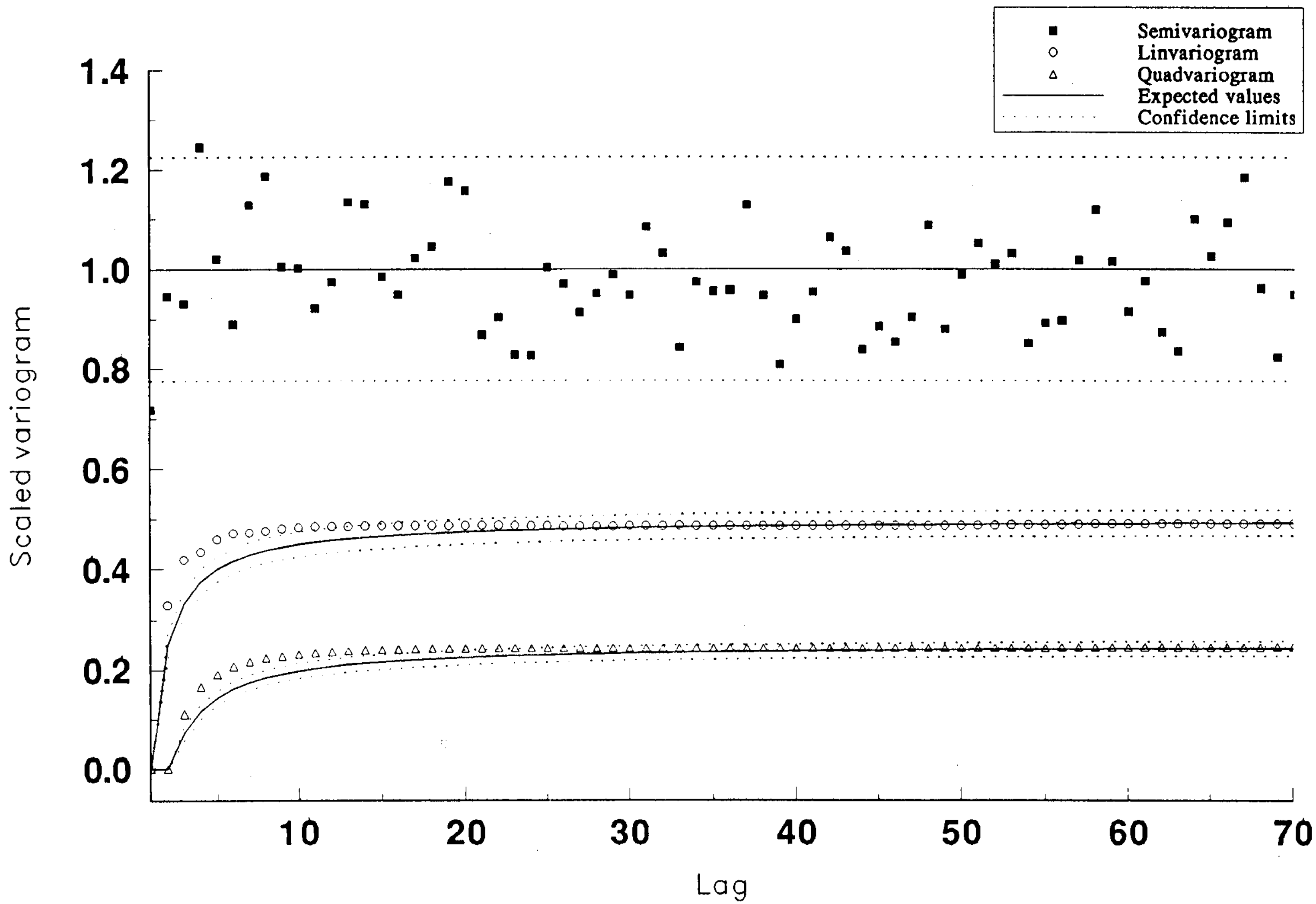

Fig.1.4.1. Plot of scaled variograms for the seasonally differenced and log transformed mackerel landings with 2 standard error limits.

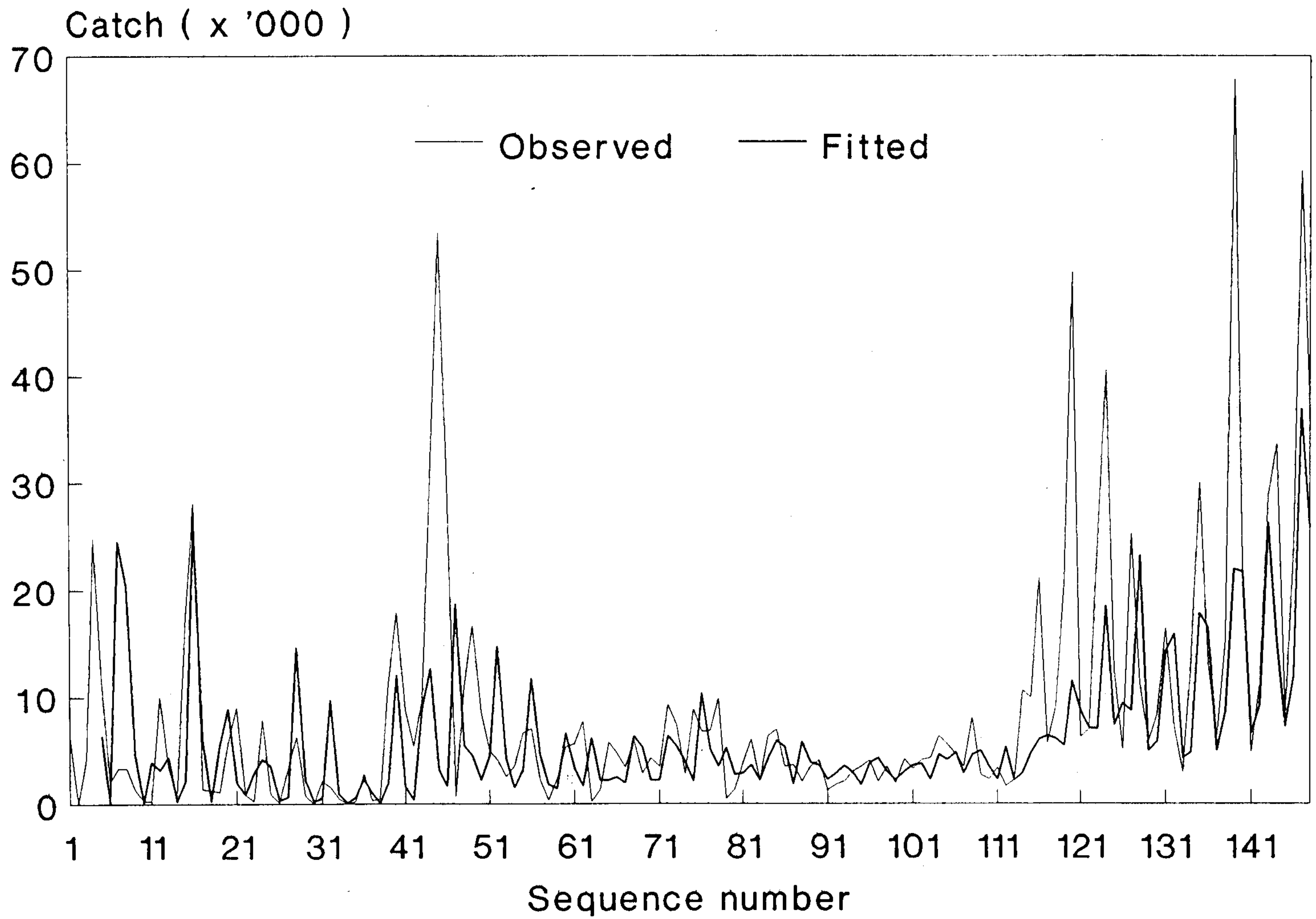

Fig.1.4.2. Plot of observed and fitted values of quarterwise landings of mackerel in Kerala during 1960-96 using the model ARIMA(1,0,0)(0,1,1)4 on logarithm of catch.

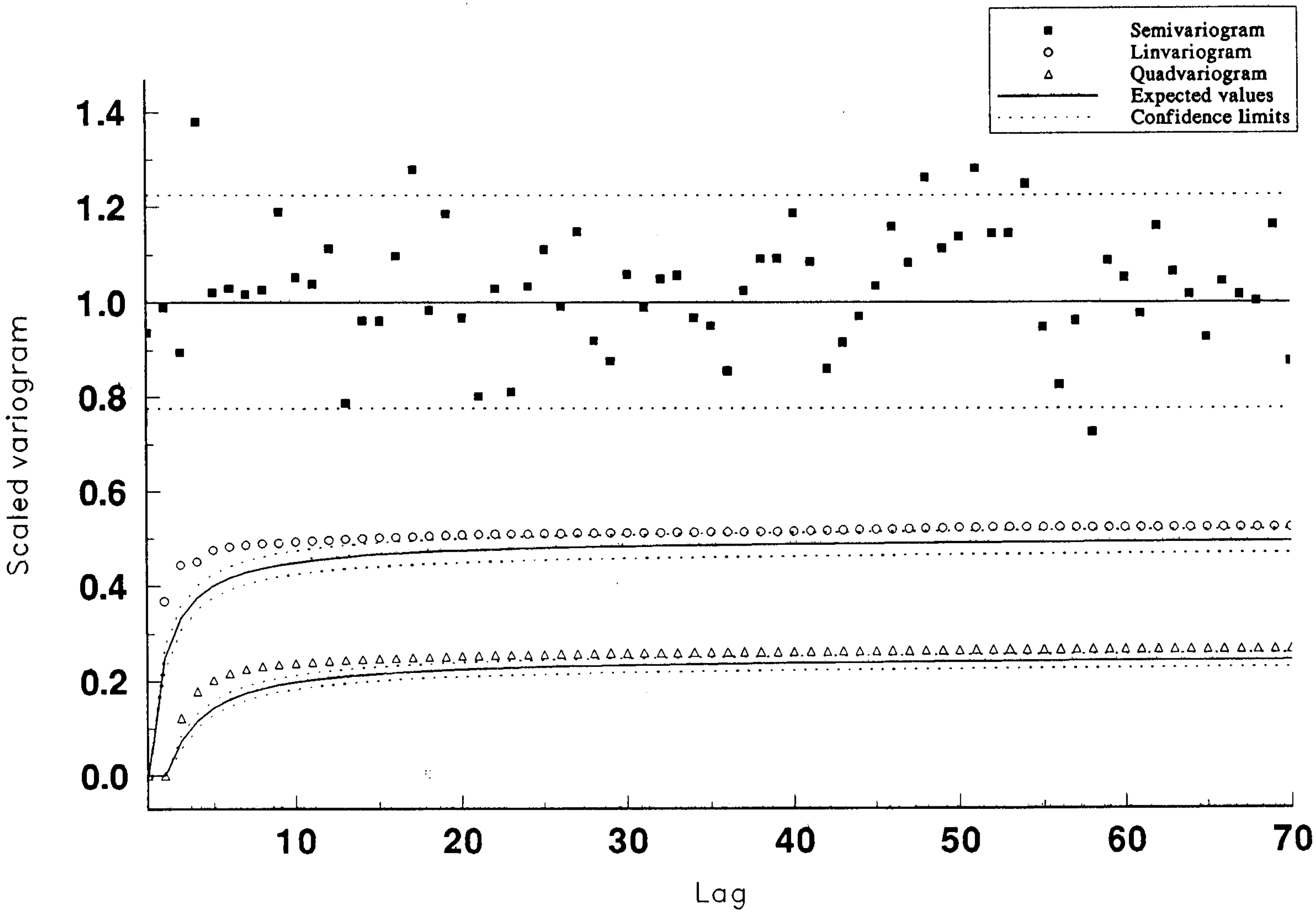

Fig.1.5.1. Plot of scaled variograms for the seasonally differenced and log transformed anchovies landings with 2 standard error limits.

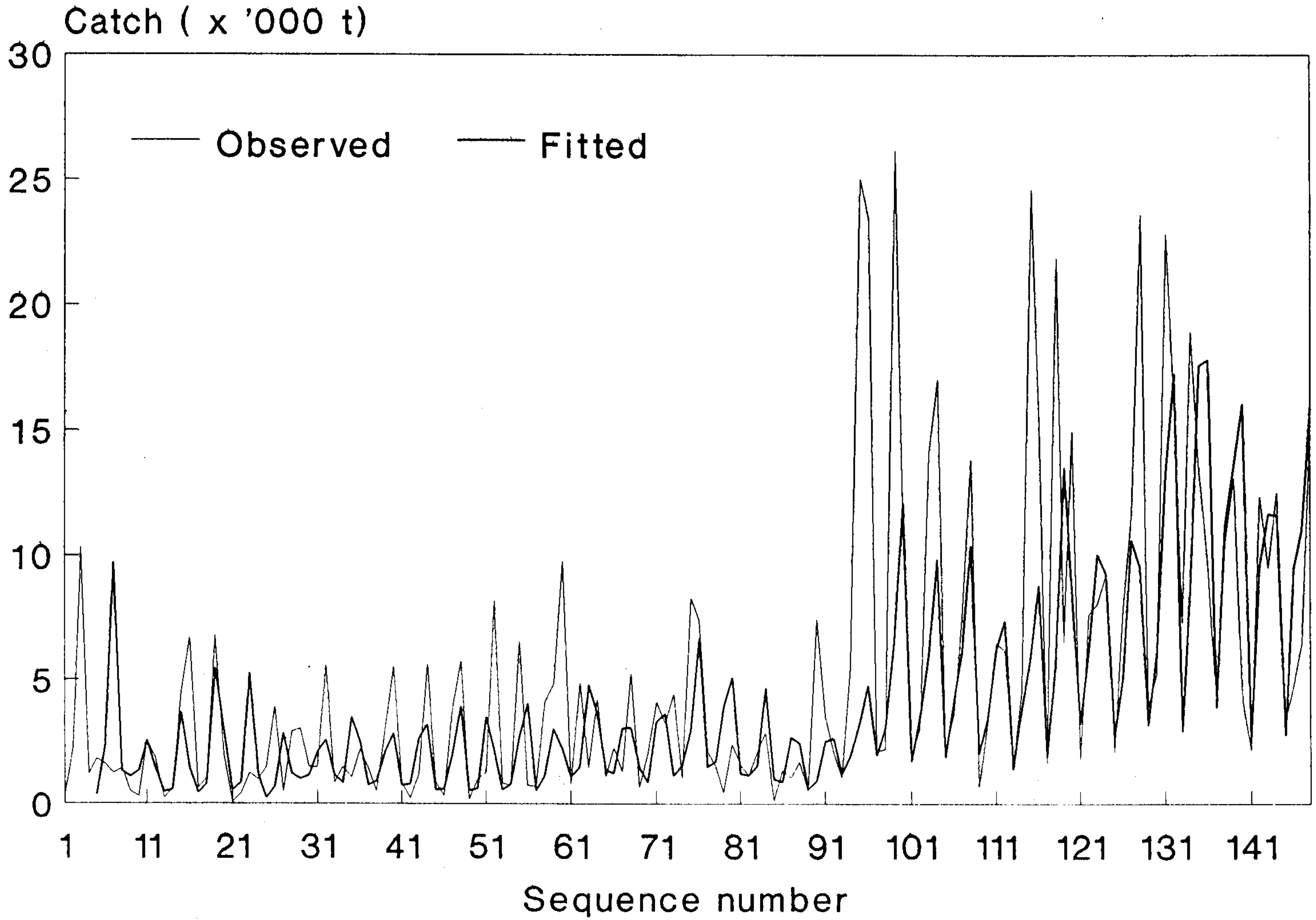

Fig.1.5.2. Plot of observed and fitted values of quarterwise landings of anchovies in Kerala during 1960-96 using the model ARIMA(4,0,0)(1,1,2)4 on logarithm of catch.

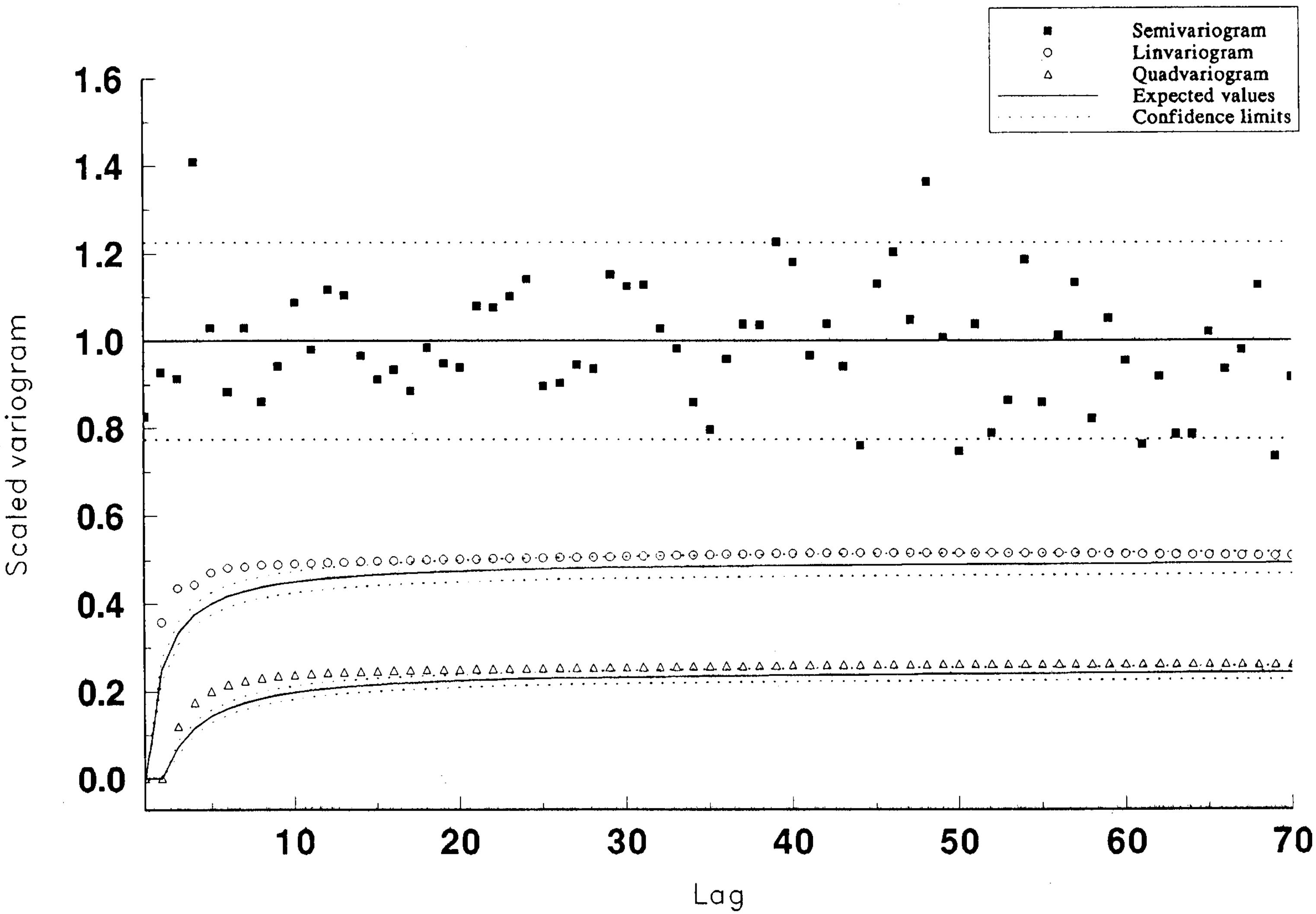

Fig.1.6.1. Plot of scaled variograms for the seasonally differenced and log transformed lesser sardine landings with 2 standard error limits.

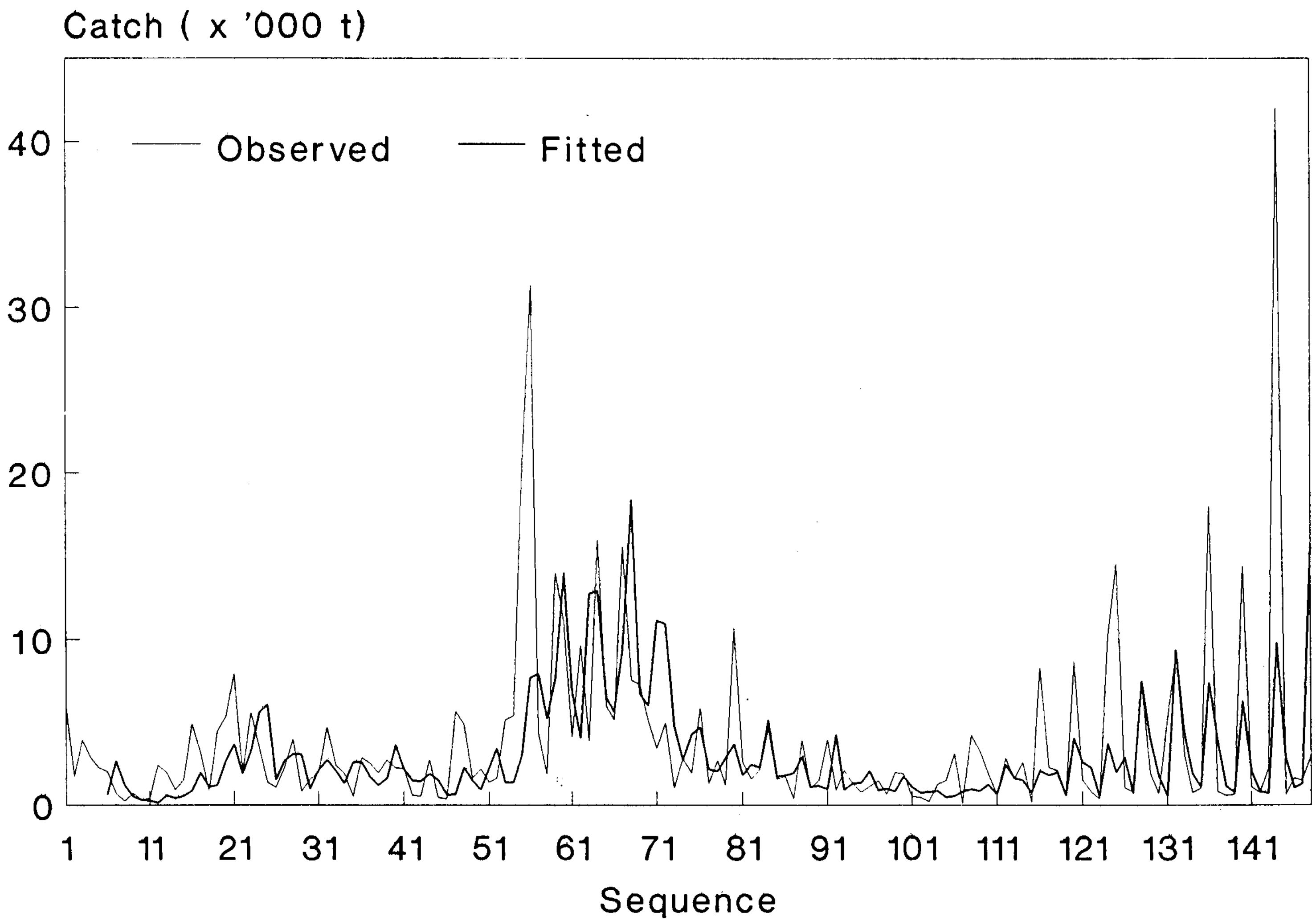

Fig.1.6.2. Plot of observed and fitted values of quarterwise landings of lesser sardines in Kerala during 1960-96 using the model ARIMA(0,1,1)(0,1,1)4 on logarithm of catch.

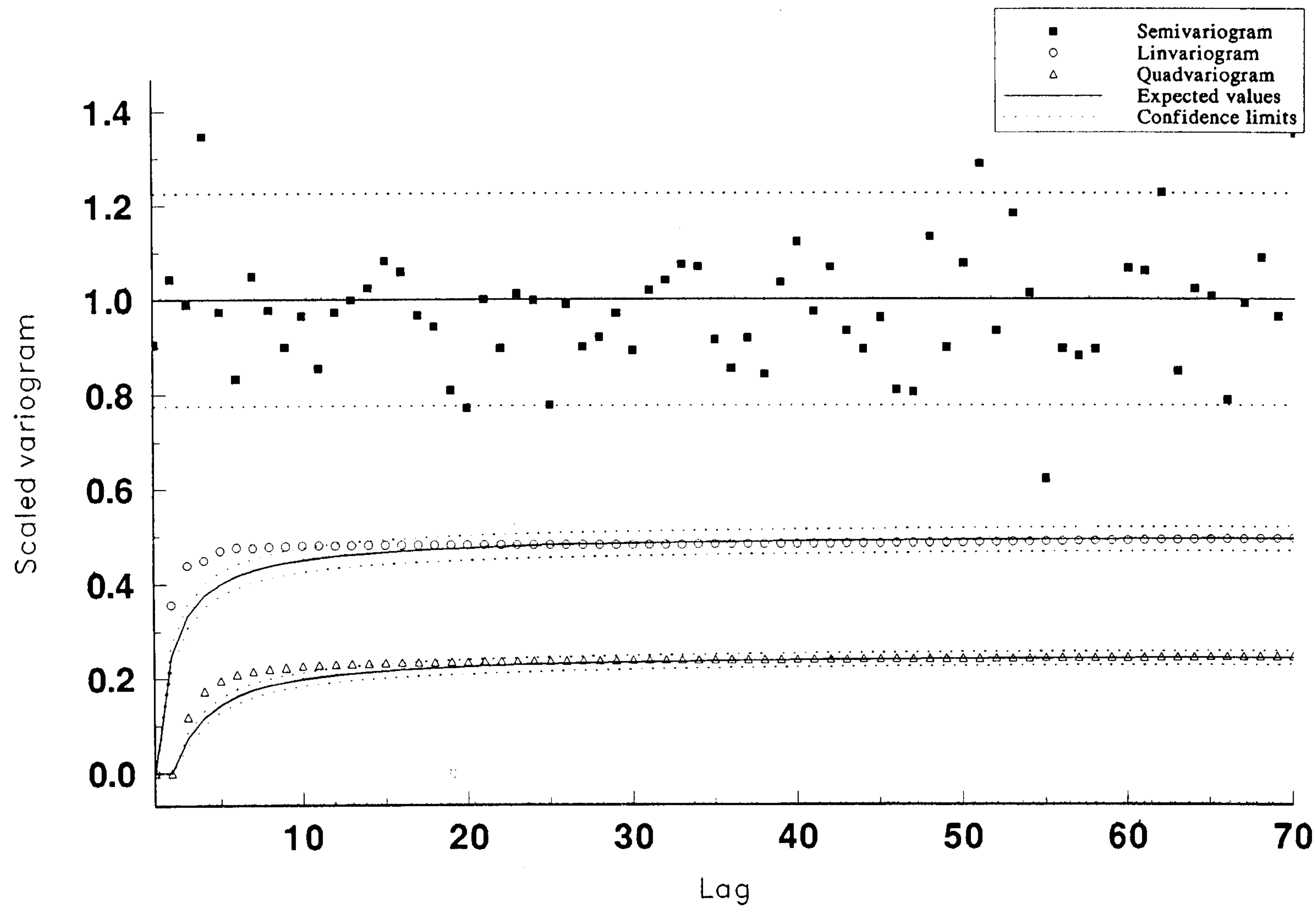

Fig.1.7.1. Plot of scaled variograms for the seasonally differenced and log transformed penaeid prawn landings with 2 standard error limits.

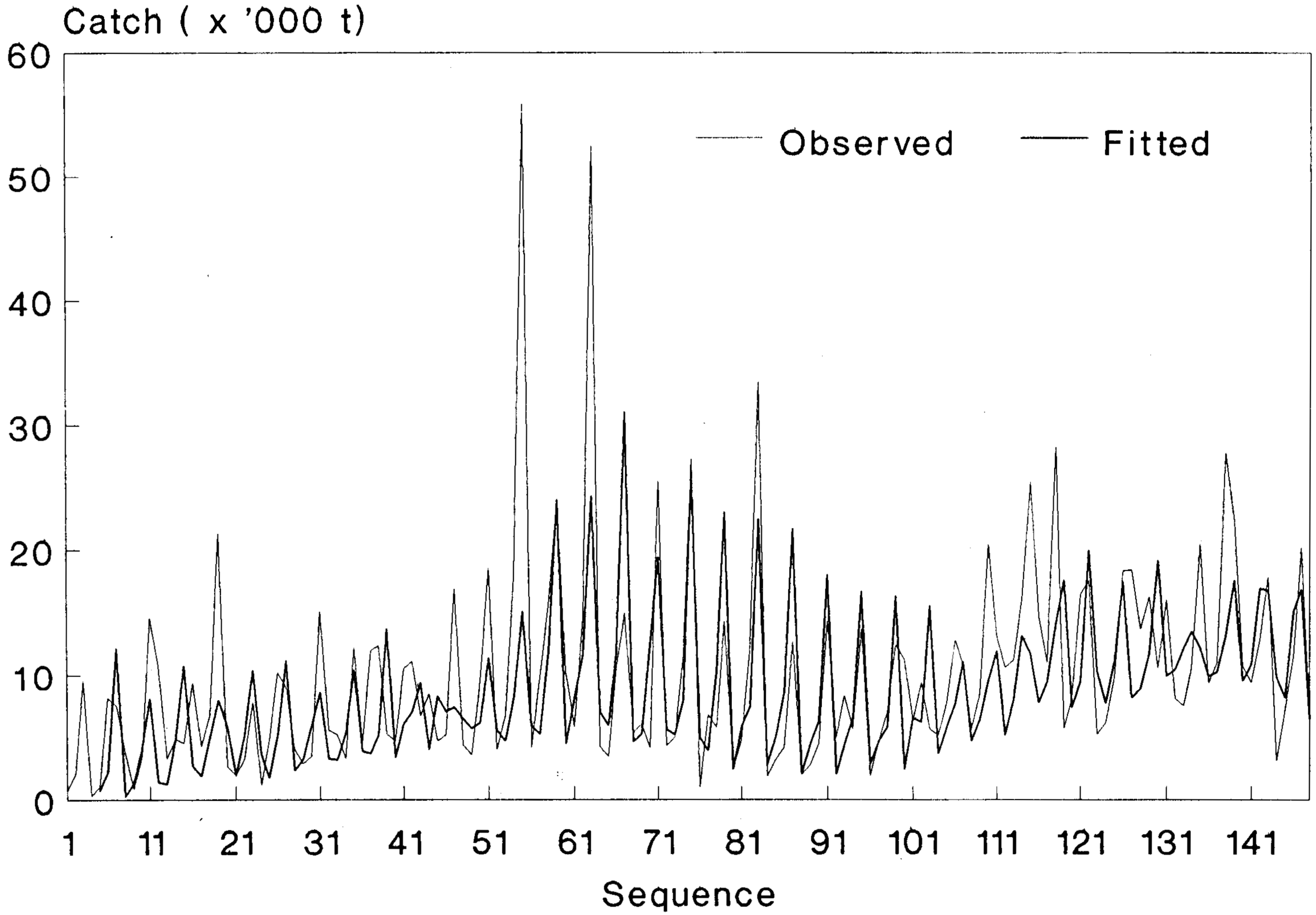

Fig.1.7.2. Plot of observed and fitted values of quarterwise landings of penaeid prawns in Kerala during 1960-96 using the model ARIMA(0,0,1)(0,1,1)4 on logarithm of catch.

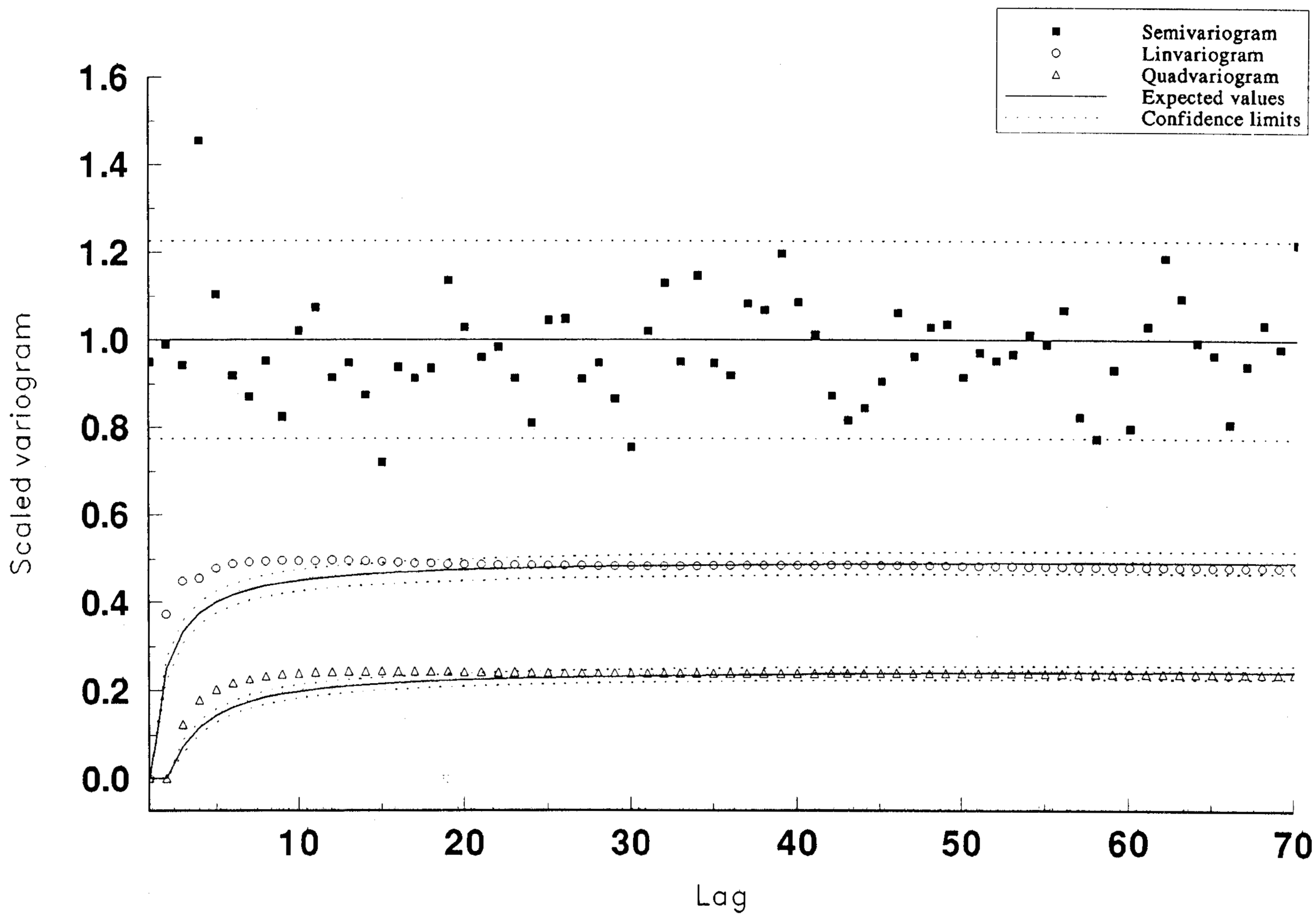

Fig.1.8.1. Plot of scaled variograms for the seasonally differenced and log transformed Tuna landings with 2 standard error limits.

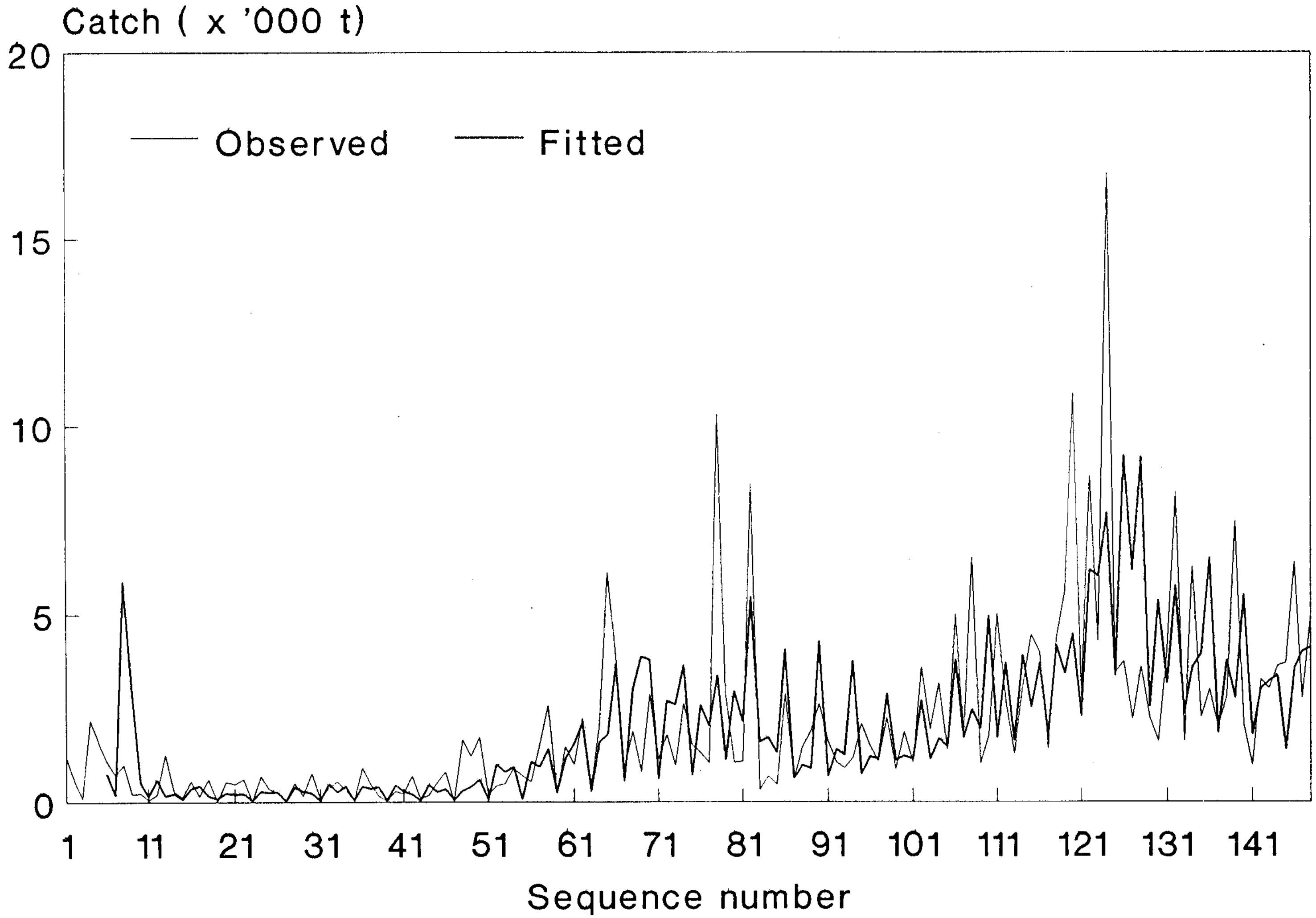

Fig.1.8.2. Plot of observed and fitted values of quarterwise landings of Tuna in Kerala during 1960-96 using the model ARIMA(0,1,1)(0,1,1)4 on logarithm of catch

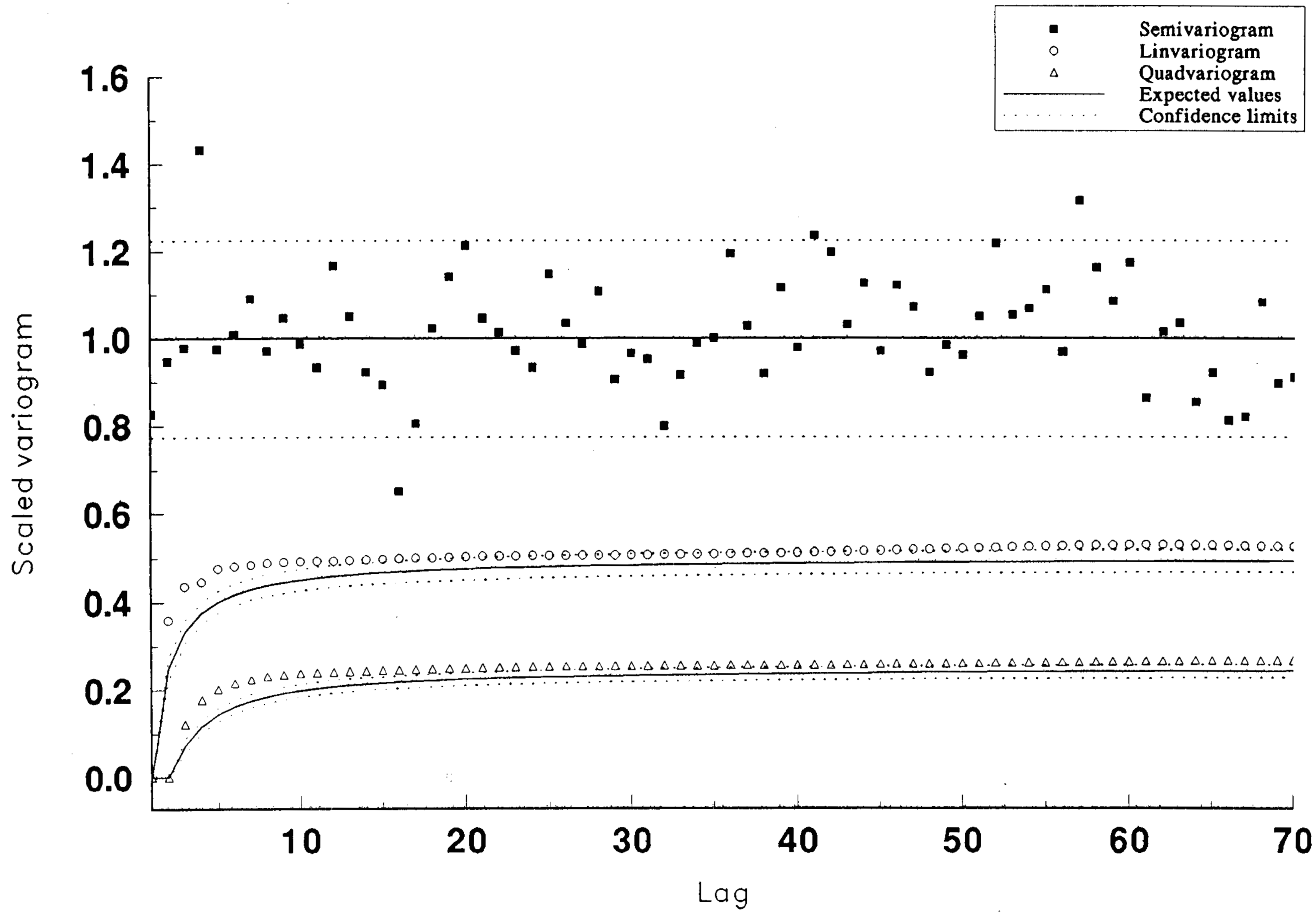

Fig.1.9.1. Plot of scaled variograms for the seasonally differenced and log transformed Thrissocles landings with 2 standard error limits.

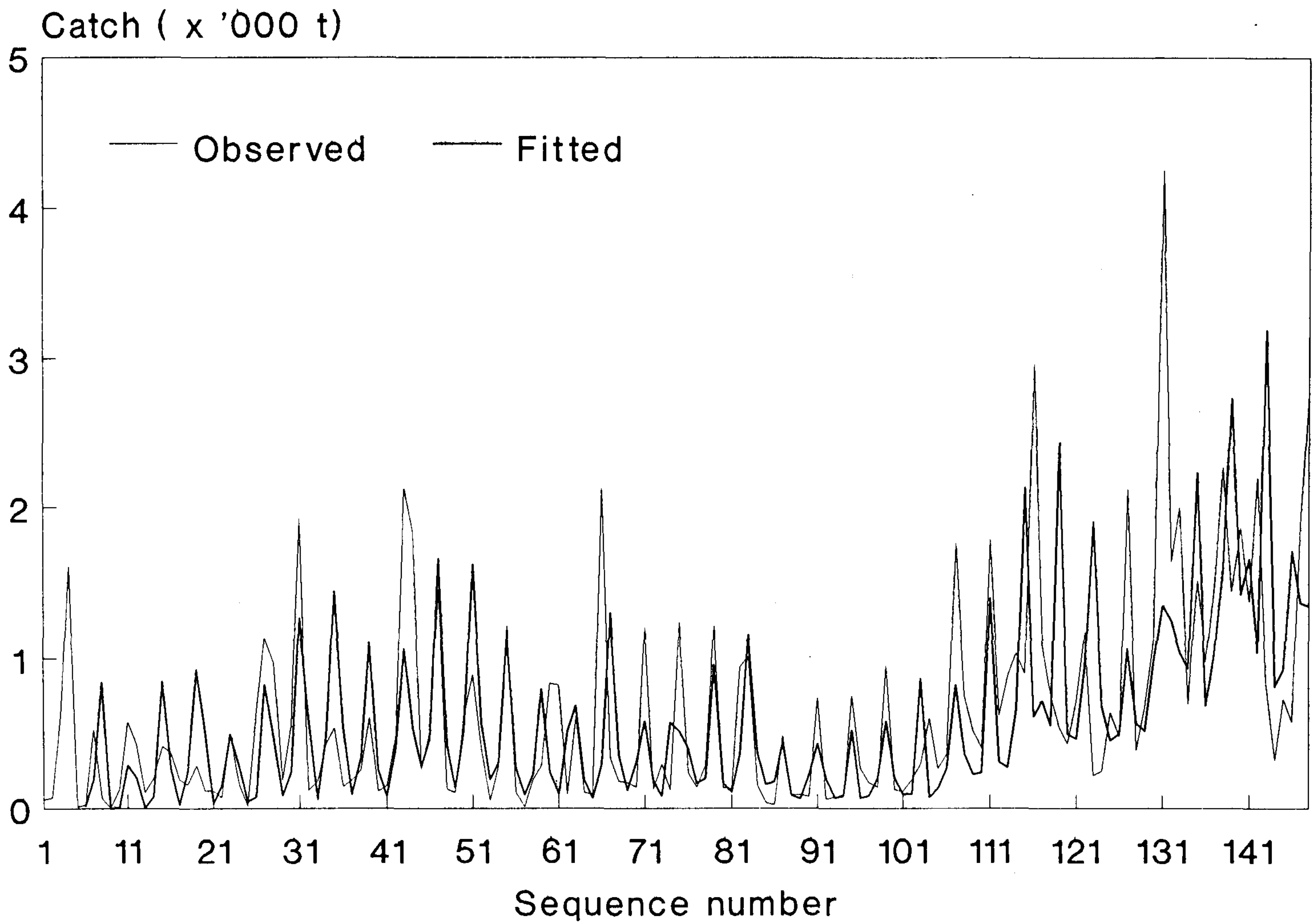

Fig.1.9.2. Plot of observed and fitted values of quarterwise landings of thrissocles in Kerala during 1960-96 using the model ARIMA(1,1,1)(1,1,2)4 on logarithm of catch.

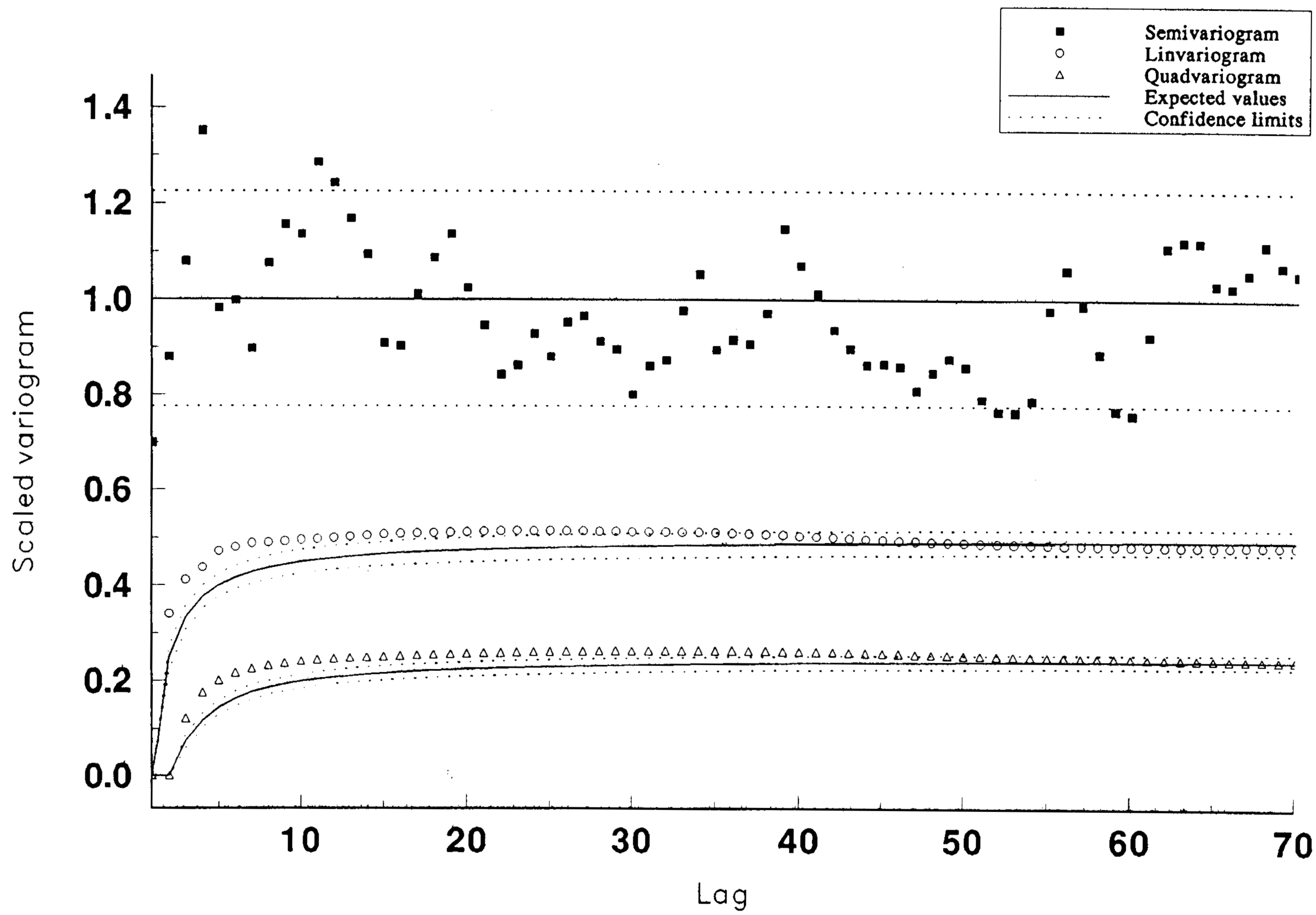

Fig.1.10.1. Plot of scaled variograms for the seasonally differenced and log transformed Ribbon fish landings with 2 standard error limits.

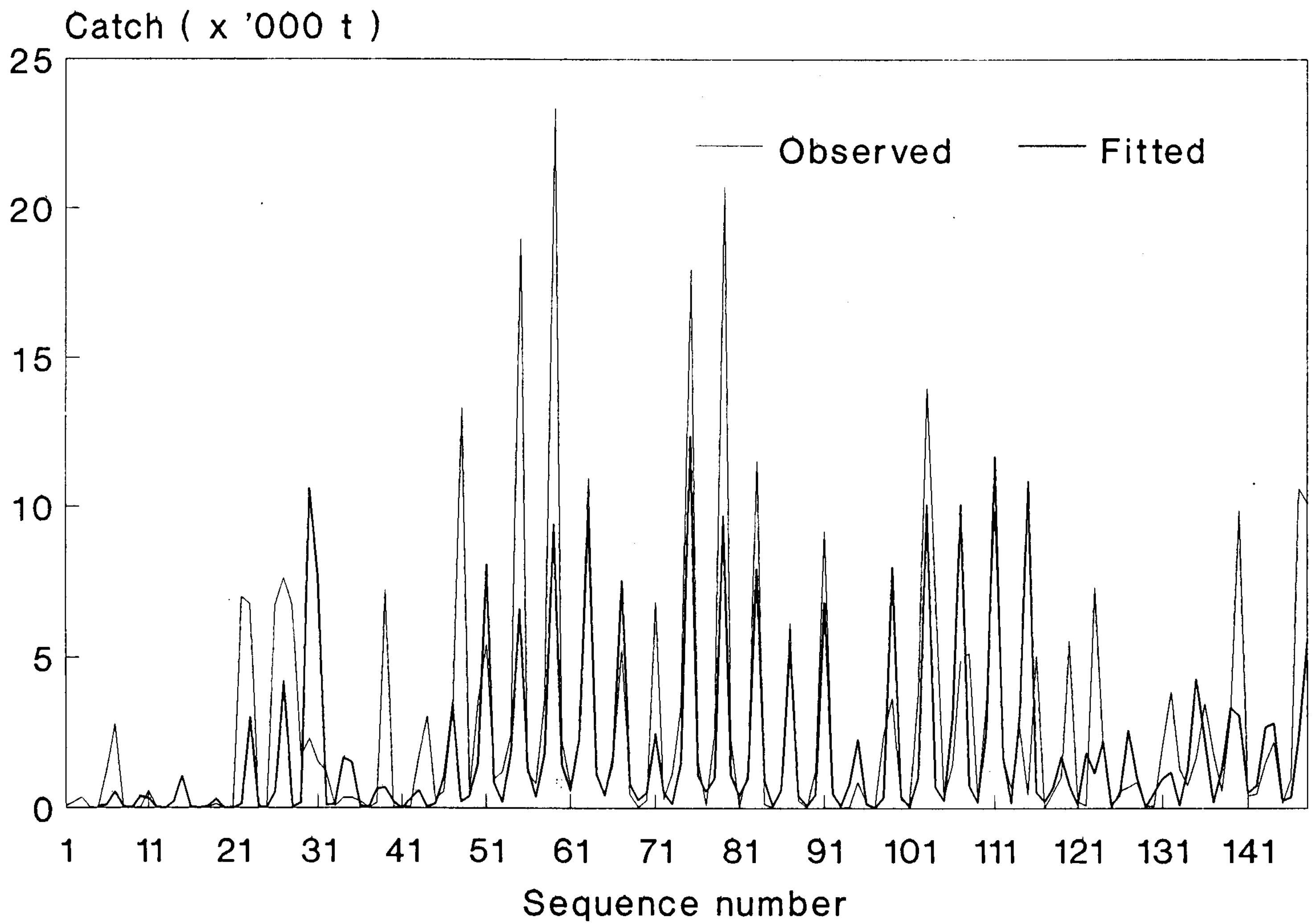

Fig.1.10.2.Plot of observed and fitted values of quarterwise landings of Ribbonfishes in Kerala during 1960-96 using the model ARIMA(1,0,1)(0,1,1)4 on logarithm of catch.

CHAPTER-2.

Vector Autoregressive modelling of marine fish landings in Kerala.

Introduction:

The major two reasons for analysing and modelling of more than one time series sequences together, known as multiple time series, are (i) to understand the dynamic relationships among the different time series components and (ii) to improve the accuracy of forecasts of one series by utilizing the information about that series contained in all other time series. Suppose there are k time series components $\{Z_{1t}\},\{Z_{2t}\},\cdots,\{Z_{kt}\}$ for $t=0,\pm 1,\pm 2,\cdots$ at equally spaced time intervals. We can represent these components by a vector $Z_t=(Z_{1t},Z_{2t},\cdots,Z_{kt})'$ which we call as a vector of time series. A vector time series process $\{Z_t\}$ is strictly stationary if the probability distribution of the random vectors $\left(Z_{t_1},Z_{t_2},\cdots,Z_{t_n}\right)'$ and $\left(Z_{t_1+l},Z_{t_2+l},\cdots,Z_{t_n+l}\right)'$ are the same for arbitrary times $t_1,t_2,\cdots,t_n$, all lags $l=0,\pm 1,\pm 2,\cdots$ and all n. That is the probability distribution of observations from a stationary process is invariant with respect to shifts in time. For all such series $E(Z_t)=\mu$ for all t where $\mu=(\mu_1,\mu_2,\cdots,\mu_k)'$ is the mean vector for the series and $E\left[(Z_t-\mu)(Z_{t+l}-\mu)'\right]=\Gamma(l)$ is known as the cross covariance matrix of lag l for $l=0,\pm 1,\cdots$ and $\Gamma(l)$ will depend only on the lag l when the series is stationary. If $\mathbf{V}=diag(\gamma_{11}(0),\gamma_{22}(0),\cdots,\gamma_{kk}(0))$ where $\gamma_{ii}(0)$ is the variance of the i^{th} component series $\{Z_{it}\}$, then $\rho(l)=\mathbf{V}^{-1/2}\Gamma(l)\mathbf{V}^{-1/2}$ is the cross correlation matrix at lag l. For a stationary vector process, the structure of the cross-covariance and cross correlation matrices provide a useful summary of information on aspects of dynamic inter relations among the component series of the process.

A stationary vector time series $\{Z_t\}$ with k components can be modeled by a vector autoregressive model of order p denoted by VAR(p), and its expression is

$$z_t = \Phi_1 z_{t-1} + \cdots + \Phi_p z_{t-p} + \varepsilon_t .$$

This can be written as

$$\Phi(B) z_t = \varepsilon_t$$

where $\Phi(B) = I - \Phi_1 B - \cdots - \Phi_p B^p$ is a matrix polynomial in the back shift operator B, $z_t = Z_t - \mu$, μ is the mean vector of the series, $\Phi_1, \Phi_2, \cdots, \Phi_p$ are $k \times k$ parameter matrices, $\varepsilon_t = (\varepsilon_{1t}, \cdots, \varepsilon_{kt})'$ are independently and identically distributed random innovation vectors having zero mean and constant covariance matrix Σ.

In the present study vector autoregressive models were attempted to fit using the quarter wise landings of selected marine fish species/species groups in Kerala. The selection of the species/groups for the analysis was made based on their commercial importance, their contribution towards the total landings in the state and biological aspects like food and feeding habits, prey-predator relation etc. The species and groups selected for the study are oil sardine, mackerel, anchovies, lesser sardines, ribbon fishes, catfish, tuna, penaeid prawns, seer fish and elasmobranchs. Since the model used is not a seasonal type model and the these time series sequences are quarterly data, the original data were transformed before analysis by taking a 4 point moving total of the logarithm and then the variances standardized to unity. Taking the 4 point moving sum will have the added advantages other than removing the seasonality present in the data that it will reduce the ill effects of outliers if any present in the data. Also the moving sum will represent yearly values as it contains values from all the four quarters.

Oil sardine (*Sardinella longiceps*) and mackerel (*Restralliger kanagurta*) are the two important species that contributes maximum towards the total marine fish landings in Kerala. They compete each other for food, both being plankton feeders and migrate to the inshore waters during the same season. Sharks, seer fish and ribbon fish are predators common to both oil sardine and mackerel. Flagillaria oceanica has been considered as favorite food of oil sardine. Other items of food are copepods, dinoflagellates, ostracods, larval prawns, larval bivalves, fish eggs and blue green algae. Juveniles of oil sardine feed on planktonic crustaceans like copepods. Dolphins and sharks are known active predators of oil sardine. *Sardinella spp.* is a common food of seer fishes (*S. commerson* and *S. guttatus*) and adults of ribbon fish also feed on *Sardinella*. Copepods, molluscan larvae, fish eggs and larvae are the major zooplankters in mackerel's food. Post larvae (5-6mm) of mackerel are herbivorous feeding on diatoms and algae and its juveniles (25mm) are omnivorous feeding on all available organisms in plankton. Adults of mackerel (35-224mm) are carnivorous feeding on larval and juvenile fish. Major predators of mackerel are sharks, seer fish (*S. guttatus*), ribbon fish and porpoise.

Major species of lesser sardine are *Sardinella jussieu*, *Sardinella fimbriata*, *Sardinella albella*, and *Sardinella sirm*. Lesser sardines are also a plankton feeder with preference for zooplanktonic organisms like diatoms, copepods, dinoflagelltes, larvel bivalves, young prawns, prawn larvae of *Acetes* and Alpheids, decapods and post larvae of anchoviella. Juveniles of lesser sardines feed on phytoplanktonic organisms. Common predators of lesser sardine are seer fishes and Sciaenids.

