D 12669 (Pages : 3) Name..................................

Reg. No.................................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016

(CCSS)

Mathematics

MAT 3E 02—OPERATIONS RESEARCH

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 4 *marks.*

1. Define a convex function and check whether the function $f(x) = 2x_1^2 + 2x_2^2 + 4x_3^2 + 2x_1x_2 + 2x_1x_3 + 4x_2x_3$ is a convex function.

2. Write the following linear programming problem in the standard form :

$$\text{Maximize } f(\mathrm{X}) = 2x_1 + x_2 - x_3$$
$$\text{subject to } \begin{aligned} 2x_1 - 5x_2 + 3x_3 &\le 4 \\ 3x_1 + 6x_2 - x_3 &\ge 2 \\ x_1 + x_2 + x_3 &= 4 \end{aligned}$$
$$x_1 \ge 0, x_2 \text{ unrestricted}, x_3 \ge 0.$$

3. Explain the terms in the context of LPP : (i) Feasible solution ; (ii) Optimal basic feasible solution ; (iii) Extreme point.

4. If the primal problem, being a maximisation one is feasible but the value of the objective function is unbounded then the dual is infeasible. Justify.

5. What is transportation problem ? Show that it can be considered as a linear programming problem.

6. Explain the concept of loop in a transportation problem.

7. Briefly explain any *one* method of solving an integer linear programming problem.

8. What is quadratic programming problem ? How does it differ from the linear programming problem ?

(8 × 4 = 32 marks)

Part B

Answer **either** *part A* **or** *part B of* **each** *question.*
Each question carries 12 *marks.*

9. A (a) Let $f(x)$ be defined in a convex domain $\mathrm{K} \subseteq \mathrm{E}_n$ and be differentiable. Then $f(\mathrm{X})$ is a convex function if and only if $f(\mathrm{X}_2) - f(\mathrm{X}_1) \ge (\mathrm{X}_2 - \mathrm{X}_1)' \nabla f(\mathrm{X}_1)$ for all $\mathrm{X}_1, \mathrm{X}_2$ in K.

Turn over

(b) A manufacturer of leather belts makes three types of belts A, B and C which are processed on three machines M_1, M_2 and M_3. But A requires 2 hours on machine M_1 and 3 hours on machine M_3. Belt B requires 3 hours on machine M_1, 2 hours on machine M_2 and 2 hours on machine M_3 and belt C requires 5 hours on machine M_2 and 4 hours on machine M_3. There are 8 hours of time per day available on machine M_1, 10 hours of time per day available on machine M_2 and 15 hours of time per day available on machine M_3. The profit gained from belt A is Rs. 3.00 per unit, from belt B is Rs. 5.00 per unit, from belt C is Rs. 4.00 per unit. What should be the daily production of each type of belts so that the profit is maximum ?

(6 + 10 = 16 marks)

Or

B (a) Prove that a vertex of the region of feasible solution S_F to a linear programming problem is a basic feasible solution.

(b) Use Simplex method to solve :

Maximise $Z = x_1 + 2x_2 + 3x_3 - x_4$

subject to the constraints

$$x_1 + 2x_2 + 3x_3 = 15 ;$$
$$2x_1 + x_2 + 5x_3 = 20 ;$$
$$x_1 + 2x_2 + x_3 + x_4 = 10,$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

(8 + 8 = 16 marks)

10. A (a) Solve graphically to show that the following problem has an unbounded solution. Write its dual and solve it to verify that it has no solution :

Maximise $3x_1 + 4x_2$

subject to $x_2 - x_1 \leq 1$;

$$x_1 + x_2 \geq 4 ;$$
$$x_1 - 3x_2 \leq 3,$$
$$x_1, x_2 \geq 0.$$

(b) Solve the following transporation problem :

	D_1	D_2	D_3	Availability
O_1	2	1	3	10
O_2	4	5	7	25
O_3	6	0	9	25
O_4	1	3	5	30
Demands	20	20	15	

(8 + 8 = 16 marks)

Or

B (a) (i) What type of problems can be solved by dual simplex method ?

