STOCHASTIC PROCESSES
IN
CELLULAR MANUFACTURING
ENVIRONMENT

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CERTIFICATE

This is to certify that the thesis entitled "**Stochastic Processes in Cellular Manufacturing Environment**" submitted by Mr. V. Madhusudanan Pillai for the award of the degree of DOCTOR OF PHILOSOPHY to the University of Calicut, is a bonafide work done by him in this department under the supervision of *Dr. M. P. Chandrasekharan.* The matter contained in the thesis has not been submitted elsewhere for any degree.

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ABSTRACT

Deterministic modelling of manufacturing systems is abundant in literature. A more realistic representation is probabilistic/stochastic, being in tune with the manufacturing environment. In this work, suitable models are developed to extract the parameters associated with stochastic events in manufacturing environment to use them in the design and planning of manufacturing systems.

An absorbing Markov chain model is developed for manufacturing of a 'component' to estimate the raw material requirement at various operating conditions, machine requirement, manufacturing lead-time, etc. under scrapping and reworking. Markov chain models are used to revise the material requirement information contained in the bill of material of product and successfully applied to the assembly and testing operations of a solenoid operated flow control valve in spacecraft propulsion. The movement of material handling devices is modelled as random process to identify certain characteristics such as relative transition and relative time matrices, which are useful for layout design decision.

Layout designs are considered under uncertainty. For Poisson arrival of demand, the relative transition matrix can be used as flow matrix, and used in a quadratic assignment formulation. Using the expected demand over all scenarios computational efforts for layout design has been considerably reduced in the case of random product mix. A sensitivity analysis is carried out using 2,40,000 problems to study the effect of demand rate variation on the output of the model. The analysis indicates the robustness of the layout formulation procedure. The significance of the observations is tested statistically using ANOVA – two-way classification.

For a probabilistic product mix there is no cell formation method that gives optimal solution. A better method, which gives optimal solution minimising the inter cell traffic, using genetic algorithm, is suggested in the present work.

It is shown that the models showing the effect of scrap and rework, and the layout models could be suitably used in cellular manufacturing systems also.

In conclusion, it can be stated that random events characterised are modelled through suitable stochastic and probabilistic models, which adequately describe the system under consideration and help in design, planning and control of the system.

CHAPTER 1

INTRODUCTION

1.1 PREAMBLE

Different types of models are used to study the manufacturing system. Generally, the models available in literature are deterministic in nature. Probabilistic models can depict the situation better as most of the events are probabilistic in nature. Many day-to-day events in a manufacturing process are random. There is a wide gap between the designed outputs and actual outputs where deterministic models are used for system modelling. Probabilistic/stochastic models take into account the variabilities and uncertainties in the events and bridge the gap between designed outputs and real outputs. In other words, realistic outputs and input-output relations can be 'designed' through a probabilistic/stochastic approach.

In this work, studying this stochastic nature, and identifying certain random events, suitable models are developed to extract the parameters associated with these events to use them in the design and planning of manufacturing systems. The present work attempts this approach in three distinctly different dimensions of manufacturing namely the effect of scrap and rework, layout design under uncertain demand and cell design for random product mix.

Initially the work identifies the stochastic processes associated with the manufacturing of a component and the associated model is used to estimate the raw material requirement, machine requirement, manufacturing lead-time, etc. Then the work identifies the effect of scrapping of in-process components on the requirement of various items to manufacture a product, which involves assembly operations. The model helps in revising the material requirement information contained in the bill of material of a product. Another stochastic process identified is concerned with the movement of material handling device. Some of the parameters obtained from this model are suitable for layout design. Further models in the work consider the random demand and they are used for system designs like layout design and part family and machine cell formation.

The present work further analyses the usefulness of these models in cellular manufacturing environment. Organisations are increasingly adopting cellular manufacturing system to become competitive.

1.2 RANDOM NATURE OF MANUFACTURING SYSTEM

Manufacturing system accepts inputs from the external environment and delivers output to the environment. The output of the system depends on the requirement of the environment and it is seldom predictable. So the uncertainties associated with the requirement should be considered while designing the system. Usually what is delivered to the environment (market) is the product of the company. As the demand for the product can be estimated probabilistically, system design must consider the parameters of the probabilistic demand. The variability has to be considered not only in the design phase but also in the operational phase. Usually the variability is considered while planning the operations of the system.

Generally, the manufacturing environment involves discrete item manufacturing, and such system responds as a series of discrete events. Evolution of such system over time depends on the complex interaction of the timing of various discrete events, such as arrival of raw work piece, completion of operation on a part by machine, failure of a machine, departure of a finished part, completion of a process plan generation in a computer of the system, etc. Systems of this nature belong to the domain of discrete event dynamic systems (DEDS). The state of DEDS changes only at discrete instants of time.

In a DEDS, operations can take place simultaneously. But events do not necessarily occur at equally spaced intervals of time. This indicates that there is randomness associated with the events in terms of the type of event and time at which the events occur. Occurrences of events over time indicate the evolution of the manufacturing system and it can be faithfully modelled as a stochastic process. As the manufacturing system responds as a series of discrete events, the state space of the process is a countable set and such process is called a chain.

To characterise a probabilistic situation, a single random variable and its distribution are sufficient, while a stochastic process can be considered as a collection of random variables. A stochastic process is represented as $\{X(t): t \geq 0\}$, where X_t is a random variable for each $t \in T$. The values assumed by X_t are called states and the set of possible values is called the state space (S). The set of possible values of the indexing parameter is called the parameter space (*T*). Index *t* is usually interpreted as time. The state space and parameter space can be either continuous or discrete.

1.3 CELLULAR MANUFACTURING SYSTEM

Manufacturing of diversified products is essential for organisations to survive in a competitive environment. As diversity can also strangle a company, it must be delimited and managed, and the waste associated with the production kept a minimum. One method to manage diversity is to use group technology (GT), which provides a means to identify and exploit the 'underlying sameness' or similarities of parts and processes. When GT is used for configuring a production system, it results in a 'cellular manufacturing system'.

The idea of GT was developed in the former Soviet Union in 1940s and 1950s. It is a means of reducing the difficulties that arise while managing large job shops where different components are to be machined and the machines are functionally grouped. Jobs of similar design and production requirement are identified to produce in a machine group (cell). Specialised cells replace the large job shop and they are less dependent on the performance of the other cells. Such a system resulted in improvement in quality, on-time delivery, set-up times, inventory, batch size reduction, material handling, throughput time, etc. The disadvantages will become evident when there are changes in product mix or overall demand that cause fluctuations in the relative workload of different cells and, in particular, poor equipment utilisation.

The essential idea of GT is the division of a large job shop or a batch production system into a number of cells with reduced product variety in each cell. The goal is to minimise the movement of jobs among the cells. The cells themselves could be organised as job shops or flow lines. The identification of similar parts to be produced in a cell is aided by a variety of methods ranging from visual, classification and coding to production flow analysis. The sociotechnical system view of the manufacturing system also aids the GT application in manufacturing.

As a sociotechnical system, the implementation of cellular manufacturing is not merely an issue of rearranging the facility for a new layout, but more importantly an issue that involves and affects the organizational and human aspects of the manufacturing firm [Wemmerlov and Johnson (1997)]. The sociotechnical system approach views a manufacturing system as consisting of a technical system (machines, materials and processes) and a social system (workers, support services such as maintenance or quality control and management). The social system is constrained by the nature of the technical system. The sociotechnical systems approach recognises the various psychological needs of the workers that should be satisfied through the work situation. The sociotechnical system implications in system design are [Buzacott and Shanthikumar (1993)]:

- Each work group has to deal with problems that arise within its boundaries.
- There should be buffers between groups to reduce the impact of disturbances that arise elsewhere.
- Each group is to be provided with meaningful goals and performance measures that are in agreement with the overall goals of the organisation.

The essential ideas of the sociotechnical systems approach are the following

- To have a production system with loosely connected cells.
- To provide motivational approach that will integrate all activities towards system goal and also create job satisfaction.

- To have a mechanism to take control and responsibilities for dealing with disturbances created within a cell without the outside managerial interaction.

It can be seen that the GT and sociotechnical systems deal with dividing of manufacturing system into cells. The cells are interconnected and the degree of interconnection is to achieve autonomous operation of the cells.

Firms employing cellular manufacturing have converted a portion or whole of the firm's manufacturing system into cells. A manufacturing cell is a cluster of functionally dissimilar machines or processes, which are placed in close proximity to one another and dedicated to the manufacturing of a set of 'similar' items. The items are similar either in their manufacturing processes required, or their geometry, or both. An item may be a part, component, subassembly or finished product. The group of items that are processed in a manufacturing cell is called a part family. In addition to family and cell formation, other design and planning activities to be performed when a facility is converted into cellular manufacturing system. Paramount among these activities are production planning, process planning, designing the material handling system, determining staffing levels, and developing the layout [Hassan (1995)]. The layout of the cellular manufacturing system is called group technology or cellular layout.

1.4 THE OBJECTIVE AND SCOPE OF THE WORK

The domain of study involves cellular manufacturing environment where discrete parts are manufactured. Such an environment is laden with a series of random discrete events that characterise the system. Occurrences of events over time indicate the evolution of the manufacturing system that can be faithfully modelled as a stochastic process. This study attempts the characterisation of the system based on the events, and uses this characteristic information in design, planning and operation of the system. The random events identified in the study are associated with the following: (i) The outcome after inspection in various stages of production of an item (ii) Random arrival of production order (demand) of various products to the shop floor. The work developed stochastic as well as probabilistic models. The models associated with the random events, outcome after inspection, are used to study the effect of scrap and rework on the performance of the system. The models associated with the second random events mentioned above are used to design the system, in particular, layout design, and part family and cell formation. The objectives set for the thesis are as follows:

➢ To analyse and develop suitable models for performance measures under rework and scrapping of in-process items in a manufacturing system.

This objective leads to the following sub-objectives:

- To develop certain performance measures for a discrete part serial manufacturing system.
- To develop a model to determine the raw material required to meet a given service level for the finished goods.
- To develop a raw material requirement model, which will minimise the total cost consists of production cost and under stock cost.
- To modify the quantity of material represented in the bill of material of a product.

➢ To model and analyse the layout problem under random demand of products

The sub-objectives are:

- To critically analyse the layout formulation method and to study the layout formulation when the demand is known probabilistically.
- To develop a stochastic model to obtain input (flow matrix) required for a layout problem when the order arrival is considered as Poisson process.
- To study the sensitivity of the layout model when the flow matrix obtained in the above objective is used.

➢ To develop suitable model and analyse the formulation of cell and part family formation under random product-mixes manufactured.

1.5 ORGANISATION OF THE THESIS

Including the introduction, there are nine chapters in this thesis. Chapter 2 presents an extensive survey of the literature in the direction of the present work. This chapter provides a survey of literature on facilities design and performance modelling, especially using Markovian models, of manufacturing system. The survey reveals the gap in existing knowledge that leads to the present work.

Chapter 3 presents an absorbing Markov chain modelling of flow of material through a production system. The model characterises the system under uncertainty due to scrapping and reworking. The output of the model is used for determining raw material requirement.

The modelling of assembly and testing operation for manufacturing a product is the subject of chapter 4, using the concept of absorbing Markov chain modelling of serial discrete manufacturing system. The modelling procedure leads to the modification of bill of material of the product. This chapter also contains a case study, which describes the effect of scrapping of material on material requirement of a real life problem of assembly and testing operation of solenoid operated flow control valve used for spacecraft propulsion.

The modelling of material handling process as a stochastic process is presented in chapter 5. Three models of material handling are described. In each model the material handling process is first modelled as a continuous time Markov chain. Then the discrete time Markov chain embedded in the continuous time Markov chain is identified. Some of the results of the model are useful for layout problem modelling.

Chapter 6 discusses modelling uncertainties in layout planning for manufacturing systems. It contains the layout modelling method when the demand of each product is an independent random variable. It also contains the sensitivity analysis when flow matrix obtained by modelling of material handling process is used for layout planning.

Manufacturing cell formation under probabilistic product mix is the subject matter of chapter 7. The description contains cell formation using genetic algorithm.

Chapter 8 describes that the models showing the effect of scrap and rework, and the layout models can be suitably used in cellular manufacturing systems.

The last chapter of the thesis summarises the contents presented earlier and its limitations. It also shows the directions for further investigations.

Certain details from chapters 6 and 7 are relegated to Annexure.

1.6 SUMMARY

This chapter is an introduction that details out the research problem, a brief outline of the approach and the contributions made. This also outlines the organisation of the thesis.

CHAPTER 2

REVIEW OF LITERATURE

2.1 PREAMPLE

Cellular manufacturing is the application of group technology concepts in manufacturing. Earlier developments were in decomposition of manufacturing system into cells and part families such that they are fully processed within a machine group called cells. This chapter provides a survey of literature on facility design with particular importance to design of cellular manufacturing system and layout design. Rest of the chapter considers stochastic models on discrete part manufacturing and the literature on performance evaluation of manufacturing system with a section on process yield and material planning

2.2 FACILITY DESIGN

It is a recognised fact that an optimal manufacturing layout will lead to reduced material handling and management effort, shorter production cycles, simplified material flow and increased quality. An optimal layout is possible only for a static environment. But, the manufacturing environment is dynamic in nature as changes in product mix, product volume, process and product design are the basic characteristics of the environment. A job shop production layout has better flexibility. However, the above said factors of optimal layout cannot be fully achieved. Perhaps, CMS is one which combines flexibility and factors of optimal layout to an extent. Flexibility in facility design is defined as the capability of a layout to react to disturbances, caused by changes [Webster and Tyberghein (1980)].

A comprehensive facility design of CMS involves three stages [Proth and Vernadat (1991) and Harhalakis and Minis (1991)]. The first stage, called manufacturing cell design, is concerned with grouping of physical facilities to form cells. Second, intra cell location, is addressing the layout of physical facilities within the cells. Finally, the cell location stage consists of placing the cells on the shop-floor surface. The manufacturing cell design involves grouping of physical facilities into cells in order to optimise one or more criteria taking into account several constraints. A vast majority of the researches on CMS are directed towards manufacturing cell design. Certain research publications of this kind lead to concurrent formation of machine cell and part family. For instance, the algorithm proposed by Chandrasekharan and Rajagopalan (1987) - called ZODIAC, is a research work of this type. A good account of contributions on cell design and CMS design methodology are available in survey papers of Kusiak and Chow (1988), Singh (1993), Heragu (1994) and, Offodile *et al.* (1994). Generally, the objective of this kind of work is to minimise/reduce material handling (inter cell movement of parts).

All the above work is deterministic in nature, since the parameters required for the design must be known with certainty at the time of formulation of the problem. When we look into the dynamics of the manufacturing environment, there is a great deal of uncertainty in most cell design problems. Often, when the cell is being designed there is only a general idea about the products it would produce. This may further be augmented when the product life cycle become shorter due to increased global competition, rapid changes in technology and the necessity to respond to cost and quality conscious customer.

The above discussions illustrate the need for looking onto the dynamics of manufacturing environment when facility design is carried out. CMS design problem is a subset of the general facility design problem and the objective of both the design problems is generally to minimise the material handling cost, maximise an adjacency measure or optimise some combination of the two. Hence, the literature review focuses its attention on the dynamic aspect of layout design. But, static models are also included to know the current research scenario.

A facility design is usually prepared for a planning horizon. In the planning horizon the facilities may or may not be relocated or rearranged. Based on this, the design problem is classified as (i) Static facility layout problem (SFLP) (ii) Dynamic facility layout problem (DFLP). If the planning horizon contains one period or more, it could be a basis for classification: i.e., (i) Single period facility layout problem (SPFLP) (ii) Multi-period facility layout problem (MPFLP). The planning horizon may be any convenient unit of time. It can be measured in month, quarter or year. A SPFLP can have time horizon as years if the manufacturing flow characteristics are stable over the years of planning. Then this layout problem is also called SFLP, as there is no rearrangement over the planning horizon. An MPFLP also can have planning horizon as years, but it contains a number of periods, as the flow characteristic is different at different periods over the planning horizon. Hence, rearrangement of facility from period to period may result in considerable benefit in material handling cost compared to rearrangement cost. Thus, MPFLP is none other than DFLP.

2.2.1 Static Facility Layout Problem

Flow characteristics with fairly constant product mix and volume lead to SFLP. Variations that normally arise are handled in a facility by making suitable changes in material handling and flow, schedules and routings, etc. Majority of research publications assume deterministic flow parameters as in the case of traditional layout problems. Comparison and discussion of these can be found in Kusiak and Heragu (1987). However, some recent publications employ probabilistic parameters for the facility design. Optimal or heuristic methodology is used to get the facility design. Solution through optimal method depends on the size of the problem. A general layout problem may be formulated as a quadratic assignment problem (QAP).

Static Facility Layout Problem - Probabilistic Models

The basic input to a layout problem is 'from-to' flow matrix. In a deterministic situation only one such matrix exists. A typical manufacturing environment consists of probabilistic demand for the items and hence arriving at a single 'from-to' matrix is not easy. Each possible change in demand involves the determination of the joint probabilities of various demand states, and such an approach may not be practical. Alternative combinations of levels of items would result in a different flow matrix and may result in a different layout. Rosenblatt and Lee (1987) suggested a method to reduce the combination of levels. They proposed three levels of demand estimates similar to the one used for the time estimate of PERT model. For n number of items there are 3n possible combinations of demand states. Then, layouts are generated for various demand states and preferred layouts are identified from the generated layouts. Robustness is the criterion used to identify (flexible) layout that can cope with changes in demand. Robustness of an alternative is the number of times that its solution lies within a pre-specified percentage of the optimal objective, for different sets of scenarios. Certain approaches that consider some kind of probabilistic distributions are discussed in the following part.

A few of the papers deal with probabilistic layout design problem. Rosenblatt and Kropp (1992) models layout problems considering the different scenarios of demand and its probability of occurrences. Each scenario has a flow characteristic, which can be estimated, in the form of 'from-to' flow matrix. Now, each scenario can have an optimum layout by modelling it as a QAP. This optimum layout is an outcome of a demand scenario, which has a probability of occurrence. Similarly, for each scenario there is an optimum layout and a probability. Now, the problem becomes a selection of layout, which has minimum expected cost of material handling. They proved that for a given probability distribution (discrete and tabular) of scenario, the best layout will be obtained when the problem is formulated with expected flow matrix. Expected flow matrix is the weighted-average flow matrix. The robustness of the procedure is tested using sensitivity analysis to variations in the probabilities of occurrence.

Under the above modelling procedure, it is good to take deviation to inventory control problem such as static inventory problem under risk. Starr and Miller (1986) provide a detailed account of the above inventory problem analysis. Nature of the inventory problem and the layout problem discussed above are more or less similar. Hence the basics of the solution procedure are not different. Skill involved in this is in adapting the basic procedure to a different kind of problem. However, in the layout problem, considering the special nature, the optimum layout corresponds to the expected flow matrix and not the one from a given scenario like in the inventory problem.

Another application of the above basic concept is to cellular manufacturing system (CMS) design, as attempted by Seifoddini (1990). It is one of the first of its kind in a probabilistic machine-cell formation. A CMS design involves machine cell and part family formation in which some kind of similarity measure is used to form the cells and part families. The basic data required for the CMS design is the machine-part incidence matrix. Often, while designing CMS a deterministic product mix is considered. But there is a considerable scope for product mix change during the effective life of the CMS. The method of redesigning the system as and when change happens in the product mix is not practical. It is possible to incorporate all the expected changes during the initial design phase. In such a case, it is necessary to identify the probability function of occurrence of product mix. Such a function, in its simplest form, can be a set of product mixes with their associated probabilities of occurrence. Each possible product mix has a machine-part incidence matrix, and the corresponding best grouping (cellular design) for the given incidence matrix can be developed. Evaluate the cellular design using some performance measure such as intercellular material handling cost. Now, it is necessary to select a grouped system from among those available, which can perform best under all the product mixes. That is, calculate the expected value of the performance measure when each grouped system is subjected to the product mix changes. The grouped system, which has minimum expected performance measure is the best CMS design. When the product mix is the basic information to prepare the machine-part incidence matrix, (which is the data grouped to get CMS design), the best CMS design will be obtained when all the possible product mixes are considered and compressed in a single machine-part incidence matrix. That is, Seifoddini's design may be suboptimal.

Demand variations can have effect on the performance of CMS. A CMS design should be robust to the random variations of demand. Harhalakis *et al.* (1994) provides a solution methodology for the robust design of CMS under random demand statistics of the independent demand and when the resource capacity is explicitly considered for the design. The criterion used to evaluate the CMS design is the intercellular material handling cost. For a given capacity and statistics of independent demand, the mean of the production volume, which maximises the profit is determined. This mean value of the feasible production volume is used to obtain the near optimum grouping.

Facility layout problem is generally formulated as QAP. When probabilistic flow matrix is used such problems are called stochastic QAP. Li and Smith (1995) suggest a heuristic algorithm suitable for quadratic and stochastic quadratic assignment problem. The algorithm uses simple and clever sampling properties, it is both efficient and effective. Although it will not guarantee an optimal solution, it provides very acceptable sub-optimal solutions in a very reasonable amount of computing time.

Arzi *et al.* (2001) present a new multi-objective approach for cell formation problem in a lumpy demand environment. The periodic lumpy demand is modelled as a random variable. The effect of lumpiness on the capacity requirement is the prime aim of the work. The new approach suggests that the correlation between the demand of produced part types should be evaluated and then taken into consideration in the cell formation. Consequently, much better cell formation solutions are obtained in terms of lower capacity requirements.

The operational characteristic of a manufacturing system should be taken into account while designing a layout. Castillo and Peters (2002) present a formulation and solution procedure that integrated unit load and material handling consideration in facility layout design. The integration is based on a stochastic model that captures the operational characteristics of the manufacturing system and a non-linear mixed integer program that incorporates the assignment of machines to departments and facility layout. The methodology evaluates the expected WIP for a particular manufacturing system scenario and prescribes the assignment of machines to department locations, the unit load sizes between departments, and the number of material handling devices used in the system to minimise the total cost of the expected WIP in the system.

Static Facility Layout Problem - Deterministic Models

A few deterministic and static approaches are reviewed here in order to get a comprehensive idea of the research work in the facility design. A vast majority of papers reported for the CMS design are deterministic, and those generally contain grouping of machines and parts alone. But some authors recently reported grouping as well as spatial arrangement.

Irani, *et al.* (1991) propose a method for layout design of a CMS that would simultaneously generate grouping of machines unique to a part family and machines shared by several cells to be located together in a functional section. This layout procedure differs from conventional CMS design since the procedure allows inter cell moves. However, the overall flows are simplified to get the advantages of CMS. This is achieved by analysing the complex interactions between the critical subproblems in the cell formation – machine grouping, part family formation, distribution and utilisation of shared machines, intra cell layout, inter cell layout and material handling. A graph theoretic approach is used for this. This CMS design procedure has more flexibility under the dynamics of the manufacturing environment than the conventional method of cell and part family formation.

Several characteristics of the production system design affect the performance of CMS. One such factor is utilisation of functionally identical machines. For such cases, Harhalakis and Minis (1991) propose a two-stage CMS design. The first stage determines cell composition and the next stage is for intra cell layout. Both the stages aim at reducing and simplifying material flow in the system. The cell formation heuristic considers an environment consisting of both unique machines and groups of functionally identical machines with finite capacity. Furthermore, it accounts for many important system parameters, such as part production volumes, batch sizes and pallet sizes. The grouping is based on pre-specified generic part production routings. This algorithm is suitable for make-to-stock environment, in which the product mix does not vary greatly with time. The output of the cell formation is the input to the intra cell layout, which is based on the method of simulated annealing. The layout algorithm is similar to the conventional layout procedure. It generates randomly an initial layout and improves upon it. But the layout algorithm is insensitive to initial conditions. The authors hint that this algorithm with appropriate enhancement may be used for inter cell layout.

A CMS design involves not only the cell and part family formation but also spatial arrangement of machines within cells and locating cells on the shop floor. Proth and Vernadat (1991) suggest some method in this direction during their effort to develop a comprehensive software called COALA (Computer-Aided Manufacturing Layout Design) to the design of CMS. Their approach consists of three stages. All these stages use simulated annealing to arrive solutions. First stage provides several equivalent solutions. It allows the user to make his final choice on the basis of qualitative criteria. The second stage – intra cell location stage consists of two step approach: in the first step, a knowledge based approach is used to find the best layout configuration type and the appropriate handling system for a cell; the second step involves mathematical programming to compute physical location of the machines in the cell. The last stage consists of optimally locating the cells on the shop-floor surface taking into account the physical limitation of the shop-floor such as size, configuration, process constraints, reserved positions, proximity constraints and cell input and output areas. It is a comprehensive model covering all facets of CMS design especially the layout design.

Hilger (1991) proposes an algorithm for cell formation. The basic input required for the algorithm is sequence of operations. The size of the manufacturing cells is one of the outputs of algorithm and not a parameter of the algorithm. Similarity measures are used for the cell formation heuristic.

One of the CMS design aspects is the placing of cells on the shop floor and it is the major research consideration of the work by Chandrasekharan and Rajagopalan (1993). Component families and machine cells formed in a cellular manufacturing system generally contain exceptional operations which introduce inter cell movement. The material handling consequent to inter cell moves is to be minimised while placing cells on the shop floor. The layout design procedure involves developing an ordinally ranked distance function on the basis of inter cell moves and recasting the problem as a classical problem in non-metric multidimensional scaling (MDS). The MDS output, in this case, is a plot showing the configuration of cells in a plane (or straight line if desired) with a table of their co-ordinate positions. That is, the output can be used for designing two-dimensional and one-dimensional layout. This procedure is amenable for process layout design also.

Several aspects of the cellular layout are similar to both the machine and block layout problems, and, thus, benefit from their concepts, models and solution procedures. Hassan (1995) suggests that though the cellular layout has several aspects in common with the general facility layout problem, the unique characteristics of cellular layout necessitate a separate treatment of its layout problems. He highlights several layout factors and design issues as they pertain to cellular manufacturing system.

The work of Akturk and Balkose (1996) provides grouping of machines and parts, and layout of machines within the cell. They consider several factors for the design such as similarities in terms of the design and manufacturing attributes and operation sequence, machine investment, workloads within cells and between cells and number of skipping – as a measure of material handling activity. The resultant spatial arrangement is the modified flow shop where each part will move in only one direction (backtracking is not allowed), but not necessarily processed by all of the machines.

A manufacturing system contains unique machines and multiple identical machines. One of the problems while grouping is the assigning of operations of a particular part to specific machines within the multiple identical machines. This problem discussed by Harhalakis and Minis (1991), and Akturk and Balkose (1996) also takes into account this fact by considering investment in multiple machines of specific type. Wu and Salvendy (1999) suggest a graphical model to solve the cell formation and assignment of identical machines concurrently.

A method for the intra cell layout, based on machine sequence required for operation of parts of a part family, is suggested by George Varghese (1998). He defines two matrices called position eligibility matrix and successor preference matrix. If there are m machines required for a part family, m positions are available for these to be laid. Each machine may be a candidate for any of the position. Now, number of times a machine becomes eligible for a particular position can be found out from the route sheet of the part family and it is called the "position eligibility of the machine" for that position. A best layout may maximise the position eligibility and hence this can be modelled as an assignment problem – a special case of linear programming problem. Solution of this is a linear arrangement of machines. The route sheet of the part family provides another information on the number of times a machine becomes successor (immediate or distant successor) to another machine. This is defined as successor preference. A best layout may maximise the successor preference. Here again this problem is modelled as an assignment problem. The output of this problem is successor preference pairs. Now, the two solutions may not match. Thus, a final layout may be prepared considering the information from the above solutions. But the work is deficient in arriving a generalised algorithm to the final layout preparation.

A multi-objective model to generate the layout designs for machines and cells in a cellular manufacturing environment is the area of work of Bazargan-Lari (1999). The model addresses a number of issues related to the practical implementation of cellular manufacturing structures such as closeness relationships, location restrictions/preferences, machine/cell orientations and aisles. The approach of this paper allows to generate alternative distinct layout designs, thus providing the decision maker with wider selections considering tangible and intangible factors.

A class of layout problem called machine layout problem (MLP) differs from the classical facility layout problem, which is generally formulated as a QAP. The major disadvantage of QAP is that they require location sites to be known a priori. MLP has machines of different sizes and hence the distances between the sites are not fixed. Factors such as width of the material handling carrier path, clearance between machines, orientation of machines, load/unload points, etc, are considered while determining the layout in MLP.

MLP is particularly suitable for flexible manufacturing system (FMS). An FMS is usually equipped with automated machine tools, automated material handling systems and other sophisticated systems requires high investments. Hence, a good machine layout is needed to ensure good system performance and thereby to have the benefits necessary to justify the capital investment.

In an FMS the layout entities are actually cells, each cell containing one or more machines. The term 'machine' includes machine tools, in-process storage systems, inspection stations, etc. The load/unload point of the cell is usually located on either one of the cell axes, the exact position being determined by the cell builder. Hence, an FMS layout involves specifying the spatial co-ordinates of each cell, the orientation of each cell in either a horizontal or vertical position and the position of each cell's load/unload points.

The type of material handling device to be used in FMS is the deciding factor in determining the layout. The material handling system considered includes conveyors, handling robot, gantry robot, and automated guided vehicle (AGV). Luggen suggests four basic types of FMS layout in practice [Das, 1993]. They are

i. Progressive or in-line (single row)

ii. Closed loop (circular)
iii. Ladder (double row, multi row)
iv. Open field

The machines are arranged such that the total time required by the material handling device to transfer products, components, etc. between machines is minimised. As automated manufacturing systems and their problems are of recent origin this area got attention of researchers very recently. Some publications on static MLP are discussed in the following paragraphs.

Heragu and Kusiak (1988) present basic type of machine layout and the characteristics of material handling devices frequently used in FMS. They suggest two algorithms for solving the MLP. Their work reported in 1990 presents a knowledge-based system for machine layout. A combination of optimisation and expert system approach is used to solve the MLP. Proth and Vernadat (1991) also suggested a combined approach of expert system and optimisation techniques.

Das (1993) proposes a four-step heuristic procedure for solving an open field type layout for the MLP. The heuristic combines variable partitioning and mixed integer programming to generate the layout. The integer programming formulation is a computationally difficult program. The heuristic decomposes it partitioning the variables into cells, which can then be sequentially optimised. The procedure provided in the work model cell orientations, the position of load/unload points and fixed cell geometries.

Determining a common machine sequence for multi-products processed by different operation/machine sequence is the research topic of Chen *et al.* (2001). Linear machine sequencing is most popular due to its flow structure and its ability to arrange machines in various flow structures such as straight line, serpentine line, U-shape or a loop. The objective is to minimise the total flow distance travelled by all the products. The following constraints are considered while modelling: (i) each product must go through a subset of the common machine sequence, without back tracking, either in-sequence or by-pass. (ii) the use of duplicate machines is limited. This model is particularly suitable for intra cell layout of flow-line type.

Machine allocation (intra cell) in cellular manufacturing is a work of Chan *et al.* (2002). An algorithm developed for machine allocation considers practical constraints in machine allocation. These constraints include the part-handling factor, the flow frequency, the travelling distance and part demand, etc.

2.2.2 Dynamic Facility Layout Problem

A dynamic procedure responds to changing production requirements. As a result, material handling cost is reduced and manufacturing efficiency is improved as changes are made in the layout when new requirement arises over time. While adapting the layout to the new environment, rearrangement of facilities is done. It is hoped that the reduction in material handling costs will offset the layout rearrangement costs.

The dynamic environment existing in manufacturing field prompted researchers to focus on DFLP. However, DFLP is not the only modelling methodology. The solution suggested for the design of the manufacturing facility under dynamic environment includes virtual manufacturing cells, reconfigurable manufacturing systems and dynamic facility layouts. Most of the researchers of DFLP have resorted to quadratic assignment problem (QAP) formulation as the core mechanism. That is, the plant surface is partitioned into blocks and it is assumed that any machine can be assigned to any site [Yang and Peters (1998)]. Non QAP based formulation, does not have pre-specified layout sites (that is, the floor space is not partitioned into grid in which possible location sites are known in advance), and uses continuous floor space; which gives infinite number of candidate locations for each machine. Therefore, the machine rearrangement alternatives cannot be evaluated by simple enumeration of all possible candidate locations for each machine.

