6464 (Pages : 3) Name..................................

Reg. No..................................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2010

Mathematics

Elective—ADVANCED COMPLEX ANALYSIS

(2002 admission onwards)

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 4 marks.

1. (a) Find the region of convergence of the infinite product $\underset{R=1}{\overset{\infty}{\pi}}\left(1+Z^k\right)$.

   (b) Find the order of the entire function cos Z.

   (c) Find an analytic continuation of $\sum_{R=0}^{\infty} Z^K$ in the unit disc D to the disc D (–1, 2).

   (d) Show that if F C H (G) is normal, then $F^1 = \left\{f^1 : f \in F\right\}$ is also normal.

(4 × 4 = 16 marks)

Part B

Answer any **four** *questions without omitting any unit.*
Each question carries 16 marks.

UNIT I

2. (a) Let $\{a_j\}_{j=1}^{\infty}$ be a reference of distinct points having no finite accumulation point and let a sequence $\{k_j\}_{j=1}^{\infty}$ of positive integers be given. Then prove that there exists an entire function having roots of multiplicity k_j at a_j for all $j \in \mathbb{N}$ and nowhere else.

   (b) Prove that $\sum_{\substack{k>0 \\ k\ odd}} \frac{1}{k^2} = \frac{\pi^2}{8}$.

Turn over

3. (a) Prove : A necessary and sufficient condition for the convergence of the infinite product $\overset{\infty}{\underset{k=0}{\pi}} a_k$ is that for every $\varepsilon > 0$, there exists an integer $N = N(\varepsilon)$ such that $\left|\left(\overset{m}{\underset{k=n+1}{\pi}} a_k\right) - 1\right| < \varepsilon$ wherever $m > n \geq N$.

   (b) Prove that the convergence and the value of an absolutely convergent infinite product are independent of the order of its factors.

4. (a) Derive Jensen's formula.

   (b) Prove Wallis' formula : $\frac{2}{\pi} = \overset{\infty}{\underset{n=1}{\pi}}\left(1 - \frac{1}{(2n)^2}\right)$.

UNIT II

5. (a) Let $f(z)$ be an entire function of finite order P, $f(0) \neq 0$, having roots $a_1, a_2 \ldots a_k \ldots$, repeated according to multiplicity. Then prove that, for pH > P

$$\frac{d^p}{dz^p}\left\{\frac{f'(z)}{f(z)}\right\} = -p!\sum_{k=1}^{\infty}\frac{1}{(a_k - z)^{p+1}}.$$

   (b) Prove that an entire function of finite noninteger order assume every complex value infinctely many times.

6. (a) State Runge's theorem. Deduce Mittag-Leffler's theorem for a general region, using Runge's theorem.

   (b) Construct an entire function $f(z)$ such that

$$f(n) = n!\,(n = 0, 1, 2, \ldots).$$

7. (a) Prove that a lacunary power series has its circle of convergence as a natural boundary.

   (b) Prove Poincar-Volterna theorem : An analytic function can take at most countably many destinct values at any point in the complex plane($\mathbb{C}$).

UNIT III

8. (a) State and prove the Schwarz symmetry principle.

   (b) Suppose $F \subset C(G, \Omega)$ is equicontinuous at each point of G. Then prove that F is equicontinuous over each compact subset of G.

9. (a) State and prove Montel's theorem.

(b) Let $\{f_n\}$ be a sequence in M(G) and suppose $f_n \to f$ in $C(G, \mathbb{C}_\infty)$. Then prove that either f is meromorphic of $f \equiv \infty$.

10. Let G be a region which is not the whole plane and such that every non-vanishing analytic function on G has an analytic square root. Let $a \in G$. Then prove that there is an analytic function f on G such that

(i) $f(a) = 0$ and $f^1(a) > 0$.

(ii) f is one-one.

(iii) $f(G) = \{z : |z| < 1\}$.

(4 × 16 = 64 marks)

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Name..................................

Reg. No..............................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2010

Mathematics

PROBABILITY THEORY

(2002 admission onwards)

Time : Three Hours

Maximum : 80 Marks

Part A

Answer all the questions
Each question carries 4 marks

I (a) What are the events when four dice are thrown simultaneously. Also obtain the probability that two faces show the same number.

(b) Define the Binomial distribution. Obtain the variance of the Binomial distribution.

(c) State the Kolmogorov 0 – 1 law with necessary explanations.

(d) State the Lindeberg - Feller theorem.

