D 43034

(Pages : 8)

P.G. ENTRANCE EXAMINATION, MAY 2018

M.Sc.—MATHEMATICS

Time : Two Hours | Maximum : 200 Marks

INSTRUCTIONS

► **Question Paper:** The Question Paper contains 50 Multiple Choice Questions. Among the four options of each question given as (a), (b), (c) and (d), only one will be the most appropriate answer. Mark the bubble containing the letter corresponding to the most appropriate answer in the OMR answer sheet using ball point pen(blue or black).

► **Negative Marking:** Total four marks will be given for each correct answer. One mark will be deducted for each wrong answer.

1. Which one of the following is true for a non-empty subset A of the set of all positive integers?

   (a) A contains a largest element
   (b) A contains a smallest element
   (c) A contains a maximal element
   (d) None of these

2. If $P(S)$ denotes the powerset of a set S, then which one of the following is false?

   (a) $P(A \cup B) = P(A) \cup P(B)$
   (b) $P(A \cap B) = P(A) \cap P(B)$
   (c) If A is a subset of B, then $P(A)$ is subset of $P(B)$
   (d) None of these

3. Let f be a function from a set X to a set Y and A and B be subsets of X. Then which one of the following is true?

   (a) $f(A \cup B) = f(A) \cup f(B)$
   (b) $f(A \cap B) = f(A) \cap f(B)$
   (c) $f(A \setminus B) = f(A) \setminus f(B)$
   (d) None of these

4. The number of elements in the powerset of the powerset of the empty set is

   (a) 0
   (b) 1
   (c) 2
   (d) None of these

5. An element x of a partially ordered set $(X, \leq)$ is a maximal element in X if and only if

(a) $y \leq x$ for every $y \in X$

(b) $y \in X$ with $x \leq y$ implies that $x = y$

(c) $y \in X$ with $y \leq x$ implies that $x = y$

(d) None of these

6. Which one of the following statements is equivalent to the statement 'p implies q' ?

(a) (not p) and q

(b) p or (not q)

(c) (not p) or q

(d) None of these

7. Which one of the following is the negation of the statement:
'For every real number x, there exists a positive integer $n > x$'

(a) For every real number x, there exists no positive integer $n > x$

(b) There exists a real number x, such that there exists no positive integer $n > x$

(c) For every real number x, there exists a positive integer $n < x$

(d) None of the above

8. Which one of the following defines a function from the set of all real numbers $\mathbb{R}$ into $\mathbb{R}$?

(a) $f(x) =$ the square root of x

(b) $f(x) = ln(x)$

(c) $f(x) = \frac{1}{x-1}$

(d) None of these

9. Which one of the following defines a binary operation on $\mathbb{R}$?

(a) $a * b = \frac{a}{b^2+1}$

(b) $a * b = \frac{a}{b^2}$

(c) $a * b = \frac{a}{b^2-1}$

(d) $a * b = \frac{a}{b^3}$

10. Which one of the following is true for groups?

(a) Every finite group is abelian

(b) Every abelian group is finite

(c) Every abelian group is cyclic

(d) Every cyclic group is abelian

11. If $(32)! = 3^m k$, where $gcd\ (k, 3) = 1$, then m is equal to

(a) 12

(b) 13

(c) 14

(d) 15

12. The number $4444^{1111} + 1111^{4444}$ is

(a) divisible by both 5 and 7

(b) divisible by 5, but not by 7

(c) divisible by 7, but not by 5

(d) divisible neither by 5, nor by 7

13. If, for $x \in \mathbb{R}$, $[x]$ denotes the greatest integer less than or equal to x, then for $x \geq 1$, the sum $\sum_{n \leq x} [\frac{1}{n}]$, where the sum is taken over all positive integers less than or equal to x is equal to

(a) $[x]$ (b) 1 (c) 0 (d) $x[x]$

14. If $\phi(n)$, for $n \in \mathbb{N}$ denotes the number of positive integers not greater than n and relatively prime to n, then which one of the following is true about ϕ ?

(a) a monotonically increasing function (b) a monotonically decreasing function

(c) neither increasing nor decreasing (d) None of these

15. For any positive integer n, the number of generators of the group $\mathbb{Z}_n$ under addition modulo n is

(a) 1 (b) $n-1$ (c) $\phi(n)$ (d) n

16. The number of left cosets of the subgroup generated by the element 6 in the group $\mathbb{Z}_{120}$, under addition modulo 120 is

(a) 6 (b) 12 (c) 10 (d) 20

17. Which one of the following is a ring of characteristic 100?

(a) $\mathbb{Z}_{50} \times \mathbb{Z}_2$ (b) $\mathbb{Z}_{25} \times \mathbb{Z}_4$ (c) $\mathbb{Z}_{10} \times \mathbb{Z}_{10}$ (d) $\mathbb{Z}_5 \times \mathbb{Z}_{20}$