Important species belonging to anchovies group are *Stolephorus indicus*, *S. devisi*, *S. waitei* and *S. bataviensis*. Lesser sardines feed on post larvae of anchovies. Juveniles of ribbon fishes feed on larvae of anchovies. Adults of ribbon fishes feed on *Stolephorus spp.* Catfishes, seer fish (*S. guttatus*) and tuna (*Auxis thazard*) also feed on anchovies.

Juveniles of ribbon fish feed on copepods, other crustaceans, prawn and fish larvae. Common items of food for juveniles are calanoid copepods, larvae of anchovies, juveniles of clupeoids and carangids. Adults of ribbon fishes feed on crustaceans, Stolephorus, Thrissocles, Sphyraenea, Hemiramphus, Gardinella, Leiognanthus, octopus and squilla.

Seer fish (*Scomberomorus commerson*, *S. guttatus* and *S. lineolatus*) are carnivorous, feeding occasionally on prawns and squids. Since gill rakers are rudimentary and few even during younger stages they are forced to feed on larger organisms. Major food items of seer fish are anchovies, Sardinella spp, mackerel, Thrissocles spp., Sciaenids, Acetes, Saurida spp., Trichurus spp., Dussumiera spp. Leiognathus spp., carangids, clupeoids, perches, decapods, Lactarius, Sphyraena spp., diodontids, eel larvae, Chirocentrus, Monocanthus and tuna.

Important species of tuna are *Auxis thazard*, *Katsuwonus pelamis*, *Euthynnus affinis*, *Thunnus orientalis*, *Thunnus obesus*, and *Thunnus tongol*. They feed mainly on fishes, cephalopods and crustaceans. Anchovies and leiognathus are the major fish prey of *Axuis thazard*. Stomatopods, cephalopods, larval and juvenile reef fishes are major foods of *K. pelamis*. Squids, cuttle fish, pteropods, anchovies, carangids, stomatopod larvae and penaeid prawns are the food items of *Euthynnus affinis*. Predators of *Auxis thazard* are sail fishes, barracudas, seer fish and dolphins. Large sharks and marlins predate on oceanic skipjack and *Euthynnus affinis*. Whales, dolphins, spear fishes and large tunas predate on *Thunnus orientalis*. Many carnivorous fishes predate on juveniles of big-eye tunas.

Major species belonging to the group penaeid prawns are *Penaeus indicus*, *P.monodon*, *P. semisulcatus*, *Parapenaeopsis stylifera* and *Metapenaeus dobsoni*. *P. indicus* is omnivorous and bottom feeder. Diatoms, planktonic algae and bits of seaweed are the vegetable feed and copepods, ostracods, amphipods, larval crustaceans, molluscan larvae and polychates are the animal matter in its food. Crustaceans form the main food item of *P. monodon* and *P. semisulcatus*. Other foods are molluscan, parts of fish, polychates and vegetable matter. *M. dobsoni* feed on diatoms and other minute plant and animal organisms. Penaeid prawns are food of *Euthynnus affinis* (Tuna). Young ones of *Megalaspis cordyla* (Carangidae) feed on young prawns.

Species belonging to Carangids are *Megalaspis cordyla*, *Decapterus russelli*, Threadfin (*Alectis indica*), *Carangoides malabaricus* and *Caranx carangus*. Young ones of *M. cordyla* feed on post-larval fish, young prawns and other crustaceans and adults feed on clupeiod fish and crustaceans. Juveniles of *D. russelli* feed on acetes, copepods and other curstaceans and adults mostly feed on clupeiod fish, diatoms, copepods and other crustaceans. Stomatopode larvae, other crustaceans and polychaete worms are food of *C. carangus*. Food of *C. malabaricus* is crustaceans and fishes.

Catfishes are predacious, carnivorous and often carrion eaters. They feed mainly on fish and crustaceans. Their diet consists of anchovies, Coilia spp., sciaenids, eel elvers, Trichiurus spp., Bregmaceros spp. and Chirocentrus spp. They also eat penaeid prawns and are voracious bottom feeders.

Review of literature:

Whittle (1963) generalized the fitting of AR schemes of successively increasing order to that of multivariate autoregressions and for schemes with rational spectral densities. He showed that the AR fitted through Yule-Walker relations has stability properties which holds in multivariate case also. Priestley (1964) considered the analysis of two-dimensional stationary processes. He explained the general considerations for spectral analysis of two-dimensional data and detection of signals, amplitude and frequency estimation for points of discontinuities, estimation of autocovariance spectrum etc. Clearbout (1966) gave a spectral factorization of multiple time series. He has shown how polynomials with matrix coefficients can be factorized and how to apply this to the factorization of a known rational spectral density matrix. Hannan (1967) described a technique for estimating coefficient matrices in the regression of vector time series to lagged values of a second vector time series. This technique was based on computation of spectra and cross-spectra. Brillinger (1969) examined the asymptotic properties of second order spectral estimates of stationary vector time series. He derived expressions for first and second order moments and asymptotic distributions of matrix of second order periodograms, matrix of sample spectral measurements, matrix of sample spectral densities and matrix of covariances.

Nicholls (1977) used Tensor products to show that in the case of vector linear time series models the estimates obtained by the application of Newton-Raphson procedure are identical to those derived by him (1976) and hence the estimates so obtained are asymptotically normal and efficient. MacNeill (1977) discussed tests for determining the presence of common periodicities in components of a multiple time series. He obtained distributional results for white noise, independent normal series. Box and Tiao (1977) proposed a canonical transformation of a vector AR process and ordered the components of the transformed process from least to most predictable. According to them the least predictable components can reflect stable contemporaneous relationships among the original variables and the most predictable components represents the dynamic growth characteristics of the series. Baillie (1979) derived the asymptotic mean squared error of multi-step prediction error of a vector AR process. He also derived results for regression models with autoregressive errors where the set of exogenous variables follow a VAR process. Troutman (1979) examined properties of a periodic AR process by considering a related stationary multivariate AR process. Hsiao (1979) suggested a sequential procedure based on FPE criterion and Grangers concept of causality to fit multiple autoregressions. This allows each variable to enter with different time lag and also provides a powerful test for exogenity or causality. He demonstrated this by using Canadian money and income data.

Obrien (1980) proposed a procedure for testing the hypothesis that a sequence of vector valued random variables are mutually independent and he found that the test statistic as a close approximation to F distribution. Abraham (1980) presented a general model to encapsulate interventions in multiple time series and discussed the estimation of model parameters. Yamamoto (1981) gave a simple formula for multiperiod predictions of multivariate autoregressive moving average models as a function of suitably defined parameter matrices and observation vector. Tiao and Box (1981) proposed an approach to the modelling and analysis of multiple time series that consisted of tentative specification, estimation and diagnostic checking. Geweke (1981) compared tests of independence of two covariance stationary multivariate time series and had shown that the approximate slopes of regression tests are at least as great as those based on residuals of univariate ARIMA models. Thisted and Wecker (1981) provided some principles for shrinkage estimators and they examined estimation and prediction of multiple time series through shrinkage estimators and applied the results to the problem of demand estimation in an inventory control setting. Poskitt (1982) considered the development and application of diagnostic checks for vector linear time series models and developed a test procedure based on Lagrangian multiplier principle and its distribution. Portmanteau tests for model adequacy were also examined by him.

Sakai (1983) had shown that the autocovariance matrices of a stationary multivariate time series could be uniquely characterized by a sequence of normalized partial autocorrelation matrices having singular values less than unity. Reinsel (1983) introduced methods for modelling multivariate AR time series in terms of a smaller number of index series that were chosen to provide complete summary. He discussed maximum likelihood method of estimation, asymptotic properties of the estimators of the coefficients that determine the index variable and the corresponding AR coefficients. Geweke (1984) provided measures for linear dependence and feed back for two multiple time series conditional on a third. Estimates of these measures provided by him are straight forward. Li (1985) derived the asymptotic distribution of residual autocorrelations in multivariate autoregressive index models and discussed about a generalization of the classical portmanteau statistic for checking model adequacy. Velu *et. al.* (1986) investigated reduced rank coefficient models for multiple time series. He studied multivariate AR processes that have a structure to that of classical multivariate reduced rank regression and derived estimation methods. Koch and Yang (1986) developed a method for testing the independence of two time series that account for a pattern in the cross correlation function. Pena and Box (1987) described how to identify hidden factors in multivariate time series processes. They developed a methodology for the identification of the number of factors and to build a simplifying transformation to represent the series using eigen vectors of the covariance matrices. Hannan (1987) considered the problem of approximating the structure of a stationary vector time series by rational transfer functions. He discussed the structure and co-ordination of such systems together with some deterministic approximation theory. He also gave algorithms for real time calculation of the estimates.

Ahn and Reinsel (1988) considered nested reduced rank AR models in order to simplify and provide a more detailed description about the structure of the multivariate time series and to reduce parameters in time series modelling. They suggested a canonical variable transformation that produces simpler structure in the model and illustrated how different components of the vector series depend on past lagged values. Ahn (1988) derived the asymptotic distribution with structured parameterization, which is a form employed in multivariate time series modelling to achieve parsimony. Eakin (1988) considered estimation and testing of vector AR coefficients in panel data and applied the technique to analyze the dynamic relationships between wages and hours worked in two samples of American males. Fountis and Dickey (1989) developed a test procedure based on the largest eigen value of the matrix equation used to determine stationarity of a multiple AR time series. Li and Hui (1989) proposed a robust estimation procedure for multiple time series models by robustifying the residual autocovariances in the estimation equation. They derived asymptotic distribution of these estimators and a portmanteau statistic for diagnostic checking. Roy (1989) gave a proof of the asymptotic joint normality of finite set of serial correlations of a multivariate second order stationary time series. He derived a formula for the asymptotic covariance between two serial correlations. Liang and Zeger (1989) proposed a logistic model for multivariate binary time series and used it to describe a Markovian model for the vector of time series. Thompson *et. al.*(1989) used parametric curves to approximate the annual age-specific fertility rate and a multivariate time series model to forecast the curve parameters, that yield forecasts of future fertility curves. They used these forecasted fertility curves to compute age-specific fertility rate forecasts. Ali (1989) presented a modified test statistic to test autocorrelations and randomness in multiple time series. He obtained these statistics when the means and covariances in the standardization were replaced by the exact means and covariances and under the assumption that the time series is Gaussian.

Degerine (1990) suggested a definition for partial autocorrelation function for a multivariate stationary time series through a canonical analysis of forward and backward innovations. Tsay and Tiao (1990) considered the asymptotic properties of a non-stationary multivariate time series with characteristic roots on the unit circle. He derived the limiting distributions of certain statistics that are useful in understanding a non-stationary process and used it to establish the consistency properties of ordinary least square estimates of various autoregressions of vector processes. Stergiou (1991) used vector autoregressions to describe and forecast sardine anchovy complex in the eastern Mediterranean. Using a VAR(6) model with two variables he could explain 98% and 72% of variability in the catches of anchovy and sardines respectively. Grubb (1992) analyzed an index of monthly price of flour at three cities in US using multivariate time series models to explore the relationships between them and to discover the structure responsible for their movements. Based on the method suggested by Tsay and Tiao they fitted a VAR(1) model for the data.

Materials and Methods:

In general for a stationary autoregressive model of order p, that is VAR(p) model, the auto and cross correlations will decay gradually to zero as the lag increases. Often models of low orders provide adequate approximations. Sample estimate of elements of the lag l cross correlation matrix based on a sample of size T is computed as

$$\hat{\rho}_{ij}(l) = \frac{\sum_{t=1}^{T-l}(Z_{it}-\bar{Z}_i)(Z_{j(t+l)}-\bar{Z}_j)}{\sqrt{\left\{\sum_{t=1}^{T-l}(Z_{it}-\bar{Z}_i)^2\right\}\left\{\sum_{t=1}^{T-l}(Z_{jt}-\bar{Z}_j)^2\right\}}} \quad \text{for } i,j=1,\cdots,k;\ l=0,\pm1,\pm2,\cdots,$$

where $\bar{z}_i$ is the sample mean of the i^{th} component series. The sample cross correlations can be used to identify low order vector moving average models. For a large sample size T, under white noise assumption, $\hat{\rho}_{ij}(l)$'s are expected to be distributed as normal deviate with mean zero and approximate variance $1/T$ and this property was used to test the significance of individual sample cross correlations. Also, we can form a sequence of matrices for different lags *0,1,2,...* with the sign '+' or '-' or '.' at the $(i,j)^{th}$ location of the l^{th} matrix if $\hat{\rho}_{ij}(l) > 2/\sqrt{T}$ or $\hat{\rho}_{ij}(l) < -2/\sqrt{T}$ or $-2/\sqrt{T} \le \hat{\rho}_{ij}(l) \le 2/\sqrt{T}$ respectively and these matrices can be used as a useful tool for the identification of a small order VMA(q) model. Under white noise assumption to test the combined significance of the elements of sample cross-correlation matrix $\hat{\rho}(l)$ for different lags l, the χ^2 statistic defines as $\chi^2_{k^2} = T^2(T-l)^{-1}tr\left\{\hat{\rho}(l)\hat{\rho}(0)^{-1}\hat{\rho}(l)'\hat{\rho}(0)^{-1}\right\}$ having k^2 degrees of freedom was used.

The conditions required for stationarity of a VAR(p) model can be brought out by considering an equivalent VAR(1) representation (Reinsel, 1993). By repeated substitution for $z_1, z_2, \cdots, z_t$ in the VAR(1) model $z_t = \Phi\, z_{t-1} + \varepsilon_t$, the model can be restructured into the form

$$z_t = (I + \Phi + \Phi^2 + \cdots + \Phi^{t-1})\mu + \Phi^t z_0 + \sum_{i=0}^{t-1} \Phi^i \varepsilon_{t-i}$$

Hence the process vectors $Z_1, Z_2, \cdots, Z_t$ are uniquely determined by Z_0 the initial value of the process and the sequence of innovation vectors. If all eigen values of Φ have modulus less than unity, then the process $\{Z_t\}$ is a well defined stochastic process and Z_t can be expressed as $Z_t = \delta + \sum_{i=0}^{\infty} \Phi^i \varepsilon_{t-i}$ for $t = 0, \pm 1, \pm 2, \cdots$ where $\delta = (I - \Phi)^{-1}\mu$. This condition is equivalent to $det(I - \Phi x) \neq 0$ for $|z| \leq 1$. Hence the condition for stability/stationarity of a VAR(1) process is that the roots of the determinental equation $det(I - \Phi x) = 0$ have all its roots out side the unit circle or equivalently all the eigen values of Φ are less than one in absolute value. The general VAR(p) model $Z_t = \delta + \Phi_1 Z_{t-1} + \cdots + \Phi_p Z_{t-p} + \varepsilon_t$ can be brought to an equivalent VAR(1) model of a *kp* dimensional process as $Y_t = \eta + \Phi Y_{t-1} + \gamma_t$ where $Y_t = (Z_t', Z_{t-1}', \cdots, Z_{t-p+1}')'$, $\eta = (\mu', 0', \cdots, 0')'$, $\gamma_t = (\varepsilon_t', 0', \cdots, 0')'$ which are column vectors of length *kp* and Φ is a *kp*$\times$ *kp* square matrix given by

$$\Phi = \begin{pmatrix} \Phi_1 & \Phi_2 & \cdots & \Phi_{p-1} & \Phi_p \\ \mathrm{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathrm{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathrm{I} & \mathbf{0} \end{pmatrix}$$

Based on this representation of VAR(p) model the condition for stationarity of the model is that all eigen values of the above matrix have modules value less than unity. Equivalently $det(\mathrm{I} - \Phi x) \neq 0$ for $|z| \leq 1$. Since $det(\mathrm{I} - \Phi x) = det(\mathrm{I} - \Phi_1 x - \cdots - \Phi_p x^p)$ the condition for stationarity of the VAR(p) model is that the determinantal polynomial $det(\mathrm{I} - \Phi_1 x - \cdots - \Phi_p x^p) = 0$ have all its roots out side the unit circle.

In terms of the back shift operator B, the VAR(p) model can be written as $\Phi(B)Z_t = \mu + \varepsilon_t$ where $\Phi(B) = \mathrm{I} - \Phi_1 B - \cdots - \Phi_p B^p$. Now consider a function $\psi(B) = \Phi(B)^{-1}$ so that $\psi(B)\Phi(B) = \mathrm{I}$ and $\psi(B) = \sum_{j=0}^{\infty} \psi_j B^j$. Operating this function on the VAR(p) model gives, $\psi(B)\Phi(B)Z_t = \psi(B)\mu + \psi(B)\varepsilon_t$ which reduces to $Z_t = \psi(B)\mu + \psi(B)\varepsilon_t$. But $\psi(B)\mu = \sum_{j=0}^{\infty} \psi_j B^j \mu = (\sum_{j=0}^{\infty} \psi_j)\mu$ because $B^j \mu = \mu$. If we write $\delta = \sum_{j=0}^{\infty} \psi_j$ then we can write the VAR(p) model as $Z_t = \delta + \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j}$ which can exists only when the sequence $\{\psi_j\}$ converges which is the condition for invertibility of the process. The relation between these coefficient matrices and parameter matrices can be obtained by expanding $\psi(B)\Phi(B) = I$ and then equating coefficients of powers of B, as $\psi_j = \Phi_1\psi_{j-1} + \Phi_2\psi_{j-2} + \cdots + \Phi_p\psi_{j-p}$ with $\psi_0 = I$ and $\psi_j = 0$ for $j < 0$.

Similar to the univariate case the covariance matrices of a VAR(p) model satisfy the following Yule-Walker relations.

$$\Gamma(l) = \sum_{j=1}^{p} \Gamma(l-j)\Phi_j' \text{ for } l = 1,2,\cdots$$

with $\Gamma(0) = \sum_{j=1}^{p} \Gamma(-j)\Phi_j' + \Sigma$ where $\Sigma = E(\varepsilon_t \varepsilon_t')$. These equations for l=0,1,...,p can be used to calculate the theoretical cross-covariance matrices $\Gamma(l)$ in terms of the parameter matrices $\Phi_1,\cdots,\Phi_p$ and Σ of the VAR(p) model. Conversely, the estimates of the parameter matrices can be calculated using the estimates of cross-covariance matrices. Let $\Gamma_{(p)} = (\Gamma(1)', \Gamma(2)', \cdots, \Gamma(p)')'$, $\Phi = (\Phi_1, \Phi_2, \cdots, \Phi_p)'$ and Γ_p a $kp \times kp$ matrix with $\Gamma(i-j)$ as its $(i,j)^{th}$ block element. Then the solution for Φ is obtained as $\Phi = \Gamma_p^{-1}\Gamma_{(p)}$ and $\Sigma = \Gamma(0) - \Phi'\Gamma_p\Phi$.

In the present study the parameter matrices of VAR(p) models are estimated by generalized least square method (Reinsel 1993). The vector AR(p) model $(Z_t - \mu) = \sum_{j=1}^{p} \Phi_j (Z_{t-j} - \mu) + \varepsilon_t$ can be expressed as $(Z_t - \mu) = \Phi_{(p)}\widetilde{X}_t + \varepsilon_t$ where $\widetilde{X}_t = ((Z_{t-1} - \mu)', \cdots, (Z_{t-p} - \mu)')'$ and $\Phi_{(p)} = (\Phi_1, \cdots, \Phi_p)'$. An equivalent form of the model is $Z_t = \delta + \sum_{j=1}^{p} \Phi_j Z_{t-j} + \varepsilon_t$ with $\delta = (I + \sum_{j=1}^{p} \Phi_j)\mu$ and the model can be rewritten as $Z_t = B' X_t + \varepsilon_t$ where $X_t = (1, Z'_{t-1}, \cdots, Z'_{t-p})'$ and $B = (\delta, \Phi_1, \cdots, \Phi_p)'$. If T is the sample size and $n = T\text{-}p$, then define an $n \times k$ matrices $Z = (Z_{p+1}, \cdots, Z_T)'$ and $\varepsilon = (\varepsilon_{p+1}, \cdots, \varepsilon_T)'$. Let X be a matrix of order $n \times (pk+1)$ with its t^{th} row as $Z'_{t,p} = (1, Z'_{t-1}, \cdots, Z'_{t-p})$ for $t = p+1, \cdots, T$. Then we have the relation $Y = XB + \varepsilon$ which is in the general form of a multivariate linear model and can be solved for **B** as $\hat{B} = (X'X)^{-1} X'Y$. The estimate of innovation dispersion matrix Σ is obtained as $\hat{\Sigma} = S_m / (n - \overline{kp+1})$ where $S_m = \sum_{t=p+1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_t$ and $\hat{\varepsilon}_t = Z_t - \hat{\delta} - \sum_{j=1}^{p} \hat{\Phi}_j Z_{t-j}$. For large sample sizes under stationarity and Gaussian assumption the approximate large sample distribution of $\hat{\phi}$ which is the estimate of $\phi = vec(\Phi_{(p)})$ is $N\left(\phi, \hat{\Sigma} \otimes \left(\tilde{X}'\tilde{X}\right)^{-1}\right)$ and this property was used to compute the standard errors of the estimates and for further testing.

For the selection of the order parameter p of vector autoregressive models different order selection criteria are used. If $\tilde{\Sigma}_{(p)}$ is the maximum likelihood estimator of the innovation dispersion matrix Σ obtained by fitting a VAR(p) model to the data, then the Final Prediction Error criterion (FPE criterion) is given by

$FPE(p) = \left(\frac{T + kp + 1}{T - kp - 1}\right)^k det(\tilde{\Sigma}_{(p)})$. The information criterion introduced by Akike is defined by $AIC_r = \frac{-2\log(maximized\ likelihood)}{T} + 2r$ where T is the sample size and r

is the number of parameters estimated for the model and it is approximated as $AIC_r \approx \log(|\tilde{\Sigma}_r|) + \frac{2r}{T} + c$ where $\tilde{\Sigma}_r$ is the maximum likelihood estimate of the innovation dispersion matrix Σ and c is a constant. Since there are $(pk^2 + k)$ parameters in the case of a general VAR(p) model the minimum AIC criterion used was calculated by $AIC(p) = \ln(|\tilde{\Sigma}_{(p)}|) + 2(pk^2 + k)/T$. Other two criteria used are the Baysean information criterion BIC suggested by Schwarz's (also known as SC criterion) and the HQ criterion proposed by Hannan and Qunin. These are given defined as $BIC_r = \log(|\tilde{\Sigma}_r|) + \frac{r\log(T)}{T}$ and $HQ_r = \log(|\tilde{\Sigma}_r|) + \frac{2r\log(\log(T))}{T}$. In the case of a general VAR(p) model these criterion were calculated using the formula $HQ(p) = \ln(|\tilde{\Sigma}_{(p)}|) + 2\ln(\ln(T))(pk^2 + k)/T$ and $SC(p) = \ln(|\tilde{\Sigma}_{(p)}|) + \ln(T)(pk^2 + k)/T$ respectively. The orders that yield minimum value for these criteria were selected as the suitable order for the model.

A likelihood ratio test was used for testing the null hypothesis $H_0 : \Phi_{p-i+1} = 0$ of the VAR(p) model against $H_1 : \Phi_{p-i+1} \neq 0$ given that $\Phi_{p-i+2} = \cdots = \Phi_p = 0$ and the test statistic is $\Lambda_{LR}(i) = T\left(\ln|\tilde{\Sigma}(p-i)| - \ln|\tilde{\Sigma}(p-i+1)|\right)$ where $\tilde{\Sigma}(p)$ denote the maximum likelihood estimate of Σ when a VAR(p) model was fitted to the vector time series of length T. This test statistic has an asymptotic χ^2 distribution with k^2 degrees of freedom.

A vector time series sequence $\{y_t\}$ with k components can be represented by a suitable vector autoregressive model of order m and as a solution of the Yule-Walker equations

$$\Gamma(l) = \sum_{j=1}^{m} \Gamma(l-j)\Phi'_{jm} \text{ for } l = 1,\cdots,m$$

we can estimate the coefficient matrices $\Phi_{1m},\cdots,\Phi_{mm}$ by approximating the vector time series by a VAR(p) model. The coefficient matrix Φ_{mm} is then known as partial autoregressive matrix of lag m. The sequence of *partial autoregressive m*atrices Φ_{mm} for $l = 1,\cdots,m$ has the characteristic property that if the process is a vector AR process of order p, then $\Phi_{pp} = \Phi_p$ and $\Phi_{mm} = 0$ for all $m > p$ so that these matrices will have the cut off property which is useful at the identification stage. To test for the order of a VAR model based on the partial autoregression matrices by testing $H_0 : \Phi_m = 0$, the Wald statistic is approximately equal to

$$Ttr\{\hat{\Phi}_{mm}\hat{\Sigma}^{*}_{m-1}\hat{\Phi}'_{mm}\hat{\Sigma}^{-1}_{m-1}\}$$

and it will have an approximate χ^2 with k^2 degrees of freedom. An asymptotically equivalent likelihood ratio test (LR test) statistic for this test is

$$M_m = -[N - mk - 1 - \tfrac{1}{2}]\log(U_m)$$

where $N = T - m$, $U_m = |S_m|/|S_{m-1}|$ and $S_m = \sum_{t=m+1}^{T} \hat{\varepsilon}_t\hat{\varepsilon}'_t$ is the residual sum of squares matrix.

For a vector time series sequence $\{y_t\}$ based on k elements, the *partial cross correlation* between the vectors y_t and y_{t-m} given the in between vectors $y_{t-1},\cdots,y_{t-m+1}$ is the cross correlation matrix between the elements of vectors y_t and y_{t-m}, after adjustment of both for their dependence on the elements of intervening vectors $y_{t-1},\cdots,y_{t-m+1}$ and it is denoted by P_m. For a stationary vector process, the partial cross correlation matrix P_m can be approximated as follows. Consider the error vector U_{mt} resulting from approximating the process by a VAR model of order *(m-1)*, given by

$$U_{mt} = y_t - \sum_{j=1}^{m-1} \Phi_{j(m-1)} y_{t-j} = y_t - \Phi'_{(m-1)} Y_{(m-1),(t-1)}$$

where $\Phi_{(m-1)} = (\Phi_{1(m-1)}, \Phi_{2(m-1)}, \cdots, \Phi_{(m-1)(m-1)})'$ and $Y_{(m-1),(t-1)} = (y'_{t-1}, \cdots, y'_{t-m+1})'$.

Similarly by considering a backward VAR model of order *(m-1)* as

$$y_{t-m} = \sum \Phi^*_{j(m-1)} y_{t-m+j} + U^*_{m,t-m}$$

we can obtain the backward AR coefficient matrices $\Phi^*_{1(m-1)}, \cdots, \Phi^*_{(m-1)(m-1)}$ as a solution from the set of equations

$$\Gamma(-l) = \sum_{j=1}^{m-1} \Gamma(j-l) \Phi^*_{j(m-1)} \quad \text{for} \quad l = 1, \cdots, (m-1)$$

and the backward error vectors $U^*_{m,t-m}$ can be obtained as

$$U^*_{m,t-m} = y_{t-m} - \sum_{j=1}^{m-1} \Phi^*_{j(m-1)} y_{t-m+j} = y_{t-m} - \Phi^{*\prime}_{(m-1)} Y_{m-1,t-1}$$

Partial cross correlation matrix at lag m is then defined as

$$P_m = Corr(U^*_{m,t-m}, U_{m,t}) = V_*(m)^{-1/2} E(U^*_{m,t-m} U'_{m,t}) V(m)^{-1/2}$$

where $V_{*}(m) = diag(\Sigma^{*}_{m-1})$, $V(m) = diag(\Sigma_{m-1})$,

$$\Sigma_{m-1} = Cov(U_{m,t}) = \Gamma(0) - \sum_{j=1}^{m-1} \Gamma(-j)\Phi'_{j(m-1)},$$

$$\Sigma^{*}_{m-1} = Cov(U^{*}_{m,t-m}) = \Gamma(0) - \sum_{j=1}^{m-1} \Gamma(m-j)\Phi^{*\prime}_{(m-j)(m-1)} \text{ and}$$

$$E(U^{*}_{m,t-m}U'_{m,t}) = \Gamma(m) - \Gamma^{*\prime}_{(m-1)}\Phi'_{(m-1)} = \Gamma(m) - \Phi^{*\prime}_{(m-1)}\Gamma_{(m-1)}$$

where $\Gamma_{(m-1)} = (\Gamma'(1), \cdots, \Gamma'(m-1))'$ and $\Gamma^{*}_{(m-1)} = (\Gamma(m-1), \cdots, \Gamma(1))'$. Under VAR($p$) model assumption, the elements of sample partial cross correlation matrix $\hat{P}_m$ are approximately normally distributed with zero means and variances $1/T$ for $m > p$ and this property was used to test their significance.

The *partial canonical correlation* at lag m of a stationary vector time series process $\{y_t\}$, with k elements, denoted by $1 \geq \rho_1(m) \geq \rho_2(m) \geq \cdots \geq \rho_k(m)$, are the canonical correlation between vectors y_t and y_{t-m} after adjusting for the dependence of these variables on the intervening values $y_{t-1}, \cdots, y_{t-m+1}$. Hence these are the canonical correlations between the residual series $U_{m,t}$ and $U^{*}_{m,t-m}$. The squared partial canonical correlations $\rho_i^2(m)$ are the eigen values of the matrix

$$\{Cov(U_{m,t})\}^{-1} E(U_{m,t}U^{*\prime}_{m,t-m})\{Cov(U^{*}_{m,t-m})\}^{-1} E(U^{*}_{m,t-m}U'_{m,t})$$

$$= \Sigma^{-1}_{m-1}\Phi_{mm}\Sigma^{*}_{m-1}\Phi'_{mm}$$

$$= \Phi^{*\prime}_{mm}\Phi'_{mm}$$

where Φ^{*}_{mm} is the coefficient matrix of y_t in the backward autoregression,

$$y_{t-m} = \sum_{j=1}^{m} \Phi^{*}_{jm} y_{t-m+j} + U^{*}_{m+1,t-m} \quad \text{of } y_{t-m} \text{ on } y_{t-m+1}, \cdots, y_t .$$

The examination of partial canonical correlations is useful in determination of certain reduced rank structures. The LR test statistic for testing the order of a VAR model can also be formulated in terms of the sample partial canonical correlations under the null hypothesis that $p < m$ as

$$M_m = -[N - mk - 1 - \tfrac{1}{2}] \sum_{i=1}^{k} \log(1 - \hat{\rho}_i^2(l))$$

The minimum mean square error (MSE) predictor for forecast horizon h at forecast origin t of a VAR(p) process $y_t = \delta + \Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} + \varepsilon_t$ is the conditional expected value $E_t(y_{t+h}) = \delta + \Phi_1 E_t(y_{t+h-1}) + \cdots + \Phi_p E_t(y_{t+h-p})$. To get the forecast the parameters of the model are replaced with their respective estimates so that we get

$$\hat{y}_t(h) = \hat{\delta} + \hat{\Phi}_1 \hat{y}_t(h-1) + \cdots + \hat{\Phi}_p \hat{y}_t(h-p)$$

where $\hat{y}_t(j) = y_{t+j}$ for $j \le 0$. The forecast error is then

$$y_{t+h} - \hat{y}_t(h) = \sum_{i=0}^{h-1} \Psi_i \varepsilon_{t+h-i} + [y_t(h) - \hat{y}_t(h)]$$

The MSE matrix of the forecast $\hat{y}_t(h)$ is then given by

$$\Sigma_{\hat{y}} = \sum_{i=0}^{h-1} \Psi_i \Sigma_\varepsilon \Psi_i' + \sum_{i=0}^{h-1} \sum_{j=0}^{h-1} tr\left[(B')^{h-i-1} \Gamma^{-1} B^{h-j-1} \Gamma\right] \Psi_i \Sigma_\varepsilon \Psi_j'$$

where $\Gamma = E(Z_t Z_t')$, $Z_t = (1, y_t', \cdots, y_{t-p+1}')'$, Ψ_i's are the coefficient matrices of the canonical representation of y_t and B is a $(kp+1) \times (kp+1)$ matrix defined by

$$B = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ \delta & \Phi_1 & \Phi_2 & \cdots & \Phi_{p-1} & \Phi_p \\ 0 & I_k & 0 & \cdots & 0 & 0 \\ 0 & 0 & I_k & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & I_k & 0 \end{pmatrix}$$

The matrices Ψ_i for $i = 0, 1, \cdots$ can be calculated recurssively using the relation

$$\Psi_i = \Phi_1 \Psi_{i-1} + \Phi_2 \Psi_{i-2} + \cdots + \Phi_p \Psi_{i-p}$$

$\Phi_1 \Psi_{i-1}$

where $\Psi_0 = I$ and $\Psi_i = 0$ for $i < 0$.

Results:

2.1. <u>VAR modelling of oil sardine, mackerel, anchovies and lesser sardine</u>

Quarterwise landings of oil sardine, mackerel, anchovies and lesser sardines during 1960-96 were used to estimate a suitable vector autoregressive model of appropriate order. These series were initially transformed by taking a 4 point moving sum of natural logarithm of quarterwise landings and then divided by corresponding standard deviations to standardize the variance. All the four species/groups are competitors for the same food all being mainly feeding on plankton. As a preliminary analysis, to examine the inter relations between the series, cross-correlations up to lag 24 and the covariance matrix S were computed for this vector time series. The cross correlation matrices are shown in table 2.1.1. and their significance is indicated using the notations suggested by Tiao and Box (1981) in brackets.

The maximum cross correlation observed at lag 0, was 0.509 between mackerel and anchovies. At lag 0, the cross correlation between oil sardine and anchovies was –0.412 which is also high. Cross correlation between oil sardine and mackerel were significant at all lags from 0 to 8. The maximum cross correlations between these two species observed were –0.382 at lag 4 and –0.534 at lag –12. Between oil sardine and anchovies, the maximum cross correlation observed were –0.412 at lag 0 and –0.633 at lag –10. Between oil sardine and lesser sardine maximum cross correlation observed was 0.223 at lag –4 and other significant cross correlations were at lags –8 and –2. Between mackerel and anchovies the highest cross correlations found were 0.509 at lag 0 and 0.598 at lag –2. Among the significant cross correlations between mackerel and lesser sardines the maximum observed value was 0.238 at lag 10. Significant cross correlations between anchovies and lesser sardines were at lags 11,12 and 13 with maximum of –0.185 at lags 11 and 12 and other significant cross correlations were for lags –15 to –24 with a maximum of –0.316 at lag 24. To test the combined significant of these cross correlation matrices at different lags the combined significance χ^2 statistic with 16 degrees of freedom were computed and these are given in table.2.1.2. From this table it can be seen that the combined significance χ^2 values are significant for all lags both at 5% and 1% levels of significance, which shows that there is significant inter relations between the landings of these species even at higher lags.

Partial cross correlations up to lag 16 were calculated for this vector time series and their significance were tested. The partial cross correlation matrices along with the significance of elements in bracket are shown in the table.2.1.3. From the partial cross correlation matrices it was seen that there is significant partial cross correlation between the landings of oil sardine, mackerel and anchovies at lags 1 and -1 but no significant partial cross correlation was found between the landings of these species with that of lesser sardine. The partial cross correlation between oil sardine and mackerel were found significant at lags -2, -1, 1 and 15. Oil sardine landings were found to have significant partial cross correlation with that of anchovies at lags -4, -1 and 1. The only significant partial cross correlation between the landings of oil sardine and lesser sardine was at lag 5. Mackerel landings had significant partial cross correlation with the landings of anchovies at lags -9 and -6. There was no significant partial cross correlation between the landings of mackerel and lesser sardine. The only significant partial cross correlation between the landings of anchovies and lesser sardine was at lag -12.

The squared partial canonical correlation coefficients up to lag 10 were calculated for the vector time series consisting of transformed landings of these four species along with the LR test statistic, which is a χ^2 with 16 degrees of freedom, for testing the significance of partial canonical correlations. These are given in table.2.1.4. The partial canonical correlations at lags 1,2,4,5 and 8 were found significant for this vector time series.

To select a suitable vector autoregressive model for the vector time series different minimization criteria were computed by estimating the residual dispersion matrices for different VAR(p) models for $p = 0, 1, \cdots\cdots, 10$. The estimate of residual dispersion determinant and the values of different criteria for different values of the order parameter p are given in table.2.1.5. As the order of the model is increased, it was found that the *FPE* and *AIC* criterion values were going on decreasing. The *HQ* criterion was found to have least values for orders 2 and 5 and the *SC* criterion have minimum value for order 2. Hence VAR(2) was tentatively fixed as the suitable model for the series.

A computer software was developed in C for the estimation of parameters of VAR(p) models, the source which is given in Appendix-1. Using this software the constant vector δ, coefficient matrices Φ_1, Φ_2 and the innovation dispersion matrix Σ which are the parameters of the VAR(2) model, $y_t = \delta + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \varepsilon_t$ identified for the series, were estimated. The estimates of the parameter matrices and the standard errors of the elements of the matrices were made and it is given below.

$$\hat{\delta} = (1.1824 \quad 0.8430 \quad 0.6068 \quad 1.9616)', SE(\hat{\delta}) = (0.4810 \quad 0.5652 \quad 0.5647 \quad 0.7290)$$

$$\hat{\Phi}_1 = \begin{pmatrix} 1.3958 & -0.0206 & 0.1105 & -0.0216 \\ 0.0951 & 1.2156 & 0.1884 & -0.0082 \\ -0.1550 & 0.0629 & 1.0348 & -0.0653 \\ -0.0021 & -0.1038 & 0.0448 & 1.1040 \end{pmatrix}, SE(\hat{\Phi}_1) = \begin{pmatrix} 0.0720 & 0.0658 & 0.0693 & 0.0553 \\ 0.0846 & 0.0774 & 0.0815 & 0.0649 \\ 0.0845 & 0.0773 & 0.0814 & 0.0649 \\ 0.1091 & 0.0998 & 0.1051 & 0.0838 \end{pmatrix}$$

$$\hat{\Phi}_2 = \begin{pmatrix} -0.4971 & 0.0082 & -0.1576 & 0.0459 \\ -0.1303 & -0.3233 & -0.1359 & -0.0026 \\ 0.1436 & -0.0542 & -0.0994 & 0.0708 \\ -0.0459 & 0.0889 & -0.0646 & -0.2067 \end{pmatrix} SE(\hat{\Phi}_2) = \begin{pmatrix} 0.0718 & 0.0654 & 0.0706 & 0.0551 \\ 0.0844 & 0.0769 & 0.0829 & 0.0647 \\ 0.0843 & 0.0768 & 0.0829 & 0.0647 \\ 0.1088 & 0.0991 & 0.1070 & 0.0835 \end{pmatrix}$$

$$\hat{\Sigma} = \begin{pmatrix} 0.0675 & -0.0008 & 0.0024 & 0.0131 \\ -0.0008 & 0.0932 & 0.0021 & 0.0114 \\ 0.0024 & 0.0021 & 0.0930 & 0.0035 \\ 0.0131 & 0.0114 & 0.0035 & 0.1550 \end{pmatrix}$$

Among the elements of the constant vector $\hat{\delta}$ the first and last elements were found to be significant. In $\hat{\Phi}_1$ all diagonal elements and the element in the second row and third column were the only significant estimates. Significant elements of $\hat{\Phi}_2$ were the diagonal elements except the third diagonal element and the element in its first row and third column. Hence the estimated model suggests that the landings of anchovies is significantly influenced by the lagged landings of oil sardine. To examine whether the estimated model satisfy the necessary stationary conditions eigen values of the VAR(1) equivalent characteristic matrix were evaluated for the model and these are given in table.2.1.6. It was found that all the eigen values of the characteristic matrix have absolute value less than unity and hence the estimated model is stationary. To examine the suitability of the fitted VAR model residual analysis was carried out by computing the cross correlation matrices of the residual series for lags $l = 1,\cdots,24$ and their combined significance were tested by calculating the appropriate χ^2 statistics with 16 degrees of freedom. The values of these χ^2 statistics for different lags are given in table.2.1.2. Among individual residual cross correlations it was found that 5 out of the 16 cross correlations at lags 3 and 4 were significant and only very few values were significant at all other lags. By examining the estimate of innovation dispersion matrix, it was found that this model could explain 93% of variations in oil sardine landings, 91% each of the variations in mackerel and anchovies landings and 84% of the variations in lesser sardine landings. Observed values, forecasts made by the fitted VAR(2) model and standard error of forecasts are given in table.2.6.1. for different quarter of 1997 and 1998. Original forecasts obtained through retransformation are given in table.2.6.6.

2.2. VAR modelling with landings of anchovies, lesser sardine, Ribbon Fish and Catfish.

Quarterwise landings of anchovies, lesser sardines, ribbon fish and catfish in Kerala during the period 1960-96 were used to fit a suitable vector autoregressive model after transforming these series by taking a 4 point moving sum of natural log of landings and then standardized by dividing each time series by respective standard deviations. Among these four groups of marine fishes, anchovies and lesser sardine being plankton feeders compete each other for food and the other two groups prey up on anchovies and lesser sardines. The cross correlation matrices calculated for this vector time series are given in table.2.2.1.

From the cross correlation matrices it was seen that cross correlations between all these series were significant at lag zero except for that between anchovies and lesser sardine series. The maximum cross correlation observed at lag zero was –0.424 between anchovies and catfishes. Anchovies series was found to have significant negative cross correlation with catfishes series at lags –8 to 24 and the maximum cross correlation observed was –0.600 at lag 13. Cross correlations of anchovies series with ribbon fish series were negative and significant for the lags from –24 to 11 and the highest cross correlation was 0.347 at lag –11. Between lesser sardine and ribbon fish series cross correlations were found to be significant and positive at lags –11 to 10. The maximum cross correlation between this two series was 0.352 at lag –6. Lesser sardine series was found to have significant positive cross correlations with catfish series at lags –8 to 10 and 18 to 24. The maximum cross correlation between these series was 0.335 at lag 24. Significant positive cross correlations were found between ribbon fish and catfish series at lags –8 to 4 and the maximum value was 0.199 at lag –5. Out of the 384 elements in the cross correlation matrices of different lags 206 were found to be significant. The combined significance of cross correlation matrices at lags 0,1,...,24 were tested by computing the combined significance χ^2 with 16 degrees of freedom and these are given in the table.2.2.2. It was found that these χ^2 values are highly significant for all the lags from 0 to 24. This shows that there is considerable inter relation at different lags between these series.

Since the cross correlations at different lags will not be free from the influence of in-between values, partial cross correlation matrices were also computed up to lag 16 for this time series. Partial cross correlation matrices of different lags for this vector time series are given in table.2.2.3. indicating the significance of its elements using standard notations. It was found that at lag 1 all the partial cross correlations are significant expect for that between anchovies and lesser sardines and the maximum was –0.441 between anchovies and catfishes. Between anchovies and ribbon fishes the partial cross correlations were significant at lags 1,-1,-2,-3 and –13 with maximum of 0.312 at lag –1. At lag –3 the partial cross correlation was negative and at all other significant lags it was positive. Between anchovies and lesser sardines partial cross correlations were negative and significant at lags 8 and –15 with a maximum of –0.243 at lag 8. Partial cross correlation between anchovies and catfishes were significant and negative at lags 1, -1 and 16. The maximum partial cross correlation between these two groups was –0.441 at lag 1. Lesser sardines had significant partial cross correlation with ribbon fishes at lags 1 and –1 both being positive with maximum of 0.250 at lag –1. Lesser sardines and catfishes had significant and positive partial cross correlations at lags 1, -1 and 15 with 0.227 as the maximum at lag 1. Ribbon fishes and catfishes had significant and positive partial cross correlations at lags 1, -1 and 9 with maximum of 0.247 at lag 9. Out of 256 elements in the partial cross correlation matrices for lags 1 to 16 of this vector time series 33 elements were found significant. Squared partial canonical correlations up to lag 10 were also computed for this vector time series and it is given in table.2.2.4. It was found that except for lags 3, 4 and 7 these canonical correlations were significant at all lags.

Selection of an appropriate order of the required VAR model for the vector time series consisting transformed landings of anchovies, lesser sardine, ribbon fish and catfish was carried out by computing FPE, AIC, HQ and SC order selection criteria by estimating different VAR(p) models for values of $p = 1,\cdots,10$ and it is given in table.2.2.5. The FPE criterion was found to have a decreasing nature as the order of the model is increased so that the minimum FPE was corresponding to the model VAR(10). The AIC criterion also behaved in a similar manner and its minimum value was corresponding to the model VAR(10). The minimum value for HQ criterion was for the model VAR(2) and the SC criterion had the minimum value for VAR(1) model. These two models were then estimated and compared for final selection.

The expression for the VAR(2) model is $y_t = \delta + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \varepsilon_t$ and its parameter matrices are δ, Φ_1, Φ_2 and Σ, where Σ is the dispersion matrix for the innovation series $\{\varepsilon_t\}$. The estimates of parameters and their standard errors are given below.

$\hat{\delta} = (0.8239, 1.4387, -0.3973, 0.6180)'$, $SE(\hat{\delta}) = (0.4882, 0.6319, 0.5123, 0.3159)'$

$$\hat{\Phi}_1 = \begin{pmatrix} 1.0225 & -0.0835 & -0.1093 & -0.0062 \\ 0.0447 & 1.0943 & -0.0648 & 0.1948 \\ -0.1312 & -0.0713 & 1.2619 & 0.0171 \\ 0.0420 & 0.0198 & 0.0221 & 1.1617 \end{pmatrix}, \; SE(\hat{\Phi}_1) = \begin{pmatrix} 0.0815 & 0.0636 & 0.0727 & 0.1264 \\ 0.1055 & 0.0824 & 0.0941 & 0.1636 \\ 0.0855 & 0.0668 & 0.0763 & 0.1327 \\ 0.0527 & 0.0412 & 0.0470 & 0.0818 \end{pmatrix}$$

$$\hat{\Phi}_2 = \begin{pmatrix} -0.1001 & 0.0677 & 0.1613 & -0.0122 \\ -0.0682 & -0.2164 & 0.1180 & -0.1973 \\ 0.1918 & 0.0808 & -0.3739 & 0.0214 \\ -0.0943 & -0.0148 & -0.0037 & -0.2061 \end{pmatrix}, SE(\hat{\Phi}_2) = \begin{pmatrix} 0.0823 & 0.0635 & 0.0732 & 0.1265 \\ 0.1065 & 0.0822 & 0.0947 & 0.1637 \\ 0.0864 & 0.0666 & 0.0768 & 0.1327 \\ 0.0533 & 0.0411 & 0.0474 & 0.0818 \end{pmatrix}$$

$$\hat{\Sigma} = \begin{pmatrix} 0.0914 & -0.0004 & 0.0058 & -0.0077 \\ -0.0004 & 0.1532 & 0.0168 & 0.0003 \\ 0.0058 & 0.0168 & 0.1007 & 0.0091 \\ -0.0077 & 0.0003 & 0.0091 & 0.0383 \end{pmatrix}$$

The significance of elements of the coefficient matrices were tested using Students 't' and it was found that only the diagonal elements are significant in $\hat{\Phi}_1$ and in $\hat{\Phi}_2$ all diagonal elements and the elements at positions (1,3) and (3,1) are significant. Stationarity of the estimated model was tested by computing the eigen values of the VAR(1) equivalent characteristic matrix generated using the estimates of the AR coefficient matrices and these are given in table.2.2.6. From the table it can be seen that the absolute value of all the eight eigen values are less than unity which is the required condition for stationarity of the model. Hence the estimated model is stationary. To examine the suitability of the fitted VAR model, the residual analysis was carried out by computing the innovation vectors based on the fitted model and then calculating the cross correlation matrices up to lag 24 for the series of innovation vectors. The combined significance of these cross correlation matrices were tested using the approximate χ^2 with 16 degrees of freedom and these values are given in the table.2.2.2. Among these χ^2 for the residuals those at lags 4, 18 and 22 were found significant. Out of 386 elements of the cross correlation matrices of residuals 25 were found significant. This model could explain 90.86% of the variations in anchovies series, 84.68% of the variations in lesser sardine series, 89.93% of the variations in ribbon fish series and 96.17% of the variations in catfish series.

The VAR(1) model is $y_t = \delta + \Phi_1 y_{t-1} + \varepsilon_t$ and the estimates of parameter vector δ and parameter matrices Φ_1 and Σ with standard errors of elements of these vector and matrices are given below.

$\hat{\delta} = (0.6570, 1.1179, 0.1221, 0.5198)'$, $SE(\hat{\delta}) = (0.4872, 0.6357, 0.5442, 0.3167)'$

$$\hat{\Phi}_1 = \begin{pmatrix} 0.9404 & -0.0165 & 0.0320 & -0.0027 \\ -0.0182 & 0.9011 & 0.0445 & 0.0026 \\ 0.0244 & -0.0080 & 0.9259 & 0.0165 \\ -0.0502 & 0.0066 & 0.0159 & 0.9668 \end{pmatrix}, \; SE(\hat{\Phi}_1) = \begin{pmatrix} 0.0317 & 0.0270 & 0.0298 & 0.0314 \\ 0.0414 & 0.0352 & 0.0389 & 0.0409 \\ 0.0354 & 0.0302 & 0.0333 & 0.0350 \\ 0.0260 & 0.0176 & 0.0194 & 0.0204 \end{pmatrix}$$

$$\hat{\Sigma} = \begin{pmatrix} 0.0963 & -0.0002 & -0.0023 & -0.0073 \\ -0.0002 & 0.1640 & 0.0085 & 0.0031 \\ -0.0023 & 0.0085 & 0.1202 & 0.0074 \\ -0.0073 & 0.0031 & 0.0074 & 0.0407 \end{pmatrix}$$

Among the elements of the parameter matrices all elements of $\hat{\delta}$ are non-significant and in $\hat{\Phi}_1$ all diagonal elements and element in position (4,1) are significant. Stationarity of the estimated model was tested by calculating the eigen values of $\hat{\Phi}_1$ and it was found that all the eign values have modulus less than unity so that the estimated model is stationary. The eigen values are given in table.2.2.6. Using this model residual vectors were evaluated and cross correlation matrices up to lag 24 were calculated for the residual vector series. Among 384 elements of these matrices 28 elements were found to be significant. Combined significance of the elements of the residual cross correlation matrices were tested by calculating the χ^2 with 16 degrees of freedom and these are given in table.2.2.2. It was found that these χ^2 statistics were significant for lags 1, 4, 6, 13 and 22. The estimated VAR(1) model could explain 90.37% of variations in anchovies series. 83.60% of variations in lesser sardine series, 87.98% of variations in ribbon fish series and 95.93% of variations in catfish series. When the two models VAR(1) and VAR(2) fitted for this vector time series were compared, it was found that the VAR(2) model behaves better in terms of its capability to explain the variations in the component series and the significance of residual cross correlation matrices. Hence VAR(2) model was chosen as the best vector model to represent this vector time series. Quarterwise forecasts and standard errors of forecasts along with observed values for 1997 and 1998 are given in table.2.6.2. Original re-transformed values of the forecasts are given in table.2.6.6.

2.3. VAR modelling of mackerel, anchovies, tuna and penaeid prawns.

Time series data on quarterwise landings of mackerel, anchovies, tuna and penaeid prawns in Kerela during the period 1960-96 were initially transformed by taking a 4 point moving sum of natural log and then standardized by dividing each series with corresponding standard deviations. Mackerel, anchovies and penaeid prawns are plankton feeders and they compete each other for food where as tuna predate on these three group of marine species. The vector time series consisting of transformed landings of these species were then used to fit a suitable vector autoregressive model. Cross correlation matrices up to lag 24 were computed for this sample vector time series and these are given in table.2.3.1.

It was found from the cross correlation matrices that the cross correlations between these four species were positive at all lags and out of 384 elements in these matrices 353 were significant. At lag zero, there was significant, high and positive cross correlation between all the species and the maximum was 0.754 between anchovies and tuna series. Cross correlations between the landings of these two groups were positive, significant and high at all lags from –24 to 24 and the minimum cross correlation was 0.329 at lag 24. Between mackerel and anchovies series also the cross correlations were significant at all lags with 0.598 as the maximum cross correlation at lag –12 and the minimum was 0.183 at lag 24. Between mackerel and tuna cross correlations at all lags from –24 to 24 were significant and the maximum cross correlation was 0.566 at lag 0 and minimum was 0.189 at lag 24. Cross correlations between mackerel and penaeid prawns were found to be significant at lags from –24 to 16 and the maximum cross correlation observed was 0.597 at lag –7. Cross correlations between anchovies and penaeid prawns were significant at all lags except at lag 24. The highest observed cross correlation between them was 0.593 at lag –3. Between tuna and penaeid prawns the cross correlations were significant at lags from –24 to 15 with a maximum of 0.675 at lag 7. The combined significance of these cross correlation matrices were tested by computing the appropriate χ^2 statistic with 16 degrees of freedom and these are given in table.2.3.2. All these χ^2 values were found to be highly significant and hence there is significant inter-relation between different series considered in the vector time series.

To examine the independent lagged correlations that exist between component series belonging to the vector time series, the partial cross correlation matrices up to lag 16 were also computed and these matrices are given in table.2.3.3. indicating the significance of individual elements of the partial cross correlation matrices. Out of the 256 elements in the partial cross correlation matrices 39 were found significant. Partial cross correlations at lag 1 were found to be significant, high and positive between all the four series. The maximum partial cross correlation observed at this lag was 0.761 for the anchovies series with that of tuna. Between mackerel and anchovies series the partial cross correlations found significant are at lags 1, -1, –6, -9, -13 and 14. At lags 1, -1, –6 and –9 these partial cross correlations were positive and for other significant lags these were negative. Partial cross correlations between mackerel and tuna found significant were for lags 1, -1 and –5 and all these partial cross correlations were positive. Mackerel series had significant partial cross correlations with penaeid prawn series at lags 1, -1, –3, -7 and –13 and all these values except that at lag –13 were positive. The partial cross correlations of anchovies series with tuna series that were found significant are for lags 1 and –1 and these cross correlations were positive. The partial cross correlations that were found significant for the anchovies series with the penaeid prawn series were for lags 1, -1, –4, -8, and –10 out of which those at lags 1, -1 and –10 were positive. Partial cross correlations that were found significant between tuna and penaeid prawn series are for lags 1, -1, -2, -4 and –9. Among these significant values those at lags 1, -1 and -2 were positive and the rest negative.

For selection of the appropriate vector autoregressive model for this vector time series the FPE , AIC, HQ and SC order selection criteria were calculated for different values of the order parameter $p = 0, 1, \cdots, 10$ by estimating the residual dispersion matrix for different models and the values of these criteria for different models are given in table.2.3.4. It was found that both the FPE and AIC criterion values go on decreasing as the order is increased so that they suggest VAR(10) as a suitable model. The HQ criterion has minimum value for the VAR(6) model and VAR(1) model yielded the next minimum. For the SC criterion the minimum value was corresponding to the VAR(1) model. Hence VAR(1) was selected as the suitable model by considering parsimony and the HQ and SC criteria.

The expression for the VAR(1) model is $y_t = \delta + \Phi_1 y_{t-1} + \varepsilon_t$ with parameter vector δ and parameter matrices Φ_1 and Σ where Σ is the dispersion matrix for the independent and identically distributed innovation series $\{\varepsilon_t\}$. These parameters were estimated and the estimated vector and matrices and their standard errors are given below.

$\hat{\delta} = (-0.7531, 0.3099, -0.3981, 2.0793)'$, $SE(\hat{\delta}) = (0.5069, 0.4778, 0.3307, 0.5545)'$

$$\hat{\Phi}_1 = \begin{pmatrix} 0.9196 & 0.0771 & -0.0546 & 0.0550 \\ -0.0182 & 0.8862 & 0.0831 & 0.0220 \\ 0.0176 & 0.0617 & 0.9196 & 0.0100 \\ -0.0196 & 0.0243 & 0.0544 & 0.8697 \end{pmatrix}, \quad SE(\hat{\Phi}_1) = \begin{pmatrix} 0.0338 & 0.0445 & 0.0435 & 0.0323 \\ 0.0318 & 0.0419 & 0.0410 & 0.0305 \\ 0.0220 & 0.0290 & 0.0284 & 0.0211 \\ 0.0370 & 0.0487 & 0.0476 & 0.0354 \end{pmatrix}$$

$$\hat{\Sigma} = \begin{pmatrix} 0.1060 & 0.0040 & 0.0181 & -0.0134 \\ 0.0040 & 0.0942 & -0.0036 & 0.0243 \\ 0.0181 & -0.0036 & 0.0451 & -0.0160 \\ -0.0134 & 0.0243 & -0.0160 & 0.1268 \end{pmatrix}$$

In these estimates the last element of the constant vector $\hat{\delta}$ was significant and other elements were non-significant. In the coefficient matrix $\hat{\Phi}_1$, all diagonal elements and the elements in position (2,3) and (3,2) were significant. To examine whether the estimated VAR(1) model is stationary or not, eigen values of the estimated parameter matrix $\hat{\Phi}_1$ was evaluated and these are given in table.2.3.6. Since all these eigen values have absolute value less than unity, it satisfies the required stationary condition and hence the estimated model is stationary. Using the estimated model the residual series were generated and cross correlation matrices up to lag 24 were calculated for the residual series. Out of 384 elements of these matrices only 33 elements were found significant. To test the combined significance of the residual cross correlation matrices the approximate χ^2 statistic with 16 degrees of freedom were calculated for different lags and these are given in table.2.3.5. These statistics were significant for lags 1, 4, 6, 8, 13 and 19. The estimated model could explain 89.40% of variations in mackerel landings, 90.58% of the variations in anchovies landings, 95.49% of the variations in tuna landings and 87.32% of the variations in penaeid prawns landings. Observed values, forecasts and standard errors of forecasts computed using the estimated VAR(1) model are given in table.2.6.3 for different quarters of 1997 and 1998. Original values of forecasted obtainged through retransformation are given in table.2.6.6.

2.4. VAR modelling with the landings of oil sardine, anchovies, tuna and penaeid prawns.

Time series data on quarterwise landings of oil sardine, anchovies, tuna and penaeid prawns in Kerala during the period 1960-96 was initially transformed by taking a 4 point moving sum of natural logarithm of the landings and then standardized by dividing with corresponding standard deviations. The vector time series consisting of transformed landings of these four marine species/groups was then used to fit a suitable vector autoregressive model. Cross correlation matrices up to lag 24 were calculated for this sample vector time series and these are given in table.2.4.1. Among these species/groups tuna is the only carnivorous type of species group feeding on young ones of the other three where as the other species/groups are plankton feeders and compete each other for the same food resources.

From the cross correlation matrices it was found that at lag zero the maximum cross correlation was 0.754 between anchovies and tuna landings. Oil sardine series was found to have significant cross correlations with anchovies at lags from –24 to 9 and also at lag 21. The maximum cross correlation between these two series was –0.633 for lag –10 and all the significant cross correlations were negative. Oil sardine and tuna series had significant cross correlations between them at lags from –24 to 15 with a maximum of –0.505 at lag 18 and all the significant cross correlations were negative. The significant cross correlations of oil sardine series with penaeid prawns series were all negative and these are for lags from –24 to 6. The maximum cross correlation observed between these two series was –0.413 at lag –1. Out of 384 elements in these cross correlation matrices 315 elements were found to be significant. Combined significance of the cross correlation matrices were tested using the approximate χ^2 with 16 degrees of freedom values which for different lags are given in table.2.4.2. It was found that for all lags these χ^2 values are highly significant.

Partial cross correlations up to lag 16 were also computed for this vector time series and these are given in table.2.4.3. In these partial cross correlation matrices 39 elements out of a total of 256 elements were found to be significant. The maximum partial cross correlation observed was 0.761 at lag 1 between anchovies and tuna series.

Significant partial cross correlations that the oil sardine series had with anchovies series were –0.397 at lag 1 and –0.420 at lag –1. Oil sardine and tuna series had significant partial cross correlations at lags 1, -1, 2, 5, 13 and 15 with –0.374 as the maximum which is at lag –1. Significant partial cross correlations of oil sardine series with penaeid prawn series were 1, -1, -3 and -12 with maximum partial cross correlation of –0.413 at lag –1. Squared partial canonical correlations were also calculated for different lags using this vector time series data and these are given in table.2.4.4. along with the LR statistic for testing their significance. From the table it can be seen that these χ^2 values are significant for all lags from 1 to 9 except for that at lag 3. These analysis evidently show that there exist strong inter-relation between the series considered in the vector which can be exploited by modelling them together using vector time series models.