Dynamic Facility Layout Problem - Deterministic Models

Researchers have traditionally looked into the facility layout problem as static one. Recent approaches focus on the dynamic aspect. Rosenblatt (1986) presents the general dynamic problem, introducing dynamic programming approach to solve the multi-period layout selection problem. At each period, it solves the static layout problem to propose a number of potential layout alternatives. The objective is to select the sequence of layout, which minimises the overall sum of material handling costs and rearranging costs, both provided as input to the model.

Batta (1987) suggests that when the same layout is to be used over all the periods, the dynamic layout problem can be solved as a static layout problem in which the flow matrix is obtained by adding the flows in each period of the planning horizon. This method gives the best possible upper bounds for dynamic layout problem.

Strategic interpolative design concept is employed for the DFLP by Montreuil and Venkatadri (1991). Every organisation has a life cycle and its maturity phase is strategically envisaged. The methodology first estimates, at maturity level, the possible scenarios of the system requirements, strategically designs the layout for the maturity phase for the given set of scenarios, then tactically interpolates intermediate layouts backward to the facility plan required now. This is a proactive methodology on a perspective that the present is the past of a future, and the organisation can be used to push the strategy towards its realisation rather than be pulled along by it.

Lacksonen and Enscore (1993) formulate the DFLP as a QAP and suggest algorithms to solve the QAP form of the DFLP. Five existing algorithms for the QAP form of the SFLP are modified to solve the new formulation.

Kochhar and Heragu (1998) propose an algorithm called Dynamic Heuristically Operated Placement Evolution (DHOPE) for multi-floor DFLP. It is concerned with the design of a dynamic facility over two consecutive planning periods. Given the flow information for two consecutive periods at the initial design stage, DHOPE finds a facility layout in two iterations. In the first iteration desired number of solutions are generated considering the first period problem as a SFLP. For each of these first period layouts, the algorithm generates corresponding second period layout that minimises rearrangement costs and material flow costs. Thus the best combination of layouts for two periods is found out. They also propose that as the distant future facility planning is associated with more uncertainties, the robustness based facility planning is not desirable. Instead, the alternative strategy could be an easily adaptable facility, which may be appropriately reconfigured to suit to the requirements.

A cellular manufacturing system is designed to perform best for a specific product mix. Seifoddini and Djassemi (1996) suggest a simulation model to evaluate the rate of change of performance as product mix changes. The performance measures such as mean flow time and work-in-process are used to determine the changes in the performance of the system. The same authors suggest in another paper (1997) a flexibility range representing the capability of the system in dealing with these changes. The procedure can be used to evaluate the capability of a cellular manufacturing system in dealing with product mix variations.

CMS performance is sensitive to the fluctuations in the demand and the product mix. The dynamic nature of the production environment is addressed by Wicks and Reasor (1999) for the CMS design. The design allows the composition of part families and machine cells to change as the product mix and the demand change in the planning horizon. The forecast of product mix and demand for various periods of planning horizon are basic input to the design. In addition to this, many other system parameters such as operation sequence of parts and capacity of the resources, and constraints such as number of cells and part families to be formed, minimum number of machines per cell and minimum number of parts in a part family, are included. The objectives of the design methodology are the minimisation of inter cell movements, the minimisation of machine duplication and the minimisation of the system reconfiguration. It also provides comparison of this multi-period approach with a cellular design obtained under the assumption of constant demand and product mix. Single period approach considers the initial part population in the design of the system and does not consider reallocation of part families and relocation of machines between cells as time evolves.

Wang *et al.* (2001) present a model that minimises the total material handling cost under variable demand and solves both inter and intra cell facility layout problems in CMS simultaneously. This model is suitable for product whose demand rate varies over the product life cycle. A simulated annealing algorithm for the problem modelled as biquadratic assignment is developed.

Huang *et al.* (2003) suggest a method to manage the product mix variation in a dynamic environment and thus to get a better performance. They suggest that to achieve efficient and economical production, planners and/or designers of production system must frequently evaluate manufacturing process similarities for products that they intend to produce. The proposed algorithm can be applied to optimise decisions on capacity exchange between rush orders and prescheduled orders, a growing practice in manufacturing in response to increasingly varied customer requirements and market demand, on better performance.

A facility design alternative to CMS under frequent product mix changes is virtual cellular manufacturing system (VCMS). In traditional cellular manufacturing system, the shop floor configuration is fixed, whereas; in VCMS the shop floor configuration changes in response to change in product mix over time. Therefore, the life of a given shop floor configuration continues as long as the product mix remains relatively unchanged. Furthermore, in traditional cellular manufacturing system, a machine cell occupies a contiguous region of the shop floor, whereas; the same is not necessarily true with virtual cells. Virtual manufacturing cells are logical instead of physical; machines in a cell can be at any location on the shop floor. Ko and Egbelu (2003) present a method for designing VCMS. Cell sharing among parts or part families is a feature used in the design. The algorithm developed works directly with job routing without the need to pre-specify the number of cells required.

Chan *et al.* (2004) suggest an intra cell machine layout model, which exploits varying periodic demand in a cellular manufacturing environment. It minimises the total part travelling cost by taking into consideration the closeness of machines, the handling efficiency, the part travelling distance, the material flow frequency and the machine rearrangement cost in a dynamic environment. First, it will assign machines within a cell effectively based on several factors like the operation sequence, the part flow frequencies, and the customer demands of a part family in order to determine the required closeness of machines in the layout. Then, the total travelling score will be calculated by accumulating values of all related pair-wise machines, each of which incorporates information such as the distance, the frequency of part flow, and the part handling efficiency. The dynamic machine cellular layout is obtained by having a good balance between the total machine rearrangement cost and the total part travelling cost.

Dynamic Facility Layout Problem - Probabilistic Models

The literature reviewed till now on DFLP assumed that all the future scenarios are known or deterministic. Kouvelis, *et al.* (1992) provide a "robust approach" to layout problems when considerable uncertainty exists, both in terms of products to be produced and their production volumes for single and multi-period problems. The inaccuracy of the input data may affect the layout design objective. In such situations, the use of 'optimality' with respect to a design objective is insufficiently discriminating. They suggest that robustness criterion is more acceptable for such decisions. The work suggests simple modifications to standard branch and bound procedure for QAP formulation to generate the list of robust layouts. A systematic approach is suggested for efficient generation of robust layouts for medium size problems (less than 15 machines) of multi-period layouts, which contain monument-like structures difficult to relocate.

Palekar *et al.* (1992) present a stochastic dynamic facility layout problem. A quadratic integer program formulation is provided to determine a set of layouts, one for each period, to minimise the expected cost of material handling and relocation. A heuristic solution procedure is suggested as the formulation is NP-hard. The uncertainties that can arise in the layout design input are described. It is assumed that the system is described by finite sets of flow matrices, which represent states in a Markov chain, and one step-transition probabilities. Based on these assumptions the probability associated with each flow matrix (state) is determined. These probabilities and flow matrices are the major input to the formulation of the problem.

Afentakis *et al.* (1990) investigate the relative effectiveness of different layout strategies is the case of dynamic layout design associated with automated manufacturing system for changing part-mix with unidirectional closed loop type material handling system. They examine the extent to which system performance was affected as different layout strategies are used. The result indicates a decrease of up to 30% in performance capability for some relayout strategy when compared to the best strategy.

Yang and Peters (1998) suggest an open-field type stochastic flexible machine layout for a set of unequal size machines over a planning horizon by optimising the trade-offs between cost of material handling and machine rearrangement as the production requirements change over time. The approach involves the determination of robust machine layouts for a set of planning time windows, which cover the total planning horizon. This means a rearrangement of machines at the beginning of the time window, which consists of one or more periods. The flexible strategy chooses the time windows to minimise the total cost, and include robust and dynamic strategies. The number and length of time windows are found out based on the trade-off between material handling costs and machine rearrangement costs. This trade-off is similar to the one used in the lot-sizing policy in material requirement planning.

For an uncertain and dynamic environment either a robust layout or an agile layout may be better. A robust layout is a unique layout able to behave efficiently, even when mix and/or volume fluctuate. In an agile layout approach frequent layout modifications are carried out at each productive period affected by significant variations. From an industrial point of view, the adoption of an agile layout also implies the availability of 'agile' resources, e.g. machine tools that can be easily relocated. Some manufacturers proposing innovative machines have preferred this manufacturing philosophy, as have been particularly evident in recent years. It also suits the needs of assembly manufacturing units. On the other hand, the accurate definition of a robust layout becomes an obligatory approach when the manufacturing cycle involves resources that are not 'agile' at all (furnaces, presses etc). Braglia *et al.* (2003) suggest indices that will help in identifying the strategies (robust or agile layout) to be preferred. A formulation method under probabilistic situation is suggested which is congruent with the Rosenblatt and Kropp (1992) model.

2.3 PERFORMANCE MODELLING

Current manufacturing scenario is different from the yester years and the rate of change is fast in recent years. The changes are in accordance with the requirement placed by customers and the competition. The manufacturing management is responding to these factors by going for high performance automated manufacturing systems and it helps the management to be increasingly customer responsive. These systems are necessarily flexible and highly capital-intensive. Modelling and performance evaluation play a vital role in the design and operation of such manufacturing systems and have received widespread attention of researchers of recent years. The capital-intensive systems can produce high-quality, low cost products that are competitive in world markets. To be competitive, the manufacturing systems must be designed effectively and efficiently, and right operation strategies must be employed. Mathematical modelling provides a systematic way of decision making in the design and operation of manufacturing systems. Often used analytical tools for the performance modelling are Markov chains, queuing networks and stochastic Petri nets [Jungnitz and Desrochers (1991), Viswanadham and Narahari (1992)]. Performance evaluation models involving Markov chain are discussed in the present work.

A stochastic process whose future is independent of the past but depends on the current state can be modelled as a special class of stochastic process called Markov process. The discrete state space Markov process is called Markov chain.

Markov chain constitutes the basis of the most analytical modelling tools used for the performance modelling of manufacturing systems. The general analytical tools used for the performance modelling are Markov processes, queue networks and stochastic Petri nets. In fact, Markov chain is a natural modelling tool for discrete event dynamic systems because of the notion of discrete states and state transitions. The underlying stochastic process of the high level models such as queuing networks and stochastic Petri nets turns out to be a Markov chain [Viswanadham and Narahari (1992)].

2.3.1 Role of Performance Modelling

Decisions are involved in the planning and design phase, and operation phase of manufacturing management. Performance modelling acts as an effective aid to this decision-making. Typical decisions during the planning and design stages include best possible layout, number and type of machines, number of material handling devices, manufacturing system configuration, machine grouping, size of pallet pool and number of fixtures, number of buffers, tool storage capacity, line balancing, production batch determination, scheduling policies, etc.

Decisions during operational phase are related to finding the best routes in the event of breakdowns, obtaining optimum schedules when part mix or demand changes; and in the event of machine failure, predicting the effect of adding or withdrawing resources, etc. Performance modelling can also be used to take decisions on selection of alternatives such as push production versus pull production, shared resources versus distributed resources, effect of various levels of flexibility, etc.

2.3.2 Performance Measures

Performance modelling can be used to compute the performance measures of a given system, which in turn aids decision-making. Certain performance measures used for the automated manufacturing system are: manufacturing lead time, work-in-process, machine utilisation, capacity, flexibility, throughput (rate of production), quality and performability (combined analysis of both performance and reliability; performance means how well the system performs, provided it is correct and reliability means the probability of performing successfully [Meyer (1982)]). These performance measures are indicative of the competitive status of a manufacturing system [Viswanadham and Narahari (1992)]. Groover (1989) provides generic performance measures computation procedure for deterministic environment.

When parts are processed according to a fixed sequence by a series of machines possibly separated by a finite buffer it is called a serial production system [Lim, Meerkov and Top (1990)]. Performance of this type of system has been studied in manufacturing science for a long time. Literature on performance evaluation of serial production system varies in content basically on the factors influencing the performance included in the research. Parameters influencing the serial system considered in the literature are number of stages, machine reliability, processing time randomness, intermediate buffers and its capacity, possible scraping of units when machine fails, control (computer) and communication overhead, switch control system used to control the in-process inventory, reliability of transfer device, reworking of the work material, scraping of work material as not meeting specification, Bottleneck effect, demand variability, number of kanban and configuration of inspection stations.

White (1970) models a manufacturing system with scrap and rework at various stages of the system. He provides expected value and variance for the resources consumed by the process when the process is modelled as absorbing Markov chain. These results are used to determine the amount of raw material to be scheduled through the production system to meet a known demand. This is a profit maximisation model.

Gershwin and Berman (1981) present a Markov process model of a two-stage system with finite intermediate buffer. The machines are characterised by exponential service, failure and repair processes. The model can be used for average in-process inventory and throughput calculations.

Shanthikumar and Tien (1983) demonstrate the performance of a two-stage synchronised automatic transfer line where work piece transfer at all stages is synchronised to occur at the same time epoch and with intermediate buffer. Finite, unlimited buffer and no buffer cases are analysed when the failure and repair times are geometrically distributed and that the units may be scrapped with a certain probability when the machine processing it fails. The model analyses the throughput, and number of parts in the buffer based on the intermediate storage consideration.

Markovian modelling of serial production systems loses its analytical tractability when number of stages/buffers increases due to the enumerative nature of state in the Markov model. Gershwin and Schick (1983) suggest a method to mitigate this problem. They applied the method to two- and three-stage system and hope that this method can be extended to longer lines. It can be used for performance measures such as production rate, down times and expected in-process inventory when the source of randomness is the unreliability of the machines or workstations.

Work material, passing through the machines in serial production sometimes acquires defects. The defective unit may be either rectified by reworking or scrapped. The effect of reworking and scrapping on system performance is analysed using Markov chain in the research paper of Davis and Kennedy (1987). It illustrates relationships of design and control issues in production system modelling. The work analyses the scrapping and reworking effect on the requirements of material, capacity and buffer.

Manufacturing systems of these days are integrated with computers to have better performance. There is a controller associated with each machine and at every cell level. The cell controller can communicate with the machine controller and vice-versa. The cell controller is able to provide processing details when it receives appropriate message from the machine controller. The processing speed of cell controller may affect the throughput of the system. Ammar (1987) suggests a Markov chain model to study this effect when the cell contains two-stage serial system and part processing time of machines and message processing time of cell controller are random variables.

A manufacturing system where production operations are initiated on the basis of a customer demand is called a pull system. Such systems generally use kanban-controlled system for regulating the production operations. Deleersnyder, *et al.* (1989) model an N-stage serial production system as a Markov chain to analyse the effect of number of kanbans, the machine reliability, the demand variability and safety stock requirements on the performance of the kanban controlled pull system. It is demonstrated for a two- and three-stage manufacturing system.

The throughput models discussed in literature are generally applicable for less complicated serial production system. A serial production may become complex for analysis when the system contains more number of stages, processing time variability, and buffers between stations. Blumenfeld (1990) suggests an analytical formula for throughput of a general n-stage serial production system when all the machines have the same processing rate under the above complexity factors.

In a serial production system inventory may vary in the inter-stage buffer due to unbalanced machines in the line, randomness in processing time and machine breakdown. The control of in-process inventory plays a vital role in the manufacturing system. Hwang and Koh (1992) suggest a model for optimum control parameters for a full work system. The basic model involved follows the Markov process. A full work system contains switch control system to control the in-process inventories in serial automated production system. They present a two-stage full work system.

Generally, serial production systems are analysed using continuous time Markov chain. The analysis becomes complicated when the transition rate matrix of the chain becomes large in size. Hong and Seong (1993) present a simple, accurate and fast algorithm that favourably compares with the currently available methods for a two-stage serial production system. The system analysed is characterised as one consisting of two unreliable machines with random processing times and a finite buffer storage.

Bottleneck machines impede the performance of production lines. It is the objective of analysis by Chiang *et al.* (1998). They defined the bottlenecks in Markovian production system as the partial derivatives of the system production rate with respect to machines' up- and down-time. The Markovian production system is the one where the state of a machine in a cycle (The time necessary to process one part) is determined by a conditional probability, with the condition being the state of the machine in the previous cycle. They considered the bottlenecks in one- and two-machine production line when the machines are unreliable and an interstorage buffer of finite capacity exists for a two-machine case.

The process yield can have dramatic effect on the economics of the product. The first effect is on product cost. Yet measuring costs can substantially underestimate the importance of yield improvement. Bohn and Terwiesh (1999) show the importance of process yield analysis in their work. They show that yields are especially important in periods of constrained capacity. The analysis involves numerical example taken from hard disk drive manufacturing. The economic model developed by them was used to compare the economic value of wage reduction, yield improvement and automation. They found that the effect of yield improvement in increasing contribution and profit is very strong.

An automated production line has material handling device as an integrated part of the system. The reliability of the material handling device can affect the performance of the system. This is analysed in the work of Suliman (2000). He modelled as a Markov process a two-stage manufacturing system, which consists of an inter-stage buffer of finite capacity, and a transfer robot for loading, unloading and moving parts into and out of the buffer. The throughput of the system is analysed under the condition that both the stages and the transfer robot are unreliable.

Machine failure and its effect on the economic production quantity is the subject of the paper by Abboud (2001). The analysis situation is a single machine production inventory system that produces a single item, and the production and demand rates are assumed to be known and constant. The failure time and repair time of the machine is assumed as geometric random variable. While the machine is being repaired, demand is met from on-hand inventory if available; otherwise, demand is backordered up to a maximum limit. A Markov chain model of the situation is developed and the result from the model is used to develop an efficient algorithm to compute the average cost, which in turn, can be used to find the economic manufacturing quantity.

Rupe and Kuo (2001) modelled an FMS as a Markov process to obtain the performability of the system by dividing the FMS into subsystems with independent failure and repair processes. Spare parts inventory is also considered while modelling. The subsystem's results are combined to obtain system results and to get the performability measure for the entire FMS. This modelling methodology makes the FMS modelling less complicated.

A serial production system with inspection station modelled for evaluating the optimum inspection configuration was the subject of work by Cochran and Erol (2001). The work material that comes to the inspection station may have multiple defect types acquired by the production units from the raw material stores and during processing operations. Analytical models at steady-state are developed to calculate throughput rate, scrap rate and outgoing quality level.

Hadjinicola and Soteriou (2003) address the management's concern of how to allocate a budget to the various production stages of a multi stage system in order to improve the yield of various stages while minimising the annual cost incurred from defects observed in all stages. They provide a mathematical model for managerial decision making on the above operational problem. The model considers four factors namely, the current mean yield of the production stages, the cost of achieving the yield improvement at each production stage, the cost incurred to the firm from a defect observed at each production stage and the annual number of products processed at each production stage. The formulation results in a budget allocation tool that allows managers to consider tradeoffs on the aforementioned factors across all stages. A sensitivity analysis identifies cases where a particular production stage is more likely to have a higher yield improvement compared to other stages.

2.3.3 Process Yield and Material Planning

The material requirement in a production process is affected by the process yield. Not every unit of material that starts into the production process makes it to the end as a sellable, high quality product. Problems of various kinds affect the "fall-out" of material along the way. Often, some of the fall-out can be reworked, but always a fraction of it must be scrapped. This means that the effect of process yield on the material requirement should be incorporated during material planning. Also it may be noted that when production schedules of the components are coordinated using material requirement planning (MRP) logic, most of the demand uncertainty is eliminated. But the complication arises from scrap in manufacturing.

New and Map (1984) describe the operation yield and its effect in material planning under MRP logic. The analysis involves three methods of coping with yield rate. They are: (i) fixed quantity of buffer stock (ii) safety lead time (iii) increase in requirement forecast/bill of material. They also considered the market environment factor for taking the material requirement decision.

The primary goal of MRP is to meet the master production schedule (MPS). Accomplishing this objective with minimum inventory is its second objective. The cost of not being able to meet the MPS requirements and the cost of component inventory are inversely related. Usually scrap allowances are considered in MRP. Kurtulus and Pentico (1988) study the effect of scrap allowance policy on the above objective. The study develops simulation models to study the performance of the lot-size adjustment (considering scrap) rules studied by previous researchers and modifications suggested by managers for their implementation.

Murthy and Ma (1991) analyses many forms of uncertainty, which affect the MRP and advocate different approaches for MRP with uncertainty. They mainly discussed MRP planned manufacturing with quality variation in the production process. Two scenarios of defectiveness were examined: individual items being defective or not and a batch of parts, with quality denoting either the number of scrap or the fraction of defective parts in the batch. The approaches suggested are modelling quality variation, overplanning approach, overplanning approach with lot sizing and overplanning approach with inspection.

Wein (1992) study a production process with multi-stage having random yield at various stages. The marketing environment considered is make-to-order where an order is met from a single production lot size. The objective of the study is to characterise optimal rework and scrap policies in order to understand how the rework option affects the planning yield decisions. The study illustrates the need to determine the underlying yield density function at each stage in a production process.

Kurtulus (1996) studies in his paper, manufacturing systems with non-linear, multi-stage process that produces multiple lots. He analyses the effect of process yield on MPS requirements and component inventory planned using MRP. The study applies the popular lot-size adjustment rules, borrowed from the single-stage reject allowance problem, in an MRP environment. The effects of the rules on the component inventory and their ability to meet the MPS requirement have been analysed.

Yeung *et al.* (1998) reviewed the important parameters, which affect the effectiveness of manufacturing resource planning system. They could identify that most of the previous researches dealt with one kind of uncertainty, that is demand uncertainty, in this system. They suggest that in the real-world there are many other uncertainties facing the users of the system. Process yield is one such uncertainty.

Agnihotri *et al.* (2000) investigate a single stage manufacturing system with random yield. They examine particularly the impact of yield uncertainty on lot size decisions when there is a tardiness cost for not meeting the customer specified due date. It is assumed that the demand, as well as the customer specified due date, is known. Optimal lot sizes are derived when the yield rate has some specific distributions. They carried out some distribution free analysis to examine the robustness of the optimal lot size under the assumption of a specific yield rate distribution.

An extensive literature review on uncertainty under MRP-planned manufacturing is available in the paper of Koh *et al.* (2002). The authors refer to the use of MRP, MRP II and ERP as a production planning and scheduling system within manufacturing enterprise as MRP-planned manufacture. The paper provides a structure within which directions can be given for future research and the result of past research work are summarised to give an indication to practitioners of how to cope with uncertainty. An uncertainty categorisation structure has been developed using system theory to categorise uncertainty into input and process uncertainties. Buffering and dampening approaches are proposed and experimental methods used, are incorporated in the structure. The input uncertainty is defined as the uncertainty that occurs in the external supply or demand. Process uncertainty is defined as the uncertainty that occurs at internal supply or demand. The uncertainty due to yield loss is coming in the category of internal demand, which is defined as the uncertainties occurring at the demand chain within the manufacturing cycle of MRP-planned manufacture. For example quality variation in the manufacturing process could render part shortages.

The process yield usually affects supply chain inventory management. Khouja (2003) models a two-stage supply chain in which the quality of the output deteriorates with increased lot sizes. The supply chain consists of a supplier who produces a product in a single machine and delivers it to a retailer who in turn sells it to the final customer. The number of acceptable quality items in a lot is used to measure quality. It has been shown that the quality considerations can lead to significant reduction in production lot sizes. In addition, the models show that most benefits to the supply chain are attained from the suppliers producing on a just-in-time basis rather than delivering to their customers just-in-time.

2.4 CLOSURE

An efficient facility design involves identification of specific input factors of the manufacturing system. As a part of literature review the general environment of manufacturing system is identified. Facility designs as the conglomeration of various problems such as layout (static, dynamic and machine) and CMS design are discussed. Actually the CMS design involves both decomposition of manufacturing system to form part family and cell, layout to arrange facilities within the cell and placing cell on the shopfloor. Emphasis has been, generally, given for the identification of probabilistic models. In the course of the survey, effort has been made to identify dynamic models whereas traditional models are deterministic and static in nature. Dynamic models provide a better picture of the manufacturing situation than static models.

A portion of this chapter is concerned with performance evaluation of manufacturing system that produces discrete parts. Such systems can be considered as discrete event dynamic systems. Discrete state space stochastic process could be used for the faithful representation of such systems. This type of manufacturing system shows some of sort dependence and a first order dependence is generally considered. First order dependence means that the future depends on the current state and not on the past states. Such system evolution can be modelled as Markov process to study the performance. Performance evaluation of manufacturing system provides lot of information to management for proper decision-making. The literature reviewed provides various performance measures considering the different factors influencing the performance of the system.

Usually yield of the production system may not be hundred percent due to scrapping. The yield loss has significant impact on the material requirement. The literature survey pays attention to this aspect also.

The literature reviewed shows the current scenario of research in the areas of facility design and performance measures. The limitations of the procedures developed by researchers in the specific areas of effect of scrap and rework on the performance of the manufacturing system, layout design especially stochastic aspect of layout design and cell design under probabilistic environment are identified. In the literature survey, special emphasis was given for Markov models as it can faithfully model the discrete manufacturing system.

CHAPTER 3

APPLICATION OF AN ABSORBING MARKOV CHAIN TO PRODUCTION SYSTEM WITH SCRAP AND REWORK

3.1 INTRODUCTION

This chapter describes the modelling of production flow of components as an absorbing Markov chain. As all materials that start from raw material store do not reach the finished component stage due to scrapping and reworking, deterministic modelling is unrealistic. We adopt a stochastic process of a type called absorbing Markov chain. The data required for such a model are (i) the relative frequency with which the item goes from one stage of production to another, and (ii) relative frequency of rework and scrap at various stages.

A discrete manufacturing system, where a work-part moves through the system and comes out as a component is modelled as Absorbing Markov Chain (AMC). The work-part is a raw material or semifinished part before the production operations. At every stage of production the part is subjected to inspection; if it does not conform to specifications, it is either scrapped or reworked. The reworked component undergoes inspection again. It is assumed that nonconforming items are produced randomly at each stage. Figure 3.1 shows a manufacturing process that requires three serial operations modelled as an AMC.

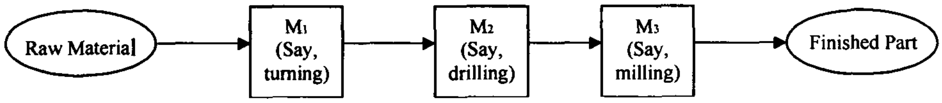


Figure 3.1 Manufacturing stages

The process is observed when the part transits from one state to another. Here, a state may represent raw material, particular normal operation, rework operation, scrap or finished goods. At any epoch[1], the part occupies a state, which is a discrete random variable X. As the parameter t (representing time) changes, the random variable X generates a random process $\{X(t): t \geq 0\}$. This stochastic process with discrete state space and discrete values of the parameter t becomes a discrete time (first order) Markov chain when the transition from one state to the next depends only on the current state. Among the states some are transient and the remaining are absorbing. A Markov chain with one or more absorbing states is known as absorbing Markov chain. An absorbing state is, as the name implies, one that endures. In other words, when a work-part reaches such a state, it never leaves the state. A scrapped work-part remains scrapped, and a finished work-part remains with no further changes.

The states, together with the transition probability matrix form the Markov chain model. The transition probability p_{ij} is the probability that a work-part transits from state i to state j in one step. The statistical data available from the manufacturing system can be used for developing the transition probability matrix (P). The hypothetical statistical data related with scrap rate and rework rate for the above example are given in the table 3.1 and the transition probability matrix generated from the data is shown in figure 3.2. The model may also be represented as a graph (transition diagram) as shown in figure 3.3.

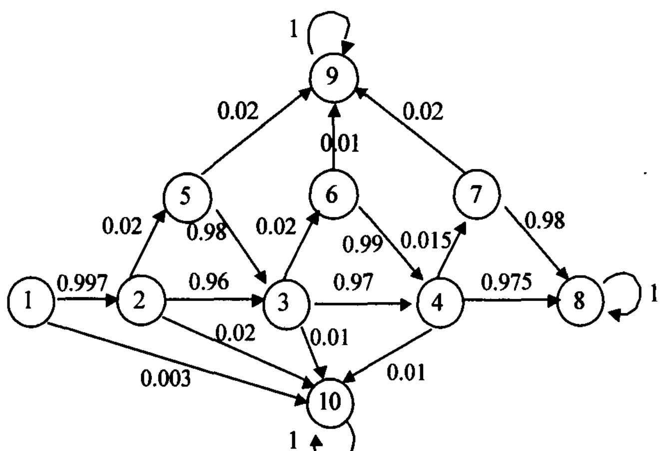
Process	*Scrap rate in %*	*Rework rate in %*
Incoming material	0.3	-
Turning	2.0	2.0
Rework turning	2.0	-
Drilling	1.0	2.0
Rework drilling	1.0	-
Milling	1.0	1.5
Rework milling	2.0	-

Table 3.1 Input data for the Markov model

[1] The points of time at which the system is observed are called epochs.

$$
P = \begin{bmatrix}
- & 0.997 & - & - & - & - & - & - & - & 0.003 \\
- & - & 0.96 & - & 0.02 & - & - & - & - & 0.02 \\
- & - & - & 0.97 & - & 0.02 & - & - & - & 0.01 \\
- & - & - & - & - & - & 0.015 & 0.975 & - & 0.01 \\
- & - & 0.98 & - & - & - & - & - & 0.02 & - \\
- & - & - & 0.99 & - & - & - & - & 0.01 & - \\
- & - & - & - & - & - & - & 0.98 & 0.02 & - \\
- & - & - & - & - & - & - & 1 & - & - \\
- & - & - & - & - & - & - & - & 1 & - \\
- & - & - & - & - & - & - & - & - & 1
\end{bmatrix}
$$

Figure 3.2 Transition probability matrix



1- Raw material state; 2, 3, 4 - Turning, drilling & milling states; 5, 6, 7 - Rework states; 8 - Finished part state; 9, 10 -Scrap states

Figure 3.3 Transition diagram

The probabilities shown in the transition probability matrix and in the transition diagram can be explained with the help of table 3.1. The first data of the table shows the scrap rate of the incoming material after inspection, which is 0.3 percent. This indicates that, an item from the incoming material state (state 1) transits to the scrap state (state 10) with a probability of 0.003 and to the state representing normal turning (state 2) with a probability of 0.997. The data related with normal turning indicates that from state 2 there are three transitions to states 3, 5 and 10 with probabilities 0.96, 0.02 and 0.02 respectively, the sum of probabilities adding up to 1. The rework turning data indicates that there are two transitions from rework turning state (state 5), to reworked scrap state (state 9) and to normal drilling state (state 3) with probabilities 0.02 and 0.98 respectively. Similarly the other data contained in the table can be interpreted.

The AMC of the production process can be modelled with two or three absorbing states. In an AMC with two absorbing states, one absorbing state is for finished goods and the other is for scrap. In an AMC with three absorbing states (as in the above example), one absorbing state stands for finished goods and the others for scrap generated from normal and rework operations. These two types of models have been used extensively in literature [White (1970), Davis and Kennedy (1987), and Viswanadham and Narahari (1994)]. An AMC with three absorbing states model of a production system is superior to a model of two absorbing states as the former model provides more information such as the amount of material reworked that became scrap. This information is useful for decisions related to inspection and quality control.

The present work identifies an anomaly in the estimation of equipment requirement available in literature [Davis and Kennedy (1987)] and presents another method of estimation. It also concerns with methods for determining the manufacturing lead-time and the amount of raw material to be scheduled through the production system to meet finished component demand with certain service level and the material to be scheduled to minimise the inventory cost, under scrapping and reworking.

3.2 NOTATIONS

The symbols used in the study are given below.

s – number of transient states

r – number of absorbing states

Q – matrix of transition probabilities between transient states

R – matrix of transition probabilities from transient states to absorbing states

E – matrix of expected number of times in (transient) state j, given starting state i

A – matrix of probabilities of absorption in (absorbing) state j, given starting state i

G – matrix of expected resource consumed in state j before absorption, given starting state i

T – diagonal matrix of resource consumed when in state i

B – column vector of expected resource consumed before absorption, given starting state i

Z – column vector of 1's.

e – a vector which is the first row of the matrix E

a – absorption probability that the process gets absorbed at finished goods state when the process starts from the first transient state. This is the first row first column element of A.

w – vector of unit operation time at state j

h – vector of number of hours available per unit time (day) for state j

k – vector of maintenance time for state j

t^{rc} – vector of duration of tool replacement and calibration for state j

u – vector of maintenance schedule for state j which indicate the number of parts to be produced before the next maintenance, and tool replacement and calibration.

x – number of finished components required per unit time (day)

c – vector of cost per unit at state j. c_1 represents cost of raw material. $c_j, j = 2,...s$ represent processing cost

l – number of production stages

φ – vector of expected number of machines required at state j

ψ – vector of expected number of machines required due to the non-operation time such as time required per unit time (day) for maintenance, and tool replacement and calibration for state j to produce given quantity of finished components

v – vector of expected number of machines required at state j when time required for maintenance, and tool replacement and calibration is also taken into account. $v = \varphi + \psi$

d – expected direct production cost to manufacture x units of components

t^m – move time between machines for a work-part

q – quantity of raw material required to produce x units of components

y – cycle time; time between two consecutive component coming out of the production system when the system has line layout for the component to produce.