(ii) Solve by dual simplex method :

$$\text{Minimize } Z = x_1 + 3x_2 + 2x_3$$

$$\text{subject to } \begin{aligned} 4x_1 - 5x_2 + 7x_3 &\le 8 \\ 2x_1 - 4x_2 + 2x_3 &\ge 2 \\ x_1 - 3x_2 + 2x_3 &\le 2 \\ x_1, x_2 &\ge 0. \end{aligned}$$

(b) Solve the following transportation problem :

	W_1	W_2	W_3	Supply
O_1	16	20	12	200
O_2	14	8	18	160
O_3	26	24	16	90
Demand	180	120	150	

(8 + 8 = 16 marks)

11. A (a) Distinguish between pure integer and mixed integer linear programming problem. Give one example for each.

(b) Solve the following problem by cutting plane algorithm :

$$\text{Maximize } Z = 3x_1 + x_2 + 3x_3$$

$$\text{subject to } \begin{aligned} -x_1 + 2x_2 + x_3 &\le 4 \\ 4x_2 - 3x_3 &\le 2 \\ x_1 - 3x_2 + 2x_3 &\le 2 \\ x_1, x_2, x_3 \ge 0 &\text{ and integers.} \end{aligned}$$

(4 + 12 = 16 marks)

B (a) Write the Kuhn-Tucker conditions for the following problem :

$$\text{Minimize } f(X) = (x_1 - 2)^2 + x_2^2$$

$$\text{subject to } x_1^2 + x_2 - 1 \le 0$$

$$x_1, x_2 \ge 0.$$

(b) Solve the following quadratic programming problem :

$$\text{Minimize } 3x_1 + 6x_2 - 4x_1x_2 - 3x_1^2 - 2x_2^2$$

$$\text{subject to } \begin{aligned} 3x_1 + 2x_2 &\le 4 \\ x_1 + x_2 &\le 4 \\ x_1, x_2 &\ge 0. \end{aligned}$$

(6 + 10 = 16 marks)

[3 × 16 = 48 marks]

D 12668 (Pages : 2) Name.................................

Reg. No.................................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016

(CCSS)

Mathematics

MAT 3E 01—ADVANCED TOPOLOGY

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 4 *marks.*

1. Does there exist an one-one, continuous function of the open interval (0, 1) onto (0, 1) which is not a homeomorphism ? Justify your claim.
2. Let S′ be the unit circle in R^2 centred at O and $f : S' \to R$ be continuous. Show that there exists a point $x_0 \in S'$ such that $f(x_0) = f(-x_0)$.
3. Give an example of a Tychonoff space which is not normal.
4. Let X and Y be T_2 spaces. Is the projection map $P_X : X \times Y \to X$ is an open map ? Justify your claim. Here X × Y is equipped with the product topology.
5. Prove that a closed subset of a complete metric space is complete with respect to the induced topology.
6. Let $\{x_n\}$ be a sequence on a topological space X having a cluster point x in X. If X is first countable at x, then show that there is a subsequence of $\{x_n\}$ that converges to x.
7. Prove that every contraction is a weak contraction. Is the converse true ? Justify your answer.
8. Let X be a topological space and $x \in X$. Prove that the intersection of all filters on X converging to x is precisely the neighbourbood system of x in X.

(8 × 4 = 32 marks)

Part B

Answer **either** *A* **or** *B of* **each** *question.*
Each question carries 12 *marks.*

9. A. State and prove Teitze extension theorm of normality.

Or

B. Let X be a topological space and (Y, d) be a metric space. Let $\{f_n\}$ be a sequence of continuous functions from X to Y which converges to the function $f : X \to Y$ uniformly. Show that f is continuous.

Turn over

10. A. Introduce contor ternary set. Show that this set is compact, totally disconnected and homeomorphic to the space $\mathbb{Z}_2^{\mathbb{N}}$.

Or

B. Show that the product of spaces is locally connected if and only if each co-ordinate space is locally connected and all except finitely many of them are connected.

11. A. For a topological space X, show that the following statements are equivalent :

(i) X is compact.

(ii) Every net in X has a cluster point in X.