For identification of a suitable VAR model for this vector time series data, the different order selection criteria namely FPE, AIC, HQ and SC were calculated by estimating the residual dispersion matrix for different values of the order parameter $p = 1, 2, \cdots, 10$ and these are given in table.2.4.5. From the table it was found that the FPE and AIC criteria does not yield any suitable order as these criteria were found to go on decrease as the order is increased. The HQ criterion had minimum value for $p=5$ and the SC criterion had minimum value for $p=1$. But both HQ and SC criterion had second least values for $p=2$. Based on these results three models namely, VAR(1), VAR(2) and VAR(5) were considered for estimation and final selection was made based on their properties.

For the VAR(1) model $y_t = \delta + \Phi_1 y_{t-1} + \varepsilon_t$ the estimates of parameter vector δ, parameter matrix Φ_1, innovation dispersion matrix Σ and the standard errors of their elements were

$\hat{\delta} = (1.3549, 0.3452, -0.5359, 2.0756)'$, $SE(\hat{\delta}) = (0.6469, 0.6519, 0.4513, 0.7575)'$

$$\hat{\Phi}_1 = \begin{pmatrix} 0.9311 & -0.0001 & -0.0358 & -0.0264 \\ -0.0051 & 0.8827 & 0.0754 & 0.0191 \\ 0.0100 & 0.0655 & 0.9274 & 0.0155 \\ -0.0025 & 0.0213 & 0.0462 & 0.8668 \end{pmatrix}, \quad SE(\hat{\Phi}_1) = \begin{pmatrix} 0.0284 & 0.0420 & 0.0388 & 0.0305 \\ 0.0286 & 0.0423 & 0.0391 & 0.0307 \\ 0.0198 & 0.0293 & 0.0271 & 0.0213 \\ 0.0332 & 0.0491 & 0.0454 & 0.0357 \end{pmatrix}$$

$$\hat{\Sigma} = \begin{pmatrix} 0.0929 & -0.0013 & 0.0122 & -0.0157 \\ -0.0013 & 0.0943 & -0.0038 & 0.0246 \\ 0.0122 & -0.0038 & 0.0452 & -0.0161 \\ -0.0157 & 0.0246 & -0.0161 & 0.1272 \end{pmatrix}$$

The significant elements of estimates of parameter matrices were, the last element of constant vector $\hat{\delta}$ and all diagonal elements and the element in position (3,2) of $\hat{\Phi}_1$. Eigen values of the coefficient matrix $\hat{\Phi}_1$ are given in table.2.4.6. The absolute value of all the eigen values were less than unity and hence the estimated VAR(1) model is stationary. Combined significance of the residual cross correlation matrices up to lag 24 were tested using χ^2 with 16 degrees of freedom and those found significant were for lags 1, 4, 6 and 19. These χ^2 values are given in table.2.4.2. Out of a total of 384 elements in the residual cross correlation matrices 25 elements were found significant. The estimated VAR(1) model could explain 90.71% of variations in oil sardine landings, 90.57% of variations in anchovies landings, 95.48% of variations in tuna landings and 87.28% of variations in penaeid prawn landings.

The estimates of parameter vector δ, parameter matrices Φ_1, Φ_2 and the innovation dispersion matrix Σ for the VAR(2) model with the expression $y_t = \delta + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \varepsilon_t$ and standard errors of the estimates were

$\hat{\delta} = (1.4690, 0.3532, -0.6318, 2.2002)'$, $SE(\hat{\delta}) = (0.5586, 0.6430, 0.4416, 0.7490)'$

$$\hat{\Phi}_1 = \begin{pmatrix} 1.3893 & 0.1355 & -0.0094 & -0.0752 \\ -0.1475 & 1.0256 & 0.0103 & -0.0909 \\ 0.1507 & 0.0573 & 0.9446 & -0.0450 \\ -0.2147 & 0.0018 & 0.1873 & 1.0064 \end{pmatrix}, \ SE(\hat{\Phi}_1) = \begin{pmatrix} 0.0724 & 0.0717 & 0.1052 & 0.0627 \\ 0.0834 & 0.0825 & 0.1210 & 0.0722 \\ 0.0573 & 0.0567 & 0.0831 & 0.0496 \\ 0.0971 & 0.0961 & 0.1410 & 0.0841 \end{pmatrix}$$

$$\hat{\Phi}_2 = \begin{pmatrix} -0.4857 & -0.1658 & -0.0172 & 0.0682 \\ 0.1493 & -0.1645 & 0.0819 & 0.1117 \\ -0.1463 & 0.0086 & -0.0249 & 0.0707 \\ 0.2183 & 0.0281 & -0.1419 & -0.1535 \end{pmatrix}, \ SE(\hat{\Phi}_2) = \begin{pmatrix} 0.0714 & 0.0730 & 0.1040 & 0.0611 \\ 0.0822 & 0.0840 & 0.1197 & 0.0703 \\ 0.0564 & 0.0577 & 0.0822 & 0.0483 \\ 0.0957 & 0.0978 & 0.1395 & 0.0819 \end{pmatrix}$$

$$\hat{\Sigma} = \begin{pmatrix} 0.0672 & 0.0028 & 0.0045 & -0.0049 \\ 0.0028 & 0.0890 & -0.0021 & 0.0231 \\ 0.0045 & -0.0021 & 0.0420 & -0.0116 \\ -0.0049 & 0.0231 & -0.0116 & 0.1208 \end{pmatrix}$$

Significant elements in the estimate of parameters of the VAR(2) model were, first and last elements of $\hat{\delta}$, diagonal elements and elements in positions (3,1) and (4,1) of $\hat{\Phi}_1$ and elements in positions (1,1), (1,2), (3,1) and (3,4) of $\hat{\Phi}_2$. The estimated model was stationary as the absolute values of all the eigen values of the characteristic VAR(1) equivalent matrix were less than unity. The egien values of the characteristic matrix are given in table.2.4.6. For the residual cross correlation matrices up to lag 24 computed for this model, the combined significance χ^2 statistic were found significant for lags 3, 4, 6, 19 and 21. The values of the χ^2 statistic for different lags are given in table.2.4.2. Among 384 elements in these residual cross correlation matrices 28 elements were found significant. The estimated VAR(2) model explained 93.28% of the variations in oil sardine landings, 91.10% of variations in anchovies landings, 95.80% of variations in tuna landings and 87.92% of variations in penaeid prawn landings.

The expression for the vector autoregressive model of order 5, VAR(5) is $y_t = \delta + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \Phi_3 y_{t-3} + \Phi_4 y_{t-4} + \Phi_5 y_{t-5} + \varepsilon_t$ with parameters, the constant vector δ, coefficient matrices $\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5$ and innovation dispersion matrix Σ. Estimates of these parameters and their standard errors were made and these are given below.

$$\hat{\delta} = (1.2038, 0.2332, -1.0425, 1.8105)', \quad SE(\hat{\delta}) = (0.5598, 0.6376, 0.4223, 0.7825)'$$

$$\hat{\Phi}_1 = \begin{pmatrix} 1.2890 & 0.1108 & -0.0698 & -0.0802 \\ -0.1575 & 1.0813 & -0.0452 & -0.0854 \\ 0.0605 & 0.0045 & 0.9657 & -0.0524 \\ -0.2573 & 0.0546 & -0.0119 & 1.0579 \end{pmatrix}, \; SE(\hat{\Phi}_1) = \begin{pmatrix} 0.0786 & 0.0692 & 0.0977 & 0.0553 \\ 0.0895 & 0.0788 & 0.1112 & 0.0630 \\ 0.0592 & 0.0522 & 0.0737 & 0.0417 \\ 0.1098 & 0.0967 & 0.1365 & 0.0773 \end{pmatrix}$$

$$\hat{\Phi}_2 = \begin{pmatrix} -0.2477 & -0.1292 & -0.0156 & -0.0162 \\ 0.2346 & -0.0680 & 0.3475 & 0.1037 \\ -0.0775 & 0.1305 & -0.0019 & 0.0880 \\ 0.3452 & -0.0022 & -0.0511 & -0.2578 \end{pmatrix}, SE(\hat{\Phi}_2) = \begin{pmatrix} 0.1254 & 0.0972 & 0.1326 & 0.0813 \\ 0.1428 & 0.1107 & 0.1510 & 0.0926 \\ 0.0946 & 0.0733 & 0.1000 & 0.0613 \\ 0.1753 & 0.1358 & 0.1853 & 0.1137 \end{pmatrix}$$

$$\hat{\Phi}_3 = \begin{pmatrix} 0.0452 & 0.0546 & 0.0099 & 0.1116 \\ 0.0352 & 0.0330 & -0.3806 & 0.0849 \\ 0.1432 & -0.0901 & 0.0264 & -0.1153 \\ -0.2252 & 0.0455 & -0.0223 & 0.1787 \end{pmatrix}, \; SE(\hat{\Phi}_3) = \begin{pmatrix} 0.1291 & 0.0951 & 0.1328 & 0.0826 \\ 0.1471 & 0.1083 & 0.1513 & 0.0940 \\ 0.0974 & 0.0718 & 0.1002 & 0.0613 \\ 0.1805 & 0.1330 & 0.1857 & 0.1154 \end{pmatrix}$$

$$\hat{\Phi}_4 = \begin{pmatrix} -0.4944 & -0.0050 & 0.1284 & -0.0496 \\ -0.1403 & -0.5088 & 0.3284 & -0.0145 \\ -0.1899 & -0.0478 & -0.5099 & 0.0952 \\ 0.2698 & -0.0855 & 0.2977 & -0.4498 \end{pmatrix}, SE(\hat{\Phi}_4) = \begin{pmatrix} 0.1280 & 0.0932 & 0.1337 & 0.0827 \\ 0.1458 & 0.1062 & 0.1522 & 0.0942 \\ 0.0966 & 0.0703 & 0.1008 & 0.0624 \\ 0.1789 & 0.1303 & 0.1868 & 0.1156 \end{pmatrix}$$

$$\hat{\Phi}_5 = \begin{pmatrix} 0.3211 & -0.1234 & -0.0466 & 0.0571 \\ 0.0194 & 0.3473 & -0.1827 & -0.0605 \\ 0.0776 & 0.0506 & 0.4495 & 0.0334 \\ -0.1572 & -0.0080 & -0.1995 & 0.3823 \end{pmatrix}, \ SE(\hat{\Phi}_5) = \begin{pmatrix} 0.0805 & 0.0700 & 0.0940 & 0.0578 \\ 0.0917 & 0.0797 & 0.1071 & 0.0658 \\ 0.0607 & 0.0528 & 0.0709 & 0.0436 \\ 0.1125 & 0.0978 & 0.1314 & 0.0807 \end{pmatrix}$$

$$\hat{\Sigma} = \begin{pmatrix} 0.0513 & -0.0017 & 0.0039 & -0.0086 \\ -0.0017 & 0.0665 & 0.0010 & 0.0160 \\ 0.0039 & 0.0010 & 0.0292 & -0.0058 \\ -0.0086 & 0.0160 & -0.0058 & 0.1002 \end{pmatrix}$$

Among the parameter estimates three elements of the constant vector $\hat{\delta}$ were significant. The significant elements in the estimates of coefficient matrices were, all diagonal elements and the element in position (4,1) of $\hat{\Phi}_1$; elements in positions (1,1), (2,3), (4,1), (4,4) of $\hat{\Phi}_2$; element in position (2,3) of $\hat{\Phi}_3$; elements in positions (1,1), (2,2), (2,3), (3,1), (3,3) and (4,4) of $\hat{\Phi}_4$; all the diagonal elements of $\hat{\Phi}_5$. Eigen values of the characteristic VAR(1) equivalent matrix were evaluated to examine whether the model is stationary or not and these are given in table.2.4.6. All the 20 eigen values were found to have absolute value less than unity and hence the estimated VAR(5) model is stationary. Suitability of this model was tested by evaluating the residual vectors and their cross correlations up to lag 24. Out of 384 elements in the residual cross correlation matrices only 17 elements were found to be significant. Combined significance of the residual cross correlation matrices were tested by calculating the χ^2 with 16 degrees of freedom for different lags and these are given in table.2.4.2. These χ^2 values were found to be significant for lags 8, 19 and 21. The VAR(5) model fitted to the vector time series was found to explain 94.87%, 93.35%, 97.08% and 89.98% respectively of the variations in the component series belonging to the vector time series studied.

By comparing these three models for their properties it was seen that though VAR(5) model has more number of parameters it behaves better than the other two models in terms of lesser number of significant cross correlations, both for the individual elements of residual cross correlation matrices and their combined significance. Also it is capable of explaining the variations in the component series more efficiently than other models. Hence this model was selected as the final model for representing this vector time series. Quarterwise forecasts and observed values for 1997 and 1998 along with standard errors of forecasts are given in table.2.6.4. Original re-transformed values of forecasts are given in table.2.6.6.

2.5. VAR modelling with landings of elasmobranchs, oil sardine, mackerel and seer fish.

Time series data on quarterwise landings of these species/groups in Kerala during 1960-96 period were initially subjected to transformation by taking a 4 point moving sum of natural logarithm of the landings and then standardized by dividing each series with corresponding standard deviations. Among these species/groups oil sardine and mackerel compete each other for the same food resources both being plankton feeders. Elasmobranchs and seer fishes are carnivorous type of fishes and they predate on oil sardine and mackerel. The vector time series consisting of transformed landings of elasmobranchs, oil sardine, mackerel and seer fishes were then used for the analysis. Cross correlation matrices up to lag 24 were calculated for the vector time series and it is given in table.2.5.1. Out of 384 elements in the cross correlation matrices 271 elements were found to be significant. For testing the combined significance of elements of cross correlation matrices the χ^2 statistic with 16 degrees of freedom were calculated for each lag and it is given in table.2.5.2. All these χ^2 statistics were found to be highly significant which indicate that the component series are well inter-related.

The maximum cross correlation observed at lag zero was 0.421 between mackerel and seer fish series. Among all the elements of these cross correlation matrices the maximum value observed was 0.504 which corresponds to the cross correlation of seer fish series with that of mackerel for lag –9. Elasmobranchs had positive and significant cross correlations with oil sardine at lags –24 to –17 and –7 to 22. The maximum cross correlation between these two was 0.315 at lags –3 and 6. Cross correlations of elasmobranchs series with mackerel series were found to be significant and negative for lags –24 to –16, -7 to –2 and 0 to 17, the maximum cross correlation being –0.382 at lag 5. Significant cross correlations of elasmobranchs series with that of seer fish were for lags –24 to –1 and these cross correlations were negative. The maximum cross correlation observed between these two groups was –0.398 at lag –12. Oil sardine series have significant cross correlations with that of mackerel at algs –24 to 12 and all these cross correlations were negative. The maximum cross correlation observed between oil sardine and mackerel series was –0.382 at lag 4. Cross correlations of oil sardine with seer fishes were found to be negative and significant at lags –24 to 10 and the maximum cross correlation observed between them was –0.454 at lag –7. Mackerel series was found to have significant and positive cross correlation with seer fish series at all lags and the maximum of these cross correlations was 0.504 for lag –9.

Partial cross correlation matrices up to lag 16 were also computed for this vector times series and these matrices are given in table.2.5.3. In these 16 partial cross correlation matrices 34 elements were found to be significant out of a total of 256 elements and 14 of the significant ones belong to lag 1 partial cross correlation matrix. Significant partial cross correlations of elasmobranchs with oil sardine were for lags –10, -1 and 1 with a maximum of 0.264 at lag –1. All these partial cross correlations were positive. Elasmobranchs and mackerel series had negative and significant partial cross correlations for lags –7, -2, 1, 2 maximum being –0.204 for lag 1. Partial cross correlations of elasmobranchs with seer fish were negative and significant for lags –7, -1 and 8 with –0.186 as the maximum at lag –1. At lags –2, -1, 1 and 9 the partial cross correlation of oil sardine with mackerel were significant and the maximum was –0.354 for lag 1. Oil sardine and seer fish series had significant partial cross correlations at lags –1, 1 and 7. The maximum partial cross correlation between them was –0.348 at lag –1. Significant values of the partial cross correlations between mackerel and seer fish series were found for lags 1 and –1, the maximum being 0.418 at lag –1. Squared partial canonical correlations up to lag 10 were also computed for this sample vector time series and these are given in table.2.5.4. along with the LR statistic for testing the significance of these partial canonical correlations. It was found that the LR statistic which is a χ^2 with 16 degrees of freedom were significant for lags 1, 2, 5, 7 and 9. These analysis show that there exist strong inter-relation between the components of the vector time series considered.

For selection of a suitable order for the VAR model to be fitted for this vector time series, the FPE, AIC, HQ and SC order selection criteria were calculated by estimating the residual dispersion matrix for different values of the order parameter $p = 1, 2, \cdots, 10$. The values of these criteria for different values of the order parameter are given in table.2.5.5. From the table it was seen that the FPE and AIC criterion values keep on decreasing as the order of the model is increased and hence these two criteria do not yield any suitable model. For both the HQ and SC criterion the minimum values of the criterion were corresponding to the value of the order parameter $p = 2$ so that VAR(2) model was taken as the suitable model to represent this vector time series. The parameters δ, Φ_1, Φ_2 and Σ of the VAR(2) model having the expression $y_t = \delta + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \varepsilon_t$ were estimated and these estimates are given below along with standard errors of each parameter estimates.

$$\hat{\delta} = (4.7990, 0.7564, 2.1302, -0.2324)', \quad SE(\hat{\delta}) = (1.1716, 0.6609, 0.7476, 0.6151)$$

$$\hat{\Phi}_1 = \begin{pmatrix} 1.1640 & -0.0239 & 0.2116 & -0.1492 \\ -0.0438 & 1.3883 & -0.0375 & 0.1751 \\ 0.1382 & 0.1308 & 1.2025 & -0.2406 \\ 0.1162 & 0.1365 & -0.0546 & 1.0567 \end{pmatrix}, \quad SE(\hat{\Phi}_1) = \begin{pmatrix} 0.0803 & 0.1261 & 0.1172 & 0.1589 \\ 0.0453 & 0.0711 & 0.0661 & 0.0896 \\ 0.0512 & 0.0805 & 0.0748 & 0.1014 \\ 0.0422 & 0.0662 & 0.0615 & 0.0834 \end{pmatrix}$$

$$\hat{\Phi}_2 = \begin{pmatrix} -0.3741 & 0.0867 & -0.2167 & 0.0980 \\ 0.0680 & -0.4852 & 0.0162 & -0.2136 \\ -0.1881 & -0.1667 & -0.3063 & 0.2757 \\ -0.0900 & -0.1619 & 0.0775 & -0.1025 \end{pmatrix}, \quad SE(\hat{\Phi}_2) = \begin{pmatrix} 0.0791 & 0.1269 & 0.1173 & 0.1599 \\ 0.0446 & 0.0716 & 0.0662 & 0.0902 \\ 0.0505 & 0.0810 & 0.0749 & 0.1020 \\ 0.0415 & 0.0666 & 0.0616 & 0.0839 \end{pmatrix}$$

$$\hat{\Sigma} = \begin{pmatrix} 0.2097 & -0.0098 & -0.0032 & 0.0267 \\ -0.0098 & 0.0667 & 0.0062 & -0.0042 \\ -0.0032 & 0.0062 & 0.0854 & 0.0102 \\ 0.0267 & -0.0044 & 0.0102 & 0.0578 \end{pmatrix}$$

When the estimates of elements of the parameter matrices were tested using their respective standard errors it was found that in the estimate of the constant vector $\hat{\delta}$ the first and third elements were significant. All the diagonal elements and elements in positions (3.1), (3,4), (4,1) and (4,2) were found significant in the estimate of first coefficient matrix $\hat{\Phi}_1$. In $\hat{\Phi}_2$ also all the diagonal elements were found significant. Other significant elements of $\hat{\Phi}_2$ were those in positions (3,1), (3,2), (3,4), (4,1) and (4,2). To test the stationarity of the estimated model eigen values of the VAR(1) equivalent characteristic matrix were calculated which are given in table.2.5.6. All the eight eigen values were found to have absolute value less than unity which is the required condition for stationarity of the model. Hence the estimated VAR(2) model is stationary. To examine the adequacy of the fitted model the residual vector series were evaluated using the estimated VAR(2) model and cross correlation matrices up to lag 24 were calculated for the residual vector series. Among the elements of the residual cross correlation matrices only 19 elements out of a total of 384 elements were significant. The combined significance of these residual cross correlation matrices were tested using χ^2 with 16 degrees of freedom. The computed values of this χ^2 for different lags are given in table.2.5.2. These combined significance χ^2 statistics were found to be significant for lags3, 4, 11,12 and 14. The estimated VAR(2) model could explain 79.06% of the variations in elasmobrancs landings, 93.41% of the variations in oil sardine landings, 91.47% of the variations in mackerel landings and 94.23% of the variations in seer fish landings. Using the fitted VAR(2) model, quarterwise forecasts were made and these are given in table.2.6.5. along with observed values and standard errors of forecasts. The original forecasts obtained through retransformation are given in table.2.6.6.

Discussion

From the VAR(2) model fitted to the vector time series consisting of transformed landings of oil sardine, mackerel, anchovies and lesser sardines we arrive at the individual models for these series as,

$$y_{1t} = 1.1824 + 1.3958\, y_{1,t-1} - 0.0206\, y_{2,t-1} + 0.1105\, y_{3,t-1} - 0.0216\, y_{4,t-1} - 0.4971\, y_{1,t-2} + 0.0082\, y_{2,t-2} - 0.1576\, y_{3,t-2} + 0.0459\, y_{4,t-2} + \varepsilon_t,$$

$$y_{2t} = 0.8430 + 0.0951\, y_{1,t-1} + 1.2156\, y_{2,t-1} + 0.1884\, y_{3,t-1} - 0.0082\, y_{4,t-1} - 0.1303\, y_{1,t-2} - 0.3233\, y_{2,t-2} - 0.1359\, y_{3,t-2} - 0.0026\, y_{4,t-2} + \varepsilon_t,$$

$$y_{3,t} = 0.6068 - 0.1550\, y_{1,t-1} + 0.0629\, y_{2,t-1} + 1.0348\, y_{3,t-1} - 0.0653\, y_{4,t-1} + 0.1436\, y_{1,t-2} - 0.0542\, y_{2,t-2} - 0.0994\, y_{3,t-2} + 0.0708\, y_{4,t-2} + \varepsilon_t,$$

and

$$y_{4t} = 1.9616 - 0.0021\, y_{1,t-1} - 0.1038\, y_{2,t-1} + 0.0448\, y_{3,t-1} + 1.1040\, y_{4,t-1} - 0.0459\, y_{1,t-2} + 0.0889\, y_{2,t-2} - 0.0646\, y_{3,t-2} - 0.2067\, y_{4,t-2} + \varepsilon_t.$$

The significant coefficients in the model for oil sardine series were for $y_{1,t-1}$, $y_{1,t-2}$ and $y_{3,t-2}$. Here $y_{1,t-1}$ and $y_{1,t-2}$ correspond to the lagged values of oil sardine series itself and $y_{3,t-2}$ corresponds to the lagged value of the series on anchovies. Hence there is significant influence of anchovies series on that of oil sardine series at lag 2. Also, the oil sardine series was autocorrelated up to lag 2. Among the coefficients of the model for the mackerel series, the significant ones were for $y_{2,t-1}$, $y_{3,t-1}$ and $y_{2,t-2}$ in which $y_{2,t-1}$ and $y_{2,t-2}$ are the lagged values of mackerel series and $y_{3,t-1}$ is lagged value of the series on anchovies. Hence the time series on mackerel landings significantly depend on its own past up to lag 2 and also on the series on anchovies with lag l. In the model derived for the series on anchovies the only coefficient found significant was that for $y_{3,t-1}$. Thus the time series corresponding to the landings of anchovies mainly depend on its own past at lag 1 and the other three series does not have any significant influence on this series. In the model for lesser sardine series the coefficients found significant were for $y_{4,t-1}$ and $y_{4,t-2}$. Hence this model suggests that the series on lesser sardine landings depend on its own past up to lag 2 and the other three series does not have any significant effect on the lesser sardine landings series.

The influence of each components of the vector time series on other components were studied by excluding one series at a time from the VAR(2) model and recalculating the model parameters and innovation dispersion matrix. When the mackerel series was excluded from the model the estimate of residual variance of oil sardine increased only by 0.15%. When the series on anchovies was excluded from the model there was an increase of 4.44% in the residual variance of oil sardine series and the increase was 1.19% when lesser sardine series was excluded from the model. Hence compared to mackerel and lesser sardines, the series that was capable of explaining some portion of the variation in oil sardine landings was the anchovies series. A similar analysis for the mackerel landings revealed that the increase in the estimate of its residual variance was 2.25% when oil sardine was excluded from the model, 4.94 % when anchovies was excluded from the model and 0.11% when lesser sardine was excluded from the model. Here also the presence of the series on anchovies in the model was more effective in explaining the variations in mackerel landings. The influence of oil sardine on mackerel landings was quite high compared to that of lesser sardines. In the case of the series on anchovies similar analysis showed that the increase in the estimate of residual variance were 2.37%, 0.54% and 0.86% respectively when the series on oil sardine, mackerel and lesser sardine were excluded from the model. This shows that oil sardine series had comparatively high influence on the variations in anchovies series. In the case of lesser sardine that the percentage increase in the estimate of its residual variance were 1.16 %, 0.77 % and 0.39 % respectively when the series on oil sardine, mackerel and anchovies were removed.

Individual models for the component series on landings of anchovies, lesser sardines, ribbon fishes and catfishes derived from the estimated VAR(2) model are given below.

$$y_{1t} = 0.8239 + 1.0225\, y_{1,t-1} - 0.0835\, y_{2,t-1} - 0.1093\, y_{3,t-1} - 0.0062\, y_{4,t-1} \\ - 0.1001\, y_{1,t-2} + 0.0677\, y_{2,t-2} + 0.1613\, y_{3,t-2} - 0.0122\, y_{4,t-2} + \varepsilon_t$$

$$y_{2t} = 1.4387 + 0.0447\, y_{1,t-1} + 1.0943\, y_{2,t-1} - 0.0648\, y_{3,t-1} + 0.1948\, y_{4,t-1} \\ - 0.0682\, y_{1,t-2} - 0.2164\, y_{2,t-2} + 0.1180\, y_{3,t-2} - 0.1973\, y_{4,t-2} + \varepsilon_t$$

$$y_{3t} = -0.3937 - 0.1312\, y_{1,t-1} - 0.0713 y_{2,t-1} + 1.2619\, y_{3,t-1} + 0.0171\, y_{4,t-1} \\ + 0.1918\, y_{1,t-2} + 0.0808\, y_{2,t-2} - 0.3739\, y_{3,t-2} + 0.0214\, y_{4,t-2} + \varepsilon_t$$

$$y_{4t} = 0.6180 + 0.0420\, y_{1,t-1} + 0.0198\, y_{2,t-1} + 0.0221\, y_{3,t-1} + 1.1617\, y_{4,t-1} \\ - 0.0943\, y_{1,t-2} - 0.0148\, y_{2,t-2} - 0.0037\, y_{3,t-2} - 0.2061\, y_{4,t-2} + \varepsilon_t$$

In this set, the coefficients found significant in the model for anchovies were for that of $y_{1,t-1}$, y_{1t-2} and $y_{3,t-2}$. This indicate that the series on ribbon fish landings had significant positive effect on the landings of anchovies with lag 2. In the model corresponding to lesser sardine landings, the significant coefficients were for $y_{2,t-1}$ and $y_{2,t-2}$. Thus the series on landings of lesser sardines significantly depend on its own past up to lag 2 and the other three series does not have any significant effect on lesser sardine landings. In the model for ribbon fish landings the coefficients found significant were for

$y_{3,t-1}$, $y_{3,t-2}$ and $y_{1,t-2}$. Hence apart from the dependence on its own past up to lag 2, the landings of ribbon fishes had significant and positive dependence on the landings of anchovies with lag 2. The model corresponding to the catfish series had significant coefficients for $y_{4,t-1}$ and $y_{4,t-2}$. So the catfish landings series depend mainly on its own past up to lag 2 and other series does not have any significant effect on catfish landings. Influence of individual components of the vector time series on others were also examined by excluding components one at a time and then examining the change in the residual variances. Percentage increase in the residual variance of the series on anchovies were 0.33%, 4.60% and 1.31% respectively when the series on catfishes, ribbon fish and lesser sardines were excluded from the model. Similar values for the series on lesser sardines were 1.04%, 1.89% and 0.39% respectively after excluding series on catfishes, ribbon fishes and anchovies. When the series on landings of catfishes, lesser sardines and anchovies were excluded from the model, the percentage increase in residual variance of the series on ribbon fish landings were 0.79%, 0.99% and 4.47% respectively. The percentage increase in the residual variance of catfish landings series by excluding the series on landings of ribbon fishes, lesser sardines and anchovies form the model were 0.26%, 0.26% and 5.38% respectively. This analysis showed that ribbon fish landings had maximum influence on anchovies and lesser sardine landings and the series on anchovies had maximum influence on ribbon fish and catfish landings.

From the VAR(1) model estimated for the vector time series consisting of landings of mackerel, anchovies, tuna and penaeid prawns as components, we obtain the individual models for the components as,

$$y_{1t} = -0.7531 + 0.9196\, y_{1,t-1} + 0.0771\, y_{2,t-1} - 0.0546\, y_{3,t-1} + 0.0550\, y_{4,t-1} + \varepsilon_t\,,$$

$$y_{2t} = 0.3099 - 0.0182\, y_{1,t-1} + 0.8862\, y_{2,t-1} + 0.0831\, y_{3,t-1} + 0.0220\, y_{4,t-1} + \varepsilon_t,$$

$$y_{3t} = -0.3981 + 0.0176\, y_{1,t-1} + 0.0617\, y_{2,t-1} + 0.9196\, y_{3,t-1} + 0.0100\, y_{4,t-1} + \varepsilon_t,$$

and $$y_{4t} = 2.0793 - 0.0196\, y_{1,t-1} + 0.0243\, y_{2,t-1} + 0.0544\, y_{3,t-1} + 0.8697\, y_{4,t-1} + \varepsilon_t.$$

In the model for the series on mackerel the only significant coefficient was of $y_{1,t-1}$. Significant coefficients in the model corresponding to anchovies series were for $y_{2,t-1}$ and $y_{3,t-1}$. Hence the mackerel series are autocorrelated and other series did not have any significant effect on mackerel series. The series on anchovies had significant dependence on the series on landings of tuna in addition to its dependence on its own past values. In the model corresponding to the landings of tuna the coefficients found significant were for $y_{2,t-1}$ and $y_{3,t-1}$. Thus the series on landings of tuna depend on that of anchovies and it is autocorrelated. The coefficient found significant in the model for penaeid prawns was for $y_{4,t-1}$. This indicates that all other components of the vector time series does not have any significant effect on landings by penaeid prawns. When the three components, corresponding to anchovies, tuna and penaeid prawns were excluded, one at a time, from the estimated VAR(1) model, the residual variance of mackerel series increased by 1.98%, 1.04% and 1.98% respectively. Similarly, when the series corresponding to landings of mackerel, tuna and penaied prawns were excluded from the model the percentage increase in the residual variance of anchovies series were 0.32%, 2.76% and 0.32% respectively. By excluding the series corresponding to the landings of mackerel, anchovies and penaeid prawns, the percentage increase observed in the residual variance of tuna series were 0.44%, 3.10% and 0.22% respectively. Similar values for the penaeid prawn series were 0.16%, 0.32% and 0.87% respectively when the series excluded were mackerel, anchovies and tuna. From this analysis we find that the series on landings of anchovies and penaeid prawns had almost equal effects on the mackerel series. The series on tuna landings had maximum influence on the anchovies landings and the anchovies series had maximum influence on tuna series. Tuna series had comparatively higher influence on the series for penaeid prawns.

From the VAR(5) model fitted for the vector time series consisting of landings of oil sardine, anchovies, tuna and penaeid prawns as components we obtain models for the components as,

$$\begin{aligned} y_{1t} = {} & 1.2038 + 1.2890\, y_{1,t-1} + 0.1108\, y_{2,t-1} - 0.0698\, y_{3,t-1} - 0.0802\, y_{4,t-1} \\ & - 0.2477\, y_{1,t-2} - 0.1292\, y_{2,t-2} - 0.0156\, y_{3,t-2} - 0.0162\, y_{4,t-2} \\ & + 0.0452\, y_{1,t-3} + 0.0546\, y_{2,t-3} + 0.0099\, y_{3,t-3} + 0.1116\, y_{4,t-3} \\ & - 0.4944\, y_{1,t-4} - 0.0050\, y_{2,t-4} + 0.1284\, y_{3,t-4} - 0.0496\, y_{4,t-4} \\ & + 0.3211\, y_{1,t-5} - 0.1234\, y_{2,t-5} - 0.0466\, y_{3,t-5} + 0.0571\, y_{4,t-5} + \varepsilon_t \end{aligned}$$

$$\begin{aligned} y_{2t} = {} & 0.2332 - 0.1575\, y_{1,t-1} + 1.0813\, y_{2,t-1} - 0.0452\, y_{3,t-1} - 0.0854\, y_{4,t-1} \\ & + 0.2346\, y_{1,t-2} - 0.0680\, y_{2,t-2} + 0.3475\, y_{3,t-2} + 0.1037\, y_{4,t-2} \\ & + 0.0352\, y_{1,t-3} + 0.0330\, y_{2,t-3} - 0.3806\, y_{3,t-3} + 0.0849\, y_{4,t-3} \\ & - 0.1403\, y_{1,t-4} - 0.5088\, y_{2,t-4} + 0.3284\, y_{3,t-4} - 0.0145\, y_{4,t-4} \\ & + 0.0194\, y_{1,t-5} + 0.3473\, y_{2,t-5} - 0.1827\, y_{3,t-5} - 0.0605\, y_{4,t-5} + \varepsilon_t \end{aligned}$$

$$\begin{aligned} y_{3t} = {} & -1.0425 + 0.0605\, y_{1,t-1} + 0.0045\, y_{2,t-1} + 0.9657\, y_{3,t-1} - 0.0524\, y_{4,t-1} \\ & - 0.0775\, y_{1,t-2} + 0.1305\, y_{2,t-2} - 0.0019\, y_{3,t-2} + 0.0880\, y_{4,t-2} \\ & + 0.1432\, y_{1,t-3} - 0.0901\, y_{2,t-3} + 0.0264\, y_{3,t-3} - 0.1153\, y_{4,t-3} \\ & - 0.1899\, y_{1,t-4} - 0.0478\, y_{2,t-4} - 0.5099\, y_{3,t-4} + 0.0952\, y_{4,t-4} \\ & + 0.0776\, y_{1,t-5} + 0.0506\, y_{2,t-5} + 0.4495\, y_{3,t-5} + 0.0334\, y_{4,t-5} + \varepsilon_t \end{aligned}$$

$$
\begin{aligned}
y_{4t} = 1.8105 &- 0.2573\, y_{1,t-1} + 0.0546\, y_{2,t-1} - 0.0119\, y_{3,t-1} + 1.0579\, y_{4,t-1} \\
&+ 0.3452\, y_{1,t-2} - 0.0022\, y_{2,t-2} - 0.0511\, y_{3,t-2} - 0.2578\, y_{4,t-2} \\
&- 0.2252\, y_{1,t-3} + 0.0455\, y_{2,t-3} - 0.0223\, y_{3,t-3} + 0.1787\, y_{4,t-3} \\
&+ 0.2698\, y_{1,t-4} - 0.0855\, y_{2,t-4} + 0.2977\, y_{3,t-4} - 0.4498\, y_{4,t-4} \\
&- 0.1572\, y_{1,t-5} - 0.0080\, y_{2,t-5} - 0.1995\, y_{3,t-5} + 0.3823\, y_{4,t-5} + \varepsilon_t
\end{aligned}
$$

In the model for oil sardine series, the coefficients found significant were for $y_{1,t-1}$, $y_{1,t-2}$, $y_{1,t-4}$ and $y_{1,t-5}$. Significant coefficients in the model for the anchovies series were for $y_{2,t-1}$, $y_{3,t-2}$, $y_{3,t-3}$, $y_{2,t-4}$, $y_{3,t-4}$ and $y_{2,t-5}$. The coefficients found significant in the model for tuna series were for $y_{3,t-1}$, $y_{1,t-4}$, $y_{3,t-4}$ and $y_{3,t-5}$. In the model for the series on penaeid prawns the significant coefficients were for $y_{1,t-1}$, $y_{4,t-1}$, $y_{1,t-2}$, $y_{4,t-2}$, $y_{4,t-4}$ and $y_{4,t-5}$. From these results we see that all the four series significantly depend on their own past values up to lag 5. The series on tuna landings had significant influence on the anchovies series at lags 2, 3 and 4. Oil sardine series had significant influence on tuna landings with lag 4 and penaeid prawn series was also influenced by oil sardine series at lags 1 and 2. Percentage increase in the residual variance of oil sardine series were 10.53%, 1.95% and 5.65% when series on landings of anchovies, tuna and penaeid prawns respectively were excluded from the VAR(5) model. The percentage increase observed in the residual variance of anchovies series were 3.61%, 10.68% and 5.56% respectively by excluding series on landings of oil sardine, tuna and penaeid prawns. In the case of series on tuna landings, the percentage increase observed in its residual variance by excluding series on landings of oil sardine, anchovies and penaeid prawns from the model were 4.79%, 5.82% and 10.62% respectively. Percentage increase in the residual variance of penaeid prawns series were 5.19%, 0.30% and 1.30% respectively when series on landings of oil sardine, anchovies and tuna were excluded from the model. These results revealed that the maximum effect on the oil sardine series was by the series on anchovies landings and that on anchovies series was by tuna landings series. Maximum effect on tuna series was by penaeid prawn series and on penaeid prawn series the maximum effect was due to the series on oil sardine landings.

From the VAR(2) model fitted to the vector time series with series on landings of elasmobranchs, oil sardine, mackerel and seer fish as components, the individual models for these components were obtained as,

$$\begin{aligned} y_{1t} = {} & 4.7990 + 1.1640\, y_{1,t-1} - 0.0239\, y_{2,t-1} + 0.2116\, y_{3,t-1} - 0.1492\, y_{4,t-1} \\ & - 0.3741\, y_{1,t-2} + 0.0867\, y_{2,t-2} - 0.2167\, y_{3,t-2} + 0.0980\, y_{4,t-2} + \varepsilon_t \end{aligned}$$

$$\begin{aligned} y_{2t} = {} & 0.7564 - 0.0438\, y_{1,t-1} + 1.3883\, y_{2,t-1} - 0.0375\, y_{3,t-1} + 0.1751\, y_{4,t-1} \\ & + 0.0680\, y_{1,t-2} - 0.4852\, y_{2,t-2} + 0.0162\, y_{3,t-2} - 0.2136\, y_{4,t-2} + \varepsilon_t \end{aligned}$$

$$\begin{aligned} y_{3t} = {} & 2.1302 + 0.1382\, y_{1,t-1} + 0.1308\, y_{2,t-1} + 1.2025\, y_{3,t-1} - 0.2406\, y_{4,t-1} \\ & - 0.1881\, y_{1,t-2} - 0.1667\, y_{2,t-2} - 0.3063\, y_{3,t-2} + 0.2757\, y_{4,t-2} + \varepsilon_t \end{aligned}$$

$$\begin{aligned} y_{4t} = {} & -0.2324 + 0.1162\, y_{1,t-1} + 0.1365\, y_{2,t-1} - 0.0546\, y_{3,t-1} + 1.0567\, y_{4,t-1} \\ & - 0.0900\, y_{1,t-2} - 0.1619\, y_{2,t-2} + 0.0775\, y_{3,t-2} - 0.1025\, y_{4,t-2} \end{aligned}$$

Significant coefficients in the model for the elasmobranchs series were for $y_{1,t-1}$ and $y_{1,t-2}$. In the model for oil sardine series the significant coefficients were for $y_{2,t-1}$, $y_{2,t-2}$ and $y_{4,t-2}$. The coefficients found significant in the model corresponding to mackerel series were for $y_{1,t-1}$, $y_{3,t-1}$, $y_{4,t-1}$, $y_{1,t-2}$, $y_{2,t-2}$, $y_{3,t-2}$ and $y_{4,t-2}$. In the model for the series on seer fish landings, the significant coefficients were for $y_{1,t-1}$, $y_{2,t-1}$, $y_{4,t-1}$, $y_{1,t-2}$ and $y_{2,t-2}$. From these results it was found that the series on landings of elasmobranchs, oil sardine and mackerel are autocorrelated up to lag 2 where as the series on landings of seer fishes depend on its own past values up to lag 1 only. Also, the oil sardine landings were significantly influenced by the series on landings of seer fishes with lag 2. The series on landings of mackerel had significant dependence on that of elasmobranchs and seer fishes at lags 1 and 2. Oil sardine series also had significant influence on landings of mackerel with lag 2. The series on landings of seer fish had significant dependence on that of elasmobranchs and oil sardine at lags 1 and 2. The percentage increase observed in the residual variance of elasmobranchs series by excluding one each of the series on oil sardine, mackerel and seer fish from the VAR(2) model were 1.62%, 2.38% and 1.38% respectively. In a similar manner, by excluding one series each at a time from the vector model, among series on elasmobranchs, mackerel and seer fish, the percentage increase in residual variance of oil sardine series were 1.95%, 0.60% and 4.80% respectively. The percentage increase observed in the residual variance of mackerel series by excluding series on ealsmobranchs, oil sardine and seer fish were 10.19%, 3.51% and 5.50% respectively. Similar values for seer fish series were 5.36%, 4.33% and 1.38%, when the series on elasmobranchs, oil sardine and mackerel respectively were excluded from the model. Thus, it was found that mackerel series had comparatively higher influence on the landings of elasmobranchs. The series on oil sardine landings had comparatively high dependence on the series on seer fish landings. The influence of series on elasmobranchs on mackerel landings was almost double that of seer fish series and three times that of oil sardine series. The series on landings of elasmobranchs and oil sardine had comparatively higher influence on the landings of seer fishes.

Appendix-II (Tables)

Table.2.1.1. Cross correlation matrices of the vector time series composed of transformed landings of oil sardine, mackerel, anchovies and lesser sardines. The notation '(+)' indicates positive and significant, '(-)' indicates negative and significant and '(.) indicates not significance.

$C(0)=\begin{pmatrix} 1.000(-) & -0.350(-) & -0.412(-) & 0.115(\cdot) \\ -0.350(-) & 1.000(+) & 0.509(+) & -0.027(\cdot) \\ -0.412(-) & 0.509(+) & 1.000(+) & -0.077(\cdot) \\ 0.115(\cdot) & -0.027(\cdot) & -0.077(\cdot) & 1.000(+) \end{pmatrix}$	$C(1)=\begin{pmatrix} 0.948(+) & -0.354(-) & -0.397(-) & 0.059(\cdot) \\ -0.343(-) & 0.929(+) & 0.476(+) & -0.029(\cdot) \\ -0.420(-) & 0.524(+) & 0.946(+) & -0.071() \\ 0.151(\cdot) & -0.027(\cdot) & -0.079(\cdot) & 0.911(+) \end{pmatrix}$
$C(2)=\begin{pmatrix} 0.852(+) & -0.268(-) & -0.366(-) & 0.004(\cdot) \\ -0.334(-) & 0.826(+) & 0.443(+) & -0.025(\cdot) \\ -0.437(-) & 0.522(+) & 0.887(+) & -0.072(\cdot) \\ 0.186(+) & -0.047(\cdot) & -0.077(\cdot) & 0.800(+) \end{pmatrix}$	$C(3)=\begin{pmatrix} 0.732(+) & -0.380(-) & -0.338(-) & -0.030(\cdot) \\ -0.319(-) & 0.726(+) & 0.414(+) & -0.004(\cdot) \\ -0.453(-) & 0.501(+) & 0.826(+) & -0.087(\cdot) \\ 0.213(+) & -0.071(\cdot) & -0.075(\cdot) & 0.679(+) \end{pmatrix}$
$C(4)=\begin{pmatrix} 0.597(+) & -0.382(-) & -0.312(-) & -0.046(\cdot) \\ -0.301(-) & 0.623(+) & 0.394(+) & 0.031(\cdot) \\ -0.480(-) & 0.488(+) & 0.751(+) & -0.105(\cdot) \\ 0.223(+) & -0.089(\cdot) & -0.072(\cdot) & 0.548(+) \end{pmatrix}$	$C(5)=\begin{pmatrix} 0.483(+) & -0.369(-) & -0.284(-) & -0.032(\cdot) \\ -0.295(-) & 0.547(+) & 0.376(+) & 0.083(\cdot) \\ -0.518(-) & 0.482(+) & 0.713(+) & -0.127(\cdot) \\ 0.231(+) & -0.096(\cdot) & -0.077(\cdot) & 0.496(+) \end{pmatrix}$
$C(6)=\begin{pmatrix} 0.375(+) & -0.346(-) & -0.263(-) & -0.001(\cdot) \\ -0.296(-) & 0.475(+) & 0.366(+) & 0.127(\cdot) \\ -0.551(-) & 0.492(+) & 0.679(+) & -0.147(\cdot) \\ 0.221(+) & -0.096(\cdot) & -0.085(\cdot) & 0.454(+) \end{pmatrix}$	$C(7)=\begin{pmatrix} 0.272(+) & -0.314(-) & -0.235(-) & 0.016(\cdot) \\ -0.303(-) & 0.388(+) & 0.365(+) & 0.164(\cdot) \\ -0.583(-) & 0.519(+) & 0.645(+) & -0.139(\cdot) \\ 0.195(+) & -0.104(\cdot) & -0.105(\cdot) & 0.399(+) \end{pmatrix}$
$C(8)=\begin{pmatrix} 0.180(+) & -0.285(-) & -0.211(-) & 0.037(\cdot) \\ -0.317(-) & 0.314(+) & 0.378(+) & 0.199(+) \\ -0.609(-) & 0.535(+) & 0.614(+) & -0.147(\cdot) \\ 0.168(+) & -0.103(\cdot) & -0.115(\cdot) & 0.355(+) \end{pmatrix}$	$C(9)=\begin{pmatrix} 0.097(\cdot) & -0.249(-) & -0.181(-) & 0.044(\cdot) \\ -0.327(-) & 0.263(+) & 0.395(+) & 0.222(+) \\ -0.626(-) & 0.561(+) & 0.584(+) & -0.151(\cdot) \\ 0.141(\cdot) & -0.097(\cdot) & -0.116(\cdot) & 0.291(+) \end{pmatrix}$
$C(10)=\begin{pmatrix} 0.026(\cdot) & -0.220(-) & -0.156(\cdot) & 0.043(\cdot) \\ -0.342(-) & 0.218(+) & 0.409(+) & 0.238(+) \\ -0.633(-) & 0.582(+) & 0.568(+) & -0.160(\cdot) \\ 0.112(\cdot) & -0.095(\cdot) & -0.112(\cdot) & 0.222(+) \end{pmatrix}$	$C(11)=\begin{pmatrix} -0.027(\cdot) & -0.196(-) & -0.149(\cdot) & 0.050(\cdot) \\ -0.351(-) & 0.177(+) & 0.416(+) & 0.236(+) \\ -0.630(-) & 0.588(+) & 0.557(+) & -0.185(-) \\ 0.080(\cdot) & -0.081(\cdot) & -0.098(\cdot) & 0.171(+) \end{pmatrix}$
$C(12)=\begin{pmatrix} -0.066(\cdot) & -0.173(-) & -0.142(\cdot) & 0.053(\cdot) \\ -0.354(-) & 0.138(\cdot) & 0.412(+) & 0.226(+) \\ -0.616(-) & 0.598(+) & 0.546(+) & -0.185(-) \\ 0.047(\cdot) & -0.069(\cdot) & -0.101(\cdot) & 0.121(\cdot) \end{pmatrix}$	$C(13)=\begin{pmatrix} -0.095(\cdot) & -0.153(\cdot) & -0.148(\cdot) & 0.051(\cdot) \\ -0.343(-) & 0.103(\cdot) & 0.405(+) & 0.229(+) \\ -0.582(-) & 0.589(+) & 0.544(+) & -0.171(-) \\ 0.013(\cdot) & -0.079(\cdot) & -0.112(\cdot) & 0.095(\cdot) \end{pmatrix}$
$C(14)=\begin{pmatrix} -0.117(\cdot) & -0.124(\cdot) & -0.153(\cdot) & 0.051(\cdot) \\ -0.326(-) & 0.086(\cdot) & 0.385(+) & 0.229(+) \\ -0.535(-) & 0.563(+) & 0.521(+) & -0.149(\cdot) \\ -0.014(\cdot) & -0.073(\cdot) & -0.140(\cdot) & 0.089(\cdot) \end{pmatrix}$	$C(15)=\begin{pmatrix} -0.136(\cdot) & -0.087(\cdot) & -0.147(\cdot) & 0.039(\cdot) \\ -0.317(-) & 0.087(\cdot) & 0.357(+) & 0.228(+) \\ -0.481(-) & 0.525(+) & 0.491(+) & -0.111(\cdot) \\ -0.019(\cdot) & -0.068(\cdot) & -0.191(-) & 0.084(\cdot) \end{pmatrix}$
$C(16)=\begin{pmatrix} -0.152(\cdot) & -0.045(\cdot) & -0.147(\cdot) & 0.025(\cdot) \\ -0.319(-) & 0.088(\cdot) & 0.345(+) & 0.213(+) \\ -0.431(-) & 0.485(+) & 0.459(+) & -0.094(\cdot) \\ -0.019(\cdot) & -0.066(\cdot) & -0.230(-) & 0.068(\cdot) \end{pmatrix}$	$C(17)=\begin{pmatrix} -0.153(\cdot) & -0.009(\cdot) & -0.143(\cdot) & 0.030(\cdot) \\ -0.332(-) & 0.083(\cdot) & 0.324(+) & 0.164(\cdot) \\ -0.393(-) & 0.448(+) & 0.432(+) & -0.094(\cdot) \\ -0.001(\cdot) & -0.047(\cdot) & -0.268(-) & 0.044(\cdot) \end{pmatrix}$

Table.2.1.1. Continued

$C(18)=\begin{pmatrix} -0.142(\cdot) & 0.016(\cdot) & -0.148(\cdot) & 0.031(\cdot) \\ -0.340(-) & 0.085(\cdot) & 0.313(\cdot) & 0.106(\cdot) \\ -0.359(-) & 0.419(+) & 0.425(+) & -0.087(\cdot) \\ 0.018(\cdot) & -0.028(\cdot) & -0.293(-) & 0.002(\cdot) \end{pmatrix}$	$C(19)=\begin{pmatrix} -0.123(\cdot) & 0.036(\cdot) & -0.157(\cdot) & 0.040(\cdot) \\ -0.338(-) & 0.091(\cdot) & 0.315(+) & 0.047(\cdot) \\ -0.332(-) & 0.410(+) & 0.415(+) & -0.089(\cdot) \\ 0.028(\cdot) & -0.010(\cdot) & -0.295(-) & -0.043(\cdot) \end{pmatrix}$
$C(20)=\begin{pmatrix} -0.087(\cdot) & 0.035(\cdot) & -0.060(\cdot) & 0.054(\cdot) \\ -0.327(-) & 0.117(\cdot) & 0.294(+) & -0.001(\cdot) \\ -0.304(-) & 0.410(+) & 0.416(+) & -0.081(\cdot) \\ 0.048(\cdot) & 0.012(\cdot) & -0.299(-) & -0.094(\cdot) \end{pmatrix}$	$C(21)=\begin{pmatrix} -0.039(\cdot) & 0.030(\cdot) & -0.172(-) & 0.051(\cdot) \\ -0.316(-) & 0.157(\cdot) & 0.269(+) & -0.030(\cdot) \\ -0.279(-) & 0.406(+) & 0.412(+) & -0.060(\cdot) \\ 0.057(\cdot) & 0.017(\cdot) & -0.304(-) & -0.147(\cdot) \end{pmatrix}$
$C(22)=\begin{pmatrix} 0.010(\cdot) & 0.022(\cdot) & -0.170(-) & 0.054(\cdot) \\ -0.309(-) & 0.180(+) & 0.243(+) & 0.059(\cdot) \\ -0.265(-) & 0.409(+) & 0.398(+) & -0.045(\cdot) \\ 0.067(\cdot) & 0.014(\cdot) & -0.309(-) & -0.181(-) \end{pmatrix}$	$C(23)=\begin{pmatrix} 0.068(\cdot) & 0.002(\cdot) & -0.162(\cdot) & 0.061(\cdot) \\ -0.297(-) & 0.194(+) & 0.211(+) & -0.080(\cdot) \\ -0.256(-) & 0.408(+) & 0.391(+) & -0.492(\cdot) \\ 0.078(\cdot) & 0.009(\cdot) & -0.316(-) & -0.201(-) \end{pmatrix}$
$C(24)=\begin{pmatrix} 0.119(\cdot) & -0.021(\cdot) & -0.157(\cdot) & 0.078(\cdot) \\ -0.272(-) & 0.196(+) & 0.183(+) & -0.111(\cdot) \\ -0.242(-) & 0.394(+) & 0.364(+) & -0.066(\cdot) \\ 0.086(\cdot) & -0.016(\cdot) & -0.307(-) & -0.202(-) \end{pmatrix}$	$S=\begin{pmatrix} 1.000 & -0.350 & -0.412 & 0.115 \\ -0.350 & 1.000 & 0.509 & -0.027 \\ -0.412 & 0.509 & 1.000 & -0.077 \\ 0.115 & -0.027 & -0.077 & 1.000 \end{pmatrix}$

Table.2.1.2. The χ^2 statistic for testing combined significance of cross correlation matrices calculated for the original and residual series of VAR(2) model using the vector time series of transformed landings of oil sardine, mackerel, anchovies and lesser sardines.

Lag	χ^2 statistic		Lag	χ^2 statistic	
	Original series	Residual series		Original series	Residual series
1	496.18	4.26	13	163.85	21.37
2	393.66	11.07	14	154.74	22.27
3	299.55	33.57	15	140.70	17.89
4	212.29	74.85	16	130.05	29.42
5	173.33	12.28	17	120.24	26.28
6	146.85	31.30	18	111.02	18.78
7	128.67	14.36	19	104.94	23.99
8	127.42	18.82	20	97.02	35.26
9	133.50	10.61	21	90.15	23.68
10	144.91	25.86	22	87.36	15.06
11	156.10	21.28	23	84.60	23.96
12	162.86	27.92	24	76.21	19.42

Table.2.1.3. Partial cross correlation matrices for the vector time series of transformed landings of oil sardine, mackerel, anchovies and lesser sardines.

$$\hat{\rho}(1)=\begin{pmatrix} 0.948(+) & -0.354(-) & -0.397(-) & 0.059(\cdot) \\ -0.343(-) & 0.929(+) & 0.476(+) & -0.029(\cdot) \\ -0.420(-) & 0.524(+) & 0.946(+) & -0.071(\cdot) \\ 0.151(\cdot) & -0.027(\cdot) & -0.079(\cdot) & 0.911(+) \end{pmatrix}$$

$$\hat{\rho}(2)=\begin{pmatrix} -0.454(-) & -0.110(\cdot) & 0.152(\cdot) & -0.058(\cdot) \\ 0.006(\cdot) & -0.280(-) & -0.024(\cdot) & 0.036(\cdot) \\ -0.126(\cdot) & -0.104(\cdot) & -0.066(\cdot) & -0.062(\cdot) \\ 0.036(\cdot) & -0.128(\cdot) & 0.021(\cdot) & -0.159(\cdot) \end{pmatrix}$$

$$\hat{\rho}(3)=\begin{pmatrix} -0.170(-) & 0.029(\cdot) & -0.101(\cdot) & 0.108(\cdot) \\ 0.053(\cdot) & 0.005(\cdot) & 0.021(\cdot) & 0.091(\cdot) \\ 0.019(\cdot) & -0.078(\cdot) & -0.075(\cdot) & -0.111(\cdot) \\ -0.026(\cdot) & 0.002(\cdot) & 0.004(\cdot) & -0.108(\cdot) \end{pmatrix}$$

$$\hat{\rho}(4)=\begin{pmatrix} -0.131(\cdot) & 0.032(\cdot) & -0.026(\cdot) & -0.010(\cdot) \\ -0.008(\cdot) & -0.150(\cdot) & 0.067(\cdot) & 0.073(\cdot) \\ -0.190(-) & 0.119(\cdot) & -0.172(-) & -0.079(\cdot) \\ -0.079(\cdot) & 0.034(\cdot) & -0.005(\cdot) & -0.134(\cdot) \end{pmatrix}$$

$$\hat{\rho}(5)=\begin{pmatrix} 0.282(+) & 0.102(\cdot) & -0.016(\cdot) & 0.167(+) \\ -0.169(-) & 0.163(\cdot) & -0.011(\cdot) & 0.121(\cdot) \\ -0.144(\cdot) & 0.122(\cdot) & 0.307(+) & -0.070(\cdot) \\ 0.078(\cdot) & 0.090(\cdot) & -0.106(\cdot) & 0.434(+) \end{pmatrix}$$

$$\hat{\rho}(6)=\begin{pmatrix} -0.224(-) & -0.075(\cdot) & 0.074(\cdot) & -0.046(\cdot) \\ -0.016(\cdot) & -0.159(\cdot) & 0.048(\cdot) & -0.049(\cdot) \\ -0.030(\cdot) & 0.167(+) & -0.033(\cdot) & -0.058(\cdot) \\ -0.043(\cdot) & -0.057(\cdot) & 0.030(\cdot) & -0.093(\cdot) \end{pmatrix}$$

$$\hat{\rho}(7)=\begin{pmatrix} -0.108(\cdot) & 0.093(\cdot) & 0.060(\cdot) & -0.078(\cdot) \\ -0.069(\cdot) & -0.107(\cdot) & 0.053(\cdot) & 0.081(\cdot) \\ 0.017(\cdot) & 0.094(\cdot) & 0.023(\cdot) & 0.117(\cdot) \\ -0.010(\cdot) & -0.045(\cdot) & -0.134(\cdot) & -0.160(\cdot) \end{pmatrix}$$

$$\hat{\rho}(8)=\begin{pmatrix} -0.130(\cdot) & -0.041(\cdot) & -0.136(\cdot) & 0.127(\cdot) \\ -0.082(\cdot) & 0.016(\cdot) & 0.162(\cdot) & 0.042(\cdot) \\ -0.070(\cdot) & -0.046(\cdot) & -0.122(\cdot) & -0.228(-) \\ 0.000(\cdot) & 0.145(\cdot) & 0.094(\cdot) & -0.064(\cdot) \end{pmatrix}$$

$$\hat{\rho}(9)=\begin{pmatrix} 0.048(\cdot) & 0.084(\cdot) & 0.122(\cdot) & -0.085(\cdot) \\ -0.100(\cdot) & 0.093(\cdot) & -0.036(\cdot) & -0.028(\cdot) \\ -0.058(\cdot) & 0.301(+) & 0.131(\cdot) & -0.069(\cdot) \\ 0.074(\cdot) & 0.043(\cdot) & -0.022(\cdot) & 0.048(\cdot) \end{pmatrix}$$

$$\hat{\rho}(10)=\begin{pmatrix} -0.135(\cdot) & -0.126(\cdot) & -0.083(\cdot) & -0.003(\cdot) \\ -0.136(\cdot) & -0.094(\cdot) & 0.037(\cdot) & -0.032(\cdot) \\ -0.021(\cdot) & -0.020(\cdot) & 0.130(\cdot) & -0.143(\cdot) \\ -0.062(\cdot) & -0.133(\cdot) & 0.067(\cdot) & -0.065(\cdot) \end{pmatrix}$$

$$\hat{\rho}(11)=\begin{pmatrix} -0.013(\cdot) & 0.083(\cdot) & -0.031(\cdot) & -0.024(\cdot) \\ 0.030(\cdot) & -0.054(\cdot) & -0.045(\cdot) & -0.161(\cdot) \\ -0.005(\cdot) & 0.100(\cdot) & 0.052(\cdot) & 0.034(\cdot) \\ -0.100(\cdot) & 0.058(\cdot) & -0.040(\cdot) & -0.078(\cdot) \end{pmatrix}$$

$$\hat{\rho}(12)=\begin{pmatrix} -0.063(\cdot) & -0.005(\cdot) & 0.037(\cdot) & 0.009(\cdot) \\ -0.063(\cdot) & 0.022(\cdot) & 0.066(\cdot) & 0.072(\cdot) \\ -0.041(\cdot) & -0.019(\cdot) & -0.009(\cdot) & 0.036(\cdot) \\ 0.003(\cdot) & 0.057(\cdot) & -0.170(-) & 0.050(\cdot) \end{pmatrix}$$

$$\hat{\rho}(13)=\begin{pmatrix} -0.150(\cdot) & 0.144(\cdot) & -0.006(\cdot) & -0.098(\cdot) \\ 0.139(\cdot) & 0.028(\cdot) & 0.000(\cdot) & 0.152(\cdot) \\ 0.153(\cdot) & -0.015(\cdot) & 0.048(\cdot) & 0.134(\cdot) \\ 0.092(\cdot) & -0.135(\cdot) & -0.016(\cdot) & 0.049(\cdot) \end{pmatrix}$$

$$\hat{\rho}(14)=\begin{pmatrix} -0.098(\cdot) & 0.060(\cdot) & 0.102(\cdot) & -0.045(\cdot) \\ -0.134(\cdot) & -0.059(\cdot) & -0.136(\cdot) & -0.029(\cdot) \\ 0.084(\cdot) & -0.165(\cdot) & -0.095(\cdot) & -0.090(\cdot) \\ -0.002(\cdot) & 0.15(\cdot) & -0.098(\cdot) & -0.014(\cdot) \end{pmatrix}$$

$$\hat{\rho}(15)=\begin{pmatrix} -0.153(\cdot) & 0.174(+) & 0.026(\cdot) & -0.080(\cdot) \\ -0.144(\cdot) & 0.105(\cdot) & 0.024(\cdot) & -0.036(\cdot) \\ 0.012(\cdot) & -0.060(\cdot) & -0.008(\cdot) & 0.003(\cdot) \\ 0.162(\cdot) & -0.033(\cdot) & -0.154(\cdot) & -0.043(\cdot) \end{pmatrix}$$

$$\hat{\rho}(16)=\begin{pmatrix} 0.013(\cdot) & 0.043(\cdot) & -0.093(\cdot) & 0.064(\cdot) \\ -0.136(\cdot) & -0.054(\cdot) & 0.161(\cdot) & -0.060(\cdot) \\ -0.141(\cdot) & 0.127(\cdot) & -0.058(\cdot) & -0.109(\cdot) \\ 0.023(\cdot) & 0.106(\cdot) & 0.074(\cdot) & -0.150(\cdot) \end{pmatrix}$$

Table.2.1.4. Calculated values of squared partial canonical correlations and test statistic for the vector time series of transformed landings of oil sardine, mackerel, anchovies and lesser sardines.

