450195

D 54094 **(Pages : 3)** **Name.......................................**

Reg. No..................................

FIRST SEMESTER P.G. DEGREE EXAMINATION NOVEMBER 2023

(CCSS)

Mathematics

MAT 101—ALGEBRA—I

(2022 Admissions)

Time : Three Hours Maximum : 50 Marks

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find all sub-groups of $Z_2 \times Z_2 \times Z_4$ that are isomorphic to the Klein 4-group.
2. Describe all symmetries of a point in the plane R^2.
3. Establish that no group of order p^r for $r > 1$ is simple, where p is a prime.
4. Let H be a sub-group of a group G. Show that $G_H : \{g \in G : gHg^{-1} = H\}$ is a sub-group of G.
5. Find a basis for Z_4.
6. How many non-zero polynomials are there of degree ≤ 3 in $Z_2[x]$?
7. Check whether the polynomial $8x^3 + 6x^2 - 9x + 24$ is irreducible in $Z[x]$.
8. Let $G\{e, a, b\}$ be a cyclic group of order 3 with identity elements e. Write the elements $(2e + 3a + 0b)(4e + 2a + 3b)$ in the group algebra $Z_5(G)$ in the form $re + sa + tb$ for $r, s, t \in Z_5$.

(8 × 1 = 8 marks)

Turn over

450195

Part B

Answer any **six** *questions.*
Each question carries 3 marks.

9. Let H be a normal sub-group of a group G, and let $m = (G : H)$. Show that $a^m \in H$ for every $a \in G$.

10. Show that a sub-group M of a group G is maximal in G iff G/M is simple.

11. Let G be a finite group and X a finite G-set. Show that if r is the number of orbits in X under G, then $r \cdot |G| = \sum_{g \in G} |X_g|$.

12. Let K and L be normal sub-groups of a group G with $K \vee L = G$ and $K \cap L = \{e\}$. Show that $G/K \simeq L$ and $G/L \simeq K$.

13. Show that every group G′ is a homomorphic image of a free group G.

14. Show that the polynomial $x^2 - 2$ has no zeros in the field of rational numbers.

15. Let F be a field. Show that if $p(x)$ is irreducible in $F[x]$ and $p(x)$ divides the product $r_1(x) \ldots\ldots r_n(x)$ for $r_i(x) \in F[x]$, then $p(x)$ divides $r_i(x)$ for atleast one i.

16. Let R be a commutative ring and let $a \in R$. Show that $I_a = \{x \in R \mid ax = 0\}$ is an ideal of R.

17. Show that if R is a ring with unity 1, then the map $\phi : Z \to R$ given by $\phi(n) = n \cdot 1$ for $n \in Z$ in a homomorphism of Z into R.

(6 × 3 = 18 marks)

Part C

Answer any **three** *questions.*
Each question carries 8 marks.

18. (a) Let H be a sub-group of a group G. Show that the left coset multiplication is well defined by the equation $(a\mathrm{H})(b\mathrm{H}) = ab\mathrm{H}$ iff H is a normal sub-group of G.

(b) Show that if H and K are normal sub-groups of a group G such that $\mathrm{H} \cap \mathrm{K} = \{e\}$, then $hk = kh$ for all $h \in \mathrm{H}$ and $k \in \mathrm{K}$.

19. (a) Let G be a group such that p divides the order of G. Show that G has an element of order p.

(b) Let p be a prime number. Prove that a finite group G is a p group iff $|\mathrm{G}| = p^n$ for some integer $n \geq 0$.

20. (a) Let H be a sub-group of a group G and let N be a normal sub-group of G. Show that $\frac{(\mathrm{HN})}{\mathrm{N}} \simeq \mathrm{H}/\mathrm{H} \cap \mathrm{N}$.

(b) Let P_1 and P_2 be Sylow p-sub-groups of a finite group G. Show that P_1 and P_2 are conjugate sub-groups of G.