(iii) Every net in X has a convergent subset in X.

Or

B. Let X and Y be topological spaces, $x \in$ X and $f : X \to Y$ a function. Then show that f is continuous at x, iff whenever a filter $\mathfrak{F}$ converges to x in X, its image filter $f_{\#}(\mathfrak{F})$ converges to $f(x)$ in Y.

12. A. (a) Show that a metric space is compact iff it is complete and totally bounded.

(b) Give an example of a bounded set which is not totally bounded in a suitable metric space.

Or

B. (a) Let (Y, d) be a completemetric space. Then show that a subset of first category in X cannot have any interior point.

(b) Show that a complete metric space with no isolated points is uncountable.

(4 × 12 = 48 marks)

D 12667 (Pages : 2) Name.................................

Reg. No...............................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016

(CCSS)

Mathematics

MAT 3C 12—P.D.E. AND INTEGRAL EQUATIONS

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 4 marks

1. Obtain a first order partial differential equation satisfied by surfaces of the form $F(u, v) = 0$ where u and v are functions of the independent co-ordinates x, y and z.

2. If $\overline{X}$ is a smooth vector field in R^3 and $\mu \neq 0$, show that $\overline{X}.\ \text{curl}\,\overline{X} = 0$ if and only if $\mu\overline{X}.\,\text{curl}\left(\mu\overline{X}\right) = 0$.

3. Solve the first order p.d.e, $p^2x + q^2y = z$ by Charpit's method.

4. Find the integral surface of the equation $(p^2 + q^2)x = pz$ containing the curve $C : x_0 = 0, \quad y_0 = s^2$, $z_0 = 2s$.

5. Find an integral surface of the p.d.e $p^2 + qy - x = 0$ containing the initial line $y = 1, x + z = 0$ by the method of characteristics.

6. Solve the following I.B.V.P :

$$\frac{\partial^2 y}{\partial^2 x} = \frac{1}{e^2}\frac{\partial^2 y}{\partial^2 t}, 0 \le x \le l, t > 0.$$
$$y(x, 0) = u(x), y_t(x, 0) = v(x)$$
$$y(0, t) = y_t(0, t) = 0 ; y(l, t) = y_t(l, t) = 0.$$

7. Prove that a harmonic function in a bounded domain attain its maximum and minimum on the boundary. State Dirichlet problem for Laplace's equation and show that if a solution for the problem exists, it must be unique.

8. Find a bounded harmonic function u in the region $r > a$ satisfying $u(a, \theta) = f(\theta)$ on the circle $r = a$.

9. Solve $\nabla^2 u = 0, r < a$ subject to the conditions (i) $\frac{\partial u}{\partial r} = f(\theta)$ on $r = a$ and (ii) $\int_0^{2\pi} f(\theta)\, d\theta = 0$.

Turn over

10. prove that :

$$\int_a^x \int_a^{x_n} \cdots \int_a^{x_3} \int_a^{x_2} f(x_1)\, dx_1\, dx_2 \ldots dx_{n-1}\, dx_n = \frac{1}{(n-1)!} \int_a^x (x-\xi)^{n-1} f(\xi)\, d\xi.$$

11. Prove that the : equation $\frac{1}{\pi}\int_0^{2\pi} \sin(x+\xi)\, y(\xi)\, d\xi + x$ Possesses no solution.

12. Determine the resolvent kernel associated with $K(x,\xi) = x\xi$ in the interval (0, 1) in the form of a power series in λ, obtaining .the first three terms.

(12 × 4 = 48 marks)

Part B

Answer **either** *part (a)* **or** *part (b) of each question.*
Each question carries 8 *marks.*

13. (a) When do you say that two first order p.d.e's are compatible on a domain D in R^2 ? State and prove a necessary and sufficient condition for the same.

(b) Define Pfaffian equation in n variables. Show that the Pfaffian equation $P(x, y, z)\, dx + Q(x, y, z)\, dy + R(x, y, z)\, dz = 0$ is integrable if and only if $\overrightarrow{X}.\text{cur}\,\overrightarrow{X} = 0$. Where $\overrightarrow{X} = (P, Q, R)$.