18. The system of equations $2x + 2y = 1$; $4x + 4y = 3$ has

(a) no solution (b) a unique solution (c) two solutions (d) infinitely many solutions

19. The inverse of the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

20. The eigenvalues of the matrix $\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$ are

(a) 4, 4 (b) 1, 1 (c) 3, 5 (d) −3, −5

21. The matrix $\begin{bmatrix} 8 & 4 & 2 \\ x & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}$ is singular if x is

(a) 0 (b) 1 (c) 2 (d) 4

22. Which one of the following is correct about vector spaces?

(a) A one-dimensional vector space has exactly one basis
(b) There exists no vector space containing only a finite number of elements
(c) A one-dimensional vector space has no proper nonzero subspace
(d) None of the above

23. Which one of the following is not a linear transformation from the vector space $\mathbf{R}^2$ into $\mathbf{R}^2$?

(a) $T(x_1, x_2) = (x_1 + x_2, 0)$ (b) $T(x_1, x_2) = (x_1 - x_2, 0)$
(c) $T(x_1, x_2) = (x_1 x_2, 0)$ (d) None of these

24. The nullspace of the linear transformation $T : \mathbf{R}^2 \to \mathbf{R}$ defined by $T(x_1, x_2) = \frac{x_1}{2}$ is

(a) $\{(x_1, x_2) : x_1 = 2\}$ (b) $\{(x_1, x_2) : x_1 = 0\}$
(c) $\{(x_1, x_2) : x_2 = 0\}$ (d) $\{(x_1, x_2) : x_2 = 2\}$

25. Which one of the following lines is perpendicular to the line $2x + y = 1$?

(a) $2x + y = -1$ (b) $2x - y = 1$ (c) $x + 2y = 1$ (d) $x - 2y = 1$

26. Consider the interval $(-1, 1)$ and a sequence $\{a_n\}$ of elements in it. Then

(a) every limit point of $\{a_n\}$ is in $(-1, 1)$
(b) The limit points of $\{a_n\}$ can only be in $\{-1, 1, 0\}$
(c) every limit point of $\{a_n\}$ is in $[-1, 1]$
(d) The limit points of $\{a_n\}$ cannot be in $\{-1, 1, 0\}$

27. The sequence $\{n^{1/n}\}$ is

(a) is convergent and converges to 0 (b) monotonically decreasing

(c) monotonically increasing (d) neither monotonically increasing or decreasing

28. The set $C = \bigcap_{n=1}^{\infty}[-1 - \frac{1}{n},\ 1 + \frac{1}{n}]$ is

(a) [-1,1] (b) (-1,1) (c) [-1,1) (d) (-1,1]

29. Let X be a countably infinite subset of $\mathbf{R}$ and A be a countably infinite subset of X. Then the set $X \setminus A = \{x \in X : x \notin A\}$

(a) is empty (b) is a finite set (c) can be countably infinite (d) can be uncountable

30. The period of the function $f(x) = \sin^{100} x + \cos^{100} x$ is

(a) π (b) $\pi/2$ (c) $\pi/4$ (d) none of the above

31. Let $x_1 = 0, x_2 = 1$ and for $n \geq 3$ define $x_n = \frac{x_{n-1}+x_{n-2}}{2}$. Then which of the following is true?

(a) $\{x_n\}$ is a divergent sequence (b) $\lim_{n\to\infty} x_n = \frac{1}{2}$

(c) $\lim_{n\to\infty} x_n = \frac{2}{3}$ (d) $\{x_n\}$ is a Cauchy sequence

32. The sequence of real valued functions $f_n(x) = x^n,\ x \in [0,1] \cup \{5\}$ is

(a) pointwise convergent but not uniformly convergent (b) uniformly convergent

(c) bounded but not pointwise convergent (d) not bounded

33. The function $f(x) = a_0 + a_1|x| + a_2|x|^2 + a_3|x|^3$ is differentiable at $x = 0$

(a) for no values of a_0, a_1, a_2, a_3 (b) for any value of a_0, a_1, a_2, a_3

(c) only if $a_1 = 0$ (d) only if both $a_1 = 0$ and $a_2 = 0$

34. If $u = sin^{-1}\left(\frac{x+2y+3z}{x^8+y^8+z^8}\right)$, then $x\,\frac{\partial u}{\partial x} + y\,\frac{\partial u}{\partial y} + z\,\frac{\partial u}{\partial z}$ is

(a) $7\tan u$ (b) $-7\tan u$ (c) 0 (d) $\tan u$

35. The ordinary differential equation: $\frac{dy}{dx} = \frac{2y}{x}$ with the initial condition $y(0) = 0$, has

(a) infinitely many solutions
(b) no solution
(c) more than one but only finitely many solutions
(d) unique solution