Lag	Squared Partial Canonical Correlation				LR Statistic
	(i)	(ii)	(iii)	(iv)	
1	0.9304	0.8491	0.8290	0.7898	1142.97
2	0.2780	0.0911	0.0293	0.0105	66.93
3	0.0697	0.0359	0.0058	0.0000	16.62
4	0.1504	0.0284	0.0127	0.0014	29.87
5	0.2568	0.1045	0.0915	0.0154	75.19
6	0.1087	0.0484	0.0106	0.0000	25.43
7	0.0596	0.0389	0.0233	0.0027	18.47
8	0.1148	0.0713	0.0219	0.0112	33.25
9	0.1319	0.0232	0.0119	0.0010	25.79
10	0.1059	0.0451	0.0032	0.0021	23.70

Table.2.1.5. Computed values of different order selection criteria for the vector time series of transformed landings of oil sardine, mackerel, anchovies and lesser sardines.

p	$\lvert\tilde{\Sigma}\rvert$	FPE	AIC	HQ	SC
1	0.000154	0.000165	-8.5606	-8.4271	-8.2321
2	0.000088	0.000100	-8.8942	-8.6273	-8.2373
3	0.000076	0.000091	-8.8171	-8.4167	-7.8317
4	0.000061	0.000077	-8.8271	-8.2932	-7.5132
5	0.000030	0.000040	-9.3262	-8.6588	-7.6838
6	0.000024	0.000034	-9.3254	-8.5246	-7.3546
7	0.000019	0.000028	-9.3443	-8.4100	-7.0450
8	0.000013	0.000021	-9.4747	-8.4070	-6.8470
9	0.000009	0.000016	-9.5884	-8.3872	-6.6322
10	0.000009	0.000013	-9.6390	-8.3043	-6.3543

Table.2.1.6. Eigen values of the charecteristic matrix of the VAR(2) model estimated for the vector time series of transformed landings of oil sardine, mackerel, anchovies and lesser sardines.

No.	Real Part	Imaginary Part	Absolute value
1	0.1987	-0.1072	0.2258
2	0.1987	0.1072	0.2258
3	0.3315	0.0000	0.3316
4	0.9237	0.0000	0.9237
5	0.7993	0.0000	0.7993
6	0.6796	-0.0258	0.6801
7	0.6796	0.0258	0.6801
8	0.9389	0.0000	0.9389

Table.2.2.1. Cross correlation matrices of the vector time series of transformed landings of anchovies, lesser sardines, ribbon fish and catfish.

$$C(0)=\begin{pmatrix} 1.000(+) & -0.077(\cdot) & 0.309(+) & -0.424(-) \\ -0.077(\cdot) & 1.000(+) & 0.229(+) & 0.229(+) \\ 0.309(+) & 0.229(+) & 1.000(+) & 0.189(+) \\ -0.424(-) & 0.229(+) & 0.189(+) & 1.000(+) \end{pmatrix}$$

$$C(1)=\begin{pmatrix} 0.946(+) & -0.071(\cdot) & 0.297(+) & -0.441(-) \\ -0.079(\cdot) & 0.911(+) & 0.210(+) & 0.227(+) \\ 0.312(+) & 0.250(+) & 0.928(+) & 0.198(+) \\ -0.384(-) & 0.217(+) & 0.192(+) & 0.963(+) \end{pmatrix}$$

$$C(2)=\begin{pmatrix} 0.887(+) & -0.073(\cdot) & 0.301(+) & -0.465(-) \\ -0.077(\cdot) & 0.800(+) & 0.200(+) & 0.238(+) \\ 0.338(+) & 0.274(+) & 0.816(+) & 0.195(+) \\ -0.346(-) & 0.198(+) & 0.197(+) & 0.916(+) \end{pmatrix}$$

$$C(3)=\begin{pmatrix} 0.826(+) & -0.087(\cdot) & 0.310(+) & -0.487(-) \\ -0.075(\cdot) & 0.679(+) & 0.189(+) & 0.247(+) \\ 0.347(+) & 0.305(+) & 0.693(+) & 0.187(+) \\ -0.318(-) & 0.179(+) & 0.198(+) & 0.869(+) \end{pmatrix}$$

$$C(4)=\begin{pmatrix} 0.751(+) & -0.105(\cdot) & 0.298(+) & -0.504(-) \\ -0.072(\cdot) & 0.548(+) & 0.200(+) & 0.249(+) \\ 0.337(+) & 0.329(+) & 0.585(+) & 0.172(+) \\ -0.287(-) & 0.170(+) & 0.198(+) & 0.820(+) \end{pmatrix}$$

$$C(5)=\begin{pmatrix} 0.713(+) & -0.127(\cdot) & 0.288(+) & -0.524(-) \\ -0.077(\cdot) & 0.496(+) & 0.223(+) & 0.252(+) \\ 0.314(+) & 0.349(+) & 0.518(+) & 0.160(\cdot) \\ -0.264(-) & 0.167(+) & 0.199(+) & 0.782(+) \end{pmatrix}$$

$$C(6)=\begin{pmatrix} 0.679(+) & -0.147(\cdot) & 0.277(+) & -0.546(-) \\ -0.085(\cdot) & 0.454(+) & 0.243(+) & 0.242(+) \\ 0.280(+) & 0.352(+) & 0.446(+) & 0.145(\cdot) \\ -0.238(-) & 0.171(+) & 0.197(+) & 0.742(+) \end{pmatrix}$$

$$C(7)=\begin{pmatrix} 0.645(+) & -0.139(\cdot) & 0.263(+) & -0.558(-) \\ -0.105(\cdot) & 0.399(+) & 0.263(+) & 0.228(+) \\ 0.259(+) & 0.340(+) & 0.364(+) & 0.131(\cdot) \\ -0.208(-) & 0.172(+) & 0.186(+) & 0.701(+) \end{pmatrix}$$

$$C(8)=\begin{pmatrix} 0.614(+) & -0.147(\cdot) & 0.247(+) & -0.574(-) \\ -0.115(\cdot) & 0.355(+) & 0.260(+) & 0.218(+) \\ 0.251(+) & 0.322(+) & 0.266(+) & 0.121(\cdot) \\ -0.184(-) & 0.170(+) & 0.173(+) & 0.667(+) \end{pmatrix}$$

$$C(9)=\begin{pmatrix} 0.584(+) & -0.151(\cdot) & 0.224(+) & -0.586(-) \\ -0.116(\cdot) & 0.291(+) & 0.241(+) & 0.199(+) \\ 0.247(+) & 0.297(+) & 0.174(+) & 0.123(\cdot) \\ -0.156(\cdot) & 0.162(\cdot) & 0.161(\cdot) & 0.634(+) \end{pmatrix}$$

$$C(10)=\begin{pmatrix} 0.568(+) & -0.160(\cdot) & 0.199(+) & -0.591(-) \\ -0.112(\cdot) & 0.222(+) & 0.200(+) & 0.175(+) \\ 0.246(+) & 0.257(+) & 0.101(\cdot) & 0.124(\cdot) \\ -0.130(\cdot) & 0.148(\cdot) & 0.139(\cdot) & 0.608(+) \end{pmatrix}$$

$$C(11)=\begin{pmatrix} 0.557(+) & -0.185(-) & 0.168(+) & -0.597(-) \\ -0.098(\cdot) & 0.171(+) & 0.141(\cdot) & 0.146(\cdot) \\ 0.245(+) & 0.200(+) & 0.046(\cdot) & 0.123(\cdot) \\ -0.103(\cdot) & 0.140(\cdot) & 0.128(\cdot) & 0.579(+) \end{pmatrix}$$

$$C(12)=\begin{pmatrix} 0.546(+) & -0.185(-) & 0.145(\cdot) & -0.599(-) \\ -0.101(\cdot) & 0.121(\cdot) & 0.076(\cdot) & 0.113(\cdot) \\ 0.232(+) & 0.128(\cdot) & 0.022(\cdot) & 0.123(\cdot) \\ -0.073(\cdot) & 0.133(\cdot) & 0.122(\cdot) & 0.542(+) \end{pmatrix}$$

$$C(13)=\begin{pmatrix} 0.544(+) & -0.171(-) & 0.128(\cdot) & -0.600(-) \\ -0.112(\cdot) & 0.095(\cdot) & 0.007(\cdot) & 0.096(\cdot) \\ 0.233(+) & 0.053(\cdot) & 0.017(\cdot) & 0.118(\cdot) \\ -0.047(\cdot) & 0.123(\cdot) & 0.121(\cdot) & 0.500(+) \end{pmatrix}$$

Table.2.2.1. Continued

$C(14)=\begin{pmatrix} 0.521(+) & -0.149(\cdot) & 0.099(\cdot) & -0.596(-) \\ -0.140(\cdot) & 0.089(\cdot) & -0.042(\cdot) & 0.091(\cdot) \\ 0.239(+) & 0.000(\cdot) & 0.023(\cdot) & 0.118(\cdot) \\ -0.024(\cdot) & 0.126(\cdot) & 0.128(\cdot) & 0.456(+) \end{pmatrix}$	$C(15)=\begin{pmatrix} 0.491(+) & -0.111(\cdot) & 0.068(\cdot) & -0.589(-) \\ -0.191(-) & 0.084(\cdot) & -0.079(\cdot) & 0.107(\cdot) \\ 0.250(+) & -0.018(\cdot) & 0.031(\cdot) & 0.116(\cdot) \\ -0.001(\cdot) & 0.129(\cdot) & 0.135(\cdot) & 0.418(+) \end{pmatrix}$
$C(16)=\begin{pmatrix} 0.459(+) & -0.094(\cdot) & 0.035(\cdot) & -0.586(-) \\ -0.230(-) & 0.068(\cdot) & -0.110(\cdot) & 0.132(\cdot) \\ 0.271(+) & -0.026(\cdot) & 0.025(\cdot) & 0.100(\cdot) \\ 0.0186(\cdot) & 0.123(\cdot) & 0.140(\cdot) & 0.387(+) \end{pmatrix}$	$C(17)=\begin{pmatrix} 0.432(+) & -0.094(\cdot) & 0.002(\cdot) & -0.576(-) \\ -0.268(-) & 0.044(\cdot) & -0.130(\cdot) & 0.161(\cdot) \\ 0.273(+) & -0.018(\cdot) & 0.006(\cdot) & 0.071(\cdot) \\ 0.033(\cdot) & 0.123(\cdot) & 0.141(\cdot) & 0.361(+) \end{pmatrix}$
$C(18)=\begin{pmatrix} 0.425(+) & -0.087(\cdot) & -0.025(\cdot) & -0.561(-) \\ -0.293(-) & 0.002(\cdot) & -0.149(\cdot) & 0.188(+) \\ 0.265(+) & -0.011(\cdot) & -0.005(\cdot) & 0.031(\cdot) \\ 0.040(\cdot) & 0.107(\cdot) & 0.143(\cdot) & 0.336(+) \end{pmatrix}$	$C(19)=\begin{pmatrix} 0.415(+) & -0.089(\cdot) & -0.050(\cdot) & -0.546(-) \\ -0.295(-) & -0.043(\cdot) & -0.155(\cdot) & 0.199(+) \\ 0.247(+) & -0.022(\cdot) & 0.005(\cdot) & -0.020(\cdot) \\ 0.040(\cdot) & 0.080(\cdot) & 0.142(\cdot) & 0.310(+) \end{pmatrix}$
$C(20)=\begin{pmatrix} 0.416(+) & -0.081(\cdot) & -0.075(\cdot) & -0.527(-) \\ -0.299(-) & -0.094(\cdot) & -0.131(\cdot) & 0.215(+) \\ 0.223(+) & -0.020(\cdot) & 0.037(\cdot) & -0.063(\cdot) \\ 0.042(\cdot) & 0.063(\cdot) & 0.141(\cdot) & 0.282(+) \end{pmatrix}$	$C(21)=\begin{pmatrix} 0.412(+) & -0.060(\cdot) & -0.102(\cdot) & -0.514(-) \\ -0.304(-) & -0.147(\cdot) & -0.105(\cdot) & 0.239(+) \\ 0.216(+) & -0.030(\cdot) & 0.078(\cdot) & -0.107(\cdot) \\ 0.043(\cdot) & 0.045(\cdot) & 0.143(\cdot) & 0.254(+) \end{pmatrix}$
$C(22)=\begin{pmatrix} 0.398(+) & -0.045(\cdot) & -0.126(\cdot) & -0.512(-) \\ -0.309(-) & -0.181(-) & -0.064(\cdot) & 0.261(+) \\ 0.226(+) & -0.046(\cdot) & 0.112(\cdot) & -0.141(\cdot) \\ 0.049(\cdot) & 0.031(\cdot) & 0.147(\cdot) & 0.224(+) \end{pmatrix}$	$C(23)=\begin{pmatrix} 0.391(+) & -0.049(\cdot) & -0.137(\cdot) & -0.509(-) \\ -0.316(-) & -0.201(-) & -0.019(\cdot) & 0.299(+) \\ 0.243(+) & -0.033(\cdot) & 0.128(\cdot) & -0.153(\cdot) \\ 0.049(\cdot) & 0.036(\cdot) & 0.147(\cdot) & 0.197(+) \end{pmatrix}$
$C(24)=\begin{pmatrix} 0.368(+) & -0.066(\cdot) & -0.140(\cdot) & -0.505(-) \\ -0.307(-) & -0.202(-) & 0.010(\cdot) & 0.336(+) \\ 0.249(+) & -0.024(\cdot) & 0.131(\cdot) & -0.165(\cdot) \\ 0.058(\cdot) & 0.037(\cdot) & 0.147(\cdot) & 0.165(\cdot) \end{pmatrix}$	$S=\begin{pmatrix} 1.000 & -0.077 & 0.309 & -0.424 \\ -0.077 & 1.000 & 0.229 & 0.229 \\ 0.309 & 0.229 & 1.000 & 0.189 \\ -0.424 & 0.229 & 0.189 & 1.000 \end{pmatrix}$

Table.2.2.2. The χ^2 statistic for testing combined significance of cross correlation matrices calculated for the original series and residual series of VAR(1) and VAR(2) models using the vector time series of transformed landings of anchovies, lesser sardines, ribbon fish and catfish.

Lag	χ^2 statistic			Lag	χ^2 statistic		
	Original series	Residual series of VAR(2)	Residual series of VAR(1)		Original series	Residual series of VAR(2)	Residual series of VAR(1)
1	497.82	2.49	39.87	13	133.24	24.24	28.54
2	397.87	11.45	13.17	14	127.21	16.28	19.75
3	314.55	25.65	15.14	15	125.62	19.71	14.78
4	249.68	83.82	73.74	16	125.45	14.00	18.78
5	224.81	10.78	7.86	17	120.96	19.34	21.03
6	205.03	26.14	26.50	18	113.68	29.41	24.74
7	186.41	21.52	16.97	19	102.31	24.20	21.28
8	175.27	10.84	13.20	20	96.19	11.69	14.56
9	167.64	18.88	18.29	21	98.70	13.34	16.30
10	160.68	12.89	18.65	22	104.78	38.99	36.04
11	152.69	20.65	22.30	23	111.00	18.53	18.92
12	141.69	16.72	18.97	24	112.17	19.94	21.97

Table.2.2.3. Partial cross correlation matrices for the vector time series of transformed landings of anchovies, lesser sardines, ribbon fish and catfish.

$\hat{\rho}(1)=\begin{pmatrix} 0.946(+) & -0.071(\cdot) & 0.297(+) & -0.441(-) \\ -0.079(\cdot) & 0.911(+) & 0.210(+) & 0.227(+) \\ 0.312(+) & 0.250(+) & 0.928(+) & 0.198(+) \\ -0.384(-) & 0.217(+) & 0.192(+) & 0.963(+) \end{pmatrix}$	$\hat{\rho}(2)=\begin{pmatrix} -0.062(\cdot) & -0.058(\cdot) & 0.138(\cdot) & -0.133(\cdot) \\ 0.016(\cdot) & -0.173(-) & 0.046(\cdot) & 0.121(\cdot) \\ 0.204(+) & 0.057(\cdot) & -0.341(-) & -0.070(\cdot) \\ 0.013(\cdot) & -0.072(\cdot) & 0.023(\cdot) & -0.140(\cdot) \end{pmatrix}$
$\hat{\rho}(3)=\begin{pmatrix} -0.044(\cdot) & -0.095(\cdot) & 0.039(\cdot) & -0.035(\cdot) \\ 0.001(\cdot) & -0.123(\cdot) & -0.039(\cdot) & -0.043(\cdot) \\ -0.181(-) & 0.099(\cdot) & -0.066(\cdot) & 0.006(\cdot) \\ -0.082(\cdot) & 0.006(\cdot) & -0.044(\cdot) & 0.007(\cdot) \end{pmatrix}$	$\hat{\rho}(4)=\begin{pmatrix} -0.192(-) & -0.043(\cdot) & -0.156(\cdot) & -0.030(\cdot) \\ -0.035(\cdot) & -0.131(\cdot) & 0.154(\cdot) & -0.046(\cdot) \\ -0.088(\cdot) & 0.000(\cdot) & 0.004(\cdot) & 0.023(\cdot) \\ 0.054(\cdot) & 0.081(\cdot) & -0.001(\cdot) & -0.044(\cdot) \end{pmatrix}$
$\hat{\rho}(5)=\begin{pmatrix} 0.301(+) & -0.059(\cdot) & 0.105(\cdot) & -0.077(\cdot) \\ -0.053(\cdot) & 0.439(+) & 0.042(\cdot) & 0.009(\cdot) \\ -0.051(\cdot) & 0.087(\cdot) & 0.147(\cdot) & 0.085(\cdot) \\ -0.052(\cdot) & 0.057(\cdot) & 0.026(\cdot) & 0.169(+) \end{pmatrix}$	$\hat{\rho}(6)=\begin{pmatrix} 0.002(\cdot) & -0.043(\cdot) & 0.066(\cdot) & -0.145(\cdot) \\ -0.003(\cdot) & -0.094(\cdot) & -0.012(\cdot) & -0.039(\cdot) \\ 0.093(\cdot) & -0.061(\cdot) & -0.281(-) & -0.073(\cdot) \\ 0.109(\cdot) & 0.004(\cdot) & -0.018(\cdot) & -0.089(\cdot) \end{pmatrix}$
$\hat{\rho}(7)=\begin{pmatrix} 0.002(\cdot) & 0.132(\cdot) & 0.029(\cdot) & 0.063(\cdot) \\ -0.116(\cdot) & -0.228(-) & -0.001(\cdot) & -0.047(\cdot) \\ -0.018(\cdot) & 0.022(\cdot) & -0.110(\cdot) & 0.046(\cdot) \\ -0.020(\cdot) & -0.048(\cdot) & -0.106(\cdot) & 0.006(\cdot) \end{pmatrix}$	$\hat{\rho}(8)=\begin{pmatrix} -0.072(\cdot) & -0.243(-) & -0.092(\cdot) & -0.098(\cdot) \\ 0.091(\cdot) & -0.036(\cdot) & -0.071(\cdot) & -0.002(\cdot) \\ 0.071(\cdot) & 0.067(\cdot) & -0.162(\cdot) & 0.110(\cdot) \\ -0.039(\cdot) & 0.042(\cdot) & 0.019(\cdot) & 0.079(\cdot) \end{pmatrix}$
$\hat{\rho}(9)=\begin{pmatrix} 0.128(\cdot) & -0.017(\cdot) & 0.032(\cdot) & -0.015(\cdot) \\ -0.033(\cdot) & 0.037(\cdot) & -0.005(\cdot) & -0.077(\cdot) \\ -0.069(\cdot) & -0.061(\cdot) & 0.077(\cdot) & 0.247(+) \\ 0.065(\cdot) & -0.051(\cdot) & 0.051(\cdot) & 0.036(\cdot) \end{pmatrix}$	$\hat{\rho}(10)=\begin{pmatrix} 0.167(+) & -0.109(\cdot) & 0.104(\cdot) & -0.053(\cdot) \\ 0.096(\cdot) & -0.123(\cdot) & -0.155(\cdot) & -0.100(\cdot) \\ 0.140(\cdot) & -0.158(\cdot) & -0.158(\cdot) & -0.074(\cdot) \\ 0.001(\cdot) & -0.007(\cdot) & -0.156(\cdot) & 0.070(\cdot) \end{pmatrix}$
$\hat{\rho}(11)=\begin{pmatrix} 0.033(\cdot) & 0.019(\cdot) & 0.047(\cdot) & -0.081(\cdot) \\ -0.059(\cdot) & -0.074(\cdot) & -0.098(\cdot) & -0.046(\cdot) \\ 0.015(\cdot) & -0.074(\cdot) & -0.045(\cdot) & 0.080(\cdot) \\ 0.036(\cdot) & 0.058(\cdot) & 0.114(\cdot) & -0.097(\cdot) \end{pmatrix}$	$\hat{\rho}(12)=\begin{pmatrix} -0.044(\cdot) & 0.053(\cdot) & 0.065(\cdot) & -0.106(\cdot) \\ -0.077(\cdot) & -0.027(\cdot) & -0.013(\cdot) & 0.018(\cdot) \\ -0.103(\cdot) & 0.099(\cdot) & 0.097(\cdot) & 0.132(\cdot) \\ 0.031(\cdot) & 0.099(\cdot) & 0.030(\cdot) & -0.050(\cdot) \end{pmatrix}$
$\hat{\rho}(13)=\begin{pmatrix} 0.104(\cdot) & 0.156(\cdot) & 0.086(\cdot) & -0.013(\cdot) \\ -0.011(\cdot) & 0.105(\cdot) & 0.004(\cdot) & 0.141(\cdot) \\ 0.186(+) & 0.062(\cdot) & 0.162(\cdot) & 0.151(\cdot) \\ -0.024(\cdot) & -0.049(\cdot) & 0.033(\cdot) & -0.057(\cdot) \end{pmatrix}$	$\hat{\rho}(14)=\begin{pmatrix} -0.155(\cdot) & 0.003(\cdot) & -0.111(\cdot) & 0.069(\cdot) \\ -0.065(\cdot) & 0.012(\cdot) & 0.084(\cdot) & 0.025(\cdot) \\ 0.005(\cdot) & 0.153(\cdot) & -0.171(-) & 0.086(\cdot) \\ -0.010(\cdot) & 0.072(\cdot) & -0.032(\cdot) & -0.072(\cdot) \end{pmatrix}$
$\hat{\rho}(15)=\begin{pmatrix} 0.004(\cdot) & 0.056(\cdot) & 0.023(\cdot) & -0.117(\cdot) \\ -0.239(-) & 0.016(\cdot) & -0.009(\cdot) & 0.204(+) \\ 0.006(\cdot) & 0.166(\cdot) & -0.102(\cdot) & 0.104(\cdot) \\ 0.055(\cdot) & 0.019(\cdot) & 0.026(\cdot) & 0.067(\cdot) \end{pmatrix}$	$\hat{\rho}(16)=\begin{pmatrix} -0.109(\cdot) & -0.110(\cdot) & -0.054(\cdot) & -0.171(-) \\ 0.054(\cdot) & -0.109(\cdot) & -0.052(\cdot) & 0.069(\cdot) \\ 0.007(\cdot) & -0.017(\cdot) & -0.190(-) & 0.024(\cdot) \\ -0.022(\cdot) & -0.124(\cdot) & -0.002(\cdot) & 0.031(\cdot) \end{pmatrix}$

Table.2.2.4. Calculated values of squared partial canonical correlations for the vector time series of transformed landings of anchovies, lesser sardines, ribbon fish and catfish.

Lag	Squared partial canonical correlation				LR Statistic
	(i)	(ii)	(iii)	(iv)	
1	0.9356	0.9057	0.8280	0.7402	1190.75
2	0.2191	0.0823	0.0315	0.0000	52.96
3	0.0617	0.0300	0.0044	0.0002	14.31
4	0.0691	0.0562	0.0052	0.0034	20.02
5	0.2133	0.0922	0.0593	0.0087	58.96
6	0.1239	0.0411	0.0120	0.0020	27.31
7	0.0871	0.0302	0.0040	0.0000	18.24
8	0.1201	0.0478	0.0113	0.0010	27.47
9	0.1109	0.0389	0.0012	0.0004	23.02
10	0.1557	0.0420	0.0137	0.0001	32.77

Table.2.2.5. Computed values of different order selection criteria for the vector time series of transformed landings of anchovies, lesser sardines, ribbon fish and catfish.

p	$\lvert\tilde{\Sigma}\rvert$	**FPE**	**AIC**	**HQ**	**SC**
1	0.000075	0.000080	-9.2773	-9.1438	-8.9488
2	0.000051	0.000057	-9.4493	-9.1824	-8.7924
3	0.000046	0.000055	-9.3342	-8.9338	-8.3488
4	0.000039	0.000049	-9.2738	-8.7399	-7.9599
5	0.000021	0.000029	-9.6522	-8.9849	-8.0099
6	0.000016	0.000023	-9.6980	-8.8972	-7.7272
7	0.000012	0.000018	-9.7701	-8.8358	-7.4708
8	0.000010	0.000016	-9.7168	-8.6491	-7.0891
9	0.000008	0.000013	-9.7728	-8.5716	-6.8166
10	0.000005	0.000009	-9.9618	-8.6271	-6.6771

Table.2.2.6. Eigen values of the characteristic matrices of the VAR(1) and VAR(2) models estimated for the vector time series of transformed landings of anchovies, lesser sardines, ribbon fish and catfish.

No.	VAR(2) model			VAR(1) model		
	Real part	Imaginary part	Absolute value	Real part	Imaginary part	Absolute value
1	-0.0181	0.0000	0.0181	0.9665	-0.0078	0.9665
2	0.2342	-0.0818	0.2481	0.9665	0.0078	0.9665
3	0.2342	0.0818	0.2481	0.8878	0.0000	0.8878
4	0.6521	-0.2350	0.6932	0.9135	0.0000	0.9135
5	0.6521	0.2350	0.6932			
6	0.8600	0.0000	0.8600			
7	0.9630	-0.0140	0.9631			
8	0.9630	0.0140	0.9631			

Table.2.3.1. Cross correlation matrices of the vector time series of transformed landings of mackerel, anchovies, tuna and penaeid prawns

$C(0)=\begin{pmatrix} 1.000(+) & 0.509(+) & 0.566(+) & 0.416(+) \\ 0.509(+) & 1.000(+) & 0.754(+) & 0.584(+) \\ 0.566(+) & 0.754(+) & 1.000(+) & 0.483(+) \\ 0.416(+) & 0.584(+) & 0.483(+) & 1.000(+) \end{pmatrix}$	$C(1)=\begin{pmatrix} 0.929(+) & 0.476(+) & 0.558(+) & 0.374(+) \\ 0.524(+) & 0.946(+) & 0.761(+) & 0.556(+) \\ 0.535(+) & 0.743(+) & 0.969(+) & 0.474(+) \\ 0.450(+) & 0.570(+) & 0.493(+) & 0.886(+) \end{pmatrix}$
$C(2)=\begin{pmatrix} 0.826(+) & 0.443(+) & 0.548(+) & 0.331(+) \\ 0.522(+) & 0.887(+) & 0.767(+) & 0.529(+) \\ 0.512(+) & 0.737(+) & 0.937(+) & 0.461(+) \\ 0.483(+) & 0.577(+) & 0.519(+) & 0.764(+) \end{pmatrix}$	$C(3)=\begin{pmatrix} 0.726(+) & 0.414(+) & 0.550(+) & 0.292(+) \\ 0.501(+) & 0.826(+) & 0.764(+) & 0.497(+) \\ 0.495(+) & 0.721(+) & 0.903(+) & 0.455(+) \\ 0.527(+) & 0.593(+) & 0.546(+) & 0.665(+) \end{pmatrix}$
$C(4)=\begin{pmatrix} 0.623(+) & 0.394(+) & 0.549(+) & 0.282(+) \\ 0.488(+) & 0.751(+) & 0.757(+) & 0.451(+) \\ 0.482(+) & 0.709(+) & 0.870(+) & 0.449(+) \\ 0.573(+) & 0.577(+) & 0.601(+) & 0.561(+) \end{pmatrix}$	$C(5)=\begin{pmatrix} 0.547(+) & 0.376(+) & 0.545(+) & 0.274(+) \\ 0.482(+) & 0.713(+) & 0.749(+) & 0.427(+) \\ 0.494(+) & 0.688(+) & 0.859(+) & 0.432(+) \\ 0.592(+) & 0.565(+) & 0.646(+) & 0.539(+) \end{pmatrix}$
$C(6)=\begin{pmatrix} 0.475(+) & 0.366(+) & 0.542(+) & 0.272(+) \\ 0.492(+) & 0.679(+) & 0.726(+) & 0.417(+) \\ 0.507(+) & 0.667(+) & 0.854(+) & 0.403(+) \\ 0.593(+) & 0.550(+) & 0.660(+) & 0.512(+) \end{pmatrix}$	$C(7)=\begin{pmatrix} 0.388(+) & 0.365(+) & 0.525(+) & 0.272(+) \\ 0.519(+) & 0.645(+) & 0.702(+) & 0.417(+) \\ 0.513(+) & 0.663(+) & 0.842(+) & 0.365(+) \\ 0.597(+) & 0.510(+) & 0.675(+) & 0.456(+) \end{pmatrix}$
$C(8)=\begin{pmatrix} 0.314(+) & 0.378(+) & 0.506(+) & 0.265(+) \\ 0.535(+) & 0.614(+) & 0.675(+) & 0.418(+) \\ 0.520(+) & 0.664(+) & 0.825(+) & 0.329(+) \\ 0.567(+) & 0.468(+) & 0.663(+) & 0.417(+) \end{pmatrix}$	$C(9)=\begin{pmatrix} 0.263(+) & 0.395(+) & 0.493(+) & 0.279(+) \\ 0.561(+) & 0.584(+) & 0.645(+) & 0.418(+) \\ 0.519(+) & 0.672(+) & 0.804(+) & 0.310(+) \\ 0.530(+) & 0.433(+) & 0.627(+) & 0.393(+) \end{pmatrix}$
$C(10)=\begin{pmatrix} 0.218(+) & 0.409(+) & 0.470(+) & 0.306(+) \\ 0.582(+) & 0.568(+) & 0.626(+) & 0.415(+) \\ 0.514(+) & 0.676(+) & 0.777(+) & 0.299(+) \\ 0.495(+) & 0.402(+) & 0.612(+) & 0.379(+) \end{pmatrix}$	$C(11)=\begin{pmatrix} 0.177(+) & 0.416(+) & 0.446(+) & 0.325(+) \\ 0.588(+) & 0.557(+) & 0.611(+) & 0.401(+) \\ 0.518(+) & 0.664(+) & 0.757(+) & 0.292(+) \\ 0.451(+) & 0.387(+) & 0.589(+) & 0.369(+) \end{pmatrix}$

Table.2.3.1. Continued

$C(12)=\begin{pmatrix} 0.138(\cdot) & 0.412(+) & 0.428(+) & 0.328(+) \\ 0.598(+) & 0.546(+) & 0.590(+) & 0.375(+) \\ 0.511(+) & 0.643(+) & 0.740(+) & 0.275(+) \\ 0.451(+) & 0.390(+) & 0.574(+) & 0.333(+) \end{pmatrix}$	$C(13)=\begin{pmatrix} 0.103(\cdot) & 0.405(+) & 0.407(+) & 0.312(+) \\ 0.589(+) & 0.544(+) & 0.573(+) & 0.348(+) \\ 0.506(+) & 0.615(+) & 0.722(+) & 0.246(+) \\ 0.458(+) & 0.402(+) & 0.578(+) & 0.298(+) \end{pmatrix}$
$C(14)=\begin{pmatrix} 0.086(\cdot) & 0.385(+) & 0.396(+) & 0.272(+) \\ 0.563(+) & 0.521(+) & 0.554(+) & 0.320(+) \\ 0.494(+) & 0.585(+) & 0.701(+) & 0.212(+) \\ 0.439(+) & 0.405(+) & 0.569(+) & 0.271(+) \end{pmatrix}$	$C(15)=\begin{pmatrix} 0.087(\cdot) & 0.357(+) & 0.393(+) & 0.234(+) \\ 0.525(+) & 0.491(+) & 0.533(+) & 0.296(+) \\ 0.477(+) & 0.569(+) & 0.676(+) & 0.179(+) \\ 0.422(+) & 0.419(+) & 0.564(+) & 0.255(+) \end{pmatrix}$
$C(16)=\begin{pmatrix} 0.088(\cdot) & 0.345(+) & 0.379(+) & 0.196(+) \\ 0.485(+) & 0.459(+) & 0.523(+) & 0.285(+) \\ 0.466(+) & 0.560(+) & 0.644(+) & 0.155(\cdot) \\ 0.379(+) & 0.431(+) & 0.554(+) & 0.274(+) \end{pmatrix}$	$C(17)=\begin{pmatrix} 0.083(\cdot) & 0.324(+) & 0.363(+) & 0.150(\cdot) \\ 0.448(+) & 0.432(+) & 0.509(+) & 0.267(+) \\ 0.452(+) & 0.550(+) & 0.611(+) & 0.143(\cdot) \\ 0.336(+) & 0.435(+) & 0.536(+) & 0.288(+) \end{pmatrix}$
$C(18)=\begin{pmatrix} 0.085(\cdot) & 0.313(+) & 0.346(+) & 0.103(\cdot) \\ 0.419(+) & 0.425(+) & 0.487(+) & 0.250(+) \\ 0.438(+) & 0.540(+) & 0.579(+) & 0.131(\cdot) \\ 0.324(+) & 0.423(+) & 0.515(+) & 0.292(+) \end{pmatrix}$	$C(19)=\begin{pmatrix} 0.091(\cdot) & 0.315(+) & 0.324(+) & 0.062(\cdot) \\ 0.410(+) & 0.415(+) & 0.461(+) & 0.240(+) \\ 0.419(+) & 0.524(+) & 0.550(+) & 0.117(\cdot) \\ 0.315(+) & 0.387(+) & 0.487(+) & 0.285(+) \end{pmatrix}$
$C(20)=\begin{pmatrix} 0.117(\cdot) & 0.294(+) & 0.302(+) & 0.033(\cdot) \\ 0.410(+) & 0.416(+) & 0.428(+) & 0.226(+) \\ 0.395(+) & 0.506(+) & 0.526(+) & 0.108(\cdot) \\ 0.305(+) & 0.350(+) & 0.455(+) & 0.245(+) \end{pmatrix}$	$C(21)=\begin{pmatrix} 0.157(\cdot) & 0.269(+) & 0.282(+) & -0.004(\cdot) \\ 0.406(+) & 0.412(+) & 0.390(+) & 0.213(+) \\ 0.368(+) & 0.493(+) & 0.505(+) & 0.085(\cdot) \\ 0.305(+) & 0.296(+) & 0.428(+) & 0.175(+) \end{pmatrix}$
$C(22)=\begin{pmatrix} 0.180(+) & 0.243(+) & 0.249(+) & -0.034(\cdot) \\ 0.409(+) & 0.398(+) & 0.362(+) & 0.203(+) \\ 0.341(+) & 0.493(+) & 0.481(+) & 0.069(\cdot) \\ 0.303(+) & 0.248(+) & 0.410(+) & 0.123(\cdot) \end{pmatrix}$	$C(23)=\begin{pmatrix} 0.194(+) & 0.211(+) & 0.214(+) & -0.070(\cdot) \\ 0.408(+) & 0.391(+) & 0.342(+) & 0.175(+) \\ 0.318(+) & 0.497(+) & 0.457(+) & 0.063(\cdot) \\ 0.311(+) & 0.215(+) & 0.394(+) & 0.085(\cdot) \end{pmatrix}$
$C(24)=\begin{pmatrix} 0.196(+) & 0.183(+) & 0.189(+) & -0.117(\cdot) \\ 0.394(+) & 0.368(+) & 0.329(+) & 0.144(\cdot) \\ 0.293(+) & 0.496(+) & 0.431(+) & 0.053(\cdot) \\ 0.328(+) & 0.171(+) & 0.376(+) & 0.056(\cdot) \end{pmatrix}$	$S=\begin{pmatrix} 1.000 & 0.509 & 0.566 & 0.416 \\ 0.509 & 1.000 & 0.754 & 0.584 \\ 0.566 & 0.754 & 1.000 & 0.483 \\ 0.416 & 0.584 & 0.483 & 1.000 \end{pmatrix}$

Table.2.3.2. The χ^2 statistic for testing combined significance of cross correlation matrices calculated for the original series and residual series of VAR(1) model using the vector time series of transformed landings of of mackerel, anchovies, tuna and penaeid prawns.

Lag	χ^2 statistic		Lag	χ^2 statistic	
	Original series	Residual series		Original series	Residual series
1	464.14	29.29	13	166.46	30.61
2	345.69	18.30	14	157.57	20.95
3	262.64	18.96	15	145.15	15.09
4	196.45	76.77	16	132.57	16.02
5	176.14	23.83	17	122.42	22.45
6	162.05	33.67	18	112.89	20.82
7	152.77	12.22	19	106.97	41.09
8	150.11	26.54	20	97.09	20.37
9	152.25	25.62	21	97.24	21.80
10	159.71	22.92	22	106.32	23.11
11	160.97	15.86	23	112.62	19.62
12	164.02	18.52	24	121.29	13.50

Table.2.3.3. Partial cross correlation matrices for the vector time series of transformed landings of mackerel, anchovies, tuna and penaeid prawns.

$\hat{\rho}(1)=\begin{pmatrix} 0.929(+) & 0.476(+) & 0.558(+) & 0.374(+) \\ 0.524(+) & 0.946(+) & 0.761(+) & 0.556(+) \\ 0.535(+) & 0.743(+) & 0.969(+) & 0.474(+) \\ 0.450(+) & 0.570(+) & 0.493(+) & 0.886(+) \end{pmatrix}$	$\hat{\rho}(2)=\begin{pmatrix} -0.265(-) & -0.040(\cdot) & 0.010(\cdot) & -0.048(\cdot) \\ -0.082(\cdot) & -0.094(\cdot) & 0.046(\cdot) & -0.010(\cdot) \\ 0.046(\cdot) & 0.076(\cdot) & -0.062(\cdot) & -0.026(\cdot) \\ 0.065(\cdot) & 0.166(\cdot) & 0.168(+) & -0.113(\cdot) \end{pmatrix}$
$\hat{\rho}(3)=\begin{pmatrix} 0.045(\cdot) & 0.004(\cdot) & 0.163(\cdot) & -0.011(\cdot) \\ -0.076(\cdot) & -0.092(\cdot) & -0.057(\cdot) & -0.054(\cdot) \\ -0.004(\cdot) & -0.135(\cdot) & -0.054(\cdot) & 0.051(\cdot) \\ 0.176(+) & 0.084(\cdot) & 0.063(\cdot) & 0.018(\cdot) \end{pmatrix}$	$\hat{\rho}(4)=\begin{pmatrix} -0.100(\cdot) & 0.027(\cdot) & -0.056(\cdot) & 0.139(\cdot) \\ 0.109(\cdot) & -0.246(-) & -0.006(\cdot) & -0.109(\cdot) \\ 0.015(\cdot) & 0.086(\cdot) & -0.014(\cdot) & -0.010(\cdot) \\ 0.080(\cdot) & -0.212(-) & 0.302(+) & -0.134(\cdot) \end{pmatrix}$
$\hat{\rho}(5)=\begin{pmatrix} 0.191(+) & -0.043(\cdot) & 0.033(\cdot) & -0.066(\cdot) \\ 0.083(\cdot) & 0.303(+) & 0.015(\cdot) & 0.094(\cdot) \\ 0.290(+) & -0.091(\cdot) & 0.408(+) & -0.066(\cdot) \\ 0.057(\cdot) & 0.083(\cdot) & 0.039(\cdot) & 0.310(+) \end{pmatrix}$	$\hat{\rho}(6)=\begin{pmatrix} -0.136(\cdot) & 0.044(\cdot) & 0.096(\cdot) & 0.043(\cdot) \\ 0.184(+) & -0.048(\cdot) & -0.116(\cdot) & 0.056(\cdot) \\ 0.026(\cdot) & 0.075(\cdot) & 0.118(\cdot) & -0.086(\cdot) \\ 0.098(\cdot) & -0.011(\cdot) & -0.071(\cdot) & -0.092(\cdot) \end{pmatrix}$
$\hat{\rho}(7)=\begin{pmatrix} -0.101(\cdot) & 0.040(\cdot) & -0.048(\cdot) & -0.067(\cdot) \\ 0.030(\cdot) & -0.006(\cdot) & -0.063(\cdot) & 0.074(\cdot) \\ -0.004(\cdot) & 0.125(\cdot) & -0.137(\cdot) & -0.057(\cdot) \\ 0.175(+) & -0.149(\cdot) & 0.100(\cdot) & -0.126(\cdot) \end{pmatrix}$	$\hat{\rho}(8)=\begin{pmatrix} -0.011(\cdot) & 0.120(\cdot) & -0.078(\cdot) & 0.055(\cdot) \\ -0.036(\cdot) & -0.105(\cdot) & 0.003(\cdot) & -0.035(\cdot) \\ -0.035(\cdot) & 0.124(\cdot) & -0.180(-) & 0.002(\cdot) \\ -0.125(\cdot) & -0.189(-) & 0.018(\cdot) & -0.041(\cdot) \end{pmatrix}$
$\hat{\rho}(9)=\begin{pmatrix} 0.057(\cdot) & -0.003(\cdot) & -0.102(\cdot) & 0.084(\cdot) \\ 0.218(+) & 0.102(\cdot) & 0.026(\cdot) & 0.160(\cdot) \\ -0.026(\cdot) & 0.015(\cdot) & 0.065(\cdot) & -0.004(\cdot) \\ -0.023(\cdot) & 0.037(\cdot) & -0.257(-) & 0.137(\cdot) \end{pmatrix}$	$\hat{\rho}(10)=\begin{pmatrix} -0.173(-) & -0.040(\cdot) & -0.127(\cdot) & 0.085(\cdot) \\ -0.075(\cdot) & 0.115(\cdot) & 0.011(\cdot) & 0.040(\cdot) \\ 0.017(\cdot) & -0.026(\cdot) & 0.010(\cdot) & -0.009(\cdot) \\ -0.080(\cdot) & 0.182(+) & 0.015(\cdot) & 0.020(\cdot) \end{pmatrix}$

Table.2.3.3. Continued

$\hat{\rho}(11)=\begin{pmatrix} -0.153(\cdot) & -0.039(\cdot) & -0.087(\cdot) & -0.014(\cdot) \\ 0.047(\cdot) & 0.038(\cdot) & 0.031(\cdot) & -0.018(\cdot) \\ 0.072(\cdot) & -0.065(\cdot) & 0.001(\cdot) & -0.005(\cdot) \\ 0.086(\cdot) & -0.005(\cdot) & -0.062(\cdot) & -0.083(\cdot) \end{pmatrix}$	$\hat{\rho}(12)=\begin{pmatrix} -0.018(\cdot) & -0.047(\cdot) & 0.082(\cdot) & -0.019(\cdot) \\ -0.003(\cdot) & -0.174(-) & -0.139(\cdot) & -0.068(\cdot) \\ -0.047(\cdot) & 0.035(\cdot) & -0.048(\cdot) & -0.073(\cdot) \\ 0.125(\cdot) & -0.098(\cdot) & 0.101(\cdot) & -0.100(\cdot) \end{pmatrix}$
$\hat{\rho}(13)=\begin{pmatrix} 0.032(\cdot) & -0.042(\cdot) & -0.115(\cdot) & -0.005(\cdot) \\ -0.173(-) & 0.051(\cdot) & -0.084(\cdot) & -0.008(\cdot) \\ 0.128(\cdot) & -0.055(\cdot) & -0.032(\cdot) & -0.029(\cdot) \\ -0.171(-) & 0.070(\cdot) & -0.023(\cdot) & 0.027(\cdot) \end{pmatrix}$	$\hat{\rho}(14)=\begin{pmatrix} -0.080(\cdot) & -0.191(-) & -0.052(\cdot) & -0.088(\cdot) \\ -0.092(\cdot) & -0.197(-) & -0.054(\cdot) & 0.074(\cdot) \\ -0.143(\cdot) & -0.145(\cdot) & -0.107(\cdot) & -0.052(\cdot) \\ 0.013(\cdot) & -0.111(\cdot) & 0.045(\cdot) & 0.037(\cdot) \end{pmatrix}$
$\hat{\rho}(15)=\begin{pmatrix} -0.062(\cdot) & -0.024(\cdot) & -0.026(\cdot) & 0.028(\cdot) \\ -0.086(\cdot) & -0.058(\cdot) & -0.140(\cdot) & 0.064(\cdot) \\ 0.041(\cdot) & 0.088(\cdot) & -0.034(\cdot) & -0.066(\cdot) \\ 0.053(\cdot) & 0.144(\cdot) & 0.020(\cdot) & 0.050(\cdot) \end{pmatrix}$	$\hat{\rho}(16)=\begin{pmatrix} -0.162(\cdot) & 0.144(\cdot) & -0.087(\cdot) & -0.066(\cdot) \\ 0.105(\cdot) & -0.011(\cdot) & 0.032(\cdot) & 0.059(\cdot) \\ 0.079(\cdot) & 0.086(\cdot) & -0.054(\cdot) & 0.002(\cdot) \\ 0.023(\cdot) & -0.099(\cdot) & -0.124(\cdot) & 0.031(\cdot) \end{pmatrix}$

Table.2.3.4. Calculated values of squared partial canonical correlations and test statistic for the vector time series of transformed landings of mackerel, anchovies, tuna and penaeid prawns.

	Squared Partial Canonical Correlation				LR
Lag	(i)	(ii)	(iii)	(iv)	Statistic
1	0.9555	0.8435	0.6934	0.6866	1059.66
2	0.1586	0.0904	0.0121	0.0005	40.63
3	0.0931	0.0446	0.0129	0.0023	22.99
4	0.1611	0.0809	0.0480	0.0001	44.86
5	0.2484	0.1045	0.0881	0.0388	76.50
6	0.1600	0.0619	0.0052	0.0001	35.32
7	0.0993	0.0533	0.0319	0.0013	28.00
8	0.1197	0.0186	0.0059	0.0002	22.08
9	0.1745	0.0613	0.0058	0.0004	37.88
10	0.0847	0.0564	0.0006	0.0000	21.35

Table.2.3.5. Computed values of different order selection criteria for the vector time series of transformed landings of mackerel, anchovies, tuna and penaeid prawns.

P	$\lvert\tilde{\Sigma}\rvert$	FPE	AIC	HQ	SC
1	0.000048	0.000051	-9.7265	-9.5930	-9.3980
2	0.000036	0.000041	-9.7808	-9.5138	-9.1238
3	0.000031	0.000037	-9.7160	-9.3156	-8.7306
4	0.000024	0.000030	-9.7641	-9.2303	-8.4503
5	0.000012	0.000016	-10.2410	-9.5736	-8.5986
6	0.000008	0.000011	-10.4012	-9.6004	-8.4304
7	0.000007	0.000010	-10.3977	-9.4634	-8.0984
8	0.000005	0.000008	-10.3873	-9.3196	-7.7596
9	0.000003	0.00006	-10.6167	-9.4155	-7.6605
10	0.000002	0.000004	-10.7960	-9.4613	-7.5113

Table.2.3.6. Eigen values of the characteristic matrix of the VAR(1) model estimated for the vector time series of transformed landings of mackerel, anchovies, tuna and penaeid prawns.

No.	Real Part	Imaginary part	Absolute Value
1	0.8809	-0.0434	0.8820
2	0.8809	0.0434	0.8820
3	0.8463	0.0000	0.8463
4	0.9869	0.0000	0.9869

Table.2.4.1. Cross correlation matrices of the vector time series of transformed landings of oil sardine, anchovies, tuna and penaeid prawns

$C(0)=\begin{pmatrix} 1.000(+) & -0.412(-) & -0.359(-) & -.400(-) \\ -0.412(-) & 1.000(+) & 0.754(+) & 0.584(+) \\ -0.359(-) & 0.754(+) & 1.000(+) & 0.483(+) \\ -0.400(-) & 0.584(+) & 0.483(+) & 1.000(+) \end{pmatrix}$	$C(1)=\begin{pmatrix} 0.948(+) & -0.397(-) & -0.347(-) & -0.371(-) \\ -0.420(-) & 0.946(+) & 0.761(+) & 0.556(+) \\ -0.374(-) & 0.743(+) & 0.969(+) & 0.474(+) \\ -0.413(+) & 0.570(+) & 0.493(+) & 0.886(+) \end{pmatrix}$
$C(2)=\begin{pmatrix} 0.852(+) & -0.366(-) & -0.350(-) & -0.324(-) \\ -0.437(-) & 0.887(+) & 0.767(+) & 0.529(+) \\ -0.392(-) & 0.737(+) & 0.937(+) & 0.461(+) \\ -0.404(-) & 0.577(+) & 0.519(+) & 0.764(+) \end{pmatrix}$	$C(3)=\begin{pmatrix} 0.733(+) & -0.338(-) & -0.357(-) & -0.284(-) \\ -0.453(-) & 0.826(+) & 0.764(+) & 0.497(+) \\ -0.408(-) & 0.721(+) & 0.903(+) & 0.455(+) \\ -0.358(-) & 0.593(+) & 0.546(+) & 0.665(+) \end{pmatrix}$
$C(4)=\begin{pmatrix} 0.597(+) & -0.312(-) & -0.359(-) & -0.241(-) \\ -0.480(-) & 0.751(+) & 0.757(+) & 0.451(+) \\ -0.418(-) & 0.709(+) & 0.870(+) & 0.449(+) \\ -0.313(-) & 0.577(+) & 0.601(+) & 0.561(+) \end{pmatrix}$	$C(5)=\begin{pmatrix} 0.482(+) & -0.284(-) & -0.351(-) & -0.205(-) \\ -0.518(-) & 0.713(+) & 0.749(+) & 0.427(+) \\ -0.425(-) & 0.688(+) & 0.859(+) & 0.432(+) \\ -0.270(-) & 0.565(+) & 0.646(+) & 0.539(+) \end{pmatrix}$
$C(6)=\begin{pmatrix} 0.375(+) & -0.263(-) & -0.331(-) & -0.177(-) \\ -0.551(-) & 0.679(+) & 0.726(+) & 0.417(+) \\ -0.430(-) & 0.667(+) & 0.854(+) & 0.403(+) \\ -0.234(-) & 0.550(+) & 0.660(+) & 0.512(+) \end{pmatrix}$	$C(7)=\begin{pmatrix} 0.272(+) & -0.235(-) & -0.298(-) & -0.155(\cdot) \\ -0.583(-) & 0.645(+) & 0.702(+) & 0.417(+) \\ -0.435(-) & 0.663(+) & 0.842(+) & 0.365(+) \\ -0.209(-) & 0.510(+) & 0.675(+) & 0.456(+) \end{pmatrix}$
$C(8)=\begin{pmatrix} 0.180(+) & -0.211 & -0.270(-) & -0.140(\cdot) \\ -0.609(-) & 0.614(+) & 0.675(+) & 0.418(+) \\ -0.440(-) & 0.664(+) & 0.825(+) & 0.329(+) \\ -0.194(-) & 0.468(+) & 0.663(+) & 0.417(+) \end{pmatrix}$	$C(9)=\begin{pmatrix} 0.097(\cdot) & -0.181(-) & -0.247(-) & -0.106(\cdot) \\ -0.626(-) & 0.584(+) & 0.645(+) & 0.418(+) \\ -0.441(-) & 0.672(+) & 0.804(+) & 0.310(+) \\ -0.198(-) & 0.433(+) & 0.627(+) & 0.393(+) \end{pmatrix}$
$C(10)=\begin{pmatrix} 0.026(\cdot) & -0.156(\cdot) & -0.228(-) & -0.062(\cdot) \\ -0.633(--) & 0.568(+) & 0.626(+) & 0.415(+) \\ -0.440(-) & 0.676(+) & 0.777(+) & 0.299(+) \\ -0.207(-) & 0.402(+) & 0.612(+) & 0.379(+) \end{pmatrix}$	$C(11)=\begin{pmatrix} -0.027(\cdot) & -0.149(\cdot) & -0.221(-) & -0.021(\cdot) \\ -0.630(-) & 0.557(+) & 0.611(+) & 0.401(+) \\ -0.437(-) & 0.664(+) & 0.757(+) & 0.292(+) \\ -0.221(-) & 0.387(+) & 0.589(+) & 0.369(+) \end{pmatrix}$
$C(12)=\begin{pmatrix} -0.066(\cdot) & -0.142(\cdot) & -0.215(-) & 0.013(\cdot) \\ -0.616(-) & 0.546(+) & 0.590(+) & 0.375(+) \\ -0.432(-) & 0.643(+) & 0.740(+) & 0.275(+) \\ -0.248(-) & 0.390(+) & 0.574(+) & 0.333(+) \end{pmatrix}$	$C(13)=\begin{pmatrix} -0.095(\cdot) & -0.148(\cdot) & -0.205(-) & 0.026(\cdot) \\ -0.582(-) & 0.544(+) & 0.573(+) & 0.348(+) \\ -0.435(-) & 0.615(+) & 0.722(+) & 0.246(+) \\ -0.261(-) & 0.402(+) & 0.578(+) & 0.298(+) \end{pmatrix}$
$C(14)=\begin{pmatrix} -0.117(\cdot) & -0.153(\cdot) & -0.194(-) & 0.020(\cdot) \\ -0.535(-) & 0.521(+) & 0.554(+) & 0.320(+) \\ -0.442(-) & 0.585(+) & 0.701(+) & 0.212(+) \\ -0.267(-) & 0.405(+) & 0.569(+) & 0.271(+) \end{pmatrix}$	$C(15)=\begin{pmatrix} -0.136(\cdot) & -0.147(\cdot) & -0.177(-) & 0.017(\cdot) \\ -0.481(-) & 0.491(+) & 0.533(+) & 0.296(+) \\ -0.460(-) & 0.569(+) & 0.676(+) & 0.179(+) \\ -0.281(-) & 0.419(+) & 0.564(+) & 0.255(+) \end{pmatrix}$

Table.2.4.1. Continued

$$C(16)=\begin{pmatrix} -0.152(\cdot) & -0.147(\cdot) & -0.159(\cdot) & 0.015(\cdot) \\ -0.431(-) & 0.459(+) & 0.523(+) & 0.285(+) \\ -0.484(-) & 0.560(+) & 0.644(+) & 0.155(\cdot) \\ -0.269(-) & 0.431(+) & 0.554(+) & 0.274(+) \end{pmatrix} \quad C(17)=\begin{pmatrix} -0.153(\cdot) & -0.143(\cdot) & -0.143(\cdot) & 0.009(\cdot) \\ -0.393(-) & 0.432(+) & 0.509(+) & 0.267(+) \\ -0.499(-) & 0.550(+) & 0.611(+) & 0.143(\cdot) \\ -0.262(-) & 0.435(+) & 0.536(+) & 0.288(+) \end{pmatrix}$$

$$C(18)=\begin{pmatrix} -0.142(\cdot) & -0.148(\cdot) & -0.120(\cdot) & 0.005(\cdot) \\ -0.359(-) & 0.425(+) & 0.487(+) & 0.250(+) \\ -0.505(-) & 0.540(+) & 0.579(+) & 0.131(\cdot) \\ -0.263(-) & 0.423(+) & 0.515(+) & 0.292(+) \end{pmatrix} \quad C(19)=\begin{pmatrix} -0.123(\cdot) & -0.157(\cdot) & -0.092(\cdot) & -0.004(\cdot) \\ -0.332 & 0.415(+) & 0.461(+) & 0.240(+) \\ -0.493 & 0.524(+) & 0.550(+) & 0.117(\cdot) \\ -0.234 & 0.387(+) & 0.487(+) & 0.285(+) \end{pmatrix}$$

$$C(20)=\begin{pmatrix} -0.087(\cdot) & -0.160(\cdot) & -0.062(\cdot) & -0.012(\cdot) \\ -0.304(-) & 0.416(+) & 0.428(+) & 0.226(+) \\ -0.469(-) & 0.506(+) & 0.526(+) & 0.108(\cdot) \\ -0.282(-) & 0.350(+) & 0.455(+) & 0.245(+) \end{pmatrix} \quad C(21)=\begin{pmatrix} -0.039(\cdot) & -0.172(-) & -0.037(\cdot) & -0.015(\cdot) \\ -0.279(-) & 0.412(+) & 0.390(+) & 0.213(+) \\ -0.444(-) & 0.493(+) & 0.505(+) & 0.085(\cdot) \\ -0.303(-) & 0.296(+) & 0.428(+) & 0.175(+) \end{pmatrix}$$

$$C(22)=\begin{pmatrix} 0.010(\cdot) & -0.170(-) & -0.023(\cdot) & -0.020(\cdot) \\ -0.265(-) & 0.398(+) & 0.362(+) & 0.203(+) \\ -0.420(-) & 0.493(+) & 0..481(+) & 0.069(\cdot) \\ -0.322(-) & 0.248(+) & 0.410(+) & 0.123(\cdot) \end{pmatrix} \quad C(23)=\begin{pmatrix} 0.068(\cdot) & -0.162(\cdot) & -0.018(\cdot) & -0.025(\cdot) \\ -0.256(-) & 0.391(+) & 0.342(+) & 0.175(+) \\ -0.403(-) & 0.497(+) & 0.457(+) & 0.063(\cdot) \\ -0.341(-) & 0.215(+) & 0.394(+) & 0.085(\cdot) \end{pmatrix}$$

$$C(24)=\begin{pmatrix} 0.119(\cdot) & -0.157(\cdot) & -0.020(\cdot) & -0.029(\cdot) \\ -0.242(-) & 0.368(+) & 0.329(+) & 0.144(\cdot) \\ -0.392(-) & 0.496(+) & 0.431(+) & 0.053(\cdot) \\ -0.356(-) & 0.171(+) & 0.376(+) & 0.056(\cdot) \end{pmatrix} \quad S=\begin{pmatrix} 1.000 & -0.412 & -0.359 & -0.400 \\ -0.412 & 1.000 & 0.754 & 0.584 \\ -0.359 & 0.754 & 1.000 & 0.483 \\ -0.400 & 0.584 & 0.483 & 1.000 \end{pmatrix}$$

Table.2.4.2. The χ^2 statistic for testing combined significance of cross correlation matrices calculated for the original series and residual series of VAR(1), VAR(2) and VAR(5) models using the vector time series of transformed landings of oil sardine, anchovies, tuna and penaeid prawns.