14. (a) Find the two characteristic strips of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ passing through the x-axis and hence find the two solutions containing x-axis.

(b) Derive the cannonical form for an elliptic second order linear p.d.e.

15. (a) Define stable solution for an I.V.P. Give an example of a solution that is not stable. Show that Dirichelet problem for the Laplace's equation in the upper half plane has stable solutions.

(b) Use the method of seperation of variables to solve the B.V.P :

$$u_{xx} + u_{yy} = 0\,;\, 0 < x < a,\, 0 < y < b,$$
$$u(x,0) = f(x),\, u(x,b) = 0\,;\, 0 \le x \le a,$$
$$u(0,y) = 0,\, u(a,y) = 0\,;\, 0 \le y \le b.$$

16. (a) Transform the B.V.P, $y'' + xy = 1, y(0) = 0, y(l) = 1$ into an integral equation by Green's function method.

(b) Show that any solution of the equation,

$$y(x) = \lambda \int_0^1 (1 - 3x\xi)\, y(\xi)\, d\xi + F(x).$$

can be expressed as the sum of F(x) and some linear combination of the characteristic functions.

D 12666 (Pages : 3) Name.....................................

Reg. No.....................................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016

(CCSS)

Mathematics

MAT 3C 11—FUNCTIONAL ANALYSIS

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 4 *marks.*

1. Let Y be a subset of a separable metric space. Prove that Y is separable in the induced metric.
2. Let X be a compact metric space. Prove that every sequence in X has a convergent subsequence.
3. Let Y be a closed subspace of a normed space X. For $x + \mathrm{Y} \in \mathrm{X/Y}$, let :

   $$|||x + \mathrm{Y}||| = \inf\left\{\|x + y\| : y \in \mathrm{Y}\right\}.$$

   Prove that $|||x + \mathrm{Y}|||$ is a norm on X/Y.
4. Let Y be a subspace of a normed space X. Prove that $\mathrm{Y}^{\circ} \neq 0$ if and only if Y = X.
5. Prove that the norms $\| \|_1, \| \|_2$ and $\| \|_\infty$ on K^n are equivalent.
6. Let X and Y be normed spaces over K. If X is finite dimensional, then prove that every linear map from X to Y is continuous.
7. Let $\mathrm{X} = \mathbb{R}^2$ with $\| \|_1$ and let $\mathrm{Y} = \{(x(1), x(2)) : x(2) = 0\}$ be a subspace of X. Find a Hahn-Banach extension of the linear functional g on Y defined by $g(x(1), x(2)) = x(1)$.
8. Let X be a normed space over K, Y be a subspace of X and $a \in \mathrm{X}$. If $a \notin \overline{\mathrm{Y}}$, then prove that there is some $f \in \mathrm{X}'$ such that $f|_{\mathrm{Y}} = 0, f(a) = \mathrm{Dist}(a, \overline{\mathrm{Y}})$ and $\|f\| = 1$.
9. Let Y be a proper dense subspace of a Banach space X. Prove that Y is not a Banach space in the induced norm.
10. Let X be a normed space over K and E be a subset of X. Prove that E is bounded if and only if $f(\mathrm{E})$ is bounded in K for every linear functional f on X.

Turn over

11. Let F be a bijective closed map from a normed space X to a normed space Y. Prove that F^{-1} is also a closed map.

12. Let Z be a closed subspace of a normed space X. Prove that the quotient mapping $Q : X \to X/Z$ is continuous.

(12 × 4 = 48 marks)

Part B

Answer **either** A **or** B *of each question.*
Each question carries 8 *marks.*

13. (A) (a) For $1 \le p < \infty$, prove that l^p is separable.

(b) Let x and y be measurable functions on a measurable subset E of $\mathbb{R}$. If $1 < p < \infty$ then $\frac{1}{p} + \frac{1}{q} = 1$, prove that :

$$\int_E |xy|\, dm \le \left(\int_E |x|^p \, dm \right)^{\frac{1}{p}} \left(\int_E |y|^q \, dm \right)^{\frac{1}{q}}.$$

(B) (a) Prove that the metric space l^p is complete, where $1 \le p < \infty$.

