10/01/18

C 31786 (Pages : 3) Name

C 31789

(Pages : 2)

Name..

Reg. No.....................................

THIRD SEMESTER P.G. DEGREE EXAMINATION, DECEMBER 2017

(CCSS)

Mathematics

MAT 3E 02—OPERATION RESEARCH

Time : Three Hours | Maximum : 80 Marks

Part A

Answer **all** *the questions.*
Each question carries 4 marks.

1. Write an example for a linear programming problem in two dimension.
2. Define a convex function. Write an example for a convex function.
3. What is meant by degeneracy in linear programming problems. Explain.
4. What is meant by assignment problem in optimization methods. How is it related to transportation problem ?
5. Define artificial variables. Describe the uses of artificial variables in solving linear programming problems.
6. Write the general form of a transportation problem.
7. Write the general form of an integer linear programming problem. Discuss the significance of integer linear programming.
8. Describe the concept of primal and dual problems in optimization theory.

($8 \times 4 = 32$ marks)

Part B

Answer **either** *Part A* **or** *Part B of each question.*
Each question carries 16 *marks.*

9. A. (i) Prove that a basic feasible solution of a linear programming problem is a vertex of the convex set of feasible solutions.

(ii) Solve graphically the linear programming problem :

Minimize $Z = x_1 + 3x_2$

subject to $x_1 + x_2 \geq 3, -x_1 + x_2 \leq 2, x_1 - 2x_2 \leq 2,$

$x_1 \geq 0, x_2 \geq 0.$

Or

Turn over

B. (i) If $f(x)$ is minimum at more than one of the vertices of the set of basic feasible solutions, then prove that it is minimum at all those points which are the convex linear combinations of these vertices.

(ii) Use simplex method to solve the problem :

Maximize $f(x) = 5x_1 + 3x_2 + x_3$

subject to the constraints $2x_1 + x_2 + x_3 = 3, -x_1 + 2x_3 = 4,$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$

10. A. (i) Prove that the transportation problem has a triangular basis.

(ii) Solve the transportation problem for minimum cost starting with the degenerate solution $x_{12} = 30, x_{21} = 40, x_{32} = 20, x_{43} = 60$.

	D1	D2	D3	
O1	4	5	2	30
O2	4	1	3	40
O3	3	6	2	20
O4	2	3	7	60
	40	50	60	

Or

B. (i) Solve the following integer linear programming problem :

Maximize $\phi(X) = 3x_1 + 4x_2$;

subject to $2x_1 + 4x_2 \leq 13, -2x_1 + x_2 \leq 2, 2x_1 + 2x_2 \geq 1, 6x_1 - 4x_2 \leq 15, x_1, x_2 \geq 0, x_1$ and x_2 are integers.

(ii) Describe cutting plane method in solving integer linear programming problems.

11. A. (i) Solve by quadratic programming :

Minimize $-6x_1 + 2x^2{}_1 - 2x_1x_2 + 2x^2{}_2$

subject to $x_1 + x_2 \leq 2, x_1 \geq 0, x_2 \geq 0.$

(ii) Use Kuhn-Tucker conditions to find maxima and minima of $(x_1 - 4)^2 + (x_2 - 3)^2$ subject to $36 (x_1 - 2)^2 + (x_2 - 3)^2 \leq 9.$

Or

B. (i) Solve the geometric programming problem :

Minimize $f(x) = \dfrac{c_1}{x_1x_2x_3} + c_2x_2x_3$

subject to $g_1(x) = c_3x_1x_2 + c_4x_1x_2 = 1, ci \geq 0, xj \geq 0, i = 1,\ldots,4, j = 1, 2, 3.$

(ii) Describe the branch and bound method in integer programming.

(3 × 16 = 48 marks)

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THIRD SEMESTER P.G. DEGREE EXAMINATION, DECEMBER 2017

(CCSS)

Mathematics

MAT 3E 01—ADVANCED TOPOLOGY

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 4 marks.