τ – vector of expected process time at state j to produce a good component

l – expected lead time to produce x good units of components

s_l – desired service level

ω – production level of component

n_t – number of trials (quantity of raw material)

$p_r(\omega)$ – Binomial distribution function

c_p – production cost when one unit of raw material is scheduled through the production system. $c_p = d$, when $n_t = q = 1$.

c_u – cost of understocking one unit of good item.

T_c – expected total cost when n_t units are scheduled through the system. It is the sum of production and understock cost

All the vectors mentioned above are of size s.

3.3 PROPERTIES OF ABSORBING MARKOV CHAIN

As an AMC has a mix of absorbing states and transient states, it will be advantageous to rearrange the transition probability matrix into the following form to obtain certain useful results [White (1970), and Ravindran *et al.* (1987)].

$$p = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix},$$

where

$$\begin{aligned} Q &= s \times s \text{ matrix} \\ R &= s \times r \text{ matrix} \\ 0 &= r \times s \text{ zero matrix} \\ I &= r \times r \text{ identity matrix} \end{aligned}$$

The results provided by White (1970) are as shown below.

$$E = [I - Q]^{-1} \qquad \cdots[3.1]$$

$$A = ER \qquad \cdots[3.2]$$

$$G = ET \qquad \cdots[3.3]$$

$$B = GZ \qquad \cdots[3.4]$$

In a general AMC any one of the transient states can be the starting state. In the matrices *E*, *A* and *G* the rows represent starting states and the columns represent transient states. The first element of the column vector *B* is the total resource consumed when one work-part started from the first transient state. The second element represents resource consumed when the process starts from second transient state and so on. In the case of production system the resource may be of the type time, money, labour, etc.

3.4 SOME RESULTS FOR PRODUCTION SYSTEM

When a production system is modelled as an AMC, there is little or no advantage in obtaining values for the above results for states other than the first transient state as starting state. This is due to the property that always a work-part starts from first transient state, which is usually raw material state. The results to be derived in this paper consider this property.

3.4.1 Material Requirement

A work-part enters the production system in the form of raw material and comes out as either finished part or scrap. This process can be considered as a Bernoulli trial. Each work-part processing can be considered as an independent trial. To get x number of finished parts, the number of trials to be conducted is to be decided. Series of independent Bernoulli trials constitute a Binomial experiment. The number of trials is the quantity of raw material (q), which can be determined from the equation of Binomial average. Here, x represents the expected number of success in the Binomial experiment. That is, it is the Binomial average. The probability of success is the absorption probability a. Thus

$$q = \frac{x}{a} \quad \ldots [3.5]$$

3.4.2 Number of Machines

An element of vector e represents the number of times in state j once the process starts from first state. As the state j except the first state is representing an operation, e_j stands for number of times the operation j to be carried out on a work-part that starts from raw material state. Hence, operation time required at state j for q quantity of raw material is qe_jw_j. (This can also be understood from the equation 3.3.) When the time required for scheduled maintenance, and tool replacement and calibration is included, the total time required at state j, for q units of raw material started from first state, is $q\left(e_j w_j + \frac{e_j}{u_j}\left(t_j^{rc} + k_j\right)\right)$. Thus,

$$v_j = \frac{q\left(e_j w_j + \frac{e_j}{u_j}\left(t_j^{rc} + k_j\right)\right)}{h_j}$$

$$\text{ie,} \quad v = \varphi + \psi \qquad \ldots [3.6]$$

where

$$\varphi_j = \frac{q e_j w_j}{h_j} \qquad \ldots [3.7]$$

$$\psi_j = \left(\frac{q e_j}{u_j}\right)\left(\frac{t_j^{rc} + k_j}{h_j}\right) \qquad \ldots [3.8]$$

Davis and Kennedy (1987), computes the expected number of machines (v) as follows:

$$v_j = \frac{q e_j w_j}{h_j - \delta_j} \qquad \cdots [3.9]$$

where,

$$\delta_j = \frac{q e_j\left(t_j^{rc} + k_j\right)}{u_j}$$

On comparison with equation 3.6, it can be seen that the estimate of Davis and Kennedy (1987) gives a larger number of machines. This can be explained as follows. To meet a given demand per unit time (day), more than one machine may be needed at a given state. If the non-operation (maintenance, and tool replacement and calibration) time required for the needed quantity of the work-part at state j is δ_j it has to be distributed among all the machines used to produce the needed quantity at that state. But it has not been distributed in the estimation procedure of Davis and Kennedy (1987). Instead they accounted δ_j for each machine and making the effective time available for a machine $(h_j - \delta_j)$, leading to the underestimation.

3.4.3 Production Cost

From equation 3.3 it is understood that the resource consumed at state *j* when the process starts from first state is the product of e_j and resource required per unit at state *j*. Here, the resource is cost. The total cost of one work-part to process is the sum of the costs over all transient states. Hence,

$$d = q\sum_{j=1}^{S} e_j c_j \quad \ldots [3.10]$$

3.4.4 Manufacturing Lead-time

From equation 3.3, it is known that when a work-part starts from first state, the time required to process at state *j* is $e_j w_j$. To get a finished work-part, the number of units to start from first state is $\frac{1}{a}$. Thus,

$$\tau_j = \frac{e_j w_j}{a} \quad \ldots [3.11]$$

If the work-part is produced from a manufacturing system with line layout and the time to move the work-part between machines is assumed to be constant, the cycle time can be written as

$$y = t^m + \max(\tau_j) \quad \ldots [3.12]$$

When *x* units of finished work-part is to be produced in a system with a line layout, after the first part, an item may be obtained in every interval of *y*. Hence the time required to produce *x* units is the sum of time to produce first unit and $y(x-1)$. Thus,

$$l = (\iota - 1)t^m + \sum_{j=1}^{S} \tau_j + y(x-1) \quad \ldots [3.13]$$

When x is sufficiently large the equation 3.13 can be approximated as

$$l = yx \quad \ldots [3.14]$$

The equation 3.13 is particularly useful in a cellular manufacturing system where a family of parts is processed in a cell. Linear sequencing of machines is useful to attain most of the advantages of cellular manufacturing and enables implementation of concepts like just-in-time [Akturk and Balkose (1996), Nicholas (1998)]. A linear machine sequence provides simple and efficient flow structure that can be configured as a straight line, U-shape line or serpentine line [Chen *et al.* (2001)]. Such a shop in which the entire processing takes place in the forward direction is called a flowshop if the processing is consecutive, and 'modified flowshop' if it is non-consecutive [Akturk and Balkose (1996)].

As the cell has to produce a family of parts, each part may be produced in small quantities intermittently. Hence, equation 3.13 may be suitable for the calculation of manufacturing lead-time in a flowshop type cell. But, only rarely do all the items belonging to a part family use all the machines of the cell. So a cell may be of flowshop type for some of the items of the part family whereas for other items it may be a modified flowshop. For the modified flowshop, cycle time y should be revised considering the time to move between two consecutive stages. It is not necessary that the distance between consecutive operation stages be a constant, as some production stages of the cell are not used. For the work-parts, which use the first and last stages of the cell, the number of stages of the cell may be used as ι in the equation 3.13. For other work-parts ι should be modified appropriately.

Manufacturing lead-time can be calculated for systems with other types of layout provided parameters such as set-up time, waiting time, other non-operation time, etc are defined. Groover (1989) provides these types of calculations for various types of manufacturing systems, without considering the effect of scrap and rework. The results available in Groover (1989) can be modified to include these effects using the above results.

3.5 ILLUSTRATION

The solution procedure is illustrated through the example given in figure 3.1. The matrices E and A are shown in figure 3.4 and 3.5 respectively. An element E_{ij} of matrix E represents the mean number of times a transient state j is occupied for the initial state i. An element A_{ij} of the matrix A is the absorption probability which shows the fraction of the work-part that starts in the state i and ends in the absorbing state j. In the case of production system, the element A_{11} is sufficient to estimate the parameters and it is represented as a. The input vectors (data) required for calculation of production system parameters and the vectors showing the results are given as columns of the table 3.2. Other inputs required are x = 600 units and t^m = 0.5 minutes. Certain results other than those available in the table 3.2 are d = \$15701, q = 627.13 units, y = 0.0375 hours and l = 22.56 hours.

$$E = \begin{bmatrix} 1.0000 & 0.9970 & 0.9767 & 0.9667 & 0.0199 & 0.0195 & 0.0145 \\ 0 & 1.0000 & 0.9796 & 0.9696 & 0.0200 & 0.0196 & 0.0145 \\ 0 & 0 & 1.0000 & 0.9898 & 0 & 0.0200 & 0.0148 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 & 0.0150 \\ 0 & 0 & 0.9800 & 0.9700 & 1.0000 & 0.0196 & 0.0146 \\ 0 & 0 & 0 & 0.9900 & 0 & 1.0000 & 0.0148 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

Figure 3.4 Matrix of expected number of times in transient state

$$A = \begin{bmatrix} 0.9567 & 0.0009 & 0.0424 \\ 0.9596 & 0.0009 & 0.0395 \\ 0.9796 & 0.0005 & 0.0199 \\ 0.9897 & 0.0003 & 0.0100 \\ 0.9600 & 0.0205 & 0.0195 \\ 0.9798 & 0.0103 & 0.0099 \\ 0.9800 & 0.0200 & 0 \end{bmatrix}$$

Figure 3.5 Absorption probability matrix

$1/w$ Units/hr.	h Hrs.	k Minutes	t^{rc} Minutes	u Parts/change	c \$/unit	ψ No.	φ No.	v No.	τ Hrs.
0	8	0	0	0	15.0000	0	0	0	0
40	8	10	4	60	3.0000	0.3039	1.9539	2.2578	0.0261
35	8	15	4	70	4.0000	0.3463	2.1875	2.5338	0.0292
40	8	15	4	120	3.0000	0.2000	1.8945	2.0945	0.0253
15	8	10	4	60	4.5000	0.0061	0.1042	0.1103	0.0014
10	8	15	4	70	5.0000	0.0069	0.1531	0.1600	0.0020
20	8	15	4	120	3.5000	0.0030	0.0568	0.0598	0.0008

Table 3.2 Data and different estimates

The following comparison of results with Davis and Kennedy (1987) illustrates the difference. The vector v according into Davis and Kennedy (1987) is

[0 2.8071 3.3465 2.3681 0.1048 0.1542 0.0570].

When it is rounded off to the nearest integer, the values of v are

[0 3 4 3 1 1 1] ... Davis and Kennedy (1987)

When the v values of table 3.2 are rounded off to the nearest integer, the result is

[0 3 3 3 1 1 1]

This reveals that estimate of Davis and Kennedy (1987) gives higher value even after rounding off.

The example considered here assumes that normal operation and rework operation are carried out on the same machine. Hence, the number of machines for the various stages is the sum of the respective normal operation state and rework operation state of vector v.

That is, the expected number of machines at stage 1 = $v_2 + v_5$

The expected number of machines at stage 2 = $v_3 + v_6$

and the expected number of machines at stage 3 = $v_4 + v_7$

Thus the expected number of machines (rounded off to the nearest integer) at stages 1, 2 and 3 are 3, 3 and 3 respectively.

It can be seen that equation 3.14, which is comparable with the production rate equation of continuous production system, is a good approximation to equation 3.13 in the case of large batch sizes, and results in large errors in the case of small batches (For example when equation 3.14 is used for batch size 5, the error is 27.79 percent and for batch size 500 it is 0.38 percent).

In the absence of statistical data, the production costs are to be estimated deterministically based on the number of units to be produced. This leads to deviation from the actual costs, which would be higher because of scrapping and reworking. Production cost estimation can be done accurately by using the stochastic model.

The present work suggests models to determine the amount of raw material to be scheduled through the production system to meet finished component demand. Some of the parameters for the models are derived from the absorbing Markov chain model. Models for two criteria namely service level and minimal cost are suggested.

3.6 DETERMINATION OF RAW MATERIAL REQUIREMENT

3.6.1 Service Level Criteria

Obviously, the production process can be considered as a Binomial experiment. Probability of success of the Binomial experiment is the probability that the work-part that started from the raw material state gets absorbed at finished work-part (component). (Raw material state represents the first transient state of the Markov chain of the production system). The probability of success is denoted in the previous section as *a*.

For a given number of products, the quantity of components required can be worked out through Material Requirement Planning (MRP). Quantity of raw material needed to produce the required quantity of components can be determined in the Binomial experiment only probabilistically. Under such circumstances managers are interested in meeting the requirement with certain service level, by laying down a policy that the probability of out of stock would not be more than an acceptable percent, say one percent. The number of 'trials' required in the Binomial experiment indicates the quantity of raw material required. A 'trial' is the set of manufacturing operations required to convert input material to finished component, assuming that each trial is independent.

The $p_r(\omega)$ can be written as

$$p_r(\omega) = \binom{n_t}{\omega} a^{\omega}(1-a)^{n_t-\omega}$$

The probability that x or more is produced is

$$1-\sum_{\omega=1}^{x-1} p_r(\omega)$$

This probability is equal to the service level. That is,

$$s_l = 1-\sum_{\omega=1}^{x-1} p_r(\omega) \qquad \ldots[3.15]$$

From this relation, n_t is to be determined through trial and error. However, the following algorithm helps in easily arriving at the correct value of n_t. If x is considered as expected value of the Binomial experiment with parameter a and n_1, then n_1 can be determined from the equation 3.5 as

$$n_1 = \frac{x}{a}$$

Now let,

$$p_c^{n_1} = \sum_{\omega=1}^{x-1} \binom{n_1}{\omega} a^{\omega} (1-a)^{n_1-\omega}$$

Besides n_1 and $p_c^{n_1}$, a variable p_c is also used in the algorithm. The variable p_c is appropriately defined at various places of the algorithm.

3.6.1.1 Algorithm

Start

Read s_l, x, a;

$n_1 = \frac{x}{a}$;

$p_c^{n_1} = \sum_{\omega=1}^{x-1} \binom{n_1}{\omega} a^{\omega} (1-a)^{n_1-\omega}$;

$p_c = 0$;

$n_t = 0$;

If $s_l > 1 - p_c^{n_1}$

For i = 1 to n_1

$p_c = \sum_{\omega=1}^{x-1} \binom{n_1+i}{\omega} a^{\omega} (1-a)^{n_1+i-\omega}$;

If $s_l \le 1 - p_c$

$n_t = n_1 + i$;

Break from the for loop

End of if block

End of for block

Else if $s_l = 1 - p_c^{n_1}$

$n_t = n_1$;

Else

For i = 1 to n_1

$$p_c = \sum_{\omega=1}^{x-1} \binom{n_1 - i}{\omega} a^{\omega} (1-a)^{n_1 - i - \omega} ;$$

If $s_l > 1 - p_c$

$n_t = n_1 - i + 1$;

Break from the for loop

Else

If $s_l = 1 - p_c$

$n_t = n_1 - i$;

Break from the for loop

End of if block

End of if …else block

End of for block

End of if …else block.

End of algorithm

For the above example, for a service level of 95 percent and 600 units of finished components, the raw material required is 636.

3.6.2 Minimisation Criteria

So far we did not consider the cost in the computations. It is possible to find the optimum level of raw material to be scheduled through the manufacturing system under the existing cost.

The expected total cost can be shown as the sum of production cost and cost of understock. That is,

$$T_c = n_t c_p + c_u \sum_{\omega=0}^{x} (x-\omega) \binom{n_t}{\omega} a^{\omega} (1-a)^{n_t-\omega} \qquad \ldots[3.16]$$

The optimum is value of n_t for which the function T_c has minimum value. The optimality exists only when T_c is a convex function. T_c is strictly convex if

$$\Delta^2(T_c) > 0$$

Since, T_c is strictly convex (White 1970), the optimum value of n_t is the smallest value of n_t satisfying the condition, $\Delta T_c > 0$, or

$$\sum_{\omega=1}^{x-1} p_r(\omega) < \frac{c_p}{a c_u} \qquad \ldots [3.17]$$

When $c_p = c_u$, the optimality condition is

$$\sum_{\omega=1}^{x-1} p_r(\omega) < 1$$

Although the ratio $\frac{c_p}{a c_u}$ is greater than 1 the above condition is true as the cumulative probability cannot be more than 1. This condition is satisfied when $n_t = x$. That is, the optimum value of n_t can be as low as x. In all practical situations c_u will be greater than c_p. Hence the optimum value of n_t will be greater than or equal to x.

As the understock cost (c_u) is an intangible, it is difficult to quantify. Therefore it is preferable to estimate the raw material requirement that satisfies certain service level. Thus, the condition 3.17 can be used to determine the understock cost ascribed to the service level. The following condition gives an estimate of the understock cost.

$$c_u < \frac{c_p}{a(1-s_l)} \qquad \ldots [3.18]$$

In the case of above example, c_p can be obtained from equation 3.10 by substituting $q = 1$ and is \$ 25.0359. When c_u = \$ 30 and x = 600 units, the raw material required is 621 units and the total cost is \$15731. This is the case when all the relevant costs are available.

When the relevant (understock) cost is not available the cost imputed for a given service level can be computed using the above criterion. It was seen earlier that, for a service level of 95 percent, the raw material requirement is 636 units. Now, the imputed under stock cost estimated using condition 3.18 works out to \$ 523.3573.

3.7 CLOSURE

The Markov model adequately describes the production system under uncertainties due to scrapping and reworking. The analysis shows clearly the interaction between design and control decisions, and it provides an opportunity for the management to analyse quality-related problems. It reveals the necessity of collecting more data on quality control and operational issues. The information for planning and design such as MRP, capacity requirement planning and system design could be obtained from the analysis. It also gives a lot of insight to managers on certain manufacturing situations such as (i) the amount of reworked material that is subsequently scrapped, and (ii) the effect of scrapping and reworking on production cost. Thus, a complete characterisation of the production system under rework and scrapping is possible through the Markov model.

Raw material determination model shows the quantity of raw material required to meet a known demand of finished components both when relevant cost is available and not available. The raw material required is greater than the finished component requirement (demand) as there is scrapping of material during production. The demand may be derived from MRP process or from a customer order. The raw material requirement calculation involves the use of the Binomial formula associated with the total cost function identifying the number of trials n_t that result in the minimum total cost. While searching for n_t, the range of search can be reduced based on understanding of the ratio of under stock cost (c_u) to production cost c_p. That is, if the ratio is close to 1, the optimum value of n_t will be close to x; and away otherwise.

CHAPTER 4

MODIFICATION OF BILL OF MATERIAL UNDER REJECTION AND REWORK

4.1 INTRODUCTION

Planning of an activity should identify and estimate all appropriate factors that affect the activity. An important planning activity in the production planning is Material Requirements Planning (MRP). It is mainly concerned with providing the right quantity of the right part at the right time to meet the schedules for the products being planned. To accomplish this, one of the inputs used is Bill Of Material (BOM). BOM provides the details on the components and assemblies required for making the product. It contains the information such as, in what sequence these items (An item may be component, subassembly or final assembly.) are required, how many units of components are required to produce the parent item together with the lead-time. The accuracy of planning depends on the correctness of these inputs.

The number of units of components required to make a unit of a parent item is one of the basic information obtained from the BOM. This means that, the number of units specified in the BOM is the quantity of material upon which operations are carried out to obtain a higher-level item in the bill. This is true when there is no rejection during assembly, machining or testing operation. Under rejection this information provided in the BOM should be modified so that it could be used for material planning. This can be achieved by splitting the BOM into several subsystems and modelling the subsystem as an absorbing Markov chain. The subsystem results may be used to modify the information contained in the BOM. Subsystem production operations consist of sequence of operations, which may contain assembly, machining and testing operations.

4.2 FORMING SUBSYSTEMS FROM BOM

A typical BOM, also called the product structure, is shown in figure 4.1. This diagram shows only the sequence in which various items are used to create the final product. It does not contain the information such as quantity of each component required per unit parent item and the lead-time of making various items. It has levels up to four; level zero represents final product, which requires 25 items for its production, and hence the BOM contains 26 items. The items in the BOM are coded to permit easy modelling. (Actually this product structure is generated to show the complexity of subsystem identification and coding.) In the modelling procedure the BOM is separated into a number of subsystems. The letter M in the item code stands for a subsystem; that item may be a subassembly or a component. Later each subsystem is modelled as a Markov chain. The letters S and C stand for subassembly and component respectively and they could be part of the subsystems or the final assembly (F). The final assembly (F) is also considered as a subsystem. (Appropriate letter in the item code is assigned after the subsystem identification, described below.) There is a unique number associated with the code of an item. The numbering starts from the final item that is zeroth level, down to the lowest level in the BOM; the numbering sequence in a level is from left to right. A subsystem is identified in such a way that it is viable for Markovian modelling. A subsystem contains group of items and it is known in the name of highest-level item in the group. The subsystem identification procedure starts from the highest level in the BOM. The highest level in the BOM has the lowest level number. The notations used for the subsystem identification are as follows:

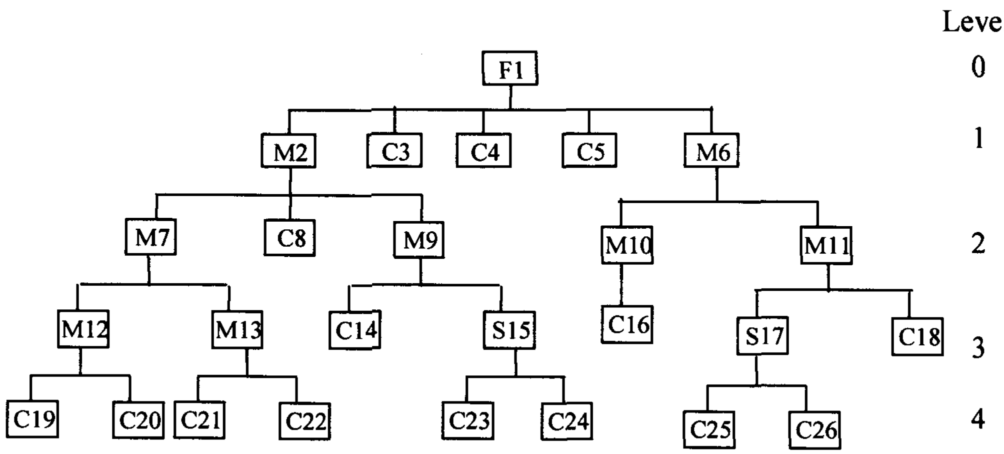


Figure 4.1 A typical product structure

N – Number of items in a BOM

L – Number of levels of a BOM

Z_{ij} - Set of items required for producing an item i of the level j. The items in the set belong to the level $j+1$.

$W_{i\,j+1}$ - Set of items required for producing an item i and the items in the set are produced from some other items from the lower level.

i – Unique number given while coding of an item in a BOM.

j – Level number

Y – Set of subsystems

Each item in the BOM has sets Z and W. Since a unique number can identify an item, the elements of the sets Z, W and Y are represented using this number. For every item in the BOM there are two sets Z_{ij} and $W_{i\,j+1}$. From the definition of $W_{i\,j+1}$ it is clear that $W_{i\,j+1} \subset Z_{ij}$. The set Y can be defined as

$$Y = \{\bigcup W_{i\,j+1} / \text{size of } W_{i\,j+1} > 1, i = 1,2,...N, j = 0,1,2,...L-1\} \cup \{1\}$$

Each element in the set Y is associated with the name of a subsystem. That is, if $i \in Y$ for $i \neq 1$, the subsystem name is Mi. When $i = 1$, the subsystem name is F1. The items belonging to a subsystem is identified in the following way: First, identify $W_{i\,j+1}$ for each element in set Y. If size of the set $W_{i\,j+1}$ is greater than 1, the subsystem contains the item i and elements of the set Z_{ij}. If the size of the set $W_{i\,j+1}$ is equal to 1, there exists a set $T_i^1 = \{i,...,k\}$ such that

i. The elements of the set represent items (that is, $i,\ldots,k$ represent the unique code numbers of the items) and to make the first item i of the set, requires the remaining items.

ii. The code numbers have the relationship, $i < ... < k$ and if the level number of items is represented as $j,...,r$, then these level numbers have the relationship $j \leq ... \leq r$.

iii. Each item in the set T^1 has the set Z, which is non-null. The last item k, whose set W will be either null, or with two or more elements. Only the last item k has this property. That is, while identifying items belonging to the set T^1, the process can be stopped when this property is met.

The subsystem Mi contains items, which constitute a union of certain sets, and it can be written as

$$\text{Mi} = \{i\} \cup Z_{i\,j} \cup ... \cup Z_{k\,r}.$$

Now, identify the elements in set Y whose set $W_{i\,j+1}$ is null. The subsystem in this case contains the item i and elements of the set Z_{ij}.

For the above BOM, the set Y and the items in the subsystem can be found out. The sets of Z and W for each item in the BOM are:

$$Z_{1,0} = \{2,3,4,5,6\},\quad W_{1,1} = \{2,6\},\quad Z_{2,1} = \{7,8,9\},\quad W_{2,2} = \{7,9\}$$
$$Z_{7,2} = \{12,13\},\quad W_{7,3} = \{12,13\},\quad Z_{12,3} = \{19,20\},\quad W_{12,4} = \Phi$$
$$Z_{6,1} = \{10,11\},\quad W_{6,2} = \{10,11\},\quad Z_{10,2} = \{16\},\quad W_{10,3} = \Phi$$
$$Z_{9,2} = \{14,15\},\quad W_{9,3} = \{15\},\quad Z_{13,3} = \{21,22\},\quad W_{13,4} = \Phi$$
$$Z_{11,2} = \{17,18\},\quad W_{11,3} = \{17\},\quad Z_{15,3} = \{23,24\},\quad W_{15,4} = \Phi,\quad Z_{17,3} = \{25,26\},\quad W_{17,4} = \Phi$$

The sets of Z of the remaining items are null and hence their sets of W is also null. The set Y is the union of W_{ij} having size two or more and the set $\{1\}$. It can be written as

$$Y = \{1,2,6,7,9,10,11,12,13\}$$

The items in the various subsystems are to be identified as given below.

Identify the elements of set Y whose set W has a size of two or more. Such elements are 1, 2, 6 and 7. The items belonging to the corresponding subsystem are as follows

$$\text{F1} = \{1,2,3,4,5,6\},\ \text{M2} = \{2,7,8,9\},\ \text{M6} = \{6,10,11\},\ \text{M7} = \{7,12,13\}.$$

Identify the elements of set Y whose set W has a size equal to one. Such elements are 9 and 11. The set T^1 for these items are $T_9^1 = \{9,15\}$ and $T_{11}^1 = \{11,17\}$. The items belonging to the corresponding subsystem are as follows

$$\text{M9} = \{9,14,15,23,24\} \text{ and } \text{M11} = \{11,17,18,25,26\}.$$

Identify the elements of set Y whose set W is null. Such elements are 10, 12 and 13. The items belonging to the corresponding subsystem are as follows

$$\text{M10} = \{10,16\},\ \text{M12} = \{12,19,20\},\ \text{M13} = \{13,21,22\}$$

4.3 MARKOVIAN MODELLING OF SUBSYSTEMS

The subsystems are modelled as Markov chain, with the following assumptions: i) The components required for making the parent are first assembled, upon which various operations are carried out to get the parent. ii) When rejection occurs, an assembly as a whole is rejected.

Now the subsystems, which are the elements of the set Y, are to be modelled. A subsystem looks like a product structure, although it is only a part of the product structure represented independently. Usually, more than one item from a particular level is assembled to get the higher-level item. Further details on the production and modelling of a subsystem are explained with the help of the subsystem M9. The subsystem product structure is shown in the figure 4.2.

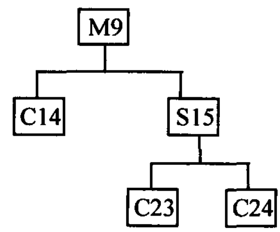


Figure 4.2 Product structure of subsystem M9

It is assumed that to make one unit of final item M9, two units of S15 and one unit of C14 are required. To make one unit of S15, two units of C23 and three units of C24 are required. The production process required for producing the final item, of the subsystem product structure, can be considered as a series of operations starting from the lowest level items. Let there be four operations as shown in figure 4.3.

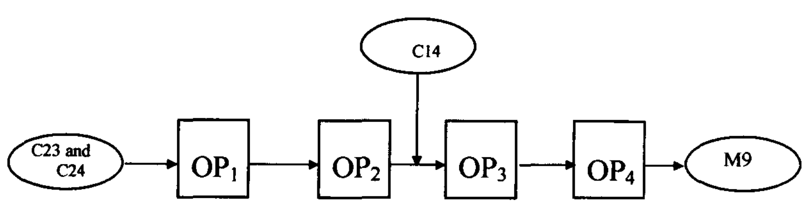


Figure 4.3 Operation stages

After each operation (the operations are designated as OP_1, OP_2, OP_3 and OP_4), there is an inspection to ascertain the quality. After assembling the items C23 and C24, certain operations are carried out and these activities are represented by OP_1. The assembled item goes through the second operation. After the second operation, item S15 is ready which is assembled with C14 and some other activities are carried out at the third operation. The item, M9 is ready by the fourth operation. During inspection, any one of the following decisions is taken: (i) send the item to next operation stage or finished item stage, (ii) rework the item if the dimensions indicate that reworking is possible, or (iii) scrap the item if rework cannot salvage the item.

As an item (assembly of C23 and C24) progresses through various stages, observations are made when the item transits from one state to another. Here, a state may represent a particular operation, scrap or finished goods. At any observation epoch, the item occupies a state, which is a discrete random variable X. As the parameter t (representing time) changes, the random variable X generates a random process $\{X(t): t \geq 0\}$. This stochastic process with discrete state space and discrete values of the parameter t becomes a discrete time (first order) Markov chain when the transition from one state to the next depends only on the current state. Among the states some of them are transient and the remaining are absorbing. Since some of the states are absorbing, the Markov chain is an Absorbing Markov chain. The characteristics of the absorbing Markov chain are explained in the chapter 3, which describes modelling of a serial discrete manufacturing system, which is applicable here also. But the model considered here is an absorbing Markov chain with two absorbing states.

The absorbing probability matrix of the model provides the probability of an item from a transient state reaching an absorbing state. Every subsystem model contains two absorbing states; one for the finished item and the other for scrapped item. For the subsystem M9, the finished item is M9. Usually, it is required to know the probability of an item reaching an intermediate state (next higher level), from a lower level. For instance, items C23 and C24 are at level 4 of the BOM in figure 4.1 and the probability of an assembly of C23 and C24 reaching level 3 is needed, to determine the quantity of the assembly required to make one unit of item S15. This is the probability of an item starting from a transient state and reaching another transient state. Now, these two transient states are to be identified. The operation stages generally represent the transient states.

It is assumed that the items (components) for the initial operation and the components used in the subsequent stages are of right quality. For instance, in the subsystem M9, the incoming items such as C14, C23 and C24 are of right quality.

Till now, the two transient states are not identified and the identification process involves the recognition of states (operations) associated with a particular level. The two transient states are the first state in the particular level and the first state in the next immediate higher level. For the subsystem M9, the starting operation is OP_1, which is carried out on assembled unit of C23 and C24 at level 4. The immediate higher level is 3 and the first operation at this level is OP_3. The probability that an item starting from state 1(OP_1) reaching state 3 (OP_3) is the required probability. This probability helps to identify the quantity of C23 and C24 required to make one unit of S15 under rework and rejection, and it is the information needed for MRP computation. This can be called *level transition probability* as it is the probability of an item starting from a lower level reaching an immediate higher-level.