Lag	χ^2 statistic				Lag	χ^2 statistic			
	Original series	Residual series of				Original series	Residual series of		
		VAR(1)	VAR(2)	VAR(5)			VAR(1)	VAR(2)	VAR(5)
1	468.72	46.70	5.71	4.59	13	170.68	19.46	15.07	13.73
2	353.18	20.93	14.04	9.87	14	158.46	13.77	18.56	11.08
3	266.94	15.39	28.40	3.36	15	150.57	16.15	17.98	13.20
4	198.14	73.93	87.03	15.34	16	148.77	22.61	25.41	19.58
5	184.18	21.05	21.15	21.85	17	146.42	24.12	17.35	13.46
6	179.81	29.42	30.96	16.55	18	142.83	21.61	15.77	18.84
7	178.36	10.49	9.83	24.20	19	138.51	31.23	28.28	26.35
8	183.23	22.66	19.22	30.88	20	128.66	18.73	23.80	22.41
9	186.54	24.07	21.64	11.95	21	131.57	23.52	26.90	28.25
10	191.55	23.33	24.11	20.13	22	136.84	22.05	23.97	17.90
11	189.11	17.49	17.14	10.57	23	136.04	12.00	19.99	19.47
12	180.39	22.77	22.27	22.42	24	140.75	14.21	17.98	11.49

Table.2.4.3. Partial cross correlation matrices for the vector time series of transformed landings of oil sardine, anchovies, tuna and penaeid prawns.

$\hat{\rho}(1)=\begin{pmatrix} 0.948(+) & -0.397(-) & -0.347(-) & -0.371(-) \\ -0.420(-) & 0.946(+) & 0.761(+) & 0.556(+) \\ -0.374(-) & 0.743(+) & 0.969(+) & 0.474(+) \\ -0.413(-) & 0.570(+) & 0.493(+) & 0.886(+) \end{pmatrix}$	$\hat{\rho}(2)=\begin{pmatrix} -0.464(-) & 0.155(\cdot) & -0.181(-) & 0.141(\cdot) \\ -0.112(\cdot) & -0.101(\cdot) & 0.052(\cdot) & -0.015(\cdot) \\ -0.071(\cdot) & 0.082(\cdot) & -0.057(\cdot) & -0.023(\cdot) \\ 0.124(\cdot) & 0.154(\cdot) & 0.174(+) & -0.120(\cdot) \end{pmatrix}$
$\hat{\rho}(3)=\begin{pmatrix} -0.172(-) & -0.092(\cdot) & 0.011(\cdot) & -0.069(\cdot) \\ 0.039(\cdot) & -0.117(\cdot) & -0.031(\cdot) & -0.075(\cdot) \\ 0.011(\cdot) & -0.128(\cdot) & -0.045(\cdot) & 0.050(\cdot) \\ 0.186(+) & 0.094(\cdot) & 0.044(\cdot) & 0.029(\cdot) \end{pmatrix}$	$\hat{\rho}(4)=\begin{pmatrix} -0.162(\cdot) & 0.008(\cdot) & 0.070(\cdot) & 0.042(\cdot) \\ -0.166(\cdot) & -0.233(-) & 0.022(\cdot) & -0.108(\cdot) \\ 0.050(\cdot) & 0.098(\cdot) & -0.033(\cdot) & 0.000(\cdot) \\ -0.095(\cdot) & -0.170(-) & 0.284(+) & -0.112(\cdot) \end{pmatrix}$
$\hat{\rho}(5)=\begin{pmatrix} 0.257(+) & -0.006(\cdot) & 0.178(+) & -0.061(\cdot) \\ -0.157(\cdot) & 0.297(+) & 0.042(\cdot) & 0.088(\cdot) \\ -0.016(\cdot) & 0.083(\cdot) & 0.385(+) & -0.074(\cdot) \\ .0.037(\cdot) & 0.029(\cdot) & 0.017(\cdot) & 0.307(+) \end{pmatrix}$	$\hat{\rho}(6)=\begin{pmatrix} -0.240(-) & 0.095(\cdot) & -0.017(\cdot) & 0.099(\cdot) \\ -0.076(\cdot) & -0.068(\cdot) & -0.157(\cdot) & 0.050(\cdot) \\ -0.028(\cdot) & 0.075(\cdot) & 0.099(\cdot) & -0.096(\cdot) \\ 0.019(\cdot) & 0.002(\cdot) & -0.038(\cdot) & -0.101(\cdot) \end{pmatrix}$
$\hat{\rho}(7)=\begin{pmatrix} -0.111(\cdot) & 0.049(\cdot) & 0.154(\cdot) & -0.053(\cdot) \\ -0.008(\cdot) & -0.003(\cdot) & -0.072(\cdot) & 0.061(\cdot) \\ -0.043(\cdot) & 0.112(\cdot) & -0.140(\cdot) & -0.065(\cdot) \\ 0.125(\cdot) & -0.142(\cdot) & 0.145(\cdot) & -0.108(\cdot) \end{pmatrix}$	$\hat{\rho}(8)=\begin{pmatrix} -0.079(\cdot) & -0.154(\cdot) & -0.117(\cdot) & 0.040(\cdot) \\ -0.100(\cdot) & -0.126(\cdot) & -0.059(\cdot) & -0.008(\cdot) \\ -0.072(\cdot) & 0.121(\cdot) & -0.156(\cdot) & -0.032(\cdot) \\ -0.101(\cdot) & -0.188(-) & -0.060(\cdot) & -0.022(\cdot) \end{pmatrix}$
$\hat{\rho}(9)=\begin{pmatrix} 0.057(\cdot) & 0.156(\cdot) & 0.134(\cdot) & 0.131(\cdot) \\ -0.101(\cdot) & 0.106(\cdot) & 0.021(\cdot) & 0.116(\cdot) \\ 0.018(\cdot) & 0.019(\cdot) & 0.101(\cdot) & 0.014(\cdot) \\ -0.136(\cdot) & 0.016(\cdot) & -0.235(-) & 0.101(\cdot) \end{pmatrix}$	$\hat{\rho}(10)=\begin{pmatrix} -0.084(\cdot) & -0.071(\cdot) & -0.049(\cdot) & 0.030(\cdot) \\ -0.002(\cdot) & 0.141(\cdot) & 0.071(\cdot) & 0.065(\cdot) \\ -0.042(\cdot) & -0.045(\cdot) & -0.014(\cdot) & -0.016(\cdot) \\ 0.040(\cdot) & 0.131(\cdot) & 0.062(\cdot) & 0.016(\cdot) \end{pmatrix}$
$\hat{\rho}(11)=\begin{pmatrix} 0.027(\cdot) & -0.022(\cdot) & -0.019(\cdot) & -0.037(\cdot) \\ -0.057(\cdot) & 0.057(\cdot) & 0.019(\cdot) & 0.033(\cdot) \\ -0.052(\cdot) & -0.032(\cdot) & 0.072(\cdot) & -0.003(\cdot) \\ 0.009(\cdot) & 0.054(\cdot) & 0.003(\cdot) & -0.006(\cdot) \end{pmatrix}$	$\hat{\rho}(12)=\begin{pmatrix} -0.074(\cdot) & 0.030(\cdot) & -0.093(\cdot) & -0.067(\cdot) \\ -0.030(\cdot) & -0.141(\cdot) & -0.113(\cdot) & -0.089(\cdot) \\ -0.034(\cdot) & 0.033(\cdot) & -0.014(\cdot) & -0.066(\cdot) \\ -0.192(-) & -0.078(\cdot) & 0.078(\cdot) & -0.130(\cdot) \end{pmatrix}$
$\hat{\rho}(13)=\begin{pmatrix} -0.132(\cdot) & -0.030(\cdot) & 0.173(+) & 0.033(\cdot) \\ 0.116(\cdot) & 0.050(\cdot) & -0.026(\cdot) & 0.076(\cdot) \\ -0.137(\cdot) & -0.035(\cdot) & 0.023(\cdot) & -0.048(\cdot) \\ -0.022(\cdot) & 0.048(\cdot) & -0.090(\cdot) & 0.092(\cdot) \end{pmatrix}$	$\hat{\rho}(14)=\begin{pmatrix} -0.037(\cdot) & 0.131(\cdot) & -0.111(\cdot) & 0.007(\cdot) \\ 0.081(\cdot) & -0.169(-) & -0.056(\cdot) & -0.046(\cdot) \\ -0.058(\cdot) & -0.119(\cdot) & -0.084(\cdot) & 0.073(\cdot) \\ 0.009(\cdot) & -0.107(\cdot) & -0.045(\cdot) & -0.037(\cdot) \end{pmatrix}$
$\hat{\rho}(15)=\begin{pmatrix} -0.206(-) & -0.025(\cdot) & 0.180(+) & 0.118(\cdot) \\ 0.073(\cdot) & 0.012(\cdot) & 0.033(\cdot) & 0.085(\cdot) \\ -0.233(-) & 0.092(\cdot) & -0.012(\cdot) & -0.005(\cdot) \\ -0.083(\cdot) & 0.160(\cdot) & 0.186(+) & 0.057(\cdot) \end{pmatrix}$	$\hat{\rho}(16)=\begin{pmatrix} 0.052(\cdot) & -0.067(\cdot) & -0.046(\cdot) & -0.080(\cdot) \\ -0.096(\cdot) & -0.073(\cdot) & 0.073(\cdot) & 0.104(\cdot) \\ -0.139(\cdot) & 0.145(\cdot) & -0.085(\cdot) & -0.028(\cdot) \\ 0.031(\cdot) & -0.076(\cdot) & -0.110(\cdot) & 0.132(\cdot) \end{pmatrix}$

Table.2.4.4. Calculated values of squared partial canonical correlations and test statistic for the vector time series of transformed landings of oil sardine, anchovies, tuna and penaeid prawns.

	Squared Partial Canonical Correlation				LR
Lag	(i)	(ii)	(iii)	(iv)	Statistic
1	0.9595	0.8686	0.6938	0.6884	1099.91
2	0.2957	0.0945	0.0203	0.0021	68.51
3	0.1051	0.0271	0.0095	0.0012	21.66
4	0.1787	0.0601	0.0180	0.0022	40.48
5	0.2296	0.1354	0.0760	0.0569	78.86
6	0.1185	0.0535	0.0107	0.0003	27.88
7	0.1279	0.0438	0.0210	0.0002	29.45
8	0.1284	0.0632	0.0036	0.0003	29.95
9	0.1392	0.0655	0.0048	0.0004	32.31
10	0.0538	0.0118	0.0007	0.0001	9.85

Table.2.4.5. Computed values of different order selection criteria for the vector time series of transformed landings of oil sardine, anchovies, tuna and penaeid prawns.

p	$\lvert\tilde{\Sigma}\rvert$	**FPE**	**AIC**	**HQ**	**SC**
1	0.000044	0.000047	-9.8219	-9.6885	-9.4935
2	0.000028	0.000031	-10.0518	-9.7848	-9.3948
3	0.000023	0.000028	-10.0137	-9.6133	-9.0283
4	0.000019	0.000024	-10.0018	-9.4679	-8.6879
5	0.000009	0.000012	-10.4872	-9.8199	-8.8449
6	0.000007	0.000010	-10.5465	-9.7457	-8.5757
7	0.000005	0.000008	-10.6036	-9.6693	-8.3043
8	0.000004	0.000006	-10.6521	-9.5843	-8.0243
9	0.000003	0.000005	-10.7252	-9.5240	-7.7690
10	0.000002	0.000004	-10.7697	-9.4350	-7.4850

Table.2.4.6. Eigen values of the characteristic matrices of the VAR(1), VAR(2) and VAR(5) models estimated for the vector time series of transformed landings of oil sardine, anchovies, tuna and penaeid prawns.

No.	VAR(1) model			VAR(2) model			VAR(5) model		
	Real part	Imaginary part	Absolute value	Real part	Imaginary part	Absolute value	Real part	Imaginary part	Absolute value
1	0.9851	0.0000	0.9851	0.0635	-0.1051	0.1228	-0.6161	-0.5011	0.7942
2	0.8344	0.0000	0.8344	0.0635	0.1051	0.1228	-0.6161	0.5011	0.7942
3	0.9333	0.0000	0.9333	0.5218	0.0000	0.5218	-0.5322	-0.6942	0.8747
4	0.8552	0.0000	0.8552	0.3281	0.0000	0.3281	-0.5322	0.6942	0.8747
5				0.7829	-0.1072	0.7902	-0.5442	-0.5651	0.7845
6				0.7829	0.1072	0.7902	-0.5442	0.5651	0.7845
7				0.9840	0.0000	0.9840	-0.4381	-0.6279	0.7656
8				0.8392	0.0000	0.8392	-0.4381	0.6279	0.7656
9							0.5194	-0.6553	0.8362
10							0.5194	0.6553	0.8362
11							0.6741	-0.5281	0.8563
12							0.6741	0.5281	0.8563
13							0.6197	-0.5169	0.8070
14							0.6197	0.5169	0.8070
15							0.7050	-0.3463	0.7855
16							0.7050	0.3463	0.7855
17							0.7912	0.0000	0.7912
18							0.9870	0.0000	0.9870
19							0.9203	-0.0245	0.9206
20							0.9203	0.0245	0.9206

Table.2.5.1. Cross correlation matrices of the vector time series of transformed landings of elasmobranchs, oil sardine, mackerel and sear fish.

$C(0)=\begin{pmatrix} 1.000(+) & 0.240(+) & -0.172(-) & -0.120(\cdot) \\ 0.240(+) & 1.000(+) & -0.350(-) & -0.345(-) \\ -0.172(-) & -0.350(-) & 1.000(+) & 0.421(+) \\ -0.120(\cdot) & -0.345(-) & 0.421(+) & 1.000(+) \end{pmatrix}$	$C(1)=\begin{pmatrix} 0.855(+) & 0.236(+) & -0.204(-) & -0.075(\cdot) \\ 0.264(+) & 0.948(+) & -0.354(-) & -0.340(-) \\ -0.153(\cdot) & -0.343(-) & 0.929(+) & 0.418(+) \\ -0.186(-) & -0.348(-) & 0.401(+) & 0.963(+) \end{pmatrix}$
$C(2)=\begin{pmatrix} 0.623(+) & 0.245(+) & -0.259(-) & -0.051(\cdot) \\ 0.295(+) & 0.852(+) & -0.368(-) & -0.347(-) \\ -0.171(-) & -0.334(-) & 0.826(+) & 0.416(+) \\ -0.243(-) & -0.360(-) & 0.391(+) & 0.914(+) \end{pmatrix}$	$C(3)=\begin{pmatrix} 0.370(+) & 0.266(+) & -0.323(-) & -0.034(\cdot) \\ 0.315(+) & 0.733(+) & -0.380(-) & -0.354(-) \\ -0.205(-) & -0.319(-) & 0.726(+) & 0.420(+) \\ -0.277(-) & -0.379(-) & 0.398(+) & 0.864(+) \end{pmatrix}$
$C(4)=\begin{pmatrix} 0.125(\cdot) & 0.293(+) & -0.357(-) & -0.044(\cdot) \\ 0.317(+) & 0.597(+) & -0.382(-) & -0.354(-) \\ -0.248(-) & -0.301(-) & 0.623(+) & 0.416(+) \\ -0.293(-) & -0.402(-) & 0.412(+) & 0.817(+) \end{pmatrix}$	$C(5)=\begin{pmatrix} 0.021(\cdot) & 0.311(+) & -0.382(-) & -0.059(\cdot) \\ 0.285(+) & 0.482(+) & -0.369(-) & -0.350(-) \\ -0.281(-) & -0.295(-) & 0.547(+) & 0.414(+) \\ -0.282(-) & -0.428(-) & 0.442(+) & 0.797(+) \end{pmatrix}$

Table.2.5.1. Continued

C(6) =	0.024(·)	0.315(+)	-0.379(-)	-0.069(·)	C(7) =	0.079(·)	0.297(+)	-0.356(-)	-0.084(·)
	0.231(+)	0.375(+)	-0.346(-)	-0.334(-)		0.182(+)	0.272(+)	-0.314(-)	-0.307(-)
	-0.279(-)	-0.296(-)	0.475(+)	0.406(+)		-0.237(-)	-0.303(-)	0.388(+)	0.382(+)
	-0.253(-)	-0.447(-)	0.467(+)	0.785(+)		-0.249(-)	-0.454(-)	0.482(+)	0.771(+)
C(8) =	0.151(·)	0.268(+)	-0.339(-)	-0.104(·)	C(9) =	0.229(+)	0.246(+)	-0.306(-)	-0.112(·)
	0.129(·)	0.180(+)	-0.285(-)	-0.273(-)		0.084(·)	0.097(·)	-0.249(-)	-0.233(-)
	-0.162(·)	-0.317(-)	0.314(+)	0.369(+)		-0.092(·)	-0.327(-)	0.263(+)	0.374(+)
	-0.263(-)	-0.452(-)	0.499(+)	0.757(+)		-0.300(-)	-0.439(-)	0.504(+)	0.748(+)
C(10) =	0.264(+)	0.243(+)	-0.264(-)	-0.113(·)	C(11) =	0.257(+)	0.252(+)	-0.221(-)	-0.114(·)
	0.070(·)	0.026(·)	-0.220(-)	-0.190(-)		0.056(·)	-0.027(·)	-0.196(-)	-0.151(·)
	-0.046(·)	-0.342(-)	0.218(+)	0.389(+)		-0.015(·)	-0.351(-)	0.177(+)	0.419(+)
	-0.355(-)	-0.424(-)	0.500(+)	0.729(+)		-0.388(-)	-0.412(-)	0.489(+)	0.714(+)
C(12) =	0.249(+)	0.262(+)	-0.184(-)	-0.089(·)	C(13) =	0.253(+)	0.263(+)	-0.169(-)	-0.056(·)
	0.056(·)	-0.066(·)	-0.173(-)	-0.124(·)		0.068(·)	-0.095(·)	-0.153(·)	-0.105(·)
	-0.024(·)	-0.354(-)	0.138(·)	0.450(+)		-0.062(·)	-0.343(-)	0.103(·)	0.456(+)
	-0.398(-)	-0.394(-)	0.470(+)	0.699(+)		-0.388(-)	-0.384(-)	0.449(+)	0.677(+)
C(14) =	0.254(+)	0.246(+)	-0.188(-)	-0.026(·)	C(15) =	0.265(+)	0.225(+)	-0.212(-)	0.017(·)
	0.074(·)	-0.117(·)	-0.124(·)	-0.096(·)		0.096(·)	-0.136(·)	-0.087(·)	-0.095(·)
	-0.090(·)	-0.326(-)	0.086(·)	0.460(+)		-0.141(·)	-0.317(-)	-0.087(·)	0.458(+)
	-0.360(-)	-0.375(-)	0.427(+)	0.660(+)		-0.343(-)	-0.374(-)	0.413(+)	0.641(+)
C(16) =	0.280(+)	0.204(+)	-0.222(-)	0.057(·)	C(17) =	0.248(+)	0.183(+)	-0.214(-)	0.083(·)
	0.132(·)	-0.152(·)	-0.045(·)	-0.087(·)		0.171(+)	-0.153(·)	-0.009(·)	-0.074(·)
	-0.173(-)	-0.319(-)	0.088(·)	0.446(+)		-0.173(-)	-0.332(-)	0.083(·)	0.443(+)
	-0.330(-)	-0.382(-)	0.396(+)	0.614(+)		-0.319(-)	-0.390(-)	0.379(+)	0.590(+)
C(18) =	0.192(+)	0.170(+)	-0.180(-)	0.091(·)	C(19) =	0.120(·)	0.178(+)	-0.150(·)	0.072(·)
	0.215(+)	-0.142(·)	0.016(·)	-0.059(·)		0.250(+)	-0.123(·)	0.036(·)	-0.041(·)
	-0.188(-)	-0.340(-)	0.085(·)	0.429(+)		-0.193(-)	-0.338(-)	0.091(·)	0.397(+)
	-0.327(-)	-0.398(-)	0.372(+)	0.561(+)		-0.332(-)	-0.393(-)	0.368(+)	0.535(+)
C(20) =	0.047(·)	0.179(+)	-0.124(·)	0.031(·)	C(21) =	0.019(·)	0.172(+)	-0.092(·)	-0.000(·)
	0.271(+)	-0.089(·)	0.035(·)	-0.028(·)		0.274(+)	-0.039(·)	0.030(·)	-0.026(·)
	-0.189(-)	-0.327(-)	0.117(·)	0.358(+)		-0.198(-)	-0.316(-)	0.157(·)	0.313(+)
	-0.341(-)	-0.386(-)	0.371(+)	0.513(+)		-0.352(-)	-0.383(-)	0.371(+)	0.497(+)
C(22) =	0.047(·)	0.167(+)	-0.061(·)	-0.013(·)	C(23) =	0.107(·)	0.140(·)	-0.036(·)	-0.004(·)
	0.251(+)	0.010(·)	0.022(·)	-0.029(·)		0.216(+)	0.068(·)	0.002(·)	-0.038(·)
	-0.202(-)	-0.309(-)	0.182(+)	0.258(+)		-0.199(-)	-0.297(-)	0.194(+)	0.219(+)
	-0.353(-)	-0.381(-)	0.365(+)	0.483(+)		-0.354(-)	-0.386(-)	0.349(+)	0.472(+)
C(24) =	0.142(·)	0.119(·)	-0.023(·)	0.029(·)	S =	1.000	0.240	-0.172	-0.120
	0.172(+)	0.119(·)	-0.021(·)	-0.045(·)		0.240	1.000	-0.350	-0.345
	-0.211(-)	-0.272(-)	0.196(+)	0.193(+)		-0.172	-0.350	1.000	0.421
	-0.367(-)	-0.390(-)	0.330(+)	0.461(+)		-0.120	-0.345	0.421	1.000

Table.2.5.2. The χ^2 statistic for testing combined significance of cross correlation matrices calculated for the original series and residual series of the VAR(2) model using the vector time series of transformed landings of of elasmobranchs, oil sardine, mackerel and sear fish.

Lag	χ^2 statistic		Lag	χ^2 statistic	
	Original series	Residual series		Original series	Residual series
1	488.54	4.57	13	152.39	22.14
2	362.35	10.78	14	148.18	35.30
3	261.88	31.60	15	146.11	15.07
4	197.22	61.94	16	145.88	25.71
5	171.25	12.67	17	144.43	18.23
6	150.62	25.40	18	137.62	16.67
7	130.82	13.22	19	126.97	20.27
8	121.77	18.16	20	112.99	18.21
9	125.31	20.72	21	98.82	15.39
10	135.22	15.64	22	86.02	24.88
11	145.87	29.53	23	75.55	22.26
12	152.93	29.38	24	71.49	10.89

Table.2.5.3. . Partial cross correlation matrices for the vector time series of transformed landings of elasmobranchs, oil sardine, mackerel and sear fish.

$\hat{\rho}(1)=\begin{pmatrix} 0.855(+) & 0.236(+) & -0.204(-) & -0.075(\cdot) \\ 0.264(+) & 0.948(+) & -0.354(-) & -0.340(-) \\ -0.153(\cdot) & -0.343(-) & 0.929(+) & 0.418(+) \\ -0.186(-) & -0.348(-) & 0.401(+) & 0.963(+) \end{pmatrix}$	$\hat{\rho}(2)=\begin{pmatrix} -0.397(-) & 0.095(\cdot) & -0.174(-) & -0.094(\cdot) \\ 0.089(\cdot) & -0.472(-) & -0.097(\cdot) & -0.154(\cdot) \\ -0.169(-) & 0.009(\cdot) & -0.278(-) & 0.032(\cdot) \\ -0.024(\cdot) & -0.118(\cdot) & 0.063(\cdot) & -0.139(\cdot) \end{pmatrix}$
$\hat{\rho}(3)=\begin{pmatrix} -0.156(\cdot) & 0.087(\cdot) & -0.077(\cdot) & 0.070(\cdot) \\ -0.034(\cdot) & -0.160(\cdot) & 0.051(\cdot) & 0.041(\cdot) \\ 0.013(\cdot) & 0.037(\cdot) & 0.052(\cdot) & 0.075(\cdot) \\ 0.004(\cdot) & -0.030(\cdot) & 0.104(\cdot) & 0.016(\cdot) \end{pmatrix}$	$\hat{\rho}(4)=\begin{pmatrix} -0.164(\cdot) & 0.057(\cdot) & 0.093(\cdot) & -0.154(\cdot) \\ -0.023(\cdot) & -0.192(-) & 0.087(\cdot) & 0.093(\cdot) \\ -0.051(\cdot) & -0.016(\cdot) & -0.129(\cdot) & -0.092(\cdot) \\ -0.031(\cdot) & -0.006(\cdot) & -0.018(\cdot) & 0.028(\cdot) \end{pmatrix}$
$\hat{\rho}(5)=\begin{pmatrix} 0.377(+) & -0.001(\cdot) & -0.142(\cdot) & 0.082(\cdot) \\ -0.136(\cdot) & 0.254(+) & 0.136(\cdot) & 0.009(\cdot) \\ 0.046(\cdot) & -0.131(\cdot) & 0.156(\cdot) & 0.135(\cdot) \\ 0.074(\cdot) & -0.021(\cdot) & 0.162(\cdot) & 0.380(+) \end{pmatrix}$	$\hat{\rho}(6)=\begin{pmatrix} 0.055(\cdot) & 0.022(\cdot) & 0.015(\cdot) & 0.009(\cdot) \\ -0.010(\cdot) & -0.266(-) & -0.030(\cdot) & 0.101(\cdot) \\ 0.044(\cdot) & -0.002(\cdot) & -0.145(\cdot) & -0.038(\cdot) \\ 0.083(\cdot) & 0.062(\cdot) & -0.076(\cdot) & -0.055(\cdot) \end{pmatrix}$
$\hat{\rho}(7)=\begin{pmatrix} -0.074(\cdot) & -0.024(\cdot) & -0.069(\cdot) & -0.028(\cdot) \\ 0.087(\cdot) & -0.155(\cdot) & 0.150(\cdot) & 0.177(+) \\ 0.169(+) & -0.002(\cdot) & -0.201(-) & -0.110(\cdot) \\ -0.174(-) & 0.084(\cdot) & -0.047(\cdot) & -0.123(\cdot) \end{pmatrix}$	$\hat{\rho}(8)=\begin{pmatrix} -0.134(\cdot) & 0.083(\cdot) & -0.043(\cdot) & -0.178(-) \\ -0.099(\cdot) & -0.138(\cdot) & -0.038(\cdot) & 0.132(\cdot) \\ 0.088(\cdot) & 0.024(\cdot) & -0.017(\cdot) & 0.119(\cdot) \\ -0.073(\cdot) & 0.003(\cdot) & 0.051(\cdot) & 0.011(\cdot) \end{pmatrix}$
$\hat{\rho}(9)=\begin{pmatrix} 0.237(+) & 0.155(\cdot) & 0.063(\cdot) & 0.077(\cdot) \\ -0.045(\cdot) & 0.044(\cdot) & 0.187(+) & 0.145(\cdot) \\ -0.060(\cdot) & -0.036(\cdot) & 0.090(\cdot) & 0.149(\cdot) \\ -0.071(\cdot) & 0.093(\cdot) & -0.091(\cdot) & 0.161(\cdot) \end{pmatrix}$	$\hat{\rho}(10)=\begin{pmatrix} 0.056(\cdot) & 0.132(\cdot) & -0.047(\cdot) & -0.092(\cdot) \\ 0.192(+) & -0.083(\cdot) & -0.133(\cdot) & 0.065(\cdot) \\ -0.071(\cdot) & -0.088(\cdot) & -0.105(\cdot) & 0.052(\cdot) \\ 0.002(\cdot) & -0.004(\cdot) & -0.093(\cdot) & -0.160(\cdot) \end{pmatrix}$

Table.2.5.3. Continued

$\hat{\rho}(11)=\begin{pmatrix} 0.007(\cdot) & -0.001(\cdot) & -0.054(\cdot) & -0.065(\cdot) \\ 0.003(\cdot) & -0.008(\cdot) & 0.022(\cdot) & 0.123(\cdot) \\ 0.141(\cdot) & 0.034(\cdot) & -0.013(\cdot) & 0.126(\cdot) \\ 0.106(\cdot) & 0.016(\cdot) & -0.054(\cdot) & 0.030(\cdot) \end{pmatrix}$	$\hat{\rho}(12)=\begin{pmatrix} 0.025(\cdot) & -0.038(\cdot) & -0.037(\cdot) & 0.036(\cdot) \\ -0.007(\cdot) & -0.086(\cdot) & -0.057(\cdot) & -0.102(\cdot) \\ -0.080(\cdot) & -0.086(\cdot) & -0.041(\cdot) & 0.076(\cdot) \\ -0.021(\cdot) & 0.098(\cdot) & -0.097(\cdot) & 0.051(\cdot) \end{pmatrix}$
$\hat{\rho}(13)=\begin{pmatrix} 0.327(+) & -0.031(\cdot) & -0.077(\cdot) & 0.113(\cdot) \\ -0.057(\cdot) & -0.107(\cdot) & 0.047(\cdot) & 0.068(\cdot) \\ -0.083(\cdot) & 0.111(\cdot) & 0.090(\cdot) & -0.153(\cdot) \\ -0.011(\cdot) & -0.005(\cdot) & -0.048(\cdot) & -0.060(\cdot) \end{pmatrix}$	$\hat{\rho}(14)=\begin{pmatrix} 0.046(\cdot) & 0.106(\cdot) & -0.170(-) & -0.012(\cdot) \\ 0.025(\cdot) & -0.068(\cdot) & 0.019(\cdot) & 0.034(\cdot) \\ 0.091(\cdot) & -0.106(\cdot) & -0.016(\cdot) & 0.069(\cdot) \\ 0.091(\cdot) & 0.083(\cdot) & -0.132(\cdot) & 0.046(\cdot) \end{pmatrix}$
$\hat{\rho}(15)=\begin{pmatrix} 0.096(\cdot) & 0.027(\cdot) & 0.048(\cdot) & 0.136(\cdot) \\ 0.102(\cdot) & -0.076(\cdot) & 0.131(\cdot) & 0.000(\cdot) \\ -0.127(\cdot) & -0.099(\cdot) & 0.081(\cdot) & 0.024(\cdot) \\ -0.093(\cdot) & -0.110(\cdot) & 0.084(\cdot) & -0.028(\cdot) \end{pmatrix}$	$\hat{\rho}(16)=\begin{pmatrix} 0.107(\cdot) & 0.069(\cdot) & 0.074(\cdot) & 0.090(\cdot) \\ 0.072(\cdot) & 0.048(\cdot) & 0.016(\cdot) & 0.084(\cdot) \\ 0.072(\cdot) & -0.072(\cdot) & -0.102(\cdot) & 0.072(\cdot) \\ 0.070(\cdot) & 0.028(\cdot) & -0.116(\cdot) & -0.072(\cdot) \end{pmatrix}$

Table.2.5.4. Calculated values of squared partial canonical correlations and test statistic for the vector time series of transformed landings of elasmobranchs, oil sardine, mackerel and sear fish.

Lag	Squared Partial Canonical Correlation				LR Statistic
	(i)	(ii)	(iii)	(iv)	
1	0.9454	0.8817	0.8222	0.6967	1154.63
2	0.3121	0.2488	0.0676	0.0004	105.93
3	0.0731	0.0295	0.0098	0.0026	17.14
4	0.0683	0.0448	0.0216	0.0026	20.46
5	0.1976	0.1556	0.0646	0.0164	68.53
6	0.0843	0.0272	0.0045	0.0003	17.48
7	0.1085	0.0879	0.0087	0.0008	31.39
8	0.0796	0.0442	0.0081	0.0035	20.27
9	0.0921	0.0900	0.0494	0.0035	35.56
10	0.0768	0.0592	0.0190	0.0085	24.46

Table.2.5.5. Computed values of different order selection criteria for the vector time series of transformed landings of elasmobranchs, oil sardine, mackerel and sear fish.

p	$\lvert\tilde{\Sigma}\rvert$	FPE	AIC	HQ	SC
1	0.000145	0.000155	-8.6196	-8.4861	-8.2911
2	0.000062	0.000070	-9.2431	-8.9762	-8.5861
3	0.000055	0.000066	-9.1390	-8.7386	-8.1536
4	0.000048	0.000061	-9.0609	-8.5270	-7.7470
5	0.000026	0.000034	-9.4733	-8.8060	-7.8310
6	0.000021	0.000029	-9.4589	-8.6581	-7.4881
7	0.000016	0.000024	-9.5159	-8.5817	-7.2167
8	0.000013	0.000021	-9.4870	-8.4193	-6.8593
9	0.000009	0.000015	-9.6571	-8.4559	-6.7009
10	0.000006	0.000012	-9.7455	-8.4108	-6.4608

Table.2.5.6. Eigen values of the characteristic matrices of the VAR(2) model estimated for the vector time series of transformed landings of elasmobranchs, oil sardine, mackerel and sear fish.

No.	Real Part	Imaginary part	Absolute Value
1	-0.0786	0.0000	0.0786
2	0.3165	0.0000	0.3165
3	0.6712	-0.3347	0.7500
4	0.6712	0.3347	0.7500
5	0.7058	-0.1558	0.7228
6	0.7058	0.1558	0.7228
7	0.8322	0.0000	0.8322
8	0.9875	0.0000	0.9875

Table.2.6.1. Forecasts with standard errors for the vector time series consisting of transformed landings of oil sardine, mackerel, anchovies and lesser sardine using the fitted VAR(2) model.

Year	Quarter	Oil sardine			Mackerel		
		Observed	Forecasted	SE	Observed	Forecasted	SE
1997	I	8.3564	8.1038	0.2598	10.5004	10.1696	0.3053
	II	8.9392	8.1562	0.4475	10.2735	10.0773	0.4852
	III	9.0120	8.1672	0.5832	10.1245	9.9693	0.6093
	IV	9.2357	8.1585	0.6757	9.9324	9.8561	0.6943
1998	I	8.9451	8.1475	0.7380	9.6394	9.7481	0.7534
	II	8.8739	8.1428	0.7809	9.6173	9.6510	0.7962
	III	8.9840	8.1474	0.8120	9.6238	9.5665	0.8287
	IV	8.9954	8.1612	0.8363	9.5270	9.4938	0.8547
Year	Quarter	Anchovies			Lesser sardine		
		Observed	Forecasted	SE	Observed	Forecasted	SE
1997	I	10.9587	10.8919	0.3050	11.7177	11.1246	0.3937
	II	10.9741	10.8497	0.4410	11.5152	11.1684	0.5855
	III	11.0627	10.7914	0.5339	10.9002	11.2437	0.7059
	IV	10.8624	10.7376	0.6046	11.4506	11.3185	0.7866
1998	I	10.9498	10.6898	0.6618	11.4429	11.3884	0.8436
	II	11.1438	10.6462	0.7095	11.6955	11.4531	0.8849
	III	11.0740	10.6055	0.7500	10.8220	11.5122	0.9154
	IV	11.1145	10.5670	0.7847	10.9244	11.5654	0.9379

Table.2.6.2. Forecasts with standard errors for the vector time series consisting of transformed landings of anchovies, lesser sardine, ribbon fish and catfish using the fitted VAR(2) model.

Year	Quarter	Anchovies			Lesser sardine		
		Observed	Forecasted	SE	Observed	Forecasted	SE
1997	I	10.9587	10.9121	0.3024	11.7177	11.1628	0.3914
	II	10.9741	10.9406	0.4341	11.5152	11.2580	0.5796
	III	11.0627	10.9672	0.5239	10.9002	11.3948	0.6971
	IV	10.8624	10.9868	0.5881	11.4506	11.5200	0.7734
1998	I	10.9498	10.9949	0.6366	11.4429	11.6173	0.8259
	II	11.1438	10.9909	0.6756	11.6955	11.6849	0.8647
	III	11.0740	10.9766	0.7084	10.8220	11.7273	0.8947
	IV	11.1145	10.9549	0.7370	10.9244	11.7505	0.9184
Year	Quarter	Ribbon fish			Catfish		
		Observed	Forecasted	SE	Observed	Forecasted	SE
1997	I	6.0280	5.6722	0.3173	3.1677	3.9499	0.1957
	II	6.2939	5.5799	0.5088	2.8621	3.9988	0.3000
	III	6.1958	5.4331	0.6416	2.6690	4.0389	0.3765
	IV	6.1316	5.2840	0.7278	2.7261	4.0723	0.4378
1998	I	6.0040	5.1569	0.7823	3.0627	4.0988	0.4902
	II	5.5221	5.0593	0.8177	3.2491	4.1190	0.5369
	III	5.6453	4.9896	0.8424	3.1171	4.1344	0.5793
	IV	5.5083	4.9424	0.8613	3.1187	4.1464	0.6180

Table.2.6.3. Forecasts with standard errors for the vector time series consisting of transformed landings of mackerel, anchovies, tuna and penaeid prawns using the fitted VAR(1) model.

Year	Quarter	Mackerel			Anchovies		
		Observed	Forecasted	SE	Observed	Forecasted	SE
1997	I	10.5004	10.1589	0.3255	10.9587	10.8394	0.3068
	II	10.2735	10.0791	0.4409	10.9741	10.8275	0.4107
	III	10.1245	10.0044	0.5195	11.0627	10.8190	0.4783
	IV	9.9324	9.9348	0.5793	10.8624	10.8132	0.5275
1998	I	9.6394	9.8703	0.6272	10.9498	10.8096	0.5655
	II	9.6173	9.8109	0.6670	11.1438	10.8076	0.5962
	III	9.6238	9.7563	0.7006	11.0740	10.8070	0.6217
	IV	9.5270	9.7064	0.7293	11.1145	10.8074	0.6436
Year	Quarter	Tuna			Penaeid prawns		
		Observed	Forecasted	SE	Observed	Forecasted	SE
1997	I	7.8524	7.9458	0.2124	20.2447	19.7818	0.3561
	II	7.8221	7.9531	0.2891	20.3667	19.7802	0.4721
	III	7.8610	7.9577	0.3426	20.1132	19.7806	0.5441
	IV	7.8340	7.9600	0.3846	20.3892	19.7824	0.5932
1998	I	7.7659	7.9606	0.4197	20.3828	19.7853	0.6285
	II	7.6598	7.9598	0.4503	20.5662	19.7890	0.6546
	III	7.6158	7.9580	0.4777	20.6265	19.7934	0.6744
	IV	7.5407	7.9554	0.5028	20.3538	19.7981	0.6899

Table.2.6.4. Forecasts with standard errors for the vector time series consisting of transformed landings of oil sardine, anchovies, tuna and penaeid prawns using the fitted VAR(5) model.

Year	Quarter	Oil sardine			Anchovies		
		Observed	Forecasted	SE	Observed	Forecasted	SE
1997	I	8.3564	7.9372	0.2265	10.9587	10.8250	0.2579
	II	8.9392	7.5849	0.3719	10.9741	10.9849	0.3790
	III	9.0120	7.4091	0.4983	11.0627	11.0803	0.4722
	IV	9.2357	7.3217	0.6197	10.8624	11.0943	0.5578
1998	I	8.9451	7.3062	0.6874	10.9498	11.0812	0.5976
	II	8.8739	7.4624	0.7240	11.1438	10.9844	0.6268
	III	8.9840	7.5972	0.7434	11.0740	10.8933	0.6477
	IV	8.9954	7.7164	0.7549	11.1145	10.8371	0.6607
Year	Quarter	Tuna			Penaeid prawns		
		Observed	Forecasted	SE	Observed	Forecasted	SE
1997	I	7.8524	7.7908	0.1708	20.2447	19.9420	0.3165
	II	7.8221	7.6729	0.2407	20.3667	20.0907	0.4719
	III	7.8610	7.6003	0.2926	20.1132	20.2066	0.5581
	IV	7.8340	7.5425	0.3424	20.3892	20.0970	0.6382
1998	I	7.7659	7.5981	0.3584	20.3828	20.0497	0.6614
	II	7.6598	7.6698	0.3714	20.5662	19.9595	0.6708
	III	7.6158	7.7548	0.3813	20.6265	19.8291	0.6838
	IV	7.5407	7.8221	0.3923	20.3538	19.8102	0.6922

Table.2.6.5. Forecasts with standard errors for the vector time series consisting of transformed landings of elasmobranchs, oil sardine, mackerel and seerfish using the fitted VAR(2) model.

Year	Quarter	Elasmobranchs			Oil sardine		
		Observed	Forecasted	SE	Observed	Forecasted	SE
1997	I	22.6104	23.0553	0.4579	8.3564	7.9137	0.2583
	II	22.6569	23.2519	0.6985	8.9392	7.8537	0.4421
	III	22.5766	23.2553	0.8294	9.0120	7.8358	0.5808
	IV	22.2715	23.1572	0.8870	9.2357	7.8544	0.6762
1998	I	22.3049	23.0283	0.9069	8.9451	7.8942	0.7378
	II	22.4139	22.9141	0.9133	8.8739	7.9394	0.7763
	III	21.9812	22.8362	0.9179	8.9840	7.9786	0.8007
	IV	22.1759	22.7981	0.9232	8.9954	8.0058	0.8169
Year	Quarter	Mackerel			Seerfish		
		Observed	Forecasted	SE	Observed	Forecasted	SE
1997	I	10.5004	10.4499	0.2922	8.1530	8.2848	0.2404
	II	10.2735	10.4642	0.4588	8.0764	8.3356	0.3611
	III	10.1245	10.3765	0.5742	8.2773	8.3840	0.4527
	IV	9.9324	10.2401	0.6487	8.1423	8.4258	0.5228
1998	I	9.6394	10.0975	0.6966	8.1580	8.4593	0.5771
	II	9.6173	9.9740	0.7296	8.2203	8.4840	0.6198
	III	9.6238	9.8802	0.7542	8.3186	8.5004	0.6537
	IV	9.5270	9.8164	0.7736	8.4636	8.5100	0.6813

Table.2.6.6. Quarterwise forecasts (in tonnes) of marine fish landings in Kerala for selected species/groups computed using different Vector Autoregressive models fitted.

Year	Quarter	Model-1: VAR(2)				Model-2: VAR(2)			
		OILSRD	MACKRL	ANCHOV	LESSRD	ANCHOV	LESSRD	RIBFSH	CATFSH
1997	I	10091	6077	3779	424	4032	468	203	192
	II	1835	16690	4235	1822	5321	2081	602	68
	III	11401	38699	5386	1822	7080	2136	4798	110
	IV	11034	23517	12637	3608	16014	4110	4519	123
1998	I	9619	3961	3239	508	4139	602	102	217
	II	1799	11360	3680	2154	5252	2479	355	75
	III	11631	27685	4724	2123	6761	2383	3290	118
	IV	11712	17630	11162	4139	14935	4365	3501	130

Year	Quarter	Model-3: VAR(1)				Model-4: VAR(5)			
		MACKRL	ANCHOV	TUNA	PPRAWN	OILSRD	ANCHOV	TUNA	PPRAWN
1997	I	5824	3190	3893	7155	4899	3045	2026	9656
	II	17540	4671	6617	11466	317	8128	3903	15191
	III	44155	6323	2841	20246	5071	8839	2053	25131
	IV	27954	14755	5047	7228	7842	15728	3915	5868
1998	I	4510	3152	3903	7194	4580	2919	2561	8838
	II	13858	4641	6596	11547	625	5949	5280	12832
	III	35567	6310	2819	20411	9102	6589	2938	19689
	IV	22939	14773	4991	7293	13150	13122	5198	5665

Year	Quarter	Model-5: VAR(2)			
		ELASMO	OILSRD	MACKRL	SERFSH
1997	I	1644	4424	18453	1646
	II	1295	1127	25470	1446
	III	831	10060	41936	633
	IV	1473	12422	21456	2132
1998	I	1402	5257	10485	1846
	II	1125	1371	15609	1573
	III	754	11924	28915	670
	IV	1405	13978	16662	2203

CHAPTER-3

MODELLING MARINE FISH LANDINGS USING VARMA MODELS

Introduction

Consider a vector time series y_t represented by Vector Autoregressive model of order p given by $y_t = \delta + \Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} + \varepsilon_t$ where ε_t's are independently and identically distributed random vectors with $E(\varepsilon_t) = 0$ and $E(\varepsilon_t \varepsilon_t') = \Sigma$ for all t, which are otherwise known as white noise series. Instead, if we assume that the error terms are autocorrelated up to a certain lag say q, then ε_t can be replaced by an auto correlated term $a_t - \Theta_1 a_{t-1} - \cdots - \Theta_q a_{t-q}$ where the a_t's are independently and identically distributed white noise series and $\Theta_1, \cdots, \Theta_q$ are matrices of order $k \times k$. By making this substitution for ε_t, we get the corresponding model as

$$y_t = \delta + \Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} + a_t - \Theta_1 a_{t-1} - \cdots - \Theta_q a_{t-q}$$

and this model is known as Vector Autoregressive Moving Average model with orders (p,q) denoted by VARMA(p,q). This can be rewritten as $\Phi(B) y_t = \delta + \Theta(B) a_t$ where $\Phi(B) = I - \Phi_1 B - \Phi_2 B^2 - \cdots - \Phi_p B^p$, $\Theta(B) = I - \Theta_1 B - \Theta_2 B^2 - \cdots - \Theta_q B^q$, I is an identity matrix of order k and B is the back shift operator such that $B^i y_t = y_{t-i}$. A vector time series can be modelled by a VARMA model which is a multivariate analog of the univariate ARMA model. The conditions required for stationarity and invertibility of a VARMA(p,q) process is that the zeros of the determinental polynomials $|\Phi(B)|$ and $|\Theta(B)|$ in B lie outside the unit circle.

In the earlier analysis for fitting a suitable VAR(p) model to the vector time series consisting of transformed landings of oil sardine, anchovies, tuna and penaeid prawns, the model found suitable was a VAR(5) model and this model was estimated for the vector time series. Such a model with a higher order will not be parsimonious as there are 90 elements in the parameter matrices which are to be estimated. Hence it is advisable to go for an alternative model with lesser number of parameters and have almost the same amount of accuracy. It is known that the mixed VARMA(p,q) models with low orders can replace a higher order VAR(p) model (Reinsel 1993). There fore an attempt is made here to find a suitable VARMA(p,q) model with low orders for the vector time series consisting of transformed landings of oil sardine, anchovies, tuna and penaeid prawns.

Review of literature.

Hannan (1969) derived a necessary and sufficient condition for unique identification of a VARMA process and extended it to deal the problem for continuous processes. He also developed a method using Fourier Transform for the estimation of parameters of univariate ARMA models and extended it for the vector moving average process. Tuan (1978) showed that there is a one-to-one correspondence between each ARMA process and a member of a class of special Markovian representation. He used this property to fit multivariate processes of the ARMA type. Phadke (1978) proposed three methods for computing exact likelihood function of a multivariate moving average process. These methods utilize the structure of the covariance matrices in different ways. He compared these estimates through Monte Carlo simulation. Nicholls and Hall (1979) derived an expression for exact likelihood function of a stationary vector ARMA process through tensor products using concentrated maximum likelihood techniques. In this process they also derived an expression for the covariance function of the process in terms of the coefficients of the model. Hillmer and Tiao (1979) developed procedures for the estimation of parameters in multivariate ARMA models by assuming Gaussian errors. They developed an exact maximum likelihood estimation procedure for pure moving average models and an approximate procedure for the estimation of stationary ARMA models.

Yamamoto (1981) gave a simple formula for multiperiod predictions of multivariate autoregressive moving average models as a function of suitably defined parameter matrices and observation vector. Tiao and Box (1981) proposed an approach to the modelling and analysis of multiple time series consisting of tentative specification, estimation and diagnostic checking. They also discussed the properties of a class of vector ARMA models. Lutkepohl (1982) proposed modified polynomial lag models for multiple time series analysis, which he considered as a compromise between modelling pure AR and ARMA processes. Ansley and Kohn (1983) pointed out that using Kalman filter with non-constant coefficients the exact likelihood of a vector ARMA process with observed noise can be computed when some of the observations are missing. Spliid (1983) presented a fast and simple algorithm for estimation of the parameters in multivariate time series and distributed lag models that is useful at the identification stage. His method is purely based on regression estimation and can also be used for estimation of parameters in VARMAX models. Poskitt and Tremayne(1984) considered the application of hypothesis testing procedures to vector ARMAX systems. They suggested a Lagrangian multiplier principle to obtain miss specification tests appropriate for these models. They also investigated the structure of the information matrix to discuss the identifiability of alternative models and used it to show the existence of a set of admissible alternate models that are locally equivalent to one another.

Stoffer (1986) presented a method for modelling and fitting multivariate spatial time series data based on current spacial methodology coupled with the parameterisation of ARMAX models. McLeod and Hipel (1987) considered bivariate ARMA time series models with diagonal parameter matrices for AR and MA components. They derived an efficient maximum likelihood algorithm for parameter estimation and compared it with standard multivariate procedures. Pukkila (1987) gave methods for the identification of nonzero elements in VARMA(1,1) models and have shown how to use these methods for higher order models. Hannan and Poskit (1988) had shown that in the case of stationary vector ARMA processes, the number of linearly independent pairs is the number of zeroes of the determinant of transfer functions from innovations to outputs that lie on the unit circle. Phillips (1988) introduced the concept of a near integrated vector random process and developed a general asymptotic regression theory for multiple time series in which some series may be ARIMA type and the rest stable ARMA processes. Ahn and Reinsel (1988) considered nested reduced rank AR models in order to simplify and provide a more detailed description about the structure of the multivariate time series and to reduce parameters in time series modelling. They suggested a canonical variable transformation that produces simpler structure in the model and illustrated how different components of the vector series depend on past lagged values. Shea (1989) gave a

FORTRAN algorithm for computing the exact likelihood of a vector ARMA model based of Chandrasekar algorithm. Li and Hui (1989) proposed a robust estimation procedure for multiple time series models, which is based on robustifying the residual autocovariances in the estimation equation. They derived asymptotic distribution of these estimators and a portmanteau statistic for diagnostic checking.

Grubb (1992) analyzed an index of monthly price of flour at three cities in US using multivariate time series models to explore the relationships between them and to discover the structure responsible for their movements. They tried to build VARMA models based on the method suggested by Tsay and Tiao and fitted a VAR(1) model for the data. Luceno (1994) developed a fast and numerically efficient algorithm for calculating the exact likelihood function for stationary and partially non-stationary vector autoregressive moving average process. The method proposed by him does not require differencing of the series and it avoids complications caused by over differencing and related identification problems. Mauricio (1995) presented a new efficient procedure for evaluating the exact likelihood function of vector autoregressive moving average model. Based on this procedure he developed an algorithm for exact maximum likelihood estimation of VARMA models. In the algorithm he has described steps to check for stationarity and invertibility of the model at different stages of evaluation of a concentrated log likelihood function. Yap and Reinsel (1995) considered Gaussian estimation of partially non-stationary vector autoregressive moving average models and derived an asymptotic distribution of the likelihood ratio statistic for testing the number of unit roots.

Materials and Methods

The conditions required for stationarity of a VARMA model is that of the corresponding vector autoregressive model and the conditions for invertibility of the model is same as that for the corresponding vector moving average model. These conditions are $det(I - \Phi_1 B - \cdots - \Phi_p B^p) \neq 0$ for $|B| \leq 1$ for stationarity and $det(I - \Theta_1 B - \cdots - \Theta_q B^q) \neq 0$ for $|B| \leq 1$ for invertibility. A general VARMA(p,q) model can have a VAR(1) representation using which it is possible to test the stationary condition in terms of eigen values of a characteristic matrix (Lutkepohl, 1993). The form of the AR coefficient matrix for such a representation in terms of the parameter matrices is

$$\Phi = \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix}$$

a matrix of order $(p+q)k \times (p+q)k$ where the component matrices are given by

$$\Phi_{11} = \begin{pmatrix} \Phi_1 & \Phi_2 & \cdots & \Phi_{p-1} & \Phi_p \\ I_k & 0 & \cdots & 0 & 0 \\ 0 & I_k & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_k & 0 \end{pmatrix} \text{ a matrix of order } kp \times kp,$$

$$\Phi_{12} = \begin{pmatrix} \Theta_1 & \Theta_2 & \cdots & \Theta_{q-1} & \Theta_q \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix} \text{ a matrix of order } kp \times kq,$$

Φ_{21} is a matrix of order $kq \times kp$ with all its elements zero and

$$\Phi_{22} = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ I_k & 0 & \cdots & 0 & 0 \\ 0 & I_k & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_k & 0 \end{pmatrix} \text{a matrix of order } kq \times kq$$

If all the eigen values of matrix Φ are less than one in absolute value then the VARMA(p,q) model is stationary. In particular for a VARMA(1,1) model with the expression $y_t - \Phi\, y_{t-1} = \varepsilon_t - \Theta\, \varepsilon_{t-1}$, the condition for stationarity is that all eigen values of the coefficient matrix Φ are less than one in absolute value and the condition for invertibility of the model is that all eigen values of the coefficient matrix Θ are less than one in absolute value.

Selection of the order parameters p and q are made by using order selection criteria like AIC, BIC and HQ. The Akaike's information criterion is approximated by $AIC_r \approx \log(|\tilde{\Sigma}_r|) + \frac{2r}{T} + c$, the Baysean information criterion given by Schwarz (1978) is $BIC_r = \log(|\tilde{\Sigma}_r|) + \frac{r\log(T)}{T}$, and the criterion proposed by Hannan and Quinn (1979) is $HQ_r = \log(|\tilde{\Sigma}_r|) + \frac{2r\log(\log(T))}{T}$ where $\tilde{\Sigma}_r$ is the maximum likelihood estimate of the innovation dispersion matrix Σ, r is the number of parameters estimated, T is the sample size and c is a constant. The orders that yield minimum value for these criterion is selected as the required order for the model. For a VARMA(p,q) model the number of parameters to be estimated is $(p+q)k^2$ so that we get these criterion for selection of orders p and q of a VARMA(p,q) model as

$$AIC(p,q) = \log(|\tilde{\Sigma}|) + \frac{2(p+q)k^2}{T},$$

$$BIC(p,q) = \log(|\tilde{\Sigma}|) + \frac{(p+q)k^2 \log(T)}{T} \text{ and}$$

$$HQ(p,q) = \log(|\tilde{\Sigma}|) + \frac{2(p+q)k^2 \log(\log(T))}{T}$$

For the estimation of parameters of VARMA(p,q) model, the approximate likelihood method proposed by Wilson (1973) which is iterative in nature was used. Consider a stationary vector time series $\{y_t\}$ with k components modelled by a VARMA(p,q) model. Assuming zero mean vector for the series, the expression for this model is $y_t = \Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} + a_t - \Theta_1 a_{t-1} - \cdots - \Theta_q a_{t-q}$ where $\Phi_1,\cdots,\Phi_p,\Theta_1,\cdots,\Theta_q$ are $k \times k$ matrices, $a_t = (a_{1t},\cdots,a_{kt})'$ are independently and identically distributed random variables with zero mean vector and innovation dispersion matrix D. The unknown parameters of the model are the (p+q+1) matrices $\Phi_1,\cdots,\Phi_p,\Theta_1,\cdots,\Theta_q$ and D. Denote by K the total number of unknown parameters of the model and let $\beta = (\beta_1,\cdots,\beta_K)'$ is the vector of these parameters in some standard order. Since the parameter matrices must satisfy the conditions for stationarity and invertibility, we can assume that β lies within a parameter space Ω determined by the conditions $|\Phi(z)| \neq 0$ and $|\Theta(z)| \neq 0$ for $|z| \leq 1$. For a proposed estimate $\dot{\beta}$ of the parameter vector β, let $\dot{a}_t$ is the estimate of a_t based on a sample of size T which is obtained as $\dot{a}_t = y_t - \dot{\Phi}_1 y_{t-1} - \cdots - \dot{\Phi}_p y_{t-p} + \dot{\Theta}_1 \dot{a}_{t-1} + \cdots + \dot{\Theta}_q \dot{a}_{t-q}$ and this can be used recursively to evaluate all the vectors $\dot{a}_t$ for $t = 1,\cdots,T$. Here $\dot{\Phi}_1,\cdots,\dot{\Phi}_p,\dot{\Theta}_1,\cdots,\dot{\Theta}_q$ are the estimates of $\Phi_1,\cdots,\Phi_p,\Theta_1,\cdots,\Theta_q$ respectively corresponding to the estimate $\dot{\beta}$ of β. Under normality assumption of a_t, the likelihood of the parameters is

$L(\dot{\beta},\dot{D}) = (2\pi)^{-\frac{1}{2}kT}|\dot{D}|^{T/2}\exp[-\frac{1}{2}(\sum_{t=1}^{T}\dot{a}_t'\dot{D}^{-1}\dot{a}_t)]$ and the loglikelihood is given by

$LL(\dot{\beta},\dot{D}) = -\frac{1}{2}kT\log(2\pi) - \frac{1}{2}TF(\dot{\beta},\dot{D})$ where $F(\dot{\beta},\dot{D}) = \log|\dot{D}| + \frac{1}{T}\sum_{t=1}^{T}\dot{a}_t'\dot{D}^{-1}\dot{a}_t$.

