STUDY ON INVENTORY CONTROL UNDER UNCERTAINTY IN SUPPLY CHAIN PROCESSES

Thesis submitted to the University of Calicut

for the award of the degree of

DOCTOR OF PHILOSOPHY

IN

STATISTICS

under the Faculty of Science

by

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under the guidance of

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CERTIFICATE

I hereby certify that, this thesis entitled **"Study on inventory control under uncertainty in supply chain processes"** is a bonafide record of research work carried out by **Mr. Praveen V P.**, Research Scholar, Department of Statistics, University of Calicut, under my supervision and guidance for the award of the Degree of Doctor of Philosophy in Statistics, of the University of Calicut. The work reported herein does not form part of any other thesis or dissertation submitted previously for the award of any degree or diploma of any other university or institution. Also certified that the contents of the thesis have been checked using anti-plagiarism data base and no unacceptable similarity was found through the software check.

University of Calicut
Date: 11/10/2024

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DECLARATION

I, Praveen V P., hereby declare that this thesis entitled **"Study on inventory control under uncertainty in supply chain processes"** submitted to the University of Calicut for the award of the degree of Doctor of Philosophy in Statistics is a bonafide record of the work done by me under the guidance and supervision of Dr.M.Manoharan, Senior Professor (Retd.), Department of Statistics, University of Calicut. This thesis contains no material which has been accepted for the award of any degree or diploma of any university or institution and to the best of my knowledge and belief, it contains no material previously published by any other person.

11th October 2024 **Praveen V P**

ACKNOWLEDGEMENTS

I would like to express my deepest gratitude to everyone who supported and guided me throughout the completion of this thesis. I feel immense pleasure in expressing my sincere gratitude to my research supervisor, Dr. M. Manoharan, Retd. Senior Professor, Department of Statistics, University of Calicut, for his unwavering support, mentorship, and kindness throughout my journey. He granted me the freedom to explore my research while gently ensuring I stayed on track, exemplifying an unwavering commitment to perfection, passion, courage, and conviction. I am deeply thankful for his guidance and honored to be associated with such an exceptional individual.

I am very thankful to my teachers in the Department of Statistics, Dr. S.D. Krishnarani (Head of the Department), Dr. K. Jayakumar (Senior Professor), Dr. Dileep Kumar (Assistant Professor), Dr. C. Chandran (Retd. Sr. Professor), and Dr. N. Raju (Retd. Sr. Professor) for facilitating the whole process. Additionally, I am thankful to the librarian and the non-teaching staff of the Department of Statistics at the University of Calicut, N. P. Jamsheer(CHMK) and N. U. Sajeev for their help and cooperation.

It is my previlage to thank my colleagues Vidya, Aparna, Kavya, Fasna, Prasily, Arya, Jeena, Jumna, Salima, Anjale, Jiji, Safwana, Anand, Greeshma, Rehna, Amjish, Nimisha, Shabeer, Devanand, and Anjali for the cooperation and warmth they showed me. My friends from All Kerala Research Scholars Association, Calicut University and hostel friends deserve special mention for their unwavering support and care, sustaining me throughout my research tenure. They stood by me during both joyful and challenging moments, pushing, and motivating me to keep going.

A very special thanks to my parents, Prasanna and Narayanan, brother Naveen and Athulya for their blessings and supports on me. I would like to extend my heartfelt thanks to my extended family members and friends for their unwavering support and assistance throughout this research journey.

11th October 2024 **Praveen V P**

Abstract

Inventory control and supply chain processes are essential components in efficiently managing the flow of goods and materials within a business. Effective inventory management aims to strike a balance, avoiding overstock and stock shortages while ensuring sufficient inventory to support smooth organizational operations. In supply chain processes, coordination between suppliers, manufacturers, and distributors is vital in meeting customer demand while maintaining operational efficiency. This research focuses on various inventory control and supply chain management models, addressing critical aspects like deterioration, demand fluctuations, and financing strategies. The study explores by proposing a model for inventory control for items with a general deterioration rate, emphasizing the optimization of costs over time. To provide insights into balancing profitability and credit terms, the research develops an EOQ (Economic Order Quantity) model for deteriorating items, factoring in two-level trade credit financing with expiration dates. The study investigates inventory decisions for items experiencing both deterioration and amelioration, incorporating partial backlogging and time-varying demand. The research examines an integrated supply chain model with manufacturer and retailer where product demand is influenced by factors like price, freshness, and advertisement strategies, allowing for more adaptive approaches in highly competitive markets. Finally, the study concludes with a summary of findings and outlines future research directions, particularly in extending these models to address emerging challenges in global supply chains. This research also includes numerical examples and sensitivity analysis to validate and illustrate the proposed models. Through these analyses, we aim to demonstrate the practical applicability and effectiveness of the models in addressing real-world inventory and supply chain challenges.

Keywords: inventory, supply chain processes, deterioration, amelioration, expiration date

സംഗ്രഹം

ചരക്കുനിയന്ത്രണവും വിതരണ ശൃംഖലാ പ്രകിയകളും ഒരു വ്യവസായത്തിനുള്ളിലെ ചരക്കുകളുടെയും വസ്തുക്കളുടെയും ഒഴുക്ക് കാര്യക്ഷമമായി കൈകാര്യം ചെയ്യുന്നതിനുള്ള അവശ്യ ഘടകങ്ങളാണ്. കാര്യക്ഷമമായ ചരക്കുകൈകാര്യം, ശേഖരക്കൂടുതലും ശേഖരക്കുറവും ഒഴിവാക്കിക്കൊണ്ടും മതിയായ ചരക്കുകൾ ഉറപ്പാക്കിക്കൊണ്ടും സുഗമമായ സംരംഭക സംഘാടനത്തിന് ഒരു സന്തുലിതാവസ്ഥ കൊണ്ടുവരാൻ ശ്രമിക്കുന്നു. വിതരണ ശൃംഖല പ്രക്രിയകളിൽ, ഉപഭോക്താക്കളുടെ ആവശ്യകത നിറവേറ്റുന്നതിലും പ്രവർത്തന കാര്യക്ഷമത നിലനിർത്തുന്നതിലും, മൊത്തവിതരണക്കാർ, ഉൽപാദകർ, ചില്ലറവിൽപ്പനക്കാർ എന്നിവർക്കിടയിലുള്ള ഏകോപനം പ്രധാനമാണ്. ഈ ഗവേഷണം ചരക്കുനിയന്ത്രണത്തിന്റെയും വിതരണ ശൃംഖലാ കൈകാര്യത്തിന്റെയും വിവിധ മാതൃകകളിൽ ശ്രദ്ധ കേന്ദ്രീകരിക്കുന്നു, ഒപ്പം ചരക്കുകളുടെ ഉപയോഗനാശം, ആവശ്യകതാ വ്യതിയാനം, സാമ്പത്തികനയങ്ങൾ എന്നിവയിലും. ഉപയോഗനാശം വരുന്ന ചരക്കുകളുടെ നിയന്ത്രണത്തിന്റെ പരമാവധി ചിലവിന് ഊന്നൽ നൽകിക്കൊണ്ട്, ഈ പഠനം, അത്തരം ചരക്കുകളുടെ ഒരു നിയന്ത്രണമാതൃക മുന്നോട്ടുവയ്ക്കുന്നു. അടുത്തൊരു മാതൃകയിലൂടെ, ലാഭത്തിന്റെയും ധനനയത്തിന്റെയും സന്തുലനം വഴി കാലാവധിയുള്ള വസ്തുക്കളുടെ മതിയായ ഓർഡറിന്റെ അളവ് കണ്ടെത്തുന്നു. തുടർന്ന് ഈ പഠനത്തിൽ കാലാവധിയുള്ളതും അഭിവൃദ്ധിഗുണമുള്ളതുമായ വസ്തുക്കളുടെ ചരക്കുനിയന്ത്രണ നയങ്ങളും കൂടാതെ, അവയുടെ കരുതൽവിതരണം (Backlogging), സമയാനുസൃതമായി മാറ്റംവരുന്ന ആവശ്യകത എന്നിവയും അന്വേഷിക്കുന്നു. ശേഷം ഈ പഠനം പരിശോധിക്കുന്നത് നിർമ്മാതാക്കളും വിൽപ്പനക്കാരും അടങ്ങിയ ഒരു വിതരണശൃംഖലാ മാതൃകയാണ്. ഈ മാതൃകയിൽ വസ്തുക്കളുടെ ആവശ്യകത അതിന്റെ വില, പുതുമ, പരസ്യനയങ്ങൾ എന്നിവയുമായി ബന്ധപ്പെട്ടിരിക്കുന്നു. അവസാനമായി, കണ്ടെത്തലുകൾ ക്രോഡീകരിക്കുന്നതോടൊപ്പം ആഗോളവിതരണശൃംഖലയിൽ ഉയർന്നുവരുന്ന വെല്ലുവിളികളെ സംബോധന ചെയ്യുന്നതിന് ഉതകുന്ന രീതിയിലുള്ള ഭാവിഗവേഷണത്തിനുള്ള ദിശകൾ അടയാളപ്പെടുത്തുകയും ചെയ്യുന്നു. ഈ മാതൃകകൾ സാധൂകരിക്കുന്നതിനും ചിത്രീകരിക്കുന്നതിനും നിരവധി ഉദാഹരണങ്ങളും പ്രതികരണ വിശകലനങ്ങളും (Sensitivity Analysis) പഠനത്തിൽ ഉൾപ്പെടുത്തിയിരിക്കുന്നു. ഇത്തരം വിശകലനങ്ങളിലൂടെ ഈ പഠനം ലക്ഷ്യമാക്കുന്നത്, ചരക്കുനിയന്ത്രണത്തിന്റെയും വിതരണശൃംഖലയുടെയും മേഖലയിലെ വെല്ലുവിളികൾ സംബോധനചെയ്യുന്നതിൽ ഈ മാതൃകകളുടെ ഫലപ്രാപ്തിയും പ്രയോഗക്ഷമതയും സമർത്ഥിക്കുവാനാണ്.

സൂചകപദങ്ങൾ: ചരക്ക്, വിതരണ ശൃംഖല പ്രക്രിയകൾ, തകർച്ച, മെച്ചപ്പെടുത്തൽ, കാലഹരണ തീയതി

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Chapter 1

INTRODUCTION AND OVERVIEW

1.1 Introduction

This thesis deals with the problem of inventory control under uncertainty in supply chain (SCP). The main objective of the study is to develop and analyze inventory models and supply chain models for deteriorating and ameliorating items under varying operational conditions, including trade credit financing, lead-time control, and time-dependent demand patterns. The study aims to incorporate realistic business scenarios by proposing and solving inventory models by optimizing cost, profit, and service levels. The study of science rises out of the practical necessity alongside a shared interest in convergence in a few lessons of issues of human beings. Operations research, where use of advanced analytical methods to reform the finding and improve decision making, is no exception. Operations research in its early stage is came into act after the military forces contribution. During the second world war a joint work of of scientists with military persons of united kingdom were developed to met different issues of arrangements and allocations. Since it was a war time high need of using any kind of objects in the field of operation was required. The objective of identifying uncommon resources and their maintenance, distribution to concerned area was appropriate task. The result consolidated the best conceivable utilization of newly invented radar, assign-ment of British Airforce planes to missions and encourage assurance of best patterns for looking for submarines. The enabling eventual outcomes of such endeavours lead to the advancement of more bunches in British armed organizations and the diagram of such experimental social occasions was a colossal commitment to western Allies. Soon after the war numerous of the scientists who were active in the military groups diversified their consideration for applying comparative approaches to civilian-problems like business, industry, agribusiness, aeronautical building, administration science, financial matters, genetic engineering etc. to explore complicated genuine world frameworks with a point of making strides optimum solution to ideal level. As an application of operations research the generation framework was evolved in genuine world issue. The most vital thing is the stocking of things from a smaller retailer to a bigger generation firm or shop to meet up the customers request as distant as possible. Again, stocking of things depends on distinctive variables such as deterioration, amelioration, demand, replenishment of order. The taking care of such sort of issues is known as inventory management.

Inventory is the physical stock of items that a trade proceeds with the goal of ensuring the smooth and satisfactory running of its operations. Inventory can be said to consist of economically valuable assets like labour, goods or property. It can also be considered as a kind of wasted asset which is not used. Inventory is also described as items that are purchased, monitored and stored for everyday use of the business system. It can be viewed as a collection of necessary goods, for example components used in the production process, work in progress materials, final products ready for distribution to customers, and human resources such as financial resources available for operations or workers not directly hired. The inventory model works on the stock began in the 1920s. In the initial stages, it had simple models that utilized only a few parameters to capture the key elements. Later, these models were developed further to represent more points of interest by including additional parameters. Step by step probabilistic models were created in 1950s to capture the impacts of unconventional areas and lead times. Since these models managed with as it were one item at a time that was suffered as a limitation to the area. The actual stock standing up to various people was to direct on managing a wide range of things, with sufficient interrelations among them to speak to an organization issue. Consequently, a partitioned information handling oriented subject called inventory control or inventory management advanced. Here the major concern was for organizing and keeping up records. Gradually the optimizing execution has been enhanced. This chapter contains a brief account of preliminary concepts in inventory management system and its developments, importance, decision making aspects and supply chain management.

1.2 Operations Research in Production process

Organizations and affiliations much of the time confront testing operational issues whose effective course of action requires certain arrangement in associated estimations, advancement, stochastic displaying, or a mix of these ranges. To show this, an organization might require to plan an assessing course of action so as to meet specific quality control objectives. In an assembling situation, operations that look for the same resources must be arranged in a way that due dates are not abused. The chief of a market must choose what number of checkout lines to keep open at diverse times in the midst of the day and night with the objective that clients are not senselessly delayed or as a final test, the extent of the locales held for putting absent work in method must be settled so that a smooth stream of work comes about, indeed at the busiest era times. At the present time, the field of OR is enormously effective and continuously developing. To give a few cases of the modern research wanders, force work in OR tries to make programming for fabric stream examination and layout of adaptable assembling workplaces utilizing plan affirmation and chart speculation calculations. In the period preceding the industrial revolution, most trade and industry comprised of small enterprises, each coordinated by a single boss who did the purchasing, arranged and supervised production, sold the item, contracted and terminated individual etc. Such mechanization of production led to quick development of mechanical endeavours that it got to be strange for one man to perform all these administrative works took put. Inventory control is one such result. The administration of each financial segment picked up intrigued after World War II to study inventory management framework due to much hazard calculate and uncertainty. Subsequently, the study of inventory model literature has been broadened day by day and different models have been distributed in distinctive journals. Arrow et al. (1951) built up a result on mathematical investigation of inventory models on Optimal inventory Policy. Whitin (1954) published an EOQ model on stochastic form.. Later, Peston and Whitin (1958) analyzed a review of the inventory system framework on The Theory of Inventory Management. The collection and processing of vast amount of information are required in each inventory problem.

1.3 Inventory management Systems

Inventory is essentially stock of physical states having a few financial values. It is an idle resource as long as it is not utilized. It may be utilized as those merchandise that are put away and utilized for day to day working of an organization. Centuries back, inventories were seen as measures of riches and control of a country or an individual. In later past it was moreover seen as a degree of trade failure. Businessmen subsequently, have begun to put bigger accentuation on the liquidation for quick turnover. Now a days due to quick progression of innovations, inventories are seen as a large potential chance or maybe than a degree of riches. In this way it is required to utilize experimental techniques in organization of inventories known as 'Inventory Control'. It is the system of looking after stock things at pined for level. Thus inventory management administration framework is the means by which materials of rectify quality and quantity is made accessible as and when required with respect to economy in the manufacturing costs, purchase costs, market capital, shortage costs and set up costs. In common inventory management takes the obligation of the accessibility of right quantity at right time at least cost. In expansion to the above, the control and maintenance of any inventory is a common problem to all organizations and financial segments. The inventory management concerns about different deals associated with the process of replenishment lead time management, handling various costs, forecasting of inventory model, valuation of the model, possible physical space, quality check and classifying defective products. The task of inventory management includes maintain certain amounts of stock during the period of supplier to user transactions in the supply chain. The uncertainties that may present in the demand, supply and transportation of items need to be maintained for the smooth functioning. The responsibility of keeping right amount at right instant require bulk buying and storing which tends to lot of costs that brings the economies of scale in inventory management. For instance, inventories are kept up in Industry, Commerce, Military, and Agribusiness etc. A few of the benefits of keeping of the inventories are as follows:

1. It makes a difference in legitimate and productive running of organization.
2. It gives the quick benefit to the customer.
3. It minimizes the chance of misfortune due to alter in cost of items.
4. It makes a difference in minimizing the loss due to the deterioration, out of date quality and damages.
5. It employs as a buffer stock when crude materials are delay to get or due to delay in supply to the market.
6. It minimizes the item cost due to an advantage of batching.
7. It keeps up the economy by persevering a portion of the deviations when the intrigued of a thing varies.
8. It coordinates how to utilize of accessible money in a most proficient way and maintain a strategic distance from an unnecessary use on high inventories.

In wide sense term stock might be apportioned into two sorts

Direct inventories

These inventories incorporate those things which expect a basic portion in the production and turned into a fundamental piece of completed stock. For illustration, crude material stock, work in development stock and completed items stock.

Indirect inventories

Such inventories include different things which are fundamental for fabricating the component of finished production such as oil, grease, lubricants, etc. Inventories are utilized to meet the regular variance in demand economically. Production of specialized things like crackers well before festivals, fans and coolers some time recently on set of summers, natural products and vegetables are gathered a few months in each year to meet demand for the whole year. Consequently inventory control is the forms to keep up the stock inside desired limits. It moreover standardizes and centralized data on stock levels. It has opportune records of inventories of all items. In inventory management framework, Inventory models contain the diverse decision variables such as amount requesting and stock. At that point there emerges three fundamental questions such as

1. How much amounts of an item require to be ordered?

2. What is the right time that the requesting to be placed?

3. How do you keep up the completion arrange of stocked items?

But in real business, it is very troublesome to decide a suitable securing policy. An inventory management issue is a choice making problem which answers the above questions. On the other hand the point of an inventory model is to get an ideal order request which minimizes the add up to inventory expenses. i.e., an inventory problem deals with decisions that maximizes the total profit accomplished while assembly the customers demand.

1.4 Basics Concepts and Inventory Terminology

Knowledge of the fundamental concepts and terminology associated with the effectiveness of inventory management is better for controlling and optimizing inventory levels. This section covers key terms that are essential for managing inventory and its smooth functioning. Familiarization of these concepts in turn useful for anticipating challenges and make informed decisions to ensure efficient supply chain.

1.4.1 Deterioration

The degradation of the products is a vital one in the study of inventory. Product fails to perform its operation on a regular basis is termed as deterioration. The perishable nature of inventory can be seen in different categories. The goods such as fruits, vegetable, food, dairy products etc. which are quickly unusable is one among them. Occurrence of the physical reduction for the items like alcohol and gasoline is another category. The negative spoiling, radiation changing and failure of effectiveness in inventory for medicine and electronic components are another example for deteriorating products. The deterioration can be treated in another way by considering time and utility. Examples like some liquid drug shows constant utility as time passes the usage period. An increasing nature of utility can be seen in a few intoxicating drinks as the time increases. Similarly, some items such as fruits, vegetables and brand-new foods etc. shows decreasing usefulness as the time passes the usage period.

1.4.2 Amelioration

The growing items during inventory management are a vital field in the study of inventory control. Some stocks are there where some improvement or growth is seen in a company. Specific improvement leads to amelioration. In other words, amelioration occurs when the worth and the stock size proliferate gradually. Yet there are many amelioration problems because they exist in the real world, such as the farming, brewery products, fishery, and poultry industries. The speedy budding animals like ducks, pigs, and broilers in poultry farms, hybrid fishes in ponds and cultivated vegetables and fruits in farms are typical field applications. Amelioration is quite different from the deteriorating items and deserves a comprehensive study. Therefore, the current research has also considered the ameliorating items.

1.4.3 Demand

Optimal inventory policy is most of the time directly involve with customers demand, i.e., size of demand, rate of demand, and the patterns of demand for a given item.

The dealing with demand of a product is the consideration of number of units taken from its inventory and the quantity required to satisfy the demand for inventory. The characteristic of demand may sometime known and unknown hence the methods of utilizing demand is treated accordingly as deterministic and probabilistic. Different situations of inventory system can be obtain where demand depends on various parameters namely time, stock, shelf space and price etc, probabilistic with known or unknown distribution or demand can considered as constant.

1.4.4 Lead Time

When the stock level diminishes the order for next replenishment placed. The time to deliver this order can be deterministic, stochastic or constant. This time between the placing of order and delivering into inventory is called lead time. The lead time having the combination of different time components such as the time taken for initial order processing, preparing time required for the supplier, transportation time from supplier to the buyer, inspection time for checking the defectiveness. Understanding and maintaining lead time for avoiding stockouts and managing timely delivery of products to customers are essential for optimal inventory levels.