When the absorption probability matrix is available, the determination of level transition probability and quantity of components required to make one unit of a parent, are discussed here. The following symbols are used for this purpose.

a_{ij} - Probability that an item gets absorbed at state j given that the initial state (transient state) is i, where j is the finished item state, which is an absorbing state.

$b_{l\,l-1}$ - Level transition probability. Where l is a level number; $l=1,2,...L$

ρ_{ik} - Probability of an item that started from a transient state i reaching another transient state k.

t_u^l - Quantity of an item u, which is at level l, required to make one unit of a parent when rejection does not exist.

q_u^l - Quantity of an item u, which is at level l, required to make one unit of a parent under rejection. Where $u=1,2,...N$

Now the absorption probability a_{ij} for the Markov chain model considered here can be written as

$$\begin{aligned} a_{ij} &= \rho_{ik}\, a_{kj} \\ &= \rho_{ik}\, \rho_{k\,k+x}\, a_{k+x\,j} \qquad \ldots [4.1] \end{aligned}$$

Where, $k+x$ is a transient state, which is x number of states away (in down stream direction) from the transient state k. The absorption state j represents the finished item state and the relationship is valid only for this absorption state. Equation 4.1 is evident from the independent property of Markov process.

If i is an initial state of the Markov chain, which is at level l and is also the first operation state of the subsystem, and k is the first operation state at level $l-1$, then $\rho_{ik} = b_{l\,l-1}$. Since a_{ij} and a_{kj} are available from the absorption probability matrix, $b_{l\,l-1}$ can be determined from the equation 4.1. If k is the first operation state at level $l-1$ and $k+x$ is the first operation state at level $l-2$, then $\rho_{k\,k+x} = b_{(l-1)(l-2)}$ which can also be determined from the equation 4.1. If the subsystem represents a single level BOM, then $a_{1j} = b_{l\,l-1}$. A single level BOM shows the different components, all belonging to a level that are required to make the parent.

The splitting of a BOM into several subsystems can be done in two ways. One way is as explained above. The other way is to split the BOM into number of single level BOMs in which case the subsystem identification gets a little simplified. But the number of subsystems and hence the number of Markov models may be higher than the other method. For instance, the BOM in figure 4.1 can be split into several single level BOMs. The number of single level BOMs in this case is 11 and hence 11 Markov chain models have to be developed. But, when the subsystem identification procedure discussed in this chapter is followed, the number of Markov chain models to be developed is 9. The number of models to be developed using this method may be smaller than single level BOM method. Another example, which is to be described in the later part of this chapter as a case study, requires 4 models while the single level BOM method requires 11 models.

The following is a concise method for calculation of the level transition probability. For a subsystem, assume that $i_l, i_{l-1}, i_{l-2}, ..., i_r$ are some of the states corresponding to levels $l, l-1, l-2, ..., r$. The states $i_l, i_{l-1}, i_{l-2}, ..., i_{r-1}$ are the first transition states in the levels $l, l-1, l-2, ..., r-1$. The level r is the highest level of the subsystem. The state i_r is the finished item state, which is also an absorption state. This state is usually represented as j. Now, the various level transition probability equations can be written as

$$
\begin{aligned}
a_{i_l\,j} &= b_{l\,l-1}\; a_{i_{l-1}\,j} \\
a_{i_{l-1}\,j} &= b_{l-1\,l-2}\; a_{i_{l-2}\,j} \\
\ldots &\qquad \ldots \\
\ldots &\qquad \ldots \\
a_{i_{r-1}\,j} &= b_{r-1\,r}\; a_{j\,j}
\end{aligned}
$$

The left side of the system of equations can be represented in a column vector form, which can be called *level jump probability vector* as this vector shows the probability of the items reaching the highest level of the subsystem from various levels. The rows represent the levels of the subsystem from lowest to second highest. The column stands for the highest level. This vector can be obtained from the absorption probability matrix. The system of equations can be written in the matrix multiplication form as given below. The following are the symbols used for this purpose.

A^L – Level jump probability vector

B^L – Diagonal matrix of level transition probability. The rows of the matrix represent the levels of the subsystem from lowest to second highest. The columns of the matrix represent levels of the subsystem from second lowest to highest.

C^L – Column vector generated from A^L. This vector is obtained by eliminating the first element of the vector A^L and by adding $a_{j\,j}$ as last element, which is equal to 1.

Now, the system of equations can be represented as

$$A^L = B^L C^L$$

The diagonal element of the B^L matrix can be computed using the equation

$$b_{l\,l-1} = \frac{a_{i_l\,j}}{a_{i_{l-1}\,j}} \quad \ldots [4.2]$$

The discussions till now were on level transition probability, which is used for estimation of the material requirement. The quantity of an item u required can be estimated as

$$q_u^l = \frac{t_u^l}{{}_u b_{l\,l-1}} \quad \ldots [4.3]$$

where, ${}_u b_{l\,l-1}$ is the level transition probability $b_{l\,l-1}$ for a subsystem, which contain the item u.

The equation 4.3 can be derived as follows. The item u and some other items are assembled upon which operations are carried out to obtain the higher-level item. The number of units of the first assembly (before any operation starts) required is $\frac{1}{{}_u b_{l\,l-1}}$. The number of units of the item u required for making each first assembly is t_u^l. Hence, $q_u^l = \frac{t_u^l}{{}_u b_{l\,l-1}}$.

4.4 CASE STUDY

This section presents the modelling of the assembly and testing operations of a solenoid operated flow control valve used for spacecraft propulsion. Since the product is intended for space application, several quality assurance tests are performed at each level of the assembly operations. The acceptance, rework, or rejection of the subassembly/assembly depends on the outcome of these tests. Almost all the components of the product except a few are fabricated outside the organisation. The components fabricated either outside or in-house are individually inspected and the accepted items alone are sent to assembly.

4.4.1 Bill of Material of the Product

The bill of material (BOM) of the solenoid operated flow control valve is shown in figure 4.4. There are 6 levels, from 0 to 6, in the BOM and there are 34 items (an item in the BOM may be a component, subassembly or an assembly). The items in the BOM are coded as per the coding procedure explained earlier. Besides the item code, the number in the bracket indicates the quantity of item required to make one unit of the parent item when rejection does not exist. The unit of measurement of each item in the BOM is as follows.

Item Measured in Nos.	: F1, M2, M4, M5, C3, C6, C10, C11, C12, C14, C15, C17, C19, C20, C21, C23, C24, C29, C31, C32, C33, C34, S8, S13, S16, S18, S22, S26, S30.
Items Measured in cm. or m.	: C25 & C27 in m. and C28 in cm.
Items Measured in grams	: C7 and C9.

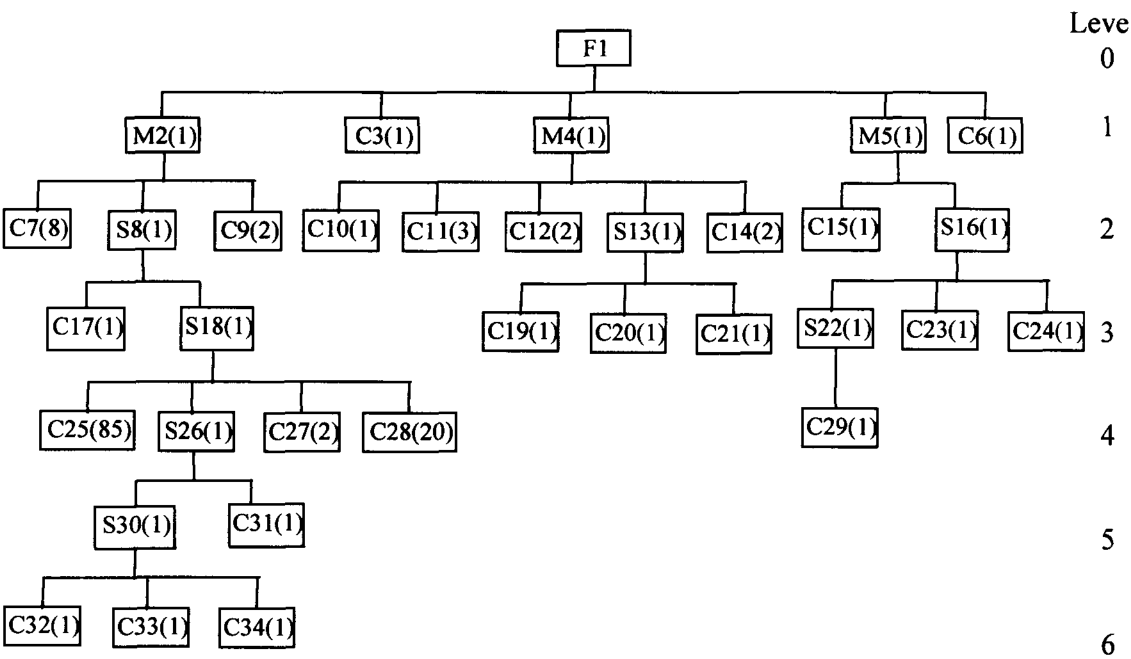


Figure 4.4 Bill of material of flow control valve

The code of certain items contains the letter M. Usually this M is added to the code of items belonging to the set *Y* except for the item 1 to which the letter added is F. That is, the elements of set Y have to be identified to decide which items are to be coded with letter M. Y is the union of two sets Z and W defined as follows.

$Z_{1,0} = \{2,3,4,5,6\}$, $W_{1,1} = \{2,4,5\}$, $Z_{2,1} = \{7,8,9\}$, $W_{2,2} = \{8\}$, $Z_{4,1} = \{10,11,12,13,14\}$, $W_{4,2} = \{13\}$, $Z_{5,1} = \{15,16\}$, $W_{5,2} = \{16\}$, $Z_{8,2} = \{17,18\}$, $W_{8,3} = \{18\}$, $Z_{13,2} = \{19,20,21\}$, $W_{13,3} = \Phi$, $Z_{16,2} = \{22,23,24\}$, $W_{16,3} = \{22\}$, $Z_{18,3} = \{25,26,27,28\}$, $W_{18,4} = \{26\}$, $Z_{22,3} = \{29\}$, $W_{22,4} = \Phi$, $Z_{26,4} = \{30,31\}$, $W_{26,5} = \{30\}$, $Z_{30,5} = \{32,33,34\}$, $W_{30,6} = \Phi$

For the remaining items, the sets of Z are null and hence the corresponding sets of W are also null. Now, Y is the union of all Ws (whose size is two or more) and the set {1}, and it is

$$Y = \{1,2,4,5\}$$

The codes of the items to which the letter M is added are 2, 4 and 5. Since the set Y has 4 elements, the BOM is split into 4 subsystems namely F1, M2, M4 and M5. As the set W of item 1 contains more than one element, the items belonging to the subsystem F1 can be shown as

$$F1 = \{1,2,3,4,5,6\}$$

The set W of the items 2, 4 and 5 contains only one element. Hence, the set T_i^1 for each item is to be identified. The sets are

$$T_2^1 = \{2,8,18,26,30\}, \quad T_4^1 = \{4,13\}, \quad T_5^1 = \{5,16,22\}$$

The items belonging to the subsystem M2, M4 and M5 are as follows

M2 = {2,7,8,9,17,18,25,26,27,28,30,31,32,33,34}, M4 = {4,10,11,12,13,14,19,20,21} and M5 = {5,15,16,22,23,24,29}

4.4.2 Assembly Operations

We have seen that the BOM is split into four subsystems, which is tantamount to dividing the production process into four subsystems. One subsystem produces the product F1. The figure 4.4 indicates that to produce the item F1, the items M2, M4 and M5 are used along with some other components. The other three subsystems produce items M2, M4 and M5. Now, the assembly and testing operations required to produce the items F1, M2, M4 and M5 are to be identified. The schematic diagram of total assembly and testing operations of the product is shown in figure 4.5. It contains the production process of the four subsystems. The diagram also shows the rejection and rework, if it exists in any operation. For ease of representation, the operations are named by a code. A code contains three pieces of information. They are the subsystem name, operation number and level number, embedded in the code in the same sequence. The operation number is unique for a subsystem.

As seen in the assembly operation sequence diagram, each subsystem contains definite sequence of operations. It also shows that the items M2, M4 and M5 are used to produce the product F1. Consider the subsystem M2 where 10 operations are involved. The operations 1, 2 and 3 are carried out at level 6 of the BOM. That is, the items C32, C33 and C34 are assembled, upon which the operations 1, 2 and 3 are carried out. The operations 4 to 7 are carried out at level 5 on items S30 and C31. The figure 4.5 indicates that, there may be rejection and rework during operation 4, while there may be only rejection during operation 6. The levels starting from 4 to 2 have one operation each. Item M2 is ready by operation 10. Every other subsystem shown in the assembly operation sequence diagram can likewise be explained.

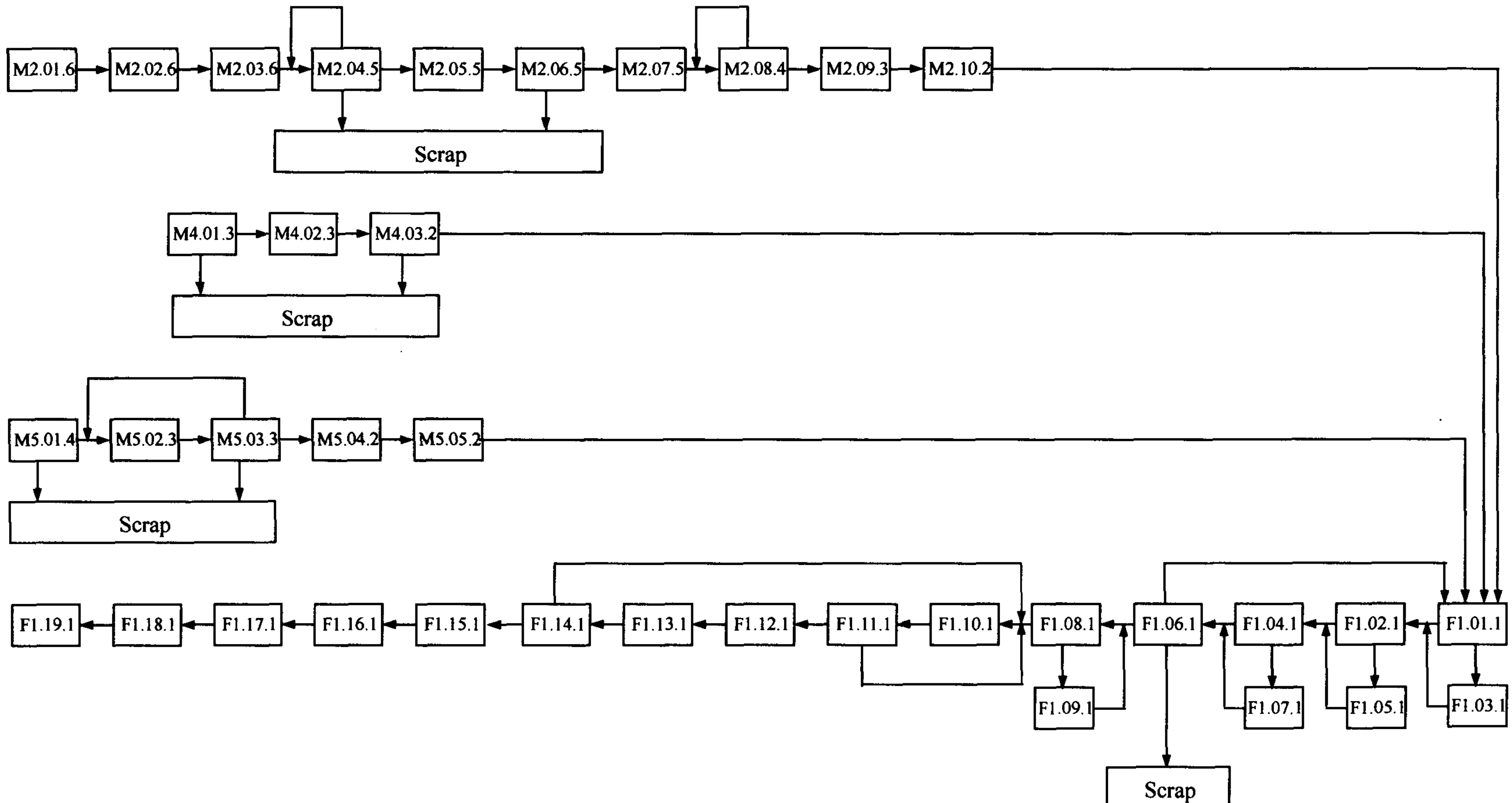


Figure 4.5 Assembly operation sequence diagram

4.4.3 Modelling Procedure

To develop the Markov model, the basic inputs required are the states and the transition probabilities. The states can be identified from the operation sequence diagram (figure 4.5). The transition probability can be obtained when rejection, rework and pass rates are available. These details can be obtained from the statistics available for 200 units of the flow control valve produced so far by the organisation. The statistics for the four subsystems are shown in the tables 4.1 – 4.4.

Operation	Pass Rate in %	Rejection Rate in %	Rework Rate in %
01	100	0	0
02	100	0	0
03	100	0	0
04	74.4	7.3	18.3
05	100	0	0
06	98.9	1.1	0
07	100	0	0
08	99	0	1
09	100	0	0
10	100	0	0

Table 4.1 Statistics for subsystem M2

Operation	Pass Rate in %	Rejection Rate in %	Rework Rate in %
01	97.3	2.7	0
02	100	0	0
03	98.4	1.6	0

Table 4.2 Statistics for subsystem M4

Operation	Pass Rate in %	Rejection Rate in %	Rework Rate in %
01	75	25	0
02	100	0	0
03	80.1	6.6	13.3
04	100	0	0
05	100	0	0

Table 4.3 Statistics for subsystem M5

Operation	Pass Rate in %	Rejection Rate in %	Rework Rate in %
01	33.4	0	66.6
02	99	0	1
03	100	0	0
04	75	0	25
05	100	0	0
06	97	1	2
07	100	0	0
08	90	0	10
09	100	0	0
10	100	0	0
11	99	0	1
12	100	0	0
13	100	0	0
14	99	0	1
15	100	0	0
16	100	0	0
17	100	0	0
18	100	0	0
19	100	0	0

Table 4.4 Statistics for subsystem F1

4.4.3.1 State Identification Procedure

The operation sequence diagram shows the sequence of operations required and the levels at which these operations are performed. In Markovian modelling, each state may represent a particular operation or a set of operations. In order to reduce the number of states, as many operations as possible are clubbed to form a single state satisfying the following conditions. (i) All the operations associated with a single state should be performed at the same level. (ii) There may not be any rejection or rework in any of the operations clubbed (iii) In the case of rejection in the operations clubbed, only the last operation of the club can contain the rejection. (iv) In the case of rework all the operations in the state are repeated.

The clubbing of activities can be illustrated using the subsystem M2. The subsystem consists of 10 operations performed at different levels ranging from 6 to 2 (see figure 4.5). The first three operations are performed at level 6 and there is no rejection or rework between these operations. So these three operations are put together to form a single state, the state 1. The next operation 4, which is at level 5, has rework and rejection and hence it is taken as a single state, the state 2. The operations 5 and 6 are done at level 5 and there exists a scope of rejection during operation 6. Now, these two can be combined, as a chance for rejection exists for the last operation in the club, to form a single state, the state 3. The operations 7 and 8 are done at different levels. So these two operations cannot be put together and can be considered as separate states viz. state 4 and state 5 respectively. The operations 9 and 10 are done at different levels. So they can also be considered as different states. There are two absorbing states in each subsystem, corresponding to the finished item and the scrap. For the subsystem considered, the finished item is M2. The last state of each subsystem represents the scrap.

4.4.3.2 The Markov Model

All the inputs required to develop the Markov model are available from the tables 4.1 to 4.4 and from figure 4.5. The Markov model for each subsystem can be represented either in transition diagram form or in transition probability matrix form. Both the forms may be used for better understanding. The absorption probability matrix obtained from the model is used for generating the level jump probability matrix, used for the computation of level transition probability. The rows of the level jump probability matrix represent the levels of the subsystem and the column (the matrix has only one column) stands for the highest level of the subsystem. The Markov model, the absorption probability matrix, the level jump probability matrix and the level transition probability matrix of the subsystem M2, M4, M5 and F1 are provided in the following pages. In the case of subsystem F1, the level jump probability, the level transition probability and the absorption probability from the first state to the finished item are the same, as the system is equivalent to a single level BOM. Hence, only its absorption probability matrix is shown here.

The earlier discussion indicated that the subsystem M2 has 9 states (7 transition states and two absorption states). Now, the various transition possibilities are to be identified along with the probabilities. For the subsystem M2, table 4.1

and figure 4.5 will assist in doing this. The pass rate is the transition probability from a state to the succeeding state. The rejection rate is the transition probability from a state (which has rejection) to the scrap state. If there is rework existing in a state, then, there is transition from that state to the same state as long as the same normal operation is used for the rework operation. If the rework operation is different from the normal operation, then, there is a state to represent this rework operation. (See figure 4.5) Wherever this type of rework exists, a state is required. The rework rate is also a transition probability. The pass rate from the last transition state is the probability of transition from that state to the finished item state. The transition diagram showing these transition probabilities for the subsystem is given in figure 4.6 and the corresponding transition probability matrix is shown in figure 4.7.

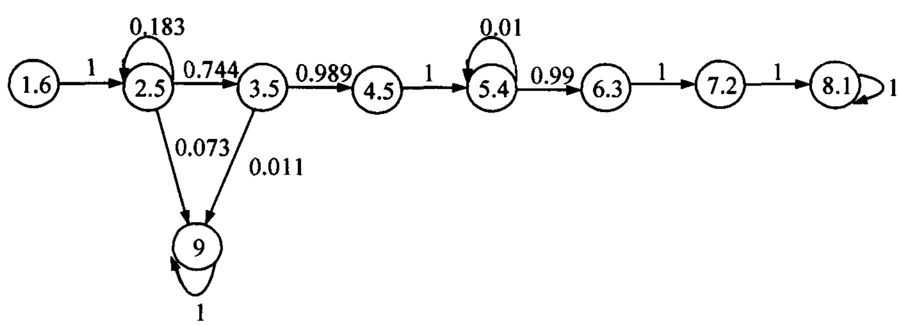


Figure 4.6 Transition diagram of subsystem M2

The first state in figure 4.6 is 1.6. It represents three operations M2.01.6, M2.02.6 and M2.03.6 shown in the figure 4.5. As there is no rejection or rework at any of these operations (also see table 4.1), an item from this state transits to next state, 2.5 with probability 1. As seen in the transition diagram the states are identified with two numbers; the first number shows the position of the state among all the states of the subsystem and the second number stands for level at which operations associated with that state are performed. From the state 2.5 an item transits to itself with probability 0.183, transits to scrap state with probability 0.073 and transits to the next state 3.5 with probability 0.744. Similarly other transitions with the help of the table 4.1 can be explained. For the subsystem F1, the states are identified with only one number as all the operations are performed at level, 1.

$$
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.183 & 0.744 & 0 & 0 & 0 & 0 & 0 & 0.073 \\
0 & 0 & 0 & 0.989 & 0 & 0 & 0 & 0 & 0.011 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.01 & 0.99 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

Figure 4.7 Transition probability matrix of subsystem M2

$$
\begin{bmatrix}
0.9006 & 0.0994 \\
0.9006 & 0.0994 \\
0.9890 & 0.0110 \\
1.0000 & 0 \\
1.0000 & 0 \\
1.0000 & 0 \\
1.0000 & 0
\end{bmatrix}
$$

Figure 4.8 Absorption probability matrix of subsystem M2

From Level	To Level 1
6	0.9006
5	0.9006
4	1
3	1
2	1

Figure 4.9 Level jump probability vector of subsystem M2

	5	4	3	2	1
6	1	0	0	0	0
5	0	0.9006	0	0	0
4	0	0	1	0	0
3	0	0	0	1	0
2	0	0	0	0	1

Figure 4.10 Level transition probability matrix of subsystem M2

Markov model and output of subsystem M4

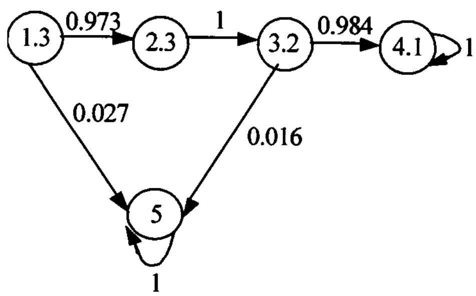


Figure 4.11 Transition diagram of subsystem M4

$$\begin{bmatrix} 0 & 0.973 & 0 & 0 & 0.027 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.984 & 0.016 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 4.12 Transition probability matrix of subsystem M4

$$\begin{bmatrix} 0.9574 & 0.0426 \\ 0.9840 & 0.0160 \\ 0.9840 & 0.0160 \end{bmatrix}$$

Figure 4.13 Absorption probability matrix of subsystem M4

From To 1

$$\begin{matrix} 3 \\ 2 \end{matrix} \begin{bmatrix} 0.9574 \\ 0.9840 \end{bmatrix}$$

Figure 4.14 Level jump probability vector of subsystem M4

$$\begin{matrix} & \begin{matrix} 2 & & 1 \end{matrix} \\ \begin{matrix} 3 \\ 2 \end{matrix} & \begin{bmatrix} 0.9730 & 0 \\ 0 & 0.9840 \end{bmatrix} \end{matrix}$$

Figure 4.15 Level transition probability matrix of subsystem M4

Markov model and output of subsystem M5

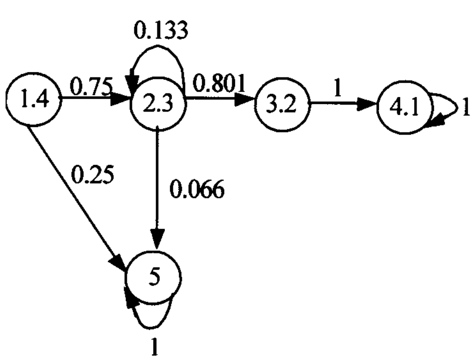


Figure 4.16 Transition diagram of subsystem M5

$$\begin{bmatrix} 0 & 0.75 & 0 & 0 & 0.25 \\ 0 & 0.133 & 0.801 & 0 & 0.066 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 4.17 Transition probability matrix of subsystem M5

$$\begin{bmatrix} 0.6929 & 0.3071 \\ 0.9239 & 0.0761 \\ 1.0000 & 0 \end{bmatrix}$$

Figure 4.18 Absorption probability matrix of subsystem M5

From	To 1
4	0.6929
3	0.9239
2	1

Figure 4.19 Level jump probability vector of subsystem M5

	3	2	1
4	0.75	0	0
3	0	0.9239	0
2	0	0	1

Figure 4.20 Level transition probability matrix of subsystem M5

Markov model and absorption probability of subsystem F1

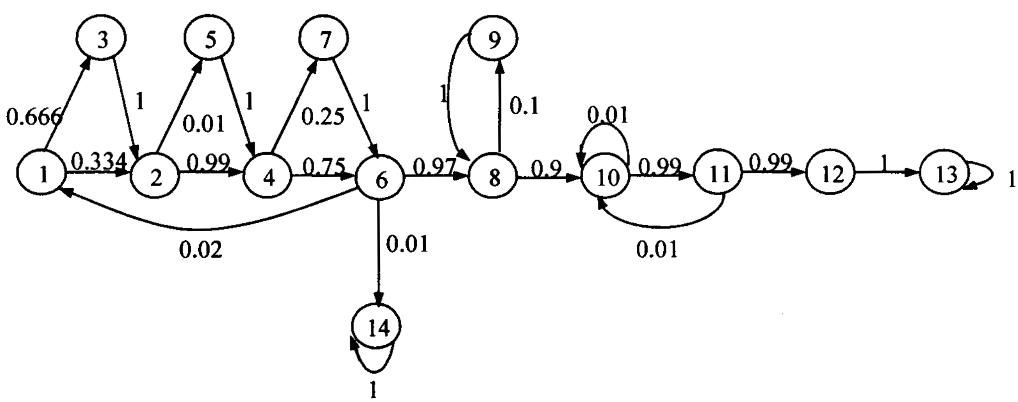


Figure 4.21 Transition diagram of subsystem F1

$$\begin{bmatrix}
0 & 0.334 & 0.666 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.99 & 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.75 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.02 & 0 & 0 & 0 & 0 & 0 & 0 & 0.97 & 0 & 0 & 0 & 0 & 0 & 0.01 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0.9 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & 0.99 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & 0 & 0.99 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

Figure 4.22 Transition probability matrix of subsystem F1

$$\begin{bmatrix}
0.9898 & 0.0102 \\
0.9898 & 0.0102 \\
0.9898 & 0.0102 \\
0.9898 & 0.0102 \\
0.9898 & 0.0102 \\
0.9898 & 0.0102 \\
0.9898 & 0.0102 \\
1.0000 & 0 \\
1.0000 & 0 \\
1.0000 & 0 \\
1.0000 & 0 \\
1.0000 & 0
\end{bmatrix}$$

Figure 4.23 Absorption probability matrix of subsystem F1

4.4.3.3 Estimation of Material Requirement

Since the level transition probability of every item required to make the product is available, the quantity of each item required can be computed using equation 4.3. The level transition probability assimilated from different subsystems and the quantities estimated are shown in table 4.5. These quantities can be used in the place of materials requirement given in the bill of material.

u	l	$_{u}b_{l\,l-1}$	t_u^l	q_u^l
M2	1	0.9898	1	1.0103
C3	1	0.9898	1	1.0103
M4	1	0.9898	1	1.0103
M5	1	0.9898	1	1.0103
C6	1	0.9898	1	1.0103
C7	2	1	8	8
S8	2	1	1	1
C9	2	1	2	2
C10	2	0.98400	1	1.0163
C11	2	0.9840	3	3.0488
C12	2	0.9840	2	2.0325
S13	2	0.9840	1	1.0163
C14	2	0.9840	2	2.0325
C15	2	1	1	1
S16	2	1	1	1
C17	3	1	1	1
S18	3	1	1	1
C19	3	0.9730	1	1.0277
C20	3	0.9730	1	1.0277
C21	3	0.9730	1	1.0277
S22	3	0.9239	1	1.0824
C23	3	0.9239	1	1.0824
C24	3	0.9239	1	1.0824
C25	4	1	85	85
S26	4	1	1	1
C27	4	1	2	2
C28	4	1	20	20
C29	4	0.75	1	1.3333
S30	5	0.9006	1	1.1104
C31	5	0.9006	1	1.1104
C32	6	1	1	1
C33	6	1	1	1
C34	6	1	1	1

Table 4.5 Material requirements under rejection

4.5 SUMMARY

A raw material that is converted through a series of operations using a serial production system is modelled as a Markov model. The same concept is used for the assembly operation. In this case, the modelling is possible only when the operations are performed after assembling the items. Moreover, when assembly operations are carried out in stages, the initial assembly progresses through various stages while more items join the initial assembly during the course.

This modelling concept is useful in analysing the entire manufacturing process that involves assembly of parts and many subassemblies. It can also estimate the quantity of various materials required to produce the product when rejection and rework exist in subassembly and assembly preparations. The bill of material contains the details of various items required to manufacture a product. One piece of information contained in the bill of material is the quantity of each item required to manufacture the parent item. Through the method of analysis, the quantity of each item required is revised to take an account of the scrapping and rework effect. Hence, the modelling explained in this chapter involves modifying the bill of material, on the basis of probable scrapping and rework.

CHAPTER 5

STOCHASTIC MODELLING OF MATERIAL HANDLING PROCESS (SMMHP)

5.1 INTRODUCTION

In a manufacturing system the production machines and Material Handling Devices (MHD) should be physically laid out and functionally matched so that together they act as a single integrated unit. Though MHDs have a significant influence on the manufacturing system, they have received little attention by way of analysis. The material handling activity generally depends on the route sheet and job arrival process to the shop floor. The arrival of jobs to the shop floor is, usually random, that can be characterised as Poisson process. This means that, the movement of materials and hence the movement of material handling device between machines can also be characterised as a random process. This chapter describes the modelling of material handling as a stochastic process. The motivation for this modelling stems from the description of a random process explained, with the help of a limousine service, in Ravindran *et al.* (1987). The example describes a limousine operation, as taxi service between the airport, downtown, and a cluster of suburban hotels, as a stochastic process. The system to be modelled here is also similar to the limousine service modelling.