Part B

Answer any four questions without omitting any unit
Each question carries 16 marks

UNIT I

II (a) Prove that continuous real valued functions on the set of real numbers are Borel functions.

(b) Prove that the class of random variables is equal to the class of uniform (finite) limits of sequences of elementary random variables.

III (a) Define the Hyper geometric distribution. Illustrate the distribution using an example.

(b) Prove that the distribution function F_X of a random variable X is non-decreasing continuous on the right with $F_X(-\infty) = 0$ and $F_X(+\infty) = 1$. Conversely prove that every function F with these properties is the distribution function of a random variable on some probability space.

Turn over

IV (a) Prove that if $\{A_n\}$ is a sequence of random variables that converges to A, then $\{P(A_n)\}$ converges to $P(A)$.

(b) State and prove Jordan Decomposition theorem.

UNIT II

V (a) Define expectation of a random variable. Prove that expectation is a measure of central tendency.

(b) State and prove the Holder's inequality for the expectation function.

VI (a) Define moment generating function of a random variable. For the Cauchy distribution, prove that the moment generating function does not exist.

(b) If $f(x)$ is a continuous real valued function and X_n converges to X in probability, then prove that $f(X_n)$ converges to $f(X)$ in probability.

VII (a) Prove that if a sequence of random variables $\{X_n\}$ is such that $|X_n| \leq Y$, Y-integrable, then X_n converges to X almost surely implies EX_n converges to EX.

(b) Define characteristic function of distribution functions. Prove that characteristic function of a general distribution function is continuous.

UNIT III

VIII (a) When are two events independent ? Give example of independent events. Prove that sub-classes of independent classes are independent.

(b) State and prove the Multiplication theorem on independent random variables.

IX (a) State and prove the theorem on Borel almost sure criterion.

(b) If X_n's are uniformly bounded and $\sum X_n$ converges a.s., then prove that $\sum \sigma_n^2$ and $\sum EX_n$ converge.

X (a) Define the Radon-Nikodym derivative.Prove that Radon-Nikodym derivative is unique up to sets of $P-$measure zero.

(b) State and prove Kronecker's lemma.

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Name..................................

Reg. No...............................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2010

Mathematics

MEASURE AND INTEGRATION

(2002 admission onwards)

Time : Three Hours Maximum : 80 Marks

Part A

Answer all questions. Each question carries 4 marks

I (a) Let f be a continuous function and g a measurable function. Show that the composite function $f \circ g$ is measurable.

(b) Let $\mathcal{M}$ be a σ-algebra in a set X and μ be a positive measure on $\mathcal{M}$. Let $A, B \in \mathcal{M}$, $A \subset B$ and $f \geq 0$. Prove that $\int_A f\, d\mu \leq \int_B f\, d\mu$.

(c) Give an example of a measure which is not complete.

(d) Show that every compact subset of $\mathbb{R}^1$ is the support of a Borel measure.

(e) Let μ, λ be measures on a σ-algebra $\mathcal{M}$ with μ positive. If $\lambda \ll \mu$, then prove that $|\lambda| \ll \mu$.

($4 \times 4 = 16$ marks)

Part B

Answer any four questions without omitting any unit.
Each question carries 16 marks.

Unit I

II (a) Let X be a measurable space and let $f = u + iv$ be a complex measurable function on X. Prove that u, v and $|f|$ are real measurable functions on X.

(b) Let X be a measurable space and let $f : X \to [0, \infty]$ be a measurable function. Prove that there exist simple measurable functions s_n on X such that $0 \leq s_1 \leq s_2 \leq \cdots \leq f$ and for every $x \in X$, $s_n(x) \to f(x)$ as $n \to \infty$.

III (a) Let X be a measurable space, the functions $f_n : X \to [-\infty, \infty]$ be measurable for $n = 1, 2, \ldots$ and let $f = \sup_{n \geq 1} f_n$. Prove that f is measurable.

Turn over

(b) Let X be a measurable space and $f_n : X \to [0, \infty]$ be a measurable function, for each positive integer n. Prove that

$$\int_X (\liminf_{n\to\infty} f_n)\, d\mu \leq \liminf_{n\to\infty} \int_X f_n\, d\mu.$$

IV (a) Let $f : X \to [0, \infty]$ be measurable, E be a measurable set and $\int_E f\, d\mu = 0$. Prove that $f = 0$ *a.e.*.

(b) State and prove Lebesgue's dominated convergence theorem.

UNIT II

V (a) Let X be a locally compact Hausdorff space in which every open set is σ-compact. Let λ be any positive Borel measure on X such that $\lambda(K) < \infty$ for every compact set K. Prove that λ is regular.