1.4.5 Time Horizon

The specific period of time over which the activities and projections of inventory level monitored is referred to as time horizon. In the optimization procedure of inventory management time horizon plays a role as the duration of the horizon in turn connected to different parameters. The cost is taken to consideration using proper discount factors for long time period while short time effect perishability or obsolescence. Hence the model should rely with a suitable time horizon to cover the optimum cost over the specific period of time.

1.4.6 Safety Stock

The uncertainties in supply and demand may lead to the risk of stockouts. Keeping extra inventory known as safety stock to overcome such situation mitigate the risk and act as a buffer. This safety storage can meet customer demand at the duration of unexpected delays in supply or increase in the demand. Safety stock is a critical component of inventory control ensuring efficiency in meeting customer needs when company is not sure about demand or lead time for the goods.

1.4.7 Trade Credit

A payment system in inventory management is an important criterion to free up cash flow and finance growth. The buyer's perspective was considered in the traditional Economic Order Quantity (EOQ) model with assumptions such as no stockouts, constant demand rate, unlimited storage space, instantaneous replenishment rate, and instant settlement upon delivery of merchandise. In the practical situation, these assumptions are only sometimes possible. Consider transactions in the industry sector where the supplier grants the buyer an extended period to enhance demand without penalty. The other functions will continue smoothly before the end of this payment grace period. Buyers can sell items and earn interest from deposited revenue. The trade-credit policy charges interest if the payment is not made during the extended period. Therefore, it makes financial sense for the buyer to delay payment until the end of the allowed period provided by the supplier. Consequently, the assumption that the buyer must pay for the items immediately upon receipt is debatable. The impact of the supplier's trade credit policy on inventory management has attracted the attention of many researchers.

1.4.8 Partial Backlogging

A supply chain system experiences a situation where some customer orders are fulfilled partially during a stockout period. During this period, the supplier will deliver a portion that is not entirely satisfying and backlog the remaining items at the next refill of the stock. The model must manage lost sales and back orders to maintain the shortage concern. Considering some sales, such as trendy goods or high-tech items with short product life cycles, some customers are unwilling to wait for the next refill. Even though they are not waiting, this approach helps to manage inventory levels without completely losing the goodwill. Based on the duration of the wait period, backlogging costs will decrease. Hence, balancing customer goodwill with efficiency and inventory costs requires careful examination.

1.4.9 Inflation

The growth of prices of merchandise and services in the economy over time is inflation. When prices rise, both money and assets buy fewer goods and services. Inflation makes money buy less over time, reducing its value to purchase things and measure economic worth. It is usually measured as an annual price change, like the Consumer Price Index (CPI). Inflation has many effects that can simultaneously be good or bad for the economy. Bad effects include: Money and other assets lose value over time. Uncertainty about future prices can discourage spending and investing. People put money into stocks or gold instead of businesses that make things. This can hurt the economy by making it harder to get the resources needed to update businesses. High inflation can also cause shortages of goods if people rush to buy them before prices change. However, inflation can also help by easing economic downturns and lowering the absolute level of debt. Economists think high inflation comes from printing too much money. Low to moderate inflation can come from changes in how much people want things or how much there is to buy, like during shortages. But most agree that long periods of high inflation happen when there's more money than the economy can grow with.

1.4.10 Warehouse

A warehouse is a facility used for the storage and management of merchandise. It acts as the central location where merchandise is collected from suppliers, kept in bulk quantity until needed for the next stage and then distributed to meet the demand or retail locations. Therefore, warehouses are usually placed close to the market in favor of a fast flow of finished items to the consumers. The warehouse is essential in the inventory study to ensure that the inventory is available to customers while optimizing storage space and minimizing cost.

1.4.11 Re-order Level

The merchandise, whether raw materials, items in production, or finished products to sell, is constantly depleted depending on the demand. The order has to be replenished to meet the customer's satisfaction at the time of a reduction in stock level. When placing an order, the rate at which the customers use inventory has to be noted. This inventory level at which a new order is placed to add the stock level is called the re-order level. Hence, the order point has to be maintained precisely to ensure enough inventory during the lead time and smooth functioning at the depletion time.

1.5 Supply Chain

A supply chain is a typical network of facilities and distribution options that performs the functions of buying of materials, conversion of these materials into intermediate and finished products and the supply of these final products to customers. It exists in both manufacturing and service organizations, albeit the complexity of the chain vastly vary among industries. The functioning of company-to-company relationships and overseeing the flow of goods into and through the company are part of supply chain management. It encompasses various functions including demand planning, procurement, production, quality control, fulfilment, warehousing, and customer service. Inventory management is crucial for ensuring an efficient supply chain and tracking trends and orders within the company or its divisions. It ensures goods flow smoothly into, through, and out of warehouses, providing visibility across all inventory-related activities. Goods reach customers through various supply chains, some more complex and extensive than others. When individuals in the supply chain make business decisions without considering the impact on other members, it can lead to sub-optimization, increased costs, and longer waiting times across the supply chain. This ultimately results in higher end product prices, lower service levels, and reduced customer satisfaction. Therefore, effective supply chain management requires strategic coordination and alignment of traditional business functions across all entities within the supply chain. Hence, the aim is to improve the sustained performance of individual organizations and the supply chain. The supply chain's functions are capturing and fulfilling customer demand, such as product development, marketing, operations, distribution, finance, and customer service.

Supply chain models play an essential role in the economy of todays competitive environment as they provide an integrated networking system between suppliers, manufacturers, retailers, and customers. Again, it informs how to survive the current competition through collaboration between suppliers, manufacturers, retailers, and customers. A supply chain model connects suppliers, manufacturers, distributors, and customers to a network that executes a series of interrelated business processes in order to have:

- the most appropriate purchase of raw materials from nature
- transportation of raw-materials into the warehouse
- manufacturing of goods in production cell and distribution of finished goods to retailer for sale to customers.

The supply chain management is an essential and often critical element to operational efficiency. It can be applied to protect the goodwill of the customers and company success as well as within societal settings, including disaster relief operations, medical missions and other kind of emergencies, in order to help improve quality of life.

1.6 Inventory: Problem classification

Inventory problems can be categorized in several ways to improve management and efficiency. One way is based on repetitiveness, where inventory is classified into single-order items purchased once and not reordered. Repeated orders are regularly restocked according to predefined guidelines as they are consumed. Another classification is by supply source, distinguishing between items sourced from external suppliers and those produced internally by the organization.

Knowledge of demand also plays a crucial role in inventory classification. Constant demand items have consistent demand patterns over time, while variable demand items experience fluctuations in demand levels. Additionally, demand can be categorized as independent or dependent. Independent demand items have demand patterns unrelated to other items, whereas dependent demand items' demand is directly related to the demand for different products or components. Understanding lead time is another crucial classification factor. Inventory with constant lead times has consistent times for procurement, whereas inventory with variable lead times experiences fluctuations in procurement times. Lastly, inventory systems vary in how they manage stock. Perpetual inventory systems continuously monitor stock levels and reorder when inventory reaches predefined levels. On the other hand, periodic inventory systems review stock levels regularly and place orders based on those reviews. These classifications help organizations tailor inventory management strategies to optimize operations, minimize costs, and effectively meet customer demand.

1.6.1 Deterministic model and Stochastic inventory models

In the deterministic model, demand is considered static, meaning it is assumed to be fixed and fully predetermined. Such models are often called economic lot size models. The inventory model under this framework operates without uncertainty, assuming that items are withdrawn at a fixed rate from the inventory. A fixed quantity Q defines lot sizes, and the lead time is either zero or constant. Because of its simplicity, optimal solutions to deterministic models can be derived under several operational assumptions.

In real-world scenarios, the requirements of future demand are often uncertain. Managing this uncertainty is a major challenge for businesses, hence, they adopt stochastic inventory models. In the new approach the system monitors stock levels continuously. A new order is placed when the stock falls to a predetermined reorder point. This approach is also called modern inventory management. Computers are frequently used to monitor inventory, especially for items crucial enough to warrant a formal inventory policy. This model is similar to the Economic Order Quantity (EOQ) model but incorporates the demand and lead time variability. The critical difference is that the timing and quantity of withdrawals from inventory may be uncertain, and the probability distribution of demand is either estimated or known.

1.7 Various Inventory control policies

Inventory control is exercised by introducing various measures and policies for smooth operations. Suitable inventory control policies must be applied to determine when and how much inventory should be ordered or produced. Different measures of inventory policies are mentioned below to exercise such control.

1.7.1 Economic Order Quantity (EOQ) Policy

This policy determines the optimal order quantity that minimizes the total cost, including ordering, holding and stock out costs. EOQ is calculated based on constant demand, fixed lead time, and no stock shortages.

1.7.2 Reorder Point (ROP) Policy

Inventory control is associated with the choice of placing an order. Control policies are necessary to determine when the inventory must be added before the inventory drops to a comfortable level. This point is commonly known as the reorder point. Reorder point policies determine the point at which the stock will be purchased.

1.7.3 Just in Time (JIT) Inventory Policy

JIT aims to reduce holding costs by producing or ordering inventory just in time for use. It minimizes excess stock and focuses on eliminating waste throughout the production process. This policy requires strong supplier relationships and accurate demand forecasting.

1.7.4 ABC Analysis

Inventory is classified into three categories: A, B, and C based on importance and value. High value and low quantity items are categorized as A items, medium value items as B items, and low-value and high quantity items as C. This categorization helps prioritize management efforts and resource allocation.

1.7.5 (s, S) Policy

This is a type of periodic review system where orders are placed when inventory falls below a certain level s to raise the stock level to a maximum S. It is effective for managing seasonal demand and uncertain supply conditions.

1.8 Various Types of Inventory Cost

Managing inventory ensures the right amount of resources is available in the right place, at the right time, and at the lowest cost possible. Inventory costs are directly linked to the operation of inventory systems and can significantly impact the performance and success of an organization in implementing these systems. These costs form the fundamental financial factors in any inventory decision-making model, influencing various system design and implementation aspects. Key considerations typically include:

1.8.1 Carrying Cost or Holding Cost

The cost associated with keeping inventory on hand is known as carrying or holding costs. It is typically expressed as an annual cost per unit of inventory held. This cost depends directly on the quantity and duration for which inventory is stored. It includes expenses like insurance, storage facility rent, handling and movement costs, and the risk that inventory items may become obsolescence, theft, damage, or depreciation. Carrying cost is often calculated as a percentage of the inventory's value.

1.8.2 Order/Setup Cost

Order/setup cost refers to the expenses incurred when placing an order with an external supplier or setting up production internally. This cost tends to increase with the number of orders or setups made rather than the size of each order. It includes generating purchase orders, verifying suppliers, preparing paperwork, receiving goods, and inspecting materials. Setup costs involve preparing production schedules, organizing work, conducting pre-production checks, and ensuring quality standards are met.

1.8.3 Purchase Cost

The purchase cost is the price paid for acquiring inventory from an external source or the cost of manufacturing it internally. This cost should always reflect the actual cost of the item as it is placed into inventory. For purchased items, it includes the purchase price plus any shipping costs. Internally manufactured items include direct labour, materials, and factory overhead costs.

1.8.4 Stock-Out Cost

Stock-out cost refers to the financial impact of running out of inventory. External stock-outs occur when customer orders cannot be fulfilled, while internal stock-outs occur when orders within the organization cannot be met. External stock-outs may lead to costs such as backorder expenses, loss of current and potential sales, and damage to customer goodwill. Internal stock-outs can result in lost production time, idle resources, and deadline delays. These costs may include expenses for overtime, rush orders at higher costs, loss of goodwill, and missed opportunities to sell products. The severity of these costs depends on how customers react to the out-of-stock situation. If demand remains high, the financial impact can vary from delayed shipments to cancelled orders. Generally, businesses prioritize urgent backorders and may incur additional costs for expedited handling, shipping, or service fees.

1.8.5 Sales revenue

Sales revenue represents the money a company makes from selling goods or providing services. It's crucial for several reasons such as estimating profits accurately, making investment decisions, qualifying for loans or contracts assessing the company's overall value. Additionally, sales revenue is a primary metric for measuring business performance and sustainability in the marketplace.

1.9 Methodology Used

The challenges of functioning inventory management have to be addressed systematically. The methodology includes a thorough examination of the inventory problem and its surrounding environment and a detailed analysis and definition of the problem. This structured process ensures a comprehensive understanding of the issues at hand, enabling the development of practical solutions and strategies for optimal inventory management.

1.9.1 Examination of the inventory problem and its environment

The inventory models and their optimum values are associated with different parameters, such as other costs, nature and distributions of factors such as demand, deterioration, amelioration, trade credit, etc. Such vital factor fluctuations affect inventory optimization techniques. The analysis of the optimal solution to each model depends on the element that impacts inventory management and the company's operational capabilities.

1.9.2 Analysis and definition of the problem

The effectiveness of inventory control models and optimization depends on the methods used to develop them. Identifying the appropriate methods and solution techniques is crucial for determining the level at which inventory needs to be replenished for different cycles and the quantity replenished during each cycle within the given planning horizon. Quantifying the extent of the problem and identifying the root causes and challenges due to fluctuations must be clearly defined. When formulating these issues into mathematical problems, we follow these steps:

1. Justify the nature of the inventory system.

2. Deduce and derive formulas for the inventory control system.

3. Formulate a mathematical model of the proposed system.

4. Propose the optimal solution for the inventory system.

Various computational and analytical tools were employed to facilitate this analysis, including MATHEMATICA for complex mathematical modelling, Microsoft tools for data management and analysis, MATLAB for numerical computing and simulations, and R programming for statistical analysis and data visualization.

1.10 General methods for solving inventory models

We use various methods depending on the nature of the problem to optimize decision variables to enhance total profitability and minimize costs. For analytical problems, classical optimization techniques such as calculus and graphing tools are used to find optimal solutions. Complex problems with many parameters and constraints are evaluated using numerical methods and programming software. This approach provides solutions based on possible parameter values verified through sensitivity analysis by adjusting these values. We construct and formulate a numerical mathematical model based on decision variables. The values of the decision variables were obtained to assess the model's validity. These decision variables depend on non-governable variables that cannot be directly controlled by the system, leading to certain restrictions. After resolving any conflicts, the implementation of obtained solutions ensures smooth system functioning to meet organizational requirements.

1.11 Motivation for proposing inventory models

In inventory system three types of cost, viz. carrying, shortage and replenishing are significant and subject to control by appropriate decision. These three types of cost are usually closely associated. When one price is reduced or increased one of the other two cost and from time to time still both may increase there is thus the problem of controlling the cost so that their sum will be lowest. It is challenging question to control the inventory. Many concepts and techniques were proposed by mathematicians for controlling the inventory effectively. It should consider two questions mainly, first is when should inventory be replenished? and second is how much should be added to inventory? Thus, the time component and amount component are variable so as to one subject to manage inventory system. Inventory problem is for finding the special cost of the functions that reduce the overall value and minimum value is obtain when the carrying cost and replenishing cost are literally balanced. Analysis of an inventory system involves of the following phases:

1. Assessments of the properties of an organization.
2. Development of formulation of inventory problem.
3. Development of a model.
4. Derivation of a solution.

For the entire inventory cycle, few models assumed that the holding cost is constant. It is not practical in realistic business. In the proposed study have considered the inventory policy has stock-level dependent demand rates and a storage-time dependent holding cost. The holding cost per unit of the item per unit time has taken as a function of the time spent in storage. Different time-dependent holding cost step functions have considered, such as Retroactive and Incremental. Procedures have developed for determining the optimal order quantity and the optimal cycle time for both cost structures. Since then, on top of the conversation one can conclude that there is scope as well as the need to make the model more general and more flexible. When the decision maker has more control over the inventory policy, he is able to manipulate it so as to maximize the profit of the organization. Better optimization can be achieved when all the relevant information about demand through Visa and other quantities such as holding cost, production cost and selling price are properly defined. Consideration of realistic events such as inflation, trade credit, selling prices and stock-dependent demand makes inventory control policy better informed. The present thesis provides optimum strategies to deal with inventory models having various realist situations such as perishable products, time dependent demand, deteriorating items, delay in payment is permitted, trade credit, replenishment, shortages, stock-dependent demand and holding cost.

1.12 Literature review

A brief literature survey about inventory control models and its different assumptions were carried out to get an interaction with the area of study. Here we present some of the research works in this field and found out research gaps and motivated from this to fill such gaps. Harris (1990) and later Wilson (1934) were known as first developed inventory models and proposed EOQ model under certain assumptions regarding demand, replenishment, different costs and lead time. The journey in this area continued and several text books were written to popularise this topic. In this literature survey we report different inventory models that are extended Harris Wilsons assumptions regarding the various parameters of models such as demand, replenishment, shortages, warehouse facility, different costs, trade credit financing, advance payment, effects of deterioration and amelioration, discount facilities, preservation technology, inflation and impreciseness of different parameters. In the competitive business scenario due to the globalization of market economy the effect of factors like selling price, quality and brand value of a product play a vital role. Customers buying behaviour is related with such factors. Selling price is directly connected with popularity as selling price decreases, the intention to buy increases. Likewise, popularity is more for item with less selling price than the same item with nearer quality but higher selling price. Hence to deal with the selling price, a retailer has to take appropriate decision in ordering and storage of products. With this assumption, Kotler (1971) proposed an inventory model includes the relation between pricing and order quantity, with the concept of marketing policy. The gap of effect of advertisement on demand is become a part of assumptions in inventory studies. Subramanyam and Kumaraswamy (1981), Urban (1992) and Abad (1996) investigated and developed EOQ model considering various marketing policies. Price and advertisement related demand is discussed by Luo (1998) and time-dependent partial backlogging inventory model with pricing and quantity discount facilities for deteriorating item by Papachristos and Skouri (2003). Chen (2009) studied model by considering price dependent demand and use inventory centralization games with the fact of quantity discount for the customers. Selling price dependent demand and Weibull rate of replenishment is proposed by Sridevi et al. (2010). Non-instantaneous deteriorating item with the effect of joint pricing is studied by Maihami and Abadi (2012). They included partial backlogging and demand is considered as depend on time and selling price. Later, several inventory models for deteriorating item considering selling price dependent demand can seen in the studies of Giri and Bardhan (2011), Pal et al. (2012), Maihami and Kamalabadi (2012). The assumption of quantity-dependent trade credit with price-dependent demand for deteriorating items is introduced by Annadurai (2013). A Weibull distributed deterioration effect is studied by Bhunia and Samanta (2014), who introduced an inventory model by considering and selling price-dependent demand. Later, involvement of payment relaxation for the inventory system of ameliorating items with consideration of price dependent demand was introduced by Mahata (2016). Several developments in this area can referred from the works of Bhunia and Shaikh (2016), Shaikh (2017a), Hsieh and Dye (2017), Shaikh (2017b), Xie et al. (2021), Mishra et al. (2023) and Pando et al. (2023). The demand and its rate are essential in any inventory control system since this directly involves in business management. Initial studies of inventory models dealt with constant demand and several researchers have developed inventory, supply chain, and production models in this regard. The real-life situation assumes, customer demand as the function of various factors such as time period, selling price, stock level in the showroom, freshness index, advertisement effort, etc. other than constant demand which is unrealistic. Teng and Chang (2005) introduced a production inventory model for items that deteriorate and demand depends on stock level and selling price. Stock dependent demand rate taking variable holding cost is the theme of the inventory model developed by Alfares (2007). Considering quantity discount and price dependent demand, Chen (2009) studied inventory centralization games. Sana (2011) introduced stock and price dependent demand in an inventory system. Multi-item inventory model is considered Pal et al. (2012) with price sensitive demand under price break facility. Later, a two-storage inventory model for deteriorating item is developed by Agrawal et al. (2013) considering a ramp-type variable demand. Sanni and Chukwu (2013) investigated inventory problem with ramp type demand and affect of three-parameter Weibull distributed deterioration. Considering selling price sensitive demand, Bhunia et al. (2014) proposed an inventory model for deteriorating good with three-parameter Weibull distributed deterioration rate. Palanivel and Uthayakumar (2015) considered advertisement and selling price dependent demand in formulating a finite time horizon inventory model for deteriorating item (non instantaneous). Sanni and Chukwu (2016) developed an inventory model where the demand is quadratic and the deterioration rate follows three-parameter Weibull distributions. Manna et al. (2017) proposed an imperfect production model with advertisement dependent demand and production rate sensitive defective rate. Khan et al. (2020b) developed an inventory model for decaying item/good with advertisement and selling price dependent demand under advanced payment. Udayakumar et al. (2020) further studied non-instantaneous deteriorating inventory model with advertisement and price sensitive demand rate under permissible delay-in-payment policy. Considering nonlinear stock, price and stock sensitive demand rate, Halim et al. (2021) formulated a production inventory model for decaying item/good. Then, Palanivel and Suganya (2021) developed an inventory model considering partial backlogging with price and stock level dependent demand under quantity discounts facility. In this area, the works of San-Jos et al. (2019), Tripathi et al. (2019), Kuraie et al. (2021), Taleizadeh et al. (2022), Mahata and Debnath (2022), Van Zyl and Adetunji (2022), Salas-Navarro et al. (2023), Guo and Wang (2023) and Jauhari et al. (2023) are worth mentioning. In the literature of inventory, it is observed that several investigators drew their attention to investigate the impact of trapezoidal type demand rate on the different inventory systems. The idea of trapezoidal type demand in the modelling of an inventory control problem proposed by Cheng and Wang (2009) and Cheng et al. (2011) expanded the model with the help of partially backlogged shortages and also the effect of deterioration. After that, Chuang et al. (2013) and Singh and Pattnayak (2013) formulated inventory models considering trapezoidal type demand for deteriorating item. Xu et al. (2020a) studied an inventory model for non-perishable item with trapezoidal type demand and partial backlogging shortages. Kumar (2021a) investigated a fuzzy inventory model with trapezoidal demand and time-varying holding costs under shortages. Several researchers developed different inventory models based on type of demand. Among these works, some worth mentioning works were Singh et al. (2021), Xu et al. (2020b), andShah and Shroff (2022).