The present work models a manufacturing system consisting of machines and a MHD. The MHD is used to transport materials from one machine to another. This transfer of materials is modelled as a stochastic process. Three types of models, namely SMMHP1, SMMHP2 and SMMHP3, are proposed. The following sections, except the next one, describe these models and their usefulness. The next section describes the determination of transition rate under Poisson arrival of jobs to the production shop.

5.2 DETERMINATION OF TRANSITION RATE

Consider a manufacturing system that consists of m machines and a material-handling device. Let the material to be processed in the system first arrive at a distribution station from where it is distributed to machines according to the route sheet information. The materials that complete their process requirements are received at the receiving station. As the present work is mainly concerned with the movement of materials between the machines, it is assumed that the MHD transports materials only between machines and not from the distribution station to the machines or from the machines to the receiving station.

Let there be n types of jobs (set A) to process in m machines (set B).

$$A = \{1,2,3,...,n\} \text{ and } B = \{1,2,3,...,m\}$$

Let, C_k $(k \in A)$ denote a kind of job and M_i $(i \in B)$ a machine. The arrival pattern of each type of jobs at the distribution station is assumed to be a Poisson process. That is, the inter-arrival time of jobs is, an independent and identically distributed random variable. The arrival rate of the variable is λ_k. It is also assumed that different types of jobs arrive at the distribution station as Poisson processes in an independent way. As the superposition of different Poisson processes result in a Poisson process, the rate of the pooled arrival stream is the sum of the rates of the individual Poisson streams. Let λ be the rate of the Poisson process after superposition.

The jobs are assigned to various machines from the distribution station according to an independent probability distribution $(p_1, p_2,..., p_m)$, with $p_1 + p_2 + ... + p_m = 1$. The individual streams after decomposition are also Poisson processes with rates $\lambda p_1, \lambda p_2, ..., \lambda p_m$. The superposition and decomposition processes are depicted in figure 5.1.

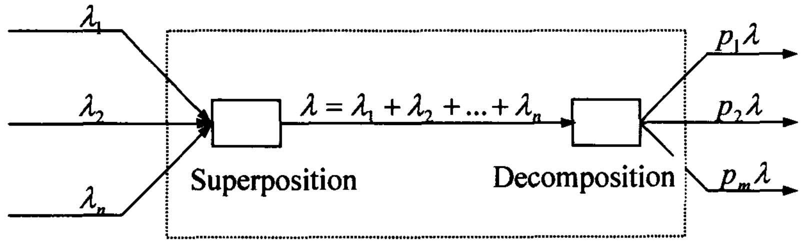


Figure 5.1 Superposition and decomposition of Poisson processes

The concept of superposition and decomposition of processes in the case of the distribution station is also applicable to each workstation. For all $k, l, p \in A$, and $i, j \in B$, let a machine M_i be processing items C_k, C_l and C_p. The subsequent operation for C_k and C_l is in machine M_j. If the requirement rates (the arrival rates) for C_k, C_l and C_p are λ_k, λ_l and λ_p, then the transition rate of the MHD from M_i to M_j is λ_{ij}, which is equal to $\lambda_k + \lambda_l$. The time between transfers (from M_i to M_j) is independent and identically distributed with a rate $\lambda_k + \lambda_l$.

5.3 SMMHP1

As stated earlier, MHD is used to transfer materials from machine *i* to machine *j*. A state space is defined to model the evolution of the movement of MHD as a stochastic process. The MHD is said to be in state *i* when it starts waiting at machine M_i to pick up the material, until it delivers the material at machine M_j; for all $i, j \in B$. The state space *S* is countable.

$$S = \{1, 2, \ldots, m\}$$

The following assumptions are made to model the stochastic process.

1. There are items processed by machine at each state that wait for MHD to transit them to the next machine for further processing.
2. The transition of MHD to the subsequent state depends on the availability of material at the current state for want of further processing at any other machine, and is not dependent on the material waiting for MHD at any other state.

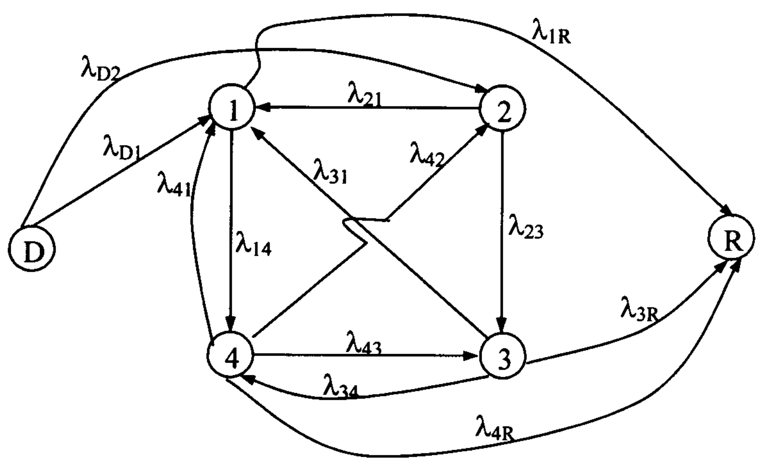
3. If more than one type of job waits at a state when the MHD reaches that state, it picks a job randomly for the next transition.
4. The transition probabilities are stationary in nature.

At any point of time, the MHD will be at any one of the states. It is possible to represent the operation of MHD as a process $\{X(t): t \geq 0\}$, where $X(t)$ is a random variable for each $t \in T$ in state space S. T is an index set of the process and any $t \in T$ is known as index or parameter. Here, the index set T stands for the time. The process $\{X(t): t \geq 0\}$ is a Continuous Time Markov Chain (CTMC).

The modelling can be explained with an example. Consider a manufacturing cell, which consists of 4 machines to process 6 components. The route sheet and the requirement rates of components are shown in the table 5.1. The route sheet indicates the sequence in which machines are used to process the components. The interaction element of row C_i and column M_j in the table denotes the process sequence at which the machine M_j is used for the component C_i. It is assumed that items flow from one machine to another as per the requirement rate indicated in table 5.1. That is, when an item k flows from machine M_i to machine M_j, the rate of flow is λ_k. For the above route sheet the associated rate of flow between machines is shown in figure 5.2. This figure also shows the flow from the distribution station to machines and from machines to the receiving station. In this problem some values of λ_{ij} are zeroes. The rate at which MHD moves between machines is shown in figure 5.3. This flow diagram is a representation of continuous time Markov chain in the form of transition diagram.

	M_1	M_2	M_3	M_4	Requirement rate λ_i
C_1	1	-	3	2	5
C_2	2	1	-	3	7
C_3	1	3	4	2	8
C_4	3	1	2	4	6
C_5	-	1	2	3	10
C_6	4	1	2	3	4

Table 5.1. Route sheet and demand rate



D – Distribution Station   R – Receiving Station

Figure 5.2 Flow rate diagram

The various rates of flow shown in the figure 5.2 is provided below in terms of the requirement rate of the components.

$\lambda_{21} = \lambda_2$
$\lambda_{23} = \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6$
$\lambda_{34} = \lambda_5 + \lambda_6$
$\lambda_{31} = \lambda_4$
$\lambda_{42} = \lambda_3$
$\lambda_{41} = \lambda_6$
$\lambda_{14} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$
$\lambda_{43} = \lambda_1$
$\lambda_{D1} = \lambda_1 + \lambda_3$
$\lambda_{D2} = \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6$
$\lambda_{1R} = \lambda_6$
$\lambda_{3R} = \lambda_1 + \lambda_3$
$\lambda_{4R} = \lambda_2 + \lambda_4 + \lambda_5$

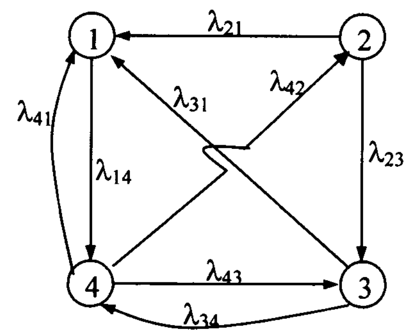


Figure 5.3 Transition diagram

The above CTMC is ergodic, since it is irreducible and finite with positive 'aperiodic' recurrent states. Therefore, there exists a steady state probability vector. To calculate the steady state probability, let the steady state probability vector be $\Pi = [\pi_i]$, and the transition rate matrix be $Q = \lfloor q_{ij} \rfloor$. For a continuous time Markov chain at steady state, $\Pi Q = 0$.

$$Q = \begin{bmatrix} -\lambda_{14} & 0 & 0 & \lambda_{14} \\ \lambda_{21} & -(\lambda_{21}+\lambda_{23}) & \lambda_{23} & 0 \\ \lambda_{31} & 0 & -(\lambda_{31}+\lambda_{34}) & \lambda_{34} \\ \lambda_{41} & \lambda_{42} & \lambda_{43} & -(\lambda_{41}+\lambda_{42}+\lambda_{43}) \end{bmatrix}$$

The steady state probability vector Π, is obtained as a solution of

$$\Pi Q = 0$$

$$\sum_j \pi_j = 1$$

$$\pi_j \geq 0, j \in S$$

The steady state probability π_j is independent of the initial state and is stationary. The steady state probability can be interpreted as the long-run proportion of residence time in state j.

5.3.1 Embedded Markov Chain in the CTMC

If the evolution of CTMC is examined at instants when a state change occurs, then it is possible to identify a discrete time process $\{X_N : N = 0,1,2,\ldots\}$

where X_N denotes the state reached by the CTMC after N transitions. The discrete time process is a discrete time Markov chain known as an Embedded Markov Chain (EMC) of the CTMC.

Parameters of the EMC can be determined as follows: The state space of EMC is same as the state space S of the CTMC. Let $Q = \lfloor q_{ij} \rfloor$ be the infinitesimal generator of the CTMC and the typical states are indicated with indices i, j and k. Also let the transition probability of the EMC from state i to state j be p_{ij}. If the random variable $T_{ij}, i \neq j$, denotes the time to reach state j from state i, then T_{ij} is exponentially distributed with rate q_{ij}.

Now, for $i \neq j$,

$$p_{ij} = \frac{q_{ij}}{\sum_{k \neq i} q_{ik}}$$

since, T_{ik}'s are mutually independent exponential random variable with rates q_{ik}. As the system is observed at the instants of change of state, $p_{ii} = 0, i \in S$. The transition probabilities of the EMC are,

$$p_{ij} = 0, \quad \text{if } i = j$$

$$p_{ij} = \frac{q_{ij}}{\sum_{k \neq i} q_{ik}} \quad \text{if } i \neq j$$

The transition diagram of the EMC is shown in figure 5.4.

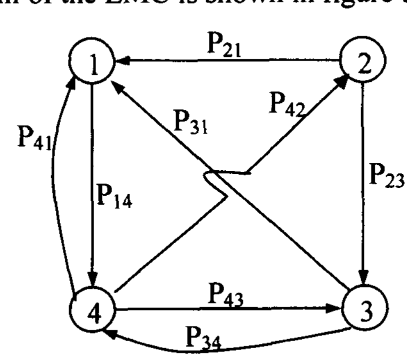


Figure 5.4 Transition diagram of the EMC of the CTMC

The transition probability matrix of the EMC is,

$$P=\begin{bmatrix} 0 & 0 & 0 & 1 \\ \lambda_{21}/(\lambda_{21}+\lambda_{23}) & 0 & \lambda_{23}/(\lambda_{21}+\lambda_{23}) & 0 \\ \lambda_{31}/(\lambda_{31}+\lambda_{34}) & 0 & 0 & \lambda_{34}/(\lambda_{31}+\lambda_{34}) \\ \lambda_{41}/(\lambda_{41}+\lambda_{42}+\lambda_{43}) & \lambda_{42}/(\lambda_{41}+\lambda_{42}+\lambda_{43}) & \lambda_{43}/(\lambda_{41}+\lambda_{42}+\lambda_{43}) & 0 \end{bmatrix}$$

It can be shown that a CTMC and its EMC have the same communication class [Viswanadham and Narahari (1992)]. That is, the CTMC is irreducible and positive recurrent, if the EMC is irreducible and positive recurrent, and vice versa. Therefore the EMC is ergodic as the CTMC is ergodic. An ergodic EMC has a unique steady state probability vector $Y=[y_i]$ given by,

$$YP = Y$$

$$\sum_j y_j = 1$$

$$y_j \geq 0 \quad \forall j$$

The probability y_j is to be interpreted as the visit ratio to state *j* or the relative number of visits to state *j* in the long run.

5.3.2 Relative Transition Matrix (RTM)

Let $R=[r_{ij}]$ be a probability matrix, where r_{ij} is the probability with which the MHD makes a transition from state *i* to state *j*, at steady state. In the long run, r_{ij} represents the relative transition made by the MHD from state *i* to state *j* and hence the matrix *R* can be called **'relative transition matrix'**. Consider the steady state vector *Y* of EMC as a column vector. An element in matrix *R* is the product of the steady state probability of state *i* and transition probability from a given state *i* to state *j*. That is, the r_{ij} can be obtained as,

$$r_{ij} = y_i p_{ij}$$
$$r_{ij} \geq 0 \quad \forall\, i, j \in S$$
$$\sum_i \sum_j r_{ij} = 1$$

This shows that r_{ij} is a probability distribution. This fact can be verified from following statements.

$$\sum_i \sum_j r_{ij} = \sum_i \sum_j y_i p_{ij}$$
$$= \sum_i y_i \quad \text{as } \sum_j y_i p_{ij} = y_i$$
$$= 1 \qquad \text{because } y_i \text{ is a probability distribution.}$$

The r_{ij} can be interpreted as the relative number of transits made by MHD from machine M_i to machine M_j. It also denotes the relative rate of flow from machine M_i to machine M_j.

5.4 SMMHP2

It is possible to model the material handling process as a CTMC with state space different from the earlier model. Two more state spaces are identified and these state spaces are the basis of the models SMMHP2 and SMMHP3. The model SMMHP2 is described in this section. The state space description of the model is as follows:

i – If the MHD is with any of the machines M_i, $i = 1,2,3,\ldots$

(j,k) - If the MHD is transferring materials from machine M_j to machine M_k, $j,k = 1,2,3,\ldots$ for $j \neq k$.

If m is the number of available machines, the maximum number of states possible is m^2. The state space for the example (table 5.1) considered is:

$$S = \{1,2,3,4,(1,4),(2,1),(2,3),(3,1),(3,4),(4,1),(4,2),(4,3)\}$$

From the route sheet it is observed that there is no material flow from certain machines to some other machines, and hence the number of state is only 12. The CTMC of the model is shown in figure 5.5.

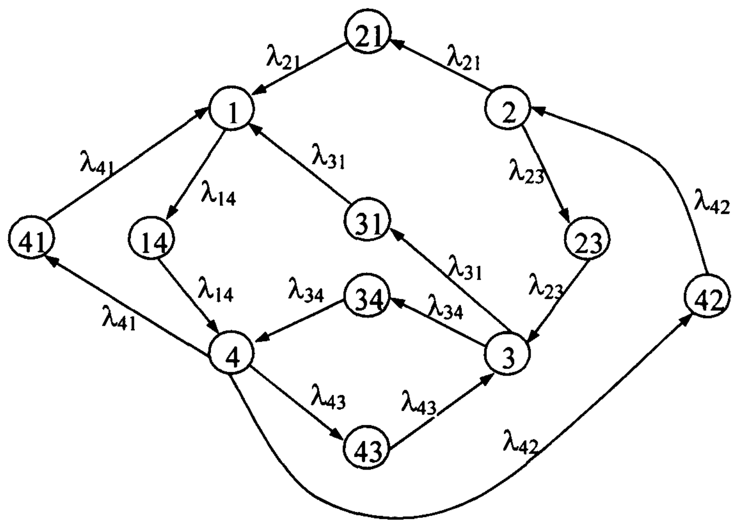


Figure 5.5 Transition diagram

The transition rate can be computed as in the earlier model. Since the CTMC has 12 states, the size of the steady state vector Π is also 12. The first m elements in the vector show the relative residence time of the MHD with machines, and hence the sum of these is proportion of the time the MHD idles. Hence, the proportion of time effectively utilized is (1-idle time). This can be considered as a measure of utilisation of the MHD.

5.4.1 Relative Time Matrix

It is possible to calculate the time spent by the MHD for transferring material from one machine to another, relative to the total time spent for material transfer alone. The relative time spent in each material transfer state is obtained on dividing the probability (steady state) of each state in material transfer by the sum of the probability of the all-material transfer state. This relative time can be represented as elements of a matrix of size $m \times m$ called **relative time matrix**. A material transfer state in the state space is being represented as (j,k). The interaction element of row j and column k of the relative time matrix is the relative time of the state (j,k). (Let the size of the state space be a.) For instance, the fifth state in the state space is (1,4) whose steady state probability is represented as π_5, the first row fourth column element of the relative time matrix is $\frac{\pi_5}{\sum_{i=m+1}^{a}\pi_i}$. As there are $(a-m)$ states that are in material transfer state, $(a-m)$ elements of the matrix have positive values and all other elements are zero. Let the relative time matrix be represented as B and an element of it be b_{jk}. Now the b_{jk} can be determined as

$$
\begin{aligned}
b_{jk} &= \frac{\pi_i}{\sum_{i=m+1}^{a}\pi_i} \quad \text{for } (j,k)\in S \text{ and } S_i=(j,k) \\
&= 0 \quad \text{for } (j,k)\notin S
\end{aligned}
$$

The b_{jk} is a probability distribution, hence

$$\sum_j \sum_k b_{jk} = 1$$

5.4.2 Embedded Markov Chain in the CTMC

The EMC of the CTMC can now be developed. The transition diagram of the EMC is similar to the CTMC diagram except that in places of transition rate, transition probability is used. The transition probability from the transition rate can be determined as explained in the earlier model.

On analysis of the EMC, it is possible to observe the following,

1. The system is being observed at epochs when the MHD is "picking" and "placing" the material. Hence, the number of "picking" is equal to the number of "placing".
2. Under the above condition, sum of the probabilities of first four states is equal to the sum of the probabilities of the remaining states and it should be equal to 0.5. This can be verified from the steady state probability vector provided in section 5.6 for the example problem.

Following points can be noted while comparing the models SMMHP1 and SMMHP2.

1. The number of epochs at which the system is observed in the case of the discrete time modelling in SMMHP2 is twice the number of epochs at which the system is observed in the discrete time modelling in SMMHP1.
2. The first four states in SMMHP2 are comparable with the states in the SMMHP1. The steady state probability of states in the EMC of SMMHP1 is equal to twice the steady state probability of corresponding state in the EMC of SMMHP2.

5.5 SMMHP3

The state space description for the model is as follows:

i – If the MHD is with any of the machines M_i, $i = 1,2,3,\ldots,m$

$m+1$ - If the MHD is in material transferring condition.

The transition diagram of the CTMC for the example (table 5.1) is shown below.

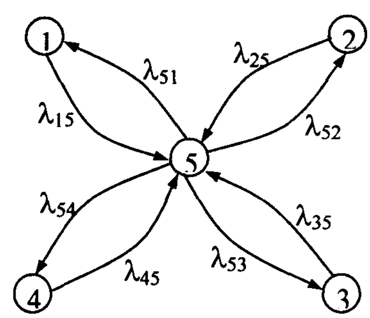


Figure 5.6 Transition diagram

In this case also, the discrete time Markov chain embedded in the CTMC can be generated. The transition rate and the transition probability calculation are similar to the earlier models. The steady state behaviour of the continuous time and discrete time Markov chain can also be worked out as in the earlier cases.

5.6 NUMERICAL RESULTS

The route sheet, of 6 components to be produced in a shop with 4 machines, is available in table 5.1. This table contains the demand rate for the components also. Using the data from the table, various results obtained for the models are shown below.

Numerical results of model SMMHP1

$$\begin{bmatrix} -26 & 0 & 0 & 26 \\ 7 & -35 & 28 & 0 \\ 6 & 0 & -20 & 14 \\ 4 & 8 & 5 & -17 \end{bmatrix}$$

Figure 5.7 Transition rate matrix

$$\begin{bmatrix} 0.1617 & 0.1065 & 0.2657 & 0.4661 \end{bmatrix}$$

Figure 5.8 Steady state vector of the CTMC

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0.2 & 0 & 0.8 & 0 \\ 0.3 & 0 & 0 & 0.7 \\ 0.24 & 0.47 & 0.29 & 0 \end{bmatrix}$$

Figure 5.9 Transition probability matrix of DTMC embedded in the CTMC

$$\begin{bmatrix} 0.1986 & 0.1761 & 0.2510 & 0.3743 \end{bmatrix}$$

Figure 5.10 Steady state vector of DTMC

$$\begin{bmatrix} 0 & 0 & 0 & 0.1986 \\ 0.0352 & 0 & 0.1409 & 0 \\ 0.0753 & 0 & 0 & 0.1757 \\ 0.0881 & 0.1761 & 0.1101 & 0 \end{bmatrix}$$

Figure 5.11 Relative transition matrix

Numerical results of model SMMHP2

$$\begin{bmatrix}
-26 & 0 & 0 & 0 & 26 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -35 & 0 & 0 & 0 & 7 & 28 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -20 & 0 & 0 & 0 & 0 & 6 & 14 & 0 & 0 & 0 \\
0 & 0 & 0 & -17 & 0 & 0 & 0 & 0 & 0 & 4 & 8 & 5 \\
0 & 0 & 0 & 26 & -26 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 0 & 0 & 0 & 0 & -7 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 28 & 0 & 0 & 0 & -28 & 0 & 0 & 0 & 0 & 0 \\
6 & 0 & 0 & 0 & 0 & 0 & 0 & -6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 14 & 0 & 0 & 0 & 0 & -14 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 0 & 0 \\
0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -8 & 0 \\
0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5
\end{bmatrix}$$

Figure 5.12 Transition rate matrix

$$\begin{bmatrix} 0.0489 & 0.0322 & 0.0804 & 0.1411 & 0.0489 & 0.0322 & 0.0322 & 0.0804 & 0.0804 & 0.1411 & 0.1411 & 0.1411 \end{bmatrix}$$

Figure 5.13 Steady state vector of CTMC

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.20 & 0.80 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.30 & 0.70 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.24 & 0.47 & 0.29 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Figure 5.14 Transition probability matrix of DTMC embedded in the CTMC

$$\begin{bmatrix}0.0993 & 0.0881 & 0.1255 & 0.1871 & 0.0993 & 0.0176 & 0.0705 & 0.0376 & 0.0878 & 0.0440 & 0.0881 & 0.0550\end{bmatrix}$$

Figure 5.15 Steady state vector of DTMC

$$\begin{bmatrix}0 & 0 & 0 & 0.0702\\ 0.0462 & 0 & 0.0462 & 0\\ 0.1153 & 0 & 0 & 0.1153\\ 0.2023 & 0.2023 & 0.2023 & 0\end{bmatrix}$$

Figure 5.16 Relative time matrix

Numerical results of model SMMHP3

$$\begin{bmatrix}-26 & 0 & 0 & 0 & 26\\ 0 & -35 & 0 & 0 & 35\\ 0 & 0 & -20 & 0 & 20\\ 0 & 0 & 0 & -17 & 17\\ 17 & 8 & 33 & 40 & -98\end{bmatrix}$$

Figure 5.17 Transition rate matrix

$$\begin{bmatrix}0.1111 & 0.0388 & 0.2804 & 0.3998 & 0.1699\end{bmatrix}$$

Figure 5.18 Steady state vector of CTMC

$$\begin{bmatrix}0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 1\\ 0.17 & 0.08 & 0.34 & 0.41 & 0\end{bmatrix}$$

Figure 5.19 Transition probability matrix of DTMC embedded in the CTMC

$$\begin{bmatrix}0.0867 & 0.0408 & 0.1684 & 0.2041 & 0.5000\end{bmatrix}$$

Figure 5.20 Steady state vector of DTMC

5.7 CLOSURE

The modelling procedure characterises the movement of material handling device under stochastic environment of Poisson arrival of items to the manufacturing system. Three models are suggested. In each model the material handling process is first modelled as a continuous time Markov chain. Then the discrete time Markov chain embedded in the continuous time Markov chain is identified. The steady state characteristics of the continuous and discrete models are determined. Two characteristics identified as relative transition and relative time matrices are useful for layout design decision. The relative transition matrix is equivalent to a flow matrix used as input to layout design, which actually provides the flow of material among machines relative to the total flow. This flow is generated under stochastic flow of material due to the stochastic arrival nature of items. The relative time matrix shows the time spent in material handling from machine i to machine j relative to the total time in material handling among machines.

CHAPTER 6

MODELLING OF UNCERTAINTIES IN LAYOUT PLANNING FOR MANUFACTURING SYSTEMS

6.1 INTRODUCTION

Manufacturing system design is critically important for an efficient system operation. Physical layout of manufacturing facilities is one of the most important design issues that should be addressed in the early stages of the system design. A well designed layout results in reduced material handling and management effort, shorter production cycles, simplified material flow, reduction in work-in-process, smaller queues and so on. Layout planning is mainly concerned with the determination of relative location of given number of facilities in the given plant area that allows smooth flow of materials, equipment and personnel. Generally, the layout is designed to facilitate the flow of raw materials, parts, subassemblies, assemblies and other associated materials between facilities, facilitating the smooth operation of the material handling function. The material handling cost is proportional to the volume of flow and the distance between facilities. A well designed layout minimises the material handling cost, the main objective of the layout design problem.

The volume of flow depends on the level of production of each product and the product mix. As the market demand is the driving factor in deciding the level of production it is beyond the control of the planner. Through a good layout design it is possible to control the distance between the facilities to an extent. A good layout design places facilities of greater interaction (volume of flow) nearer to each other. As a result, the sum of the products of the volume of flow and the distance between facilities is brought to a minimum. Usually, the cost of carrying a unit quantity of material a unit distance is assumed to be constant. Hence, the

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objective function of the layout problem can be considered as minimisation of the sum of the products of the volume of flow and the distance between facilities. The volume of flow between facilities is determined by the level of products to be produced within the facility and the routes (that is, sequence of machines) that these products have to follow. The traditional approach is to formulate the layout problem as Quadratic Assignment Problem (QAP).

The layout design problem is usually treated as static deterministic problem, which implies that all the inputs over the planning horizon are known with certainty at the time of formulation. That is, the level of demand of each product is known at the time of the problem formulation. Usually, manufacturing systems operate in an uncertain environment where the demand of each product can be estimated only probabilistically. Certain environment may not necessarily be static, which means that the input information for layout problem can change from period to period. Under such circumstances, the facility planner must determine the layout to be used in each period that minimises some measure of the material handling cost, simultaneously minimising relocation cost caused by layout changes. The layout problem where flow varies over time is usually referred to as dynamic plant layout problem, whereas, the case in which the flow is nearly constant from period to period is called static plant layout problem. The discussion in this chapter is mainly concerned with static plant layout problem.

As indicated earlier a layout problem is traditionally formulated as QAP. The formulation generally considers deterministic environment. However, the discussion of this chapter provides the QAP formulation under probabilistic and stochastic environment. The layout problem where inputs are known probabilistically assumes that, demand is known with certain probability distribution or, the discrete product mixes to be produced is known with probability. The stochastic environment assumes that the characteristics of the demand can be expressed in terms of a family of random variables. The discussion to follow explains the general QAP formulation.

6.2 MODELLING OF LAYOUT PROBLEM

The relationship among facilities to be located on the shop floor has an impact on the location decision. The relationship is measured as volume of flow. While deciding the locations the objective is to minimise the material handling cost. The optimum assignment of facilities to locations can be modelled as QAP. A general QAP formulation [Francis *et al.* (1999)] for deterministic environment is discussed below.

6.2.1 Deterministic Case

The inputs for the problem are, the flow of materials between facilities and the distance between locations. The flow of materials between facilities can be determined when various items to be processed in the facilities and the routes required for the processing of the items are known. To find the distance the location configuration of the production shop is required. The distance measure usually used is either rectilinear (Manhattan) or Cartesian (Euclidean). The following symbols are used for the formulation of the problem.

m - number of facilities

f_{jk} - flow of material from facility j to facility k

d_{rs} - distance between locations r and s

$x_{jr} = 1$ if facility j is assigned to location r

$\quad\;\; = 0$ otherwise

As every facility must be assigned to a location and every location must be assigned to a facility, the problem formulation is

$$\text{Minimise } z = \sum_{j=1}^{m} \sum_{k=1}^{m} \sum_{r=1}^{m} \sum_{s=1}^{m} f_{jk}\, d_{rs}\, x_{jr}\, x_{ks} \qquad \ldots[6.1]$$

subjected to

$$\sum_{r=1}^{m} x_{jr} = 1, \qquad j = 1,2,\ldots,m$$

$$\sum_{j=1}^{m} x_{jr} = 1, \qquad r = 1,2,\ldots,m$$

$$x_{jr} \in \{0,1\}, \qquad j = 1,2,\cdots,m; \quad r = 1,2,\ldots,m$$

Since the objective function contains the products of pairs of location decision variables, it has a *quadratic* form. Also, the constraints are similar to the classical *assignment* problem. Thus, this problem, in general, is called as *quadratic assignment problem.* This is suitable for a wide variety of other real problems such as back plane wiring in computers, design of control panel or keyboards, architectural space planning, VLSI design, etc. This discrete optimisation problem is NP-complete [Kusiak and Heragu (1987)] and hence the size of the problem optimally solved is limited. Thus, heuristic methods are more suitable for solution procedure and there are a number of algorithms available in literature. This work does not go into the solution procedure of QAP.

Usually, the determination of flow of material from facility j to facility k is based on the items that use facility k for processing, after the operations in facility j. The flow from facility j to facility k is the sum of the demand of all such items. That is, the flow can be stated in terms of the demand of items. However, the demand data available at the initial system design stage is, the demand of the end products and not the demand of various items (raw material, parts, subassemblies) used to make the products. Hence it is more suitable to represent the flow of materials using the demand of the end products. But, the bill of material of each product is required for this purpose. The information useful here that can be obtained from the bill of material is the list of items required to make the product and the quantities of each item required per parent.

Usually, the QAP formulation in the literature assumes that f_{ij} is available. But as already explained above, the basic data available is the demand of products. Thus the derivation below shows the formulation in terms of this demand. The symbols used for this purpose in addition to the symbols used above are as follows.

n - number of products produced.

D_i - demand of the product i; $i = 1,2,\ldots,n$.

ϕ_{i1} - finished product (final assembly) i.

u_i - number of make items contained in the bill of material of product ϕ_{i1}.

ϕ_{ib} -an item required for making the product ϕ_{i1}; $b = 2,3,\ldots,u_i$.

q_{ib} - quantity of item ϕ_{ib} required for making one unit of the product ϕ_{i1}.

$y_{ib}(j,k)$ = number of times the item ϕ_{ib} is transported from facility j to facility k; $i = 1,2,\ldots,n$ and $b = 1,2,\ldots,u_i$. Usually the value of $y_{ib}(j,k)$ is 1 or 0. Its value can be greater than one when the item ϕ_{ib} uses facilities j and k more than one time and in the route sheet of the item ϕ_{ib}, the facilities j and k appears consecutively as facility j first and facility k second more than one time.

$f_i(j,k)$ - volume of flow of final product ϕ_{i1} or any of its components ϕ_{ib}; $b = 2,3,\ldots,u_i$ between facilities j and k.

Now, $f_i(j,k)$ can be estimated as

$$f_i(j,k) = D_i \sum_{b=1}^{u_i} q_{ib}\, y_{ib}(j,k)$$

The flow of materials from facility j to facility k due to the entire products manufactured by the system is

$$f_{jk} = \sum_{i=1}^{n} D_i \sum_{b=1}^{u_i} q_{ib}\, y_{ib}(j,k)$$

$$= \sum_{i=1}^{n}\sum_{b=1}^{u_i} D_i\, q_{ib}\, y_{ib}(j,k)$$

Substitute this f_{jk} in the objective function of the QAP formulation and it can be written as

$$z = \sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{r=1}^{m}\sum_{s=1}^{m}\sum_{i=1}^{n}\sum_{b=1}^{u_i} D_i\, q_{ib}\, y_{ib}(j,k)\, d_{rs}\, x_{jr}\, x_{ks}$$

$$= \sum_{i=1}^{n}\left[\sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{r=1}^{m}\sum_{s=1}^{m}\sum_{b=1}^{u_i} q_{ib}\, y_{ib}(j,k)\, d_{rs}\, x_{jr}\, x_{ks}\right] D_i$$

For the given products and layout configuration, the term contained in the square bracket is a constant and hence, the function z can be written as

$$z = \sum_{i=1}^{n} \alpha_i\, D_i$$

where the constant

$$\alpha_i = \sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{r=1}^{m}\sum_{s=1}^{m}\sum_{b=1}^{u_i} q_{ib}\, y_{ib}(j,k)\, d_{rs}\, x_{jr}\, x_{ks}$$

For the given products to be manufactured, this constant depends on the layout configuration.