21. (a) Let F be a field of quotients of an integral domain D and let L be any field containing D. Show that there exists a map $\psi : \mathrm{F} \to \mathrm{L}$ that gives an isomorphism of F with a sub field of L such that $\psi(a) = a$ for $a \in \mathrm{D}$.

(b) Describe the field F of quotients of the integral domain $\mathrm{D} = \{n + m\sqrt{5} \mid n, m \in \mathrm{Z}\}$ of R.

22. (a) Show that if G is a finite sub-group of the multiplicative group $(\mathrm{F}^*, \cdot)$ of a field F, then G is cyclic.

(b) Let F be a field. Show that every ideal in $\mathrm{F}[x]$ is principal.

(3 × 8 = 24 marks)

D 54090 **(Pages : 3)** Name.................................

Reg. No...............................

FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2023

(CCSS)

Mathematics

MAT 1C 02—LINEAR ALGEBRA

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 2 *marks.*

1. Show that the vectors $\alpha_1 (1, 0, -1), \alpha_2 (1, 2, 1), \alpha_3 (0, -3, 2)$ forms a basis for R^3.
2. Describe the linear transformation T from F^2 into F^2 such that T (1, 0) = (a, b), T (0, 1) = (c, d).
3. Let V be the space of all polynomial function from R into R of the form $f(X) = C_0 + C_1X + C_2X^2 + C_3X^3$ then if D is the differential operator, find $[D]_B$ where B is the set of f_1, f_2, f_3, f_4 defined by $f_j(X) = X^{j-1}$.
4. Find two linear operators T and U on R^2 such that TU = 0 but UT $\neq$ 0.
5. Prove that characteristic space of T associated with c is the nullspace of T$- cI$.
6. Characterise a diagonalizable matrix.
7. Let E be a projection. Let R be the range of E on vector space and N be its nullspace.Then prove that the vector β is in the range R of E if and only if E $(\beta) = \beta$.
8. If V is an inner product space ,then for any vectors α, β in V and any scalar c, prove that $\|\alpha + \beta\| \leq \|\alpha\| + \|\beta\|$.

Part B

Answer any **four** *questions.*
Each question carries 4 *marks.*

9. Let S be linearly independent subset of a vector space V. Suppose β is a vector in V which is not in the subspace spanned by S.Then prove that the set obtained by adjoining β to S is linearly independent.

Turn over

10. Show that $F^{m \times n}$ is isomorphic to F^{mn}.

11. Let V be the finite dimensional vector space over the field F and W be a subspace of V then prove that dim W + dim W^0 = dim V.

12. Let A be an $m \times n$ matrix over the field F .Then prove that the row rank of A is equal to the column rank of A.

13. Let T be a linear operator on an n dimensional vector space V. Prove that the characteristic and minimal polynomial for T have the same roots,except for multiplicities.

14. Let V be a vector space and E be a projection of V .Let R be the range of E on vector space V and N be its nullspace.Then prove that $V = R \oplus N$.

Part C

Answer A **or** B *of the following questions.*

Each question carries 12 *marks.*

15. (A) (a) If W is a subspace of a finite dimensional vector space V, every linearly independent subset of W is finite and is part of a basis for W.

 (b) If W is a proper subspace of a finite-dimensional vector space V ,then W is finite-dimensional and dim W < dim V .

 (B) State and prove Rank-Nullity theorem.

16. (A) Let $g, f_1, f_2, \ldots f_r$ be linear functional on vector space V with respective nullspaces $N_1, N_2, \ldots N_r$. Then show that g is a linear combination of $f_1, f_2, \ldots f_r$ iff N contains the intersection $N_1 \cap N_1 \cap \ldots \cap N_r$.

 (B) Let V and W be finite dimensional vector space over the field F. Let B be an ordered basis of V with dual basis B* and let B' be an ordered basis for W with dual basis (B') *.Let T be a linear transformation from V into W and let A be the matrix of T relative to B, B' and let C be the matrix of T^t relative to (B')*, B*.Then show that $C_{ij} = A_{ji}$.