Stock-out period in a business is a situation when the stock is empty but the customers demand for the items are available. Due to deterioration effect of the items/goods, uncertain customers demand, offering of discount facility to the customers, etc., this situation may occur. The stock-out time interval is known as shortage period. In storage period, two types of customers behaviour occur. These are:(i) all the customers whose demands are not fulfilled immediately are interested to wait to receive the goods until next lot arrives, (ii) a part of the customers are willing to wait to receive the goods until the arrival of next lot. The first and second cases are known as fully and partial backlogged shortages respectively. So, the consideration of partially/fully backlogged shortages/backordering is another important feature in formulating inventory model. Agrawal and Banerjee (2011) formulated a two-warehouse inventory model with partially backordering and ramp type demand. Maihami and Kamalabadi (2012) developed an inventory model for non-instantaneously deteriorating item and studied the pricing effect on the optimal policy under permissible delay in payments and partially backlogged situation. Pandey and Pandey (2014) studied a partial backlogged inventory model for exponentially deteriorating item considering multi variate consumption rate. Bhunia et al. (2015) investigated a two-warehouse deteriorated inventory problem considering partial backlogged shortages and variable demand. In this connection, the works of Tripathi and Pandey (2015), Palanivel et al. (2016), Sahoo et al. (2019), Singh et al. (2019), Khan et al. (2020a), Shaikh et al. (2021), SanJos et al. (2021), Adak and Mahapatra (2020), Manna and Bhunia (2022), Hajialirezaei and Pasandideh (2023) and others are worth mentioning. There are several physical goods, viz. vegetables, fruits, volatile liquids, foodgrains, pharmaceuticals, blood, chemicals & radioactive chemicals, alcohols, meat, milk, bakery items, many kinds of electrical items etc. which lose their weights and freshness after a certain time and these effects are continued with time. After a certain time period from the receiving of order quantity or production of goods, the natural phenomena of the loss of weights and freshness of a product are known as non-instantaneous deterioration. Large number of products are there lose their freshness during the product period. Implementation of preservation facility, maintenance facility of warehouse and length of stocking time of the product are some remedies to overcome the situation. Hence this effect has an impact on the analysis of inventory control system. The deterioration concept was first introduced by Ghare and Schrader (1963) and formulated an inventory model. After them, Covert and Philip (1973) extended this concept by considering two-parameter Weibull distributed deterioration. Thereafter, a lot of works were carried out by several researchers considering fixed or variable deterioration. Geetha and Uthayakumar (2010) formulated an inventory model for non-instantaneous deteriorating item with trade credit policy. Maihami and Kamalabadi (2012) derived the inventory control models with partially backlogged shortages for non instantaneous deteriorating items. Dye (2012) proposed a non-instantaneous decaying inventory model considering preservation facility. Mahata (2012) developed a supply chain inventory model considering exponential deteriorating rate and retailer partial trade credit scheme. Shah et al. (2013) considered generalized deterioration rate in formulating their inventory model. Jaggi et al. (2013) studied ordering policy of a two-warehouse inventory system for deteriorating item under inflationary conditions. Singh and Rathore (2015) proposed a trade credit policy oriented inventory model for deteriorating item under preservation facility. Tayal et al. (2016) investigated a production inventory model with a preservation facility for a deteriorating item. Tiwari et al. (2016) studied the effects of inflation on retailers order policy and also the credit facility of deteriorating item. Shaikh et al. (2017) developed a stock and price-related inventory model for non-instantaneous decaying item. Mishra et al. (2017) proposed an inventory model with a preservation facility for deteriorating item under a trade credit facility. Shah et al. (2018) proposed a non-instantaneous decaying model assuming price sensitive demand and considering learning effects. Palanivel et al. (2017) studied the optimal policy of an inventory system of non-instantaneous deterioration product under promotional effort and inflation. Mishra et al. (2018) formulated a deteriorated inventory model taking the impact of decaying reduction technology investment. They considered stock and price-dependent demand with shortages in their model. Tiwari et al. (2018) formulated a sustainable EOQ model considering the effect of imperfect quality and deterioration of the products in demand with carbon emission policy. Udayakumar and Geetha (2017) studied the effect of deterioration to the optimal policy of an EOQ model. Chen et al. (2019) developed an optimal pricing inventory model by taking stock-level, price, and time-dependent demand for decaying item. Mahmoodi (2019) introduced the concept of duopoly retailers and formulated a deteriorating inventory model with a linear trend in demand. Khakzad and Gholamian (2020) introduced an advance payment-related inventory model with the effect of the inspection rate of deterioration. Xu et al. (2020b) studied the strategy of inventory control for deteriorating item with time-varying demand and carbon emission regulations. Khan et al. (2020b) discussed the effect of non-instantaneous deterioration in a two-warehouse system under advance payment and shortages. Nath and Sen (2021) showed the effect of deterioration on the optimal decision of an inventory model with exponential demand and fully backlogged shortages. Sharma and Kaushik (2021) studied a deteriorated inventory problem with delay payment facility. Duary et al. (2021) considered Weibull distributed deterioration in their proposed inventory model. Considering carbon emission, Singh et al. (2021) investigated a sustainable inventory problem considering the effect of non instantaneous deterioration. Mahdavisharif et al. (2022) solved a non-instantaneous deteriorating inventory problem by considering Stackelberg game theory approach. Besides these, a lot of deteriorating inventory models were developed by several researchers, viz. Tiwari et al. (2017), Ahmad and Benkherouf (2018), Chakrabarty et al. (2018), Li et al. (2019), Panda et al. (2019), Chen et al. (2019), Gupta et al. (2020), Khan et al. (2020b), Sundararajan et al. (2019), Rout et al. (2021) and Duary et al. (2022). In the literature of inventory control, it is observed that not much works was done for the inventory of ameliorating items. These items are fast growing animals or hybrid fishes. When these are stock-in situation at firm or in big pond or at distribution centre or at the centre of retail sale, the stock increases as well as decreases simultaneously due to growth and death respectively. Hence this natural phenomenon cannot be ignored from the analysis of inventory of ameliorating items. In this area, considering the amelioration effect only, the first inventory model for fast growing animals was reported by Hwang (1997). Again, he developed two different inventory models, one for ameliorating and another for deteriorating item under LIFO and FIFO issuing policies considering constant demand and prescribed cycle length. After that, considering the simultaneous effect of deterioration and amelioration, Mondal et al. (2003) reported another inventory model with price dependent demand. Again, taking into the account of the effect of time value of money and inflation, Moon et al. (2005) developed an inventory model for ameliorating item with variable demand over a finite planning horizon. After that, several inventory models for ameliorating item were reported in this area. In this connection, the works of Mahata and De (2016), Vandana and Srivastava (2017), Ahmad and Benkherouf (2018), Hatibaruah and Saha (2021), Khedlekar and Singh (2021) are worth mentioning. In the present competitive market scenario, various marketing policies have been introduced to attract retailers attention to manufacturer. In a collaborative business operation between a retailer and a customer, it is generally assumed that customer makes the payment after receiving the item. However, to finalize an appropriate deal with the customer, sometimes the corresponding retailer demands a percentage of whole payment in advance. In the current business situation, advanced payment policy is widely applied. In this connection, some existing works are reviewed. Cachon (2004) introduced the concept of discount facility for advance payment of purchase cost in an inventory model. Maiti et al. (2009) studied optimal policy of an inventory model incorporating advance payment facilities, customers demand as a decreasing function of selling price and stochastic lead time. Further, Gupta et al. (2009) developed an inventory model with advance payment policy by taking all the inventory cost components as interval-valued. Chen et al. (2013) studied the effect of payment schemes on inventory decisions. Taleizadeh (2014) described an inventory model permitting multiple prepayments scheme instead of a one-time prepayment policy. Taleizadeh and Nematollahi (2014)Taleizadeh (2014) developed two inventory models considering the multiple prepayments. Zhang et al. (2016) introduced advance payment scheme for stable supply capacity in a two-stage supply chain inventory model. Lashgari et al. (2015) proposed partial advanced payment policy in a three-level supply chain model. Taleizadeh et al. (2017) developed an advanced payment-based inventory model under planned partial backordering. Palanivel et al. (2017), Khan et al. (2019) developed various inventory models under advanced payment policy. Khakzad and Gholamian (2020) studied optimal policies of inventory models for deteriorating item considering advanced payment facility. Further, citediabat-2017, Khan et al. (2019), (Khan et al., 2020a,b), Khan et al. (2021), Duary et al. (2021), Rahman et al. (2022), Duary et al. (2022) studied various types of inventory models considering advanced payment policy. One of the most primal issues for every business organisation is to increase the efficiency of their firm by attracting customers attention to their items. However, different challenges arrive in this scenario due to the impact of competitions among these organisations. Cause of which forces a number of organisations to confer various shots of offers to their retailers or customers to attract them. Trade credit facility is one of them. When retailers purchase some items from their wholesalers/suppliers for their future business purpose, the retailers have to make the payment of the purchased items. For catching more retailers attention during this competitive market situation, supplier may allow a time duration to their retailers to make the whole payment of the purchased items under certain inconvenient agreements. This business policy is called fully trade credit financing (single level). Again, considering risk factor of suppliers side, suppliers generally demand a percentage of payment from their retailers at the time of placement of the order. This is called partial trade credit financing (single level). Similarly, the same policies may be applied by the retailers to their customers. If in a business circle, partial credit policies simultaneously applied by both the retailers and suppliers, then this policy is known as two-level partial trade credit policy. Over the last few decades, inventory problems involving credit facility were widely studied by several researchers. In this connection, some of the existing literatures are reviewed. Considering two-level trade credit policy, Huang (2006) developed an inventory model. Chung and Huang (2007) studied the optimal policy of retailer of an inventory problem for deteriorating item under trade credit financing. Few years later, Liao and Huang (2010) and Jaggi and Verma (2010) both developed two different deteriorated inventory models considering linear trend in demand along with trade credit facility. After that, Dye (2012) formulated an inventory model with trade credit policy and variable demand dependent on time. Liao et al. (2013) studied the effect of trade credit facility on the optimal policy of an inventory system of perishable items. Shastri et al. (2014) proposed a supply chain model considering bi-level trade credit financing. Palanivel et al. (2016) developed a two-warehouse inventory model considering trade credit financing and partial backlogged shortages. (Shaikh, 2017a,b), Yu (2019), Panda et al. (2019) studied various types of inventory models considering trade credit facility. Jaggi et al. (2018) studied the impact of credit financing on the optimal policy on an inventory system for Weibull-distributed deteriorating products. Mashud et al. (2020) navigated optimal policy of a sustainable inventory model considering advanced payment policy and trade-credit strategy. Succeeding them, Shaikh et al. (2020) considered trade credit facility in an inventory model for deteriorating item. Further, Tripathy et al. (2021) and Mahato and Mahata (2021b) determined the effects of credit financing on the optimal decisions on an inventory system. Das et al. (2021) proposed a deteriorated inventory model with preservation technology considering multiple credit periods-based trade credit policy. Recently, Shaikh et al. (2021), Kumar (2021b), Jiang et al. (2021), Tripathy et al. (2022), Ahmed et al. (2022) and Jani et al. (2023) developed several inventory models by considering trade credit financing. In the market of perishable items, customers perception of quality mainly depends on freshness period of items and freshness is reversely related to deterioration rate of such items. Freshness is still an important factor to the customers i.e., generally customers like to purchase fresh and healthier items. Every item which has deterioration effect also has a freshness period. Therefore, due to the effect of deterioration rate, freshness period of an item has a minor impact on suppliers optimal policy. In the current market situation, suppliers/retailers try to increase the freshness period of the item artificially by applying preservation technology, refrigeration, etc. and these are the reasons why sometimes freshness period is considered as controlling parameter. In this connection, Bai and Kendall (2008) formulated an inventory model with freshness period of the items dependent demand. Few years after, Piramuthu and Zhou (2013) established an inventory model for perishable item having demand dependent on freshness. Thereafter, Chen et al. (2016) formulated a deteriorated inventory model considering the variable demand dependent on stock and freshness. Banerjee and Agrawal (2017) studied an inventory model for deteriorating item with freshness period dependent demand rate. Pal (2018) described an optimal replenishment policy under preservation technology and random deterioration start time for non-instantaneously perishable items. Shah et al. (2018) proposed an inventory model with preservation investment. Mashud et al. (2020) developed an inventory model for non-instantaneous deteriorating item under preservation technology along with the joint effects of trade credit and advertisement policy. Besides them, several works were accomplished considering freshness period of item in which the works of Mohanty et al. (2018), Mishra et al. (2018), Li et al. (2019), Bardhan et al. (2019), Gautam et al. (2020), Zhang and Wang (2020), Sebatjane and Adetunji (2020), Rana et al. (2021), Sepehri et al. (2021), Mashud et al. (2021), Priyamvada et al. (2021), Shah et al. (2021), Mahata and Debnath (2022), Mahato and Mahata (2022) and Gautam et al. (2023) are noteworthy.

1.13 Organization of Thesis

In the proposed thesis, different models of inventory problems related to real life situations are considered and solved with the effect of inventory parameters in different environments. The proposed thesis has been divided into five chapters as follows:

- **Chapter 1:** Introduction and Overview

- **Chapter 2:** Analysis of inventory control model for items having general deterioration rate
- **Chapter 3:** An EOQ model for deteriorating items under two-level trade credit financing with expiration date
- **Chapter 4:** Inventory decisions for deteriorating and ameliorating items with partial backlogging and time varying demand
- **Chapter 5:** An integrated supply chain model with product demand influenced by its price, freshness and advertisement strategy
- **Chapter 6:** Summary and future research work

1.13.1 Thesis layout

Chapter 1: INTRODUCTION AND OVERVIEW

This introduction chapter discussed a detailed overview of inventory theory and models. It includes a summary of existing research to show how this study is relevant, with all related references listed at the end of the thesis. This chapter includes preliminary concepts in inventory management, its evolution over time, the role of decision making, and its importance within supply chain management.

Chapter 2: ANALYSIS OF INVENTORY CONTROL MODEL FOR ITEMS HAVING GENERAL DETERIORATION RATE

In this chapter, we developed a deterministic inventory control model with stochastic deterioration incorporated through additive Weibull distribution. This elegant approach is proposed to consider a time dependent demand in the planning process and we consider that the holding cost totally depends on time and shortages are allowed. The objective is to minimize the total inventory cost of the proposed EOQ model. Finally the formulated model is illustrated through numerical examples to determine the effectiveness of the proposed model.

Chapter 3: AN EOQ MODEL FOR DETERIORATING ITEMS UNDER TWO-LEVEL TRADE CREDIT FINANCING WITH EXPIRATION DATE

An inventory model for managing deteriorating items with expiration dates is studied in this chapter. This model accounts for both quantity and quality losses and evaluates how these factors collectively influence the best approach when implementing a trade credit policy. The consumer demand for these items depends on their selling price and their freshness condition. The objective is to analyze the inventory system for deteriorating items within the context of maximizing profits and to establish the most advantageous inventory policy. The model's effectiveness is illustrated through numerical examples. To assess the impact of variations in different inventory parameters, we conduct a sensitivity analysis.

Chapter 4: INVENTORY DECISIONS FOR DETERIORATING AND AMELIORATING ITEMS WITH PARTIAL BACKLOGGING AND TIME VARYING DEMAND

Fast growing items are an interesting area in the inventory model problems. The combination of deteriorating and ameliorating items and their impact in developing control strategies are vital in the study. A fixed planning horizon is considered in this chapter where the linearly time varying demand is considered with amelioration and deterioration impacts. We developed the model with shortages backlogged partially during the time frame. The objective is to minimize the total inventory cost of the proposed model. Using numerical examples and sensitivity analysis the model is illustrated and verified the effect of changes of model parameter on the optimal solutions.

Chapter 5: AN INTEGRATED SUPPLY CHAIN MODEL WITH PRODUCT OF THE DEMAND INFLUENCED BY ITS PRICE, FRESHNESS AND ADVERTISEMENT STRATEGY.

Effective inventory management is necessary for the supply chain process of perishable products, where the product's freshness depends on the customers' buying behavior. This chapter develops a two-echelon supply chain inventory model with product demand depending on its price, freshness and advertisement strategy. With the aim to maximize the profit of the system, separate manufacturing and retailer enterprises are taken into consideration. The objective of the model in this chapter is to find the optimal values of the advertisement frequency, selling price and ordering cycle length of the retailer with simultaneous integration of the profit of the manufacturer and retailer system. Some numerical examples illustrate the proposed model and a sensitivity analysis is conducted to verify the effect of the parameters in the model.

Chapter 6: SUMMARY AND FUTURE RESEARCH WORK

This chapter includes a summary of the thesis, its limitation and the scope of future research work.

Chapter 2

ANALYSIS OF INVENTORY CONTROL MODEL FOR ITEMS HAVING GENERAL DETERIORATION RATE

2.1 Introduction

It is reasonable to note that a product may be understood to have a lifetime which ends when utility reaches zero. Looking through the inventory models with deteriorating items extensively studied by researchers in the past shows that the deteriorating rate is considered constant or treated as some real valued functions. However, in real life situations, combinations of various factors would form the basis for modeling the inventory problem for deteriorating items. Deterioration is defined as decay, damage or spoilage such that items can't be used for intended purpose.

Mandal (2010) gave an EOQ inventory model for Weibull distributed deteriorating items under ramp type demand and shortages. Hung (2011) gave an inventory model with generalized type demand, deterioration and back order rates. Shah et al. (2013) integrated time varying deterioration and holding cost rates in the inventory model

Some contents of this chapter are based on Praveen and Manoharan (2020).

where shortages were not prohibited. Mishra et al. (2013) gave an inventory model for deteriorating items with time-dependent demand and time-varying holding cost under partial backlogging. Tripathi and Pandey (2013) presented an inventory model for deteriorating items with Weibull distributed deterioration and time-dependent demand under trade-credit policy. Yadav and Vats (2014) proposed an inventory model with constant holding cost under partial backlogging and inflation. Pervin et al. (2015) presented an inventory model in a declining demand for deteriorating items under trade-credit policy. Pervin et al. (2016) proposed an inventory model with shortage under time dependent demand and time varying holding cost including stochastic deterioration, etc. The inventory model developed in this chapter dealt with assumptions that were not discussed in any of the previous literature works. The research gaps obtained through examining previous works were filled by providing a suitable model considering different phases of deterioration rate. The developed model is validated with the help of four illustrative numerical examples. With the help of examples, we discussed the effectiveness of the proposed model. Along these lines, different sections of this chapter are developed and presented.

2.2 Motivation, research questions and gaps

Inventory model in the present chapter considered the time-dependent demand approach to represent a realistic situation perfectly. The effect of deterioration in a non-instantaneous way is used in this chapter. Shortages are allowed in the model to address more realistic circumstances. While considering deterioration rate, previous works gave the idea of various models with deterioration rates following exponential distribution, Weibull distribution, etc. Most of these works were proposed to model any of the three parts in a deterioration rate. However, only a little work has been reported on modeling situations where different phases of deterioration rate are prevalent. With the help of additive Weibull distribution, we recommend a new model that presents all three stages of deterioration in one model. The research questions that are addressed in this chapter were given below.

- To present realistic situation demand is considered as time dependent with shortages are allowed
- To discuss the three phases of deterioration using suitable distribution to propose the model

The presented model in this chapter fills the gap by addressing the above questions. The additive Weibull distribution makes the deterioration rate more realistic.

2.3 Problem Description

The inventory model in this chapter incorporates cost optimization based on general deterioration and shortages. A time varying holding cost is another realistic approach. The objective functions and decision variables were constructed to investigate the model. The optimization problem in this chapter is built under the following notations and assumptions, which have been used throughout this chapter.