6.2.2 Probabilistic Case

Usually forecasted demand is used for the layout planning, and as we know, the actual demand may be different, due to the forecast error. If the forecast model is able to incorporate the characteristics (such as trend, seasonality, cyclic factors, etc.) of the demand, the error may be due to the randomness associated with the demand. This means that the demand may be following some probability distribution and the forecast represents the mean demand. In certain cases this may not be the situation. In some cases, the various product mixes to be manufactured are known only with probability estimates. As indicated by Rosenblatt and Kropp (1992) this may be the case where the actual size of the order received is not well defined and only some probability estimates are available. In this section, the objective function formulation is described when each product demand and the product mix are random variables.

Random Product Demand Case:

When the demand is a random variable, the expected material handling cost has to be minimised. Assume that the demand D_i is a random variable with a mean μ_i. When the cost of carrying a unit quantity through a unit distance is constant, the minimisation of material handling cost is equivalent to the minimisation of the function z. Here, the expected value of z has to be minimised. Taking the expected value of the function z, we obtain

$$E(z) = E\left(\sum_{i=1}^{n} \alpha_i D_i \right)$$

As the α_i is a constant which depends on the layout configuration, the expected value can be written as

$$E(z) = \sum_{i=1}^{n} \alpha_i E(D_i)$$

$$= \sum_{i=1}^{n} \alpha_i \mu_i$$

The above relationship shows that, for the random product demand, the quadratic assignment formulation of the layout problem is equivalent to a deterministic quadratic assignment formulation. That is, the mean of the (random) product demand can be used in place of the (deterministic) demand estimate of the product in the quadratic assignment formulation. Braglia *et al.* (2003) shows the same result, but the derivation is in terms of the flow matrix. However, the result derived in the present work shows it in terms of the demand of product. Also the method of derivation of the present work suggests a simplified layout formulation procedure for discrete product mixes known probabilistically.

Random Product Mix Case:

Sometimes the manufacturing system may be subjected to various scenarios (states). Each scenario corresponds to a product mix. The occurrence of the scenarios is not known deterministically, rather known probabilistically. At the time of layout planning, the details available are the probability distribution of the scenarios and the level of demand of each product in a scenario. Now, the QAP formulation is derived with the help of the following symbols. Most of the symbols explained above are used here also. The symbols that are different in meaning compared to the above ones and the new symbols are shown below.

A - number of scenarios

D_i^a - level of demand of product i in the scenario a; $i = 1,2,\ldots,n$ and $a = 1,2,\ldots,A$.

ρ^a - probability of occurrence of scenario a.

$f_i^a(j,k)$ - flow of material from facility j and facility k due to product i in the scenario a.

$f^a(j,k)$ - flow of material from facility j and facility k due to the entire products in the scenario a.

z^a - objective function for the scenario a.

$E(z)$ - expected value of the objective function considering the entire scenarios.

Now, $f_i^a(j,k)$ can be derived as

$$f_i^a(j,k) = D_i^a \sum_{b=1}^{u_i} q_{ib}\, y_{ib}(j,k)$$

Hence, $f^a(j,k)$ is

$$f^a(j,k) = \sum_{i=1}^{n} D_i^a \sum_{b=1}^{u_i} q_{ib}\, y_{ib}(j,k)$$

$$= \sum_{i=1}^{n} \sum_{b=1}^{u_i} D_i^a\, q_{ib}\, y_{ib}(j,k)$$

Now, z^a can be written as

$$z^a = \sum_{j=1}^{m} \sum_{k=1}^{m} \sum_{r=1}^{m} \sum_{s=1}^{m} f^a(j,k)\, d_{rs}\, x_{jr}\, x_{ks}$$

Substituting the expression of $f^a(j,k)$ in the above equation, the z^a become

$$z^a = \sum_{j=1}^{m} \sum_{k=1}^{m} \sum_{r=1}^{m} \sum_{s=1}^{m} \sum_{i=1}^{n} \sum_{b=1}^{u_i} D_i^a\, q_{ib}\, y_{ib}(j,k)\, d_{rs}\, x_{jr}\, x_{ks}$$

$$= \sum_{i=1}^{n} \left[\sum_{j=1}^{m} \sum_{k=1}^{m} \sum_{r=1}^{m} \sum_{s=1}^{m} \sum_{b=1}^{u_i} q_{ib}\, y_{ib}(j,k)\, d_{rs}\, x_{jr}\, x_{ks} \right] D_i^a$$

$$= \sum_{i=1}^{n} \alpha_i\, D_i^a$$

Now, $E(z)$ can be derived as

$$E(z) = \sum_{a=1}^{A} \rho^a z^a$$

Substituting the expression of z^a in the above equation, we obtain

$$E(z) = \sum_{a=1}^{A} \rho^a \sum_{i=1}^{n} \alpha_i D_i^a$$

$$= \sum_{i=1}^{n} \alpha_i \sum_{a=1}^{A} \rho^a D_i^a$$

The above equation shows the expected cost for a layout configuration for the given products when the expected demand is considered, based on the demand of products occurring in the scenarios. Now the best layout configuration should be identified as that which minimises the expected cost for the given product under the expected demand condition. That is, the objective of the problem is

$$\text{Minimise } E(z) = \sum_{i=1}^{n} \alpha_i \sum_{a=1}^{A} \rho^a D_i^a$$

The constraints are similar to the deterministic QAP. The expected demand indicates that the demand of items of the various scenarios can be compressed into a single scenario where the demand of each item is the expected demand of the item in the various scenarios.

At this juncture the above problem (random scenario) formulation can be compared to the formulation suggested by Rosenblatt and Kropp (1992) for the same type of problem environment. They suggested that optimum layout solution to the production environment of random scenario is obtained when the quadratic assignment formulation of the problem is carried out using the weighted-average flow matrix. If F^a is the flow matrix corresponding to the scenario *a*, then the weighted-average flow matrix ($\overline{F}$) can be written as

$$\overline{F} = \sum_{a=1}^{A} \rho^a F^a$$

Now the following remark shows that both the formulations resulted in the same objective function.

Remark 6.1. The quadratic assignment formulation using the weighted average flow matrix is same as the above formulation.

Proof: An element of the matrix F^a is same as $f^a(j,k)$. Let an element of the matrix $\bar{F}$ be $\bar{f}_{jk}$. This element can be written as

$$\bar{f}_{jk} = \sum_{a=1}^{A} \rho^a f^a(j,k)$$

Substituting the expression of $f^a(j,k)$ in above equation, we get

$$\bar{f}_{jk} = \sum_{a=1}^{A} \rho^a \sum_{i=1}^{n} \sum_{b=1}^{u_i} D_i^a q_{ib} y_{ib}(j,k)$$

$$= \sum_{i=1}^{n} \sum_{b=1}^{u_i} \sum_{a=1}^{A} \rho^a D_i^a q_{ib} y_{ib}(j,k)$$

Now, taking the expected value on both sides of equation 6.1, we obtain

$$E(z) = \sum_{j=1}^{m} \sum_{k=1}^{m} \sum_{r=1}^{m} \sum_{s=1}^{m} E(f_{jk}) d_{rs} x_{jr} x_{ks}$$

where $E(f_{jk}) = \bar{f}_{jk}$

Substituting the expression of $\bar{f}_{jk}$ in above objective function, we obtain

$$E(z) = \sum_{j=1}^{m} \sum_{k=1}^{m} \sum_{r=1}^{m} \sum_{s=1}^{m} \sum_{i=1}^{n} \sum_{b=1}^{u_i} \sum_{a=1}^{A} \rho^a D_i^a q_{ib} y_{ib}(j,k) d_{rs} x_{jr} x_{ks}$$

$$= \sum_{i=1}^{n} \left[\sum_{j=1}^{m} \sum_{k=1}^{m} \sum_{r=1}^{m} \sum_{s=1}^{m} \sum_{b=1}^{u_i} q_{ib} \, y_{ib}(j,k) \, d_{rs} \, x_{jr} \, x_{ks} \right] \sum_{a=1}^{A} \rho^a \, D_i^a$$

$$= \sum_{i=1}^{n} \alpha_i \sum_{a=1}^{A} \rho^a \, D_i^a$$

QED

For every scenario, flow matrix calculation is essential for the method used by Rosenblatt and Kropp (1992) and hence as many number of flow matrixes as of the scenarios are to be estimated in addition to the weighted average flow matrix. But in the method suggested in this work, only one flow matrix is required to be computed corresponding to the expected demand.

6.2.3 Stochastic Case

Generally, production orders are despatched from the production control section. It is assumed that these orders arrive at the section independently. (Even though the demand of each item depends on the final product on which it is used, the order for making the item considers several factors such as availability of item in inventory, availability of the facility required for processing, availability of material required for producing the item, etc. and hence it can be assumed that the control section takes orders for processing (despatching) independently.) The arrival process (It is assumed that the order arrives when it is considered for despatching related purposes.) of orders can be assumed as Poison process. If this is the case, various streams of Poisson processes corresponding to the different items are superposed at the control section, then decomposed and distributed to various facilities. In this case the movement of material handling device can be modelled as a Markov process as explained in a previous chapter (the corresponding model is called SMMHP1). The discrete time Markov Chain described in the model of SMMHP1 defines a term called Relative Transition Matrix (RTM). If an element of this matrix is represented as r_{jk}, then it can be interpreted as the relative number of transits made by material handling device from facility j to facility k. It also denotes the relative rate of flow from facility j to facility k. The following remark shows that the relative rate of flow can be used in the quadratic assignment modelling of layout design.

Remark 6.2: The use of relative flow instead of actual flow in the quadratic assignment modelling does not change the quality of solution.

Proof: Suppose that the total flow between facilities is T and the corresponding relative transition matrix is $[r]$, then the real flow matrix is given by

$$[f] = T[r]$$

Now, the equation 6.1 can be written in terms of the relative flow as

$$\text{Minimise } z = \sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{r=1}^{m}\sum_{s=1}^{m} T\, r_{jk}\, d_{rs}\, x_{jr}\, x_{ks}$$

$$= T\sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{r=1}^{m}\sum_{s=1}^{m} r_{jk}\, d_{rs}\, x_{jr}\, x_{ks}$$

$$\text{Let } z' = \sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{r=1}^{m}\sum_{s=1}^{m} r_{jk}\, d_{rs}\, x_{jr}\, x_{ks}$$

The objective function z is minimised when z' is minimised. QED

That is, when the demand of various items to the despatching section is a stochastic process (Poisson process), the layout design problem can be formulated as a QAP using the relative transition matrix. The data required in this case is the Poisson demand rate and the route of each of the items. For the given products, the items (parts, subassemblies and final assembly) to be processed are known and hence the routes (sequence of machines for processing) of the items are known and fixed during the period of the layout being used. The Poisson demand rate is the mean demand of an item per unit time. The time unit can be day, week or month. There is a chance of error in estimating the demand rate. It is interesting to test the robustness of this approach against various errors in estimation of demand rate. The experimentation involves generation of random route sheets and demand rates. Some aspects related to them are discussed here.

Route sheet and demand rate

Sample route sheets and demand rates for a production shop consisting of 8 machines which process 20 items are given below. Three such route sheets and demand rates are provided. The route sheet is shown as a 20 × 8 matrix. The rows of the matrix represent the items and the columns stand for machines. The demand rate is shown as vector of size 20. The route sheet and demand rate are the input to the layout-planning problem. The flow matrix can be prepared using the demand and the route (machine sequence) the item has to follow. Several items have same machine sequence and a single item whose demand equal to the demand of all items having the same machine sequence can represent all these items. Hence the number of items represented in the route sheet will be less than the actual number of items.

Route sheet no. 1

0	2	5	0	0	1	4	3
5	6	0	4	2	1	3	7
6	4	8	3	1	7	5	2
8	3	4	2	1	7	5	6
0	2	0	0	0	1	0	3
0	0	2	0	1	4	3	0
2	0	1	4	3	5	0	0
1	0	5	0	0	2	4	3
4	1	0	3	6	5	2	0
5	3	4	6	8	7	2	1
1	8	5	7	2	6	3	4
2	0	1	4	0	0	3	0
4	2	7	0	1	6	5	3
2	6	1	3	8	7	5	4
4	3	0	1	2	6	0	5
2	0	1	4	6	5	7	3
2	5	0	6	0	3	1	4
3	0	1	0	2	0	4	0
8	4	5	2	7	1	6	3
2	0	4	1	6	5	7	3
1	8	3	7	2	4	6	5
6	8	7	4	5	3	1	2

2	0	4	0	1	0	5	3
3	4	5	1	2	8	7	6
5	3	0	4	0	2	0	1
1	8	6	7	5	3	4	2
3	4	0	5	0	0	2	1
5	1	2	4	7	6	8	3
0	3	1	0	0	4	2	0
0	4	0	3	5	0	2	1

Route sheet no. 2

3	6	4	7	8	1	2	5
0	5	2	6	3	1	7	4
0	1	0	3	0	0	0	2
3	0	2	4	1	5	0	0
6	1	3	0	2	4	7	5
2	7	1	8	3	6	4	5
5	7	8	2	3	6	1	4
3	5	1	0	7	6	4	2
0	0	5	2	1	0	3	4
0	3	1	4	0	6	5	2
2	1	5	4	0	0	0	3
0	3	0	2	0	0	1	0
5	0	4	7	1	6	3	2
5	4	3	2	8	6	7	1
6	4	1	0	2	3	5	7
0	3	0	4	6	1	5	2
7	6	5	4	8	3	1	2
4	0	5	6	1	3	0	2
0	5	1	0	4	6	3	2
8	2	3	6	7	5	4	1
1	2	8	4	6	5	7	3
8	1	7	3	4	2	5	6
0	1	5	0	3	0	2	4
2	8	6	3	7	1	4	5
8	6	3	1	2	4	7	5
0	4	1	0	2	0	0	3
0	2	4	0	0	1	0	3
4	2	1	3	5	6	7	8
1	7	4	6	5	3	2	8
6	2	7	3	4	5	8	1

Route sheet no. 3

3	6	4	0	2	7	1	5
0	3	0	5	0	4	1	2
5	1	3	0	4	0	0	2
7	1	5	6	2	8	3	4
2	4	3	8	6	1	5	7
0	1	5	7	3	2	4	6
0	7	3	5	2	1	6	4
4	0	2	0	1	0	5	3
0	0	3	1	2	0	5	4
0	3	2	5	1	7	6	4
3	0	6	1	0	5	4	2

7	8	2	4	5	6	1	3
2	6	3	0	5	0	4	1
3	2	6	4	1	5	0	0
6	1	2	0	0	3	4	5
7	2	8	4	3	5	6	1
0	0	1	3	0	2	0	0
2	3	4	0	0	0	1	0
0	0	2	0	3	0	4	1
0	3	0	0	1	0	0	2
2	0	4	3	0	0	1	0
2	3	5	6	0	1	0	4
2	1	7	5	3	4	6	8
5	0	6	1	2	4	3	0
1	8	7	5	6	4	2	3
5	3	4	1	7	8	6	2
6	2	0	7	5	1	4	3
0	2	0	1	3	0	4	5
3	6	7	4	1	2	8	5
2	1	3	0	5	0	0	4

Demand rate no. 1

63 56 63 7 35 49 49 63 49 28 35 63 56 28 21 14 14 70 28 63

Demand rate no. 2

20 30 20 100 90 100 30 10 80 80 20 80 80 70 70 20 90 60 80 40

Demand rate no. 3

56 64 24 24 48 72 8 8 40 64 56 56 56 72 24 24 24 8 8 72

The simulation experiment is coded in MATLAB. The function used for generating route sheet ensures that, at least two machines are used for an item while randomly identifying the machines required for the item to process. A feature of the function used to generate the demand rate is that the total of all the demand rates are allowed to fall within certain range, which is, between 750 and 1250. The function can satisfy this condition if the number of items to be processed is between 4 and 200. The condition related with total demand rate is necessary to ensure that the plant is functioning near to the capacity. Otherwise, randomly generated demand may show that the plant is starving due to lack of sufficient materials to process or is over loaded with materials that it can process. The sum of the above demand rate vectors are 854, 1170 and 808 respectively. All these sums fall in the prescribed range.

6.3 EXPERIMENTATION

When the layout is designed based on the Poisson process, its sensitivity to variations in the demand rate of each item is discussed here. In other words an analysis of the effect of changes in estimation of demand rate on layout performance is being carried out. The method used for the test is as follows:

1. Randomly generate the route sheet of the various items to be processed.

2. Randomly generate the initial estimate of demand rate for various items and ensure that the total demand rate is not too low or too high. This can be achieved by randomly generating the demand rate in such a way that the total demand rate falls in a certain range. The initial estimate of demand rate represents the decision maker's estimate of Poisson demand rate.

3. For the above demand rate and route sheet, develop the discrete time Markov chain and compute the relative transition matrix. Using this matrix, solve for the optimum layout. Let this layout be called as i-layout.

4. To study the performance of the i-layout under various estimation errors (estimation error occurs as a result of actual demand rate being different from the initial demand rate estimated), identify the error factors. For instance, to have the estimation error bounded within the $\pm$ 50 percent, the error factor is (1 ± 0.5). If the initial estimate of the demand rate of an item is 8 units per unit time, the actual estimate can fall within (4,12) for this error factor. (The error boundary is obtained on multiplying the initial estimate by the error factor.) Similarly, for all the items the error boundaries are to be identified. Now, the actual demand rate is estimated by randomly generating the demand rate within the boundaries. The sum of the demand rate of all the items should satisfy the condition mentioned in step 2. That is, the total demand rate of all the items should not be too low or too high.

5. This actual demand rate corresponds to a route sheet and for these demand rate and route sheet, the relative transition matrix is computed. Using this relative transition matrix, determine the solution for the layout problem.

6. Then, compare the cost performance of the optimal layout determined in step 3 with that of the optimal layout determined in step 5. In both cases, the route sheet is the same, but the demand rate is different.

The measures of effectiveness used to evaluate the performance of the i-layout under the various conditions are (i) average percentage of cost difference, (ii) percentage of situations for which i-layout is optimum, (iii) maximum percentage of cost difference and (iv) robustness indicator. The different conditions are obtained by altering the variables such as number of machines (here machines means facilities), number of items, number of routes, demand rate of components, estimation error and the degree of 'unevenness' in estimation error. Some of the terms used require explanations which are given below.

The demand rate estimation error range represents the range within which errors in estimation of demand rate is created randomly. The larger error range suggests that the prediction of demand rate is less accurate. Conversely, it is obvious that for smaller error range the actual demand rate can deviate less from the initial estimation and hence the solution procedure will perform better.

Now the percentage of cost difference (*PCD*) can be computed as follows. There is an optimum layout for actual demand rate and let the corresponding material handling cost be C_a. There is a cost when i-layout is applied for the actual demand rate and let that cost be C_e. Then the percentage cost difference can be written as

$$PCD = \frac{C_e - C_a}{C_a}$$

The percentage of situations for which i-layout is optimum indicates the number of times *PCD* is equal to zero. That is, it shows the number of actual demand rate situations in which the i-layout and the optimum layout for that situations are the same.

Robustness indicator is the number of times the *PCD* is less than or equal to a fixed percentage. The notion of layout robustness is an acceptable criterion when variation in input data affects the layout design objective. Robustness of a layout is a measure of flexibility of layout to adapt to demand changes. It is defined as the number of times the layout has the material handling cost within a prespecified percentage of the optimal solution under different demand situations. A layout designer is usually interested in choosing a layout that has the highest frequency of being closest to the optimal solution even though it may not be optimal under any demand situation. The fixed percentage (or prespecified percentage) is usually specified by the layout designer.

Different conditions are incorporated in the experimentation by randomly generating the route sheet, initial estimate and actual estimate of demand rate for different number of machines and items to be processed. Another variable mentioned above is 'unevenness' in the estimation error which is measured as the mean square error (*MSE*) of actual demand rates from the initial estimate of demand rates. It is given by

$$MSE = \frac{\sum_{i=1}^{n} (r_i - R_i)^2}{n}$$

where n is the number of items processed in the production shop, r_i is the actual demand rate of item i and R_i is the initial estimate of demand rate. When error in estimation of demand rate occurs, it is not necessary that demand rate of all the items vary with equal proportion from the initial estimate. The variation may be random and measured using the *MSE*.

Through an experimental design, the effect of *MSE* on *PCD* is analysed. A simulation experiment is carried out for a production shop containing 8 machines with a layout configuration of 2 × 4 while the shop is producing 6, 12, 20 or 30 items. During experimentation, five route sheets and three initial estimates are generated randomly for each item. An error bound of 90 percentage is used for a wide variation while randomly generating the actual estimate of demand rate. For each initial estimate of demand rate, 30 actual estimates are generated within the prescribed error bound. So, for this purpose, 450 problems are solved for each number of items. A graphical analysis where *MSE* Vs *PCD* plot is prepared and a second degree curve fitting is done for each case. The plots for a production shop with eight machines are shown in figure 6.1 to 6.4. From the plot it can be seen that there is a slight increase in *PCD* as *MSE* increases. This type of variation need not necessarily be as prominent as its proximity to zero for higher *MSE.* This is because the layout corresponding to the initial estimate may be optimum/near optimum for the actual estimate, which has a high variation.

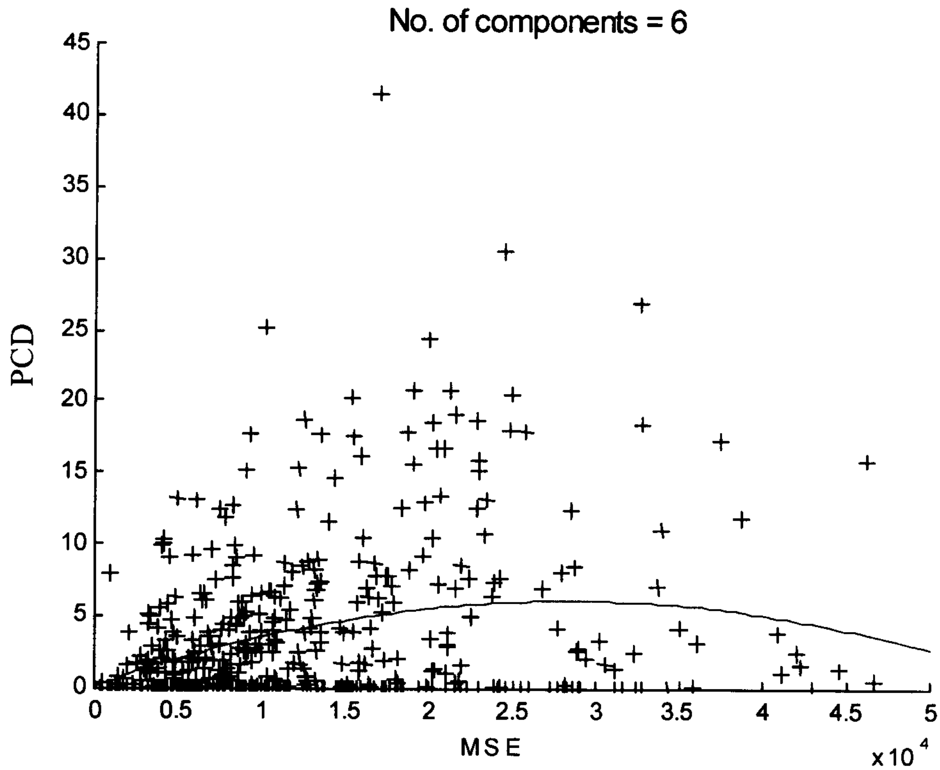

Figure 6.1 MSE Vs PCD graph for a shop with 8 machines and 6 components

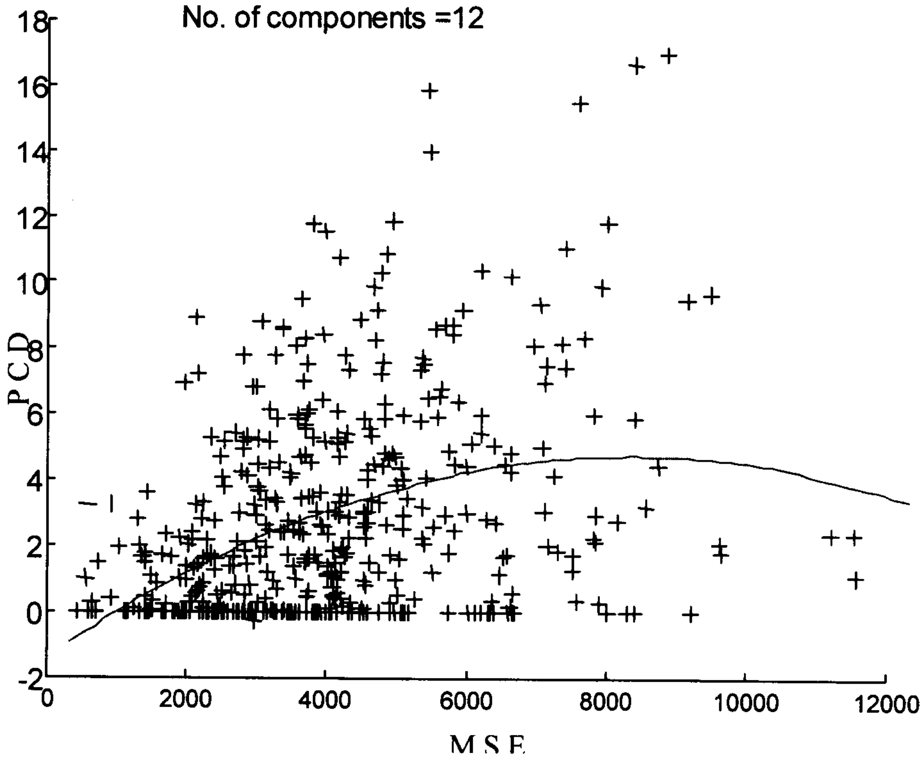

Figure 6.2 MSE Vs PCD graph for a shop with 8 machines and 12 components

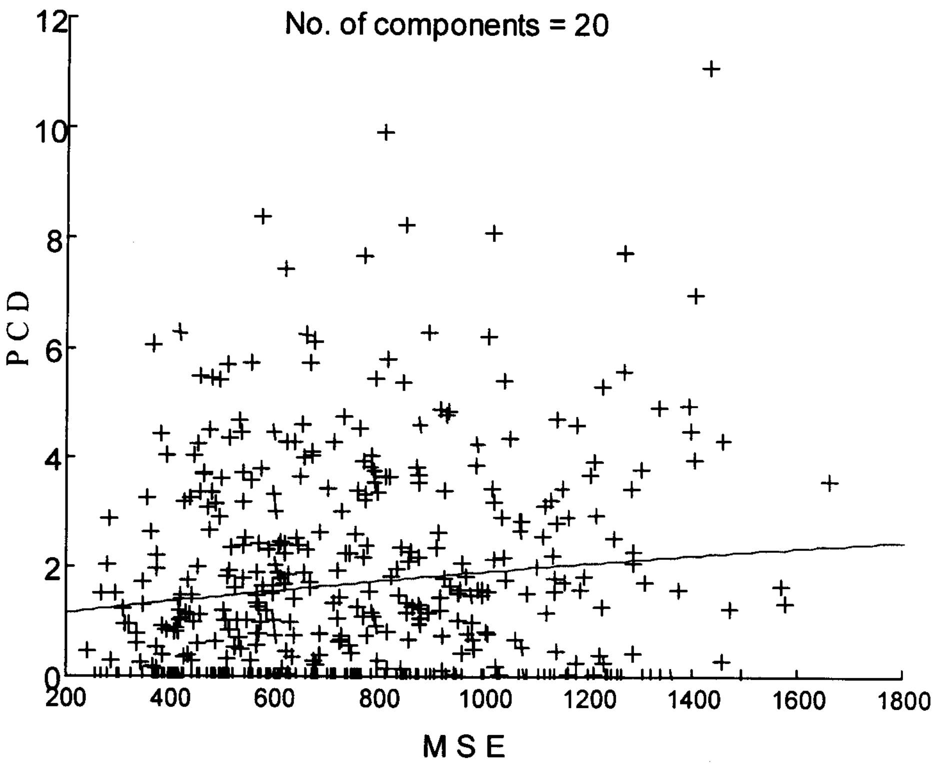

Figure 6.3 MSE Vs PCD graph for a shop with 8 machines and 20 components

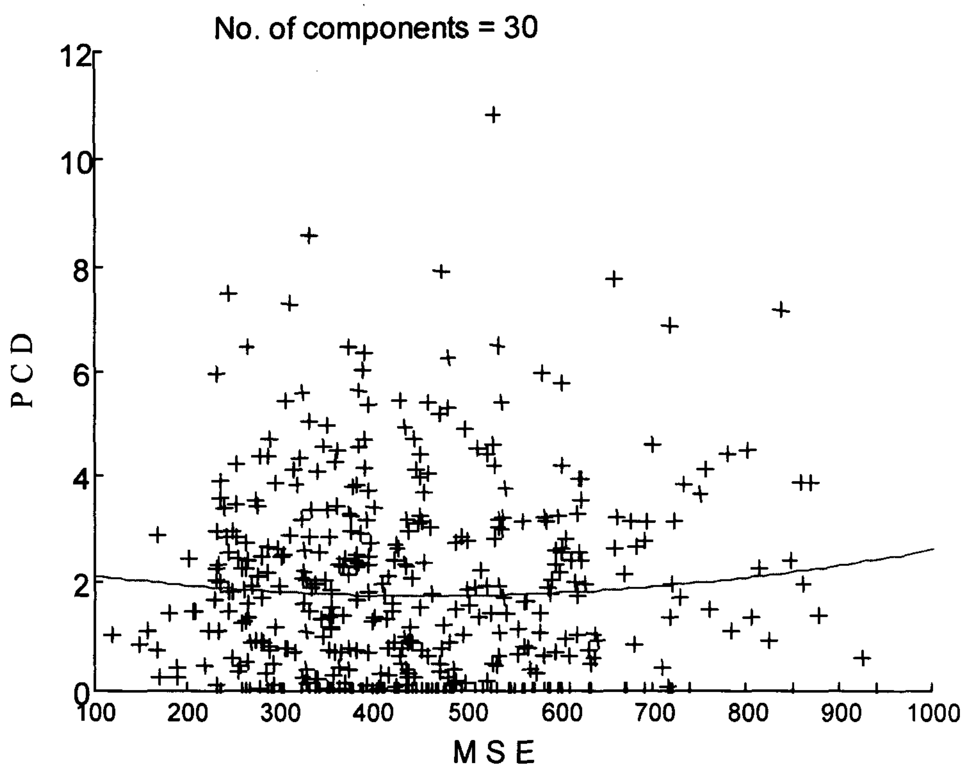

Figure 6.4 MSE Vs PCD graph for a shop with 8 machines and 30 components

Experiments are designed to study the performance measures at different error bounds. The number of machines considered are 6, 8, 9 and 10 and the corresponding layout configurations are 2 × 3, 2 × 4, 3 × 3 and 2 × 5 respectively. Each production shop of the above configuration is producing either 6, 12, 20 or 30 number of items. Number of route sheets and initial estimate of demand rates randomly generated for each shop are, 20 and 5 respectively. Actually for each randomly generated route sheet, 5 initial estimate of demand rates are also randomly generated. For each initial estimate of demand rate the various ranges of estimation error considered are $1 \pm 0.1, 1 \pm 0.3, 1 \pm 0.5, 1 \pm 0.7$ and 1 ± 0.9 . In each error range, for each initial estimate of demand rate, 30 actual demand rates are generated randomly. So a total of 2,40,000 (4*4*20*5*5*30 =2,40,000) different problems are solved.