17. (A) Let W be an invariant subspace for T. Prove that the characteristic polynomial for the restriction operator T_w divides the characteristic polynomial for T.And also prove that the minimal polynomial for Tw divides the minimal polynomial for T.

(B) Let V be a finite dimensional vector space over the field F and let T be a linear on V .Then prove that T is triangulable if and only if the minimal polynomial for T is a product of linear polynomials over F.

18. (A) If $V = W_1 \oplus ... \oplus W_k$, then prove that there exist k linear operators $E_1, ..., E_k$ on V such that :

(i) Each E_i is a projection.

(ii) $E_i E_j = 0$, if $i \neq j$.

(iii) $I = E_1 + ... + E_k$.

(iv) The range of E_i is W_i.

Conversely, if $E_i, ..., E_k$ are k linear operators on V which satisfy conditions (i), (ii) and (iii) and if we let W_i be the range of E_i, then prove that $V = W_1 \oplus ... \oplus W_k$.

(B) (a) Let V be an inner product space and let $\beta_1, ... \beta_n$ be any independent vectors in V Then prove that one may construct orthogonal vectors $\alpha_1, ...,$ in V such that for each $k = 1, 2,.., n$ the set $\{\alpha_1, ..., a_k\}$ is a basis for the subspace spanned by $\beta_1, ..., \beta_k$.

(b) Prove that every finite dimensional innerproduct space has an orthonormal basis.

D 54095 **(Pages : 4)** **Name.......................................**

Reg. No..................................

FIRST SEMESTER P.G. DEGREE EXAMINATION
NOVEMBER 2023

(CCSS)

Mathematics

MAT 102—LINEAR ALGEBRA

(2022 Admissions)

Time : Three Hours Maximum : 50 Marks

Part A

Answer **all** *questions.*
Each question carries 1 mark.

1. Show that the set of polynomials

$$\{f(x) \in \mathbb{R}[x] : f(0) = 0\}$$

is a subspace of the space $\mathbb{R}[x]$ of polynomials over $\mathbb{R}$.

2. Verify whether the set

$$\{(x, 1, y) : x, y \in \mathbb{R}\}$$

is a subspace of $\mathbb{R}^3$.

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x, y) = (x+1, 2x+1)$. Verify whether T is a linear transformation.

4. Find the null space of the linear functional f on $\mathbb{R}^2$ given by $f(x, y) = x + y$.

5. Find a characteristic value of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

Turn over

6. Let T be a linear operator on $\mathbb{R}^3$ given by $T(x, y, z) = (x + y, y, z)$. Find a one dimensional T-invariant subspace of $\mathbb{R}^3$.

7. Give an example of a projection operator on $\mathbb{R}^2$ with range $W = \{(x, x) : x \in \mathbb{R}\}$.

8. Give an example of an inner product on $\mathbb{R}^2$.

(8 × 1 = 8 marks)

Part B

Answer any **six** *questions.*
Each question carries 3 *marks.*

9. Consider the vector space $\mathbb{R}^3$ over $\mathbb{R}$. Show that $(1, 1, 2)$ is not in the span of $\{(1, 0, 0), (0, 1, 1)\}$.

10. Let V be a vector space over a field F and let $\alpha, \beta \in V$. Describe the span of $\{\alpha, \beta\}$.

11. Let S be a linearly independent subset of a vector space V. Let $\beta \in V$ such that β is not in the span of S. Show that $S \cup \{\beta\}$ is a linearly independent set.

12. Let T be a one to one linear transformation on a finite dimensional space. Show that T^{-1} is a linear transformation.

13. Let V be a finite dimensional space and V* be the dual space. Describe a basis of V* in terms of a basis of V.

14. Let T be a linear operator on a vector space V and let α, β be non zero vectors in V such that $T(\alpha) = 2\alpha$ and $T(\beta) = 3\beta$. Show that $\{\alpha, \beta\}$ is a linearly independent set.