2.3.1 Notations

Objective Function

Symbol	**Description**
TC	The total average cost

Decision Variables

Symbol	**Description**
t_1	Time at production stops
T	Length of cycle time

Cost Parameters

Symbol	Description
A	Ordering cost per order
p	Unit purchasing cost per item
s	Lost sale cost per unit
p_2	Shortage cost per unit time

Inventory Parameters

Symbol	Description
δ	Backlogging rate, $0 \leq \delta \leq 1$
$I(t)$	Inventory level at time t, t ≥ 0
I_0	Maximum inventory level during [0, T]
$I_1(t)$	Inventory level that changes with time t during the production period
$I_2(t)$	Inventory level that changes with time t during the non-production period
k	Replenishment rate which is always finite

2.3.2 Assumptions

The developed inventory model formulated upon the following assumptions:

1. The demand rate $D(t)$ at time t is a linearly increasing function of t; i.e., $D(t) = x + yt$, $0 \leq t \leq T$, where x and y are nonnegative constants

2. The distribution of the time to deterioration of an item follows the additive Weibull distribution with density function, $(abt^{b-1} + cdt^{d-1})$exp$(-at^b - ct^d)$, t$\geq$ 0, where $a > 0$ and $c > 0$ are scale parameters and $b > d > 0$ or($b < d < 0$) are shape parameters. The inventory level will change at a changing rate. Hence to present differential model we use the deterioration rate function for additive Weibull distribution θ(t).The term θ(t)=$abt^{b-1} + cdt^{d-1}$,t$\geq$ 0, where $0 \leq a, c \leq 1$ and $b, d > 0$,gives the on hand inventory deteriorates per unit time.

3. Q is the stock level reached in the cycle at the end of production period will be used in non production period.

4. The holding cost per item per time-unit is time dependent and is assumed as $h(t) = h + \gamma t$, $t \geq 0$, where $\gamma > 0$ and $h > 0$.

2.4 Mathematical formulation

In the proposed inventory model, depending on the above assumptions, the inventory system can be considered as follows. At the beginning of each inventory cycle with zero stock level, k units of products arrive at the system. Up to time t_1 due to replenishment the inventory level meets the demand in the market. Replenishment is occurring during the production period only and at time $t= t_1$ production stops and inventory level reaches the level Q which is the stock level that will be used in the non production period. The inventory level in stock during the non production period is diminishing due to those reasons of market demand and deterioration of items during the time interval [t_1,T] and shortages begin to be accumulated which are partially backlogged. Next, the inventory level is declining to its lowest position at time $t = T$. Just after the cycle period the process repeat itself with k units of products arrived at the system, so replenishment is instantaneous and lead time is zero. Figure 2.1 depicts the proposed inventory system.

Now $I(t)$ denote the inventory position at time t, $t \geq 0$,then the differential equations during the interval$[0, T]$ that describes the instantaneous state $I(t)$, where all the sudden states of the inventory level are involved, are given by

$$\frac{dI_1(t)}{dt} + \theta(t)I_1(t) = k - D(t) \ with \ I_1(0) = 0, 0 \leq t \leq t_1. \quad (2.4.1)$$

and

$$\frac{dI_2(t)}{dt} + \theta(t)I_2(t) = -D(t) \ with \ I_2(T) = 0, \ I_1(t_1) = I_2(t_1) = Q, \ t_1 \leq t \leq T. \quad (2.4.2)$$

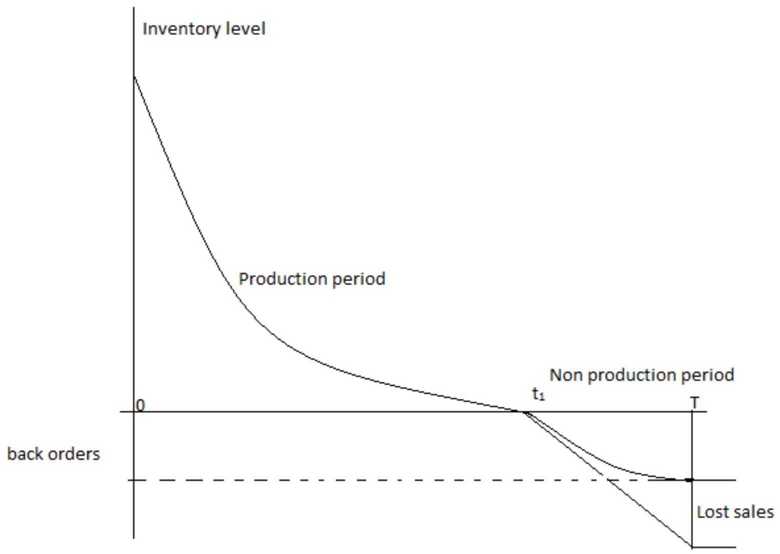

Figure 2.1: Graphical representation of our proposed model.

The solution of equation 2.4.1 using the boundary condition is

$$I_1(t) = (k-x)\Big[t - \frac{ab}{(b+1)}t^{b+1} - \frac{cd}{(d+1)}t^{d+1}\Big] - y\Big[\frac{t^2}{2} - \frac{ab}{2(b+2)}t^{b+2} - \frac{cd}{2(d+2)}t^{d+2}\Big],\ 0 \le t \le t_1. \quad (2.4.3)$$

and the solution of equation 2.4.2 using the boundary condition is

$$I_2(t) = x\Big[(t_1 - t) + (at^b + ct^d)(t - t_1) + \frac{a}{(b+1)}(t_1^{b+1} - t^{b+1}) + \frac{c}{(d+1)}(t_1^{d+1} - t^{d+1})\Big] + y\Big[(\frac{t_1^2}{2} - \frac{t^2}{2}) + (\frac{at^b}{2} + \frac{ct^d}{2})(t^2 - t_1^2) + \frac{a}{(b+2)}(t_1^{b+2} - t^{b+2}) + \frac{c}{(d+2)}(t_1^{d+2} - t^{d+2})\Big],\ t_1 \le t \le T. \quad (2.4.4)$$

Using the boundary condition $I_2(t_1) = Q$ the equation 2.4.4 becomes

$$I_2(t) = Q\Big[1 + a(t_1^b - t^b) + c(t_1^d - t^d)\Big] + x\Big[(t_1 - t) + (at^b + ct^d)(t - t_1) + \frac{a}{(b+1)}(t_1^{b+1} - t^{b+1}) + \frac{c}{(d+1)}(t_1^{d+1} - t^{d+1})\Big] + y\Big[(\frac{t_1^2}{2} - \frac{t^2}{2}) + (\frac{at^b}{2} + \frac{ct^d}{2})(t^2 - t_1^2) + \frac{a}{(b+2)}(t_1^{b+2} - t^{b+2}) + \frac{c}{(d+2)}(t_1^{d+2} - t^{d+2})\Big],\ t_1 \le t \le T. \quad (2.4.5)$$

The maximum inventory level during $[0, T]$ is given by

$$I_0 = \int_0^{t_1} I_1(t)dt = (k-x)\Big[\frac{t_1^2}{2} - \frac{ab}{(b+1)(b+2)}t_1^{b+2} - \frac{cd}{(d+1)(d+2)}t_1^{d+2}\Big] - y\Big[\frac{t_1^3}{6} - \frac{ab}{2(b+2)(b+3)}t_1^{b+3} - \frac{cd}{2(d+2)(d+3)}t_1^{d+3}\Big]. \quad (2.4.6)$$

For the proposed model a cost structure is imposed and it is analyzed by the criteria of minimization of the total expected cost per unit time. Hence for obtaining the total inventory cost we calculate the following terms:

- Annual *ordering cost*,

$$OC = \ A \tag{2.4.7}$$

.

- Total *annual stock holding cost*, HC, during time span $[0, t_1]$is defined as follows:

$$HC = \int_0^{t_1} h(t) I_1(t) dt$$

$$\begin{aligned} HC &= h(k-x)\Big[\frac{t_1^2}{2} - \frac{ab}{(b+1)(b+2)}t_1^{b+2} - \frac{cd}{(d+1)(d+2)}t_1^{d+2}\Big] \\ &\quad -hy\Big[\frac{t_1^3}{6} - \frac{ab}{2(b+2)(b+3)}t_1^{b+3} - \frac{cd}{2(d+2)(d+3)}t_1^{d+3}\Big] \\ &\quad +\gamma(k-x)\Big[\frac{t_1^3}{3} - \frac{ab}{(b+1)(b+3)}t_1^{b+3} - \frac{cd}{(d+1)(d+3)}t_1^{d+3}\Big] \\ &\quad -\gamma y\Big[\frac{t_1^4}{8} - \frac{ab}{2(b+2)(b+4)}t_1^{b+4} - \frac{cd}{2(d+2)(d+4)}t_1^{d+4}\Big]. \end{aligned} \tag{2.4.8}$$

- *Purchase cost*, PC, during time span $[t_1$,T]

$$PC = p\big[I_0 + \int_{t_1}^{T} \delta D(t) dt\big]$$

$$\begin{aligned} PC &= p(k-x)\Big[\frac{t_1^2}{2} - \frac{ab}{(b+1)(b+2)}t_1^{b+2} - \frac{cd}{(d+1)(d+2)}t_1^{d+2}\Big] \\ &\quad -py\Big[\frac{t_1^3}{6} - \frac{ab}{2(b+2)(b+3)}t_1^{b+3} - \frac{cd}{2(d+2)(d+3)}t_1^{d+3}\Big] \\ &\quad +p\delta x(T-t_1) + \frac{1}{2}p\delta y(T^2 - t_1^2). \end{aligned} \tag{2.4.9}$$

- *Deteriorating cost*, DC

$$DC = p\int_0^{t_1} k - (x + yt) dt$$

$$DC = p\big((k-x)t_1 - b\frac{t_1^2}{2}\big). \tag{2.4.10}$$

- *Shortage cost*, SC, during time span $[t_1, T]$ is expressed as:

$$SC = p_2 \int_{t_1}^{T} I_2(t)dt$$

$$\begin{aligned} SC \;=\; & Qp_2\Big[(T-t_1) + a\big(Tt_1^b - \frac{T^{b+1}+bt_1^{b+1}}{b+1}\big) + c\big(Tt_1^d - \frac{T^{d+1}+dt_1^{d+1}}{d+1}\big)\Big] \\ & +xp_2\Big[\frac{2Tt_1 - T^2 - t_1^2}{2} + \frac{a}{b+1}(T^{b+2} - T^{b+1}t_1 + Tt_1^{b+1} - t_1^{b+2}) \\ & +\frac{c}{d+1}(T^{d+2} - T^{d+1}t_1 + Tt_1^{d+1} - t_1^{d+2})\Big] \\ & +yp_2\Big[\frac{3Tt_1^2 - T^3 - 2t_1^3}{6} + \frac{a}{2}\Big(\frac{T^{b+3}-t_1^{b+3}}{b+3} - \frac{T^{b+1}t_1^2 - t_1^{b+3}}{b+1}\Big) \\ & +\frac{c}{2}\Big(\frac{T^{d+3}-t_1^{d+3}}{d+3} - \frac{T^{d+1}t_1^2 - t_1^{d+3}}{d+1}\Big) \\ & +\frac{a}{b+2}\Big[t_1^{b+2}(T-t_1) - \frac{T^{b+3}-t_1^{b+3}}{b+3}\Big] \\ & +\frac{c}{d+2}\Big[t_1^{d+2}(T-t_1) - \frac{T^{d+3}-t_1^{d+3}}{d+3}\Big]\Big]. \end{aligned} \tag{2.4.11}$$

- *Lost sale cost*, Not all customers are willing to wait for the next lot size to arrive during the shortage period $[t_1, T]$, which may cause some loss in profit. Hence *Lost sale cost*, LSC

$$LSC = s\int_{t_1}^{T}(1-\delta)D(t)dt$$

$$LSC = s(1-\delta)\big[x(T-t_1) + y\frac{T^2 - t_1^2}{2}\big]. \tag{2.4.12}$$

Hence the total average cost of the system per time unit denoted by TC defined as

$$TC = \frac{1}{T}[OC + HC + PC + DC + SC + LSC]. \tag{2.4.13}$$

where the component costs are as given in equation numbers from 2.4.7 to 2.4.12

2.5 Solution procedure and algorithm

The total average cost given by equation 2.4.13 is a highly nonlinear equation in T and t_1 and our problem is to determine the optimal values of T and t_1 that minimize the total average cost TC. The optimum values of T and t_1 is obtained by equating to zero the first order partial derivatives of total average cost(TC) with respect to T and t_1 as follows:

$$\frac{\partial TC}{\partial t_1} = 0 \tag{2.5.1}$$

and

$$\frac{\partial TC}{\partial T} = 0 \tag{2.5.2}$$

These two partial derivatives yield a minimizer (T^*, t_1^*) provided that the following second order sufficient conditions are satisfied at that point.

$$(\frac{\partial^2 TC}{\partial t_1^2})(\frac{\partial^2 TC}{\partial T^2}) - (\frac{\partial^2 TC}{\partial t_1 \partial T})^2 > 0 \tag{2.5.3}$$

and

$$(\frac{\partial^2 TC}{\partial t_1^2}) > 0, \ (\frac{\partial^2 TC}{\partial T^2}) > 0 \tag{2.5.4}$$

Equation 2.4.13 is our objective function which needs to be minimized. For this, we use the classical optimization techniques. The equations 2.5.1 and 2.5.2 obtained thereafter and are highly non-linear in the variable T and t_1. However, if we give particular values to the discrete variables, our objective function becomes the function of two variables T and t_1. We have used the mathematical software MATHEMATICA to arrive at the solution of the system in consideration. We can obtain the optimal values of different values of the time with the help of this software. With the use of these optimal values equation 2.4.13 provides minimum total average cost per unit time of the system in consideration.

We can also show the solution procedure step by step as given below:

2.5.1 Algorithm

Step 1 : Initialize the value of the variables $A, h, p, k, x, y, a, b, c, d, \gamma, \delta, Q, p_2$, and s.

Step 2 : Find t_1^* which satisfying equation 2.5.1

Step 3 : Find T^* which satisfying equation 2.5.2

Step 4 : If such t_1^* and T^* are found,then check if that t_1^* and T^* are also satisfying equation 2.5.3and equation 2.5.4

Step 5 : If every condition is satisfied ,then calculate TC from equation 2.4.13

Step 6 : The optimal solution is (T^*, t_1^*, TC^*), where t_1^* and T^* are the associated values of t_1 and T,respectively, and TC^* is the associated value of TC.

2.6 Numerical examples

The proposed model is illustrated below by considering the examples, where all associated parameters are taken in proper units. The optimal solution of the inventory system is calculated with the help of MATHEMATICA software.

Example 1: Consider an inventory system with parameters $A = 2000, h = 0.8$, $p = 20, k = 35, x = 20, y = 40, a = 0, b = 1.6, c = 0, d = 1.1, \gamma = 0.95, \delta = 0.7, Q = 25, p_2 = 6, s = 8$. Then the optimal solution is t_1^*=1.4321 and T^*=6.11341 and the minimum total average inventory cost TC^*=1132.41. The graphical representation of the total average cost in Example 1 for the proposed model is shown in Figure 2.2.

Example 2: Consider an inventory system with parameters $A = 2000, h = 0.8$, $p = 20, k = 35, x = 20, y = 40, a = 0.5, b = 0.8, c = 0.5, d = 0.4, \gamma = 0.95, \delta = 0.7, Q = 25, p_2 = 6, s = 8$. Then the optimal solution is t_1^*=1.6143 and T^*=6.4638 and the minimum total average inventory cost TC^*=1867.85. The graphical representation of the total average cost in Example 2 for the proposed model is shown in Figure 2.3.

Example 3: Consider an inventory system with parameters $A = 2000, h = 0.8$, $p = 20, k = 35, x = 20, y = 40, a = 0.5, b = 1.6, c = 0.5, d = 0.7, \gamma = 0.95, \delta = 0.7, Q = 25, p_2 = 6, s = 8$. Then the optimal solution is t_1^*=1.6472 and T^*=6.25146 and the

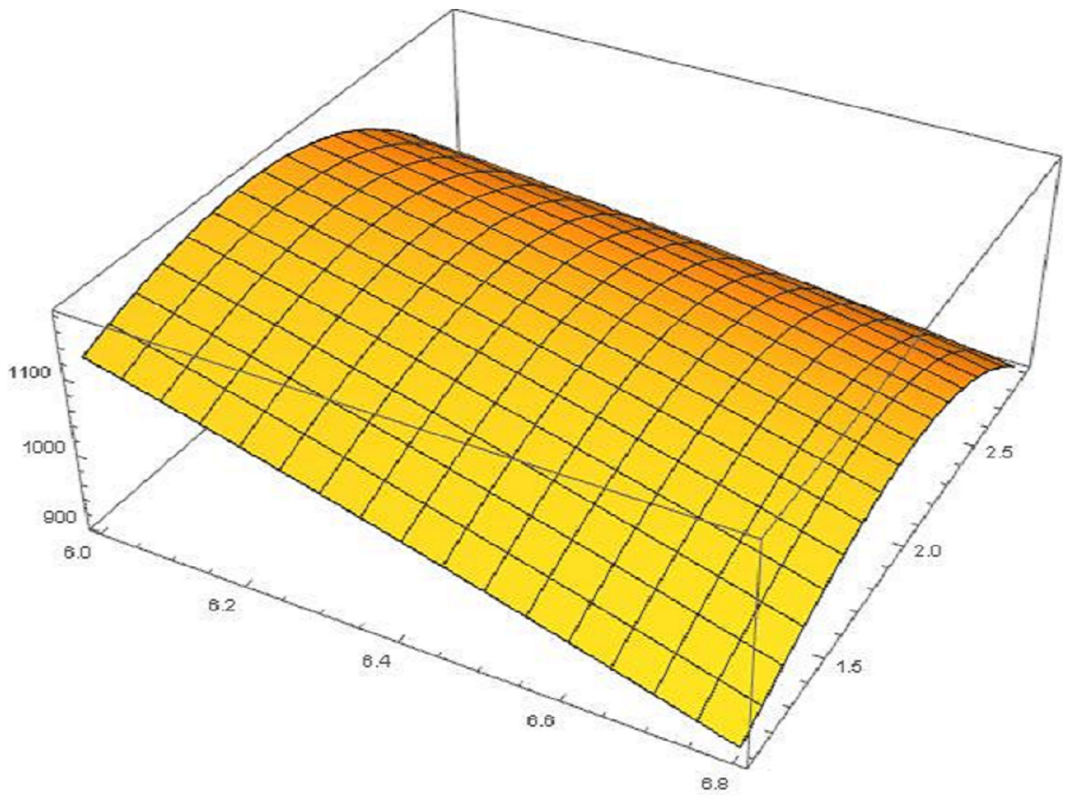

Figure 2.2: Total average cost according to various choices of parameters with t_1, T and TC along the x-axis,the y-axis and the z-axis respectively.

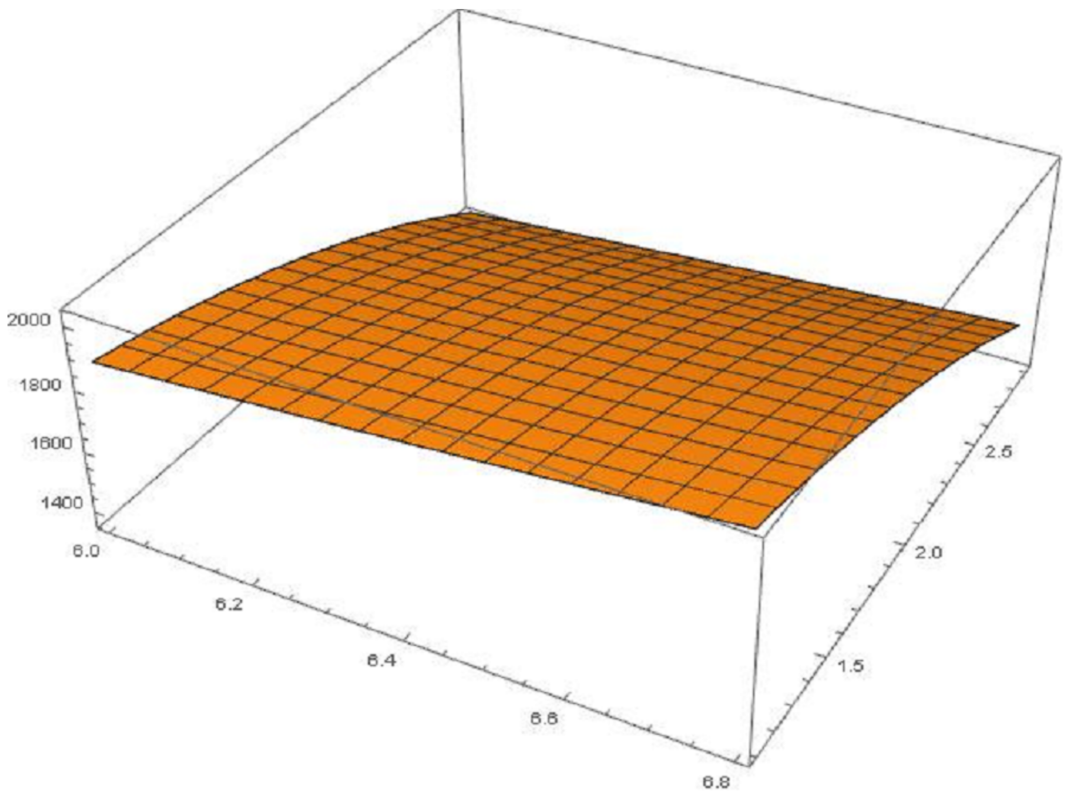

Figure 2.3: Total average cost according to various choices of parameters with t_1, T and TC along the x-axis,the y-axis and the z-axis respectively.