(b) Prove that every set of positive measure has nonmeasurable subsets.

VI (a) Let $f \in L^1(\mu)$, f be real valued and $\epsilon > 0$. Prove that there exist functions u and v on X such that $u \leq f \leq v$, u is semicontinuous and bounded above, v is lower semicontinuous and bounded below and $\int_X (v - u)\, d\mu < \epsilon$.

(b) Let μ be a complex measure on X. Prove that $|\mu|(X) < \infty$.

VII (a) State and prove the Hahn decomposition theorem.

(b) Is Hahn decomposition unique? Justify your answer.

UNIT III

VIII (a) Let μ be a complex Borel measure on $\mathbf{R}^k$ and let λ be a positive number. Prove that $m\{M\mu > \lambda\} \leq 3^k \dfrac{|\mu|(\mathbf{R}^k)}{\lambda}$.

(b) Let $f \in L^1(\mathbf{R})$ and $F(x) = \int_{-\infty}^{x} f\, dm$ where $-\infty < x < \infty$. Prove that $F'(x) = f(x)$ at every Lebesgue point of f.

IX (a) Let $f : [a, b] \to \mathbf{R}^1$ be differentiable at every point of $[a, b]$ and $f' \in L^1$ on $[a, b]$. Prove that

$$f(x) - f(b) = \int_a^x f'(t)\, dt$$

where $a \leq x \leq b$.

(b) Let $(X, \mathcal{S})$ and $(Y, \mathcal{T})$ be measure spaces. Prove that $\mathcal{S} \times \mathcal{T}$ is the smallest monotone class which contains all elementary sets.

X (a) Let ν be a positive measure on a σ-algebra $\mathcal{M}$ and let $\mathcal{M}^*$ be the completion of $\mathcal{M}$ with respect to ν. If f be an $\mathcal{M}^*$ measurable function, then prove that there exists an $\mathcal{M}$ measurable function g such that $f = g$ *a.e.* $[\nu]$.

(b) Prove that $L^1(\mathbf{R}^1)$ is closed under convolution.

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Name.................................

Reg. No..............................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JULY 2009

Mathematics—Elective

PROBABILITY THEORY

(2002 admission onwards)

Time : Three Hours

Maximum : 80 Marks

Part A

Answer all the questions.
Each Part carries 4 marks.

I (a) List the set of all events in the experiment of tossing two coins simultaneously. What is the probability that both show heads ?

(b) Obtain the variance of the Hyper geometric distribution.

(c) State the Kolmogorov three series theorem with necessary details.

(d) Prove that the distribution function F of a random variable and its characteristic function determine each other.

Part B

Answer any four questions without omitting any unit.
Each question carries 16 marks.

UNIT I

II (a) Derive the Multinomial distribution.

(b) Show that binomial distribution converges to Poisson distribution as $n \to \infty$ and $p \to 0$.

III (a) State and prove Jordan Decomposition theorem.

(b) Prove probability function defined on all intervals of the form $(a, b]$ defines uniquely an extension to the minimal field consisting all the intervals.

IV (a) Prove that the probability function is a continuous function.

(b) Prove that any Borel function of a vector random variable (X, Y) is a random variable.

UNIT II

V (a) If Z is a complex random variable, prove that $|EZ| \leq E|Z|$.

(b) If $X \geq 0$ and is integrable, then prove that X can be infinite at most on a set of probability measure zero.

VI (a) Prove the Holder's inequality for expectation function.

(b) For any characteristic function ϕ, prove that

$$|\phi(u) - \phi(u+h)|^2 \leq 2(1 - \text{Re}phi(h)).$$

VII (a) If $\{F_n\}$ is an infinite sequence of distribution functions, then prove that there exists a sub sequence that converges weakly.

(b) Obtain the Taylor series expansion for the characteristic function ϕ (u).

UNIT III

VIII (a) Define independent events. Prove that sub-classes of independent events are independent.

(b) If $X_1, X_2, \cdots, X_n$ are independent, prove that $E\left(\prod_{k=1}^{n} X_k\right) = \prod_{k=1}^{n} EX_k$, provided both sides exist.

IX (a) State the Kolmogorov inequalities with necessary explanations.

(b) Prove that if $\sum PA_n \leq \infty$, then $P(\overline{\lim} A_n) = 1$.

X (a) Discuss the weak law of large nnmbers.

(b) State and prove Kronecker's lemma.