15. Find the minimal polynomial of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

16. Let W_1, W_2 be subspaces of a vector space V and let $V = W_1 + W_2$. . Show that if $\dim V = \dim W_1 + \dim W_2$ then $V = W_1 \oplus W_2$.

17. Let $W = \{(x, 2x) : x \in \mathbb{R}\}$ be a subspace of $\mathbb{R}^2$ with usual inner product. Find a best approximation for $(1, 1)$ in W.

(6 × 3 = 18 marks)

Part C

Answer any **three** *questions.*
Each question carries 8 *marks.*

18. (a) Let V be a vector space over a field F and S be a subset of V. Describe the span of S.

(b) Show that span of S is a subspace of V.

(c) Find non zero and proper subspaces W_1 and W_2 of $\mathbb{R}^3$ such that $W_1 + W_2 = \mathbb{R}^3$.

19. (a) Describe the co-ordinate matrix $[\alpha]_B$ of a vector α in a finite dimensional vector space V relative to an ordered basis B of V.

(b) Prove that :

(i) if B and B′ are ordered bases of V then

$$[\alpha]_B = P[\alpha]_{B'}$$

for an invertible matrix P.

(ii) P is unique.

20. Let V be a finite dimensional vector space and $T : V \to V$ be a linear operator. Prove that the following are equivalent.

(a) T is one to one.

(b) $T\alpha = 0$ for $\alpha \in V$ implies $\alpha = 0$.

(c) T maps every linearly independent subset of V onto a linearly independent set.

21. (a) Define characteristic value of a linear operator.

(b) Let T be a linear operator on a finite dimensional vector space and *c* be a scalar. Prove that the following are equivalent.

(i) *c* is a characteristic value of T.

Turn over

(ii) $T - cI$ is not invertible.

(iii) $\det(T - cI) = 0$.

22. Let $W_1, W_2, ..., W_k$ be subspaces of a vector space V and let $V = W_1 \oplus W_2 \oplus ..... \oplus W_k$. Prove that there exist projections $E_i : i = 1, 2, ..., k$ on V with range W_i such that

(a) $E_i E_j = 0$ for $i \neq j$.

(b) $I = E_1 + E_2 + ... + E_k$ where I is the identity transformation.

(3 × 8 = 24 marks)

D 54098 **(Pages : 4)** **Name.......................................**

Reg. No..................................

FIRST SEMESTER P.G. DEGREE EXAMINATION NOVEMBER 2023

(CCSS)

Mathematics

MAT 105—NUMBER THEORY

(2022 Admissions)

Time : Three Hours Maximum : 50 Marks

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Let p be an odd prime. If $(a, p) = 1$, then prove that $\left(\frac{a^2}{p}\right) = 1$.

2. For any real number x prove that $[x] + \left[x + \frac{1}{2}\right] = [2x]$.

3. Convert the continued fraction $\langle -3, 2, 12 \rangle$ into a rational number.

4. Define the n^{th} convergent of an infinite continued fraction.

5. Let $T(x) = \sum_{1 \le n \le x} \log n$. For every real number $x \ge 1$, prove that $T(x) \ge x \log x - x - \log x$.

6. Define the abscissa of absolute convergence of the Dirichlet series.

7. What is a Cryptosystem ?

8. Give two disadvantages of deterministic encryption.

(8 × 1 = 8 marks)

Turn over

Part B

Answer any **six** *questions.*
Each question carries 3 *marks.*

9. Show that if p is a prime of the form $4k+1$, then the sum of the quadratic residues $(\text{mod } p)$ in the interval $[1, p)$ is $p(p-1)/4$.