Tables 6.1 to 6.4 show the results of the above simulation. These tables show the results for various combination of machines, items, routes, initial estimate of demand rates, actual demand rates and error bounds. Each cell corresponding to an error bound in a table is a measure of an effectiveness over 3000 problems. These problems correspond to different combinations of 20 route sheets, 5 initial estimate of demand rates and 30 actual demand rates. Certain features of the result shown in the cells of various tables are described below. The average *PCD* and the maximum *PCD* over 3000 problems are shown in each cell of the tables 6.1 & 6.2 respectively. Percentage of optimum situations are calculated for each initial estimate of demand rates (Corresponding to each initial estimate there are 30 actual estimates.) and it is averaged over different combinations of 20 routes and 5 initial estimates of demand rates. This value is given in a cell of table 6.3. For robustness index calculation, the prespecified percentage is 3. The robustness index shown in table 6.4 is also calculated in the same way as of the percentage of optimum situations. The observations noted while analysing these results are as follows:

Observations based on table 1.

1. In general, the results are encouraging. For the 2,40,000 problems considered, the average *PCD* of using the i-layout is 0.97.

2. As the error range becomes higher and higher the *PCD* increases. It can be seen that at lower error range the PCD is very small.

3. Production shops with higher number of machines have larger *PCD*. The *PCD* variation effect is predominant as the number of machines increases compared with the increase in the number of items.

Observations based on table 2.

1. The maximum *PCD* over all 2,40,000 problems is 57.95. It is seen from the earlier observation associated with table 6.1 that the average *PCD* for this many number of problems is 0.97. This low average indicates that, for

most of the problems for which i-layout is used, the *PCD* is much lower than 57.95. This can also be verified from the tables of percentage of optimum scenarios and robustness index.

2. The Maximum *PCD* is high at higher error range. It is in conformity with the second observation based on table 6.1.

3. The maximum *PCD* decreases with the increase in number of items processed in the shops. It can be noticed that the maximum *PCD* in all error range occurs for the lowest number of items for shops of different number of machines.

Observations based on table 3.

1. The average percentage of optimum situations over 2,40,000 problems is 53.47.

2. At higher error ranges, the percentage of optimum situations is less. As the number of items to be processed in a shop is high, the percentage of optimum situations is less.

3. The number of times the i-layout becomes optimum under demand variation for production shops with less number of machines is greater than that of shops with higher number of machines.

Observations based on table 4.

1. The average number of times the i-layout has the material handling cost within three percentage of the optimal solution under different demand situations (over 2,40,000 problems) is 89.71. This means that during most of the demand variation situations, the i-layout is very near to optimum. This is a very desirable character for a layout solution. It also indicates that the layout formulation procedure is very robust in nature.

2. The robustness of the layout is better when a shop of given number of machines processes large number of items. The robustness is poor at higher error range.

3. The robustness of the i-layout does not vary with respect to the number of machines of the shop.

Number of machines	Number of components	Error Range					
		1 ± 0.1	1 ± 0.3	1 ± 0.5	1 ± 0.7	1 ± 0.9	Average
6	6	0.03	0.20	0.64	1.31	2.32	0.90
	12	0.03	0.27	0.72	1.31	2.29	0.93
	20	0.02	0.17	0.46	0.95	1.62	0.64
	30	0.01	0.14	0.43	0.82	1.33	0.54
	Average	0.02	0.20	0.56	1.10	1.89	0.75
8	6	0.04	0.29	0.84	1.86	3.24	1.25
	12	0.03	0.29	0.85	1.77	2.99	1.19
	20	0.03	0.21	0.60	1.27	2.09	0.84
	30	0.01	0.17	0.52	1.00	1.70	0.68
	Average	0.03	0.24	0.70	1.47	2.50	0.99
9	6	0.04	0.23	0.71	1.57	3.09	1.13
	12	0.02	0.29	0.80	1.62	2.91	1.13
	20	0.03	0.22	0.64	1.24	2.09	0.84
	30	0.03	0.18	0.50	1.00	1.60	0.66
	Average	0.03	0.23	0.66	1.36	2.42	0.94
10	6	0.03	0.32	0.97	2.05	4.12	1.50
	12	0.04	0.36	1.02	2.03	3.71	1.43
	20	0.03	0.29	0.77	1.51	2.53	1.03
	30	0.03	0.23	0.64	1.28	2.08	0.85
	Average	0.03	0.30	0.85	1.72	3.11	1.20

Table 6.1 Average PCD

Number of machines	Number of components	Error Range				
		1 ± 0.1	1 ± 0.3	1 ± 0.5	1 ± 0.7	1 ± 0.9
6	6	1.65	6.30	12.56	26.57	33.03
	12	1.31	5.51	8.92	18.44	25.62
	20	0.98	4.23	6.15	9.35	12.78
	30	0.62	3.07	5.53	9.78	11.78
8	6	1.96	8.01	13.59	22.56	37.64
	12	1.40	5.18	9.34	16.44	25.10
	20	1.02	4.46	6.52	11.66	20.68
	30	0.62	2.40	5.74	7.60	13.55
9	6	2.03	6.81	11.68	17.60	32.32
	12	1.39	4.10	11.35	14.55	20.49
	20	1.02	3.12	5.64	8.72	13.55
	30	0.68	2.41	3.90	8.82	10.69
10	6	2.65	10.92	15.37	24.95	57.95
	12	1.91	4.80	11.71	19.34	26.27
	20	0.96	3.48	7.58	9.73	14.38
	30	0.76	3.05	5.30	8.16	13.86

Table 6.2 Maximum PCD

Number of machines	Number of components	Error Range					
		1 ± 0.1	1 ± 0.3	1 ± 0.5	1 ± 0.7	1 ± 0.9	Average
6	6	92.87	82.47	72.40	61.97	53.87	72.71
	12	89.23	72.07	59.93	49.90	40.97	62.42
	20	90.97	75.63	61.90	47.57	38.07	62.83
	30	91.80	73.33	59.57	48.20	39.03	62.39
	Average	91.22	75.88	63.45	51.91	42.98	65.09
8	6	89.77	73.87	59.03	45.30	36.03	60.80
	12	88.00	66.10	46.60	34.83	24.50	52.01
	20	85.80	65.93	49.80	33.43	24.27	51.85
	30	87.63	64.37	44.90	31.53	21.13	49.91
	Average	87.80	67.57	50.08	36.28	26.48	53.64
9	6	88.93	74.17	60.07	45.70	34.97	60.77
	12	88.37	62.83	46.10	34.33	23.50	51.03
	20	84.53	60.63	41.10	29.23	18.70	46.84
	30	83.27	61.93	41.90	28.00	18.70	46.76
	Average	86.27	64.89	47.29	34.32	23.97	51.35
10	6	88.03	66.27	51.23	38.37	27.73	54.33
	12	84.13	56.73	37.53	24.83	15.77	43.80
	20	79.67	50.60	32.33	19.43	12.20	38.85
	30	79.87	51.47	30.43	17.30	12.17	38.25
	Average	82.93	56.27	37.88	24.98	16.97	43.81

Table 6.3 Average percentage of optimal situations

Number of machines	Number of components	Error Range					
		1 ± 0.1	1 ± 0.3	1 ± 0.5	1 ± 0.7	1 ± 0.9	Average
6	6	100.00	98.67	91.57	83.97	74.40	89.72
	12	100.00	99.17	92.30	82.80	70.67	88.99
	20	100.00	99.93	97.43	90.47	78.73	93.31
	30	100.00	99.97	98.50	93.17	83.60	95.05
	Average	100.00	99.43	94.95	87.60	76.85	91.77
8	6	100.00	98.30	90.47	76.77	65.83	86.27
	12	100.00	99.37	92.07	76.87	60.67	85.79
	20	100.00	99.97	97.63	86.93	72.17	91.34
	30	100.00	100.00	98.87	93.73	79.90	94.50
	Average	100.00	99.41	94.76	83.57	69.64	89.48
9	6	100.00	98.93	92.23	80.43	66.50	87.62
	12	100.00	99.60	93.40	78.60	61.20	86.56
	20	100.00	99.97	97.83	88.40	72.27	91.69
	30	100.00	100.00	99.07	93.73	82.97	95.15
	Average	100.00	99.63	95.63	85.29	70.73	90.26
10	6	100.00	98.63	89.13	75.37	57.63	84.15
	12	100.00	99.23	90.50	74.10	53.70	83.51
	20	100.00	99.90	96.73	84.27	64.60	89.10
	30	100.00	99.97	98.67	90.70	73.87	92.64
	Average	100.00	99.43	93.76	81.11	62.45	87.35

Table 6.4 Average robustness index

These observations, in general, indicate the suitability of the layout formulation procedure. The statistical tests (ANOVA – two-way classification) of the data reveal that these observations are significant. The details of the statistical tests are provided in the Annexure A.

6.4 CLOSURE

A well-designed layout plays an important role on the efficient operation of the manufacturing system. The layout design problem is generally formulated as a QAP. One of the inputs, that is flow matrix, for layout design is generally assumed as available at the time of layout planning. But in practice the data available initially is the demand of products that is to be manufactured. Using this data and with the help of the route sheet, the flow matrix has to be prepared. The problem formulation provided in this work actually uses the demand of products and route sheet information along with the information from the bill of material.

Also formulations are provided when each product demand is a random variable, product mix is a random variable and when the arrival process of orders can be characterised as a Poisson process. The layout formulation shown for product mix case is superior to the existing models in terms of the computation effort. It has been seen that the layout formulation under random product demand is equivalent to a deterministic environment. Another finding is that under stochastic environment, in the place of flow matrix, the relative transition matrix can be used for the problem formulation.

The sensitivity of the layout performance under variation in estimate of the Poisson demand rate is analysed using ANOVA method. The results of the analysis are generally encouraging and they suggest the suitability of the formulation.

CHAPTER 7

MANUFACTURING CELL FORMATION UNDER PROBABILISTIC PRODUCT MIX

7.1 INTRODUCTION

A cellular manufacturing system (CMS) is one in which production shop is partitioned into production cells, each dedicated to the production of part families with similar processing requirements. Among the problems of CMS, cell formation is considered to be the foremost problem. The main objective of cell formation is to construct machine cells and identify part families so as to minimise inter cell traffic of parts.

Often, while designing a CMS deterministic environment is assumed. But there is a considerable scope for changes in environment during the effective life of the CMS. The CMS performs best under the designed environment. Changes in environment badly affect the performance of the system. Changes in environment are generally reflected in the demand of the items, which are produced in the CMS. The variation in demand can be characterised probabilistically. Two ways of characterisation of this demand are discrete random product mix and random product demand. Certain literatures, which use these types of characterisation and use the associated characteristic measures for designing CMS are discussed here. Harhalakis *et al.* (1994) and Seifoddini (1990) have demonstrated the effect of variation in demand on the performance of the CMS. Seifoddini (1990) considers the random nature of the product mix and suggests a CMS design by assigning probabilities to discrete product mixes and the associated machine part incidence matrix. Harhalakis *et al.* (1994) present a CMS design model for random product demand. Their work explicitly considers the statistical nature of independent demand and the capacity of the system resources.

The inputs for the design of CMS described here are machine sequence for the various items (component, subassembly or assembly) used in the product, product structure and demand of product. The formulation of the CMS design problem using these inputs is discussed here.

7.2 OBJECTIVE FUNCTION FOR CELL FORMATION

The CMS design problem consists of decompositioning of the manufacturing system consisting of a set of machines M={M_1, M_2, ..., M_m} into a set of cells C ={C_1, C_2, ..., C_w} so that the total inter cell traffic of parts is minimised. The shop characteristics are described below with the following symbols.

n – number of products manufactured in a CMS

D_i - demand of the product i, i=1,2, ..., n.

ϕ_{i1} - finished product (final assembly) i

u_i - number of make items contained in the bill of material of the product ϕ_{i1}. If the entire items required for the product ϕ_{i1} are not manufactured in house, the number of items in the bill of material of the product is greater than u_i. The product i, which is the final assembly, is represented in the bill of material as ϕ_{i1}.

ϕ_{ib} - a make item required for making the product ϕ_{i1}; $b = 2,3, \ldots, u_i$.

q_{ib} - quantity of item ϕ_{ib} required for making one unit of the product ϕ_{i1}.

θ_{ib} - number of operations required for the processing requirements of make item ϕ_{ib}.

$M_{(ib,k)}$ - machine required for the operation k of the item ϕ_{ib}, $k = 1,2, \ldots, \theta_{ib}$.

$C_{M_{(ib,k)}}$ - cell to which machine $M_{(ib,k)}$ is belonging

β_{ib} - inter cell traffic generated while making one unit of item ϕ_{ib}

β_i - inter cell traffic due to the final product ϕ_{i1} or any of its make components $\phi_{i2}, \phi_{i3}, \ldots, \phi_{iu_i}$. This is the total inter cell traffic generated as a result of the manufacturing of product i as per the details contained in the bill of material.

T_i - total inter cell traffic due to product i by considering its demand.

T – total inter cell traffic due to the entire product manufactured by the system

$\sigma_{M(ib,k) C_{M(ib,k+1)}} = 1$ if machine $M_{(ib,k)}$ is not belonging to the cell $C_{M_{(ib,k+1)}}$

$= 0$ otherwise

The inter cell traffic β_{ib} can be written as

$$\beta_{ib} = \sum_{k=1}^{\theta_{ib}-1} \sigma_{M(ib,k) C_{M(ib,k+1)}}$$

$$\beta_i = \sum_{b=1}^{u_i} q_{ib} \beta_{ib}$$

Now the T_i and T can be written as

$$T_i = D_i \beta_i$$

$$T = \sum_{i=1}^{n} D_i \beta_i$$

For a given cell configuration, β_i is a constant. Assume that the various products the system manufactures remain the same, but the demand of the product varies as represented as a random variable. That is, D_i can be a random variable.

(As stated earlier, changes in environment are reflected in this variable.) It is also assumed that, the product structure and sequence of machines required are fixed for each product. Another assumption is that sufficient production capacity to meet the demand is available. The objective function T can be used when the demand is deterministically known. The next section describes the application of this objective function under probabilistic environment.

7.3 PROBABILISTIC ENVIRONMENT AND CELL FORMATION

When the demand is a random variable Harhalakis *et al.* (1994) shows that minimisation of mean inter cell traffic is equivalent to minimising the traffic that corresponds to the mean demand of the finished products. When the product mixes of the system is a discrete random variable, Seifoddini (1990) suggested a method, which may not provide optimum result. A methodology is suggested in this chapter for optimum solution, which is called approach A and the method of Seifoddini (1990) is called approach B. The remark 7.1 given below shows that approach A provides optimum solution. Also approach A is simple compared to approach B.

Consider a probabilistic environment where there is a possibility of occurrence of various product mixes. A product mix corresponds to a scenario (state). The shop characteristic required (in addition to the above) to explain this environment is given below.

A – number of scenarios

D_i^a - demand of product in scenario a, $a = 1,2, \ldots, A$.

ρ^a - probability of occurrence of scenario a.

For this type of environment the approach A suggests the minimisation of inter cell traffic for a scenario with expected demand. The method is described below.

T_i^a - total inter cell traffic due to product i by considering its demand in the scenario a.

T^a - total inter cell traffic due to the entire product manufactured by the system in the scenario a.

T_e - expected inter cell traffic

$$T_i^a = D_i^a \beta_i$$

$$T^a = \sum_{i=1}^{n} \beta_i D_i^a$$

$$T_e = E\left(\sum_{i=1}^{n} \beta_i D_i^a\right)$$

$$= \sum_{i=1}^{n} \beta_i E\left(D_i^a\right)$$

T_e is representing the inter cell traffic due to a scenario which contain expected demand. That is, in this scenario demand of item i is $\sum_{a=1}^{A} \rho^a D_i^a$. To minimise the inter cell traffic, in this type of environment, it is recommended to design a CMS best suitable for a scenario containing the expected demand. The methodology described by Seifoddini (1990) (Approach B) is described below.

In this method for each scenario a best design is identified. Now the decision maker has as many numbers of alternatives as the number of scenarios. The inter cell traffics, when each alternative is used under various scenarios are determined and the corresponding expected cost is calculated. An alternative, which has minimum expected inter cell traffic is selected as the best design. The method is symbolically described below.

X_j - best design for scenario j, $j = 1,2, \ldots, A$.

T_{ja} - inter cell traffic when design X_j is used for scenario a, $j = 1, 2, \ldots, A$, $a = 1, 2, \ldots, A$. T_{ja} is calculated using the expression of T for the appropriate design.

T_j - expected inter cell traffic for the alternative design X_j

$T_{\min}$ - minimum of T_j

$$T_j = \sum_{a=1}^{A} \rho^a T_{ja}$$

$$T_{\min} = \min(T_j)$$

Now it is necessary to ascertain which approach produces the best result. The remark given below show that, approach A produces the best result.

***Remark** 7.1*: The expected inter cell traffic determined in approach A is less than or equal to the expected inter cell traffic determined in approach B. That is, $T_e \leq T_{\min}$.

Proof : To prove this remark Jensen's inequality theorem [DeGroot (1970)] is used. The theorem is as follows: Let T be a convex function on the interval (g,h) and let D be a random variable such that $p\{D \in (g,h)\} = 1$ and the expectation $E(D)$ and $E[T(D)]$ exists. Then

$$E[T(D)] \geq T[E(D)]$$

This inequality is applicable if T is a convex function. Here $T = \sum_{i=1}^{n} D_i \beta_i$ which is a linear function. A linear function is always convex.

Now the terms of this inequality can be correlated with T_e and $T_{\min}$. T_e is equivalent to $T[E(D)]$ and $T_{\min}$ is equivalent to $E[T(D)]$. This means $T_e \le T_{\min}$. QED

To prove this numerically the algorithm used for the CMS design should be an efficient one, which should provide the optimum CMS configuration. The genetic algorithm search methodology can be suitably used to identify near optimum solution for this type of problem. Hence a genetic algorithm procedure available in Godwin and Madhusudanan Pillai (2002), and Godwin (2002) is used with certain modification. The numerical verification of this method is available in Godwin (2002). But the objective function provided in these literatures is somewhat different from the objective function given in this work. The modelling also has some difference as these literatures consider every item to be processed in the system as a product as used in the Seiffoddini's (1990) work.

Earlier researchers do not consider demand of product as input to the CMS design. In this case also the approach A can be used with a modification of the random variable D_i. The value D_i can take, is based on the presence or absence of item i in the product mix. It can assume value 1 if item i is present in the product mix or 0 if not present. Also Seiffoddini (1990) has not considered demand as input. His input consists of machine part incidence matrix, which does not contain the order of machines used for processing the item. The objective function has some differences when such input is used.

When machine part incidence matrix, which does not contain the order of machines, is used for processing of items, the determination of inter cell traffic is given below. In this respect a few symbols are used which are defined below.

η_{ib} - set of cells in which machines required for the processing of the make item ϕ_{ib} belongs. $i = 1,2, ..., n$ and $b = 2,3, ..., u_i$

$|\eta_{ib}|$ - size of the set η_{ib}

Now the inter cell traffic due to an item ϕ_{ib} is

$$\beta_{ib} = |\eta_{ib}| - 1$$

and due to a product ϕ_i is

$$\beta_i = \sum_{b=1}^{u_i} q_{ib} \left(|\eta_{ib}| - 1 \right)$$

This expression of β_i is to be used in the total inter cell traffic equation of *T*.

Even though the solution of the design methodology suggested here is not an optimum design in any of the scenarios, it can be called a flexible design. (This definition is in tune with the definition of flexible layout design suggested by Rosenblatt and Kropp (1992)). The design may not be optimum in any of the scenarios because the design corresponds to the expected scenario. (The scenario with expected demand is called expected scenario.) If this design is used under various scenarios and the expected traffic is calculated, it can be seen that this expected traffic is less than or equal to the expected traffic due to an optimum design of any of the scenarios. This is a reason why it is called a flexible design. Hence it is advisable to envisage a number of scenarios with probabilities, that the system will be subject to, at the time of design of the system and use the method explained here to obtain a flexible design. On the average, the performance of this design will be better than a design based on a particular demand scenario. The robustness (flexibility) of this design can be tested with a sensitivity analysis. This analysis involves studying the performance of this design under variations in demand and probability estimates, which is not done here.

It was stated earlier that the approach A is simple. This is due to the fact that in approach B, as many number of designs as scenarios have to be prepared and then the best design, based on the expected inter cell transfer, is to be identified. In approach A only a single design corresponding to the expected demand is prepared. A genetic algorithm has been used for preparation of cell and part family formation.

7.4 GENETIC ALGORITHM

A genetic algorithm is a random search technique for global optimisation in a complex search space. It was originally inspired by an analogy with the process of natural evolution. In evolution, the problem that each species faces is one of searching for beneficial adaptations to a complicated and changing environment. The knowledge that each species has gained is embodied in the make-up of the chromosomes of its members. Genetic operations are used in the search for beneficial adaptation when the parents reproduce and evolution takes place. Likewise, a genetic algorithm combines the survival-of-the-fittest among solution structures with a structured, yet randomised, information exchange and creates offsprings. The offsprings are new sets of solutions using bits and pieces of the fittest of the old solution structures and it displaces weak solutions during each generation [Gupta and Rajamani (1996)]. Robust performance, and balance between efficiency and efficacy necessary for survival in many different environments are expected from genetic algorithms.

Genetic algorithms work by maintaining a population of possible solutions to the problem. A candidate solution (a point in the search space) is represented by a sequence of genes and is known as a chromosome. A judiciously selected set of chromosomes is called a population and the population at a given time is a generation. The algorithm begins with an initial population of solutions whose potential as solution is determined by its fitness function, which evaluates a chromosome with respect to the objective function of the optimisation problem at hand. The population size remains from generation to generation and has a significant impact on the performance of the genetic algorithm [Gupta *et al.* (1996)]. The next generation of population is obtained by first selecting parent structures (chromosomes) in proportion to their fitness from the current generation. This operation is called reproduction. By selecting fit parents it is hoped that desirable solution characteristics will be repeated in future generations while undesirable characteristics die out. The fit chromosomes are subjected to operations such as cross over and mutation. The chromosomes resulting from these three operations are often known as offsprings or children. The fitness of the children is measured and the fittest replaces the old population, and thus new generation is created. This procedure repeats until a stopping criterion is reached.

The genetic algorithm design issues such as representation, initialisation, fitness function, reproduction, cross over and mutation are discussed in the following sections.

7.4.1 Representation

Representation by coding a solution in the form of strings (chromosomes) plays a key role in the development of genetic algorithm. The bits (genes) in the strings could be binary, integers or a combination of characters. In the machine cell-part family formation problem considered here, each gene represents a cell number and the positioning of the gene in the chromosome represents the machine number [Gupta *et al.* (1996), and Moon and Kim (1999)]. The length of the chromosome represents the number of machines considered. For example, when the string '32123' represents a three-cell solution with the following machines in each cell:

Cell = 1: machine 3

Cell = 2: machines 2, 4

Cell = 3: machines 1, 5.

7.4.2 Initialisation

The initialisation step in genetic algorithm is to create an initial population. The initialisation process can be executed with either a randomly created population or a well-adapted population. In this work, an initial population of desired size is generated randomly. The appropriate population size, the length of the chromosome (number of machines), and the number of cells are to be chosen according to the design requirements. The population size depends on the length of the chromosome. In general, the population size should be at least equal to the length of the chromosome. The chromosomes in the population are decoded and it is evaluated according to the objective function.

7.4.3 Evaluation or Fitness Function

The purpose of evaluation is to measure the fitness of candidate solutions in the population with respect to the design objectives. The fitness values are used to select parent solutions to create the next generation of solutions. The specific form of evaluation function depends on the design objective being considered. The fitness of an individual solution dictates the number of copies of that solution in the mating pool. The more copies an individual receives, the greater is the probability that the characteristics will be repeated in subsequent generations.

In a genetic algorithm a fitness function value is computed for each string in the population with the objective of finding a string with maximum value. The objective of the cell formation problem is the minimisation of the inter cell traffic. However, genetic algorithm works with maximisation functions. Thus, it is necessary to map this objective function to a fitness function. The following transformation is applied here [Goldberg (1989)]:

$$F_i = T_{\max} - T_i, \text{ where } T_i < T_{\max}$$

$$= 0, \text{ otherwise}$$

where T_i is the objective function value of a string i and $T_{\max}$ is the largest objective function value in the current generation. Now, the maximisation of the fitness value indirectly means the minimisation of the objective function value.

Some of the chromosomes produced by the genetic algorithm may be illegal (infeasible) as the objective function has constraints. The illegal chromosomes are repaired to satisfy the constraints.

7.4.4 Selection and Reproduction

A generation of the genetic algorithm begins with reproduction. The reproduction operator is used to select individuals from the current population to become parents of the next generation. Parents are selected according to their fitness value. Strings with higher fitness values are selected for crossover and mutation using the selection process of remainder stochastic sampling without replacement [Goldberg (1989)]. According to this method the probability of selection is calculated as follows:

$$p_s = \frac{F_i}{\sum F_i}$$

where F_i is the fitness function of string *i*. Then the expected number of individuals for each string is e_i which is calculated as follows:

$$e_i = p_s.ppsz$$

where *ppsz* is the population size.

In the process of creating a next generation, the genetic algorithm creates an intermediate population of size equal to *ppsz*. In the intermediate population, each string from the old population has copies equal to the integer part of the e_i values, and the fractional parts of the e_i are treated as probabilities. If the intermediate population is not filled up, one by one, weighted coin tosses (Bernoulli trials) are performed using the fractional parts as success probabilities. For example, if the expected value (e_i) of a string is 2.3, the intermediate population will receive two copies of the string. This is done for all the strings of the old population till the intermediate population gets filled up. If the intermediate population is not filled up completely, the fractional part of the expected value of a string is treated as probabilities and by performing Bernoulli trials the string is copied to the intermediate population.

7.4.5 Crossover

The crossover operator creates new potential solutions by exchanging a portion of the parent solutions available in the intermediate population. The crossover is done with a probability called crossover probability. In this way, child retains some of the features of the parent solution. The chromosomes to be crossed are selected using stochastic sampling with replacement. The number of chromosomes selected is equal to population size and the crossing points are identified randomly. Here a two-point crossing technique is used. The probability of selection in the stochastic sampling with replacement is calculated as follows:

$$p_s = \frac{F_i}{\sum F_i}$$

where F_i is the fitness function for a string *i*. Calculate the cumulative value of the probability. Generate a random number and identify the chromosome corresponding to the random number. Similarly identify the next chromosome and based on the probability of crossover these two chromosomes are crossed. The crossing is explained with an example below. Let the parent identified be

Parent 1: 1 3| 2 1 2 1 3| 2 2 3

Parent 2: 2 1| 3 2 3 2 1| 2 1 3

and the randomly identified crossing points be 2 and 7. The crossing points are shown in the parent given above. The crossover operator generates two children given below.

Child 1: 1 3 3 2 3 2 1 2 2 3

Child 2: 2 1 2 1 2 1 3 2 1 3

7.4.6 Mutation

The purpose of mutation operator is to rejuvenate the search and extend it into previously unexplored areas of the solution space. Mutation prevents the value of any parameter from remaining unchanged forever. It is done with low probability called probability of mutation. For the mutation purpose two random integers r_1 and r_2 are generated such that $1 \le r_1 \le m$ and $1 \le r_2 \le w$ where w is the maximum number of cells specified and m is the number of machines. In the mutation operation the cell number corresponding to the machine specified by r_1 is replaced with the cell number r_2.

7.4.7 Replacement Strategy

After genetic operations new strings are created. Poor performing offsprings are replaced in the new generation with a replacement strategy. The offsprings are evaluated with respect to the evaluation function. The goal of the replacement strategy is to create generations of solutions that, on an average, outperform the previous generation. This is accomplished by restricting admittance to the new population to only those children that are better than members of the current population.

In this work, the fitness values of the chromosomes of the old population are compared with the chromosomes of the new population. The fittest among the two will become the old population for the next generation.

7.4.8 Terminating the Genetic Algorithm

The genetic algorithm iterates, and as the process proceeds, the generation includes chromosomes with higher fitness function values. Termination criterion is used to stop the iteration. A single criterion or a set of criteria can be used to halt the genetic algorithm. The maximum number of generations is the termination criterion used in this work.

7.4.9 Algorithm

The abbreviations used in the algorithm given below are as follows:

ppsz – Population size

xgen – Maximum number of generation

p_{cs} - Probability of crossover

P_{mut} - Probability of mutation

$POP(g)$ - Population in generation g

Step 0. Set the values for *ppsz*, *xgen*, p_{cs}, P_{mut} and constraints related parameters. Initialise the generation counter, $g = 0$

Step 1. Randomly generate an initial population, *POP(g)*.

Step 2. Evaluate the fitness of each solution with respect to the design evaluation function.

Step 3. Select individuals from the current population to become parents of the next generation according to their fitness values.

Step 4. Create children by applying the genetic operators of crossover and mutation.

Step 5. Evaluate the fitness of each child with respect to the design evaluation function.

Step 6. Increment the generation counter, $g = g + 1$.

Step 7. Form the new generation, *POP(g)*, by replacing the weak solutions in population $POP(g-1)$ with more fit children.

Step 8. If $g < xgen$ go to *step 3*. Otherwise stop.

The genetic algorithm is used only for cell formation. The part family is formed using part assignment method. An item is assigned to a cell where maximum number of machines required for the processing of the item is present. Ties are broken arbitrarily.

7.5 ILLUSTRATIVE EXAMPLE

A small order example is presented to demonstrate the CMS design method suggested for a manufacturing system subjected to discrete probabilistic product mix. The system considered produces five final assemblies and the total number of make items is 21. Seventeen functionally unique machines are used to produce these items. Since alternative production routings are not accounted, only functionally unique machines are considered in the example.

The sequence of operations for each make item is presented in the part machine incidence matrix of table 7.1. This table provides the order of operations in the part production routing. The part numbers of the end items are 1, 2, 3, 4, 5 and the part numbers of the remaining make items are 6 – 21. The machines are numbered from 1 – 17.

For simplicity, without loss of generality, each final assembly has single level bill of material and each make component is used with a quantity of one per unit parent product. Table 7.2 presents the relationship between the five final assemblies and their make components.

Three scenarios are considered for the manufacturing system. The end items to be manufactured in each scenario, its demand and the probability of occurrence of these scenarios are presented in table 7.3. The demand of the items in the expected scenario is presented in table 7.4. The expected scenario contains items with expected demand.

Part	Machine																
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	.	5	.	.	3	.	.	2	.	.	.	.	4	.	.	1	.
2	.	.	.	.	.	6	.	.	.	.	3	.	.	4	.	.	2
3	1	.	3	.	.	.	.	.	.	.	4	2	.	.	.	.	.
4	.	.	.	4	.	.	.	.	1	3	.	.	.	.	2	.	.
5	.	.	.	3	.	.	.	.	.	2	.	.	.	.	1	4	.
6	4	.	3	.	.	.	2	.	.	.	.	1	.	.	.	.	.
7	.	.	.	.	1	.	.	3	.	.	.	.	2	.	.	.	.
8	.	.	.	.	.	4	.	.	.	.	3	.	.	2	.	.	1
9	.	.	1	.	.	.	2	.	.	.	.	3	.	.	.	.	.
10	.	1	.	.	2	.	.	4	.	.	.	.	3	.	.	.	.
11	.	.	.	4	.	.	.	.	2	.	.	.	.	.	1	3	.
12	4	.	1	.	.	.	2	.	.	.	.	3	.	.	.	.	.
13	.	.	.	2	.	.	.	.	3	1	.	.	.	.	4	.	.
14	1	.	3	.	.	.	.	.	.	.	4	2	.	.	.	.	.
15	.	3	.	.	2	.	.	1	.	.	.	.	4	.	.	.	.
16	.	.	.	1	.	.	.	.	2	3	.	.	.	.	.	4	.
17	.	.	.	.	.	2	.	.	.	.	.	.	.	3	.	.	1
18	3	.	2	.	.	.	1	.	.	.	4	.	.	.	.	.	.
19	.	3	.	.	4	.	.	2	.	.	.	.	1	.	.	5	.
20	.	.	2	.	.	.	3	.	.	.	.	1	.	.	.	.	.
21	3	.	.	4	.	.	.	.	.	2	.	.	.	.	1	.	.