(b) Let E be a measurable subset of $\mathbb{R}$. If $1 \le p < \infty$, then prove that the set of all simple measurable functions on E which are zero outside subsets of finite measure is dense in $L^p(E)$.

14. (A) (a) Let Y be a finite dimensional subspace of a normed space X. Prove that Y is closed in X.

(b) Prove that a linear map F from a normed space X to a normed space Y is a homeomorphism if and only if there are $\alpha > 0, \beta > 0$ such that :

$$\beta \|x\| \le \|F(x)\| \le \alpha \|x\|$$

for all $x \in X$.

(B) (a) Let X and Y be normed spaces and $F : X \to Y$ be a linear map such that the range $R(F)$ of F is finite dimensional. Prove that F is continuous if and only if the zero space $Z(F)$ of F is closed in X.

(b) Let X and Y be normed spaces and let BL(X, Y) denote the set of all bounded linear map from X to Y. For $F \in BL(X,Y)$, define :

$$\|F\| = \sup \{ \|F(x)\| : x \in X, \|x\| \le 1 \}.$$

Prove that $\| \; \|$ is a norm on BL (X, Y). Also prove that $\|F(x)\| \le \|F\| \, \|x\|$ for all $x \in X$.

15. (A) (a) State and prove Hahn-Banach separation theorem.

(b) Let X be a normed space and Y be a closed subspace of X. Prove that X is a Banach space if and only if Y and X/Y are Banach spaces in the induced norm and the quotient norm.

(B) (a) Let X be a normed space, Y be a subspace of X and g be a continuous linear functional on Y. Prove that there is a unique Hahn-Banach extension of g to X if and only if the dual space X' is strictly convex.

(b) Prove that an infinite dimentional Banach space cannot have a denumerable Hamel basis.

16. (A) (a) Let X be a Banach space, Y be a normed space and $\mathscr{F}$ be a subset of BL (X, Y) such that for each $x \in X$, the set $\{F(x) : F \in \mathscr{F}\}$ is bounded in Y. Prove that for each bounded set E of X, the set $\{F(x) : x \in E, F \in \mathscr{F}\}$ is bounded in Y.

(b) Show by an example that completeness of X cannot be dropped from the result given in (a).

(B) State and prove closed graph theorem.

(4 × 8 = 32 marks)

D 12665

(Pages : 4)

Name..

Reg. No....................................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016

(CCSS)

Mathematics

MAT 3C 10—COMPLEX ANALYSIS

Time : Three Hours — Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 4 marks.

1. Let S be the Riemann sphere and $\mathbb{C}_\infty$ be the extended complex plane. Explain stereographic projection that connects points of S and $\mathbb{C}_\infty$.

2. Find the radius of convergence of the power series :

   (i) $\sum_{n=0}^{\infty} \frac{z^n}{n!}$.

   (ii) $\sum_{n=0}^{\infty} \frac{(-1)^n z^n}{n}$.

3. Find the fixed points of a dilation, translation and the inversion on $\mathbb{C}_\infty$.

4. Define a rectifiable path. Give an example.

5. Define $r : [0, 2\pi] \to \mathbb{C}$ by $r(t) = e^{int}$, where n is some integer (positive, negative *or* zero). Show that $\int_r \frac{1}{z} dz = 2\pi in$.

6. State and prove fundamental Theorem of Algebra.

7. Let r_0, r_1, r_2 be three closed rectifiable curves in a region G. If $r_0 \sim r_1$ and $r_1 \sim r_2$ in G, prove that $r_0 \sim r_2$ in G.

8. Let f be analytic in $B(a; R)$ and suppose that $f(a) = 0$. Show that a is a zero of multiplicity m if and only if $f^{(m-1)}(a) = f^{(m-2)}(a) = \ldots = f^{(a)} = 0$ and $f^{(m)}(a) \neq 0$.