10. Find all primes p such that $x^2 \equiv 13 \pmod{p}$ has a solution.

11. For every positive integer n, prove that $\sum_{d/n} \phi(d) = n$.

12. Prove that the value of any infinite simple continued fraction $\langle a_0, a_1, a_2, ...\rangle$ is irrational.

13. Expand $\sqrt{2}-1$ as an infinite simple continued fraction.

14. Suppose that $x \geq 2$. Prove that

$$\sum_{n \leq x} \frac{\Lambda(n)}{n} = \log x + \mathrm{O}(1).$$

15. Show that the Dirichlet series $\sum \frac{2^n}{n^s}$ diverges for all s.

16. The ciphertext "OFJDFOHFXOL" was intercepted. The ciphertext was enciphered using an affine transformation of single-letter plaintext units in the 27-letter alphabet (with blank = 26) and it is known that the first word is "I "("I " followed by blank). Determine the enciphering key, and read the message.

17. How can the communication be authenticated in public key cryptography ?

(6 × 3 = 18 marks)

Part C

Answer any **three** *questions.*
Each question carries 8 marks.

18. (a) If p is an odd prime and $(a, 2p) = 1$, then prove that $\left(\frac{a}{p}\right) = (-1)^t$ where $t = \sum_{j=1}^{(p-1)/2} \left[\frac{ja}{p}\right]$.

(b) Determine whether $x^4 \equiv 25 \pmod{1013}$ is solvable, given that 1013 is a prime.

19. (a) What is the highest power of 3 dividing 533 !.

(b) If n is any even integer, prove that $\sum_{d/n} \mu(d)\phi(d) = 0$.

20. (a) If $\langle a_0, a_1, \ldots\ldots, a_j \rangle = \langle b_0, b_1, \ldots\ldots, b_n \rangle$ where these finite continued fractions are simple, and if $a_j > 1$ and $b_n > 1$, then prove that $j = n$ and $a_i = b_i$ for $i = 0, 1, \ldots\ldots n$.

(b) Let ξ be an irrational number and let a/b be a rational number with $b > 0$. If $|\xi b - a| < |\xi k_n - h_n|$ for some $n \geq 0$, then prove that $b \geq k_{n+1}$.

21. (a) For $x \geq 2$, prove that $\pi(x) = \frac{\vartheta(x)}{\log x} + O\left(x/(\log x)^2\right)$.

(b) Let σ_a be the abscissa of absolute convergence of the Dirichlet series $A(s) = \sum_{n=1}^{\infty} a_n/n^s$.

Prove that σ_a is also the abscissa of absolute convergence of the series $B(s) = -\sum_{n=1}^{\infty} a_n (\log n)/n^s$.

Turn over

22. (a) The message "!IWGVIEX!ZRADRYD ", which was sent using a linear enciphering transformation of digraph-vectors in a 29-letter alphabet in which A-Z have numerical equivalents 0 - 25, *blank* = 26, ? = 27, ! = 28 was intercepted. The last five letters of plain text are the sender's signature "MARIA ". Find the deciphering matrix and read the message.

(b) Suppose that the following 40-letter alphabet is used for all plaintexts and ciphertxts : A – Z with numerical equivalents 0 – 25, *blank* = 26, . = 27, ? = 28, $ = 29, the numerals 0 – 9 with numerical equivalents 30 – 39. Suppose that plaintext message units are digraphs and ciphertext message units are trigraphs $\left(\text{i.e., } k = 2, l = 3, 40^2 < n_A < 40^3 \text{ for all } n_A\right)$. Send the message "GIVE $5500" to a user whose enciphering key is $(n_A, e_A) = (2047, 179)$.

(3 × 8 = 24 marks)

D 54096 **(Pages : 3)** **Name.......................................**

Reg. No...................................