Table 7.1 Part-machine incidence matrix

End Product	Make items
1	6 11 16
2	7 12
3	8 13 17 20
4	9 14 18
5	10 15 19 21

Table 7.2 End product and its components

Scenario	1	2	3
Product	1,2,4,5	1,2,3,4,5	1,2,3,4,5
Demand	10,15,8,20	7,11,12,15,5	5,8,7,20,10
Probability	0.3	0.4	0.3

Table 7.3 Demand of end products at various scenarios with probability

Product	1	2	3	4	5
Expected Demand	7.3	11.3	6.9	14.4	11

Table 7.4 Demand of product at expected scenario

As per the methodology suggested in this work the CMS design must be prepared for the expected scenario. The inputs for the design are machine sequence required for each item in the expected scenario and the expected demand. Generally the expected scenario will contain every item, which has demand in the scenarios considered. Table 7.1 contains the machine sequence of all such items. Hence table 7.1 is one of the inputs and the expected demand of make items is the other. The latter can be determined from the table 7.4 with the help of table 7.2. The problem is solved for 3 cells using genetic algorithm. The inter cell traffic for the design is 40.3. The machines in each cell and the corresponding part families are given in table 7.5.

The genetic algorithm parameters for solving this problem are discussed here. The probability of crossover and mutation are chosen as 0.5 and 0.2 respectively. The population size is chosen as 25. To ensure best solution for the problem the genetic algorithm is applied three times. The number of runs and the termination criterion of maximum number of generations at each time are 10 runs of 100 generations each, 10 runs of 500 generations each and 5 runs of 1000 generations each. From the total 25 runs the number of designs obtained is 25. A best design from the 25 is selected as the solution for the problem.

Cell No.	1	2	3
Machines	1,3,6,7,11,12,14,17	2,5,8,13	4,9,10,15,16
Parts	2,3,6,8,9,12,14,17,18,20	1,7,10,15,19	4,5,11,13,16,21

Table 7.5 Machine cell and part family for the expected scenario

To compare the method with the method of Seifoddini (1990) the solution obtained for the same problem using his method is given in the Annexure B.

7.6 CLOSURE

The manufacturing cell formation under random product mix has been discussed. An objective function formulation for cell formation has been suggested which uses finished product demand, product structure and sequence of machines used for processing of items contained in the product structure of finished product. A method is suggested for random product mix situations, which considers minimisation of inter cell traffic for a scenario, which contains expected demand. The method is simple compared to Seifoddini (1990) as the method considers cell formation for a single scenario only. Also it leads to an optimum cell formation for random product mix situations. For this type of environment, the method provides a flexible design of cells as the expected inter cell traffic of this designs is less than or equal to the expected inter cell traffic of any other alternative design. Genetic algorithm is used for the solution procedure. The method is demonstrated for a simple problem and it is compared with the approach of Seifoddini (1990).

CHAPTER 8

SOME FEATURES OF CELLULAR MANUFACTURING SYSTEM

8.1 INTRODUCTION

In previous chapters, the discussions were mainly on some stochastic processes in manufacturing environment. Specifically the problem of cell formation under probabilistic environment also was discussed in a previous chapter. All characteristics of stochastic processes in the general manufacturing environment are applicable to cellular systems also because demand variations and defects are common to all systems. But, how the system responds to uncertainty due to defects or demand depends on the policies and practices followed in managing the system. Hence this chapter discusses the policies and practices used in managing cellular manufacturing system.

Firms employing cellular manufacturing have converted a portion or whole of the firm's manufacturing system into cells. A manufacturing cell is a cluster of functionally dissimilar machines or processes, which are placed in close proximity to one another and dedicated to the manufacture of a set of items. The items are similar in their manufacturing processes required, items' geometry or both. An item may be a part, component, subassembly or finished product. The group of items that are processed in a manufacturing cell is called a part family.

8.2 PRODUCTION SCHEDULING AND CONTROL

Cellular manufacturing generally aims to reduce set-up times and flow times, and thus to reduce inventories and market response times. The set-up-time reduction methodologies available in Nicholas (1998), Steudel and Desruelle (1992) generally help in simplifying the machine set-up procedure. However, the cellularisation further assists in improving set-up-time reduction by using part family tooling and sequencing. The flow time encompasses times related with set-up, process, move and wait (including storage). In a cell machines or processes are located in close proximity, and it assists in simplifying production schedule and control, and implementation of visual production control procedure like kanban. The imperative of all these is the reduction of various times, except the process time, associated with the flow time. Also, this indicates that cellular configuration is suitable for a repetitive production environment. In addition, cells are like sociological units conducive for teamwork. All these indicate that cellular manufacturing and JIT go hand in hand. That is, cellular manufacturing is an important part of JIT systems.

In a JIT system of manufacturing, cell uses pull production methods. Pull production is a way of controlling a process and reacting quickly to changes without relying on inventory. In a pull system, each stage of a process produces exactly what the immediate downstream stage requests; in effect, material is pulled through the process by each stage producing only what is demanded of it from the next stage. This contrast to the push production wherein every stage produces according to a preplanned schedule, then pushes material to the next stage, whether or not that next stage is ready.

Pull production requires repetitive manufacturing, that is, fairly smooth continuous production of somewhat standardised items. Production volume does not have to be large, but it must be somewhat uniform and stable. This does not mean that daily scheduled amount does not vary. This type of system has a *daily base schedule*[1], which may be different from period to period. It allows relatively small adjustments, which depends on how much variation upstream operations are capable of absorbing on short notice. For example, Toyota[2] production system allows daily adjustment limited to ±10% of the base scheduled amount [Nicholas

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[1] For instance, in a daily base schedule (It is a level schedule), the production volume for each product is set at 1/25 of the monthly requirement (assuming 25 working days/month). If the demand variation within the month is large, then half month demands can be used instead, and the daily production level for each half-month can be set at 1/12.5 of the half-month amount.

[2] Toyota production system is a pioneer in pull production system

(1998)]. These discussions point out that pull systems are not suitable for demands that have very large standard deviation and do not have continuity in demand. But it is suitable for products with continued demand with small variation.

Products whose demand is highly variable and unstable, that require lengthy set-ups, trials, adjustments, or testing, or products that are uniquely made to customer requirements cannot be produced using pull production. Cells in these situations can use push systems and conventional planning and control systems using MRP.

Survey papers on issues related with cellular manufacturing implementation [Wemmerlov and Hyer (1989), Olorunniwo (1996), Wemmerlov and Johnson (1997)] indicate that organisations use MRP, kanban or a hybrid of these two for planning and control of production in cells. MRP has been widely used in a general production environment for planning and control even after implementation of cellular manufacturing. Its use has declined after implementation in preference to kanban system and hybrid [Olorunniwo (1996)].

Unlike traditional systems where jobs are scheduled and tracked at every machine, planning of production in a cellular manufacturing focuses on what goes into the cell and what comes out of it; and not on what is happening within cells. This reduces the amount of production schedule and control information generated since the material is tracked only at two places – where it enters and leaves the cell.

Although a work-cell is a shop floor entity, its implementation must be integrated with planning and control mechanisms beyond the shop floor. Organisations implementing work-cells usually have centralised planning and control systems already in place, and their problem is to adapt them to the control procedures of work-cells. Centralised planning and control systems like MRP focus on order releases and completion dates for all individual components and higher-level subassemblies/assemblies in a product. This feature when adapted for work cells is not necessary for every item represented in the bill of material.

Though cell operators might be responsible for short-term capacity planning and job scheduling, the size and frequency of jobs arriving at the cell is determined elsewhere by the centralised system, which forecasts demand, accumulates job orders, performs rough-cut capacity planning, and prepares and coordinates master production schedules. To adapt the MRP system for releasing orders to work-cells, the product bill of material must be restructured. (Some aspects of the bill of material discussed in a previous chapter are concerned with the modification of bill of material details under uncertainty due to rejection and rework. This modification provides a more accurate input for planning in MRP system. But it is not concerned with restructuring.)

8.3 RESTRUCTURING OF BILL OF MATERIAL

In work-cells materials move directly from one workstation to the next and is held so briefly between stations that it need not be tracked. In other words, control in a cellular environment seeks to focus on the cell input and output, and not on the input and output of every machine. This leads to the simplification of the bill of material of the product being manufactured. It is equivalent to saying that, for the purpose of scheduling and control, all records at intermediate levels of the bill of material are unnecessary. Hence a restructuring of the bill of material is essential and this restructuring is especially required from the scheduling and controlling point of view. Such a restructuring leads to elimination of some of the items from the bill of material. (Normal representation of bill of material is essential for identifying the sequence in which items go into the production process of product and should be maintained for that purpose.) This results in flattening of the bill of material, is explained with the help of figure 4.4.

Figure 4.4 shows the bill of material structure of the flow control valve whose certain details associated with bill of material is modified under rejection and rework. (This is discussed in the chapter 4.) Assume that this product is manufactured in a cellular manufacturing system as per the descriptions given below. Item S18 is manufactured in a cell using the items C32, C33, C34, C31, C25, C27 and C28. That is, the items C32, C33 and C34 are assembled to get the item S30 first in the cell and then it is combined with C31 to get S26. Which is further combined with items C25, C27 and C28 to produce S18 in the same cell. Assume that items M2, M4 and M5 are also produced like this way in cells. In this case the bill of material structure of the flow control valve can be modified as shown in figure 8.1, for the purpose of planning and scheduling.

The flat bill of material shows only 3 levels whereas before flattening it contained 6 levels. The flat bill of material can be used for planning and scheduling purposes. Assume that the items S18, M2, M4, M5 and F1 shown in the bill of material of the flow control valve are manufactured in cells. The system generally maintains record for the input and output items of a cell. That is, the entire items shown in the flat bill of material have records. It is assumed that the items S18 and M2 are produced in different cells. If the entire production activity required for the item M2 is performed in a cell, the bill of material could be further flattened to contain only 2 levels.

8.4 MATERIAL PLANNING UNDER REJECTION AND REWORK

In an earlier section it was seen that MRP or kanban systems are used in practice for production planning and control in a cellular manufacturing environment. Production of defective items usually creates problems in production planning and control of any manufacturing system. In this section the role of uncertain production yield, in the material planning of a cellular environment, is discussed. The discussion examines whether a combination of MRP and kanban could be employed in a hybrid system, restricting their use at two different levels namely that of material planning and secondly scheduling and control

Often cellularisation of the production system leads to reduced defects and improved quality as a result of the work-cell's focused nature, employee involvement in planning and controlling activities of a cell, and cell's nature which promotes team-oriented behaviour. But better quality and reduced defectives are achieved progressively in a step-by-step manner after implementation. This means initially when cells are designed, the planning system has to incorporate certain factors to take account of the uncertainty due to rejection. Initial parameters of planning at this stage of the cell design will have to undergo modification as a result of improvement in the performance of the cells due to better understanding and corrections. (This is one of the basic rules for successful functioning of pull (kanban) system.) Sometimes the characterisation of the uncertainty due to rejection is necessary in the planning stage, as the defects level cannot be reduced significantly due to limitations in technology. Obviously the MRP environment has to accommodate the modifications in material quantities due to defects in the course of production.

As described earlier, in a cellular environment the bill of material used is somewhat different from the MRP system. The flat bill of material used for planning and scheduling in a cellular manufacturing system can be used for the material planning also with appropriate adjustment in quantities for scrapping effect. The method explained in chapter 4 to incorporate the effect of scrapping, can be used in cellular manufacturing also with appropriate translation to the flattened bill of material.

Three subassemblies required to make the flow control valve F1 are M2, M3 and M4. To make the item M2 subassemblies S18 and S8 are required. The item S18 is made in a cell where S26 and S30 are also made. That is, in the production process of S18 the two subassemblies S26 and S30 are produced. These subassemblies do not become part of the bill of material of F1, as kanban system is used. Hence these items are not stored any where in the system and not tracked. The item S30 along with the item C31 is used to make the item S26. While the item S26 is produced there are chances of quality problems and scrapping, and so the average quantity of S30 required for a unit of S26 is 1.1104 units. (This is obtained from table 4.5.)

The flat bill of material does not contain the items S26 and S30. However the entire basic components required are represented in the flat bill of material.

Thus the quantity of the items C32, C33 and C34 to be specified in the flat bill of material is 1.1104 units each instead of one unit which is the quantity shown in the normal bill of material. As there are no other stages of production with any quality problem, the quantity of all other items required to make the item S18 will remain unchanged. The quantity of items required per parent for all other items in the flat bill of material can be calculated in the same way and is given in table 8.1 below.

Item	M2	C3	M4	M5	C6	C7	C9	S18	C17	C10	C11
Quantity	1.0103	1.0103	1.0103	1.0103	1.0103	8	2	1	1	1.0163	3.0488
Item	C12	C14	C19	C20	C21	C15	C23	C24	C29	C25	C27
Quantity	2.0325	2.0325	1.0445	1.0445	1.0445	1	1.0824	1.0824	1.4432	85	2
Item	C28	C31	C32	C33	C34						
Quantity	20	1.1104	1.1104	1.1104	1.1104						

Table 8.1 Material requirements under rejection for the flat bill of material

The discussion till now was only concerned with the planning and control of production system in a cellular environment. Layout planning uncertainties were discussed in chapter 6. The appropriateness of this in the cellular environment is discussed in the next section.

8.5 LAYOUT PLANNING

In a cellular manufacturing, layout-planning need arises when machines are to be arranged within a cell and also the cells are to be placed on the factory floor. The demand of items belonging to the part family is one of the inputs for the layout planning in a cell and uncertainty may be associated with the demand estimation. When probabilistic or stochastic estimate of the demand is used, its use in layout planning as explained in the previous chapter can be used here also. While placing of cells in the factory floor, the interaction between various cells can be obtained from the items, which have inter cell movements. The demand of these items can be used to calculate the flow matrix required for placing the cells in the plant area. The flow matrix estimation under probabilistic or stochastic situation is valid in this case also.

With regard to the layout of facilities within a cell, a few points are worth mentioning. A cell configuration may be of the type: 1) line layout 2) GT (Group Technology) manufacturing work-cell or 3) GT flow line work-cell. If the production characteristics and demand characteristics of a group of items allow continuous flow processing, then line layout is the apt configuration. This type of situation arises very rarely; and the cell configuration may be manufacturing work-cell or flow line work-cell in most cases.

In a manufacturing work-cell, machines of dissimilar functional types are grouped together and dedicated to process a family of parts so that the cell is like a small dedicated job-shop. The flow line work-cell is a configuration that best realizes the full benefits of cellular manufacturing. This type of work-cell provides the advantages and efficiencies of product lines in low- or medium-volume environment. Parts in the part family are processed in the cell in low-to-medium batch sizes with piece-by-piece continuous flow through a line of general purpose, dissimilar machine types. Machine skipping (leap frogging) is typical in this type of cell while back tracking is not allowed.

8.6 SUMMARY

To get the full advantage of the cellular manufacturing system an appropriate modification of the production planning and control system is essential. The survey papers [Wemmerlov and Hyer (1989), Olorunniwo (1996), Wemmerlov and Johnson (1997)] on cellular manufacturing system implementation points out that organisations are increasingly going for a hybrid system of MRP and kanban for production planning purpose. The bill of material used in this system can be flattened, even though the number of component parts in the product and the stages in the process remains the same.

It has been seen that the modelling of uncertainties considered in the layout planning of a general manufacturing system can be appropriately applied in the present case also. The material planning under uncertainty due to rejection is suitably modelled in a cellular manufacturing system with appropriate modifications and flattening of the normal bill of material.

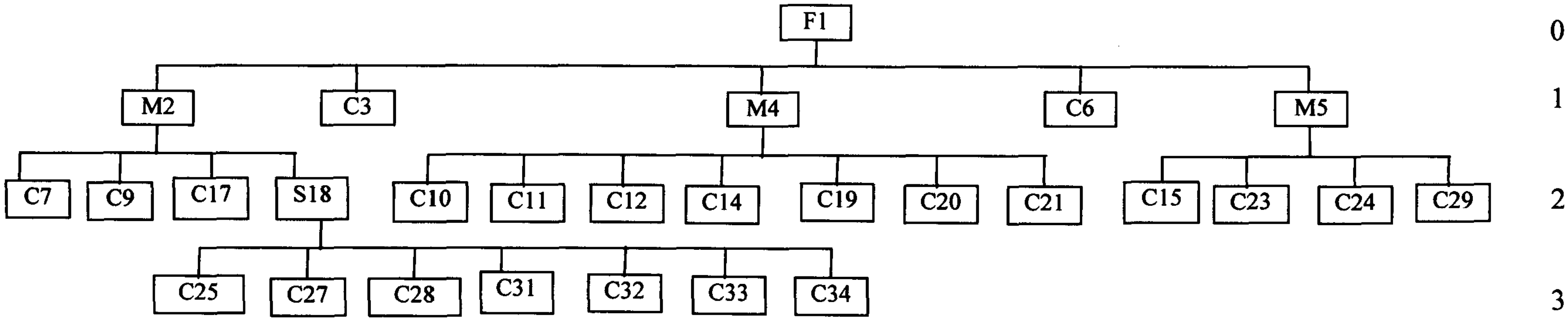

Figure 8.1 Bill of material of flow control valve after flattening

CHAPTER 9

CONCLUSIONS, LIMITATIONS AND SCOPE FOR FUTURE WORK

9.1 CONCLUSION

The work generally pertains to modelling of certain stochastic processes involved in manufacturing environment. The random nature of the material flow, due to scrapping and reworking, through the production system and the effect of the random demand on certain design aspects of production system are studied. The models in material flow capture the effect of scrapping and reworking while the models of demand variation is associated with layout design and manufacturing cell formation. The contributions of the present work could be identified in three distinct areas, is as follows:

9.1.1 Scrap and Rework

➢ When a part is produced the material takes different paths based on the decision taken during inspection at each stage of production. This process is modelled as absorbing Markov chain. The data required for the model is the statistics available from the shop floor in the form of scrap and rework rate. The characteristics of the Markov chain are used to derive various performance measures for the manufacturing system. The effect of scrap and rework is reflected in the performance measures. They are:

- Raw material requirement for the "expected quantity" of finished products
- Production cost
- Machine requirement
- Manufacturing lead-time

- Material requirement
    - Service level criteria
    - Cost minimisation criteria
- Modification of bill of material

➢ These performance measures are modelled using the output from the Markov model and various other production factors. The model of performance measures is either better than the models available in literature or uses different factors compared to the existing models. It is shown that the machine requirement model is superior to the model available in literature, which overestimates the requirement.

➢ The production cost model provides a realistic picture of cost under rework and scrap. It provides a useful input for modelling the quantity of material to be scheduled through the system for a given quantity of finished part. This could be seen in the material requirement model, which minimises the total cost.

➢ The production process of manufacturing a part is a Bernoulli trial with success probability as probability of absorption at finished part state of the Markov process. This warrants a look at the material to be scheduled because of probabilistic nature. Under this uncertainty a model is developed to meet the finished part requirement with certain service level.

➢ The Markov model concept is further applied to a more complicated manufacturing environment where a product is manufactured. The manufacturing of a product from component to the final assembly is divided into a number of subsystems, which could be modelled as absorbing Markov chain. The outputs from the Markov chains are synthesised to modify the material requirement information associated with the bill of material of the product.

➢ It has been found that the bill of material modification model could describe well the effect of scrapping of material on material requirement of a real life problem of assembly and testing operation of solenoid operated flow control valve used for spacecraft propulsion.

9.1.2 Layout Design

- The models developed considered the random nature of demand of products manufactured in a system.
- The present work models the material handling process as a stochastic process. Three models are developed. The basic data required for these models are the arrival rate and the route sheet of jobs. The models provide certain information such as relative transition matrix and relative time matrix. It is shown that the relative transition matrix could be used in the place of flow matrix that is being used in layout design. This is an entirely new approach.
- The quadratic assignment formulation of the layout problem minimises the material handling cost, which is proportional to the product of material flow and distance between facilities. The flow depends on the demand of product manufactured. The literature contains the objective function as a function of flow between facilities. It is shown in the present work that the objective function could be shown as a function of demand. The formulation uses the information from bill of material and route sheet.
- When the demands are random variables and they are independent, the quadratic assignment formulation of the layout problem could be solved deterministically using the mean value of probabilistic demands.
- It is shown that as the product mix is a random variable, the demand to be used in the quadratic assignment formulation corresponds to a scenario called expected scenario, which contains expected demand. This method simplifies the number of flow matrix calculation compared to the method available in literature.
- The layout problem is modelled using the relative transition matrix assuming that the order arrival to a production shop is a Poisson process. The effect of

variation in arrival rate on output of the model is also studied. It is seen that the model is robust in nature.

9.1.3 Manufacturing Cell Formation

- The minimisation of inter cell traffic is the main objective while grouping of machines into machine cells and parts into part families. The traffic function derived is shown as a function of demand. It uses information from bill of material and machine sequence available in route sheet.
- Changes in design variable of the cellular system will affect the performance of the system. Under product mix variation, a robust design was suggested, which is simple compared to methods employed earlier, in addition to the optimality of the solution.
- The solution obtained using the genetic algorithm shows the superiority of the problem formulation suggested in the present work compared to earlier methods.
- It is shown that the models showing the effect of scrap and rework, and the layout models could be suitably used in cellular manufacturing systems also.

9.2 LIMITATIONS

- The manufacturing systems may produce nonconforming items randomly in the production process. In such cases the models of scrap and rework can be applied. When the system constantly deteriorates in terms of the quality of the item produced, the models are not applicable.
- The Markovian modelling of assembly process described in the present work assumes that when rejection happens the assembly as a whole is discarded. That is, the model is not suitable when some of the components of the assembly alone are rejected.
- The solution of the layout models suggested shows the relative position of facilities. Due to practical limitations seldom the solutions as such are used.

- The part family and machine cell formation problem using genetic algorithm describes the effect of new way of formulation of objective function. For practical application the capacity constraints should be included. If more number of machines of particular type is available, its presence in more than one cell can be allowed and such constraints should also be included in the genetic algorithm solution procedure.

9.3 SCOPE FOR FUTURE WORK

- The models, which describe the effect of scrap and rework provide suitable information on operational characteristics of the production system. They are useful for the production planners. The initial Markov model discussed is associated with a serial production system. The output of the model can be used for lot size decision. The present work suggested a model for a producer who wants to minimise the total cost and another model, which satisfy the desired service level. Further the work can extend to study the effect of rework and scrap on the cost of a two-stage supply chain. Another view is that the models available in the literature can be analysed in the light of the information provided by the present work, if the environment of the problems are similar.
- The bill of material modification model is developed considering the quantity required at the top level of the subsystem as the expected quantity of the binomial process. The quantity of material specified in the bill of material is a piece of information used in the material requirement planning. The uncertainty associated with the material requirement planning can be reduced by considering service level criteria. Such a model has to be developed and studied.
- Conduct a simulation experiment to study the performance of the production system with the raw material requirement as discussed in the Markov model and with out such consideration.
- It has been seen in the present work that when the demand is independent random variable the mean value of it can be used in the objective function of

the quadratic assignment formulation. It does not take into account the variability of the demand. So a study will have to be conducted to analyse the robustness of the model.

- The present work contains a cell formation model under probabilistic product mix. The accurate estimation of probability associated with a product mix is difficult. This suggests that there will be variation in probability estimated from actual. How this affects the system design can be studied.

- The layout design and cellular system design models under probabilistic product mix suggested, in the present work, that the design should be made for an expected scenario. (Expected scenario is the mathematically expected scenario.) This suggests an approach for the design of the above problems with dynamic product population. A system with demand forecast available for multi-period in the planning horizon belongs to this type of problem. Compressing the information from different period into an equivalent single period and solving for the equivalent single period may give better result. This will have to be investigated.

ANNEXURE A

ANOVA - STATISTICAL TEST

The statistical test carried out here is generally called analysis of variance (ANOVA). This is usually employed to draw clear-cut answers to questions under investigation by properly planning the experiment [Johnson (2000)]. The two-way classification design is used here for the analysis. One of the variables of interest is the number of items to be processed in a production shop. What is really interesting is, the effect of number of items to be processed in a shop on the performance of the layout. The effect is usually measured using a performance measure. The effect of other (nuisance) variables will more or less be averaged out by randomisation. In other words randomisation ensures that the differences among the means of the samples corresponding to different levels of the variable, can be attributed to systematic differences due to level changes of the variable and chance variability.

In two-way classification, in addition to the randomisation, the effect of one of the nuisance variable can be controlled, by dividing the observations in each classification into blocks. This is accomplished when known sources of variability are fixed in each block, but vary from block to block. This design is also called randomised-block design, provided the observations are identified at random within each block. The blocks are defined by appropriately defining the error bounds as given in the experimentation section. There are 3000 observations corresponding to each level of the variable in each block. These observations are generated randomly as described in the experimentation section. Finally the mean (except for the performance measure of maximum *PCD*) of this many observations are calculated and used in the two-way classification. That is, each cell of the two-way classification table contains this mean. In the case of the performance measure of maximum *PCD*, each cell contains maximum *PCD* of over 3000 observations.

Now, the analysis-of-variance table can be prepared. A function called ANOVA2 available with the MATLAB is used for this purpose. The analysis-of-variance table, obtained using the function ANOVA2, contains columns representing sources of variation, sum of squares (SS), degree of freedom (df), mean square (MS) and F which is a value of random variable having the F distribution with appropriate degrees of freedom. The word 'columns' mentioned in the analysis-of-variance table stands for the levels of the variable and 'rows' mentioned in the table stands for the blocks and the term 'error' has the usual meaning associated with an analysis-of-variance table.

When the analysis-of-variance table is available, the significance test can be conducted by comparing the value of F with that of the value from the F distribution for the given level of significance and degrees of freedom. That is, reject the null hypothesis if the value of F is greater than the value from the F distribution for the given level of significance and degrees of freedom. For the variable – number of items, the number of levels considered is 4 and the number of blocks of error bound considered is 5. When the effect of the variable –number of items is considered the degrees of freedom are 3 and 12. At a significance level of 0.05 and for these degrees of freedom, the value from the F distribution is 3.49. When the block effect is analysed, the degrees of freedom are 4 and 12. In this case the value from the F distribution is 3.26 for the same significance level. These are the values used in the significance test and the conclusions (provided in the experimentation section) are noted from the tables 6.1 to 6.4 based on the analysis-of-variance tables. The analysis-of-variance tables, when the variable considered is the number of items, are given below. These tables correspond to the layout size of 6.

Average PCD, 6 m/cs

ANOVA Table

Source	SS	df	MS	F
Columns	0.5331	3	0.1777	4.577
Rows	9.146	4	2.286	58.89
Error	0.4659	12	0.03882	
Total	10.14	19		

Max PCD, 6 m/cs

ANOVA Table

Source	SS	df	MS	F
Columns	327.9	3	109.3	5.746
Rows	1047	4	261.6	13.75
Error	228.3	12	19.03	
Total	1603	19		

Percentage Optimal Situations, 6 m/cs

ANOVA Table

Source	SS	df	MS	F
Columns	388.4	3	129.5	16.67
Rows	5856	4	1464	188.5
Error	93.2	12	7.767	
Total	6338	19		

Robustness Index, 6 m/cs

ANOVA Table

Source	SS	df	MS	F
Columns	125.3	3	41.78	6.123
Rows	1506	4	376.6	55.18
Error	81.89	12	6.824	
Total	1713	19		

The conclusions drawn and provided in the experimentation section are not only based on one size of the layout but also on the different size layout problems such as 6, 8, 9 and 10. Instead of providing the analysis-of-variance tables for each size problem, the decision related to the hypothesis is provided in the following table A.1. (The null hypothesis related to the variable is that, all the means corresponding to the various levels of the variable are the same. For the blocks it can be stated that all the means of the blocks are same.) The notations

Acc and Rej shown in the table stand for the null hypothesis acceptance and null hypothesis rejection respectively.

Layout size	Average *PCD*		Maximum *PCD*		Percentage of optimum situations		Robustness Index	
	Variable	Block	Variable	Block	Variable	Block	Variable	Block
6	Rej	Rej	Rej	Rej	Rej	Rej	Rej	Acc
8	Rej	Rej	Rej	Rej	Rej	Rej	Rej	Acc
9	Acc	Rej	Rej	Rej	Rej	Rej	Rej	Acc
10	Acc	Rej	Rej	Rej	Rej	Rej	Rej	Acc

Table A.1 The decision table when the variable is the number of items

The conclusions shown in the experimentation section also depend on the ANOVA analysis when the variable considered is the number of machines (ie, the size of the layout problem). The data for the two-way classification table to be prepared can be obtained from the tables 6.1 to 6.4. The averages (shown horizontally on the tables) of the various size layout problem forms the two-way classification table for all the performance measures except the maximum *PCD*. The maximum of the values of *PCD* given in table 6.2 for various size problems is used for creating the two-way classification table for the performance measure of maximum *PCD*. For each layout size, the maximum usually occurs for the lowest number of items and here it occurs when the number of items is 6. The decisions arrived at after the ANOVA analysis is shown in table A.2 below.

Average *PCD*		Maximum *PCD*		Percentage of optimum scenarios		Robustness Index	
Variable	Block	Variable	Block	Variable	Block	Variable	Block
Rej	Rej	Acc	Rej	Rej	Rej	Acc	Rej

Table A.2 Decision table when the variable is the number of machines

ANNEXURE B

SOLUTION USING THE METHOD OF SEIFODDINI (1990)

Since there are three scenarios there will be three designs (cells) corresponding to each scenario and they are designated as design 1, design 2 and design 3 respectively. For the scenarios 2 and 3 the cells formed are the same and hence design 2 and design 3 are the same. These cells are obtained using genetic algorithm. Parts of the various scenarios are assigned to each design and thus part family is formed. The inter cell traffic is also calculated when parts are assigned to a design. The design, the corresponding part family, and inter cell traffic are presented in tables B.1 and B.2. The table B.1 contains cells formed for the first scenario, the part family formed when the parts of the various scenarios are assigned to the best cell formation of the first scenario and the inter cell traffic. The cell formed for the second and third scenarios are same and it is shown in table B.2 which also contains, the part family formed when the parts of the various scenarios are assigned to this cell formation and the inter cell traffic.

When the input for the cell formation under various scenarios is considered, it can be seen that the total number of items manufactured in first scenario is 16 as product 3 is not included in this scenario. For all other scenarios the entire items are processed in the shop. Even though the part machine incidence matrix is the same for these scenarios the demand is different. The genetic algorithm parameters for these problems are the same as the parameters provided previously in the illustrative section.

The expected cost (inter cell traffic) calculation details are shown in table B.3. In table B.3, design 4 stands for the cell formation corresponding to the expected scenario. This also shows the inter cell traffic when parts of the various scenarios are assigned to design 4. In this case also design 4 is same as the design for the second or third scenario. For this problem it can be seen that the inter cell traffic obtained when the method of Seifoddini (1990) is used is same as the method suggested in this work.

Cell Number	1	2	3
Design 1-Machines	2,5,8,13	6,11,14,17	1,3,4,7,9,10,12, 15,16
Part family corresponding to scenario 1	1,7,10,15,19	2	4,5,6,9,11,12,14, 16,18,21
	Inter cell traffic = 46		
Part family corresponding to scenario 2	1,7,10,15,19	2,8,17	3,4,5,6,9,11,12, 13,14,16,18,20
	Inter cell traffic = 54		
Part family corresponding to scenario 3	1,7,10,15,19	2,8,17	3,4,5,6,9,11,12, 13,14,16,18,20
	Inter cell traffic = 62		

Table B.1 Design (machine cells) corresponding to first scenario, part family and inter cell traffic

Cell Number	1	2	3
Design 2 or 3 – Machines	4,9,10,15,16	2,5,8,13	1,3,6,7,11,12,14, 17
Part family corresponding to scenario 1	4,5,11,16,21	1,7,10,15,19	2,5,8,12,14,18
	Inter cell traffic = 70		
Part family corresponding to scenario 2	4,5,11,13,16,21	1,7,10,15,19	2,3,6,8,9,12,14, 17,18,20
	Inter cell traffic = 22		
Part family corresponding to scenario 3	4,5,11,13,16,21	1,7,10,15,19	2,3,6,8,9,12,14, 17,18,20
	Inter cell traffic = 35		

Table B.2 Design (machine cells) corresponding to second scenario, part family and inter cell traffic

	Inter cell traffic			Expected Inter cell traffic
	Scenario 1	Scenario 2	Scenario 3	
Design 1	46	54	62	54
Design 2	70	22	35	40.3
Design 3	70	22	35	40.3
Design 4	70	22	35	40.3

Table B.3 Expected inter cell traffic

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