C 48624 (Pages 2) Name..............................

Reg. No..............................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, SEPTEMBER 2008

Mathematics (Elective)

ADVANCED COMPLEX ANALYSIS

(2002 Admissions)

Time : Three Hours | Maximum : 80 Marks

Part A

Answer **all** *the questions.*
Each question carries 4 marks.

1. (a) Prove that $\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$.

   (b) If f is an entire function such that $|f(0)| = 3$ and $M(R) \le 180$, prove that f cannot have more than 5 roots inside or on the circle of radius R.

   (c) Let $f(a) = 1/Z$ and let k be the closed annulus $\{z : 1 \le |z| \le 2\}$. Does there exist a function g analytic in D (0, 3) such that $\|f - g\|_k < 1/4$? Justify your answer.

   (d) Let $f, f_1, \ldots f_n \ldots$ be element of H(G) such that $f_n(z) \to f(z)$ uniformly for z in $\{r\}$ for every closed rectifiable curve r in G. Prove that $f_n \to f$ in H (G).

(4 × 4 = 16 marks)

Part B

Answer any **four** *questions without omitting any unit.*
Each question carries 16 marks.

Unit I

2. (a) Let $\{a_j\}_{j=1}^{\infty}$ be a sequence of distinct points having no finite accumulation point and let a sequence of positive integers $\{k_j\}_{j=1}^{\infty}$ be given. Prove that there exists an entire function having roots of multiplicity k_j at a_j for all $j \in \mathbb{N}$ and nowhere else.

   (b) Show that the function :

$$g(z) = \frac{1}{z} + \sum_{n \ne 0} \left(\frac{1}{z-n} + \frac{1}{n} \right)$$

   is analytic in $\mathbb{C}$ except at the integer points where the singularities are simple poles.

Turn over

3. (a) Prove that the infinite product $\prod_{k=1}^{\infty}(1+C_k)$ converges absolutely if and only if the series $\sum_{k=1}^{\infty} C_k$ converges absolutely.

(b) Prove Euler's identity :

$$\prod_{k=1}^{\infty} \frac{1}{(1-p_k-s)} = \sum_{n=1}^{\infty} \frac{1}{n^s} \, (s>1).$$

4. (a) Let $f(z)$ be a bounded non zero analytic function, in the unit disc D. If $\{a_n\}$ is the sequence of zeroes of f in D, each repeated according to multiplicity, prove that the product $\prod_{n=1}^{\infty} |a_n|$ is convergent.

(b) Show that sin πz has the infinite product $\sin \pi z = \pi z \prod_{n=1}^{\infty}\left(1-z^2/n^2\right)$.

UNIT II

5. (a) State and prove that Hadamand factorization theorem.

(b) Prove that a non constant entire function of finite order assumes every complex value with *one* possible exception.

6. (a) State and prove Runge's theorem.

(b) Deduce Mittag-Leffler's theorem for a general region Ω, assuming the result for $\Omega = \mathbb{C}$.

7. (a) Let Ω be a region $\subset \hat{\mathbb{C}}$. Prove that there exists a function $f(z) \in H(\Omega)$ having $\partial\Omega$, the boundary of Ω, as its natural boundary.

(b) Let $f(z) = z + z^3 + z^9 \ldots + z^{3^k}$. Prove that the unit circle is the natural boundary of this power series.

UNIT III

8. (a) State and prove that monodromy theorem.

(b) Let $\{f_n\} \subset C(G, \Omega)$ and suppose that $\{f_n\}$ is equi continuous. If $f \in C(G, \Omega)$ and $f(z) = \lim_{n\to\infty} f_n(z)$ for each z, then prove that $f_n \to f$ in $C(G, \Omega)$.

9. (a) State and prove Montel's theorem.

(b) Let $\{f_n\} \subset H(G)$ be a sequence of one-to-one functions which converge to f in H(G). Prove that f is either one-to-one or a constant function.

10. (a) State and prove the Riemann mapping theorem.

(b) Prove the complex plane $\mathbb{C}$ is homomorphic to the unit open disc $\{z \mid |z| < 1\}$.

C 48622 (Pages 2) Name..................................

Reg. No..............................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, SEPTEMBER 2008

Mathematics

PROBABILITY THEORY—(Elective)

(2002 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carry 4 marks.

I. (a) Define random variable and give an example.

(b) Prove that the binomial distribution converges to Poisson distribution as n tends to ∞ and $p \to 0$.

(c) Prove that the distribution function F of a random variable and its characteristic function determine each other.