FIRST SEMESTER P.G. DEGREE EXAMINATION NOVEMBER 2023

(CCSS)

Mathematics

MAT 103—REAL ANALYSIS—I

(2022 Admissions)

Time : Three Hours Maximum : 50 Marks

Part A

Answer **all** *the questions.*
Each question carries 1 *mark.*

1. Define closed ball in R^k. Prove that closed ball in R^k is a convex set.
2. If p is a limit point of a set E, then prove that every neighborhood of p contains infinitely many points of E.
3. If a set F is closed, then prove that its complement is open.
4. Prove that any finite set is compact.
5. If a mapping f of a metric space X into a metric space Y is continuous, then prove that for every closed set C in Y, $f^{-1}(C)$ is closed in X.
6. Prove that continuous image of a compact metric space is compact.
7. Prove that the derivative of any constant function is zero.
8. State Taylor's theorem.

(8 × 1 = 8 marks)

Part B

Answer any **six** *questions.*
Each question carries 3 *marks.*

9. Prove that a set E is open if and only if $E^o = E$ where E^o is the interior of E.
10. Define connected subset of a metrix space. State and prove a necessary condition that a subset E of the real line R^1 is connected.

Turn over

11. Define uniformly continuous mapping on a metric space. If f isa continuous mapping of a compact metric space X into a metric space Y, then prove that f is uniformly continuous on X.

12. Let f be a continuous real function on the interval $[a, b]$. If $f(a) < f(b)$ and if c is a number such that $f(a) < c < f(b)$, then prove that there exists a point $x \in (a, b)$, such that $f(x) = c$.

13. Define

$$f(x) = \begin{cases} 1 & (x \text{ rational}) \\ 0 & (x \text{ irrational}) \end{cases}.$$

Prove that f has a discontinuity of the second kind at every point x.

14. Let f be defined on $[a, b]$. If f is differentiable at a point $x \in (a, b)$, then prove that f is continuous at x.

15. State and prove the fundamental theorem of calculus.

16. Let α be monotonically increasing on $[a, b]$. Suppose $f_n \in \mathcal{R}(\alpha)$ on $[a, b]$, for $n = 1, 2, 3, \ldots..$, and suppose $f_n \to f$ uniformly on $[a, b]$. Then prove that

$$f \in \mathcal{R}(\alpha) \text{ on } [a, b] \text{ and } \int_a^b f \, d\alpha = \lim \int_a^b f_n \, d\alpha.$$

17. If the sequences $\{f_n\}$ and $\{g_n\}$ converge uniformly on a set E, prove that the sequence $\{f_n + g_n\}$ converges uniformly on E.

(6 × 3 = 18 marks)

Part C

Answer any **three** *questions.*
Each question carries 8 marks.

18. (a) Let Y be a subspace of a metric space X. Prove that a subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X.

(b) Prove that every bounded infinite subset of $\mathcal{R}^k$ has a limit point in $\mathcal{R}^k$.

19. (a) If $\{I_n\}$ is a sequence of intervals in R^1, such that $I_{n+1} \subset I_n, n = 1, 2, 3, \ldots,$ then prove that $\bigcap_1^\infty I_n$ is not empty.

    (b) Define uniformly continuous function. Prove that a uniformly continuous function of a uniformly continuous function is uniformly continuous.

20. State and prove the L'Hospital's rule.

21. (a) Define the Riemann integral of a function f over $[a, b]$.

    (b) Suppose $f \geq 0, f$ is continuous on $[a, b]$ and $\int_a^b f(x)\,dx = 0,$ then prove that $f(x) = 0$ for all $x \in [a, b]$.

22. (a) Suppose $\{f_n\}$ is a sequence of functions defined on E, and suppose $|f_n(x)| \leq M_n \ (x \in E, n = 1, 2, 3)$. Then prove that $\sum f_n$ converges uniformly on E if $\sum M_n$ converges.

    (b) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

(3 × 8 = 24 marks)

D 54097 **(Pages : 4)** **Name.......................................**

Reg. No..................................