1. Prove that the closure of a compact subset of a regular space is compact.
2. Let A be a subset of the topological space X and let $f : A \to \mathbb{R}$ be continuous. Prove that any two extensions of f to X agree on $\overline{A}$.
3. Prove that continuous image of a path connected space is path connected.
4. Prove that product of T_2 spaces is T_2.
5. Define convergence of a net. Prove that in an indiscrete space, every net converges to every other point.
6. Prove that intersection of a family of filters on a set is again filter on that set.
7. Give an example of a set of first category. Prove that the set of irrationals is of second category.
8. Let (X_1, d_1), (X_2, d_2) be complete metric spaces. Prove that $X_1 \times X_2$ is a complete metric space with respect to following metric :

$$d((x_1, y_1), (x_2, y_2)) = \max \{d_1(x_1, x_2), d_2(y_1, y_2)\}.$$

(8 × 4 = 32 marks)

Part B

Answer **either** *A* **or** *B of each question.*
Each question carries 12 marks.

9. A. (a) Prove that every compact Hausdorff space is a T_3 space.

(b) Let X be a completely regular space. If F is a compact subset of X, C is a closed subset of X and $F \cap C = \theta$, then prove that there exists a continuous function from X into the unit interval which takes the value 0 at all points of F and the value 1 at all points of C.

Or

Turn over

B. Prove that a topological space X is normal if and only if it has the property that for every two mutually disjoint, closed subsets A and B of X, there exists a continuous function $f : X \to [0, 1]$ such that $f(x) = 0$ for all $x \in A$ and $f(x) = 1$ for all $x \in B$.

10. A. (a) Prove that for a topological space X the following statements are equivalent :

(i) X is locally connected.

(ii) Components of open subsets of X are open in X.

(iii) X has a base consisting of connected sets.

(iv) For every $x \in X$ and every neighborhood N of x there exists a connected open neighborhood M of x such that $M \subset N$.

(b) Prove that every path connected space is connected.

Or

B. (a) Prove that product of topological spaces is completely regular if and only if each space is completely regular.

(b) Prove that product of topological spaces is path connected if and only if each space is path connected.

11. A. (a) Prove that a topological space is Hausdorff if and only if limits of all nets in it are unique.

(b) Prove that a subset A of a topological space X is closed if and only if limits of nets in A are in A.

Or

B. (a) Let S be a family of subsets of a set X. Prove that there exists a filter on X having S as a sub-base if and only if S has a finite intersection property.

(b) Let $\mathcal{F}$ be a filter in a topological space X and S be the associated net in X. Prove that $\mathcal{F}$ converges to $x \in X$ as a filter if and only if S converges to x as a net.

12. A. (a) Let A be a subset of the metric space (X ; d). Prove that A is totally bounded with respect to d if and only if for every $\epsilon > 0$, A can be covered by finitely many open balls with centers in A and of radii less than ϵ each.

(b) Prove that in a complete metric space the intersection of countably many open dense sets is dense.

Or

B. (a) Prove that in a locally compact Hausdorff space a subset of a first category can have no interior points.

(b) Prove that a topological space is metrically topologically complete if and only if it is an absolute G_δ.

(4 × 12 = 48 marks)

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THIRD SEMESTER P.G. DEGREE EXAMINATION, DECEMBER 2017

(CCSS)

Mathematics

MAT 3C 12—PDE AND INTEGRAL EQUATIONS

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 4 marks.

1. Show that $z = ax + \left(\frac{y}{a}\right) + b$ is a complete integral of $pq = 1$. Find the particular solution corresponding to the sub-family $b = a$.

2. Obtain the partial differential equation satisfied by the surface $F(u,v) = 0$, where $u = u(x,y,z)$ and $v = v(x,y,z)$ are known functions of x, y and z and F is an arbitrary function of u and v, having first order derivatives with respect to u and v.

3. Find the general integral of $\left(z^2 - 2yz - y^2\right)p + x(y+z)q = x(y-z)$.

4. Solve the Cauchy problem for $2z_x + yz_y = z$, when the initial data curve is :

   $C : x_0 = s,\ y_0 = s^2,\ z_0 = s, 1 \le s \le 2.$

5. Find the Monge cone at (0, 0, 0) for the equation $p^2 + q^2 = 1$.

6. Reduce the equation $4u_{xx} - 4u_{xy} + 5u_{yy} = 0$ into its Canonical form.

7. Suppose that $u(x,y)$ is harmonic in a bounded domain D and continuous in $\bar{D} = D \cup B$. Show that u attains its minimum on the boundary B of D.