(d) Describe the invariance principle in the context of Central limit theorem.

(4 × 4 = 16 marks)

Part B

Answer any **four** *questions without omitting any unit.*
Each question carries 16 marks.

UNIT I

II. (a) Derive the multinomial distribution.

(b) Prove that continuous real valued function on R, the set of real numbers, are Borel functions.

III. (a) State and prove the Jordan Decomposition theorem for distribution functions.

(b) If $\{A_n\}$ is a sequence of events such that $A_n \to A$, then prove that $P(A_n) \to P(A)$.

IV. (a) Derive the hyper geometric distribution and hence define hyper geometric random variable.

(b) If X is a standard normal random variable, show that :

$$P\left[a < X^2 < b\right] = 2P\left[-\sqrt{b} < X < -\sqrt{a}\right].$$

UNIT II

V. (a) Define expectation of a random variable. If $a \le X \le b$, almost surely, then prove that $a \le EX \le b$.

(b) If $X_1, X_2, \ldots X_n$ are such that EX_i, $i = 1, 2, \ldots n$ and $\sum_{i=1}^{n} EX_i$ exist, then prove that :

$$E(\sum X_i) = \sum EX_i.$$

Turn over

VI. (a) Obtain the mean and variance of the discrete binomial distribution.

(b) Prove that if $X_n \to X$ in probability and $Y_n \to Y$ in probability, then $X_n + Y_n \to X + Y$ in probability.

VII. (a) Define the characteristics function of the bivariate random variable (X, Y). Prove that the characteristic function of a general distribution function F is continuous.

(b) With usual notations, prove that if :

$$0 \leq X_n \uparrow X, \text{ then } EX_n \uparrow EX.$$

UNIT III

VIII. (a) Prove that Borel functions of independent random variables are independent.

(b) If $X_1, X_2, \ldots X_n$ are independent, then prove that the characteristic function of $X_1 + X_2 + \ldots + X_n$ is the product of the characteristic functions of X_k' s.

IX. (a) State and prove the Kolmogorov Three-Series theorem.

(b) If $EX_i = 0$ and $E|X_k|^{1+\delta} \leq c < \infty$ for al k and some $\delta > 0$, then with usual notations prove that $S_n / n \to 0$ in probability.

X. (a) State the Lindeberg-Feller theorem with necessary explanation.

(b) If X_n are independent random variables and $X_n \to 0$ (a.s), then prove that $\sum P\left[|X_n| \geq c\right] < \infty$, whatever be $c > 0$, finite.

(4 × 16 = 64 marks)

C 48618 (Pages 2)

Name..................................

Reg. No..................................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, SEPTEMBER 2008

Mathematics

MEASURE AND INTEGRATION—Elective

(2002 Admissions onwards)

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions in this part.*
Each question carries 4 *marks.*

I. (a) Let μ be the counting measure on $\mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ be defined by :

$$f(x) = \begin{cases} \perp & \text{for } x = 0, 1 \text{ and } 2 \\ 0 & \text{otherwise.} \end{cases}$$

Find $\int f d\mu$.

(b) Define Borel set. Verify whether the set Q of rationals is a borel set in $\mathbb{R}$.

(c) Show that if λ and μ are complex measures and if $\lambda \perp \mu$ then $|\lambda| \perp |\mu|$.

(d) Let (X, S) and (Y, τ) be measurable spaces and $E \in S \times \tau$. Show that for each $x \in X$,

$E_x = \{ y \in Y : (x, y) \in E \}$ is in τ.

(4 × 4 = 16 marks)

Part B

Answer any **four** *questions from this part without omitting any unit.*
Each question carries 16 *marks.*

Unit I

II. (a) Define σ-algebra. Let S be any collection of subsets of a set X. Show that there exists a smallest σ-algebra m^* m X such that $S \subseteq m^*$.

(b) Let f be a complex measurable function on X. Show that there exists a complex measurable function α on X such that $|\alpha| = 1$ and $f = \alpha\ |f|$.

III. (a) Define $\int_E f d\mu$ for a complex measurable function $f \in L'(\mu)$ where E is a measurable set.

(b) Show that if $f, g \in L'(\mu)$ then $f + g \in L'(\mu)$ and $\int_X (f + g)\, d\mu = \int_X f d\mu + \int_X g d\mu$.

Turn over

IV. (a) State and prove Lebesgue's dominated convergence theorem.

(b) Suppose that $f: X \to [0, \infty]$ is measurable and for a measurable set E let $\int_E f d\mu = 0$. Show that $f = 0$ *a. e* on E.