FIRST SEMESTER P.G. DEGREE EXAMINATION NOVEMBER 2023

(CCSS)

Mathematics

MAT 104—SCIENTIFIC PROGRAMMING USING PYTHON—3

(2022 Admissions)

Time : Three Hours Maximum : 50 Marks

Part A

Answer **all** *the questions.*
Each question carries 1 *mark.*

1. Write the output of the followoing Python code.

```
print (1j*1j)
```

2. Write the output of the following Python code.

```
N = 10
if N % 2 == o :
    print ( ' { } is odd' .format (N) )
else :
    print ( ' { } is even'.format (N) )
```

3. Write the output of the following Python code.

```
dic = {' a ': 1, 'b' : 2,     'c' : 3 }
for item in dic.keys ( ) :
    print (item)
```

Turn over

4. Write the output of the following Python code.

```
a, b = 0, 1
print (a)
print (b)
for x in range(5) :
    a, b = b, a + b
    print (b)
```

5. Write the output of the following Python code. import numpy as np

```
arr = np.array ( [ [ [1, 2, 3], [4, 5, 6] ], [ [7, 8, 9], [10, 11, 12] ] ] )
print (arr [0, 1, 2])
```

6. Write the output of the following Python code.

```
from sympy import symbols, expand
x, y = symbols ('x, y')
expr = (x + y) * (x – y)
print (expand (expr) )
```

7. Write the output of the following Python code.

```
from sympy import Symbol, diff, sin
x = Symbol ('x')
expr = sin (x)
print (diff (expr, x, 2 ) )
```

8. Write the output of the following Python code.

```
from sympy import symbols, integrate
x, y = symbols ('x y')
expr = x**2 + 3* x + 2
print (integrate (expr, (x, 0, y ) ) )
```

(8 × 1 = 8 marks)

Part B

Answer any **six** *questions.*
Each question carries 3 marks.

9. Write any two difference between list and tuple in Python.

10. Write a Python program to print a given string in reverse order.

11. Write a short note on 'math' module in Python.

12. Differentiate range and arange.

13. Write a Python program to print the Pythagorean triplets bellow 100.

14. Write a Python program to draw a bar chart representing following data :

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Humidity	70	70	50	70	70	75	80	85	75	70	65	70

15. Write any three functions in cmath module in Python.

16. Explain the method of including the title, axes labels and legends in a plot.

17. Explain Simpson Newton's method for finding approximate roots of equations.

(6 × 3 = 18 marks)

Part C

Answer any **three** *questions.*
Each question carries 8 marks.

18. (a) Differentiate with examples : for and while loops.

    (b) Write a Python program to check a given number is prime.

19. Consider the sequence given by the following rule :

$$a_{n+1} = \begin{cases} a_n/2 & \text{if } a_n \text{ is even} \\ 3a_{n+1} & \text{if } a_n \text{ is odd} \end{cases} \quad \text{for } n = 0, 1, \ldots..$$

Write a Python program to check whether the sequence attains the value 1 when $a_0 = 10$.

Turn over

20. Consider the function $f = x^2y - xy + xy^2$. Write a Python program to find the following :

    (a) The first order partial derivatives of f.

    (b) Put $y = 1 - x$ in f and find the derivative with respect to x.

21. Write a Python program to find the standard deviation of the following data :

x	:	20	23	26	29	32
f	:	7	12	13	11	8

22. (a) Write a Python program to solve the quadratic equation $2x^2 - 5 * x + 3 = 0$.

    (b) Write a Python program to solve the following system of linear equations :

$$x + y + z = 3$$
$$x - y + z = 1$$
$$x + y - z = 1.$$

(3 × 8 = 24 marks)

D 54091 **(Pages : 3)** **Name..**

Reg. No....................................

FIRST SEMESTER P.G. DEGREE EXAMINATION
NOVEMBER 2023

(CCSS)

Mathematics

MAT 1C 03—REAL ANALYSIS—I

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 2 marks.