Hence maximization of the likelihood is equivalent to the minimization of the function $F(\dot{\beta},\dot{D})$ with respect to the elements of the vector $\dot{\beta}$ and simultaneous zeroes of the equations of derivatives will yield the required solution. Using the notation $\dot{a}_t^m = \frac{\partial \dot{a}_t}{\partial \dot{\beta}_m}$, ($m = 1,\cdots,K$) for the first order partial derivatives and $\dot{a}_t^{ml} = \frac{\partial^2 \dot{a}_t}{\partial\dot{\beta}_m\partial\dot{\beta}_l}$ ($m,l = 1,\cdots,K$) for the second order partial derivatives we get $\frac{\partial F}{\partial\dot{\beta}_m} = \frac{2}{T}\sum_{t=1}^{T}\dot{a}_t^{m'}\dot{D}^{-1}\dot{a}_t$ and

$\frac{\partial F}{\partial\dot{Q}_{rs}} = -\dot{D}_{rs} + \frac{1}{T}\sum_{t=1}^{T}\dot{a}_{rt}\dot{a}_{st}$ where $\dot{Q} = \dot{D}^{-1}$ and hence the conditional estimate of D is

$\tilde{D} = \frac{1}{T}\sum_{t=1}^{T}\dot{a}_t\dot{a}_t'$. By equating $\frac{\partial F}{\partial\dot{\beta}_m}$ to zero we get $\frac{1}{T}\sum_{t=1}^{T}\dot{a}_t^{m'}\dot{D}^{-1}\dot{a}_t = 0$ for $m = 1,\cdots,K$. For given $\dot{D}$ we can linearise $\dot{a}_t$ by Taylors expansion as $\dot{a}_t(\beta) = \dot{a}_t(\dot{\beta}) + \sum_{l=1}^{K}\dot{a}_t^l\partial\dot{\beta} + o\|\partial\dot{\beta}\|$ where $\partial\dot{\beta} = \dot{\beta} - \dot{\beta}_0$. Ignoring the term $o\|\partial\dot{\beta}\|$ and using the above relations we get a system of linear equations $\dot{A}\partial\dot{\beta} = -g$, where the $K \times K$ matrix $\dot{A}$ has elements $\dot{A}_{ml} = \frac{1}{T}\sum_{t=1}^{T}\dot{a}_t^{m'}\dot{D}^{-1}\dot{a}_t^l$ for $m,l = 1,\cdots,K$ and the elements of vector $\dot{g}$ are given by

$\dot{g}_m = \frac{1}{T}\sum_{t=1}^{T}\dot{a}_t^{m'}\dot{D}^{-1}\dot{a}_t$ for $m = 1,\cdots,K$ both the matrix $\dot{A}$ and vector $\dot{g}$ being evaluated at $\dot{\beta}_0$. Solution of this leads to the value of $\partial\dot{\beta}$. Then a new estimate for β is $\dot{\beta}_1 = \dot{\beta}_0 + \partial\dot{\beta}$. Using this relation we have to repeat iteration by replacing $\dot{\beta}_0$ with the new estimate $\dot{\beta}_1$ and this process is continued till convergence is attained. In order to ensure convergence, the successive parameter differences are constrained as is done in non-linear least square problems. The summary of this algorithm, for simultaneous estimation of elements of vector β and matrix **D**, is given as:

i) For given starting values of $\dot{\beta}$ and $\lambda > 0$ evaluate the innovation vectors $\{a_t\}$ and set $\dot{D} = \frac{1}{T}\sum_{t=1}^{T}\dot{a}_t\dot{a}_t'$ and $\dot{Q} = \dot{D}^{-1}$.

ii) Construct matrix $\dot{A} = (\dot{A}_{ml})_{K\times K}$ with elements $\dot{A}_{ml} = \frac{1}{T}\sum_{t=1}^{T}\dot{a}_t^{m'}\dot{Q}\dot{a}_t^{l}$, vector $\dot{g}$ with elements $\dot{g}_m = \frac{1}{T}\sum_{t=1}^{T}\dot{a}_t^{m'}\dot{Q}\dot{a}_t$ and a scaling quantity $\alpha_m = \sqrt{\dot{A}_{mm}}$ for $m,l = 1,\cdots,K$.

iii) Construct a scaled matrix $\dot{B}$ with elements $\dot{B}_{ml} = \dot{A}_{ml}/(\alpha_m\alpha_l)$ and a scaled vector $\dot{h}$ with elements $\dot{h}_m = \dot{g}_m/\alpha_m$ for $m,l = 1,\cdots,K$.

iv) Set the diagonal elements of $\dot{B}$ to $(1+\lambda)$, solve for vector x in the equation $\dot{B}x = \dot{h}$ and then evaluate a new proposed set of parameters as $\beta_m = \dot{\beta}_m - x_m/\alpha_m$ for $m = 1,\cdots,K$.

v) Using this new parameter set evaluate the innovation vectors $\dot{a}_t$ and set $D = \widetilde{D}(\beta) = \frac{1}{T}\sum \dot{a}_t \dot{a}_t'$ and test whether $trace(D\dot{Q}) < m$, where m is the number of components in the multiple time series.

If the above condition is satisfied repeat iteration from, step (i) onwards, for further improvement by replacing $\dot{\beta}$ with β, $\dot{D}$ with D and by reducing the constraint parameter λ by a predetermined quantity. If the condition is not satisfied, then increase the constraint parameter λ by a predetermined quantity and repeat calculation from step (iv) onwards. When λ becomes very large or when convergence to a desired accuracy level is attained, the iteration is stopped.

The fast algorithm for estimation of vector autoregressive moving average models with exogenous variables given by Spliid (1983) was suitably modified for VARMA(p,q) model estimation and was used in the identification stage for calculation of order selection criteria. This procedure is as followed.

Consider the model $\Phi(B)y_t = \Theta(B)\varepsilon_t$ which is stationary and invertible where $\Phi(B) = I - \Phi_1 B - \cdots - \Phi_p B^p$ and $\Theta(B) = I - \Theta_1 B - \cdots - \Theta_q B^q$. Let $y_1, \cdots, y_n$ be a sample of vector realizations of the series. Define matrices

$y = (y_{ij})_{n\times k}$ for $i = 1,\cdots,n; j = 1,\cdots,k$ which is an $n \times k$ matrix of observations,

$\varepsilon = (\varepsilon_{ij})_{n\times k}$ for $i = 1,\cdots,n; j = 1,\cdots,k$ which is an $n \times k$ matrix of residuals,

$Y = (By, B^2 y, \cdots, B^p y)$ which is a lagged data matrix of order $n \times pk$,

$A = (B\varepsilon, B^2\varepsilon, \cdots, B^q\varepsilon)$ which is a lagged residual matrix of order $n \times qk$,

$U = (-A, Y)$ which is a matrix of order $n \times (p+q)k$.

Here $B^s y$ is the matrix with its $(i,j)^{th}$ element $y_{i-s,j}$ for $i = 1,\cdots,n; j = 1,\cdots,k$ and it is of order $n \times k$. Now define the matrix $\delta = (\Theta_1,\cdots,\Theta_q,\Phi_1,\cdots,\Phi_p)'$ of parameters which is of order $(p+q)k \times k$, Using these matrices the VARMA(p,q) model can be written for the sample data matrix as $y = U\delta + \varepsilon$. First make the initial estimates of residual vectors by fitting a higher order vector autoregressive model. For this construct a matrix $W = (By, B^2 y,\cdots,B^s y)$ where s is the higher order chosen for the autoregression. Then the initial residual matrix $\hat{\varepsilon}(0)$ is obtained as $\hat{\varepsilon}(0) = y - W(W'W)^{-1}W'y$. Using this residual matrix construct matrices $\hat{A}(0)$ and $\hat{U}(0)$ and compute the first estimate of δ as $\hat{\delta}(1)$ from linear regression as, $\hat{\delta}(1) = [U'(0)\,U(0)]^{-1}\,U'(0)\,y$ and initial value $\hat{\delta}(0)$ is assumed as zero.

For the j^{th} iteration to estimate $\hat{\delta}(j+1)$, we obtain the estimate from the regression $U'(j)\ U(j)\ \hat{\delta}(j+1) = U'(j)\ y$ as $\hat{\delta}(j+1) = [U'(j)\,U(j)]^{-1}\,U'(j)\,y$ and we compute the new set of residuals using the recursion $\hat{\varepsilon}_t(j+1) = y_t - \sum_{i=1}^{p} \hat{\Phi}_i(j) y_{t-j} + \sum_{i=1}^{q} \hat{\Theta}_i(j)\hat{\varepsilon}_{t-i}(j)$.

Using the new estimate of residuals and parameter matrices the iteration is continued till convergence is achieved to a satisfactory level of accuracy.

The parameters of a VARMA(p,q) model can be estimated by conditional maximum likelihood method as described by Reinsel (1993). Suppose $Y = (y_1,\cdots,y_T)'$ is the $T \times k$ matrix of sample observations and $\varepsilon = (\varepsilon_1,\cdots,\varepsilon_T)'$ is the $T \times k$ matrix of innovations.

Let B^iY and $B^i\varepsilon$ denote the $T \times k$ matrices $(y_{1-i},\cdots,y_{T-i})'$ and $(\varepsilon_{1-i},\cdots,\varepsilon_{T-i})'$ respectively. Then the model for the entire sample can be written as

$$Y - \sum_{i=1}^{p} B^i\, Y\, \Phi_i' = \varepsilon - \sum_{i=1}^{q} B^i\, \varepsilon\, \Theta_i' \quad \text{and its vector form is}$$

$$y - \sum_{i=1}^{p} (I_T \otimes \Phi_i) B^i y = e - \sum_{i=1}^{q} (I_T \otimes \Theta_i) B^i e \quad \text{where}$$

$$y = vec(Y') = (y_1',\cdots,y_T')'$$

$$e = vec(\varepsilon') = (\varepsilon_1',\cdots,\varepsilon_T')'$$

$$B^i y = vec((B^iY)') = (y_{1-i}',\cdots,y_{T-i}')'$$

$$B^i e = vec((B^i\varepsilon)') = (\varepsilon_{1-i}',\cdots,\varepsilon_{T-i}')'$$

and if we define $\phi_i = vec(\Phi_i)$ and $\theta_i = vec(\Theta_i)$ then another useful form of the equation is $y - \sum_{i=1}^{p} (B^i y \otimes I_k)\, \phi_i = e - \sum_{i=1}^{q} (B^i e \otimes I_k)\, \theta_i$. Define $\beta = (\phi_1',\cdots,\phi_p',\theta_1',\cdots,\theta_q')'$ and let β_0 is an initial estimate of β, $\widetilde{\Theta}_i, i = 1,\cdots q$ are the initial estimates of $\Theta_i, i = 1,\cdots,q$, $\widetilde{\Theta} = (I_T \otimes I_k) - \sum_{i=1}^{q} (L^i \otimes \widetilde{\Theta}_i)$ where L^i is a $T \times T$ lag matrix which has ones on the subdiagonal directly below the main diagonal and zeroes else where

$$L^i\widetilde{\varepsilon} = (0,\cdots,0,\widetilde{\varepsilon}_1,\cdots,\widetilde{\varepsilon}_{T-i})',$$

$$\widetilde{Z} = \left[(BY \otimes I_k),\cdots,(B^pY \otimes I_k),-(L\widetilde{\varepsilon} \otimes I_k),\cdots,-(L^q\widetilde{\varepsilon} \otimes I_k)\right],$$

$$\beta_0 = (\widetilde{\phi}_1',\cdots,\widetilde{\phi}_p',\widetilde{\theta}_1',\cdots,\widetilde{\theta}_q')'$$

then the Newton-Rapson algorithm which is in the form of Generalized least square is

$$\hat{\beta} = \beta_0 + \left[\widetilde{Z}'\widetilde{\Theta}^{-1'}(I_T \otimes \widetilde{\Sigma}^{-1})\widetilde{\Theta}^{-1}\widetilde{Z}\right]^{-1} \widetilde{Z}'\widetilde{\Theta}^{-1'}(I_T \otimes \widetilde{\Sigma}^{-1})\widetilde{e} \quad \text{with } \widetilde{\Sigma} = \widetilde{\varepsilon}'\widetilde{\varepsilon}/T \text{ and } \widetilde{e} = vec(\widetilde{\varepsilon}').$$

Another form of this algorithm is

$$\hat{\beta} = \beta_0 + \left[\bar{Z}'(I_T \otimes \tilde{\Sigma}^{-1})\bar{Z}\right]^{-1} \bar{Z}'(I_T \otimes \tilde{\Sigma}^{-1})\tilde{e}$$

The innovations $\tilde{\varepsilon}_t$ can be computed recursively by

$$\tilde{\varepsilon}_t = y_t - \sum_{i=1}^{p} \tilde{\Phi}_i\, y_{t-i} + \sum_{i=1}^{q} \tilde{\Theta}_i\, \tilde{\varepsilon}_{t-i} \quad \text{for} \quad t = 1,\cdots,T \quad \text{with} \quad \tilde{\varepsilon}_0 = \tilde{\varepsilon}_{-1} = \cdots = \tilde{\varepsilon}_{1-q} = 0\text{、}$$

Let $\bar{Z} = (\bar{U}_1,\cdots,\bar{U}_p,\bar{V}_1,\cdots,\bar{V}_q)$ where $\bar{U}_j = \tilde{\Theta}^{-1}(B^j Y \otimes I_k)$ and $\bar{V}_j = \tilde{\Theta}^{-1}(L^j\tilde{\varepsilon} \otimes I_k)$, then $\bar{U}_j$ and $\bar{V}_j$ can be partitioned as $\bar{U}_j = (\bar{U}'_{j1},\bar{U}'_{j2},\cdots,\bar{U}'_{jT})'$ and $\bar{V}_j = (\bar{V}'_{j1},\bar{V}'_{j2},\cdots,\bar{V}'_{jT})'$ where $\bar{U}_{jt}$ and $\bar{V}_{jt}$ are $k \times k^2$ matrices, and these matrices can be calculated as

$$\bar{U}_{jt} = \sum_{i=1}^{q} \tilde{\Theta}_i \bar{U}_{j,t-i} + (y'_{t-j} \otimes I_k) \quad \text{for } t = 1,\cdots,T \quad \text{with } \bar{U}_{jt} = 0 \quad \text{for } t \le 0.$$

$$\bar{V}_{jt} = \sum_{i=1}^{q} \tilde{\Theta}_i \bar{V}_{j,t-i} - (\tilde{\varepsilon}'_{t-j} \otimes I_k) \quad \text{for } t = 1,\cdots,T \quad \text{with } \bar{V}_{jt} = 0 \quad \text{for } t \le 0.$$

Results

The algorithm for approximate maximum likelihood estimation of parameters of a VARMA(p,q) model suggested by Wilson (1973) require computation of derivatives of the innovation vectors with respect to the parameters. Consider a zero mean VARMA(p,q) model, $y_t = \sum_{l=1}^{p} \Phi^{(l)} y_{t-l} + a_t - \sum_{l=1}^{q} \Theta^{(l)} a_{t-l}$ where $y_t = (y_{1t},\cdots,y_{kt})'$, $a_t = (a_{1t},\cdots,a_{kt})'$, $\Phi^{(l)}(l = 1,\cdots,p)$ and $\Theta^{(l)}(l = 1,\cdots,q)$ are $k \times k$ parameter matrices and $\{a_t\}$ is assumed to be independently and identically distributed white noise series with zero mean vector and constant dispersion matrix Σ. From the model we can express a_t as

$a_t = y_t - \sum_{l=1}^{p} \Phi^{(l)} y_{t-l} + \sum_{l=1}^{q} \Theta^{(l)} a_{t-l}$. This leads to the k linear equations

$$a_{it} = y_{it} - \sum_{l=1}^{p}\{\sum_{j=1}^{k}\Phi_{ij}^{(l)}y_{j,t-l}\} + \sum_{l=1}^{q}\{\sum_{j=1}^{k}\Theta_{ij}^{(l)}a_{j,t-l}\} \quad \text{for } i = 1,\cdots,k.$$

Differentiating a_{it} with respect to an element $\Phi_{rs}^{(m)}$, the element in r^{th} row and s^{th} column of the parameter matrix $\Phi^{(m)}$, we get,

$$\frac{\partial a_{it}}{\partial \Phi_{rs}^{(m)}} = -\sum_{l=1}^{p}\sum_{j=1}^{k}\frac{\partial}{\partial \Phi_{rs}^{(m)}}\left\{\Phi_{ij}^{(l)}y_{j,t-l}\right\} + \sum_{l=1}^{q}\sum_{j=1}^{k}\frac{\partial}{\partial \Phi_{rs}^{(m)}}\left\{\Theta_{ij}^{(l)}a_{j,t-l}\right\}$$

$$= -\sum_{l=1}^{p}\sum_{j=1}^{k}y_{j,t-l}\frac{\partial \Phi_{ij}^{(l)}}{\partial \Phi_{rs}^{(m)}} + \sum_{l=1}^{q}\sum_{j=1}^{k}\Theta_{ij}^{(l)}\frac{\partial a_{j,t-l}}{\partial \Phi_{rs}^{(m)}}$$

But $\frac{\partial \Phi_{ij}^{(l)}}{\partial \Phi_{rs}^{(m)}} = 1$ when $i = r, j = s$ and $l = m$ simultaneously otherwise its value is zero. Hence the first part of the derivative is zero for all terms except when $i = r, j = s$ and $l = m$ so that we get

$$\frac{\partial a_{it}}{\partial \Phi_{rs}^{(m)}} = -y_{s,t-m} + \sum_{l=1}^{q}\sum_{j=1}^{k}\Theta_{ij}^{(l)}\frac{\partial a_{j,t-l}}{\partial \Phi_{rs}^{(m)}} \text{ when } i = r \text{ and } \frac{\partial a_{it}}{\partial \Phi_{rs}^{(m)}} = \sum_{l=1}^{q}\sum_{j=1}^{k}\Theta_{ij}^{(l)}\frac{\partial a_{j,t-l}}{\partial \Phi_{rs}^{(m)}} \text{ when } i \neq r.$$

Similarly when we differentiate a_{it} with respect to $\Theta_{rs}^{(m)}$, the element in r^{th} row and s^{th} column of the parameter matrix $\Theta^{(m)}$, we get,

$$\frac{\partial a_{it}}{\partial \Theta_{rs}^{(m)}} = \frac{\partial y_{it}}{\partial \Theta_{rs}^{(m)}} - \sum_{l=1}^{p}\sum_{j=1}^{k}\frac{\partial}{\partial \Theta_{rs}^{(m)}}\left\{\Phi_{ij}^{(l)}y_{j,t-l}\right\} + \sum_{l=1}^{q}\sum_{j=1}^{k}\frac{\partial}{\partial \Theta_{rs}^{(m)}}\left\{\Theta_{ij}^{(l)}a_{j,t-l}\right\}$$

$$= \sum_{l=1}^{q}\sum_{j=1}^{k}\frac{\partial}{\partial \Theta_{rs}^{(m)}}\left\{\Theta_{ij}^{(l)}a_{j,t-l}\right\}.$$

Now $\frac{\partial}{\partial\Theta_{rs}^{(m)}}\left\{\Theta_{ij}^{(m)}a_{j,t-l}\right\}=\Theta_{ij}^{(l)}\frac{\partial a_{j,t-l}}{\partial\Theta_{rs}^{(m)}}+a_{j,t-l}\frac{\partial\Theta_{ij}^{(l)}}{\partial\Theta_{rs}^{(m)}}$ and $\frac{\partial\Theta_{ij}^{(l)}}{\partial\Theta_{rs}^{(m)}}=1$ when $r=i,s=j$ and $l=m$ simultaneously other wise $\frac{\partial\Theta_{ij}^{(l)}}{\partial\Theta_{rs}^{(m)}}=0$ so that $\frac{\partial}{\partial\Theta_{rs}^{(m)}}\left\{\Theta_{ij}^{(m)}a_{j,t-l}\right\}=\Theta_{ij}^{(l)}\frac{\partial a_{j,t-l}}{\partial\Theta_{rs}^{(m)}}+a_{s,t-m}$ when $r=i,s=j$ and $l=m$ otherwise $\frac{\partial}{\partial\Theta_{rs}^{(m)}}\left\{\Theta_{ij}^{(m)}a_{j,t-l}\right\}=\Theta_{ij}^{(l)}\frac{\partial a_{j,t-l}}{\partial\Theta_{rs}^{(m)}}$.

Hence we get $$\frac{\partial a_{it}}{\partial\Theta_{rs}^{(m)}}=\begin{cases}\sum_{l=1}^{q}\sum_{j=1}^{k}\Theta_{ij}^{(l)}\frac{\partial a_{j,t-l}}{\partial\Theta_{rs}^{(m)}}+a_{s,t-m} & when\ r=i\\ \sum_{l=1}^{q}\sum_{j=1}^{k}\Theta_{ij}^{(l)}\frac{\partial a_{j,t-l}}{\partial\Theta_{rs}^{(m)}} & otherwise\end{cases}$$

Then the required recursive relations for computing the derivatives of elements of the innovation vector with respect to the parameters are

i) $\frac{\partial a_{it}}{\partial\Phi_{is}^{(m)}}=-y_{s,t-m}+\sum_{l=1}^{q}\sum_{j=1}^{k}\Theta_{ij}^{(l)}\frac{\partial a_{j,t-l}}{\partial\Phi_{is}^{(m)}}$ for $i,s=1,\cdots,k$ $m=1,\cdots,p$ and $t=1,\cdots,T$ with initial few derivatives set to zero.

ii) $\frac{\partial a_{it}}{\partial\Phi_{rs}^{(m)}}=\sum_{l=1}^{q}\sum_{j=1}^{k}\Theta_{ij}^{(l)}\frac{\partial a_{j,t-l}}{\partial\Phi_{rs}^{(m)}}$ for $r\neq i$, $i,r,s=1,\cdots,k$ and $m=1,\cdots,p$.

iii) $\frac{\partial a_{it}}{\partial\Theta_{is}^{(m)}}=a_{s,t-m}+\sum_{l=1}^{q}\sum_{j=1}^{k}\Theta_{ij}^{(l)}\frac{\partial a_{j,t-l}}{\partial\Theta_{is}^{(m)}}$ for $i,s=1,\cdots,k$ and $m=1,\cdots,q$

iv) $\frac{\partial a_{it}}{\partial\Theta_{rs}^{(m)}}=\sum_{l=1}^{q}\sum_{j=1}^{k}\Theta_{ij}^{(l)}\frac{\partial a_{j,t-l}}{\partial\Theta_{rs}^{(m)}}$ for $r\neq i$, $i,r,s=1,\cdots,k$ and $m=1,\cdots,q$

If we denote the i^{th} row of the parameter matrix $\Theta^{(l)}$ by $\Theta_i^{(l)}$ and $a_t=(a_{1t},\cdots,a_{kt})'$ is the innovation vector, then the above recurrence relations can be re-written as

i) $\frac{\partial a_{it}}{\partial\Phi_{is}^{(m)}}=-y_{s,t-m}+\sum_{l=1}^{q}\Theta_i^{(l)}\frac{\partial a_{t-l}}{\partial\Phi_{is}^{(m)}}$

ii) $\dfrac{\partial a_{it}}{\partial \Phi_{rs}^{(m)}} = \sum_{l=1}^{q} \Theta_i^{(l)} \dfrac{\partial a_{t-l}}{\partial \Phi_{rs}^{(m)}}$ for $r \neq i$

iii) $\dfrac{\partial a_{it}}{\partial \Theta_{is}^{(m)}} = a_{s,t-m} + \sum_{l=1}^{q} \Theta_i^{(l)} \dfrac{\partial a_{t-l}}{\partial \Theta_{is}^{(m)}}$ and

iv) $\dfrac{\partial a_{it}}{\partial \Theta_{rs}^{(m)}} = \sum_{l=1}^{q} \Theta_i^{(l)} \dfrac{\partial a_{t-l}}{\partial \Theta_{rs}^{(m)}}$ for $r \neq i$.

This shows that the derivative of an element in the innovation vector a_t with respect to a parameter is the sum of inner products of the corresponding rows of the moving average parameter matrices with the vectors of derivatives of earlier innovation vectors $a_{t-1},\cdots,a_{t-q}$ with respect to the same parameter plus a term depending on whether the parameter belongs to the same row as the corresponding element of a_t.

It can be seen that if we arrange the elements of the parameter matrices $\Phi_1,\cdots,\Phi_p,\Theta_1,\cdots,\Theta_q$, in standard order, as a vector $\beta = (\beta_1,\cdots,\beta_K)'$ where $K = (p+q)k^2$, then $\Phi_{ij}^{(m)}$ is the $\{(m-1)k^2 + (i-1)k + j\}^{th}$ element of β and $\Theta_{ij}^{(m)}$ is the $\{(p+m-1)k^2 + (i-1)k + j\}^{th}$ element of β. The correspondence of elements of β with the parameter matrices can be established as follows.

To determine the l^{th} element β_l of the vector β, for $l = 1,\cdots,K$ we define

$j = l \bmod(k)$

$i = (l-j)/k \bmod(k) + 1$ and

$m = \{l - (i-1)k - j\}/k^2 + 1$

If $m \le p$, then $\beta_l = \Phi_{ij}^{(m)}$ and if $m > p$, then $\beta_l = \Theta_{ij}^{(m-p)}$. For a given set of observation vectors $y_1, y_2, \cdots, y_T$ the residuals based on a proposed set of parameter vector $\dot{\beta}$ is given by $\dot{a}_t = y_t - \dot{\Phi}_1 y_{t-1} - \cdots - \dot{\Phi}_p + \dot{\Theta}_1 \dot{a}_{t-1} + \cdots + \dot{\Theta}_q \dot{a}_{t-q}$. Hence for computing the initial few residuals $\dot{a}_1, \cdots, \dot{a}_m$, where $m = \max(p,q)$, we require past observation vectors $y_0, y_{-1}, \cdots, y_{-(p-1)}$ and past residual vectors $\dot{a}_0, \dot{a}_{-1}, \cdots, \dot{a}_{-(q-1)}$. One suggestion is to assume these values to be zeroes and another is to use observation vectors for $t = m+1, \cdots, T$ so that the initial value problem for the observation vectors can be avoided and assume the values of the initial residual vectors $\dot{a}_1, \cdots, \dot{a}_m$ as zeros. Yet another suggestion is to use an appropriate precast model to estimate initial observation vectors and residual vectors.

For selecting appropriate orders of VARMA(*p*,*q*) model to be fitted to the vector time series consisting of transformed landings of oil sardine, anchovies, tuna and penaeid prawns, the three criteria namely *AIC*, *BIC* and *HQ* were computed for different values of the order parameters *p* and *q* taking values 1 to 4. The values of these criterion were computed by estimating the model parameters of each of the resulting model using a computer software developed in C language based on the algorithm of Spliids (1983). The values of these three criteria for different orders are given in table.3.1. The minimum values for the BIC and HQ criterion were –9.1109 and –9.5009 respectively and both these values are corresponding to the model VARMA(1,1). The minimum value of AIC was –10.0140 corresponding to the model VARMA(1,3) which is of higher orders. Hence the model VARMA(1,1) was selected as the suitable model and the estimation of parameter matrices in the model was attempted by maximum likelihood method following the algorithm given by Wilson (1973). This ML estimation algorithm require initial estimates of parameter matrices as input and in this study three different initial inputs as obtained through different methods were tried. The first initial estimates used was obtained form the first step of the regression procedure of Splid's algorithm with VAR(5) model to estimate error terms. The second set of values used were obtained as the final estimates from Spliid's algorithm. The third set was obtained form the conditional maximum likelihood estimation through a Newton-Rapson algorithm in the form of a generalised least square procedure (Reinsel 1993).

The first set of inputs used for ML estimation as obtained through the first step of Spliid's algorithm were

$$\hat{\delta} = (2.2490, 2.4471, 1.6457, 4.7637)'$$

$$\hat{\Phi}_1 = \begin{pmatrix} 0.7128 & -0.0103 & -0.0316 & 0.0335 \\ 0.0060 & 0.6133 & 0.0917 & 0.0361 \\ -0.0084 & 0.0453 & 0.6919 & 0.0014 \\ 0.0579 & 0.0343 & 0.0389 & 0.6923 \end{pmatrix}$$

$$\hat{\Theta}_1 = \begin{pmatrix} -0.3051 & -0.1077 & -0.0195 & 0.0911 \\ 0.1541 & -0.2408 & 0.1790 & 0.1105 \\ -0.0528 & -0.0070 & -0.0308 & 0.0395 \\ 0.2625 & 0.0430 & -0.0172 & -0.0981 \end{pmatrix} \text{ and}$$

$$\hat{\Sigma} = \begin{pmatrix} 0.1088 & -0.0188 & -0.0171 & -0.0217 \\ -0.0188 & 0.1360 & 0.0459 & 0.0423 \\ -0.0171 & 0.0459 & 0.1056 & 0.0096 \\ -0.0217 & 0.0423 & 0.0096 & 0.1359 \end{pmatrix}$$

With the above values as input and the values of the constraint parameters of the algorithm as $\lambda = 1.0, \Delta\lambda = 0.01, \lambda_{\max} = 10.0$ the ML estimation was carried out. The log likelihood for the initial estimates was -186.6232. The algorithm failed to converge to a desired degree of accuracy of 0.001 even after 25 iterations. The difference in the successive estimates of parameters after 25 iterations was 0.0075 and the log likelihood value corresponding to the estimate after 25 iterations was –108.2975.

The second set of initial inputs used were obtained as the final output of the Spliid's method and these estimates were

$$\hat{\delta} = (0.0497, -0.0398, 0.0142, -0.0573)$$

$$\hat{\Phi}_1 = \begin{pmatrix} 0.9344 & -0.0294 & -0.0336 & 0.0550 \\ 0.0112 & 0.8371 & 0.1060 & 0.0438 \\ -0.0237 & 0.0613 & 0.9126 & 0.0092 \\ 0.0917 & 0.0648 & 0.0458 & 0.9107 \end{pmatrix}$$

$$\hat{\Theta}_1 = \begin{pmatrix} -0.4796 & -0.1369 & 0.0147 & 0.0873 \\ 0.1796 & -0.1783 & 0.1135 & 0.1315 \\ -0.1743 & 0.0036 & -0.0576 & 0.0771 \\ 0.3550 & 0.0777 & -0.0759 & -0.1781 \end{pmatrix} \text{ and}$$

$$\hat{\Sigma} = \begin{pmatrix} 0.0776 & 0.0043 & 0.0051 & 0.0003 \\ 0.0043 & 0.0902 & -0.0022 & 0.0242 \\ 0.0057 & -0.0022 & 0.0431 & -0.0128 \\ 0.0003 & 0.0242 & -0.0128 & 0.1243 \end{pmatrix}$$

The log likelihood corresponding to these estimates was –77.1709 and these estimates were used as the initial input for ML estimation of parameters. The estimation algorithm converged after 7 iterations with an accuracy of 0.001 for the parameter estimates leading to the log likelihood value of -75.83553 corresponding to the final estimates. The final estimates of the parameters obtained were

$$\tilde{\delta} = (0.0735, -0.0887, -0.0457, -0.1577)$$

$$\tilde{\Phi}_1 = \begin{pmatrix} 0.9342 & -0.0299 & -0.0348 & 0.0545 \\ 0.0124 & 0.8385 & 0.1079 & 0.0444 \\ -0.0218 & 0.0628 & 0.9146 & 0.0100 \\ 0.0945 & 0.0674 & 0.0493 & 0.9121 \end{pmatrix}$$

$$\tilde{\Theta}_1 = \begin{pmatrix} -0.4840 & -0.0953 & -0.0899 & 0.0513 \\ 0.1674 & -0.1561 & 0.1254 & 0.0883 \\ -0.1564 & 0.0195 & -0.0863 & 0.0792 \\ 0.3421 & 0.0359 & -0.1001 & -0.1834 \end{pmatrix}$$

$$\tilde{\Sigma} = \begin{pmatrix} 0.0769 & 0.0049 & 0.0050 & 0.0004 \\ 0.0049 & 0.0894 & -0.0018 & 0.0241 \\ 0.0050 & -0.0018 & 0.0428 & -0.0125 \\ 0.0004 & 0.0241 & -0.0125 & 0.1251 \end{pmatrix}$$

In these estimates the elements found significant were $\tilde{\Phi}_{11}^{(1)}$, $\tilde{\Phi}_{22}^{(1)}$, $\tilde{\Phi}_{23}^{(1)}$, $\tilde{\Phi}_{33}^{(1)}$, $\tilde{\Phi}_{41}^{(1)}$, $\tilde{\Phi}_{44}^{(1)}$; $\tilde{\Theta}_{11}^{(1)}$, $\tilde{\Theta}_{31}^{(1)}$, $\tilde{\Theta}_{41}^{(1)}$ and $\tilde{\Theta}_{44}^{(1)}$ where $\tilde{\Phi}_{ij}^{(m)}$ denote the $(i,j)^{th}$ element of matrix $\tilde{\Phi}_m$ and $\tilde{\Theta}_{ij}^{(m)}$ denote the $(i,j)^{th}$ element of matrix $\tilde{\Theta}_m$.

The third set of inputs for the ML estimation of the parameters were obtained through the conditional likelihood estimation based on generalized least square procedure described by Reinsel (1993). The conditional estimation program took 27 iterations to converge and arrive at the following estimates. The log likelihood corresponding to these estimates was –65.49831. The algebric expression of the model and estimates of its parameters are

$$y_t = \Phi\, y_{t-1} + \varepsilon_t - \Theta\, \varepsilon_{t-1}$$

$$\hat{\delta} = (2.396959, 0.116563, -0.679548, 2.274994)$$

$$\hat{\Phi}_1 = \begin{pmatrix} 0.872816 & -0.010475 & -0.051833 & -0.042331 \\ 0.008596 & 0.840623 & 0.110818 & 0.033358 \\ -0.000552 & 0.082716 & 0.896180 & 0.029064 \\ 0.022338 & 0.042814 & 0.055359 & 0.829198 \end{pmatrix}$$

$$\hat{\Theta}_1 = \begin{pmatrix} -0.528347 & -0.076779 & -0.137238 & 0.055880 \\ 0.177013 & -0.165844 & 0.140812 & 0.080439 \\ -0.159972 & 0.053174 & -0.119049 & 0.083817 \\ 0.364932 & 0.007692 & -0.130277 & -0.196914 \end{pmatrix} \text{ and}$$

$$\hat{\Sigma} = \begin{pmatrix} 0.07368 & 0.003911 & 0.006177 & -0.004617 \\ 0.003911 & 0.089200 & -0.001252 & 0.022278 \\ 0.006177 & -0.001252 & 0.042094 & -0.010020 \\ -0.004617 & 0.022278 & -0.010020 & 0.114538 \end{pmatrix}.$$

Using these estimates of parameters as input, likelihood estimation based on Wilson's algorithm was attempted. The estimation process terminated after one iteration and could not achieve further maximization of the log likelihood yielding the same set of parameter estimates with an accuracy of 0.00001. The standard errors of the estimated parameter matrices were

$$SE(\tilde{\Phi}_1) = \begin{pmatrix} 0.028521 & 0.061330 & 0.056048 & 0.029307 \\ 0.024321 & 0.053795 & 0.048411 & 0.025458 \\ 0.017175 & 0.037909 & 0.034182 & 0.017939 \\ 0.029206 & 0.064360 & 0.058064 & 0.030482 \end{pmatrix}$$

$$SE(\tilde{\Theta}_1) = \begin{pmatrix} 0.067512 & 0.074033 & 0.097519 & 0.058594 \\ 0.091552 & 0.095382 & 0.126856 & 0.077446 \\ 0.062742 & 0.065669 & 0.087247 & 0.053071 \\ 0.103102 & 0.108015 & 0.143052 & 0.087134 \end{pmatrix}$$

In the estimated model the elements of coefficient matrices found significant were $\tilde{\Phi}_{11}^{(1)}$, $\tilde{\Phi}_{22}^{(1)}$, $\tilde{\Phi}_{23}^{(1)}$, $\tilde{\Phi}_{32}^{(1)}$, $\tilde{\Phi}_{33}^{(1)}$, $\tilde{\Phi}_{44}^{(1)}$ in matrix $\tilde{\Phi}_1$; $\tilde{\Theta}_{11}^{(1)}$, $\tilde{\Theta}_{31}^{(1)}$, $\tilde{\Theta}_{41}^{(1)}$, $\tilde{\Theta}_{44}^{(1)}$ in matrix $\tilde{\Theta}_1$. The estimated model explained 92.63%, 91.08%, 95.79% and 88.5% respectively of variations in the individual component series of the vector. It was also examined whether the stationary and invertible conditions are satisfied by the estimated model. For this the eigen values of the characteristic matrices of the model were evaluated and these are given in table.3.1. All the eigen values were found to fall within the unit circle boundary and hence the model estimated is both stationary and invertible.

Discussion

To examine the suitability of the estimated model residual analysis was carried out by computing cross correlation matrices for the residual series corresponding to the estimated model. The values of χ^2 for testing the significance of elements of the residual cross correlation matrices for different lags are given in table.3.2. It was found that out of a total of 384 elements in the cross correlation matrices only 23 elements were significant. Maximum number of significant elements was 4 at lag 4 all significant elements being diagonal elements which represent autocorrelation. The combined significant χ^2 statistic was also significant at lag 4 which evidently may be due to some left out effects of seasonality present in the quarter wise data. Using the estimated model quarterwise forecasts were computed along with standard errors for the years 1997 and 1998 and these are given in table.3.4. Compared to the VAR(5) model, though the percentage of variation explained by this model is also satisfactory though it is high for the VAR(5) model. But the forecasts made using this model is more close to the observed value than that of the VAR(5) model. The expressions for the individual models for the components of the vector time series are

$$y_{1t} = 2.3970 + 0.8728\, y_{1,t-1} - 0.0105\, y_{2,t-1} - 0.0518\, y_{3,t-1} - 0.0423\, y_{4,t-1} + 0.5283\, \varepsilon_{1,t-1} + 0.0768\, \varepsilon_{2,t-1} + 0.1372\, \varepsilon_{3,t-1} - 0.0559\, \varepsilon_{4,t-1} + \varepsilon_t\,,$$

$$y_{2t} = 0.1166 + 0.0086\, y_{1,t-1} + 0.8406\, y_{2,t-1} - 0.1108\, y_{3,t-1} - 0.0334\, y_{4,t-1} - 0.1770\varepsilon_{1,t-1} + 0.1658\, \varepsilon_{2,t-1} - 0.1408\, \varepsilon_{3,t-1} - 0.0804\, \varepsilon_{4,t-1} + \varepsilon_t\,,$$

$$y_{3t} = -0.6795 - 0.0006\, y_{1,t-1} + 0.0827\, y_{2,t-1} + 0.8962\, y_{3,t-1} + 0.0291\, y_{4,t-1} + 0.1600\, \varepsilon_{1,t-1} - 0.0532\, \varepsilon_{2,t-1} + 0.1190\, \varepsilon_{3,t-1} - 0.0838\, \varepsilon_{4,t-1} + \varepsilon_t$$

and

$$y_{4t} = 2.2750 + 0.0223\, y_{1,t-1} + 0.0428\, y_{2,t-1} + 0.0554\, y_{3,t-1} + 0.8292\, y_{4,t-1} - 0.3649\, \varepsilon_{1,t-1} - 0.0077\, \varepsilon_{2,t-1} + 0.1328\, \varepsilon_{3,t-1} - 0.1969\, \varepsilon_{4,t-1} + \varepsilon_t\,.$$

In the first model for the series on transformed landings of oil sardine, significant coefficients were only for $y_{1,t-1}$ and $\varepsilon_{1,t-1}$. Hence oil sardine landings significantly depend on its own past values and residuals at lag1. Significant coefficients of the second model corresponding to anchovies were for $y_{2,t-1}$ and $y_{3,t-1}$. There fore the series on landings of anchovies is affected by the series on tuna landings and its own past values both at lag 1. In the third model corresponding to landings of Tuna the significant coefficients were for $y_{2,t-1}$, $y_{3,t-1}$ and $\varepsilon_{1,t-1}$. This shows that the series on landings of Tuna depend on the series on landings of anchovies and its own past values both with a lag of 1. It also depends on the residuals of oil sardine series with lag 1. In the fourth model corresponding to the landings of penaeid prawns the coefficients significant were for $y_{4,t-1}$, $\varepsilon_{1,t-1}$ and $\varepsilon_{4,t-1}$. According to this model the series on landings of penaeid prawns is significantly influenced by its own past values and residuals at lag 1 and also on the residual at lag 1 of oil sardine series.

Appendix-III (Tables)

Table.3.1. Values of AIC, BIC and HQ criterion for different models.

p	q	AIC	BIC	HQ
1	1	-9.7678	-9.1109	-9.5009
1	2	-9.7063	-8.7209	-9.3059
1	3	-10.0140	-8.7001	-9.4801
1	4	-9.9861	-8.3437	-9.3187
2	1	-8.7066	-7.7212	-8.3062
2	2	-8.0064	-6.6925	-7.4725
2	3	-8.0573	-6.4150	-7.3900
2	4	-5.7184	-3.7476	-4.9176
3	1	-4.6539	-3.3400	-4.1200
3	2	-5.2321	-3.5898	-4.5648
3	3	-4.6665	-2.6957	-3.8657
3	4	-7.0985	-4.7993	-6.1643
4	1	-5.7904	-4.1481	-5.1231
4	2	-2.0182	-0.0474	-1.2174
4	3	-2.9859	-0.6866	-2.0516
4	4	-0.7492	1.8785	0.3185

Table.3.2. Eigen values of characteristic matrices corresponding to the AR and MA terms in the estimated model.

	Eigen vlaues of the characteristic matrix of AR coefficient matrices			**Eigen vlaues of the characteristic matrix of MA coefficient matrices**		
Sl. No.	Real Part	Imaginary part	Absolute Value	Real Part	Imaginary part	Absolute Value
1	0.87031	0.00000	0.87031	-0.57195	0.00000	0.57195
2	0.81965	0.00000	0.81965	-0.09123	-0.13115	0.15976
3	0.98220	0.00000	0.98220	-0.09123	0.13115	0.15976
4	0.76739	0.00000	0.76739	-0.25574	0.00000	0.25574

Table.3.3. Chi-square statistic for testing the combined significance of residual cross correlation matrices.

Lag	χ^2	Lag	χ^2
1	1.693	13	15.842
2	14.534	14	18.867
3	23.273	15	15.552
4	75.696	16	23.284
5	20.305	17	14.988
6	29.832	18	17.183
7	11.245	19	24.998
8	15.904	20	24.034
9	23.196	21	23.741
10	27.458	22	23.927
11	15.793	23	13.798
12	18.885	24	16.316

Table.3.4. Forecasts and standard errors oil sardine, anchovies, tuna, penaeid prawns using VARMA(1,1) model

Year	Quarter	Oil sardine			Anchovies		
		Observed	Forecasted	SE	Observed	Forecasted	SE
1997	I	8.3564	8.0875	0.2714	10.9587	10.7984	0.2987
	II	8.9392	8.0938	0.4711	10.9741	10.8007	0.4225
	III	9.012	8.1021	0.5796	11.0627	10.7984	0.4930
	IV	9.2357	8.1119	0.6510	10.8624	10.7927	0.5405
1998	I	8.9451	8.1228	0.7012	10.9498	10.7844	0.5757
	II	8.8739	8.1346	0.7378	11.1438	10.7741	0.6037
	III	8.984	8.1469	0.7652	11.074	10.7625	0.6271
	IV	8.9954	8.1597	0.7862	11.1145	10.7498	0.6474
Year	Quarter	Tuna			Penaeid prawns		
		Observed	Forecasted	SE	Observed	Forecasted	SE
1997	I	7.8524	7.9035	0.2052	20.2447	19.8269	0.3384
	II	7.8221	7.8742	0.3015	20.3667	19.7959	0.4955
	III	7.861	7.8472	0.3603	20.1132	19.7689	0.5785
	IV	7.834	7.8220	0.4036	20.3892	19.7450	0.6296
1998	I	7.7659	7.7983	0.4389	20.3828	19.7239	0.6634
	II	7.6598	7.7757	0.4694	20.5662	19.7049	0.6870
	III	7.6158	7.7540	0.4969	20.6265	19.6877	0.7042
	IV	7.5407	7.7331	0.5220	20.3538	19.6720	0.7174

CHAPTER –4

MODELLING MARINE FISH LANDINGS WITH ENVIRONMENTAL VARIABLES

Introduction

Time series data on monthly landings of elasmobranchs, oil sardine, stolephorus and mackerel at Cochin Fisheries Harbour during the period 1988-97 were used to develop Vector Autoregressive models with environmental variables as exogenous variables (*VARX* model). The environmental time series variables considered were monthly means of maximum and minimum temperatures, lowest and highest temperature recorded in a month, total monthly rainfall, highest rainfall recorded in a month and the number of rainy days in a month, all being recorded at Cochin. The main objective of the study was to establish possible relationship between environmental variables and marine fish landings by developing suitable vector time series models.

A general vector autoregressive model with exogenous variables of orders p and r denoted by $VARX(p,r)$ is given by

$$\Phi(B)\boldsymbol{y}_t = \delta + \beta(B)\boldsymbol{x}_t + \boldsymbol{a}_t$$

where $\Phi(B) = \boldsymbol{I} - \Phi_1 B - \cdots - \Phi_p B^p$, $\beta(B) = \beta_0 + \beta_1 B + \cdots + \beta_{r-1} B^{r-1}$ are matrix polynomials in the back shift operator B of orders p and $(r\text{-}1)$ respectively, $\boldsymbol{y}_t = (y_{1t}, \cdots, y_{kt})'$ is the vector output series with k components, $\boldsymbol{x}_t = (x_{1t}, \cdots, x_{mt})'$ is the exogenous vector series with m components, $\delta = (\delta_1, \cdots, \delta_k)'$ is a constant vector of size k, $\Phi_1, \cdots, \Phi_p$ are $k \times k$ parameter matrices, $\beta_0, \beta_1, \cdots, \beta_{r-1}$ are $k \times m$ parameter matrices and $\boldsymbol{a}_t = (a_{1t}, \cdots, a_{kt})'$ is a vector of innovations. Here it is assumed that the innovation series are distributed independently and identically with zero mean vector and constant covariance matrix Σ.

Review of literature:

Murty and Edelman (1966) related the long-term fluctuations in the Indian oil sardine fishery with the strength of summer monsoon over the peninsular region of India. They found that certain range of monsoon intensity is unfavourable to the fishery and certain other range favourable. Murphy and Winkler (1984) reviewed probability forecasting methods in meteorology and described relationship between probability forecasting in meteorology and other fields. Pati (1984) studied the relationship between rainfall and coastal fishery in Indian waters. He obtained significant correlations between the fluctuations in annual rainfall and landings from drift gillnet fishery, total rainfall and total catch rate and total rainfall and catch rate of plankton. Keller (1987) used Box-Jenkins transfer function models for forecasting primary production rates. He found that the incorporation of phytoplankton biomass and hourly light as two input variables improved the fit of the models. Fogarty (1988) used Box-Jenkins transfer function models to analyse the relationship between water temperature and marine lobster catch and catch per unit effort. He found the effect as vulnerability to capture increase with water temperature and obtained a significant effect of temperature at a six-year lag. Huang and Guo (1990) considered the estimation problems of linear feed back control system described by ARMAX models. He developed an estimation algorithm for ARMAX systems in the line of Hannan and Rissanen method. Longhurst and Wooster (1990) studied the relationship between the abundance of oil sardine and upwelling on the south west coast of India. They found that the o-group recruitment to the fishery begins towards the end of the summer monsoon and its success is statistically related to sea level at Cochin just prior to the onset of monsoon. Campbell (1994) investigated the relationship between sudden infant death syndrome (SIDS) and environmental temperature. He established a strong negative relationship in which the death due to SIDS increase 2 - 5 days after the environmental temperature. Handcock and Wallis (1994) developed a random field model for mean temperature in which the stochastic structure was modelled by stationary spatial-temporal Gaussian random field. Borah and Bora (1995) modellled monthly rainfall at Guwagati using seasonal ARIMA model and used it to predict month-wise rainfall for an year ahead.

Materials and Methods:

The method of estimation of parameter matrices in *VARX* model was derived following the procedure given by Spliid (1983). Let a sample of size T is available for the input and output vector series as $y_1,\cdots y_T$ and $x_1,\cdots,x_T$. To avoid the problem of initial values for y_t and x_t, we define the data matrices for the output and input series by $y=(y_{s+1},\cdots,y_T)'$, $x=(x_{s+1},\cdots,x_T)'$ and the innovation matrix by $a=(a_{s+1},\cdots,a_T)'$ where $s=\max(p,r)$. These matrices will be of order $n\times k$, $n\times m$ and $n\times k$ respectively where $n=T-s$. Now define matrices

$Y=(By,B^2y,\cdots,B^py)$ of order $n\times pk$, $X=(x,Bx,\cdots,B^{r-1}x)$ of order $n\times rm$ and $U=(\underline{1},Y,X)$ is of order $n\times(pk+mr+1)$ where $\underline{1}$ is a column vector of size n with all elements unity. Define $\alpha=(\delta,\Phi_1,\cdots,\Phi_p,\beta_0,\beta_1,\cdots,\beta_{r-1})'$ as the parameter matrix of order $(pk+mr+1)\times k$. Then the general multivariate linear regression equivalent for the model is

$$Y = U\alpha + a\,.$$

The model representation for the t^{th} row of this equation is

$$y'_t = \delta' + y'_{t-1}\Phi'_1 + \cdots + y'_{t-p}\Phi'_p + x'_t\beta'_0 + x'_{t-1}\beta'_1 + \cdots + x'_{t-r+1}\beta'_{r-1} + a'_t$$

and transpose of this will yield the original model $\Phi(B)\,y_t = \delta + \beta(B)\,x_t + a_t$. Under this general multivariate linear regression model, the maximum likelihood estimate of the parameter matrix α is same as the least square estimate given by

$$\hat{\alpha} = (U'U)^{-1}U'y$$

and the estimate for the i^{th} column α_i of α is

$$\hat{\alpha}_i = (U'U)^{-1}U'y_i \text{ for } i = 1,\cdots,(kp+mr+1)$$

where y_i is the i^{th} column of matrix Y. The unbiased estimate of innovation covariance matrix Σ is given by

$$\hat{\Sigma} = \frac{1}{(T-m)}(Y - U\hat{\alpha})'(Y - U\hat{\alpha})$$

and the maximum likelihood estimate of Σ is

$$\tilde{\Sigma} = \frac{(T-r)}{T}\hat{\Sigma}\,.$$

The covariance matrix of the estimated parameter matrix can be estimated as

$$C\hat{o}v(\hat{\alpha}) = \hat{\Sigma}\otimes(U'U)^{-1} \text{ and } C\hat{o}v(\hat{\alpha}_i,\hat{\alpha}_j) = \hat{\sigma}_{ij}\otimes(U'U)^{-1}\,.$$

Time series data on landings of these marine fish species/groups were collected from the "National marine living resources data center" of CMFRI. The environmental time series data were collected from the India Meteorological Department. Since the time series sequences used in this study were all monthly observations, a 12 point moving average of these series were taken before analysis to remove seasonality present in the data.

Results

The relation between the four time series on landings and environmental variables series were initially examined by computing cross correlations up to lag 24 between different series. Cross correlations of the series on elasmobranchs with different environmental time series variables are given in table.4.1. The series on elasmobranchs landings did not show any significant cross correlation with that of the mean maximum temperature. The maximum cross correlation observed was 0.161 at lag 21. Cross correlations of elasmobranchs with highest temperature were significant and negative for lags 3 to 18 and for lags 15 to 24 the cross correlations were positive and significant. The maximum cross correlation observed between these two series was 0.3573 for lag 24. These cross correlations showed a cyclical pattern with low negative values for the initial lags and changed its sign after lag 8. Cross correlations of elasmobranchs with mean minimum temperature were significant and negative for lags from 0 to 12 with –0.5932 as the maximum cross correlation at lag 3. With the lowest temperature sequence elasmobranchs landings was found to have negative and significant cross correlations for all the lags, from 0 to 21. The maximum cross correlation observed between these two series was – 0.5135 at lag 2. Total rainfall series did not have any significant cross correlation with elasmobrachs for most of the initial lags. The significant cross correlations were for lags from 18 to 29 and these were negative with maximum cross correlation being –0.3145 at lag 21. Elasmobranchs and highest rainfall series had significant positive cross correlations for lags 7 to 18 and the maximum cross correlation was 0.3979 at lag 11. With the series on number of rainy days elasmobranchs landings were found to have significant negative cross correlations for all the lags from 0 to 24. The maximum cross correlation observed was –0.3795 at lag 21.

Cross correlations of the series on oil sardine landings with different series on environmental variables are given in table.4.2. Cross correlations of oil sardine series with the series on mean maximum temperature were not significant for most of the lags. The significant cross correlations were for lags 21 to 24 and were all negative. The maximum cross correlation between these two series was –0.2791 for lag 24. With the series on highest temperature oil sardine landings series had significant and positive cross correlations for lags from 4 to 18. The maximum cross correlation between them was 0.3517 at lag 12. Cross correlations of oil sardine series with mean minimum temperature series were significant for lags 0 to 5 and 20 to 24. All these cross correlations were negative and the maximum cross correlation was –0.3513 for lag 0. Cross correlations of oil sardine series with the lowest temperature were negative, significant and high for the lags from 0 to 13 and 23 to 24. The maximum cross correlation observed was -0.5294 for lag 0. For most of the initial lags the series on oil sardine landings did not have any significant cross correlation with the total rainfall series. The significant cross correlations were for the higher lags from 15 to 24 and were all positive and high, with a maximum of 0.5368 at lag 23. With highest rainfall series, the cross correlations of oil sardine landings series were positive, significant and high for all the lags from 0 to 24. The observed maximum of the cross correlations between these two series was 0.3715 at lag 1. The oil sardine landings series had high, negative and significant cross correlations with the series on number of rainy days, for all the lags from 0 to 24, with –0.7730 at lag 11 as the maximum cross correlation.

For different lags from 0 to 24, cross correlations of the series on mackerel landings with different environmental variables series are given in table.4.3. Cross correlations of the series on landings of mackerel with mean maximum temperature were positive and significant for lags 6 to 23 with maximum cross correlation of 0.4957 at lag 16. With the highest temperature series the cross correlations of mackerel series were significant at lag 0 and lags 7 to 18. All the cross correlations from lag 7 to lag 18 were positive and at lag 0 it was negative. Cross correlations of mackerel series with mean minimum temperature series were significant and positive for all the lags from 0 to 21. The highest cross correlation was 0.5017 at lag 5. With the lowest temperature series the cross correlations of mackerel were significant and positive for lags 0 to 19 with 0.5258 as the highest cross correlation at lag 9. For most of the lags mackerel landings did not have any significant cross correlation with the series on total rainfall. The significant cross correlations were for lags 0, 1 and 2, and these cross correlations were negative with –0.2258 as the maximum at lag 0. Cross correlations of the mackerel series with the highest rainfall series were also not significant for most of the lags. The cross correlations significant were for lag 0 and lags 17 to 14 which were all negative and –0.2829 was the maximum at lag 24. With the number of rainy days series cross correlations of mackerel series were positive and high for most of the lags. The significant lags were from 0 to 18 and the maximum cross correlation was 0.6514 at lag 8.

Cross correlations of the series on landings of stolephorus with different series on environmental variables are given in table.4.4. The series on stolephorus landings did not have any significant cross correlation with mean maximum temperature series for lags 0 to 21. For lags 22, 23 and 24 the cross correlations were positive and significant with 0.2598 as the maximum cross correlation at lag 24. With the series on highest temperature the cross correlations of stolephorus landings series were negative and significant for lags 5 to 17. The maximum cross correlation of stolephorus series with the highest temperature series was –0.4378 at lag 13. Cross correlations of stolephorus series with mean minimum temperature were significant and positive for lags 0 to 6 and 18 to 24. The maximum cross correlation observed between the two series was 0.3440 at lag 0. With the lowest temperature series the cross correlations of stolephorus landings series were significant for all the lags from lag 0 to lag 24 except for lags 10 and 11. All these cross correlations were positive and the maximum cross correlation between the two series was 0.3858 at lag 19. Cross correlations of stolephorus series with total rainfall were significant for lags 9 to 15 and all these cross correlations were negative. The maximum cross correlation observed between the two series was –0.2549 at lag 13. With the series on highest rainfall the cross correlation of stolephorus series were significant and negative for lags 0 to 14 and the maximum cross correlation obtained was –0.6141 at lag 6. The cross correlations of stolephorus series with the series on number of rainy days were positive, high and significant for all the lags from 0 to 24. The maximum cross correlation obtained was 0.6506 at lag 7.

Based on the above cross-correlation analysis of these time series sequences, two environmental time series variables, namely mean maximum temperature and total rainfall, were excluded from modelling as their influence was comparatively less on all the four landings series. In the present study, the four time series sequences on landings formed the output vector y_t and two time series sequences one each to represent temperature and rainfall formed the exogenous vector x_t. This resulted in six different models with same set of output vector and different pairs of environmental time series sequences as components for the exogenous vector.

For the first model considered, the components of exogenous vector were, the highest temperature series and the highest rainfall series. To identify a suitable $VARX(p,r)$ type model, order selection criteria AIC, BIC and HQ were evaluated for values of p and r ranging from 1 to 5. The model that yielded minimum AIC criterion was $VARX(3,3)$ and both BIC and HQ criteria had minimum values for the model $VARX(1,1)$. These two models were estimated and compared for final selection. The estimated $VARX(3,3)$ model could explain 98.25%, 97.56%, 94.86% and 93.17% respectively of the variations in the components of vector output series $\{y_t\}$. It was found that out of 48 elements of the estimated AR coefficient matrices, only 13 elements were significant; among 24 elements of the estimated coefficient matrices of exogenous variables only 5 elements were significant and only one element of the estimated constant vector was significant. The estimated $VARX(1,1)$ model was found to explain 97.56%, 96.63%, 94.27% and 91.06% respectively of the variations in the components of vector output series. Out of a total of 24 elements in the estimated parameter matrices of this model, 11 were found significant. Though the $VARX(3,3)$ model was slightly better than the $VARX(1,1)$ model in respect of its capability to explain variations in the components of y_t, it had too many parameters as elements of the coefficient matrices and most of them were non-significant. Hence $VARX(1,1)$ model was chosen as the suitable model for this data set. The variance covariance matrix of the sample output vector time series $\{ y_t \}$ consisting of landings of the four species/group was estimated as

$$\begin{pmatrix} 301.9829 & 899.1459 & -254.3888 & -1411.6567 \\ 899.1459 & 27435.8854 & -5554.9400 & -10480.0057 \\ -254.3888 & -5554.9400 & 2482.9746 & 568.2350 \\ -1411.6567 & -10480.0057 & 568.2350 & 27487.8107 \end{pmatrix}$$

The expression for $VARX(1,1)$ model is $y_t = \delta + \Phi_1 \, y_{t-1} + \beta_0 \, x_t + \varepsilon_t$ and the estimates of parameters of the model and standard errors of the estimates were

$$\hat{\delta} = (-66.5073, 870.5186, -274.2013, 1962.3127)'$$

$$\hat{\mathrm{SE}}(\hat{\delta}) = (39.9554, 447.6489, 175.4848, 729.5187)'$$

$$\hat{\Phi}_1 = \begin{pmatrix} 0.9491 & 0.0019 & 0.0070 & 0.0016 \\ 0.4410 & 0.9498 & -0.0645 & -0.0165 \\ -0.1911 & -0.0251 & 0.9237 & -0.0299 \\ -0.1611 & 0.0179 & 0.1622 & 0.9060 \end{pmatrix}$$

$$\hat{\mathrm{SE}}(\hat{\Phi}_1) = \begin{pmatrix} 0.0187 & 0.0025 & 0.0081 & 0.0021 \\ 0.2098 & 0.0277 & 0.0906 & 0.0231 \\ 0.0822 & 0.0109 & 0.0355 & 0.0091 \\ 0.3419 & 0.0451 & 0.1477 & 0.0377 \end{pmatrix}$$

$$\hat{\beta}_0 = \begin{pmatrix} 1.9336 & 0.0446 \\ -25.7190 & -0.3239 \\ 8.3518 & 0.4992 \\ -55.7698 & -1.8047 \end{pmatrix}, \quad \hat{\mathrm{SE}}(\hat{\beta}_0) = \begin{pmatrix} 1.2132 & 0.0453 \\ 13.5918 & 0.5071 \\ 5.3282 & 0.1988 \\ 22.1502 & 0.8263 \end{pmatrix}$$

$$\hat{\Sigma} = \begin{pmatrix} 7.3700 & 4.4179 & -2.1791 & 23.7713 \\ 4.4179 & 925.1074 & -34.3015 & 261.4268 \\ -2.1791 & -34.3015 & 142.1662 & 87.3555 \\ 23.7731 & 261.4268 & 87.355 & 2456.9138 \end{pmatrix}$$

The elements found significant in matrix $\hat{\Phi}_1$ were $\hat{\Phi}_{11}$, $\hat{\Phi}_{21}$, $\hat{\Phi}_{22}$, $\hat{\Phi}_{31}$, $\hat{\Phi}_{32}$, $\hat{\Phi}_{33}$, $\hat{\Phi}_{34}$ and $\hat{\Phi}_{44}$. In matrix $\hat{\beta}_0$, the elements found significant were $\hat{\beta}_{32}$, $\hat{\beta}_{41}$ and $\hat{\beta}_{42}$. The only significant element in the constant vector $\hat{\delta}$ was $\hat{\delta}_4$.