36. A particular solution of the differential equation $y'' - 4y' + 13y = e^{2x}\cos 3x$ is

(a) $\frac{1}{6}\,e^{2x}\,x\,\sin 3x$ (b) $\frac{1}{6}\,e^{2x}\sin 3x$ (c) $x\,\sin 3x$ (d) None of these

37. For $a,\ b, c \in \mathbb{R}$, if the differential equation $(ax^2 + bxy + y^2)\,dx + (2x^2 + cxy + y^2)\,dy = 0$ is exact, then

(a) $b = 2,\ c = 2a$ (b) $b = 4,\ c = 2$ (c) $b = 2,\ c = 4$ (d) $b = 2,\ a = 2c$

38. The Fourier series of $f(x) = \begin{cases} -x+1, & -\pi \le x \le 0 \\ x+1, & 0 \le x \le \pi \end{cases}$, $f(x) = f(x + 2\pi),\ x \in \mathbb{R}$ contains

(a) both cosine and sine terms
(b) cosine terms only
(c) sine terms only
(d) constant term and cosine terms only

39. In $\mathbb{R}^3$ the cosine of the acute angle between the surfaces $x^2+y^2+z^2-9=0$ and $z-x^2-y^2+3=0$ at the point $(2,1,2)$ is

(a) $\frac{8}{5\sqrt{21}}$ (b) $\frac{10}{5\sqrt{21}}$ (c) $\frac{8}{3\sqrt{21}}$ (d) $\frac{10}{3\sqrt{21}}$

40. The area between the curve $y^2 = \frac{4a^2(2a-x)}{x}$ and its asymptote is

(a) $4\pi a$ (b) $4\pi a^2$ (c) πa^2 (d) πa^3

41. Let $\vec{r} = x\,i + y\,j + z\,k$ and $r = |\vec{r}|$. Then $r^n\vec{r}$ is

(a) solenoidal if $n = -3$ and irrotational for all values of n

(b) solenoidal if $n \neq -3$ and irrotational for all values of n

(c) solenoidal if $n = 3$ and irrotational only for $n = -3$

(d) solenoidal if $n \neq -3$ and irrotational only for $n = 3$

42. Let S be any closed surface enclosing a volume V and $\vec{r} = x\,i + y\,j + z\,k$. Then $\int_S \nabla r^2 . d\vec{S}$ is

(a) 0 (b) V (c) $2V$ (d) $6V$

43. The value of the integral $\int_{|z|=1} e^{\frac{1}{z^2}}\, dz$ is

(a) 0 (b) 1 (c) $2\pi i$ (d) None of these

44. For the function $f(z) = e^{\frac{1}{z}}$, $z = 0$ is

(a) an essential singularity (b) a pole (c) a zero (d) None of these

45. Which of the following is not true ?

(a) $|z_1+z_2| \leq |z_1|+|z_2|$ (b) $|z_1-z_2| \leq \big||z_1| - |z_2|\big|$ (c) $|z_1z_2| = |z_1||z_2|$ (d) $\overline{z_1 + z_2} = \overline{z_1}+\overline{z_2}$

46. Let $u(x,\ y) = 2x(1-y)$ for all real x and y. Then a function $v(x,\ y)$, so that $f(z) = u(x,\ y) + i\,v(x,\ y)$ is analytic, is

(a) $x^2 - (y-1)^2$ (b) $(x-1)^2 + y^2$ (c) $(x-1)^2 - y^2$ (d) $x^2 + (y-1)^2$

47. The equation $sin^2 z + cos^2 z = 1$ is true

(a) only if z is real

(b) only if z is purely imaginary

(c) for each complex number z

(d) None of these.

48. The order of the pole at $z = 0$ of the function $\frac{\sinh z}{z^4}$ is

(a) 1 (b) 2 (c) 3 (d) 4

49. Which one of the following is not an open set ?

(a) $\{z = x+iy : r_1 < \sqrt{x^2 + y^2} < r_2\}, 0 \leq r_1 < r_2$.
(b) $\{z = x+iy : \sqrt{x^2 + y^2} > r\}, r \geq 0$.
(c) $\{z = x + iy : r_1 < x < r_2, y = 0\}, 0 \leq r_1 < r_2$
(d) None of these.

50. If $f(z) = u(x, y) + iv(x, y)$ is an analytic function, then $f'(z)$ is

(a) $\frac{\partial u}{\partial x} + i\frac{\partial u}{\partial y}$ (b) $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial y}$ (c) $\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$ (d) $\frac{\partial v}{\partial x} - i\frac{\partial u}{\partial y}$