1. Prove that a set E is closed iff $E = \bar{E}$.
2. Define connected sets and give a characterisation of the connected subsets of the real line.
3. Distinguish between continuity and uniform continuity. Give example for both.
4. If f differentiable in (a, b) and $f'(x) = 0, \forall x \in (a, b)$, then prove that f is constant
5. Evaluate $\int_0^1 x^2 \, d\left(x^2 + x\right)$.
6. Give an example of a sequence of functions such that $f_n \to f$ but $f_n' \nrightarrow f'$.
7. State Cauchy's criteria for uniform convergence of sequence of functions.
8. State Stone Weierstrass theorem.

(8 × 2 = 16 marks)

Turn over

Part B

Answer any **four** *questions.*
Each question carries 4 marks.

9. Prove that a set E is open iff its compliment is closed.

10. Prove that compact subsets of a metric space are closed.

11. State and prove the generalised Mean value theorem .

12. Prove that if P^* refinement of P then $L(P, f, \alpha) \le L(P^*, f, \alpha)$

13. Prove that every monotonic functions are Reimann Stieltjes integrable.

14. State and prove Weierstrass M- test for uniform convergence of series of functions.

(4 × 4 = 16 marks)

Part C

Answer A **or** *B of the following questions.*
Each question carries 12 marks.

UNIT 1

15. (A) (a) Prove that every bounded infinite set in R^k has a limit point in R^k.

(b) Let X be a metric space and $E \subset X$. If p is a limit point of E then show that every neighbourhood of p contains infinitely many points of E.

(B) (a) Let P be a nonempty perfect set in in R^k. Then show that P is uncountable.

(b) Let $\{F_\alpha\}$ be a collection of closed subsets of a metric space X, Prove that $\bigcap_\alpha F_\alpha$ is closed.

UNIT 2

16. (A) (a) If f is a continuous mapping from a metric space X in to a metric space Y, and if E a connected subset of X. Then show that $f(E)$ is connected in Y.

(b) Prove that continuous image of a compact metric space is compact.

(B) (a) State and prove Taylors theorem for differentiable functions.

(b) Prove that the inverse of a bijective continuous map from a compact metric space onto a compact metric space is continuous.

UNIT 3

17. (A) (a) Prove that $f \in R(\alpha)$ on $[a, b]$ iff for any $\varepsilon > 0 \exists$ a partition P of $[a, b]$ such that $U(P, f\ \alpha) - L(P, f, \alpha) < \varepsilon$.

(b) Suppose $c_n \geq 0$, for $n = 1, 2, 3....$, $\sum c_n$ converges and $\{s_n\}$ is a sequence of distinct points on (a, b) and $\alpha(x) = \sum_{n=1}^{\infty} c_n I(x - s_n)$, Let f continuous on $[a, b]$ then prove that $\int_a^b f\, d\alpha = \sum_{n=1}^{\infty} c_n f(s_n)$.

(B) (a) Suppose $f \in R(\alpha)$ on $[a, b]$, $m \leq f \leq M$, ϕ is continuous on $[m, M]$ and $h(x) = \phi(f(x))$ on $[a, b]$. Then prove that $h \in R(\alpha)$ on $[a, b]$.

(b) Define rectifiable curves and prove that If γ' is continuous on $[a, b]$, then γ rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)|\, dt$

UNIT 4

18. (A) (a) If $\{f_n\}$ is a sequence of continuous functions on E and $f_n \to f$ uniformly on E, then f is continuous on E. Prove.

(b) Show by an example that $\{f_n\}$ converges to a continuous function not imply that the convergence is uniform.

(B) (a) Let $\{f_n\}$ is a sequence of functions differentiable on $[a, b]$ and such that $f_n(x_0)$ converges for some $x_0 \in [a, b]$ and f_n' converges uniformly on $[a, b]$ then prove that $\{f_n\}$ convergence uniformly on $[a, b]$ to a function f and $f'(x) = \lim_{n \to \infty} f_n'(x)$.

(b) State and prove the theorem to establish sequence of integrable functions converge to an integrable function and $\lim \int f_n = \int \lim f_n$.

(4 × 12 = 48 marks)