With another set of exogenous vector variable consisting of the lowest temperature series and the series on number of rainy days, modelling was attempted. For this data set, the *AIC* criterion yielded minimum value for *VARX*(3,5) model where as *BIC* and *HQ* criteria had minimum values for *VARX*(1,1) model. These two models were estimated and compared to arrive at the final model. The percentage of variations in the components of output vector series that were explained by the estimated *VARX*(3,5) model were 98.29%, 97.71%, 95.47% and 93.26% respectively. Out of a total of 88 elements in the eight parameter matrices of this model only 20 elements were found significant. The estimated *VARX*(1,1) model could explain 97.17%, 96.50%, 93.89% and 90.53% of the variations in the components respectively of the output vector. In the estimates of two parameter matrices of the model, 10 out of 24 elements were found significant. Though *VARX*(3,5) model behaved better than *VARX*(1,1) model in its capability to explain variations of the components in the output vector series, it was not advisable as most of its parameter elements were not significant. Hence *VARX*(1,1) model was taken as the final model for this data set. The algebraic expression of the model, estimate of the model parameters and standard errors of the parameter estimates are as given below.

$$y_t = \delta + \Phi_1\, y_{t-1} + \beta_0\, x_t + \varepsilon_t$$

$$\hat{\delta} = (-20.3774, -80.2955, -42.0427, 1169.7475)'$$

$$\hat{\mathrm{SE}}(\hat{\delta}) = (20.3727,\ 323.0121,\ 128.3342,\ 531.5191)'$$

$$\hat{\Phi}_1 = \begin{pmatrix} 0.9534 & -0.0001 & 0.0089 & 0.0044 \\ 0.5460 & 0.9474 & -0.0638 & -0.0096 \\ -0.2136 & -0.0109 & 0.9090 & -0.0405 \\ -0.2680 & -0.0647 & 0.1795 & 0.9400 \end{pmatrix}$$

$$\hat{\mathrm{SE}}(\hat{\Phi}_1) = \begin{pmatrix} 0.0195 & 0.0028 & 0.0077 & 0.0022 \\ 0.2301 & 0.0325 & 0.0913 & 0.0262 \\ 0.0914 & 0.0129 & 0.0363 & 0.0104 \\ 0.3787 & 0.0534 & 0.1502 & 0.0431 \end{pmatrix}$$

$$\hat{\beta}_0 = \begin{pmatrix} 1.4538 & -1.1662 \\ 3.3484 & 0.6139 \\ 2.2053 & 2.2805 \\ -48.7001 & -6.0903 \end{pmatrix}, \quad \hat{\mathrm{SE}}(\hat{\beta}_0) = \begin{pmatrix} 1.2307 & 0.3513 \\ 14.5225 & 4.1459 \\ 5.7699 & 1.6472 \\ 23.8969 & 6.8221 \end{pmatrix}$$

$$\hat{\Sigma} = \begin{pmatrix} 6.9012 & 1.6133 & 0.7538 & 13.4059 \\ 1.6133 & 961.0046 & -53.0893 & 366.8509 \\ 0.7538 & -53.0893 & 151.6954 & 44.6009 \\ 13.4059 & 366.8509 & 44.6009 & 2602.1089 \end{pmatrix}$$

The elements of estimated parameter matrices found significant were $\hat{\delta}_4$ of vector $\hat{\delta}$; $\hat{\Phi}_{11}$, $\hat{\Phi}_{14}$, $\hat{\Phi}_{21}$, $\hat{\Phi}_{22}$, $\hat{\Phi}_{31}$, $\hat{\Phi}_{33}$, $\hat{\Phi}_{34}$ and $\hat{\Phi}_{44}$ in matrix $\hat{\Phi}_1$; $\hat{\beta}_{12}$ and $\hat{\beta}_{41}$ in matrix $\hat{\beta}_0$.

Another model with monthly mean minimum temperature series and the series on the number of rainy days as the two components of the vector exogenous series $\{x_t\}$ and the same output vector series $\{y_t\}$ consisting of series on landings by the four marine fish species/groups as components was also attempted. For this data set the *AIC* criterion had minimum value for *VARX*(3,1) model and both *BIC* and *HQ* criteria had minimum values for *VARX*(1,1) model. These two models were estimated and examined in detail to select the final model. The estimated *VARX*(3,1) model could explain 98.14%, 97.30%, 94.45% and 91.81% respectively of variations in the four components of output vector series. Out of 60 parameters as elements in the four parameter matrices of the model only 14 elements were found significant. When *VARX*(1,1) model was fitted to the data set it explained 97.76%, 96.53%, 93.96% and 90.42% respectively of the variations in the four components of the output vector series. Out of 24 elements belonging to two parameter matrices of the model, 7 were found significant. The *VARX*(1,1) model was selected as the final model because it was parsimonious compared to *VARX*(3,1) model and was capable of explaining the variations reasonably well. The model, estimates of parameters, estimate of standard errors of parameter estimates and estimate of innovation dispersion matrix are given below.

$$y_t = \delta + \Phi_1 \, y_{t-1} + \beta_0 \, x_t + \varepsilon_t$$

$$\hat{\delta} = (-56.7753, 469.2078, -191.1021, 1318.7337)'$$

$$\mathrm{S\hat{E}}(\hat{\delta}) = (36.8410, 436.5616, 173.4347, 726.3920)'$$

$$\hat{\Phi}_1 = \begin{pmatrix} 0.9581 & -0.0016 & 0.0055 & 0.0036 \\ 0.4252 & 0.9491 & -0.0483 & -0.0073 \\ -0.1869 & -0.0140 & 0.9003 & -0.0423 \\ -0.2031 & -0.0220 & 0.2544 & 0.9599 \end{pmatrix}$$

$$\hat{SE}(\hat{\Phi}_1) = \begin{pmatrix} 0.0192 & 0.0026 & 0.0077 & 0.0022 \\ 0.2272 & 0.0310 & 0.0918 & 0.0260 \\ 0.0903 & 0.0123 & 0.0365 & 0.0103 \\ 0.3780 & 0.0516 & 0.1527 & 0.0432 \end{pmatrix}$$

$$\hat{\beta}_0 = \begin{pmatrix} 2.8816 & -1.2260 \\ -20.0571 & 1.6271 \\ 8.3264 & 2.0192 \\ -51.4190 & -6.0321 \end{pmatrix}, \quad \hat{SE}(\hat{\beta}_0) = \begin{pmatrix} 1.5433 & 0.3502 \\ 18.2872 & 4.1496 \\ 7.2653 & 1.6485 \\ 30.4290 & 6.9045 \end{pmatrix}$$

$$\hat{\Sigma} = \begin{pmatrix} 6.7718 & 3.3401 & 0.2575 & 14.3194 \\ 3.3401 & 950.8867 & -48.3813 & 332.8231 \\ 0.2575 & -48.3813 & 150.0754 & 51.3407 \\ 14.3194 & 332.8231 & 51.3407 & 2632.5698 \end{pmatrix}$$

In the estimates of parameter matrices the elements found significant were $\hat{\Phi}_{11}, \hat{\Phi}_{22}, \hat{\Phi}_{31}, \hat{\Phi}_{33}, \hat{\Phi}_{34}$ and $\hat{\Phi}_{44}$ of matrix $\hat{\Phi}_1$ and $\hat{\beta}_{12}$ of matrix $\hat{\beta}_0$.

Using the two time series sequences on mean minimum temperature and highest rainfall series as components of the exogenous vector and the same set of output vector series on landings a fourth model was tried. The model corresponding to minimum *AIC* value was *VARX*(3,2) and both *BIC* and *HQ* criteria had minimum values for *VARX*(1,1) model. A comparison of these two models was made after estimating the model parameters to choose the final model. When the VARX(3,2) model was estimated it was found that 17 out of 68 elements of the estimated parameter matrices were significant. This model could explain 98.09%, 97.38%, 94.90% and 92.55% respectively of the variations in the four component series of the output vector y_t. The *VARX*(1,1) model was also estimated and it was found that out of 28 elements in the parameter matrices of the model 10 were significant. This model could explain 97.53%, 96.55%, 94.26% and 90.87% respectively of the variations in the four components of output vector y_t. Since this model had lesser number of parameters as elements of the parameter matrices and is equally good with regard to its capability to explain variations, the *VARX*(1,1) model was chosen to represent this data set. The algebraic expression of the model, estimates of parameters, standard error of parameter estimates and estimate of innovation dispersion matrix are given below.

$$y_t = \delta + \Phi_1 \, y_{t-1} + \beta_0 \, x_t + \varepsilon_t$$

$$\hat{\delta} = (-48.8707, 486.5004, -251.9278, 1534.6473)'$$

$$\hat{SE}(\hat{\delta}) = (38.6195, 435.2963, 168.9213, 709.0073)'$$

$$\hat{\Phi}_1 = \begin{pmatrix} 0.9525 & 0.0021 & 0.0062 & 0.0006 \\ 0.4257 & 0.9466 & -0.0605 & -0.0048 \\ -0.1654 & -0.0243 & 0.9185 & -0.0348 \\ -0.2760 & 0.0118 & 0.1864 & 0.9359 \end{pmatrix}$$

$$\hat{SE}(\hat{\Phi}_1) = \begin{pmatrix} 0.0200 & 0.0025 & 0.0082 & 0.0021 \\ 0.2257 & 0.0279 & 0.0928 & 0.0236 \\ 0.0876 & 0.0108 & 0.0360 & 0.0092 \\ 0.3676 & 0.0455 & 0.1512 & 0.0385 \end{pmatrix}$$

$$\hat{\beta}_0 = \begin{pmatrix} 1.8927 & 0.0531 \\ -19.0042 & -0.4334 \\ 10.4015 & 0.5372 \\ -57.9377 & -2.0518 \end{pmatrix}, \quad \hat{SE}(\hat{\beta}_0) = \begin{pmatrix} 1.5891 & 0.0453 \\ 17.9113 & 0.5101 \\ 6.9507 & 0.1980 \\ 29.1738 & 0.8309 \end{pmatrix}$$

$$\hat{\Sigma} = \begin{pmatrix} 7.4456 & 3.0941 & -1.9678 & 21.7650 \\ 3.0941 & 945.9179 & -38.8645 & 297.8737 \\ -1.9678 & -38.8645 & 142.4466 & 82.2113 \\ 21.7650 & 289.8737 & 82.2113 & 2509.4863 \end{pmatrix}$$

In these estimates the parameter elements found significant were $\hat{\delta}_4$ in vector $\hat{\delta}$; $\hat{\Phi}_{11}$, $\hat{\Phi}_{22}$, $\hat{\Phi}_{32}$, $\hat{\Phi}_{33}$, $\hat{\Phi}_{34}$ and $\hat{\Phi}_{44}$ in matrix $\hat{\Phi}_1$; $\hat{\beta}_{32}$, $\hat{\beta}_{41}$ and $\hat{\beta}_{42}$ in matrix $\hat{\beta}_0$.

A fifth model was attempted by replacing the vector exogenous series with another pair consisting of lowest temperature series and highest rainfall series. For this data set the *AIC* criterion suggested *VARX*(3,3) model where as both *BIC* an *HQ* criteria suggested *VARX*(1,1) model as the suitable one. To select the final model these two models were estimated and compared. Out of a total of 86 elements belonging to the parameter matrices of the estimated *VARX*(3,3) model 21 elements were found significant. This model explained 98.22%, 97.42%, 94.97% and 90.81% respectively of variations in output vector series components. For the estimated *VARX*(1,1) model these values were 97.51%, 96.52%, 94.15% and 90.82% respectively. Out of a total of 28 elements in the parameter matrices estimated for the *VARX*(1,1) model 10 elements were found significant. There fore *VARX*(1,1) model which had lesser number of parameters and was equally good in terms of variations explained was selected as the suitable model for the data set. The model, estimates of parameter vector and parameter matrices in the model, estimates of standard errors of parameter estimates and estimate of innovation dispersion matrix are given below for the VARX (1,1) model.

$$y_t = \delta + \Phi_1 \, y_{t-1} + \beta_0 \, x_t + \varepsilon_t$$

$$\hat{\delta} = (-15.7106, -106.7808, -17.0096, 1084.6822)'$$

$$\hat{\text{SE}}(\hat{\delta}) = (28.6981, 323.0731, 126.0997, 525.3455)'$$

$$\hat{\Phi}_1 = \begin{pmatrix} 0.9469 & 0.0027 & 0.0082 & 0.0012 \\ 0.5569 & 0.9507 & -0.0759 & -0.0095 \\ -0.2113 & -0.0230 & 0.9283 & -0.0319 \\ -0.2685 & -0.0286 & 0.1183 & 0.9161 \end{pmatrix}$$

$$\hat{\text{SE}}(\hat{\Phi}_1) = \begin{pmatrix} 0.0203 & 0.0027 & 0.0082 & 0.0021 \\ 0.2286 & 0.0307 & 0.0919 & 0.0233 \\ 0.0892 & 0.0120 & 0.0359 & 0.0091 \\ 0.3717 & 0.0499 & 0.1494 & 0.0378 \end{pmatrix}$$

$$\hat{\beta}_0 = \begin{pmatrix} 0.5740 & 0.0482 \\ 6.0319 & -0.4618 \\ 0.7675 & 0.5262 \\ -43.1766 & -1.7360 \end{pmatrix}, \quad \hat{\text{SE}}(\hat{\beta}_0) = \begin{pmatrix} 1.2915 & 0.0463 \\ 14.5398 & 0.5210 \\ 5.6751 & 0.2034 \\ 23.6430 & 0.8472 \end{pmatrix}$$

$$\hat{\Sigma} = \begin{pmatrix} 7.5296 & 1.9674 & -1.4487 & 19.8071 \\ 1.9674 & 954.2572 & -44.4546 & 338.8185 \\ -1.4487 & -44.4546 & 145.3757 & 67.1437 \\ 19.8071 & 338.8185 & 67.1437 & 2523.2135 \end{pmatrix}$$

The elements found significant in these estimates were $\hat{\delta}_4$ in vector $\hat{\delta}$; $\hat{\Phi}_{11}, \hat{\Phi}_{21}, \hat{\Phi}_{22}, \hat{\Phi}_{31}, \hat{\Phi}_{33}, \hat{\Phi}_{34}$ and $\hat{\Phi}_{44}$ in matrix $\hat{\Phi}_1$; $\hat{\beta}_{32}$ and $\hat{\beta}_{42}$ in matrix $\hat{\beta}_0$.

By replacing the exogenous variables with the new set consisting of time series sequences on highest temperature and number of rainy days, yet another *VARX(p,r)* type model was tried. The *AIC* criterion gave minimum value for *VARX*(3,1) model where as both *BIC* and *HQ* criteria had minimum values for *VARX*(1,1) model. The final model was selected by estimating these two models and comparing their properties. The estimated *VARX*(3,1) model could explain 98.13%, 97.33%. 94.50% and 92.06% respectively of the variations in the four components of the vector output series. Out of a total of 60 elements in 4 coefficient matrices and one constant vector only 17 elements were significant. The other model *VARX*(1,1) was also estimated and it was found that there are 12 significant elements out of a total of 28 elements. This model was found to explain 97.76%, 96.62%, 94.05% and 90.77% respectively of variations in the components of the vector output series. When these models were compared it was found that *VARX*(3,1) model could explain variations in the components of output vector series slightly better than *VARX*(1,1) model and the maximum difference in percentage was 1.29% for the fourth component. To achieve such a small improvement it was required to estimate more than double the number of parameters in *VARX*(1,1) model. Hence *VARX*(1.1) was a better compromise and this was taken as the final model for the data set. Details regarding parameter estimates of the model are given below.

$$y_t = \delta + \Phi_1 \, y_{t-1} + \beta_0 \, x_t + \varepsilon_t$$

$$\hat{\delta} = (-62.2340, 874.9945, -301.2467, 2056.4514)'$$

$$\hat{\text{SE}}(\hat{\delta}) = (38.2658, 448.4893, 179.0007, 741.5303)'$$

$$\hat{\Phi}_1 = \begin{pmatrix} 0.9509 & -0.0016 & 0.0072 & 0.0047 \\ 0.4413 & 0.9517 & -0.0557 & -0.0187 \\ -0.1991 & -0.0146 & 0.9041 & -0.0381 \\ -0.1335 & -0.0176 & 0.2321 & 0.9335 \end{pmatrix}$$

$$\hat{SE}(\hat{\Phi}_1) = \begin{pmatrix} 0.0179 & 0.0026 & 0.0076 & 0.0022 \\ 0.2102 & 0.0306 & 0.0894 & 0.0258 \\ 0.0839 & 0.0122 & 0.0357 & 0.0103 \\ 0.3475 & 0.0506 & 0.1479 & 0.0427 \end{pmatrix}$$

$$\hat{\beta}_0 = \begin{pmatrix} 2.2443 & -1.1346 \\ -26.8017 & 1.1548 \\ 9.3537 & 2.2413 \\ -59.5120 & -7.3777 \end{pmatrix}, \quad \hat{SE}(\hat{\beta}_0) = \begin{pmatrix} 1.1567 & 0.3438 \\ 13.5596 & 4.0293 \\ 5.4108 & 1.6082 \\ 22.4149 & 6.6621 \end{pmatrix}$$

$$\hat{\Sigma} = \begin{pmatrix} 6.7549 & 4.6305 & -0.0922 & 16.6624 \\ 4.6305 & 927.8973 & -41.0585 & 285.4104 \\ -0.0922 & -41.0585 & 147.8105 & 66.0920 \\ 16.6624 & 285.4104 & 66.0921 & 2536.6093 \end{pmatrix}$$

The elements found significant in the estimates of parameter vector and parameter matrices of the model were $\hat{\delta}_4$ of vector $\hat{\delta}$; $\hat{\Phi}_{11}, \hat{\Phi}_{14}, \hat{\Phi}_{21}, \hat{\Phi}_{22}, \hat{\Phi}_{31}, \hat{\Phi}_{33}, \hat{\Phi}_{34}$ and $\hat{\Phi}_{44}$ in matrix $\hat{\Phi}_1$; $\hat{\beta}_{12}, \hat{\beta}_{21}$ and $\hat{\beta}_{41}$ in matrix $\hat{\beta}_0$.

Discussion

From the cross correlation analysis it was seen that the mean maximum temperature series did not have much influence on the series on landings of oil sardine, stolephorus and elasmobranchs but had significant effects on mackerel series. The highest temperature series had lagged effects on oil sardine, mackerel and stolephorus series for almost identical lags where as the effects on elasmobranchs series were for different lags. The effects on oil sardine and mackerel series were positive but that on elasmobranchs and stolephorus series were negative. The mean minimum temperature series had more effects on mackerel and elasmobranchs series and for the initial lags it had effects on all the four landings series. For mackerel and stolephorus series the effects were positive and for oil sardine and elasmobranchs series the effects were negative. Lowest temperature series had significant effects on all the four series. The effects were positive for mackerel and stolephorus series and it was negative for oil sardine and elasmobranchs. The series on total rainfall did not have much influence on any of the four series on landings, but had little positive effects on oil sardine series at higher lags. Highest rainfall series had more effects on oil sardine series that was positive and there was effect even at higher lags. Its effects on the series of elasmobranchs landings were also positive but the influence were at higher lags. It had high and negative effects on stolephorus series at many lags, and the influence on mackerel landings was comparatively less. Its effects on mackerel series were negative and these were for higher lags. The number of rainy days series had significant influence on all the four landings series at almost all lags. Its effects on oil sardine and elasmobranchs series were negative where as its effects on mackerael and stolephorus landings were positive. The strength of influence was high for oil sardine, mackerel and stolephorus series.

In the first VARX model fitted with time series on highest temperature and highest rainfall as components of the exogenous vector, models for the third and fourth components of the output vector were having significant coefficients for the exogenous variables. These models are

$$y_{3t} = -274.2013 - 0.1911\, y_{1,t-1} - 0.0251\, y_{2,t-1} + 0.9237\, y_{3,t-1} - 0.0299\, y_{4,t-1}$$
$$+ 8.3518\, x_{1t} + 0.4992\, x_{2t} + \varepsilon_{3t}$$

and
$$y_{4t} = 1962.3127 - 0.1611\, y_{1,t-1} + 0.0179\, y_{2,t-1} + 0.1622\, y_{3,t-1} + 0.9060\, y_{4,t-1}$$
$$- 55.7698 x_{1t} - 1.8047 x_{2t} + \varepsilon_{4t}, \text{ respectively.}$$

These models correspond to the series on landings of stolephorus and mackerel respectively. All the coefficients of the model of stolephorus except that for x_{1t} were significant. Thus, apart from its dependence on lagged values of other three landings series the stolephorus series depends on the highest rainfall series also. An increase in highest rainfall is expected to cause increased landings of stolephorus. In the model for the series on landings of mackerel both the coefficients of exogenous components and the coefficient of $y_{4,t-1}$ were significant. So apart from its dependence on its own past the series on landings of mackerel depend on both highest temperature and highest rainfall series. Increase in the values of these variables are not favourable for good landings of mackerel.

In the second *VARX* model with the series on lowest temperature and number of rainy days as the components of exogenous vector, the individual univariate models with significant coefficients for exogenous vector components are

$$y_{1t} = 20.3727 + 0.9534\, y_{1,t-1} - 0.0001\, y_{2,t-1} + 0.0089\, y_{3,t-1} + 0.0044\, y_{4,t-1}$$
$$+ 1.4538\, x_{1t} - 1.1662\, x_{2t} + \varepsilon_{1t}$$

and
$$y_{4t} = 1169.7475 - 0.2680\, y_{1,t-1} - 0.0647\, y_{2,t-1} + 0.1795\, y_{3,t-1} + 0.9400\, y_{4,t-1}$$
$$- 48.7001\, x_{1t} - 6.0903\, x_{2t} + \varepsilon_{4t}.$$

In the above models, the first model is for the series on landings of elasmobranchs and significant coefficients in this model were for $y_{1,t-1}$, $y_{4,t-1}$ and x_{2t}. Therefore the series on elasmobranchs landings depend on its own past and lagged values of the series on landings of mackerel. Also, the series on number of rainy days had significant effect on the series on elasmobranchs landings and the effect is negative. According to the second model, which corresponds to the series on landings of mackerel, the significant coefficients were for $y_{4,t-1}$ and x_{1t}. Hence, the series on landings of mackerel depend on its own past and also on the series on lowest temperature. Increase in the values lowest temperature series is not favourable for increased landings of mackerel.

In the *VARX* model fitted with the third set consisting of series on mean minimum temperature and number of rainy days as components for the exogenous vector, the only model having significant relation with the exogenous vector variables was

$$y_{1t} = -56.7753 + 0.9581\, y_{1,t-1} - 0.0016\, y_{2,t-1} + 0.0055\, y_{3,t-1} + 0.0036\, y_{4,t-1}$$
$$+ 2.8816\, x_{1t} - 1.2260\, x_{2t} + \varepsilon_{1t}$$

In this model the significant coefficients were for $y_{1,t-1}$ and x_{2t}. Hence, the series on landings of elasmobranchs depend on its own past and also on the series on number of rainy days. This was established by the earlier model also.

From the VARX model fitted with series on mean minimum temperature and highest rainfall as components for the exogenous vector variable we get models with significant effect coefficients for the environmental vector components as

$$y_{3t} = -251.9278 - 0.1654\, y_{1,t-1} - 0.0243\, y_{2,t-1} + 0.9185\, y_{3,t-1} - 0.0348\, y_{4,t-1}$$
$$+ 10.4015\, x_{1t} + 0.5372\, x_{2t} + \varepsilon_{3t}$$

$$y_{4t} = 1534.6473 - 0.2760\, y_{1,t-1} + 0.0118\, y_{2,t-1} + 0.1864\, y_{3,t-1} + 0.9359\, y_{4,t-1}$$
$$- 57.9377\, x_{1t} - 2.0518\, x_{2t} + \varepsilon_{4t}$$

In the first model for the series on stolephorus landings the coefficients of $y_{2,t-1}$, $y_{3,t-1}$, $y_{4,t-1}$ and x_{2t} were significant. Hence the series on landings of stolephorus significantly depend on its own past and lagged values of the series on landings of oil sardine and mackerel. Also, the series on highest rainfall had significant and positive effect on the series on landings of stolephorus. In the second model above, for the series on mackerel landings the coefficients significant were $y_{4,t-1}$, x_{1t} and x_{2t}. Thus the series on landings of mackerel are autocorrelated with lag 1 and both the mean minimum temperature series and highest rainfall series had significant and negative effect on the landings of mackerel.

In the *VARX* model fitted with lowest temperature and highest rainfall as the component variables for the exogenous vector, the following models were obtained with significant coefficients for these environmental components.

$$y_{3t} = -17.0096 - 0.2113\, y_{1,t-1} - 0.0230\, y_{2,t-1} + 0.9283\, y_{3,t-1} - 0.0319\, y_{4,t-1}$$
$$+ 0.7675\, x_{1t} + 0.5262\, x_{2t} + \varepsilon_{3t}$$
$$y_{4t} = 1084.6822 - 0.2685\, y_{1,t-1} - 0.0286\, y_{2,t-1} + 0.1183\, y_{3,t-1} + 0.9161\, y_{4,t-1}$$
$$- 43.1766\, x_{1t} - 1.7360\, x_{2t} + \varepsilon_{4t}$$

In the first model above for the stolephorus landings series the coefficients significant were for $y_{1,t-1}$, $y_{3,t-1}$, $y_{4,t-1}$ and x_{2t}. In the second model for the series on mackerel landings, the significant coefficients were $y_{4,t-1}$ and x_{2t}. This indicate that both the series on landings of stolephorus and mackerel depend on their own past values. Both these series are effected by the series on highest rainfall but in different directions. Increased values for the highest rainfall series is expected to increase the landings of stolephorus and reduce the landings of mackerel. Stolephorus landings series was also affected by the series on landings of elasmobranchs and mackerel.

In the *VARX* model fitted with the exogenous vector consisting of series on highest temperature and number of rainy days, the models with significant values for the coefficients of the exogenous vector components are

$$y_{1t} = -62.2340 + 0.9509\, y_{1,t-1} - 0.0016\, y_{2,t-1} + 0.0072\, y_{3,t-1} + 0.0047\, y_{4,t-1} + 2.2443\, x_{1t} - 1.1346\, x_{2t} + \varepsilon_{1t}$$

$$y_{2t} = 874.9945 + 0.4413\, y_{1,t-1} + 0.9517\, y_{2,t-1} - 0.0557\, y_{3,t-1} - 0.0187\, y_{4,t-1} - 26.8017\, x_{1t} + 1.1548\, x_{2t} + \varepsilon_{2t}$$

$$y_{4t} = 2056.4514 - 0.1335\, y_{1,t-1} - 0.0176\, y_{2,t-1} + 0.2321\, y_{3,t-1} + 0.9335\, y_{4,t-1} - 59.5120\, x_{1t} - 7.3777\, x_{2t} + \varepsilon_{4t}$$

In the model for the series on elasmobranchs landings the significant coefficients were for $y_{1,t-1}$, $y_{4,t-1}$ and x_{2t}. Those significant in the model for oil sardine landings series were for $y_{1,t-1}$, $y_{2,t-1}$ and x_{1t}. For the model on landings of mackerel the coefficients found significant were for $y_{4,t-1}$ and x_{1t}. There fore all the three series on landings of elasmobranchs, oil sardine and mackerel depend on own past values at lag 1. The series on mackerel influence the series on landings of mackerel and the landings of oil sardine was influenced by elasmobranchs landings series. Both the series on landings of oil sardine and mackerel series depend on the highest temperature series where as the series on landings of elasmobranchs depends on the number of rainy days series. For all the three landings series their dependence on the two environmental variables are not positive.

Appendix-IV (Tables)

Table.4.1. Cross correlations of the series on landings of elasmobranchs with different series on environmental variables.

Lag	Mean Maximum Temperature	Highest Temperature	Mean Minimum Temperature	Lowest Temperature	Total Rainfall	Highest Rainfall	Number of Rainy days
0	0.1151	-0.0614	-0.5284	-0.4823	0.1894	0.1374	-0.3786
1	0.0613	-0.1216	-0.5636	-0.4992	0.1725	0.1413	-0.3508
2	0.0228	-0.1800	-0.5923	-0.5135	0.1404	0.1366	-0.3341
3	-0.0014	-0.2216	-0.5932	-0.5108	0.1000	0.1227	-0.3263
4	-0.0255	-0.2541	-0.5819	-0.5041	0.0633	0.1240	-0.3128
5	-0.0449	-0.2583	-0.5675	-0.4945	0.0380	0.1555	-0.2902
6	-0.0538	-0.2572	-0.5367	-0.4789	0.0066	0.1809	-0.2699
7	-0.0475	-0.2510	-0.4859	-0.4580	-0.0293	0.2257	-0.2670
8	-0.0336	-0.2222	-0.4346	-0.4447	-0.0526	0.2641	-0.2551
9	-0.0147	-0.1765	-0.3768	-0.4327	-0.0601	0.3230	-0.2441
10	0.0079	-0.1266	-0.3249	-0.4441	-0.0266	0.3914	-0.2221
11	0.0357	-0.0689	-0.2752	-0.4554	-0.0186	0.3979	-0.2172
12	0.0537	-0.0148	-0.2274	-0.4557	-0.0198	0.3879	-0.2193
13	0.0709	0.0456	-0.1860	-0.4615	-0.0263	0.3855	-0.2215
14	0.0928	0.1193	-0.1428	-0.4390	-0.0357	0.3756	-0.2235
15	0.1054	0.1616	-0.1112	-0.4253	-0.0587	0.3545	-0.2290
16	0.1229	0.2094	-0.0829	-0.4252	-0.0994	0.3190	-0.2437
17	0.1336	0.2468	-0.0680	-0.4121	-0.1334	0.2759	-0.2552
18	0.1480	0.2848	-0.0495	-0.3978	-0.1936	0.2062	-0.2897
19	0.1577	0.3055	-0.0359	-0.3682	-0.2288	0.1209	-0.3099
20	0.1591	0.3207	-0.0034	-0.3149	-0.2835	0.0282	-0.3532
21	0.1606	0.3332	0.0385	-0.2489	-0.3145	-0.0552	-0.3795
22	0.1509	0.3420	0.0720	-0.1716	-0.3144	-0.1228	-0.3786
23	0.1341	0.3487	0.1068	-0.0992	-0.3070	-0.1424	-0.3763
24	0.1345	0.3573	0.1411	-0.0442	-0.2856	-0.1624	-0.3788

Table.4.2. Cross correlations of the series on landings of oil sardine with different series on environmental variables.

Lag	Mean Maximum Temperature	Highest Temperature	Mean Minimum Temperature	Lowest Temperature	Total Rainfall	Highest Rainfall	Number of Rainy days
0	0.0058	0.0312	-0.3513	-0.5294	-0.1871	0.3569	-0.6627
1	0.0179	0.0844	-0.3181	-0.5312	-0.1641	0.3715	-0.6760
2	0.0281	0.1389	-0.2794	-0.5231	-0.1429	0.3683	-0.6873
3	0.0273	0.1888	-0.2471	-0.5100	-0.1201	0.3709	-0.7021
4	0.0204	0.2267	-0.2277	-0.5036	-0.1022	0.3541	-0.7075
5	0.0177	0.2633	-0.2058	-0.4863	-0.1054	0.3357	-0.7245
6	0.0153	0.2895	-0.1914	-0.4695	-0.0994	0.3224	-0.7411
7	0.0122	0.3156	-0.1847	-0.4629	-0.0768	0.3072	-0.7478
8	0.0040	0.3251	-0.1845	-0.4484	-0.0511	0.2951	-0.7501
9	0.0041	0.3386	-0.1765	-0.4278	-0.0438	0.2592	-0.7637
10	-0.0024	0.3412	-0.1620	-0.3742	-0.0163	0.2340	-0.7704
11	-0.0175	0.3361	-0.1473	-0.3140	0.0144	0.2192	-0.7730
12	-0.0255	0.3517	-0.1306	-0.2622	0.0603	0.2275	-0.7659
13	-0.0403	0.3436	-0.1223	-0.2064	0.1211	0.2379	-0.7476
14	-0.0598	0.3252	-0.1202	-0.1775	0.1648	0.2389	-0.7341
15	-0.0810	0.3026	-0.1330	-0.1614	0.2168	0.2357	-0.7129
16	-0.0916	0.2831	-0.1399	-0.1462	0.2402	0.2202	-0.7087
17	-0.1066	0.2502	-0.1546	-0.1474	0.2992	0.2235	-0.6985
18	-0.1273	0.2183	-0.1686	-0.1477	0.3684	0.2392	-0.6821
19	-0.1537	0.1896	-0.1861	-0.1451	0.4229	0.2655	-0.6629
20	-0.1702	0.1670	-0.2036	-0.1469	0.4740	0.2903	-0.6358
21	-0.2016	0.1386	-0.2305	-0.1570	0.5332	0.3286	-0.6020
22	-0.2226	0.1085	-0.2595	-0.1807	0.5349	0.3163	-0.5832
23	-0.2463	0.0785	-0.2882	-0.2129	0.5368	0.2995	-0.5632
24	-0.2791	0.0274	-0.3235	-0.2377	0.5348	0.2810	-0.5453

Table.4.3. Cross correlations of the series on landings of mackerel with different series on environmental variables.

Lag	Mean Maximum Temperature	Highest Temperature	Mean Minimum Temperature	Lowest Temperature	Total Rainfall	Highest Rainfall	Number of Rainy days
0	-0.1834	-0.2092	0.3674	0.2720	-0.2258	-0.2407	0.5601
1	-0.1046	-0.1306	0.4155	0.3136	-0.1955	-0.1733	0.5911
2	-0.0158	-0.0489	0.4614	0.3554	-0.1934	-0.1503	0.6104
3	0.0460	0.0109	0.4889	0.3935	-0.1865	-0.1103	0.6270
4	0.0991	0.0560	0.4898	0.4043	-0.1731	-0.0581	0.6405
5	0.1554	0.1079	0.5017	0.4408	-0.1619	-0.0257	0.6381
6	0.2048	0.1477	0.5008	0.4645	-0.1497	-0.0083	0.6310
7	0.2547	0.2096	0.4982	0.4822	-0.1291	0.0007	0.6343
8	0.2960	0.2515	0.4785	0.5094	-0.0921	0.0352	0.6514
9	0.3576	0.2878	0.4690	0.5258	-0.1425	-0.0150	0.6178
10	0.3910	0.2843	0.4466	0.5147	-0.1521	-0.0453	0.5751
11	0.4134	0.2705	0.4284	0.5007	-0.1619	-0.0771	0.5167
12	0.4280	0.2583	0.3983	0.4659	-0.1616	-0.1123	0.4661
13	0.4544	0.2573	0.3862	0.4625	-0.1490	-0.1312	0.4305
14	0.4623	0.2401	0.3680	0.4373	-0.1179	-0.1200	0.3922
15	0.4891	0.2488	0.3559	0.4072	-0.1016	-0.1454	0.3418
16	0.4957	0.2458	0.3458	0.3694	-0.0891	-0.1781	0.2941
17	0.4946	0.2344	0.3241	0.3129	-0.0727	-0.1924	0.2556
18	0.4677	0.1962	0.3006	0.2675	-0.0480	-0.1958	0.2254
19	0.4336	0.1555	0.2719	0.2170	-0.0567	-0.2163	0.1798
20	0.3944	0.1098	0.2431	0.1568	-0.0735	-0.2507	0.1318
21	0.3310	0.0457	0.1953	0.0991	-0.0529	-0.2432	0.1136
22	0.2788	-0.0196	0.1540	0.0609	-0.1027	-0.2669	0.0969
23	0.2211	-0.0931	0.1060	0.0190	-0.1466	-0.2789	0.0892
24	0.1567	-0.1721	0.0596	-0.0158	-0.1996	-0.2829	0.0670

Table.4.4. Cross correlations of the series on landings of stolephorus with different series on environmental variables.

Lag	Mean Maximum Temperature	Highest Temperature	Mean Minimum Temperature	Lowest Temperature	Total Rainfall	Highest Rainfall	Number of Rainy days
0	0.1329	0.1075	0.3440	0.3790	0.1018	-0.2853	0.4028
1	0.1189	0.0500	0.3306	0.3629	0.0740	-0.3555	0.4136
2	0.1017	-0.0064	0.3082	0.3408	0.0335	-0.4352	0.4141
3	0.0809	-0.0703	0.2773	0.3031	-0.0198	-0.5034	0.4198
4	0.0531	-0.1436	0.2450	0.2627	-0.0516	-0.5451	0.4322
5	0.0281	-0.2027	0.2148	0.2245	-0.0846	-0.5870	0.4389
6	-0.0048	-0.2579	0.1984	0.2000	-0.1197	-0.6141	0.4548
7	-0.0392	-0.3043	0.1890	0.1917	-0.1444	-0.6040	0.4793
8	-0.0651	-0.3311	0.1783	0.1976	-0.1761	-0.5971	0.5058
9	-0.0926	-0.3639	0.1642	0.1937	-0.2027	-0.5644	0.5343
10	-0.1109	-0.3836	0.1530	0.1885	-0.2219	-0.5285	0.5590
11	-0.1352	-0.4119	0.1335	0.1794	-0.2239	-0.4799	0.5837
12	-0.1476	-0.4308	0.1241	0.1938	-0.2406	-0.4364	0.5980
13	-0.1510	-0.4378	0.1216	0.2194	-0.2549	-0.3747	0.6115
14	-0.1432	-0.4267	0.1258	0.2473	-0.2298	-0.2768	0.6321
15	-0.1153	-0.3859	0.1441	0.2899	-0.1998	-0.1856	0.6373
16	-0.0749	-0.3213	0.1601	0.3226	-0.1807	-0.1154	0.6412
17	-0.0332	-0.2648	0.1711	0.3498	-0.1549	-0.0393	0.6506
18	0.0306	-0.1881	0.1930	0.3840	-0.1447	0.0119	0.6432
19	0.0820	-0.1337	0.2057	0.3858	-0.1438	0.0370	0.6226
20	0.1212	-0.0898	0.2086	0.3546	-0.1511	0.0444	0.5841
21	0.1616	-0.0446	0.2159	0.3371	-0.1608	0.0314	0.5429
22	0.1945	-0.0078	0.2188	0.3124	-0.1781	0.0075	0.5060
23	0.2292	0.0390	0.2283	0.2955	-0.1996	-0.0127	0.4761
24	0.2598	0.0771	0.2322	0.2618	-0.2160	-0.0281	0.4465

CHAPTER-5

TIME SERIES RELATIONS THROUGH CANONICAL PATH ANALYSIS AND TIME SERIES OF MOVING SUMS.

5.1. Time series relations through canonical path analysis

Introduction: Canonical correlation analysis is a well known multivariate statistical technique to form a set of paired random variables $(u_1, v_1), (u_2, v_2), \cdots, (u_p, v_p)$ which are linear combinations of elements of two given sets of random variables $X_1 = (x_{11}, \cdots, x_{1p})'$ and $X_2 = (x_{21}, \cdots, x_{2q})'$ with $p < q$ where $u_i = a_i' X_1$, $v_i = b_i' X_2$ for $i = 1, \cdots, p$ with the following properties.

i. $Cov(u_i, u_j) = \begin{cases} 1 & for\ i = j \\ 0 & for\ i \neq j \end{cases}$ and $Cov(v_i, v_j) = \begin{cases} 1 & for\ i = j \\ 0 & for\ i \neq j \end{cases}$

ii. $\rho_i = Corr(u_i, v_i) \geq 0$ and u_i and v_i are uncorrelated with u_j and v_j for $i \neq j$ and these correlations are in the order $\rho_1 \geq \rho_2 \geq \cdots \geq \rho_p$.

The quantities $\rho_1, \rho_2, \cdots, \rho_p$ are called the canonical correlations between the two sets of random variables X_1 and X_2, and the pairs of random variables (u_1, v_1), (u_2, v_2), $\cdots$, (u_p, v_p) are known as the canonical variables.

Path coefficient analysis is another popular technique in multivariate statistical analysis in which the objective is to decompose the correlations of a random variable y with another set of mutually correlated random variables $x_1, x_2, \cdots, x_p$ into direct effect of each variable on y and indirect effects of each variable through all other variables. In the present study the concepts of canonical correlations and path coefficient analysis are used to develop a strategy for the examination of inter-relations between time series sequences.

Review of literature

Hannan (1955) obtained an exact test for correlation between two time series x_t and y_t with y_t Markovian. The test statistic developed was the partial correlation between x_{2t} and y_{2t} when the effect of $(y_{2t-1}+y_{2t+1})$, x_{2t-1} and x_{2t+1} have been removed. Whittle (1963) generalized the recursive method of fitting autoregressive schemes proposed by Durbin (1960) for multivariate autoregressions and gave the approximate canonical factorization of the spectral density matrix. Gower (1966) developed a Q-technique for evaluation of canonical variables that have computational and statistical advantages over the usual R-technique. Lancaster (1966) provided an alternative proof for the derivation of canonical correlations for multivariate normal distributions and proved in bivariate case that the marginal variables have greater correlation than any other functions of the marginal variables. Chen (1971) have shown that the addition of extra variates to either set of variables can never decrease canonical correlations between two sets of variables. Kettenring (1971) considered five extensions of the classical two set theory of canonical correlation analysis to three or more sets and developed procedures for finding canonical variables associated with different approaches. Tiao and Wei (1976) considered the effect of temporal aggregation on the dynamic relationships between two discrete time series variables and found that it can lead to substantial loss in parameter estimation while the loss in prediction efficiency is less severe. Box and Tiao (1977) proposed a canonical transformation of a k dimensional stationary autoregressive process, and then ordered the components of the transformed process from least to most predictable.

Geweke (1981) derived approximate slopes of several tests of independence of two covariance stationary time series and compared these tests. He has shown that the approximate slopes of regression tests are at least as great as those based on residuals of ARIMA models. Campbell (1982) developed a robust M-estimation for canonical variate analysis based on a functional relationship model. Jewell and Bloomfield (1983) investigated the canonical correlation and canonical components of the past and future of a stationary Gaussian time series. Darroch and Mosimamn (1984) defined canonical and principal components of shape from log shape vectors and related these components to corresponding log measurements components and residual log size. They applied these results to three species of iris and red winged black birds in Florida and found that they differ strongly in shape as well as size. Geweke (1984) defined measures of linear dependence and feed back for two multiple time series conditional on a third. The measure of conditional linear dependence is the sum of linear feed back from the first to the second, conditional on the third and instantaneous linear feed back between the first and second series conditional on the third. Tsay and Tiao (1985) proposed a canonical correlation approach for tentative order determination for ARMA model building which is based on the consistency properties of certain canonical correlations.

Koch and Yang (1986) developed an asymptotic test of independence of two time series that incorporates a possible pattern in successive cross correlation coefficients.

Hannan and Poskit (1988) considered full rank VARMA processes and the situation where there are linear functions of the future and past having unit canonical correlations. They showed that the number of unit canonical correlations between future and past is the number of zeroes of the determinant of the transfer function from innovations to outputs that lie on the unit circle. Degerine (1990) suggested a definition of partial autocorrelation function for multivariate stationary time series based on the canonical analysis of forward and backward innovations.

Materials and methods:

Consider two sets of random variables $X_1 = (x_{11}, \cdots, x_{1p})'$ and $X_2 = (x_{21}, \cdots, x_{2q})'$ with $p < q$ and respective covariance matrices $D(X_1) = \Sigma_{11}$, $D(X_2) = \Sigma_{22}$ and $Cov(X_1, X_2) = \Sigma_{12}$. Then the canonical correlation analysis between these two sets is to find new sets of random variables as linear combinations $a_1'X_1, a_2'X_1, \cdots, a_p'X_1$, and $b_1'X_2, b_2'X_2, \cdots, b_p'X_2$ which are called canonical variables and the respective canonical correlations $\rho_1, \rho_2, \cdots, \rho_p$ are defined as $\rho_i = Corr(a_i'X_1, b_i'X_2) \quad for\, i = 1, \cdots, p$ so that $\rho_1 \geq \rho_2 \geq \cdots \geq \rho_p \geq 0$. These are obtained from the eigen structure solution of the two matrices $\Sigma_{11}^{-1}\,\Sigma_{12}\,\Sigma_{22}^{-1}\,\Sigma_{21}$ and $\Sigma_{22}^{-1}\,\Sigma_{21}\,\Sigma_{11}^{-1}\,\Sigma_{12}$. Non-zero eigen values of these two matrices will be identical and the number of non-zero eigen values will be equal to the rank of Σ_{12}. The eigen values of these matrices in descending order of magnitude will provide the squared canonical correlations $\rho_1^2, \rho_2^2, \cdots, \rho_p^2$. The eigen vectors of $\Sigma_{11}^{-1}\,\Sigma_{12}\,\Sigma_{22}^{-1}\,\Sigma_{21}$ corresponding to these eigen values will give the first set of linearisation vectors $a_1, a_2, \cdots, a_p$ and the eigen vectors of $\Sigma_{22}^{-1} \Sigma_{21} \Sigma_{11}^{-1} \Sigma_{12}$ corresponding to the same eigen values will be the required second set of vectors $b_1, b_2, \cdots, b_p$.

For testing the significance of canonical correlations a test procedure based on Bartlett's lambda is used. To test the hypothesis that X_1 is not related to X_2, an approximate χ^2 with pq degrees of freedom is defined as

$$\chi^2 = -[n - \frac{(p+q+1)}{2}] \ln(\Lambda)$$

where $\Lambda = \prod_{i=1}^{p} (1-\rho_i^2)$, $n = T-1$ and T is the sample size.

Once this null hypothesis is rejected, the contributions of the first r canonical correlations are tested by another χ^2 defined by

$$\chi'^2 = -[n - \frac{(p+q+1)}{2}] \ln(\Lambda')$$

where $\Lambda' = \prod_{i=r+1}^{p} (1-\rho_i^2)$ and this χ'^2 will have $(p-r)(q-r)$ degrees of freedom.

Consider a set of standardized random variables $y, x_1, x_2, \cdots, x_k$ that are correlated to each other. Since the variables are standardized we have $E(y) = E(x_i) = 0$ and $V(y) = V(x_i) = 1 \quad for\ i = 1, \cdots, k$. Let $Cov(y, x_i) = Corr(y, x_i) = r_{0i}$ and $Cov(x_i, x_j) = Corr(x_i, x_j) = r_{ij}$. Consider the linear relation of y on $x_1, x_2, \cdots, x_k$ as

$$y = \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \varepsilon$$

where ε's are independently and identically distributed random variables, uncorrelated with $x_1, x_2, \cdots, x_k$. Taking the covariance of y with each of $x_1, x_2, \cdots, x_k$ we get the set of linear equations $r_{oi}=\beta_1 r_{i1}+\beta_2 r_{i2}+\cdots+\beta_k r_{ik}$ for different values of $i=1,\cdots,k$ and collectively this can be written in matrix form as

$$\begin{pmatrix} r_{01} \\ r_{02} \\ \vdots \\ r_{0k} \end{pmatrix} = \begin{pmatrix} 1 & r_{12} & \cdots & r_{1k} \\ r_{21} & 1 & \cdots & r_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ r_{k1} & r_{k2} & \cdots & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{pmatrix}$$

We can then solve for the vector $\beta=(\beta_1,\beta_2,\cdots,\beta_k)'$ as $\beta=C^{-1}r$ where C is a $k\times k$ matrix with r_{ij} as its $(i,j)^{th}$ element and $r=(r_{01},r_{02},\cdots,r_{0k})'$. Now the correlation between y and x_i is obtained as the linear sum

$$r_{oi}=\beta_i r_{i1}+\beta_2 r_{i2}+\cdots+\beta_{i-1}r_{i,i-1}+\beta_i+\beta_{i+1}r_{i,i+1}+\cdots+\beta_k r_{ik} \text{ for } i=1,\cdots,k.$$

That is the correlation between y and x_i is composed of the direct effect of x_i on y component β_i and the effect of other *(k-1)* variables through x_i components $\beta_1 r_{i1}$, $\beta_2 r_{i2},\cdots$, $\beta_{i-1}r_{i,i-1},\beta_{i+1}r_{i,i+1},\cdots,\beta_k r_{ik}$ which are called indirect effects of the *(k-1)* components on y through x_i. Such a decomposition is available for all $i=1,\cdots,k$. The variation in y explained by these variables in terms of direct and indirect effects is given by

$$R^2=\sum_{i=1}^{k}\sum_{j=1}^{k}\beta_i\beta_j r_{ij} \quad =\beta' C \beta.$$

Results:

Consider the canonical correlations setup in a time series context with the first set consisting of only one variable $\{y_t\}$, a univariate time series sequence, and the second set a vector time series sequence $\{X_t\}$ with k components. We assume that both $\{y_t\}$ and

$\{X_t\}$ are second order stationary with $E(y_t)=0$, $E(X_t)=0$, $V(y_t)=\sigma^2$, $D(X_t)=\Sigma$, $E(X_t\, X_{t-m})=\Gamma(m)$ and $Cov(y_t, X_{t-m})=\sigma_{(m)}=(\sigma_{1(m)},\cdots,\sigma_{k(m)})'$ for $m=0,1,\cdots,p$. The variance in partitioned form for y_t and X_{t-m} can be written as

$$V(y_t : X_{t-m})=\begin{pmatrix} \sigma^2 & \sigma'_{(m)} \\ \sigma_{(m)} & \Sigma \end{pmatrix}.$$

We are interested to study the influence of X_t at different lags $0,1,\cdots,p$ on y_t as the maximum correlation possible between y_t and a linear combination of elements of X_{t-m} for $m=0,1,\cdots,p$. Since the rank of $\sigma_{(m)}$ is unity, there will be a unique linear combination corresponding to the only one non-zero canonical correlation between y_t and X_{t-m}. Since the linear combinations of y_t and X_{t-m} should have unit variances, the required combination for y_t will be $z_t = y_t/\sigma$. The linear combination $M'X_{t-m}$ will have maximum correlation, say $\rho_{(m)}$, with z_t when M is obtained as the eigen vector corresponding to the non-zero eigen value $\rho^2_{(m)}$ of the matrix $\Sigma^{-1}\sigma_{(m)}(\sigma^2)^{-1}\sigma'_{(m)} = \frac{1}{\sigma^2}\Sigma^{-1}\Delta_{(m)}$ where $\Delta_{(m)}=\sigma_{(m)}\sigma'_{(m)}$ is a $k\times k$ matrix. By considering the equation for the linear combination for y_t, which is

$$[(\sigma^2)^{-1}\sigma'_{(m)}\,\Sigma^{-1}\,\sigma_{(m)} - \rho^2_{(m)}I]=0$$

the expression for the maximum correlation $\rho_{(m)}$ between z_t and $M'X_{t-m}$ can be obtained as

$$\rho^2_{(m)}=\frac{\sigma'_{(m)}\,\Sigma^{-1}\,\sigma_{(m)}}{\sigma^2}.$$

The test based on Bartlett's lambda with k degrees of freedom given by

$$\chi^2 = -[n - \frac{(k+2)}{2}]\ln(1-\rho^2_{(m)})$$

is used to test the null hypothesis $H_0 : \rho_{(m)} = 0$ for different lags $m = 0, 1, \cdots, p$.

Let us denote the eigen vector corresponding to the eigen value $\rho^2_{(m)}$ obtained using the matrix $\frac{1}{\sigma^2}\Sigma^{-1}\Delta_{(m)}$ by $\beta_{(m)}$. For different lags of X_t we can have such linear combinations as $\beta'_{(0)}X_t, \beta'_{(1)}X_{t-1}, \cdots, \beta'_{(p)}X_{t-p}$ with corresponding canonical correlations $\rho_{(0)}, \rho_{(1)}, \cdots, \rho_{(p)}$ with z_t. Let us denote these linear correlations as $u_t = \beta'_{(0)}X_t,\ u_{t-1} = \beta'_{(1)}X_{t-1}, \cdots,\ u_{t-p} = \beta'_{(p)}X_{t-p}$. The covariance structure of these liner combinations can be obtained as follows.

$$\begin{aligned}
Cov(u_{t-i}, u_{t-j}) &= Cov(\beta'_{(i)}\, X_{t-i}, \beta'_{(j)}\, X_{t-j}) \\
&= E(\beta'_{(i)}\, X_{t-i}\ X'_{t-j}\beta_{(j)}) \\
&= \beta'_{(i)}\, E(X_{t-i}\ X'_{t-j})\beta_{(j)} \\
&= \beta'_{(i)}\, \Gamma(i-j)\beta_{(j)}
\end{aligned}$$

where $\Gamma(l)$ is the cross covariance matrix of X_t for lag l.

Then we can have a regression relation

$$z_t = \alpha_0 u_t + \alpha_1 u_{t-1} + \cdots + \alpha_p u_{t-p} + \varepsilon_t$$

where ε_t's are independently and identically distributed random variables with zero mean and constant variance σ^2_e and they are uncorrelated with $\{X_t\}$. We can then get the relations

$$\begin{array}{llllll}
\rho_{(0)} = & \alpha_0 & + \ \alpha_1 Cov(u_t, u_{t-1}) & + \cdots + & \alpha_p Cov(u_t, u_{t-p}) \\
\rho_{(1)} = & \alpha_0 Cov(u_t, u_{t-1}) + & \alpha_1 & + \cdots + & \alpha_p Cov(u_{t-1}, u_{t-p}) \\
\vdots = & \vdots & \vdots & \vdots & \vdots \\
\rho_{(p)} = & \alpha_0 Cov(u_t, u_{t-p}) + & \alpha_1 Cov(u_{t-1}, u_{t-p}) & + \cdots + & \alpha_p
\end{array}$$

and using matrix notations we can write this as

$$\begin{pmatrix} \rho_{(0)} \\ \rho_{(1)} \\ \vdots \\ \rho_{(p)} \end{pmatrix} = \begin{pmatrix} \delta_{00} & \delta_{01} & \cdots & \delta_{0p} \\ \delta_{10} & \delta_{11} & \cdots & \delta_{1p} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{p0} & \delta_{p1} & \cdots & \delta_{pp} \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_p \end{pmatrix}$$

say $P = D\alpha$ where $P = (\rho_{(0)}, \rho_{(1)}, \cdots, \rho_{(p)})'$, $D = (\delta_{ij})_{k\times k}$, $\alpha = (\alpha_0, \alpha_1, \cdots, \alpha_p)'$ and the elements of D are given by $\delta_{ij} = Cov(u_{t-i}, u_{t-j}) = \beta'_{(i)}\Gamma(i-j)\beta_{(j)}$ for $i, j = 0, 1, \cdots, p$.

Now it can be seen that

$$\begin{aligned} \delta_{ji} &= \beta'_{(j)}\Gamma(j-i)\beta_{(i)} \\ &= \beta'_{(j)}\Gamma'(i-j)\beta_{(i)} \qquad (\because\ \Gamma(-k) = \Gamma'(k)\ ) \\ &= (\beta'_{(i)}\Gamma(i-j)\beta_{(j)})' \\ &= (\delta_{ij})' \\ &= \delta_{ij} \qquad (\because\ \delta_{ij} \textit{ is scalar}\ ) \end{aligned}$$

and hence the matrix D is symmetric. We can estimate the parameter vector α as $\hat{\alpha} = \hat{D}^{-1} P$ where $\hat{D} = (\hat{\delta}_{ij})_{k\times k}$ and the estimate of elements of $\hat{D}$ is obtained as $\hat{\delta}_{ij} = \beta'_{(i)}\, C(i-j)\, \beta_{(j)}$ where $C(l)$ is the estimate of $\Gamma(l)$, the lag l cross covariance matrix for the vector time series X_t.

From the relation $P = D\alpha$ we get a decomposition of the maximum correlations $\rho_{(0)}, \rho_{(1)}, \cdots, \rho_{(p)}$ of z_t with linear combination of elements of X_t at different lags into direct and indirect effects similar to that in path coefficient analysis. Here $\rho_{(i)} = \alpha_0\delta_{i0} + \alpha_1\delta_{i1} + \cdots + \alpha_{i-1}\delta_{i,i-1} + \alpha_i + \alpha_{i+1}\delta_{i,i+1} + \cdots + \alpha_p\delta_{ip}$ for $i = 0, 1, \cdots, p$.

The coefficients $\alpha_0, \alpha_1, \cdots, \alpha_p$ are the direct effects of the respective linear combinations on z_t and the proportion of variation in z_t explained by these variables can then be estimated as

$$\hat{R}^2 = \sum_{i,j=0}^{p} \hat{\alpha}_i \, \hat{\alpha}_j \hat{\delta}_{ij} = \hat{\alpha}' \, \hat{D} \, \hat{\alpha}$$

In terms of the original variable the new relation can be written as

$$y_t = M_0' X_t + M_1' X_{t-1} + \cdots + M_p' X_{t-p} + a_t \text{ where } M_i = \sigma \, \alpha_i \, \beta_{(i)} \text{ for } i = 0,1,\cdots,p\,.$$

The information extracted through this analysis can be used for further modeling of the univariate times series $\{y_t\}$ by allowing the sequence $\{a_t\}$ to take necessary transfer function form of the innovations. The canonical path analysis described here can be successfully used as a tool for identification of time series components to be included in vector time series modelling.

Example:

To examine the influence of landings of mackerel, anchovies and lesser sardines on oil sardine landings, time series data on quarter wise landings by this species during 1960-96 in Kerala were used. All these species/groups compete each other for food, all being mainly plankton feeders. Oil sardine and mackerel are the two major fishery in Kerala. Anchovies and lesser sardines are also major contributors towards total landings in the state. Seasonality present in the data were removed by taking a 4 point moving sum of each time series and then standardized to zero mean and unit variance before subjecting to canonical path analysis. For this analysis, the standardized landings of oil sardine was treated as the univariate time series $\{y_t\}$ and the vector time series $\{X_t\}$ consist of landings of mackerel, anchovies and lesser sardines. The variance covariance matrix for the vector time series was

$$\Sigma = \begin{pmatrix} 1.000 & 0.339 & 0.139 \\ 0.339 & 1.000 & -0.007 \\ 0.139 & -0.007 & 1.000 \end{pmatrix}$$

Cross covariance matrices $C(i)$ of the vector time series $\{X_t\}$ and cross covariance vector $\sigma_{(i)}$ of $\{y_t\}$ with the elements of $\{X_t\}$ were computed up to lag 15. Canonical correlations $\rho_{(i)}$ and canonical vectors $\beta_{(i)}$ were computed for each of these lags following the method derived. The eigen value $\rho^2_{(i)}$, canonical correlation $\rho_{(i)}$, linearising vectors $\beta_{(i)}$ and the χ^2 statistic for different lags are shown in table.5.1.1. It was found that the $\rho^2_{(i)}$ values are comparatively low after lag 8 and hence for further analysis information up to lag 8 only was used. The maximum value of $\rho^2_{(i)}$ observed was 0.243667 at lag 4. The covariance matrix ***D*** of the canonical variables which is of order 9×9 was then constructed and it is given in table.5.1.2. The total effects vector P of canonical correlations, direct effects vector α obtained as the solution vector from the matrix equation $P = D\alpha$ and the total of indirect effects are given in table.5.1.3.

Among the direct effects of the linear combinations of X_t at different lags, the maximum was 0.3118 at lag 7 and other important lags in order of magnitude of direct effects were –0.21495 at lag 8, 0.21026 at lag 0, 0.20776 at lag 3 and –0.11653 at lag 2. At all other lags the direct effects were very low. The total variation in y_t explained by the linear combinations of variables in X_t up to lag 8 was 0.292223. Since $\{y_t\}$ is a time series which it self is a process that can be modelled this much variation due to the vector time series $\{X_t\}$ on $\{y_t\}$ is a valuable proportion.

The direct and indirect contributions of elements of X_t on y_t at different lags were examined by a decomposition of each canonical correlation into direct and indirect effects and this decomposition is given in table.5.1.4. The indirect effects of linear combinations of X_t on y_t were maximum for lags 0, 3 and 7 through the other lags of X_t. Out of the total proportion of 0.292223 of the variations in y_t that was explained by linear combinations of elements of X_t at different lags, 19.65% was caused by the first element of X_t, 42.48% by the second element and the remaining 37.87% by the third element. When only the direct effects were considered these percentages were 22.15%, 40.67% and 37.18% respectively due to the three elements.

5.2. Time series process generated by moving sums.

In univariate time series modelling, it may usually be required to perform some preliminary processing of the sample time series before analysing the data. When data is available in seasonal form it is a good practice to generate a new series as a moving sum or aggregate of a desired order on the original observed series before attempting to find a suitable model. This will remove any additive seasonal component present in the data and also smoothen the error component. Here in this study, the properties of the process generated as a moving sum are examined for the class of models of types autoregressive, moving average and mixed autoregressive moving average. For practical applications, Methods of estimation of mixed autoregressive moving average models based on regression procedures were suggested by Hannan and Rissanen (1982), Hannan and Kavalieris (1984) and John and Victoria (1997). These regression procedures can be suitably modified for the model on moving sum in a similar line suggested here for the autoregressive type model.