UNIT II

V. (a) Describe Lebesgue measurable sets and Lebesgue measure in $\mathbb{R}^k$.

(b) Let m be the Lebesgue measure on $\mathbb{R}^k$. Prove that :

(i) m is translation invariant.

(ii) If μ is any positive translation invariant measure on $\mathbb{R}^k$ with $\mu(k) < \infty$ for every compact K, then there is a constant C such that $\mu(E) = c.m(E)$ for all Borel sets $E \subseteq \mathbb{R}^k$.

VI. (a) Let λ be a complex measure on a σ algebra m. Show that if λ is concentrated on A then $|\lambda|$ is also concentrated on A.

(b) Describe the Lebesgue decomposition of measure λ relative a measure μ. Prove that the decomposition is unique.

VII. (a) Let μ be complex measure on a σ-algebra m in X. Show that there is a measurable function h such that $|h(x)| = 1$ for all $x \in X$ and $d\mu = hd|\mu|$.

(b) Let μ be a positive measure on m, $g \in L'(\mu)$ and let $\lambda(E) = \int_E g d\mu$ for $E \in m$. Prove that :

$|\lambda|(E) = \int_E |g| d\mu$ for all $E \in m$.

UNIT III

VIII. (a) Describe the product $S \times \tau$ of two σ-algebras S on X and τ on Y.

(b) Prove that $S \times \tau$ is the smallest monotone class which contains all elementary sets.

IX. (a) Let m be the Lebesgue measure on $\mathbb{R}$. Show that $m \times m$ is not a complete measure.

(b) Let m_r, m_s be Lebesgue measures on $\mathbb{R}^r$ and $\mathbb{R}^s$ respectively and let $k = r + s$. Show that m_k is the completion of the product measure $m_r \times m_k$.

X. Let μ be a complex Borel measure on $\mathbb{R}^k$ and m be the Lebesgue measure. Prove that :

(a) $\mu \perp m$ if and only if $(D\mu)(x) = 0$.a.e.$[m]$.

(b) $\mu << m$ if $\mu(E) = \int_E (D\mu)(x)\, dx$ for every Borel set E.

(4 × 16 = 64 marks)

C 33626 (Pages : 2)

Name..................................

Reg. No...............................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, SEPTEMBER 2007

Mathematics

Paper XVI—COMPUTER ORIENTED NUMERICAL ANALYSIS

(2002 admissions onwards)

Time : One Hour and a Half

Maximum : 30 Marks

I. Answer the following questions :—

(a) What is meant by operating system in computers ? List some of the popular operating systems.

(b) Explain the various library functions in FORTRAN programming.

(c) Write a program to transpose a 2×3 matrix.

(d) Integrate $2x^3 + 3x^2 + 8x + 1$ from $x = -1$ to $x = 1$ using Simpsons's rule with $h = 1$.

(6 marks)

Answer the following questions.
Each question carries 6 marks.

II. (a) Draw a flowchart to compute, ${}_nC_r$, the number of combinations of n things taken r at a time.

Or

(b) What is the output of the following program :

```
READ (*, 20) I, J, K
20 FORMAT (315)
WRITE (*, 40) I, J, K
40 FORMAT (15/15/15)
END
```

III. (a) Write a FORTRAN program to read the three sides of a triangle and calculate its perimeter.

Or

(b) Write a program using direct access of a file to process a banks transactions. The program should be menu driven asking for DEPOSIT, WITHDRAWAL, CURRENT BALANCE and an option to quit. The information should be updated as and when the transaction takes place.

IV. (a) Solve the following equations by Gauss-Seidel procedure. The answer should be correct to 3 significant digits.

$$9x_1 + 2x_2 + 4x_3 = 20$$
$$x_1 + 10x_2 + 4x_3 = 6$$
$$2x_1 - 4x_2 + 10\,x_3 = -15$$

Or

Turn over

(b) Given the following data, find the cubic spline equations for the four intervals :—

x	1	2	3	4	5
$f(x)$	6	–3	6	2	–6

Find the value of $f(x)$ at $x = 3.8$.

V. (a) The population of a city in a census taken once in ten years is given below. Estimate the population in the years 1925, 1975 and 1984.

Year	1921	1931	1941	1951	1961	1971	1981
Population (in thousands)	35	42	58	84	120	165	220

Or

(b) Discuss the Predictor-Corrector method for the numerical solution of differential equations.

(4 × 6 = 24 marks)