8. State the Neumann problem and show that the solution of the Neumann problem is unique upto the addition of a constant.

9. Solve :

$$u_t = u_{xx}, 0 < x < l, t > 0$$
$$u(0,t) = u(l,t) = 0$$
$$u(x,0) = x(l-x), 0 \le x \le l.$$

Turn over

10. Transform the problem :

$$\frac{d^2y}{dx^2} + y = x, y(0) = 1, y'(1) = 0$$

to a Fredholm integral equation.

11. Show that the Kernel $k(x, \xi) = 1 + \xi + 3x\xi$ has a double characteristic number associated with (–1, 1), with only one characteristic function.

12. Determine the iterated Kernel $k_2(x,\xi)$ associated with $k(x,\xi) = | x - \xi |$ in (0, 1).

(12 × 4 = 48 marks)

Part B

Answer **either** *part* A **or** *part* B *of each question.*
Each question carries 8 *marks.*

13. A Show that the Pfaffian differential equation $(y^2 + yz)\,dx + (xz + z^2)\,dy + (y^2 - xy)\,dz = 0$ is integrable and find the corresponding integral.

B Show that the equations $p^2 + q^2 = 1$ and $(p^2 + q^2)x = pz$ are compatible and find the one parameter family of common solutions.

14. A Find the complete integral of the equation $p^2x + qy = z$, and derive the equation of the integral surface containing the line $y = 1, x + z = 0$.

B (a) Derive d'Alembert's solution which describes the vibrations of an infinite string.

(b) Write short note on :

(i) Domain of dependence.

(ii) Range of influence.

15. A State the Dirichlet problem for a rectangle and solve it.

B (a) Solve the Neumann problem for the upper half plane.

(b) Sole the Dirichlet exterior problem for a circle.

16. A (a) Determine the characteristic values of λ and the corresponding characteristic functions of the equation :

$$y(x) = F(x) + \lambda \int_0^{2\pi} \cos(x + \xi)\, y(\xi)\, d\xi.$$

(b) Express the solution of the equation in part (a) in the form :

$$y(x) = F(x) + \lambda \int_0^{2\pi} \Gamma(x, \xi, \lambda) F(\xi) \, d\xi$$ when λ is not characteristic.

B (a) Explain the iterative method of solving Fredholm equation of second kind.

(b) Solve by iterative method :

$$y(x) = 1 + \lambda \int_0^1 (1 - 3x\xi) y(\xi) \, d\xi.$$

(4 × 8 = 32 marks)

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THIRD SEMESTER P.G. DEGREE EXAMINATION, DECEMBER 2017

(CCSS)

Mathematics

MAT 3C 11—FUNCTIONAL ANALYSIS

Time : Three Hours   Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 4 marks.

1. Prove *or* disprove :

   A sequence (x_n) in the metric space l^2 converges to x in l^2 iff $x_n(j) \to x(j)$ in K for each $j = 1, 2, \ldots$

2. Let X be a compact metric space. Show that every sequence in X has a convergent subsequence.

3. Show that the metric space $L^\infty([-1,1])$ is not separable.

4. Let E be a convex subset of a normed space X. Show that the interior E^0 of E and the closure $\bar{E}$ of E are also convex.

5. Show that a linear functional f on a normed space X is continuous iff its zero space $Z(f)$ is closed in X.

6. Let X and Y be normed spaces and Z be a closed subspace of X. Show that if $G \in BL\left(X/Z, Y\right)$ and we let $F(x) = G(x+Z)$ for $x \in X$, then $F \in BL(x,y)$ and $\| F \| = \| G \|$.

7. Show that there exists a linear functional f on l^∞ such that $\| f \| = 1 = f(a)$ and $f(\tau(x)) = f(x)$ for all $x \in l^\infty$, where $a = (1, 1, \ldots)$ and $\tau(x)(j) = x(j+1)$ for $j = 1, 2, \ldots$

8. Give an example of a normed linear space which is not a Banach space. Determine the Banach space in which the given normed space can be embedded as a dense subspace.