Results and Discussion:

Consider a univariate process $\{y_t\}$ generated by an autoregressive process of order 1 as, $y_t = \phi y_{t-1} + \varepsilon_t$ where $\{\varepsilon_t\}$ is an innovation sequence of independently and identically distributed random variables with zero mean and constant variance σ^2. Let $\{x_t\}$ is another process generated as a sum of s consecutive terms of $\{y_t\}$ defined by

$$\begin{aligned} x_t &= y_t + y_{t-1} + \cdots + y_{t-s+1} \\ &= \sum_{k=0}^{s-1} y_{t-k} \\ &= \sum_{k=0}^{s-1} \{\phi y_{t-k-1} + \varepsilon_{t-k}\} \\ &= \phi \sum_{k=0}^{s-1} y_{t-k-1} + \sum_{k=0}^{s-1} \varepsilon_{t-k} \\ &= \phi x_{t-1} + \sum_{k=0}^{s-1} \varepsilon_{t-k} . \end{aligned}$$

This we can write as $x_t = \phi x_{t-1} + \eta_t$, where $\eta_t = \sum_{k=0}^{s-1} \varepsilon_{t-k}$ is a new innovation sequence. Hence $\{x_t\}$ also have the same structure of an AR(1) process with the same AR coefficient parameter ϕ as that of $\{y_t\}$, but with a different innovation sequence which is the sum of s consecutive terms of the innovations ε_t of $\{y_t\}$.

Now, consider the general AR(p) representation for $\{y_t\}$ given by

$$y_t = \sum_{j=1}^{p} \phi_j y_{t-j} + \varepsilon_t$$

If we define another sequence $\{x_t\}$ as a sum of s consecutive terms of $\{y_t\}$ then we have

$$
\begin{aligned}
x_t &= \sum_{k=0}^{s-1} y_{t-k} \\
&= \sum_{k=0}^{s-1} \{ \sum_{j=1}^{p} \phi_j \, y_{t-k-j} + \varepsilon_{t-k} \} \\
&= \sum_{k=0}^{s-1} \sum_{j=1}^{p} \phi_j \, y_{t-k-j} + \sum_{k=0}^{s-1} \varepsilon_{t-k} \\
&= \sum_{j=1}^{p} \phi_j \{ \sum_{k=0}^{s-1} y_{t-j-k} \} + \sum_{k=0}^{s-1} \varepsilon_{t-k} \\
&= \sum_{j=1}^{p} \phi_j \, x_{t-j} + \eta_t \quad \text{where} \quad \eta_t = \sum_{k=0}^{s-1} \varepsilon_{t-k} \, .
\end{aligned}
$$

Hence the new process $\{x_t\}$ also have the structure of an AR(p) process with the same AR coefficients but with a different innovation sequence $\{\eta_t\}$ which is a sum of s consecutive terms of $\{\varepsilon_t\}$, the innovation sequence of the original process $\{y_t\}$.

Consider the case when the process $\{y_t\}$ has a moving average representation of order q with $\{\varepsilon_t\}$ as the innovation sequence which are independently and identically distributed random variables with zero mean and constant variance σ^2 . The MA(q) model is

$$
y_t = \sum_{j=1}^{q} \theta_j \, \varepsilon_{t-j} + \varepsilon_t
$$

Now define a new process $\{x_t\}$ generated as a moving sums of s consecutive terms of $\{y_t\}$. Then we have

$$
\begin{aligned}
x_t &= \sum_{k=0}^{s-1} y_{t-k} \\
&= \sum_{k=0}^{s-1} \{ \sum_{j=1}^{q} \theta_j \, \varepsilon_{t-k-j} + \varepsilon_{t-k} \}
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{k=0}^{s-1}\sum_{j=1}^{q}\theta_j\,\varepsilon_{t-k-j} + \sum_{k=0}^{s-1}\varepsilon_{t-k} \\
&= \sum_{j=1}^{q}\theta_j\,\{\sum_{k=0}^{s-1}\varepsilon_{t-j-k}\} + \sum_{k=0}^{s-1}\varepsilon_{t-k} \\
&= \sum_{j=1}^{q}\theta_j\,\eta_{t-j} + \eta_t \quad \text{where} \quad \eta_t = \sum_{k=0}^{s-1}\varepsilon_{t-k}
\end{aligned}
$$

which shows that the process $\{x_t\}$ also have the structure of an MA(q) process with innovation sequence $\{\eta_t\}$. Hence a process generated as the moving sum of an MA(q) process will also have an MA(q) representation and the moving average coefficients of the new process is same as that of the original process. The innovation sequence of the new process is different and it is a moving sum of the innovations of the original process.

Now consider the case when the original process $\{y_t\}$ has a mixed ARMA(p,q) representation given by

$$
y_t = \sum_{i=1}^{p}\phi_i\,y_{t-i} + \sum_{j=1}^{q}\theta_j\,\varepsilon_{t-j} + \varepsilon_t
$$

where $\{\varepsilon_t\}$ is the sequence of innovations which are independently and identically distributed with zero mean and constant variance σ^2. Let $\{x_t\}$ is a new process generated as a moving sum of s terms of sequence $\{y_t\}$. Then we have

$$
\begin{aligned}
x_t &= \sum_{k=0}^{s-1} y_{t-k} \\
&= \sum_{k=0}^{s-1}\{\sum_{i=1}^{p}\phi_i\,y_{t-k-i} + \sum_{j=1}^{q}\theta_j\,\varepsilon_{t-k-j} + \varepsilon_{t-k}\} \\
&= \sum_{k=0}^{s-1}\sum_{i=1}^{p}\phi_i\,y_{t-k-i} + \sum_{k=0}^{s-1}\sum_{j=1}^{q}\theta_j\,\varepsilon_{t-k-j} + \sum_{k=0}^{s-1}\varepsilon_{t-k}
\end{aligned}
$$

$$= \sum_{i=1}^{p} \phi_i \{\sum_{k=0}^{s-1} y_{t-i-k}\} + \sum_{j=1}^{q} \theta_j \{\sum_{k=0}^{s-1} \varepsilon_{t-j-k}\} + \sum_{k=0}^{s-1} \varepsilon_{t-k}$$

$$= \sum_{i=1}^{p} \phi_i \, x_{t-i} + \sum_{j=1}^{q} \theta_j \, \eta_{t-j} + \eta_t \text{ where } \eta_t = \sum_{k=0}^{s-1} \varepsilon_{t-k} .$$

The above expression is that of an ARMA(p,q) corresponding to a process $\{x_t\}$ with AR coefficients $\phi_1, \phi_2, \cdots, \phi_p$; MA coefficients $\theta_1, \theta_2, \cdots, \theta_q$; and innovation sequence $\{\eta_t\}$. Hence the new series $\{x_t\}$, which is a moving sum of s consecutive terms of the original series $\{y_t\}$, have the same ARMA(p,q) representation with the same AR and MA coefficients but with a new innovation sequence which is a moving sum of the innovations of $\{y_t\}$.

For estimation of parameters of the above models one will be interested to know how the new sequence $\{\eta_t\}$ will be distributed. Since the innovation sequence $\{\varepsilon_t\}$ is independently and identically distributed with zero mean and constant variance σ^2, the mean of η_t will also be zero. We shall now find the covariance structure of the sequence $\{\eta_t\}$. We have

$$\eta_t = \sum_{i=0}^{s-1} \varepsilon_{t-i} .$$

Variance of η_t can be determined as

$$\begin{aligned} V(\eta_t) &= E(\eta_t^2) \\ &= E[\{\sum_{i=0}^{s-1} \varepsilon_{t-i}\}\{\sum_{j=0}^{s-1} \varepsilon_{t-j}\}] \\ &= \sum_{i=0}^{s-1} \sum_{j=0}^{s-1} E(\varepsilon_{t-i} \, \varepsilon_{t-j}) \end{aligned}$$

$$=s\sigma^2$$

because we get nonzero terms in the above sum only for s terms when $i=j$.

Similarly we can find the covariance as

$$\begin{aligned}Cov(\eta_t,\eta_{t+k}) &= E(\eta_t\,\eta_{t+k})\\ &= E[\{\sum_{i=0}^{s-1}\varepsilon_{t-i}\}\{\sum_{j=0}^{s-1}\varepsilon_{t+k-j}\}]\\ &= \sum_{i=0}^{s-1}\sum_{j=0}^{s-1}E(\varepsilon_{t-i}\,\varepsilon_{t+k-j})\\ &= (s-k)\sigma^2 \text{ when } k<s \text{ and } 0 \text{ otherwise.}\end{aligned}$$

When the original series is first order seasonal type, its moving sum series will be free form seasonality. To examine this consider series $\{y_t\}$ and its moving sum series $\{x_t\}$ defined by

$$\begin{aligned}x_t &= y_t + y_{t-1} + \cdots + y_{t-s+1}\\ &= (1+B+\cdots+B^{s-1})\,y_t\\ &= \frac{(1-B^s)}{(1-B)}y_t\end{aligned}$$

Let $\phi(B)x_t=\theta(B)a_t$ be the suitable model to represent the moving sum series $\{x_t\}$, then the corresponding model for the original series can be obtained by substituting for x_t in terms of y_t in the model. So we get

$$\phi(B)\frac{(1-B^s)}{(1-B)}y_t=\theta(B)a_t$$

$$\therefore \phi(B)(1-B^s)\,y_t=\psi(B)a_t \text{ where } \psi(B)=(1-B)\theta(B).$$

The above representation is that of an ARMA model with first order seasonal difference applied to the series $\{y_t\}$ to represent the seasonal factor and the model for the moving sum series $\{x_t\}$ is free from this seasonal component.

If the original series $\{y_t\}$ has a linear trend apart from the ARMA representation then the moving sum series will also be affected by a linear trend with different slope and intercept. Consider the ARMA representation with linear trend for the series $\{y_t\}$ as

$$\phi(B)\, y_t = a + bt + \theta(B)\,\varepsilon_t$$

Then the corresponding model for the moving sum series $\{x_t\}$ is

$$\phi(B)x_t = \sum_{i=0}^{s-1}\{a + b(t-i)\} + \theta(B)\,\eta_t \quad \text{where} \quad \eta_t = \sum_{i=0}^{s-1}\varepsilon_{t-i}$$

$$= sa + sbt - sb(s-1)/2 + \theta(B)\,\eta_t$$

$$= c + dt + \theta(B)\eta_t \quad \text{where} \quad c = \frac{s\{2a - b(s-1)\}}{2} \text{ and } d = sb\,.$$

Hence if the original series has a linear trend, then the moving sum series will also have a linear trend with different slope and intercept.

In the case of an AR(1) process the moving sum series can be expressed as generated by a transfer function of the innovations of the original series. Consider the AR(1) representation for the original series $\{y_t\}$ as

$$y_t = \phi\, y_{t-1} + \varepsilon_t$$

Transfer function representation for this model can be obtained by writing the model using the back shift operator B as

$$(1 - \phi B)\, y_t = \varepsilon_t$$

$$\therefore \quad y_t = (1-\phi B)^{-1}\,\varepsilon_t$$
$$= (1+\phi B+\phi^2 B^2+\cdots)\,\varepsilon_t$$
$$= \sum_{j=0}^{\infty} \phi^j\,\varepsilon_{t-j}$$

so that $\frac{1}{1-\phi B}$ is the transfer function to generate the series $\{y_t\}$ from the innovations sequence $\{\varepsilon_t\}$. For the moving sum series $\{x_t\}$ we have the relation

$$x_t = \sum_{i=0}^{s-1} y_{t-i}$$
$$= \frac{(1-B^s)}{(1-B)}\,y_t$$
$$= \frac{(1-B^s)}{(1-B)(1-\phi B)}\,\varepsilon_t$$

Hence $\{x_t\}$ can also be generated from the same innovation sequence $\{\varepsilon_t\}$ using the transfer function $\frac{(1-B^s)}{(1-B)(1-\phi B)}$.

Estimation of parameters of the model require iterative algorithms for both MA and mixed ARMA type process. We shall examine the method of estimation for an AR(p) type moving sum process. Suppose that a sample realization of the AR(p) process $\{y_t\}$ of size T is available. Then we can generate the sample series of moving sums $\{x_t\}$ of size n = T-s, where s is the number of terms added. We then have the following linear equation for the moving sum sample as

$$x = X\varphi + \eta$$

where $x=(x_{p+1},\cdots,x_n)'$, $\varphi=(\phi_1,\cdots,\phi_p)'$ and $\eta=(\eta_{p+1},\cdots,\eta_n)'$ are column vectors of size *n-p*, *p* and *n-p* respectively and X is a matrix of order *(n-p)* × *p* defined as

$$X=\begin{pmatrix} x_p & x_{p-1} & \cdots & x_1 \\ x_{p+1} & x_p & \cdots & x_2 \\ \vdots & \vdots & \ddots & \vdots \\ x_{n-1} & x_{n-2} & \cdots & x_{n-p} \end{pmatrix}$$

Since the dispersion matrix of the error vector η in the above matrix equation is not $\sigma^2 I$, where I is an identity matrix of order *(n-p)*, the usual linear least square procedure can not be applied for estimating the parameter vector φ and the innovation variance σ^2. By considering the covariance structure of $\{\eta_t\}$,we get the covariance matrix of η as $\sigma^2 V$ where $V=(v_{ij})$ is square matrix of order *(n-p)* whose elements are defined by

$$v_{ij}=s-|i-j| \qquad \text{for } |i-j|<s$$

$$=0 \text{ for } |i-j|\geq s.$$

That is all the diagonal elements of V are equal to s, the off diagonal elements reduce by one as they move away from the main diagonal till it becomes one and all other elements are zeroes. Hence φ and σ^2 can be estimated through generalized least square where in a transformation of the type $z=(L')^{-1}x$ is applied using the Cholesky factor L of V to reduce the covariance of the error vector in the new matrix equation to $\sigma^2 I$ (Rao, 1973). The final solution is then

$$\hat{\varphi}=(X'V^{-1}X)^{-1}X'V^{-1}x \quad \text{and}$$

$$\hat{\sigma}^2=\frac{(x-X\hat{\varphi})'V^{-1}(x-X\hat{\varphi})}{n-p}.$$

Appendix-V (Tables)

Table.5.1.1. Cross covariance vector of y_t with elements of X_t, Cross-covariance matrix of X_t, linearising vector, squared canonical correlation and significance χ^2 for different lags.

Lag (i)	$\sigma_{(i)}$	$C(i)$	$\beta_{(i)}$	$\rho^2_{(i)}$	χ^2
0	$\begin{pmatrix}-0.197\\-0.390\\-0.211\end{pmatrix}$	$\begin{pmatrix}1.0000 & 0.3392 & 0.1391\\0.3392 & 1.0000 & -0.0067\\0.1391 & -0.0067 & 1.0000\end{pmatrix}$	$\begin{pmatrix}0.089749\\0.846954\\0.465550\end{pmatrix}$	0.19921	31.43
1	$\begin{pmatrix}-0.194\\-0.394\\-0.234\end{pmatrix}$	$\begin{pmatrix}0.9076 & 0.2999 & 0.1221\\0.3663 & 0.9371 & -0.0103\\0.1642 & -0.0141 & 0.8701\end{pmatrix}$	$\begin{pmatrix}0.066051\\0.836571\\0.504692\end{pmatrix}$	0.21250	33.80
2	$\begin{pmatrix}-0.218\\-0.383\\-0.271\end{pmatrix}$	$\begin{pmatrix}0.7747 & 0.2631 & 0.1460\\0.4048 & 0.8706 & 0.0299\\0.1211 & -0.0277 & 0.7239\end{pmatrix}$	$\begin{pmatrix}0.119215\\0.772131\\0.560709\end{pmatrix}$	0.22472	36.02
3	$\begin{pmatrix}-0.250\\-0.372\\-0.308\end{pmatrix}$	$\begin{pmatrix}0.6597 & 0.2388 & 0.1732\\0.4260 & 0.7990 & 0.0295\\0.0811 & -0.0299 & 0.5741\end{pmatrix}$	$\begin{pmatrix}0.186557\\0.696371\\0.605313\end{pmatrix}$	0.24241	39.28
4	$\begin{pmatrix}-0.288\\-0.341\\-0.328\end{pmatrix}$	$\begin{pmatrix}0.5626 & 0.2231 & 0.2123\\0.4411 & 0.7021 & 0.0388\\0.0272 & -0.0334 & 0.4194\end{pmatrix}$	$\begin{pmatrix}0.293002\\0.595936\\0.628030\end{pmatrix}$	0.24367	39.52
5	$\begin{pmatrix}-0.316\\-0.303\\-0.314\end{pmatrix}$	$\begin{pmatrix}0.4959 & 0.2069 & 0.2582\\0.4609 & 0.6484 & 0.0547\\-0.0258 & -0.0476 & 0.3953\end{pmatrix}$	$\begin{pmatrix}0.410219\\0.504544\\0.609398\end{pmatrix}$	0.22481	36.03
6	$\begin{pmatrix}-0.321\\-0.272\\-0.286\end{pmatrix}$	$\begin{pmatrix}0.4321 & 0.2011 & 0.2487\\0.4695 & 0.5913 & 0.0595\\-0.0298 & -0.0512 & 0.3779\end{pmatrix}$	$\begin{pmatrix}0.489457\\0.449945\\0.577472\end{pmatrix}$	0.19804	31.23
7	$\begin{pmatrix}-0.305\\-0.237\\-0.257\end{pmatrix}$	$\begin{pmatrix}0.3353 & 0.2039 & 0.2628\\0.4678 & 0.5441 & 0.0940\\-0.0377 & -0.0691 & 0.3501\end{pmatrix}$	$\begin{pmatrix}0.537739\\0.405464\\0.563342\end{pmatrix}$	0.16409	25.36
8	$\begin{pmatrix}-0.274\\-0.215\\-0.206\end{pmatrix}$	$\begin{pmatrix}0.2490 & 0.2089 & 0.2933\\0.4640 & 0.5171 & 0.1061\\-0.0363 & -0.0835 & 0.3473\end{pmatrix}$	$\begin{pmatrix}0.565316\\0.424028\\0.510541\end{pmatrix}$	0.12355	18.66

Table.5.1.1. Continued

Lag (i)	$\sigma_{(i)}$	$C(i)$	$\beta_{(i)}$	$\rho^2_{(i)}$	χ^2
9	$\begin{pmatrix}-0.235\\-0.210\\-0.161\end{pmatrix}$	$\begin{pmatrix}0.1867 & 0.2118 & 0.3367\\0.4583 & 0.4983 & 0.1228\\-0.0391 & -0.0971 & 0.3061\end{pmatrix}$	$\begin{pmatrix}0.532617\\0.509103\\0.456789\end{pmatrix}$	0.09314	13.83
10	$\begin{pmatrix}-0.201\\-0.210\\-0.112\end{pmatrix}$	$\begin{pmatrix}0.1416 & 0.2076 & 0.3934\\0.4563 & 0.4984 & 0.1403\\-0.0470 & -0.1225 & 0.2586\end{pmatrix}$	$\begin{pmatrix}0.491124\\0.618982\\0.352465\end{pmatrix}$	0.07174	10.53
11	$\begin{pmatrix}-0.185\\-0.219\\-0.058\end{pmatrix}$	$\begin{pmatrix}0.1242 & 0.2051 & 0.4025\\0.4586 & 0.4922 & 0.1129\\-0.0434 & -0.1384 & 0.2219\end{pmatrix}$	$\begin{pmatrix}0.465894\\0.712782\\0.170685\end{pmatrix}$	0.06369	9.31
12	$\begin{pmatrix}-0.171\\-0.223\\-0.035\end{pmatrix}$	$\begin{pmatrix}0.1105 & 0.2012 & 0.3658\\0.4480 & 0.4820 & 0.0991\\-0.0443 & -0.1516 & 0.1507\end{pmatrix}$	$\begin{pmatrix}0.423190\\0.764104\\0.089762\end{pmatrix}$	0.06060	8.84
13	$\begin{pmatrix}-0.154\\-0.235\\-0.029\end{pmatrix}$	$\begin{pmatrix}0.0991 & 0.1902 & 0.3306\\0.4378 & 0.4918 & 0.0865\\-0.0459 & -0.1631 & 0.1003\end{pmatrix}$	$\begin{pmatrix}0.323173\\0.836590\\0.078228\end{pmatrix}$	0.06176	9.02
14	$\begin{pmatrix}-0.135\\-0.240\\-0.026\end{pmatrix}$	$\begin{pmatrix}0.1052 & 0.1734 & 0.3165\\0.4213 & 0.4791 & 0.0657\\-0.0338 & -0.1659 & 0.0421\end{pmatrix}$	$\begin{pmatrix}0.234535\\0.890297\\0.080273\end{pmatrix}$	0.06117	8.93
15	$\begin{pmatrix}-0.104\\-0.239\\-0.029\end{pmatrix}$	$\begin{pmatrix}0.1251 & 0.1415 & 0.3132\\0.3974 & 0.4586 & 0.0873\\-0.0496 & -0.1720 & -0.0177\end{pmatrix}$	$\begin{pmatrix}0.089350\\0.959008\\0.113314\end{pmatrix}$	0.05834	8.51

Table.5.1.2. Covariance matrix D for the canonical variables at different lags.

	$D=\{d_{ij}=\beta'_{(i)}C(i-j)\beta_{(j)}\}$								
(i,j)	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
0	1.000000	0.927245	0.832999	0.722114	0.594237	0.536419	0.490784	0.453396	0.458934
1	0.927245	1.000000	0.918103	0.812626	0.690512	0.569745	0.525945	0.480831	0.469581
2	0.832999	0.918103	1.000000	0.915032	0.800468	0.680967	0.575557	0.535684	0.513390
3	0.722114	0.812626	0.915032	1.000000	0.909630	0.792860	0.684480	0.582986	0.566846
4	0.594237	0.690512	0.800468	0.909630	1.000000	0.910242	0.801474	0.696606	0.614656
5	0.536419	0.569745	0.680967	0.792860	0.910242	1.000000	0.916871	0.811565	0.721327
6	0.490784	0.525945	0.575557	0.684480	0.801474	0.916871	1.000000	0.920183	0.823700
7	0.453396	0.480831	0.535684	0.582986	0.696606	0.811565	0.920183	1.000000	0.923036
8	0.458934	0.469581	0.513390	0.566846	0.614656	0.721327	0.823700	0.923036	1.000000

Table.5.1.3. Table showing direct effects, indirect effects, total effects and final coefficient vector for the original variables.

Lag (i)	Direct Effects ($\alpha_{(i)}$)	Indirect Effects	Total Effects ($\rho_{(i)}$)	Vector M_i'		
0	0.210265	0.236061	0.446326	0.018871	0.178085	0.097889
1	0.056982	0.403990	0.460973	0.003764	0.047670	0.028759
2	-0.116533	0.357511	0.474043	-0.013892	-0.089979	-0.065341
3	0.207756	0.284600	0.492356	0.038758	0.144675	0.125757
4	0.089441	0.404185	0.493626	0.026206	0.053301	0.056172
5	0.084720	0.389423	0.474143	0.034754	0.042745	0.051628
6	-0.021933	0.423088	0.445020	-0.010735	-0.009868	-0.012665
7	0.311185	0.093899	0.405084	0.167336	0.126174	0.175303
8	-0.214950	0.136550	0.351500	-0.121515	-0.091145	-0.109741

Table.5.1.4. Table of direct and indirect effects of the canonical variables at different lags.

	Matrix of direct effects (diagonal elements) and indirect effects								
(i,j)	0	1	2	3	4	5	6	7	8
0	0.210265	0.052837	-0.097072	0.150024	0.053149	0.045446	-0.010764	0.141090	-0.098648
1	0.194967	0.056982	-0.106989	0.168828	0.061760	0.048269	-0.011535	0.149627	-0.100936
2	0.175150	0.052316	-0.116533	0.190103	0.071595	0.057692	-0.012623	0.166697	-0.110353
3	0.151835	0.046305	-0.106631	0.207756	0.081358	0.067171	-0.015012	0.181416	-0.121843
4	0.124947	0.039347	-0.093281	0.188981	0.089441	0.077116	-0.017578	0.216773	-0.132120
5	0.112790	0.032465	-0.079355	0.164721	0.081413	0.084720	-0.020109	0.252547	-0.155049
6	0.103195	0.029970	-0.067071	0.142205	0.071685	0.077677	-0.021933	0.286347	-0.177054
7	0.095333	0.027399	-0.062425	0.121119	0.062305	0.068756	-0.020182	0.311185	-0.198406
8	0.096498	0.026758	-0.059827	0.117766	0.054975	0.061111	-0.018066	0.287235	-0.214950

Appendix-VI

List of computer softwares developed (using C++ language)

1. Module for transposing a matrix.

2. Module for multiplication of two matrices (for both near and far type of memory allocation).

3. Module for inverting a square matrix (translated from Press *et. al.* 1992).

4. Module for Cholesky factorisation of matrix.

5. Module to compute eigen values and eigen vectors of a real symmetric matrix (translated from Press *et. al.* 1992).

6. Module for computing eigen values of a general matrix.

7. Module for evaluating the roots of a polynomial for checking stationarity of a univariate time series model.

8. Module for printing matrices.

9. Module to copy block matrices of uniform size to its partitioned matrix.

10. Module for computing the three types of variograms.

11. Program for computing cross correlation matrices of given lags for a Vector Time Series. This program also provides tests of significance of individual cross correlation coefficients and testing combined significance of cross-correlation matrices.

12. Programe for computing partial cros correlation matrices of different lags for a vector timeseries through the evaluation of forward and backward vector autoregressions.

13. Programe for computing partial canonical correlations of a vector time series. This software computes the canonical structure, proportions of varince extracted and redundancy. Test of significance of canonical correlation based on Wilk's lamda is also provided in this programe.

14. Program for the estimation of parameter matrices of a vector autoregressive model. This program will also compute the AIC, BIC, HQ and FPE order selection criteria. Standard errors of the estimated parameters also will be computed by this program.

15. Program for the estiamtion of parameters of a mixed Vector Autoregressive moving average model based on the algorithm of Spliid. This program also calculate the order selection criteria and innovation dispersion matrix.

16. Program for the estiamtion of parameters of a mixed Vector Autoregressive moving average model through maximum likelihood method. The standard errors of the estimates of parameters also will be computed in this program.

17. Program for the estiamtion of parameters of a mixed Vector Autoregressive moving average model through conditional maximum likelihood method.

18. Module for checking Stability and Invertibility conditions of an estimated VARMA(p,q) type model by computing eigen values of characteristic matrices.

19. Program for the estimation of parameter matrices and standard errors of the estimated parameters for a VARX model.

SUMMARY AND CONCLUSIONS

India is one among the top 10 fish producing countries with a coastline of about 8,129 Kilometers. Marine fish production from the country accounts for 3% of the world marine fish production. It earns a foreign exchange worth 41,500 million rupees and about 5 million people living in the coastal areas are engaged in fishing and other related activities for their livelihood. Marine fish production from Kerala contributes to almost 25% of the total marine fish production in the country though the total coast line covered is only one-tenth of the Indian coast line.

Prior information about future marine fish production can help in proper planning, storage and distribution etc. It is desirable to know about the inter-relations that exist between landings of different species as many of the marine fish species depend each other due to factors like prey-predator relation, competing for a common food resource and influenced by a common environmental condition and so on. In this study quarterwise marine fish landings in Kerala during the period 1960-96 were used to develop suitable univariate and multivariate time series models. The relation between environment and marine fish landings were also examined using multivariate time series models.

The univarate time series model used in this study was Box-Jenkins seasonal autoregressive integrated moving average models (ARIMA). Seasonal ARIMA models were fitted to the quarterwise landings of oil sardine, mackerel, anchovies, lesser sardines, penaeid prawns, tuna, thrissocles, ribbon fishes and total landings. For each of these time series autocorrelation and partial autocorrelation analysis were carried out to examine the behavior of the time series. Suitable orders for each model were selected based on AIC and SBC order selection criteria. Different models found suitable and fitted were ARIMA(1,0,0)(0,1,1)4 for oil sardine landings, ARIMA(1,0,0)(0,1,1)4 with log transformation for mackerel landings, ARIMA(4,0,0)(1,1,2)4 with log transformation for anchovies, ARIMA(0,1,1)(0,1,1)4 with log transformation for both lesser sardine and tuna landings, ARIMA(0,0,1)(0,1,1)4 with log transformation for penaeid prawn landings, ARIMA(1,1,1)(1,1,2)4 with log transformation for thrissocles, ARIMA(1,0,1)(0,1,1)4 with log transformation for ribbon fishes and ARIMA(0,1,2)(0,1,1)4 for total landings. These models fitted were then used for forecasting quarterwise landings for 1997 and 1998.

The effect of introduction of new crafts fitted with out board engines and introduction of a new fishing gear in the late eighties was examined through an intervention analysis. The model used for intervention analysis was ARIMA(0,1,2)(0,1,1)4 by treating the period 1960-87 as the pre-intervention period. This analysis revealed that on an average there is an increase of about 53,770*t* in total marine fish landings in the state by the introduction of new gears and new crafts fitted with outboard engines into the fishery.

The inter-relations between landings of selected species / groups of marine fishes were examined through cross-correlation analysis and also by using partial cross correlation matrices and partial canonical cross correlation matrices. Vector autoregressive models (VAR model) were fitted for five sets of data consisting of four times series sequences each on landings to represent a vector time series. The species/groups for each set were selected based their commercial importance and also by examining their food and feeding habits to establish prey-predator type or competeing type of relationship. Suitable orders for these VAR models were selected based order selection criteria like AIC, BIC and the HQ criterion given by Hannan and Quinn. The model parameter matrices were estimated through generalised least square method and adequacy of each fitted model was tested by computing cross correlation matrices for the residual vector series and also by testing combined significance of the residual cross correlation matrices using an approximate chi-square test. Stationarity of the fitted *VAR* models were examined by evaluating eigen values of a characteristic matrix. For the first vector time series consisting of landings of oil sardine, mackerel, anchovies and lesser sardine the model found suitable was VAR(2); for the second vector time series consisting of landings of anchovies, lesser sardine, ribbon fish and catfish the model found suitable was VAR(2) model; VAR(1) model was found suitable for the third vector time series with landings of mackerel, anchovies, tuna and penaeid prawns as components; VAR(5) model was found suitable for the vector time series consisting of landings of oil sardine, anchovies, tuna and penaeid prawns; and for the fifth vector times series with landings of elasmobranchs, oil sardine, mackerel and seer fish as components the model found suitable was VAR(2). These models were estimated and tested for their suitability through residual cross correlation analysis and also by using an appropriate chi-square test. It was found that these models are capable of explaining the variations in the component series satisfactorily, in most cases more than 90% of the variations in the data. These models were used for forecasting quarterwise landings of 1997 and 1998. These analysis revealed that

i. Compared to mackerel and lesser sardines, the series that was capable of explaining some portion of the variation in oil sardine landings was the anchovies series.

ii. The presence of the series on anchovies in the model was more effective in explaining the variations in mackerel landings

iii. The influence of oil sardine on mackerel landings was quite high compared to that of lesser sardines

iv. Oil sardine series had comparatively high influence on the variations in anchovies series

As an alternative to the higher order *VAR* model fitted to the vector time series consisting of landings of oil sardine, anchovies, tuna and penaeid prawns the mixed vector autoregressive moving average (*VARMA*) model was attempted. Here also, the orders of the model were selected based on *AIC*, *BIC* and *HQ* criterion. The model parameters were estimated by an approximate maximum likelihood method given by Wilson (1973). For the estimation algorithm, expressions were derived for computing partial derivatives of the innovation vectors which are required for the iterative computation of the maximum likelihood estimate. The fitted *VARMA* model was tested for its suitability by computing cross correlation matrices for the residual vector series and also by testing the combined significance of elements of cross correlation matrices.

Stationarity and invertibility of the fitted models were tested by evaluating the eigen values of two characteristic matrices. The estimated models were then used for forecasting quarter wise landings of each component series for 1997 and 1998.

The relationship between environmental variables and marine fish landings was examined by modelling time series data on monthly landings of elasmobranchs, oil sardine, stolephorus and mackerel at Cochin Fisheries Harbour during the period 1988-97 using vector autoregressive models with environmental variables as exogenous variables (*VARX* models). The relationship was also examined through cross correlation analysis. The environmental time series variables considered were monthly means of maximum and minimum temperatures, lowest and highest temperature recorded in a month, total monthly rainfall, highest rainfall recorded in a month and the number of rainy days in a month. Six different *VARX* models were fitted using the four landings series as output vector and two environmental time series, one each to represent temperature and rainfall, as exogenous vectors. The results of this study were

i. The stolephorus series depends on the highest rainfall series. An increase in highest rainfall is expected to cause increased landings of stolephorus.

ii. Landings of mackerel depend on highest and lowest temperatures, and highest rainfall. Increase in the values of these environmental variables are not favourable for good landings of mackerel.

iii. The series on number of rainy days had significant negative effect on the series on elasmobranchs landings.

iv. There was significant dependence of oil sardine landings on highest temperature series and the effect was negative in direction.

A new method based on canonical correlations and path coefficients analysis was developed to evaluate the relationship between time series sequences. This procedure was illustrated using data on quarterwise landings of four marine fish species/group. The properties of a univariate time series generated as a moving total of standard time series processes of *AR*, *MA* and *ARMA* type were also examined.

REFERENCES

Abraham. B. 1980. Intervension analysis and multiple time series. *Biometrika,* **67**(1): 73-78.

Abraham, B. and Ladolter, J. 1984. A note on inverse autocorrelations. *Biometrika,* **71**(3): 609-614.

Ahn, S.K. 1988. Distribution for residual autocovariances in multivariate autoregressive models with structured parameterization. *Biometrika,* 75(3): 590-593.

Ahn, S.K. 1993. Some tests for unit roots in autoregressive intergrated moving average models with deterministic trend. *Biometrika,* **80**(4): 855-868.

Ahn, S. K. and Reinsel, G.C. 1988. Nested reduced rank autoregressive models for multiple time series. *J. Amer. Statist. Assoc.* 83(403): 849-856.

Akaike, H. 1972. Use of an information theoretic quantity for statistical model identification. Proc. 5th Hawaii Inter. Conference on System Sciences, 249-250.

Ali, M.M. 1989. Tests for autocorrelation and randomness in multiple time series. *J. Amer. Statist. Assoc.*, **84**(406): 533-540.

Andersen, A. 1982. An empirical examination of Box-Jenkins forecasting. *J. R. Statist. Soc. Ser. A,* **145**(4): 472-478.

Ansley, C.F. 1979. An algorithm for the exact likelihood of a mixed autoregressive moving average process. Biometrika, **66**(1): 59-65.

Ansley, C.F. and Kohn. R. 1983. Exact likelihood of vector autoregressive moving average process with missing or aggregated data. *Biometrika,.* 70(1): 275-278.

Antony Raja, B.T. 1973. Forecasting the oil sardine fishery. *Indian J. Fish.*, **20**(2): 599-609.

Baillie, R. 1979. Asymptotic prediction mean squared error for vector autoregressive models. *Biometrika,* **66**(3): 675-678.

Bal, D.V. and Rao, K.V. 1984. *Marine Fisheries.* Tata McGraw-Hill, New Delhi, 469p.

Bhattacharyya, M.N. 1979. Effectiveness of seat belt legislation on the Queensland road toll - an Australian case study in intervention analysis. *J. Amer. Statist. Assoc.*, **74**(367): 596-603.

Borah, D.K. and Bora, P.K. 1995. Predicting the monthly rainfall around Guwahati using seasonal ARIMA model. J. Ind. Soc. Ag. Statist., 47(3): 278-287.

Box, G.E.P., Pierce, D.A. and Newbold, P. 1987. Estimating trend and growth rates in seasonal time series. *J. Amer. Statist. Assoc.*, **82** (397): 276-282.

Box, G.E.P. and Jenkins, G.M. 1976. *Time series analysis - Forecasting and control.* Holden-Day, San Francisco.

Box, G.E.P. and Tiao, C.C. 1977. A canonical analysis of multiple time-series. *Biometrika,*, **64**(2): 355-365.

Brillinger, D.R. 1969. Asymptotic properties of spectral estimates of second order. *Biometrika,* **56**(2): 375-389.

Brockett, P.L., Hinich, M.J. and Patterson, D. 1988. Bispectral based tests for the detection of Gaussianity and linearity in time series. *J. Amer. Statist. Assoc.* **83**(403): 657- 664.

Bustos, O.H. and Yohai, V.J. 1986. Robust estimates for ARMA models. *J. Amer. Statist. Assoc.*, **81**(393): 155-168.

Campbell, K. 1982. Recurssive computation of M-estimates for the parameters of a finite autoregressive process. *Ann. Statist.,* **10**(2): 442-453.

Campbell, N.A. 1982. Robust procedures in multivariate analysis - II. Robust canonical variate analysis. *Appl. Statist.,*, 31(1): 1-8.

Campbell, M.J. 1994. Time series regression for counts: an investigation into the relationship between sudden infant death syndrome and environmental temperature. J. R. Statist. Soc. Ser.A., 157(2):191-208.

Chen, C.W. 1971. On some problems in canonical correlation analysis. Biometrika, **58**(2): 399-400.

Claerbout, J.F. 1966. Spectral factorization of multiple time series. *Biometrika,* **53**(1&2): 264-267.

Clarke, B.R. 1983. An algorithm for testing goodness of fit of ARMA(p,q) models. *Appl. Statist.,*, **32**(3):335-344.

Crafts, N.F.R., S.J. Ley Hourne and T.C. Mills. 1989. Trends and cycles in British Industrial production. 1700-1913. *J. R. Statist. Ser.A*, 152(1): 43-60.

Cressie, N. 1988. A graphical procedure for determining nonstationarity in time series. *J. Amer. Statist. Assoc.*, **83**(404): 1108-1116.

Darroch, J.N. and J.E. Mosimann. 1984. Canonical and principal components of shape. *Biometrika,*, 72(2): 241-252.

Davies, N. and Petruccell, J.D. 1984. On the use of general partial autocorrelation function for order determination in ARMA(p,q) processes. *J. Amer. Statist. Assoc.* **79**(386):374-377.

Degerine, S. 1990. Canonical partial autocorrelation functions of a multivariate time series. *Ann. Statist.,* **18**(2): 961-971.

Dickey, D.A., Hasza, D.P. and Fuller, W.A. 1984. Testing for unit roots in seasonal time series. *J. Amer. Statist. Assoc.* **79**(386): 355-367.

Durbin, J. 1960. The fitting of time series models. *Rev. Int. Inst. Stat.* **28**: 233-244.

Eakin, D.H. 1988. Estimating vector autoregressions with panel data. *Econometrika,* **56**(6): 1371-1395.

Fearn, T. and Maris, P.I. 1991. An application of Box-Jenkins methodology to the control of Gluton addition in a flour mill. *Appl. Statist.,*, **40**(3): 477-484.

Fogarty, M.J. 1988. Time series models of the marine lobster fishery: the effect of temperature. Can. J. fish. Aquat. Sci. 45(7): 1145-1153.

Fountis, N.G. and Dickey, D.A. 1989. Testing for a unit root non-stationarity in multivariate autoregressive time series. *Ann. Statist.,* **17**(1): 419-428.

Franke, J. 1985. A Levinson-Durbin recursion for autoregressive moving average process. *Biometrika,* **72**(3): 573-581.

Galbraith, J.W. and Walsh, V.Z. 1994. A simple noniterative estimator for moving average models. *Biometrika,* **81**(1): 143-145.

Galbraith, J.W. and Walsh, V.Z. 1997. On some simple, autoregression based estimation and identification techniques for ARMA models. *Biometrika,* **84**(3): 685-696.

Gardner, G., Harvey, A.C. and Phillips, G.D.A. 1980. An algorithm for exact maximum likelihood estimation of autoregressive-moving average models by means of Kalman filtering. *Appl. Statist.,* **29**: 311-322.

Geweke, J.F. 1981. A comparison of tests of independence of two covariance stationary time series. *J. Amer. Statist. Assoc.*, **76**(374): 363-373.

Geweke, J.F. 1984. Measures of conditional linear dependence and feed back between time series. *J. Amer. Statist. Assoc.*, **79**(388): 907-915.

Godolphin, E.J. 1980. A method for testing the order of an autoregressive moving average process. *Biometrika,* **67**(3): 699-703.

Godolphin, E.J. and Gooijer, J.G 1982. On the maximum likelihood estimation of the parameters of a Gaussian moving average process. *Biometrika,* **69**(2): 443-451.

Godolphin, E.J. and Unwin, J.M. 1983. Evaluation of the covariance matrix for the maximum likelihood estimator of a Gaussian autoregressive moving average process. *Biometrika,* **70**(1): 279-284.

Godolphin, E.J. 1984. A direct representation for the large sample maximum likelihood estimator of a Gaussian autoregressive moving average process. *Biometrika,* **71**(2): 281-289.

Gower, J.C. 1966. A Q-technique for the calculation of canonical variates. *Biometrika* **53**(4): 588-590.

Grubb, H. 1992. A multivariate time series analysis of some flour price data. *Appl. Statist.,* **41**(1): 95-107.

Guilkey, D.K. and Schmidt, P. 1991. An interpolation test for a unit root in the presence of drift. *Biometrika,* **78**(4): 809-815.

Hall, A. 1989. Testing for a unit root in the presence of moving average errors. *Biometrika,* **76**(1): 49-56.

Hallin, M., Ingenbleck, J-Fr. And Puri, M.L. 1985. Linear serial rank tests for randomness against ARMA alternatives. *Ann. Statist.,* **13**(3): 1156-1181.

Handcock, M.S. and Wallis, J.R. 1994. An approach to statistical spacial-temporal modelling of meteorological fields. J. Amer. Statist. Assoc., **89**(426) 368-378.

Hannan. E. J., 1955. An exact test for correlation between time series. *Biometrika,* 42(0): 346-326.

Hannan, E.J. 1967. The estimation of a lagged regression relation. *Biometrika,* **54**(3&4): 409-418.

Hannan, E.J. 1969. The identification of vector mixed autoregressive moving average systems. *Biometrika,* **56**(1): 223-225.

Hannan, E.J. and Rissanen, J. 1982. Recursive estimation of mixed autoregressive moving average order. *Biometrika,* **69**(1): 81-94.

Hannan, E.J. and Kavalieris, L. 1984. A method for autoregressive moving average estimation. *Biometrika,* **71**(2): 273-280.

Hannan, E.J., Kavalieris, L. and Mckisack, M. 1986. Recursive estimation of linear systems. *Biometrika,* **73**(1): 119-133.

Hannan, E.J. 1987. Rational transfer function approximation. *Statistical Science,* **2**(2): 135-161.

Hannan, E.J. and Poskitt, D.S. 1988. Unit canonical correlations between future and past. *Ann. Statist.,* **16** (2): 784-790.

Hannan, E.J. and Quinn, B.J. 1979. The determination of order of an autoregression. *J. R. Statist. Soc. Ser.B.*, **41**: 190-195.

Harvey, A.C. and Mckenzie, C.R. 1982. Finite sample prediction from ARIMA processes. *Appl. Statist.,* **31**(2): 180-182.

Hemerly, E.M. and Davis, M.H.A. 1989. Strong consistency of the PLS criterion for order determination of autoregressive processes. *Ann. Statist.,* **17**(2): 941-946.

Hillmer, S.C. and Tiao, G.C. 1979. Likelihood function of stationary multiple autoregressive moving average models. *J. Amer. Statist. Assoc.*, 74(367):652-660.

Holden, R.T. 1987. Time series analysis of a contagious process. *J. Amer. Statist. Assoc.*, **82**(400): 1019-1026.

Hsiao, C. 1979. Autoregressive modeling of Canadian money and income data. *J. Amer. Statist. Assoc.*, **74**(367): 553-560.

Huang, D. and Guo, L. 1990. Estimation of non-stationary ARMAX models based on the Hannan-Rissanen method. *Ann. Statist.*, **18**(14): 1729-1750.

Hurvich, C.M. and Tsai, C. 1989. Regression and time series model selection in small samples. *Biometrika,* **76**(2): 297-307.

Hurvich, C.M., Shumway, R. and Tsai, C. 1990. Improved estimators of Kullback-Leibler information for autoregressive model selection in small samples. *Biometrika,* **77**(4): 709-719.

Hurvich, C.M. and Tsai, C. 1991. Bias of the corrected AIC criterion for underfitted regression and time series models. *Biometrika,* **78**(3): 499-509.

Jeffreys, H. 1961. *Theory of probability*. Oxford: Calrendon Press.

Jensen, A.L. 1976. Time series analysis and forecasting of Atlantic menhaden catch. *Chesapeake Sci.*, **17**: 305-307.

Jensen, A.L. 1985. Time series analysis and forecasting of Atlantic menhaden catch and CPUE. *North Amer. J. Fish. Management*, **5**: 78-85.

Jewell, N.P. and Bloomfield, P. 1983. Canonical correlations of past and future for time series: Definitions and theory. *Ann. Statist.,* **11**(3): 837-847.

Kavalieris, L. 1991. A note on estimating autoregressive moving average order. *Biometrika,* **78**(4): 920-922.

Kedem, B. and Slud, E. 1981. On the goodness of fit of time series models; an application of higher order crossings. *Biometrika,* **68**(2): 551-556.

Keller, A. 1987. Modelling and forecasting primary production rates using Box-Jenkins transfer functions. *Can. J. Fish. Aquat. Sci.*, **44**: 1045-1052.

Kettenring, J.R. 1971. Canonical analysis of several sets of variables, *Biometrka*, **58**(3): 433-451.

Knight, K. 1989. Consistancy of Akaike's information criterion for infinite variance autoregressive process. *Ann. Statist.,*, **17**(2): 824-840.

Koch, P. D. and S.S. Yang. 1986. A method for testing the independence of two time series that account for a potential pattern in the cross-correlation function. *J. Amer. Statist. Assoc.* 81(394): 533-544.

Kohn, R. and Ansley, C.F. 1984. A note on Kalman filtering for the seasonal moving average model. *Biometrika,*, **71**(3): 648-650.

Kreiss, J.P. 1987. On adaptive estimation in stationary ARMA processes. *Ann. Statist.,* **15**(1): 112-133.

Kreiss, J.P. 1990. Testing linear hypothesis in autoregressions. *Ann. Statist.,* **18**(3): 1470-1482 .

Lancaster. H. O., 1966. Kolmogorov's remark on the Hottelling canonical correlations. *Biometrika,* 53(4): 585-588.

Levinson, N. 1946. The Weiner Root Mean Square Error criterion filter design and prediction. *J. Math. Phy.*, **25**:261-278.

Li, W.K. 1985. Distribution of residual autocorrelations in multivariate autoregressive index models. *Biometrika,* **72**(3): 686-688.

Li, W.K. and Hui, Y.V. 1989. Robust multiple time series modelling. *Biometrika,* **76**(2): 309-315.

Li, W.K. 1991(a). Testing model adequacy for some Markov regression models for time series. *Biometrika,* **78**(1): 83-89.

Li, W.K. 1991(b). Some Lagrange multiplier tests for seasonal differencing. *Biometrika,* **78**(2): 381-387.

Liang, K.Y. and Zeger, S.L. 1989. A class of logistic regression models for multivariate binary data. *J. Amer. Statist. Assoc.*, **84**(406): 447-451.

Ljung, G.M. 1986. Diagnostic testing of univariate time series models. *Biometrika,* **73**(3): 725-730.

Ljung, G.M. and Box, G.E.P. 1978. On a measure of lack of fit in time series models. *Biometrika,*, **65**(2): 297-303.

Longhurst, A.R. and Wooster, W.S. 1990. Abundance of oil sardine fishery and upwelling on the south west coast of India. *Can. J. Aquat. Sci.*, **47**: 2407-2419.

Luceno, A. 1994. A fast algorithm for the exact likelihood of stationary and partially stationary VARMA process. *Biometrika,* **81**(3): 555-566.

Lutkepohl, H. 1982. Discounted polynomials for multiple time series building. *Biometrika,* **69**(1): 107-115.

Lutkepohl, H. 1993. *Introduction to Multiple Time Series Analysis*. Springer-Verlag, New York. 545p.

MacNeill, I.B. 1977. A test of whether several time series share common periodicities. *Biometrika,* **64**(3): 495-508.

Masarotto, G. 1987. Robust and consistant estimates of autoregressive moving average parameters. *Biometrika,* **74**(4): 791-797.

Mauricio. J. A., 1995. Exact maximum likelihood estimation of stationary vector ARMA models. *J. Amer. Statist. Assoc.*, 90(429): 282-291.

McLeod, A.I. 1984. Duality and other properties of multiplicative seasonal autoregressive moving average models. *Biometrika,* **71**(1): 207-211.

McLeod, F.C.A. and Hipel, K.W. 1987. Contemporaneous bivariate time series. *Biometrika,* **74**(1): 103-113.

McLeod, A.I. and Sales, P.R.H. 1983. An algorithm for approximate likelihood calculation of ARMA and seasonal ARMA models. *Appl. Statist.,* **32**(2): 211-223.

Melard, G. 1984. A fast algorithm for the exact likelihood of autoregressive moving average models. *Appl. Statist.,* **32**(1): 104-114.

Mendelssohn, R. 1980. Using Box-Jenkins models to forecast fishery dynamics: identification, estimation and checking. *Fish. Bull. US.*, **78**: 887-896.

Milhoj, A. 1981. A test of fit in time-series models. *Biometrika,* **68**(1): 177-187.

Mills, T.C. and Mills, G.A. 1992. Modelling the seasonal patterns in UK Macro-economic time series. *J. R. Statist. Soc. Ser. A,* **155**(1): 61-75.

Misra, R.K. and J.F. Uthe. 1987. *Can. J. Fish. Aqua. Sci.*, **44**: 859-865.

Monti, A.C. 1994. A proposal for a residual autocorrelation test in linear models. *Biometrika,* **81**(4): 776-780.

Murphy, A.H. and Winkler, R.L. 1984. Probability forecasting in meteorology. *J. Amer. Statist. Assoc.*, **79**(387): 489-500.

Murty, A.V.S. and M.S. Edelman. 1966. On the relation between the intensity of the South-West monsoon and the oil sardine fishery of India. *Indian J. Fish.,* **13**: 142-149.

Newbold, P. 1980. The equivalence of two tests of time series model adequacy. *Biometrika,* **67**(2): 463-465.

Ng, S. and Perron, P. 1995. Unit root test in ARMA models with data dependent methods for the selection of the truncation lag. *J. Amer. Statist. Assoc.*, **90**(429): 268-281.

Nicholls, D.F.A. 1976. The efficient estimation of vector linear time series models. *Biometrika*, **63**(2): 381-390.

Nicholls, D.F.A, 1977. A comparison of estimation methods for vector linear time series models. *Biometrika* **64**(1): 85-90.

Nicholls, D.F.A. and Hall, A.D. 1979. The exact likelihood function of multivariate autoregressive moving average models. *Biometrika,*, **66**(2): 259-264.

Noble, A. 1980. Is there a ten year cycle in the mackerel fishery? *Seafood Export Journal*, **12**(4): 9-13.

Noble, A. and Sathianandan, T.V. 1991. Trend analysis in all India mackerel catches using ARIMA models. *Indian J. Fish.,* **38**(2): 119-122.

Obrien, P.C. 1980. A likelihood test for multivariate serial correlation. *Biometrika,* **67**(3): 531-537.

Pati, S. 1984. Observations on the relation between rainfall and the inshore fishery along the Orissa coast. *J. Cons. Int. Explor. Mer.,* **41**: 145-148.

Pearlman, J.G. 1980. An algorithm for the exact likelihood of a high-order autoregressive moving average process. *Biometrika,* **67**(1): 232-233.

Pena, D. and Box, G.E.P. 1987. Identifying a simplifying structure in time series. *J. Amer. Statist. Assoc.,* **82**(399): 836-843.

Phadke, M.S. 1978. Computation of the exact likelihood function of multivariate moving average models. *Biometrika,* **65**(3): 511-519.

Phillips, P.C.B. 1988. Regression theory for near-integrated time series. *Econometrica* **56**(5): 1021-1043.

Pierce, D. A. 1979. R-square measures for time series. *J. Amer. Statist. Assoc.,* 74(368): 901-910.

Poskitt, D.S. and Tremayne, A.R. 1980. Testing the specification of a fitted autoregressive moving average model. *Biometrika,* **67**(2): 359-363.

Poskitt, D.S. and Tremayne, A.R. 1982. Diagnostic test for multiple time series models. *Ann. Statist.,* **10**(1): 114-120.

Poskitt, D.S. and Tremayne, A.R. 1984. Testing misspecification in vector time series models with exogenous variables. *J. R. Statist. Soc. Ser. B,* **46**(2): 304-315.

Poskitt, D.S. 1986. A Bayes procedure for the identification of univariate time series models. *Ann. Statist.,* **14**(2): 502-516.

Poskitt, D.S. 1987. A modified Hannan-Rissanen strategy for mixed autoregressive moving average order determination. *Biometrika,* **74**(4): 781-790.

Poskitt, D.S. and Tremayne, A.R. 1987. Determining a portfolio of linear time series models. *Biometrika,* **74**(1): 125-137.

Potscher, B.M. 1983. Order estimation in ARMA models by Lagrangian multiplier tests. *Ann. Statist.,* **11**(3): 872-885.

Potscher, B.M. 1989. Model selection under nonstationarity: autoregressive models and stochastic linear regression models. *Ann. Statist.*, **17**(3): 1257-1274 .

Press, W.H., Teukolsky, S.A., Vetterting, W.T. and Flannery, B.P. 1992. *Numerical receipts in FORTRAN – the art of scientific computing* . Cambridge University Press. 963p.

Priestley, M.B. 1964. The analysis of two-dimensional stationary process with continous spectra. *Biometrika,* **51**(1&2): 195-217.

Pukkila, T., Koreisha, S. and Kallinen, A. 1990. The identification of ARMA models. *Biometrika,* **77**(3): 537-548.

Pukkila, T.M. 1987. Identification of non-zero elements in the polinimial matrices of mixed VARMA processes. *J. R. Statist. Soc. Ser.B*, **49**(1): 112-126.

Rao, C.R. 1973. *Linear Statistical Inference and its Applications*. John Wiley, New York.

Reinsel, G.C. 1983. Some results on multivariate autoregressive index models. *Biometrika,* **70**(1): 145-156.

Reinsel, G.C. 1993. *Elements of Multivariate Time series Analysis*. Springer-Verlag, New York.

Rose, et. al. 1986. Analysis of long term categorical data using categorical time series regression. Can. J. Fish. Aquat. Sci. 43: 2418-2426.

Roy, R. 1989. Asymptotic covariance structure of serial correlations in multivariate time series. *Biometrika,* **76**(4): 824-827.

Said, S.E. 1984. Testing for unit roots in ARMA models of unknown order. *Biometrika,***71** (3): 599-607.

Saila, S.B., Wighbout, M. and Lermit, R.J. 1980. Comparison of some time series models for the analysis of fisheries data. *J. Cons. Int. Explor. Mer.*, **39**: 44-52.

Sakai, H. 1983. Covariance matrices characterization by a set of scalar partial autocorrelation coefficients. *Ann. Statist.,* **11**(1): 337-340.

Sathianandan, T.V. and Srinath, M. 1995. Time series analysis of marine fish landings in *India. J. mar. biol. Ass. India*, **37**: 171-178.

Sathianandan, T.V. and Alagaraja, K. 1998. Spectral decomposition of the all India landings of oil sardine, mackerel and Bombay duck. *Indian J. Fish.*, **45**(1): 13-20.

Schwarz, G. 1978. Estimating the dimensions of a model. *Ann. Statist.,* **6**: 461-464.

Shaman, P. and Stine, R.A. 1988. The bias of autoregressive coefficient estimators. *J. Amer. Statist. Assoc.* **83**(403): 842-848.

Shea, B.L. 1989. Statistical algorithm: The exact likelihood of a vector autoregressive moving average model - AS.242. *Appl. Statist.,* **38**(1): 161-184.

Solo, V. 1984. The order of differencing in ARIMA models. *J. Amer. Statist. Assoc.*, **79**(388): 916-921.

Spliid. H., 1983. A fast estimation method for the vector autoregressive moving average model with exogenous variables. *J. Amer. Statist. Assoc.*, **78**(384): 843-849.

Srinath, M. and Datta, K.K. 1985. Forecasting marine products exports - time series analysis. *Indian J. Fish.,* **32**(2): 264-267.

Stergiou, K.I. 1989. Modelling and forecasting the fishery for Pilchard (*Sardina pilchardus*) in Greek waters using ARIMA time series models. *J. Cons. int. Explor. Mer.* **46**: 16-23.

Stergiou, K.I. 1991. Describing and forecasting the sardine anchovy complex in the eastern Mediterranean using vector autoregressions. *Fish. Res.* **11**(2): 127-141.

Stergiou, K.I. and E.D. Christou. 1996. Modelling and forecasting annual fisheries catches: comparison of regression univariate and multivariate time series methods. Fish. Research. 25(2):105-138.

Stocker, M. and Hilborn. 1981. Short-term forecasting in marine fish stocks. *Can. J. Fish Aquat. Sci.,* **38**:1247-1254.

Stoffer, D.S. 1986. Estimation and identification of space-time ARMAX models in the presence of missing data. *J. Amer. Statist. Assoc.*, **81**(395): 762-772.

Taniguchi, M. 1983. On the second order asymptotic efficiency of estimators of Gaussian ARMA processes. *Ann. Statist.,* **11**(1): 157-169.

Thisted, R.A. and Wecker, W.E. 1981. Predicting a multitude of time series. *J. Amer. Statist. Assoc.*, **76**(375): 516-523.

Thompson, P.A., Bell, W.R., Long, J.F. and Miller, R.B. 1989. Multivariate time series projections of parameterrized age-specific fertility rates. *J. Amer. Statist. Assoc.* **84**(407): 689-699.

Tiao, G.C. and Tsay, R.S. 1983. Consistancy properties of least square estimates of autoregressive parameters in ARMA models. *Ann. Statist.,* **11**(3): 856-871.

Tiao, G.C. and Box, G.E.P. 1981. Modelling multiple time series with applications. *J. Amer. Statist. Assoc.* **76**(376): 802-816.

Tiao, G.C. and Wei, W.S. 1976. Effect of temporal aggregation on the dynamic relationship of two time series variables. *Biometrika,* **63**(3): 513-523.

Troutman, B.M. 1979. Some results in periodic autoregression. *Biometrika,* **66**(2): 218-228.

Tsay, R.S. 1984. Order selection in nonstationary autoregressive models. *Ann. Statist.,* **12**(4): 1425-1433.

Tsay. R. S., 1984. Consistent estimates of autoregressive parameters and extended sample autocorrelation functions for stationary and nonstationary ARMA models. *J. Amer. Statist. Assoc.*, 79(385): 84-96.

Tsay, R.S. and Tiao, G.C. 1985. Use of canonical analysis in time series model identification. *Biometrika,* **72**(2): 299-315.

Tsay, R.S. and Tiao, G.C. 1990. Asymptotic properties of multivariate nonstationary processes with applications to autoregressions. *Ann. Statist.,* **18**(1): 220-250.

Tuan, P.D. 1978. On the fitting of multivariate processes of the autoregressive moving average type. *Biometrika,* **65**(1): 99-107.

Van Winkle, W., Krik, B.L.and Rust, B.W. 1979. Periodicities in Atlantic coast striped bass (*Morone saxatillis*) commercial fisheries data. *J. Fish. Res. Bd. Can.* **36**: 54-62.

Velu, R.P., Reinsel, G.C. and Wichern, D.W. 1986. Reduced rank models for multiple time series. *Biometrika,* **73**(1): 105-118.

Wei, C.Z. 1987. Adaptive prediction by least square predictors in stochastic regression models with application to time series. *Ann. Statist.* **15**(4): 1667-1682.

Whittle, P. 1963. On the fitting of multivariate autoregressions and the approximate canonical factorization of a spectral density matrix. *Biometrika,* **50**(1): 129-134.

Wilson, G.T. 1973. The estimation of parameters in multivariate time series models. *J. R. Statist. Soc. Ser.B,* **40**: 76-85.

Woodward, W.A. and Gray, H.L. 1981. On the relationship between the S array and Box-Jenkins method of ARMA model identification. *J. Amer. Statist. Assoc.*, **76**(376): 579-587.

Yamamoto, T. 1981. Predictions of multivariate autoregressive moving average models. *Biometrika,* **68**(2): 485-492.

Yap, S.F. and Reinsel, G.C. 1995. Estimation and testing for unit root in partially non-stationary vector ARMA models. *J. Amer. Statist. Assoc.* **90**(409): 253-267.