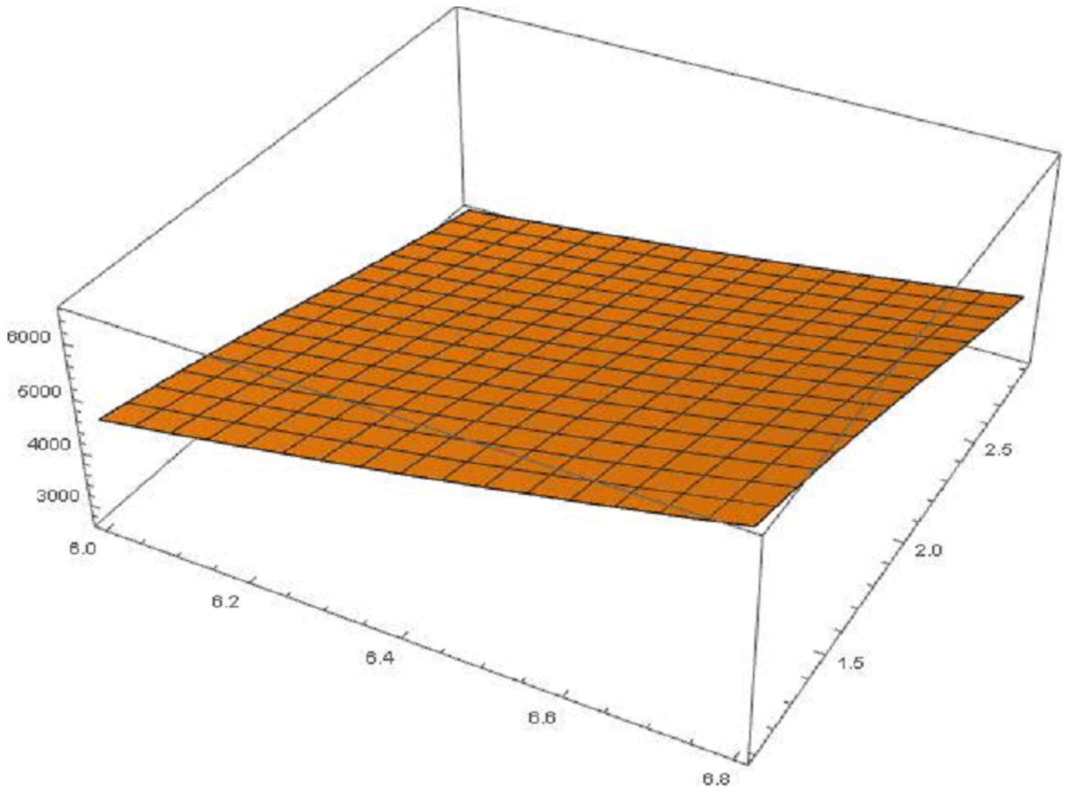

Figure 2.4: Total average cost according to various choices of parameters with t_1, T and TC along the x-axis,the y-axis and the z-axis respectively.

minimum total average inventory cost TC^*=5139.46. The graphical representation of the total average cost in Example 3 for the proposed model is shown in Figure 2.4.

Example 4: Consider an inventory system with parameters $A = 2000, h = 0.8,$ $p = 20, k = 35, x = 20, y = 40, a = 0.5, b = 1.6, c = 0.5, d = 1.1, \gamma = 0.95, \delta = 0.7, Q = 25, p_2 = 6, s = 8$. Then the optimal solution is t_1^*=1.93214 and T^*=6.14792 and the minimum total average inventory cost TC^*=5076.92. The graphical representation of the total average cost in Example 4 for the proposed model is shown in Figure 2.5.

From the above four numerical examples, if we do not consider the deterioration of the item (see Example 1), then the minimum total average inventory cost is less than that of the proposed inventory system with additive Weibull distribution deterioration. Example 3, which shows the constant rate of deterioration, has a higher total average inventory cost than the other two examples. So, our proposed model with additive Weibull distribution deterioration is more appropriate when dealing with a general

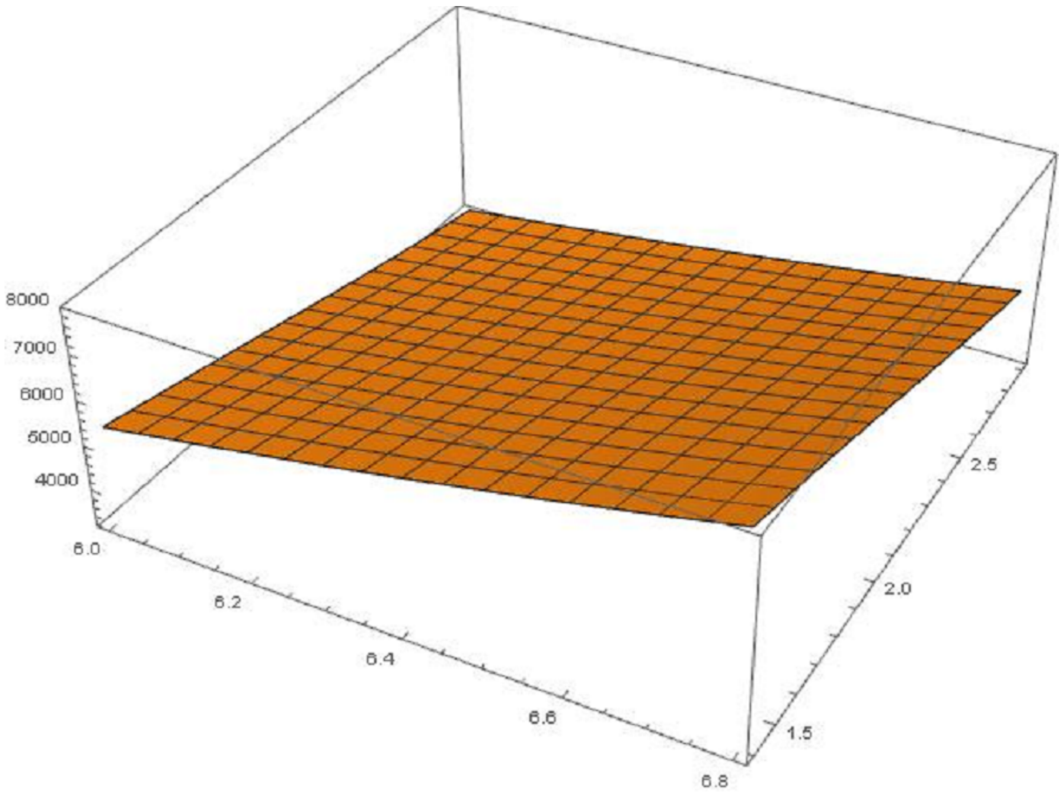

Figure 2.5: Total average cost according to various choices of parameters with t_1, T and TC along the x-axis,the y-axis and the z-axis respectively.

deterioration rate.

2.7 Conclusion

The proposed model incorporates some realistic and practical features, viz., the demand function and holding costs being time-dependent, the inventory deteriorating at a variable rate over time, and assumptions like shortages being allowed and completely backlogged. The new model is introduced with an additive Weibull deterioration rate to model different phases of deterioration rate. Four numerical assessments of the theoretical model have been done to illustrate the theory. The variations in the system statistics with a variation in system parameters have also been illustrated graphically. The solution obtained also revealed that the model was quite suitable and stable. All these facts together make this study unique and matter of fact, and we examined total inventory cost in each situation.

Chapter 3

AN EOQ MODEL FOR DETERIORATING ITEMS UNDER TWO-LEVEL TRADE CREDIT FINANCING WITH EXPIRATION DATE

3.1 Introduction

The relaxation to payment in a business transaction is vital for the smooth functioning of demand and sales. In early transactions it was assumed that retailers had to pay for received items immediately. But this assumption is no longer practical in a highly competitive market. Delayed payment has become a valuable strategy for suppliers to boost profits by encouraging more sales. It also presents a unique opportunity for retailers to reduce uncertainty in demand and the associated risks. In essence, when suppliers ship ordered units to retailers without immediate payment, they effectively shift the responsibility and costs of storage to the retailers, while bearing the risk of uncertain demand. Typically, no interest is levied on the outstanding amount if

Some contents of this chapter are based on Praveen and Manoharan (2025).

paid within the agreed-upon grace period. This allows the retailer to sell the goods, earn interest on the accumulated revenue, and delay payment until the last moment of the supplier's permitted period. However, if the payment is not made within the predefined grace period, interest is imposed in accordance with the established terms and conditions. This setup offers a financial benefit to retailers, allowing them to accrue interest from the revenue generated throughout the grace period.

Numerous businesses employ trade credit strategies to expand their operational capacity, reach a wider customer base, and secure a convenient source of short-term financing. Trade credit plays a pivotal role in enabling these organizations to procure goods and services without the immediate requirement of payment. In recent times, a growing body of researchers has devoted considerable focus to trade credit, introducing diverse policies within their inventory models to harness its potential. This research has become instrumental in enhancing business operations, customer outreach, and financial management. Kreng and Tan (2010) introduced an inventory model incorporating two-level trade credit policies, while Tayal et al. (2016) introduced an integrated inventory model that combined trade credit with investments in preservation technology. In a two-warehouse environment, Jaggi et al. (2019) examined the impact of trade credit on inventory models. Most recently, Tiwari et al. (2022) delved into the retailer's credit and inventory decisions in the context of imperfect quality and deteriorating items under a two-level trade credit policy. To mitigate the adverse effects of expiration dates on retailer order incentives, the application of a two-level trade credit system, encompassing both upstream and downstream aspects, has been a common strategy, as noted in the works of Wu et al. (2018) and Mahata et al. (2020). Tsao et al. (2023) proposed a model to determine the optimal selling price, lot size, and level of intelligent effort while maximizing the manufacturer's profit under a two-level trade credit policy.

Quantity loss pertains to the continuous removal of a portion of deteriorated quantity during the expiration date. Dye et al. (2018) and Wu et al. (2017a) have all highlighted that the deterioration rate steadily increases as the expiration date approaches, ultimately resulting in full deterioration. Consequently, more items are discarded over time, leading to a reduction in the retailer's available inventory. Conversely, quality loss reflects the decline in the freshness of deteriorating items as they near their expiration date, influenced by various factors like temperature and humidity Mahato and Mahata (2021a). Additionally, consumer preferences favor items further from their expiration date, suggesting that demand for deteriorating items is closely tied to product freshness, which can be perceived through the expiration date, see Li and Teng (2018).

Incorporating the research areas and objectives mentioned in the above works, it is found that no works were considered a combination of relaxation in the payment system demand as a function of the selling price and its expiration. Hence this chapter is developed by considering these facts and proposed appropriate model for solving this. The model is validated with the help of illustrative numerical examples. The effectiveness of the proposed model with the consideration of parameters in the study were discussed in the remaining sections of this chapter. On these lines different sections in this chapter are developed and presented.

3.2 Motivation, research questions and gaps

This chapter highlights the impact of an effective strategy that benefits economically and environmentally to meet the needs of today's business management scenario. Product expiration dates and impacts in quantity loss and quality loss are essential in depicting real-life inventory management. The literature studies show that relaxation in the payment system is beneficial for overcoming loss. It should be emphasized that not many research works have taken both quantity and quality loss into account. In practice, achieving success in inventory management necessitates a thorough consideration of deteriorating items, alongside the recognition of the interplay between demand rates and factors such as item price and freshness condition. Thus, in this chapter, we present an inventory model that takes into account these critical assumptions to offer retailers a more realistic and precise understanding of the inventory system and its processes. As motivated by these, we developed the research objectives of this chapter, summarised below.

- To study the impact of demand rates, in turn, on the dependability of selling price and freshness condition of the item.

- To get an idea of influence of purchasing decision with respect to the expiration date.

- To investigate the affect of two level trade credit system in buying decision.

- To investigate the association of various parameters in the proposed study with the maximum profit function.

An inventory management system with single type of deteriorating item with an expiration date, involving both quantity and quality losses is the theme of this chapter, The incorporation of printed expiration date, freshness index and two level payment system for deteriorating items is a significant research gap and in this chapter we developed the model to fill this gap.

3.3 Problem Description

It is widely acknowledged that the demand for fresh produce contingent significantly on its freshness, and increasing shelf space for displayed stocks often encourages more purchases. Credit policies and expiration dates frequently play a pivotal role in consumers' purchase decisions. In this chapter, we put forth an EOQ model aimed at determining the retailer's optimal credit period and cycle time. This model incorporates several key elements:

1. The supplier extending a trade credit period (M) to the retailer.

2. The retailer offering a trade credit period (N) to their customers.

3. Dealing with deteriorating items that have an expiration date (L), where the replenishment cycle time (T) doesn't exceed L.

4. The demand rate being contingent on both the selling price and the freshness index of the inventory.

5. Replenishment occurring instantaneously, with no allowances for shortages.

Within the framework of these conditions, we frame the retailer's inventory model as a profit maximization problem. To facilitate the development of our models for deteriorating items under two-level trade credit financing with an expiration date, we consider the following notations and assumptions which have used throughout this chapter.

3.3.1 Notations

Objective Function

Symbol	**Description**
$TP(s, T)$	The retailers average profit per unit time

Decision Variables

Symbol	**Description**
s	Selling price per unit
T	Recycle order time of the inventory cycle, $T \leq L$

Cost Parameters

Symbol	**Description**
A	Replenishment cost per order
p	Purchase cost per unit
h	The holding cost per unit per unit time
C_T	Fixed transportation cost
C_t	Variable transportation cost per unit
I_e	Rate of interest earned by the retailer
I_c	Rate of interest payable to the supplier

Inventory Parameters

Symbol	Description
L	Maximum lifetime (ie. time to its expiration date)in years
θ	Deterioration rate of the item
N	The customers credit period granted by retailer
M	Period of permissible delay in payments offered by the supplier
$I(t)$	Inventory level at time t
Q	Order quantity per cycle
Q_d	The total sales volume during the order cycle

3.3.2 Assumptions

The developed inventory model in this chapter is formulated upon the following assumptions:

1. The replenishment rate is assumed to be infinite, with no lead time, and shortages are not permitted.

2. The inventory system focuses on a single type of deteriorating item with an expiration date, involving both quantity and quality losses.

3. Deterioration occurs within the inventory cycle. As per references to Mahata (2016) and Wu et al. (2017b), the quantity loss rate of items during the expiration date (denoted as L) is defined as follows:
   $\theta(t) = \frac{1}{1+L-t}, 0 \leq t \leq T \leq L.$
   As time approaches the expiration date, $\theta(t)$ is tends to 1.

4. Consumer purchasing decisions are influenced by printed expiration dates. In essence, a consumer's likelihood to purchase an item decreases as the item approaches its expiration date. For simplicity and manageability, the freshness index at time t is linearly decreasing from 1 at the beginning to 0 at the maximum lifetime (L), as per references to Chen et al. (2016) and Li and Teng (2018). The freshness index ($F(t)$) is given by:
   freshness index, $F(t) = \frac{L-t}{L}$, $0 \leq t \leq T \leq L$. It is important to note that the replenishment cycle time (T) must be less than or equal to the product's lifetime (L).

5. The payment system offers flexibility for settling accounts. When $T \geq M$, the account is settled at $T = M$. The retailer pays for all units sold and retains their profits. From that point on, the retailer starts paying interest charges on the items in stock at a rate of I_p. When $T \leq M$, the account is settled at time $T = M$, and no interest charges are incurred. Additionally, if $M > N$, the retailer can accumulate revenue and earn interest during the period from N to M under the trade credit conditions, with an interest rate of I_e.

6. The demand rate is determined by both the item's selling price and its freshness index. Sales are influenced by the selling price, following observations in Mahata (2015) and Dye (2012). According to traditional marketing and economic theory, higher prices generally result in lower demand. Consequently, the demand function is a decreasing, convex function of the selling price, denoted as $d(s)$, where $d(s)$ is continuous and $d(s) > 0$.

7. The freshness of the item also impacts customer buying behavior. Initially, when the product is fresh, there is no significant effect on demand. However, as the product loses its freshness over time, demand decreases. By drawing insights from Chen et al. (2016), Feng et al. (2017), and Dobson et al. (2017), the demand rate D is considered as a freshness-dependent function for perishable products. Combining these two factors, the demand rate is expressed as:
$D(t, s) = d(s)(\frac{L-t}{L})$, $0 \leq t \leq T$

3.4 Mathematical formulation

In the inventory model proposed in this chapter, guided by the previously outlined assumptions, we start with the premise that at the commencement of each replenishment cycle, the retailer acquires an order quantity, denoted as Q, from the supplier. This inventory is then prominently displayed on the store's shelves but has a limited shelf life, commonly referred to as the expiration date. Once this date passes, the inventory becomes unsuitable for meeting consumer demands. Throughout the course of the replenishment cycle, the inventory level at any given time, denoted as t, undergoes a reduction due to the combined effects of market demand and the item's deterioration. The proposed model's inventory level is depicted in Figure 3.1.

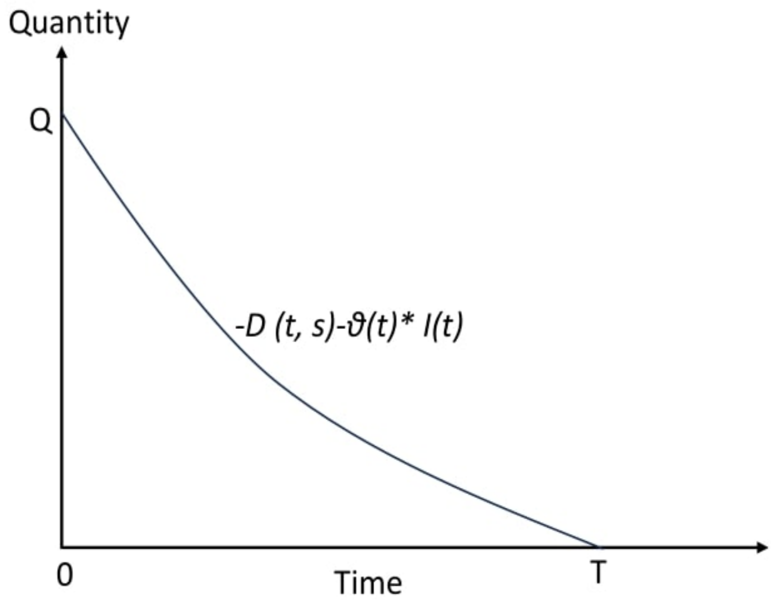

Figure 3.1: Inventory flow representation.

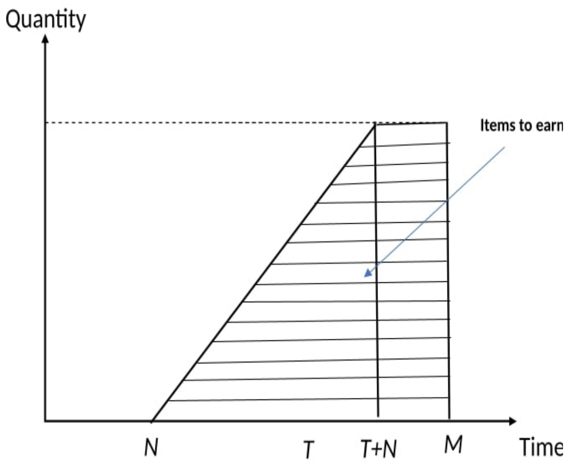

Figure 3.2: The retailers interest earned and interest charged when $T + N \leq M$.

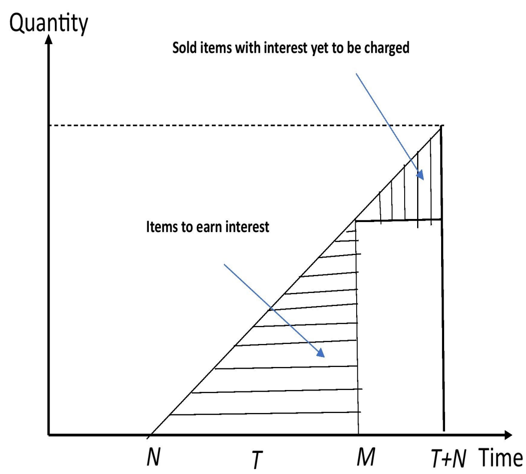

Figure 3.3: The retailers interest earned and interest charged when $T + N \geq M$.