9. Show that every finite dimensional normed space is separable.

Turn over

10. Let X and Y be Banach spaces and $F_n \varepsilon BL(X, Y), n = 1,2,\ldots$ show that if $(F_n(x))$ converges for every x in some set E whose span is dense in X and the set $\{\| F_n \| : n = 1,2,\ldots\}$ is bounded, then there is some $F \varepsilon BL(X, Y)$ such that $F_n(x) \to F(x)$ for every $x \varepsilon X$.

11. Give an example of a closed linear map which is not continuous.

12. Show that if a linear map on a normed space is open, then it is surjective.

(12 × 4 = 48 marks)

Part B

Answer **either** *part (a)* **or** *part (b) of each question.*
Each question carries 8 marks.

13. (a) State and prove Baire's theorem. (2 + 6 = 8 marks)

(b) (i) Define norm on a linear space. Show that the norm function on a normed space is continuous. (2 marks)

(ii) Let Y be a closed subspace of a normed space X. For x + Y in the quotient space X/Y, let $\|| x + Y \|| = \inf\{\| x + y \| : y \varepsilon Y\}$. Show that $\||\quad\||$ is a norm on X/Y. Further, show that a sequence $(x_n + Y)$ converges to $x + Y$ in X/Y iff there is a sequence (y_n) in Y such that $(x_n + y_n)$ converges to x in X.

14. (a) (i) Let X be a normed space. Show that the subset $\{x \varepsilon X : \| x \| \leq 1\}$ of X is compact iff X is finite dimensional.

(ii) Show that the closed unit ball in l^2 is convex, closed and bounded, but not compact.

(5 + 3 = 8 marks)

(b) (i) Show that every linear map from a finite dimensional normed space to any normed space is continuous.

(ii) Show that a linear map F from a normed space X to a normed space Y is a homeomorphism iff there are $\alpha, \beta > 0$ such that :

for $\beta \| x \| \leq \| F(x) \| \leq \alpha \| x \|$ for all $x \varepsilon X$.

(3 + 5 = 8 marks)

15. (a) (i) Let Y be a subspace of a normed space X and $g \in Y'$. Show that there is some $f \in X'$ such that $f/Y = g$ and $\| f \| = \| g \|$.

(ii) Let $\{a_1, \ldots, a_m\}$ be a linearly independent set in a normed space X. Show that there are $f_1, f_2, \ldots, f_m$ in X' such that $f_j(a_i) = \delta_{ij}, 1 \le i, j \le m$.

(6 + 2 = 8 marks)

(b) (i) Show that the dual X' of every normed space X is a Banach space.

(ii) Show that a Banach space cannot have a denumerable basis.

(5 + 3 = 8 marks)

16. (a) (i) State and prove open mapping theorem. (2 + 3 = 5 marks)

(ii) Show that the open mapping theorem may not hold if the normed spaces X and Y are not Banach spaces.

(3 marks)

(b) (i) Let X be a normed space and $P : X \to x$ be a projection. Show that P is a closed map iff the subspaces R (P) and Z (P) are closed in X.

(ii) Let Y be a finite dimensional subspace of a normed space X. Show that there is a continuous projection P defined on X such that R (P) = Y.

(4 + 4 = 8 marks)

[4 × 8 = 32 marks]

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THIRD SEMESTER P.G. DEGREE EXAMINATION, DECEMBER 2017

(CCSS)

Mathematics

MAT 3C 10—COMPLEX ANALYSIS

Time : Three Hours

Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 4 marks.

1. Let S be the Riemann sphere. Let Z, Z' be respectively points in S corresponding to z, z' in $\mathbb{C}$. If Z^* is the point in S corresponding to $z + z'$, express Z^* in terms of Z and Z'.

2. Let $a \in \mathbb{C}$, $a \neq 0$. Obtain the radius of convergence of the following series :

   (i) $\sum_{n=0}^{\infty} a^n z^n$.

   (ii) $\sum_{n=0}^{\infty} a^{n^2} z^n$.

3. Suppose f is analytic on a region G and $f' \equiv 0$ on G. Prove that f is constant on G.

4. Let $r(t) = e^{it}, 0 \leq t \leq 2\pi$. Evaluate the following integrals :

   (i) $\int_r \frac{\sin z}{z^3} dz$.