Turn over

9. If $f(z) = \frac{1}{z(z-1)(z-2)}$, give Laurent series expansion of f in the annuli ann (0 ; 1, 2).

10. Show that for $a > 1$, $\int_0^{\pi} \frac{d\theta}{5 + 3\cos\theta} = \frac{\pi}{4}$.

11. Suppose f has a simple pole at $z = a$ and let g be analytic in an open set containing a. Prove that Res $(f\,g\,; a) = g(a)$ Res $(f\,; a)$.

12. Let f and g be analytic in $\bar{B}(0\,; R)$ with $|f(z)| = |g(z)| = R$. If neither f nor g vanishes in $B(0\,; R)$, then show that there exists a constant λ, with $|\lambda| = 1$, such that $f = \lambda g$.

(12 × 4 = 48 marks)

Part B

Answer **either** *A* **or** *B of each question.*
Each question carries 8 *marks.*

13. A (a) Which subsets of the Riemann sphere S correspond to the real and imaginary axes of $\mathbb{C}$ under stereographic projection.

(b) Let f and g be analytic on open sets G and Ω, respectively. If $f(G) \subset \Omega$, prove that $g \circ f$ is analytic on G and for all z in G, $(g \circ f)'(z) = g'(f(z)).f(z)$.

(c) Show that bilinear transformations preserve cross ratios.

(2 + 3 + 3 = 8 marks)

B (a) If f is analytic on a region G and if $f'(z) = 0$ for all z in G, prove that f is constant on G.

(b) Let f be a branch of $\log z$ on a region G. Show that the totality of all branches of $\log z$ on G are the function $f(z) + 2\pi ki, k \in z$.

(c) Using Orientation Principle, prove that there exists an analytic function $f : G \to \mathbb{C}$, where $G = \{z : \text{Re}\, z > 0\}$ $f(G) = \{z : |z| < 1\}$.

(2 + 3 + 3 = 8 marks)

14. A (a) State and prove Cauchy's Integral Formula.

(b) Let f be analytic function on a region G. Prove that the following statements are equivalent :—

(i) $f \equiv 0$.

(ii) There is a point a in G such that $f^{(n)}(a) = 0$ for each $n \geq 0$.

(iii) $\{z \in G : f(z) = 0\}$ has a limit point in G.

(4 + 4 = 8 marks)

B (a) Let f be analytic in $B(a; R)$. Show that f has a power series expansion in $B(a; R)$.

(b) Let G be an open set and f be an analytic function on G. Prove that f is infinitely differentiable on G.

(c) Show that if f is analytic in a region G and a is a point with $|f(a)| \geq |f(z)|$ for all z in G, then f is a constant function.

(2 + 3 + 3 = 8 marks)

15. A (a) Let f be a function analytic on a simply connected region G. Prove that f has a primitive in G.

(b) State and prove Open Mapping theorem.

(c) Suppose f has an isolated singularity at a. Prove that a is a removable singularity of f if and only if $\lim_{z \to a} (z - a) f(z) = 0$.

(2 + 3 + 3 = 8 marks)

B (a) Let r be and closed rectifiable curve in $\mathbb{C}$. Prove that $n(r; a) = n(r; b)$ whenever a, b belong to the same component of $\mathbb{C} - \{r\}$. Also show that $n(r : a) = 0$ whenever a belongs to the unbounded component of $\mathbb{C} - \{r\}$.

(b) State and prove the theorem on Laurent series development in an annulus.

(4 + 4 = 8 marks)

Turn over

16. A (a) Prove that $\int_0^{\infty} \frac{x^2}{1+x^4}\, dx = \frac{\pi}{2\sqrt{2}}$

(b) State and prove Rouche's theorem.

(c) Justify with example. Is Maximum Modulus Principle true for unbounded regions ?

(3 + 3 + 2 = 8 marks)

B (a) State and prove Argument Principle.

(b) State and prove Residue theorem.

(c) Let G be a bounded open set in $\mathbb{C}$ and suppose f is a continuous function on G^- which is analytic in G. Then $\max\left\{ |f(z)| : z \in G^- \right\} = \max\left\{ |f(z)| : z \in \partial G^- \right\}$.

(3 + 3 + 2 = 8 marks)

[4 × 8 = 32 marks]