This change is described by the following differential equation.

$$\frac{dI(t)}{dt} = -D(t,s) - \theta(t)I(t)$$

$$\frac{dI(t)}{dt} + \frac{1}{1+L-t}I(t) = -d(s)\left(\frac{L-t}{L}\right), 0 \leq t \leq T. \tag{3.4.1}$$

Utilizing the boundary condition where $I(t) = 0$ at $t = T$, the solution to equation 3.4.1 takes the following form:

$$I(t) = d(s)\frac{1+L-t}{L}\left[\ln\left(\frac{1+L-T}{1+L-t}\right) + T - t\right],\ 0 \leq t \leq T. \tag{3.4.2}$$

By substituting $t = 0$ into equation 3.4.2 and considering the initial condition $I(0) = Q$, and rearranging the terms, we determine the order quantity, denoted as Q, at the commencement of each cycle as:

$$Q = I(0) = d(s)\frac{1+L}{L}\left[\ln\left(\frac{1+L-T}{1+L}\right) + T\right]. \tag{3.4.3}$$

In the context of the proposed model, we have established a total profit structure and examined it with the aim of maximizing the total expected profit per unit of time. To assess the total profit per replenishment cycle of the inventory system, we take into account various components, including sales revenue (SR), ordering cost (OC), transportation cost (TPC), purchase cost (PC), inventory holding cost (HC), and annual capital opportunity cost (OPC). Consequently, these components that together constitute the average profit function for the retailer are presented below.

1. The total *Sales revenue* over the entire cycle encompasses the aggregate revenue earned from satisfying the demand throughout the cycle. Within each replenishment cycle, the retailer's total sales volume is determined as:

$$Q_d = \int_0^T D(t,s)dt = \frac{d(s)T(2L-T)}{2L}.$$

Hence, the *Sales revenue* for each replenishment cycle is:

$$SR = sQ_d$$

$$SR = s\frac{d(s)T(2L-T)}{2L}. \tag{3.4.4}$$

2. *Ordering cost* for each replenishment cycle is:

$$OC = A \tag{3.4.5}$$

.

3. *Purchase cost* per cycle,

$$PC = pQ = pd(s)\frac{1+L}{L}\left[\ln\left(\frac{1+L-T}{1+L}\right)+T\right]. \tag{3.4.6}$$

4. *Inventory holding cost*, IHC, during the cycle

$$IHC = h\int_0^T I(t)dt$$

$$IHC = \frac{hd(s)}{12L}\Bigg[6(1+L)^2\ln\left(\frac{1+L-T}{1+L}\right) - T\big(2T^2-6LT-3T-6L-6\big)\Bigg]. \tag{3.4.7}$$

5. The *Transportation cost*, comprising both fixed and variable expenses, for each replenishment cycle, is:

$$TPC = c_T + c_tQ = c_T + c_td(s)\frac{1+L}{L}\left[\ln\left(\frac{1+L-T}{1+L}\right)+T\right]. \tag{3.4.8}$$

Now, in accordance with the given assumption, there are two scenarios to consider when calculating the annual capital opportunity cost. This involves computing both the interest earned and the interest paid for the items held in stock.

$Scenario$ 1: $N < M$

In this scenario, two possible cases can emerge:

$case1.1$: $T + N \leq M$ and $case1.2$: $T + N \geq M$.

Now, let's explore the comprehensive formulation of each case.

$case1.1$: $T + N \leq M$

As shown in Figure 3.2, When $T+N \leq M$, the retailer receives sales revenue for all items at time $T+N$ and can fully cover the total purchasing cost by M. Consequently, there is no interest charged, denoted as $IC_{1.1} = 0$. Conversely, during the time interval $[N, T+N]$, the retailer can earn interest on the sales revenues received from customers and on the full sales revenue during the period $[T+N, M]$. As a result, the annual interest earned per cycle is determined as:

$$IE_{1.1} = \frac{sI_e}{T}\left[\int_N^{T+N}\int_N^{t+N} D(u-N,s)dudt + (M-T-N)Qd\right].$$

$$IE_{1.1} = sI_e d(s)\left[\frac{T}{2} + N - \frac{T^2}{6L} - \frac{TN}{2L} - \frac{N^2}{2L} + (M-T-N)\frac{2L-T}{2L}\right]. \quad (3.4.9)$$

Now, the overall annual profit per unit of time is calculated as:

$$
\begin{aligned}
TP_{1.1}(T,s) = s\frac{d(s)(2L-T)}{2L} - \frac{1}{T}\Bigg\{ & A + c_T + (p+c_t)d(s)\frac{1+L}{L}\left[\ln\left(\frac{1+L-T}{1+L}\right)+T\right] \\
& + \frac{hd(s)}{12L}\left[6(1+L)^2\ln\left(\frac{1+L-T}{1+L}\right) - T\big(2T^2-6LT-3T-6L-6\big)\right]\Bigg\} \\
& + sI_e d(s)\left[\frac{T}{2}+N-\frac{T^2}{6L}-\frac{TN}{2L}-\frac{N^2}{2L}+(M-T-N)\frac{2L-T}{2L}\right].
\end{aligned}
\tag{3.4.10}
$$

Therefore, the relevant optimization problem can be expressed as follows:

Problem 1:

Maximize $TP_{1.1}(T,s)$

subject to $0 < T+N \leq M$

$case1.2$: $T+N \geq M$

In the case of $T+N \geq M$, as seen in Figure 3.3, the retailer does not receive the final payment before the allowable delay period M. Consequently, the retailer is required to finance all items sold after time $(M-N)$ until time M and settle the loan by $T+N$ at an interest rate of I_c per year. Therefore, the interest charged is calculated as follows:

$$
IC_{1.2} = \frac{pI_c}{T}\int_M^{T+N} I(t-N)dt = \frac{pI_c}{T}\int_{M-N}^{T} I(t)dt.
$$

$$
\begin{aligned}
IC_{1.2} = \frac{pI_c d(s)}{LT}\Bigg[& \frac{(1+L)T^2}{2} - \frac{T^3}{6} + \frac{T}{2}\big\{(M-N)^2 - 2(1+L)(M-N)\big\} \\
& + \frac{(M-N)^2(1+L)}{2} - \frac{(M-N)^3}{3} + \frac{(1+L-M+N)^2}{2}\ln\frac{1+L-T}{1+L-M+N} \\
& - \frac{T^2}{4} + \frac{(L+1)T}{2} + \frac{(M-N)^2-2(1+L)(M-N)}{4}\Bigg].
\end{aligned}
\tag{3.4.11}
$$

Conversely, during the period $[N, M]$, the retailer can earn interest on the sales revenues received from the delayed payments within that time frame. Hence, the annual interest earned is calculated as follows:

$$IE_{1.2} = \frac{sI_e}{T}\left[\int_N^M \int_N^{t+N} D(u-N,s)dudt\right].$$

$$IE_{1.2} = \frac{sI_e d(s)}{T}\left[\frac{M^2-N^2}{2} - \frac{M^3-N^3}{6L}\right]. \tag{3.4.12}$$

Now, the overall annual profit per unit of time is calculated as:

$$\begin{aligned}
TP_{1.2}(T,s) = s\frac{d(s)(2L-T)}{2L} - \frac{1}{T}\Bigg\{ & A + c_T + (p+c_t)d(s)\frac{1+L}{L}\left[\ln\left(\frac{1+L-T}{1+L}\right)+T\right] \\
& + \frac{hd(s)}{12L}\left[6(1+L)^2\ln\left(\frac{1+L-T}{1+L}\right) - T\big(2T^2-6LT-3T-6L-6\big)\right] \\
& + sI_e d(s)\left[\frac{M^2-N^2}{2} - \frac{M^3-N^3}{6L}\right] \\
& - \frac{pI_c d(s)}{L}\Bigg[\frac{(1+L)T^2}{2} - \frac{T^3}{6} + \frac{T}{2}\big\{(M-N)^2 - 2(1+L)(M-N)\big\} \\
& + \frac{(M-N)^2(1+L)}{2} - \frac{(M-N)^3}{3} \\
& + \frac{(1+L-M+N)^2}{2}\ln\left(\frac{1+L-T}{1+L-M+N}\right) \\
& - \frac{T^2}{4} + \frac{(L+1)T}{2} + \frac{(M-N)^2 - 2(1+L)(M-N)}{4}\Bigg]\Bigg\}.
\end{aligned} \tag{3.4.13}$$

Thus, the corresponding optimization problem can be expressed as follows:

Problem 2:

Maximize $TP_{1.2}(T,s)$

subject to $T + N \geq M$

Scenario 2: $N \geq M$

Since $N \geq M$ for the scenario 2, Interest earned is ($IE_2 = 0$). The interest charged per cycle is

$$IC_2 = \frac{pI_c}{T}\int_0^T I(t)dt + (N-M)Q_d.$$

$$IC_2 = \frac{pI_c d(s)}{2LT}\Bigg\{(N-M)T(2L-T) + \frac{1}{6}\Bigg[6(1+L)^2\ln\left(\frac{1+L-T}{1+L}\right) - T\big(2T^2 - 6LT - 3T - 6L - 6\big)\Bigg]\Bigg\}. \quad (3.4.14)$$

Now, the overall annual profit per unit of time for this scenario becomes

$$TP_2(T,s) = s\frac{d(s)(2L-T)}{2L} - \frac{1}{T}\Bigg\{A + c_T + (p+c_t)d(s)\frac{1+L}{L}\left[\ln\left(\frac{1+L-T}{1+L}\right) + T\right] + \frac{(h-pI_c)d(s)}{12L}\Bigg[6(1+L)^2\ln\left(\frac{1+L-T}{1+L}\right) - T\big(2T^2 - 6LT - 3T - 6L - 6\big)\Bigg] - \frac{pI_c d(s)}{2L}\Big((N-M)T(2L-T)\Big)\Bigg\}. \quad (3.4.15)$$

Hence, the corresponding optimization problem is as follows:

Problem 3:

Maximize $TP_2(T,s)$

subject to $N \geq M$

3.5 Optimal solutions and theoretical results

The overall profit function accounts for sales revenue, ordering cost, purchasing cost, inventory holding cost, transportation cost, interest earned and interest payable charges. The retailer's average profit per unit of time can be represented as:

$$TP(T,s) = \frac{1}{T}(SR - OC - PC - IHC - TPC + IE - IC) \tag{3.5.1}$$

$$TP(T,s) = \begin{cases} TP_{1.1}(T,s), & N \leq M, and T + N \leq M \\ TP_{1.2}(T,s), & N \leq M, and T + N \geq M \\ TP_{2}(T,s), & N \geq M \end{cases}$$

The decision variables in this chapter for the proposed model are the optimal values of s and T that maximize the overall profit function, denoted as $TP(T,s)$, for the provided data set. To achieve the maximum total profit function, the necessary conditions involve setting the first-order derivatives equal to zero with respect to the decision variables. Hence, the essential conditions are:

$$\frac{\partial TP(T,s)}{\partial s} = 0, \frac{\partial TP(T,s)}{\partial T} = 0 \tag{3.5.2}$$

Now the conditions sufficient for profit maximization are outlined as follows.

$$(\frac{\partial^2 TP(T,s)}{\partial s^2}) < 0, \ (\frac{\partial^2 TP(T,s)}{\partial T^2}) < 0 \tag{3.5.3}$$

and

$$(\frac{\partial^2 TP(T,s)}{\partial s^2})(\frac{\partial^2 TP(T,s)}{\partial T^2}) - (\frac{\partial^2 TP(T,s)}{\partial s \partial T})^2 > 0 \tag{3.5.4}$$

The optimal solution of the problem will be denoted as s^* and T^* which are obtained under various cases mentioned above. Given the highly non-linear nature of equations 3.5.2, 3.5.3, and 3.5.4, we utilized the mathematical software MATHEMATICA to compute the solution for the system under consideration. Alternatively, we have shown

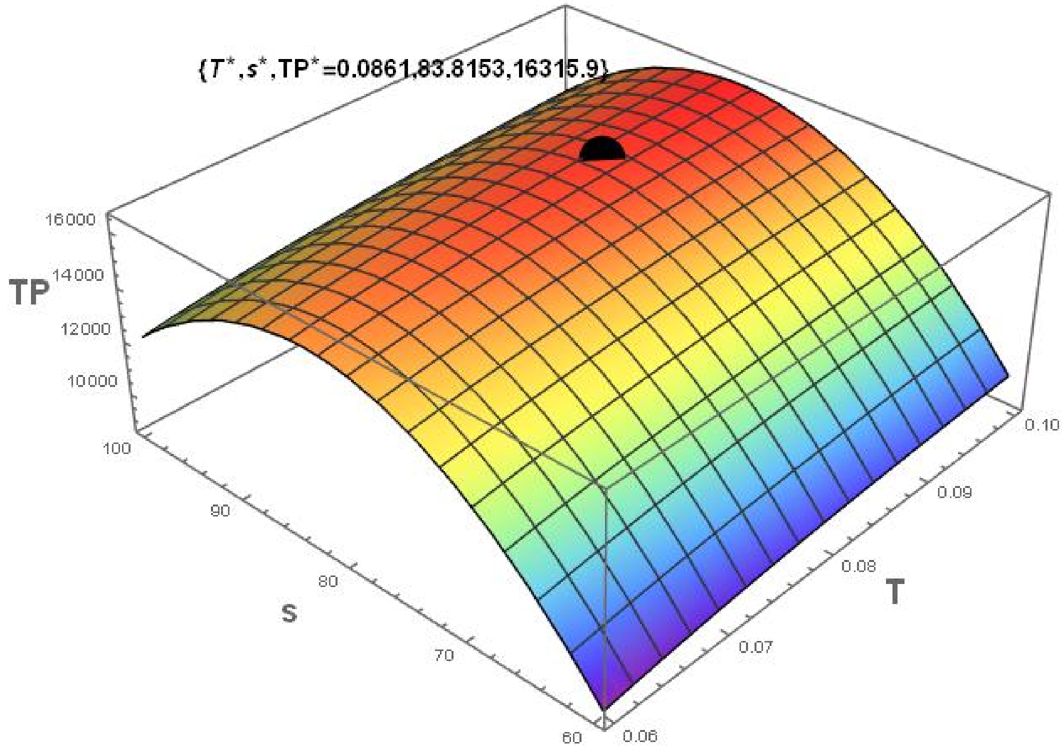

Figure 3.4: Total average profit according to various choices of parameters with s, T and TP along the x-axis,the y-axis and the z-axis respectively.

the concavity of these profit functions graphically for all the cases and sub cases. With the use of these optimal values provides maximum total average profit per unit time of the system under consideration.

3.6 Numerical examples

The proposed inventory model in this chapter is demonstrated by using three numerical examples using different parameters in proper units.

Example 1: Consider an inventory system with parameters

A=250/order, C_T=50/ship, p=40/unit, C_t=3/unit, h= 2/unit/year, L= 0.7 year, $d(s) = 1500 - 12s$, I_e=0.15/year, I_c=0.1/year, N= 0.2 year, M= 0.3 year. Then the optimal solution is s^*=83.8153 and T^*=0.0861 and the maximum average profit TP^*=16315.9. Figure 3.4 displays the graphical representation of the total average profit with respect to the decision variables for the proposed model in Example 1.

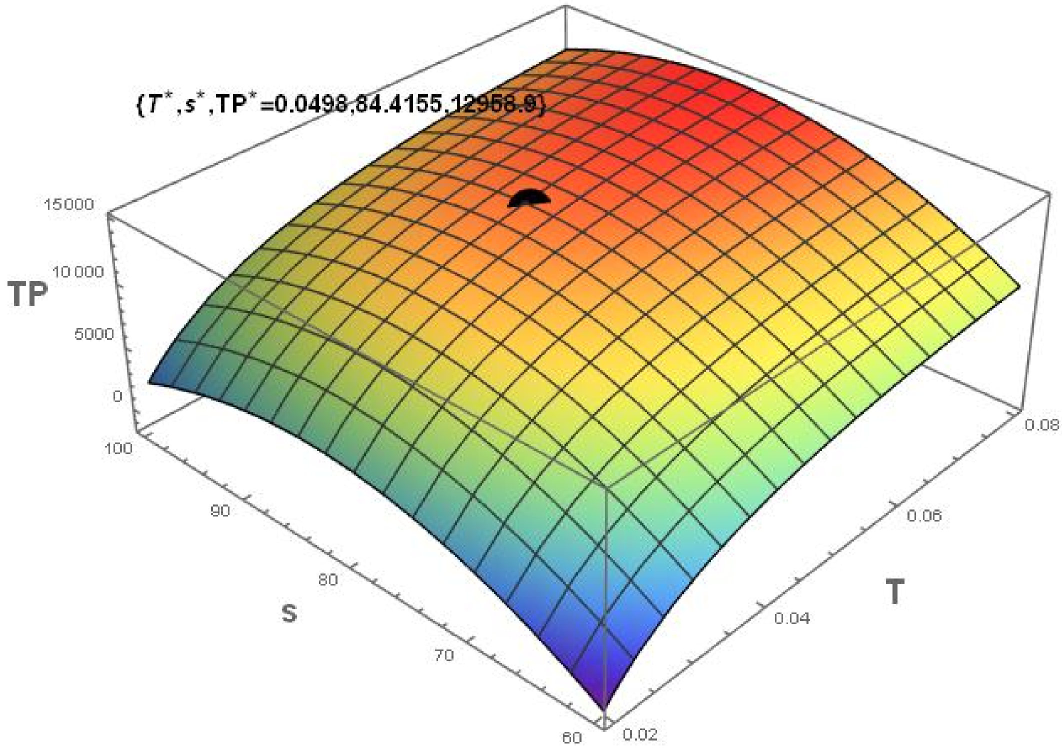

Figure 3.5: Total average profit according to various choices of parameters with s, T and TP along the x-axis,the y-axis and the z-axis respectively.

Example 2: Consider an inventory system with parameters

A=250/order, C_T=50/ship, p=40/unit, C_t=3/unit, h= 2/unit/year, L= 0.7 year, $d(s) = 1500 - 12s$, I_e=0.15/year, I_c=0.1/year, N= 0.06 year, M= 0.08 year. Then the optimal solution is s^*=84.4155 and T^*=0.0498 and the maximum average profit TP^*=12958.9. Figure 3.5 displays the graphical representation of the total average profit with respect to the decision variables for the proposed model in Example 2.

Example 3: Consider an inventory system with parameters

A=250/order, C_T=50/ship, p=40/unit, C_t=3/unit, h= 2/unit/year, L= 0.7 year, $d(s) = 1500 - 12s$, I_e=0.15/year, I_c=0.1/year, N= 0.8 year, M= 0.3 year. Then the optimal solution is s^*=83.7174 and T^*=0.0354 and the maximum average profit TP^*=11459.4. Figure 3.6 displays the graphical representation of the total average profit with respect to the decision variables for the proposed model in Example 3.

The three examples given above is worked for finding the optimal solution. The nature of the the profit functions in these examples shows the high degree of non-linearity

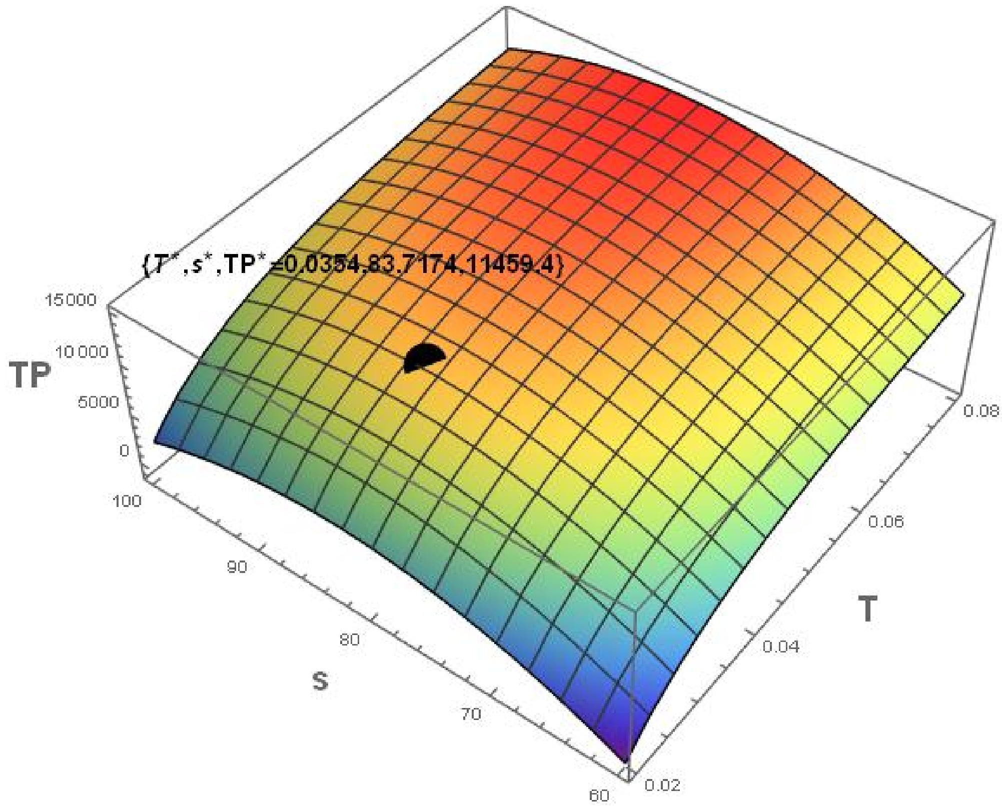

Figure 3.6: Total average profit according to various choices of parameters with s, T and TP along the x-axis,the y-axis and the z-axis respectively.

and complexity in their solutions. Therefore, their joint concavity is challenging to demonstrate. As an alternate of this we visually exhibit the concavity of these profit functions graphically using the software MATHEMATICA. (refer to Figure. 3.4, 3.5, and 3.6).