   (ii) $\int_r \frac{e^z - e^{-z}}{z^n}$, n a positive integer.

5. Let f be an entire function. Suppose there exists constants M, R > 0 and an integer $n \geq 1$ such that $|f(z)| \leq M|z|^n$ for $|z| > R$. Show that f is a polynomial of degree $\leq n$.

6. Let f be analytic in $\{z : |z| < 1\}$. Suppose $|f(z)| \leq 1$ for $|z| < 1$. Show that $|f'(0)| \leq 1$.

7. Let r be a closed rectifiable curve in a star-shaped open set G. Prove that $r \sim 0$.

8. Let f be a one-one analytic function on a region G. Show that $f'(z) \neq 0$ for any z in G.

Turn over

9. Let $f(z) = \frac{1}{z(z-1)(z-2)}$. Obtain the Laurent series expansion of f in the following regions :

(i) $\{z : 0 < |z| < 1\}$. (ii) $\{z : 1 < |z| < 2\}$. (iii) $\{z : |z| > 2\}$.

10. If f has a pole of order m at $z = a$ and if $g(z) = (z-a)^m f(z)$, show that :

$$\text{Res}(f; a) = \frac{1}{(m-1)!} g^{(m-1)}(a).$$

11. Let f be analytic in the disk $\{z : |z| \le 1\}$. Suppose $|f(z)| < 1$ for $|z| = 1$. Show that there exists a unique z with $|z| < 1$ such that $f(z) = z$.

12. Find the poles and the residues at the poles of :

(i) $\frac{z^2}{1+z^4}$. (ii) $\frac{1}{z^2 + 29z + 1}$, $a > 1$.

(12 × 4 = 48 marks)

Part B

Answer **either** (A) **or** (B) of each *question.*
Each question carries 8 *marks.*

13. A (a) Let $\sum a_n (z-a)^n$ be a power series with radius of convergence R. Then prove that :

$$R = \lim \left| \frac{a_n}{a_{n+1}} \right|, \text{ if this limit exists.}$$

(b) Let G be an open disk or whole plane $\mathbb{C}$. Show that every harmonic function on G has a harmonic conjugate.

(c) Prove that a Möbius transformation carries circles into circles.

(2 + 3 + 3 = 8 marks)

B (a) Define cross ratio. Evaluate the cross ratio $(i, \infty, 1+i, 0)$.

(b) Prove that power series is analytic in its disk of convergence.

(c) Prove that branch of the logarithm is analytic and its derivative is $1/z$.

(2 + 3 + 3 = 8 marks)

14. A (a) Prove that $\int_0^{2\pi} \frac{e^{is}}{e^{is} - z} ds = 2\pi$, if $|z| < 1$.

(b) Suppose f is analytic in a region G. If the set of zeros of f has a limit point in G, show that $f \equiv 0$.

(c) State and prove Morera's theorem.

(3 + 2 + 3 = 8 marks)

B (a) State and prove Cauchy's Integral Formula (first version).

(b) Show that a bounded entire function is constant.

(c) Let r be a closed rectifiable curve in $\mathbb{C}$. Show that $n(r\,;)$ is a constant in each component of $\mathbb{C} - \{r\}$.

(4 + 2 + 2 = 8 marks)

15. A (a) State and prove Goursat's theorem.

(b) State and prove the second version of Cauchy's theorem.

(4 + 4 = 8 marks)

B (a) State and prove the Casorati-Weierstrass' theorem on essential singularity.

(b) State and prove the theorem on Laurent series development in an annulus.

(4 + 4 = 8 marks)

16. A (a) Show that :

$$\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}.$$

(b) State and prove the Argument principle.

(c) Using Rouche's theorem, deduce the Fundamental theorem of Algebra.

(4 + 2 + 2 = 8 marks)

B (a) State and prove the Residue theorem.

(b) State and prove Maximum Modulus theorem (second version).

(c) Is the Maximum Modulus theorem true for unbounded regions. Justify.

(3 + 3 + 2 = 8 marks)

[4 × 8 = 32 marks]