3.7 Sensitivity analysis

The influence of the different parameters used in the proposed inventory model in this chapter is checked using sensitivity analysis and the results were summarised. Parameters are taken one at a time to change while others remain unchanged. The examples shown above were used for sensitivity analysis also and the changes are shown in Tables 3.5, 3.6 and 3.7.

3.7.1 Discussions on sensitivity analysis

The sensitivity analysis is performed and the Tables 3.5, 3.6 and 3.7 gives the numerical results. The solutions can be interpreted to get the influence of each of the parameter in the model.

- We observe that from sensitivity analysis table when replenishment cost per order A increases, the recycle order time of the inventory cycle T^* and selling price, s^* increased whereas the retailers average profit per unit time, TP^* decreased. Hence, the retailer's objective is to minimize the replenishment cost in order to maximize profits. In addition, with a higher ordering cost, the retailer should order for longer replenishment periods to reduce the frequency of orders, thereby diminishing the overall ordering cost.

- Sensitivity analysis of L, maximum lifetime indicates that an increased value of L leads to a reduced value of s^* but higher values of both T^* and TP^*. This signifies that as the capability to store deteriorating items for an extended duration increases, both the quantity and quality losses might decelerate slightly, which stimulates market demand to some extent and ultimately elevating the retailers performance.

Table 3.5: Variation in Total average profit with respect to different parameters of Example 1.

Changing parameter	Change	s^*	T^*	TP^*
A	175	83.7754	0.0798	17111.9
	212.50	83.7967	0.0832	16705.5
	250	83.8153	0.0861	16315.9
	287.5	83.8228	0.0873	15909.3
	325	83.8342	0.0891	15529.1
L	0.5	83.8519	0.0857	15782.0
	0.6	83.8333	0.0858	16086.9
	0.7	83.8153	0.0861	16315.9
	0.8	83.7983	0.0863	16492.6
	0.9	83.7790	0.0865	16635.5
C_T	35	82.8068	0.0849	16468.3
	42.5	83.8121	0.0856	16393.5
	50	83.8153	0.0861	16315.9
	57.5	83.8191	0.0867	16241.0
	65	83.8221	0.0872	16165.0
p	28	77.8610	0.0825	22371.8
	34	80.8396	0.0848	19249.0
	40	83.8153	0.0861	16315.9
	46	86.7935	0.0876	13595.6
	52	89.7760	0.0891	11084.2
h	1.4	83.8097	0.0872	16349.0
	1.7	83.8116	0.0865	16329.7
	2	83.8153	0.0861	16315.9
	2.3	83.8195	0.0858	16304.0
	2.6	83.8225	0.0853	16288.0
N	0.05	84.0322	0.0868	15936.2
	0.10	83.9516	0.0866	16074.2
	0.15	83.8804	0.0863	16199.5
	0.2	83.8153	0.0861	16315.9
M	0.30	83.8153	0.0861	16315.9
	0.35	83.7350	0.0854	16444.9
	0.40	83.6582	0.0850	16579.9
	0.45	83.5790	0.0844	16710.9

– We observe that the fixed transportation cost, C_T increases, then the recycle order time T^* and selling price, s^* both rises, but the average profit for the retailer decreases significantly. Consequently, maintaining only the necessary quantity in

Table 3.6: Variation in Total average profit with respect to different parameters of Example 2.

Changing parameter	Change	s^*	T^*	TP^*
A	175	84.3900	0.0436	13922.7
	212.50	84.4032	0.0469	13433.6
	250	84.4155	0.0498	12958.9
	287.50	84.4288	0.0528	12540.7
	325	84.4398	0.0552	12098.3
L	0.5	84.4432	0.0472	12369.1
	0.6	84.4297	0.0487	12709.5
	0.7	84.4155	0.0498	12958.9
	0.8	84.4034	0.0511	13196.9
	0.9	84.3940	0.0529	13458.0
C_T	35	84.4103	0.0486	13138.6
	42.5	84.4125	0.0491	13037.3
	50	84.4155	0.0498	12958.9
	57.5	84.4177	0.0503	12861.4
	65	84.4212	0.0511	12796.9
p	28	78.2988	0.0473	18834.9
	34	81.3574	0.0488	15818.5
	40	84.4155	0.0498	12958.9
	46	87.4728	0.0504	10271.9
	52	90.5318	0.0512	7820.41
h	1.4	84.4135	0.0511	13097.6
	1.7	84.4139	0.0503	13014.0
	2	84.4155	0.0498	12958.9
	2.3	84.4162	0.0491	12881.1
	2.6	84.4172	0.0485	12812.1
N	0.04	84.5084	0.0527	12613.2
	0.05	84.4673	0.0512	12943.3
	0.06	84.4155	0.0498	12958.9
	0.07	84.3550	0.0488	13027.2
M	0.075	84.3801	0.0490	12925.9
	0.08	84.4155	0.0498	12958.9
	0.085	84.4533	0.0512	13048.3
	0.09	84.4895	0.023	13101.2

the inventory would be beneficial for avoiding excessive expenses associated with inventory carrying costs.

Table 3.7: Variation in Total average profit with respect to different parameters of Example 3.

Changing parameter	Change	s^*	T^*	TP^*
A	175	8.7038	0.0331	13183.1
	212.50	83.7109	0.0343	12303.2
	250	83.7174	0.0354	11459.4
	287.5	83.7268	0.0370	10779.9
	325	83.7350	0.0384	10107.5
L	0.5	83.9829	0.0348	10865.5
	0.6	83.8588	0.0350	11147.4
	0.7	83.7174	0.0354	11459.4
	0.8	83.5577	0.0359	11790.6
	0.9	83.3782	0.0362	12083.4
C_T	35	83.7085	0.0339	11557.3
	42.5	83.7132	0.0347	11518.8
	50	83.7174	0.0354	11459.4
	57.5	83.7215	0.0361	11401.7
	65	83.7233	0.0364	11259.8
p	28	77.8091	0.0347	17415.9
	34	80.7632	0.0351	14347.0
	40	83.7174	0.0354	11459.4
	46	86.6726	0.0358	8797.2
	52	89.6299	0.0364	6380.41
h	1.4	83.7178	0.0368	11677.2
	1.7	83.7194	0.0362	11633.0
	2	83.7197	0.0354	11459.4
	2.3	83.7209	0.0349	11345.6
	2.6	83.7216	0.0346	11274.7
N	0.75	83.7681	0.0357	11447.2
	0.8	83.7174	0.0354	11459.4
	0.85	83.6678	0.0353	11484.8
	0.9	83.6176	0.0351	11619.2
M	0.25	83.6613	0.0342	11283.8
	0.3	83.7174	0.0354	11459.4
	0.4	83.8295	0.0378	11854.4
	0.5	83.9316	0.0385	11890.5

– If the Purchase cost, p considered to be increases then the both decision variables in the proposed study, s^* and T^* increases and the average profit for the retailer decreases significantly. Thus the results say that higher values of purchase cost definitely lower the average profit.

- As the holding cost increases, the selling price also rises. However, recycling order time and the average profit of the retailer decreases. This is very expected because the retailer does not want to keep too much inventory when the holding cost is high, also selling price will increase with the holding cost.

- The trade credit periods parameters, N and M offered by retailer and supplier respectively effect in the average profit of the retailer. An increase in the variables N and M results decrease of the decision variables selling price and recycle order time but in turn increases the retailers average profit.

3.8 Conclusion

This chapter introduces an inventory model tailored for deteriorating items with an expiration period to evaluate the impact of both quantity and quality losses within a trade credit policy. The demand rate is depend on the selling price and the condition of inventory freshness. A novel model is presented employing a two-level trade credit approach, allowing the supplier to grant the retailer a delay period. This policy aims to stimulate customer demand, reflecting real-world business scenarios. Numerical examples are provided to showcase the model's effectiveness and validate the stability and optimality of the solution. The analysis reveals that the $case1.1$: $T + N \leq M$ yields higher profitability compared to other cases. This scenario involves no interest charges, as the retailer can cover the total purchasing cost by M and receives sales revenue for all items at time $T + N$. The chapter highlights the sensitivity of total average cost to various demand and associated parameters. It affirms the existence of a unique and optimal solution for the retailer.

Besides these facts the other novelties are such as regarding the effect of maximum life time and trade credit period parameters on the selling price and the retailers average profit. Maximum lifetime leads to reduced selling price and in turn higher profit value. This signifies that capability of store the items will change the market demand.The payment policy applied in this model directly involve in changing average profit value.

Chapter 4

INVENTORY DECISIONS FOR DETERIORATING AND AMELIORATING ITEMS WITH PARTIAL BACKLOGGING AND TIME VARYING DEMAND

4.1 Introduction

Previous chapters explored the inventory model for deteriorating items under different conditions of inventory parameters used in it. From the literature studies various inventory models addressed the importance of having one or both of the phenomena of incurring amelioration and deterioration. The decay, spoilage, or damage of the products during the period is termed as deterioration and many researchers have included this concept in their study to connect the inventory model to real life practices. Demand for inventory items is another criterion for developing a lot of inventory models to visualize real life situations. The unmet inventory may be partially or fully backlogged with a loss depending on the customer's waiting time or loss of goodwill. So different

Some contents of this chapter are based on Praveen and Manoharan (2026).

inventory models were checked for how these scenarios were depicted in their studies. The utility of some items in the marketplace shows an increasing nature with time. These items whose quantity and utility increase with time are known as ameliorating items. We can find real life examples where amelioration occurs. In some tropical countries merchants manage big farms growing fruit items like pineapple, orange, banana, etc. They store the yield until there is a higher demand. Similarly, in farming, fast growing animals such as fish, poultry, pig, sheep, etc. and some brewage wine are examples of ameliorating items. They keep in stock which in turn increases its value.

Many researchers focused on inventory models with deteriorating items in the initial stages of economic order quantity (EOQ) model studies and little attention was given to amelioration. The study of Chowdhury et al. (2016) discussed deteriorating items replenishment policy with demand rate considered a continuous quadratic function of time and partially backlogged shortages. Rai (2018) considered the deterioration rate of ameliorating items in an increasing nature with real life business situations. The work considered two storage facilities - an own warehouse and a rented warehouse. Sebatjane and Adetunji (2019) compared exponential, linear and split linear growth rates for chickens by considering only amelioration. The optimal storage time of wine and cheese products was used by Zanoni et al. (2019) to characterize the amelioration and provide an optimal aging period. Alfares and Afzal (2021) conducted a study of growing items, focusing on reducing holding costs during the consumption period while allowing for shortages with complete backorder. In their study, Mittal and Sharma (2021) found the optimum order quantity for a certain category of chicken in the growing items inventory. An imperfect production inventory model for reworked items with non-instantaneous deteriorating items is developed by De and Narang (2021) using a genetic algorithm. De-La-Cruz-Mrquez et al. (2022) developed an optimization model for growing items in a supply chain considering the imperfect quality and shortages with full backorder. Duary et al. (2022) developed an inventory problem for deteriorating items involving the impact of advertisements on products and partial backlog with the concepts of advance and delayed payment. Marchi et al. (2022) investigated inventory management models focusing on ameliorating and deteriorating items to identify the existing literature gaps. Tiwari et al. (2022) discussed the inventory of imperfect quality items which is affected by deterioration and trade credit policy. A vendor managed inventory model for deteriorating items is introduced by Salas-Navarro et al. (2023) with a three-layer supply chain. Saraswat and Sharma (2023) developed an inventory model for the growing items with price dependent demand and deterioration. Singh and Rana (2023) derived a mathematical model for growing items with linear demand deteriorating items. Khan et al. (2024) illustrated an inventory model with a livestock farm focusing on pricing decisions. In this work, the consumption rate is adopted as a power form of both selling price and storage duration.

The theme of the inventory model in this chapter is obtained from the research gap seen in the above research works. To develop the inventory model, we highlighted real life scenarios with inventory items inherent ameliorating and deteriorating nature. The remaining sections of this chapter are presented by providing notations, assumptions used for developing the model, validation of the model with numerical examples, and variation of the decision variables concerning the change in parameters using sensitivity analysis.

4.2 Motivation, research questions and gaps

The details from the literature reviews suggest that the combined nature of deterioration and amelioration, together with some significant change in their effect, directly depends on the optimum decision. The increasing nature of associated money in certain items while kept in hand depicts real life scenarios and is a vital topic in the study of inventory control. From these, this chapter is developed and presents an inventory model with deterioration and amelioration effects where demand and shortages are considered more realistically to understand the proposed model better. Possible research questions based on this are presented below and were answered in the following sections.

- Introduce the inventory model highlighting real life scenarios inherent both amelioration and deterioration.

- Develop the model with demand is linear time varying and shortages are considered as partially backlogged.
- To study the effect of time varying deterioration rate, and amelioration rate of two parameter Weibull distribution
- To discuss the two phase dealing of deterioration and amelioration.

The system consists of only one item and the demand in the model is linear time varying and shortages are considered which is partially backlogged. Hence the model fills the gap with the contributions of time varying deterioration rate, and amelioration rate of two parameter Weibull distribution.

4.3 Problem Description

The cost optimization problem on the basis of deteriorating and ameliorating factor is incorporate in this model. A time varying deterioration effect in the first phase of the system is a key element in the study. As utility of the items considered in the model increases with time and decreases due to spoilage the amelioration of the item is taken to follow two parameter Weibull distribution. The linear time dependent demand rate is another element in the model. Another factor is a fraction of the demand is backlogged which depends on the waiting time. The objective functions and decisions variables were constructed and optimization problem in this chapter is build under the following notations and assumptions which have used throughout this chapter.

4.3.1 Notations

Objective Function

Symbol	Description
$TC(t_2, T)$	Average total cost per unit time of the system

Decision Variables

Symbol	Description
t_1	Time at maximum positive stock of the item in the system
t_2	Time up to positive stock of the item in the system
T	The cycle length (Unit of time)

Cost Parameters

Symbol	Description
A	Ordering cost per order
C_p	Purchase cost per unit
C_h	Holding cost per unit per unit time
C_d	Deterioration cost per unit
C_a	Amelioration cost per unit
C_s	Shortage cost per unit per unit time
C_o	Opportunity cost per unit

Inventory Parameters

Symbol	Description
a, b	Demand parameters
θ	Deterioration rate parameter
α, β	Amelioration rate parameters
δ	Backlogging parameter
η	Ratio between the two time phases, t_1 and t_2

4.3.2 Assumptions

The developed inventory model formulated upon the following assumptions:

1. Infinite time horizon with inventory system consists of only one item and lead time is taken as zero.

2. Deterioration occurs on the first phase of time horizon with its rate increases with time, $\theta(t) = \theta t$, $0 < \theta \ll 1$.

3. The amelioration of the items in the study follows two parameter Weibull distribution as its utility increases with time and decreases due to spoilage. The rate is $\alpha\beta t^{\beta-1}$, where α is the shape parameter ranges as $0 < \alpha \ll 1$ and β, the scale parameter, $\beta > 0$.

4. The effect of deterioration on the items under study is considered much smaller than effect of amelioration.

5. Linear time dependent demand rate, $R(t) = a + bt$, where a and b greater than zero.

6. Shortages is considered with a rate $B(t) = \frac{1}{1+\delta(T-t)}$, a fraction of demand is backlogged. Here $(T-t)$ is the waiting time and $\delta > 0$ is the constant backlogging parameter.

4.4 Mathematical formulation

The proposed model has the time horizon 0 to T which is divided into three phases. In the first phase, starting at $t = 0$ inventory level is taken as I_{ini}. Due to amelioration and deterioration, where effect of deterioration rate is lower than the amelioration, the growing up items takes the level to I_{max} at time $t = t_1$. After t_1 the second phase starts. The quantity of the matured items gradually diminishes due to demand and deterioration and at time $t = t_2$ the inventory level reaches zero. So the positive stock will be till t_2 and then the shortages occur in the third phase till T which is partially backlogged. The cycle repeats and the order quantity will be the sum of the initial inventory level and partially backlogged items. The two time points t_1 and t_2 are taken with ratio $\eta = \frac{t_1}{t_2} < 1$. The governing equations to represent the three phases of the inventory system are as follows. The equation for the first phase of growing up due to the amelioration and low deterioration is

$$\frac{dI_1(t)}{dt} + (\theta t - \alpha\beta t^{\beta-1})I_1(t) = 0, 0 \le t \le t_1. \tag{4.4.1}$$

with boundary conditions $I_1(0) = I_{ini}$ and $I_1(t_1) = I_{max}$ and the expansion for small values of x, $e^x \approx 1 + x$, the solution of equation 4.4.1 is

$$I_1(t) = I_{ini}(1 + \alpha t^{\beta} - \frac{\theta t^2}{2}). \tag{4.4.2}$$

Put the condition $I_1(t_1) = I_{max}$, the maximum inventory level obtained as

$$I_{max} = I_{ini}(1 + \alpha t_1^{\beta} - \frac{\theta t_1^2}{2}). \tag{4.4.3}$$

The equation for the second phase where inventory level decline due to deterioration and linear time dependent demand is

$$\frac{dI_2(t)}{dt} + (\theta t - \alpha\beta t^{\beta-1})I_2(t) = -a + bt, t_1 \leq t \leq t_2. \tag{4.4.4}$$

with boundary conditions $I_2(t_2) = 0$, the solution is

$$\begin{aligned} I_2(t) = a &\left[(t_2 - t) + \frac{\theta}{6}(t_2^3 - 3t^2 t_2 + 2t^3)\right. \\ &\left.+ \frac{\alpha}{\beta+1}\left((\beta+1)t_2 t^{\beta} - \beta t^{\beta+1} - t_2^{\beta+1}\right)\right] \\ &+ b\left[\frac{1}{2}(t_2^2 - t^2) + \frac{\theta}{8}(t_2^4 - 2t^2 t_2^2 + t^4)\right. \\ &\left.+ \frac{\alpha}{2(\beta+2)}\left((\beta+2)t_2^2 t^{\beta} - \beta t^{\beta+2} - 2t_2^{\beta+2}\right)\right], \quad t_1 \leq t \leq t_2. \end{aligned} \tag{4.4.5}$$

The shortage phase from t_2 to T represented by the governing equation

$$\frac{dI_3(t)}{dt} = \frac{-(a + bt)}{1 + \delta(T - t)}, \ t_2 \leq t \leq T. \tag{4.4.6}$$

with $I_3(t_2) = 0$ provide the solution as

$$I_3(t) = \left(\frac{a}{\delta} + \frac{b}{\delta}\left(T + \frac{1}{\delta}\right)\right) \ln\left(\frac{1 + \delta(T - t)}{1 + \delta(T - t_2)}\right) + \frac{b}{\delta}(t - t_2), \quad t_2 \leq t \leq T. \tag{4.4.7}$$

Put the condition $-I_3(T) = I_{back}$, the maximum partially backlogged items is

$$I_{\text{back}} = \left(\frac{a}{\delta} + \frac{b}{\delta}\left(T + \frac{1}{\delta}\right)\right)\ln\left(\frac{1}{1+\delta(T-t_2)}\right) + \frac{b}{\delta}(T-t_2). \qquad (4.4.8)$$

Hence the replenishment cycle will start with an order of quantity Q given by

$$\begin{aligned} Q &= I_{\text{ini}} + I_{\text{back}} \\ &= I_{\text{ini}} + \left(\frac{a}{\delta} + \frac{b}{\delta}\left(T + \frac{1}{\delta}\right)\right)\ln\left(\frac{1}{1+\delta(T-t_2)}\right) + \frac{b}{\delta}(T-t_2). \end{aligned} \qquad (4.4.9)$$

Total positive inventory holding during the time interval $[0, t_2]$ is

$$\begin{aligned} I_T &= \int_0^{t_1} I_1(t)\,dt + \int_{t_1}^{t_2} I_2(t)\,dt \\ &= I_{\text{ini}}\left[t_1 + \frac{\alpha t_1^{(\beta+1)}}{\beta+1} - \frac{\theta t_1^3}{6}\right] \\ &\quad + a\left[\frac{1}{2}(t_2-t_1)^2 + \frac{\theta}{6}\left(t_2 t_1^3 - t_2^3 t_1 + \frac{1}{2}(t_2^4 - t_1^4)\right)\right. \\ &\quad \left. + \frac{\alpha}{\beta+1}\left(t_1 t_2^{\beta+1} - t_1^{\beta+1} t_2 + \frac{\beta}{\beta+2}(t_1^{\beta+2} - t_2^{\beta+2})\right)\right] \\ &\quad + b\left[\frac{1}{2}\left(\frac{2t_2^3}{3} - t_2^2 t_1 + \frac{t_1^3}{3}\right)\right. \\ &\quad + \frac{\theta}{8}\left(\frac{8t_2^5}{15} - t_2^4 t_1 + \frac{2}{3}t_2^2 t_1 - \frac{t_1^5}{5}\right) \\ &\quad \left. - \frac{\alpha\beta t_2^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{\alpha}{2(\beta+2)}\left(\frac{\beta+2}{\beta+1}t_2^2 t_1^{\beta+1} + \frac{\beta}{\beta+2}t_1^{\beta+3} + 2t_2^{\beta+2}t_1\right)\right]. \end{aligned} \qquad (4.4.10)$$

The total number of deteriorated units during inventory cycle after eliminating higher powers of θ and α is given by

$$\begin{aligned} T_{\text{det}} &= \theta\left[\int_0^{t_1} tI_1(t)\,dt + \int_{t_1}^{t_2} tI_2(t)\,dt\right] \\ &= \theta\left[I_{\text{ini}}\frac{t_1^2}{2} + a\left(\frac{t_2^3}{6} + \frac{t_1^3}{3} - \frac{t_2 t_1^2}{2}\right) + \frac{b}{4}t_1^2(t_1^2 - t_2^2)\right]. \end{aligned} \qquad (4.4.11)$$

The total number of ameliorating units during inventory cycle after eliminating higher powers of θ and α is given by

$$\begin{aligned} T_{\text{ame}} &= \theta\left[\int_0^{t_1} t^{\beta-1} I_1(t)\,dt + \int_{t_1}^{t_2} t^{\beta-1} I_2(t)\,dt\right] \\ &= \alpha\left[I_{\text{ini}} t_1^{\beta} + a\left(\frac{t_2^{\beta+1}}{\beta+1} - t_2 t_1^{\beta} + \frac{\beta t_1^{\beta+1}}{\beta+1}\right)\right. \\ &\left. + \frac{b}{2}\left(\frac{2t_2^{\beta+2}}{\beta+2} - t_2^2 t_1^{\beta} + \frac{\beta t_1^{\beta+2}}{\beta+2}\right)\right]. \end{aligned} \quad (4.4.12)$$

The total number of shortages during the period $[t_2, T]$ is given by

$$\begin{aligned} T_{\text{sho}} &= -\int_{t_2}^{T} I_3(t)\,dt \\ &= \left(\frac{a}{\delta} + \frac{b}{\delta^2} + \frac{b(T+t_2)}{2\delta}\right)(T-t_2) \\ &+ \frac{1}{\delta}\left(\frac{a}{\delta} + \frac{b}{\delta^2} + \frac{bT}{\delta}\right)\ln\left(\frac{1}{1+\delta(T-t_2)}\right). \end{aligned} \quad (4.4.13)$$

The amount of lost sales during the period $[t_2, T]$ is given by

$$\begin{aligned} T_{\text{lost}} &= \int_{t_2}^{T} (a+bt)\left[1 - \frac{1}{1+\delta(T-t)}\right] dt \\ &= (a + \frac{b}{\delta} + \frac{b(T+t_2)}{2})(T-t_2) \\ &+ \left(\frac{a}{\delta} + \frac{b}{\delta^2} + \frac{bT}{\delta}\right)\ln\left(\frac{1}{1+\delta(T-t_2)}\right). \end{aligned} \quad (4.4.14)$$

Hence with respect to all these units different cost components associated with the inventory system are calculated as below.

1. The fixed cost of placing the initial order, OC = A

2. Purchase cost per cycle = *Purchase cost/unit)* $\times$ *Order quantity in one cycle*

$$\begin{aligned} PC &= C_p \times Q \\ &= C_p\left[I_{\text{ini}} + \left(\frac{a}{\delta} + \frac{b}{\delta^2}\right)\ln\left(\frac{1}{1+\delta(T-t_2)}\right) + \frac{b}{\delta}(T-t_2)\right]. \end{aligned} \quad (4.4.15)$$

3. Holding cost over the period $[0,T] = C_h \times I_T$, I_T from equation 4.4.10

4. Cost due to deterioration $CD = C_d \times T_{det}$, T_{det} from equation 4.4.11

5. The amelioration cost $AMC = C_a \times T_{ame}$, T_{ame} from equation 4.4.12

6. Cost due to shortages $CS = C_s \times T_{sho}$, T_{sho} from equation 4.4.13

7. Opportunity cost $OPC = C_o \times T_{lost}$, T_{lost} from equation 4.4.14

Combining all these costs the average total cost per unit time of the system during the cycle $[0,T]$ is obtained as

$$
\begin{aligned}
TC(t_2,T) &= \frac{1}{T}\Bigg\{OC + PC + HC + CD + AMC + CS + OPC\Bigg\} \\
&= \frac{1}{T}\Bigg\{A + C_p\Bigg[I_{\text{ini}} + \left(\frac{a}{\delta} + \frac{b}{\delta}\left(T + \frac{1}{\delta}\right)\right)\ln\left(\frac{1}{1+\delta(T-t_2)}\right) + \frac{b}{\delta}(T-t_2)\Bigg] \\
&\quad + C_h\Bigg[I_{\text{ini}}\left(t_1 + \frac{\alpha t_1^{\beta+1}}{\beta+1} - \frac{\theta t_1^3}{6}\right) + a\Bigg(\frac{1}{2}(t_2-t_1)^2 + \frac{\theta}{6}\left(t_2t_1^3 - t_2^3t_1 + \frac{1}{2}(t_2^4 - t_1^4)\right) \\
&\qquad + \frac{\alpha}{\beta+1}\left(t_1t_2^{\beta+1} - t_1^{\beta+1}t_2 + \frac{\beta}{\beta+2}(t_1^{\beta+2} - t_2^{\beta+2})\right)\Bigg] \\
&\quad + C_d\theta\left[I_{\text{ini}}\frac{t_1^2}{2} + a\left(\frac{t_2^3}{6} + \frac{t_1^3}{3} - \frac{t_2t_1^2}{2}\right) + \frac{b}{4}t_1^2(t_1^2 - t_2^2)\right] \\
&\quad + C_a\alpha\Bigg[I_{\text{ini}}t_1^{\beta} + a\left(\frac{t_2^{\beta+1}}{\beta+1} - t_2t_1^{\beta} + \frac{\beta t_1^{\beta+1}}{\beta+1}\right) \\
&\qquad + \frac{b}{2}\left(\frac{2t_2^{\beta+2}}{\beta+2} - t_2^2t_1^{\beta} + \frac{\beta t_1^{\beta+2}}{\beta+2}\right)\Bigg] \\
&\quad + \left(C_o + \frac{C_s}{\delta}\right)\Bigg[\left(a + \frac{b}{\delta} + \frac{b(T+t_2)}{2}\right)(T-t_2) \\
&\qquad + \left(\frac{a}{\delta} + \frac{b}{\delta}\left(T + \frac{1}{\delta}\right)\right)\ln\left(\frac{1}{1+\delta(T-t_2)}\right)\Bigg]\Bigg\}.
\end{aligned}
\tag{4.4.16}
$$

4.5 Solution procedure and algorithm

The cost function $TC(t_2, T)$ in equation 4.4.16 consists of the decision variables t_2 and T the optimum values of which are to be derived. The optimal values t_2^* and $T*$are obtained by the conditions,

$$\frac{\partial TC(t_2,T)}{\partial t_2} = 0, \frac{\partial TC(t_2,T)}{\partial T} = 0 \tag{4.5.1}$$

and the optimal value of t_1 is obtained by $t_1^* = \eta t_2^*$. Hence the non-linear optimisation problem becomes Minimize:

$$\begin{aligned}
&\text{Minimize } TC(t_2, T)\\
&\text{Subject to}\\
&\quad t_2 \leq T\\
&\quad 0 \leq t_2, \quad 0 \leq T
\end{aligned}$$

For minimum cost function, the necessary conditions are

$$(\frac{\partial^2 TC(t_2,T)}{\partial t_2^2}) > 0,\ (\frac{\partial^2 TC(t_2,T)}{\partial T^2}) > 0 \tag{4.5.2}$$

and

$$(\frac{\partial^2 TC(t_2,T)}{\partial t_2^2})(\frac{\partial^2 TC(t_2,T)}{\partial T^2}) - (\frac{\partial^2 TC(t_2,T)}{\partial t_2 \partial T})^2 > 0 \tag{4.5.3}$$

would be satisfied. Now the optimal values of the average total cost per unit time of the system T_c^* of $TC(t_2, T)$ is obtained by putting the value t_1=t_1^*, t_2=t_2^* and $T = T^*$in the expression given in equation 4.4.16.

4.5.1 Algorithm

To obtain the optimal solution of the proposed problem the following algorithm can be used.

Step 1: Assign values to all the parameters in the inventory system.

Step 2: Differentiate equation 4.4.16 with respect to both decision variables t_2 and T.

Step 3: To identify the critical points setting the derivatives equal to zero.

Step 4: Evaluate optimum cost of $TC(t_2, T)$ by substituting for t_2^* and T^*.

4.6 Numerical examples

The proposed model is checked for the nature of the solution under a realistic view using three numerical examples using MATHEMATICA software. An inventory system with the following parameters in their proper unit were considered in the three examples.

Example 1: Assume the parameters as A=500, C_p=8/unit, C_h=7, C_d=4, C_a=6, C_s=5, C_o=3, a=30, b=0.2, θ=0.001, α=0.003, β=2, δ=0.5, I_{ini}=50, η=0.3. Then, by solving equation 4.5.1, the values of T,t_1 and t_2 are derived as $T^* = 7.13733$, t_1^*=0.366834 and t_2^*=1.22278, which satisfy the conditions given in equation 4.5.2 and equation 4.5.3. Now, utilizing the values of T^*,t_1^* and t_2^* in equation 4.4.16, the total relevant cost of the inventory system are calculated as TC^*=236.504.

Example 2. Assume the parameters as A=500, C_p=8/unit, C_h=7, C_d=4, C_a=6, C_s=5, C_o=3, a=30, b=0.2, θ=0.001, α=0.003, β=2, δ=0.5, I_{ini}=50,η=0.2. Then, by solving equation 4.5.1, the values of T,t_1 and t_2 are derived as $T^* = 6.97162$,t_1^*=0.235512 and t_2^*=1.17756, which satisfy the conditions given in equation 4.5.2 and equation 4.5.3. Now, utilizing the values of T^*,t_1^* and t_2^* in equation 4.4.16, the total relevant cost of the inventory system are calculated as TC^*=233.77.

Example 3. Assume the parameters as A=500, C_p=8/unit, C_h=7, C_d=4, C_a=6, C_s=5, C_o=3, a=30, b=0.2, θ=0.001, α=0.003, β=2, δ=0.5, I_{ini}=50, η=0.4. Then, by solving equation 4.5.1, the values of T,t_1 and t_2 are derived as $T^* = 7.29788$, $t_1^* = 0.496024$ and t_2^*=1.24006, which satisfy the conditions given in equation 4.5.2 and equation 4.5.3. Now, utilizing the values of T^*,t_1^* and t_2^* in equation 4.4.16, the total relevant cost of the inventory system are calculated as TC^*=239.607.

From the illustrated examples the minimum total cost is obtained when t_1 takes

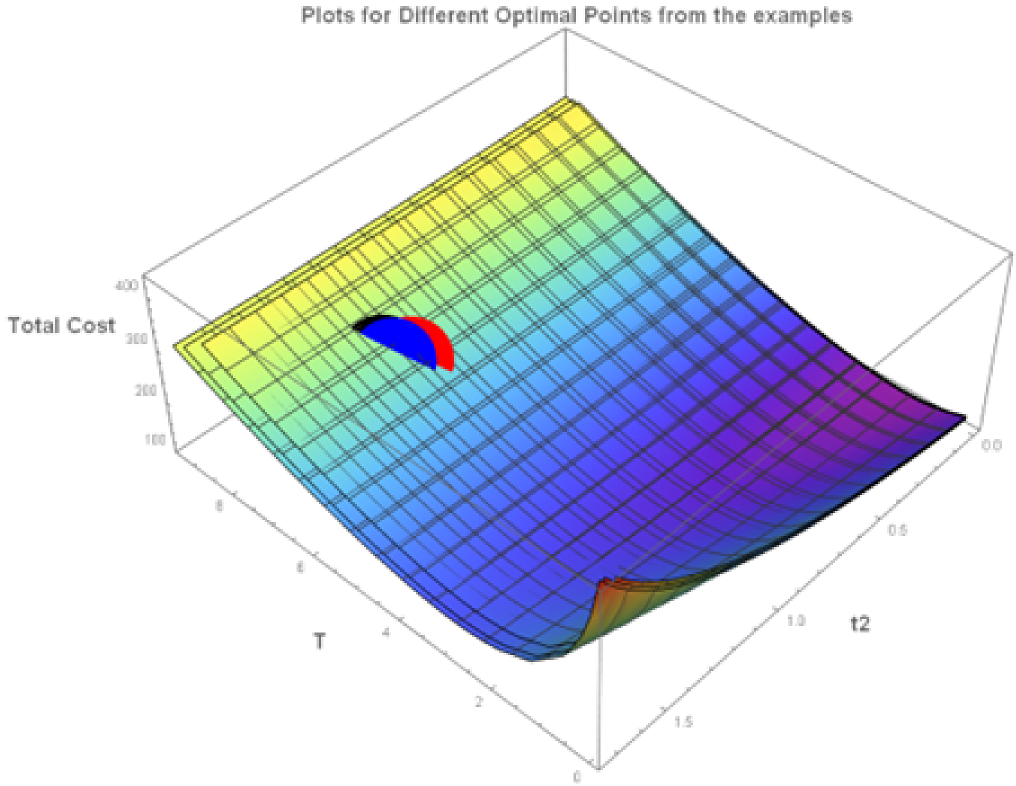

Figure 4.1: Graphical representation to show the convexity of total cost. The figure represents the total cost against T and t_2.

minimum value which means the time when maximum inventory level reaches should be minimum in order to obtain a minimum cost inventory system. The convex nature of the optimum solution is shown in Figure 4.1 where three optimum solutions from Example 1, Example 2 and Example 3 were shown respectively with blue, red and black points.

4.7 Sensitivity analysis

The selected parameters under the inventory system were checked for sensitivity analysis in order to get an idea of how the changes in these parameters effect in total cost. The response of the total cost are investigated after changing the values of selected parameters $A, C_p, C_h, a, b, \beta, \delta, I_{ini}$, namely, by changing of -20%, -10%, 0%, +10%, and +20%. From the three numerical examples we specifically focus on Example 2 since it provides the minimum cost function. The results are shown in Table 4.5 and it provide necessary conclusions regarding the effect of total cost due to the variation in the parameters.

Table 4.5: Variation in Total average cost with respect to different parameters of Example 2.

Changing parameter	Change	t_2^*	T^*	TC^*
A	400	1.0738	6.2478	218.6230
	450	1.1273	6.6081	226.4039
	500	1.1776	6.9716	233.7699
	550	1.2249	7.3388	240.7596
	600	1.2695	7.7100	247.4064
C_p	6.4	1.2304	6.7623	241.4816
	7.2	1.2042	6.8712	237.6599
	8	1.1776	6.9716	233.7699
	8.8	1.1506	7.0641	229.8183
	9.6	1.1234	7.1494	225.8110
C_h	5.6	1.5410	7.0442	227.5539
	6.3	1.3420	7.0027	230.9803
	7	1.1776	6.9716	233.7699
	7.7	1.0395	6.9472	236.0722
	8.4	0.9219	6.9271	237.9940
C_d	3.2	1.1768	6.9710	233.7717
	3.6	1.1772	6.9713	233.7708
	4	1.1776	6.9716	233.7699
	4.4	1.1780	6.9720	233.7689
	4.8	1.1785	6.9724	233.7678
C_a	4.8	1.1776	6.9716	233.7699
	5.4	1.1776	6.9716	233.7699
	6	1.1776	6.9716	233.7699
	6.6	1.1776	6.9716	233.7699
	7.2	1.1776	6.9716	233.7699
C_s	4	1.1775	6.9716	233.7699
	4.5	1.1776	6.9716	233.7699
	5	1.1776	6.9716	233.7699
	5.5	1.1776	6.9716	233.7699
	6	1.1776	6.9716	233.7699
C_o	2.4	1.1818	6.9751	233.7585
	2.7	1.1797	6.9734	233.7641
	3	1.1776	6.9716	233.7699
	3.3	1.1752	6.9697	233.7758
	3.6	1.1727	6.9677	233.7819

Table 3.5: Continued.

Changing parameter	Change	t_2^*	T^*	TC^*
a	24	1.4358	7.0714	230.5373
	27	1.2941	7.0157	232.3074
	30	1.1776	6.9716	233.7699
	33	1.0801	6.9361	234.9978
	36	0.9975	6.9068	236.0431
b	0.16	1.2651	6.9445	231.2953
	0.18	1.2214	6.9588	232.5563
	0.2	1.1776	6.9716	233.7699
	0.22	1.1335	6.9829	234.9364
	0.24	1.0892	6.9927	236.0562
θ	0.0008	1.1776	6.9716	233.7699
	0.0009	1.1776	6.9716	233.7699
	0.001	1.1776	6.9716	233.7699
	0.0011	1.1776	6.9716	233.7699
	0.0012	1.1776	6.9716	233.7699
α	0.0024	1.1776	6.9716	233.7699
	0.0027	1.1776	6.9716	233.7699
	0.003	1.1776	6.9716	233.7699
	0.0033	1.1776	6.9716	233.7699
	0.0036	1.1776	6.9716	233.7699
β	1.6	1.8191	7.4597	231.2295
	1.8	1.3855	7.1335	233.0450
	2	1.1776	6.9716	233.7699
	2.2	1.0614	6.8753	234.0367
	2.4	0.9900	6.8109	234.0851
δ	0.4	1.3696	8.1157	263.0521
	0.45	1.2729	7.5042	248.1932
	0.5	1.1776	6.9716	233.7699
	0.55	1.0833	6.5005	219.6864
	0.6	0.9897	6.0777	205.8645
I_{ini}	40	1.1806	6.3744	219.2915
	45	1.1803	6.6699	226.6935
	50	1.1776	6.9716	233.7699
	55	1.1723	7.2796	240.5395
	60	1.1648	7.5940	247.0198

4.7.1 Discussions on sensitivity analysis

From the above table 4.5 we observe the following.

- The total cost directly increases when the fixed cost of placing the initial order increases. Additionally, the time until positive stock in the system, t_2, and the cycle length, T, increases with higher ordering costs. It's better to reduce ordering costs by placing bulk orders to manage these costs effectively. Maintaining holding costs and negotiating better deals also incorporate this.

- The purchase cost per unit C_p significantly decreases the total cost function and t_2 but increases cycle length. For the optimum cost function, retailer can think about reducing purchasing cost by considering the remaining conditions.

- With the increase in holding cost, both positive stock level time and cycle time decrease, but the average cost function increases. An increase in total cost function suggests that holding cost has to be reduced. The retailer can replenish only the required number of items to maintain this holding cost.

- When the demand parameters a and b increase, the average total cost increases simultaneously. The retailer can take this as an advantage by increasing the selling price and boosting the demand.

- The steady increase in the ameliorating parameter β affects the total cost function. So, to consider a managerial approach, the effect of amelioration has to be considered according to the reduction in total cost.

- The backlogging parameter δ significantly decreases the total cost function: the cycle length and time up to the positive stock level decrease simultaneously. The retailer can think that keeping an adequate stock level is essential in optimizing the cost function.

- Initial inventory level I_{ini} directly depends on the total cost. An adequate initial inventory level must be maintained to maintain a budget inventory system, considering the amelioration and deterioration effects. Since cycle length also increases, it can be thought about reducing cycle length to decrease the average cost function.

The chnage in cost function values is shown in figures 4.2 and 4.3 .

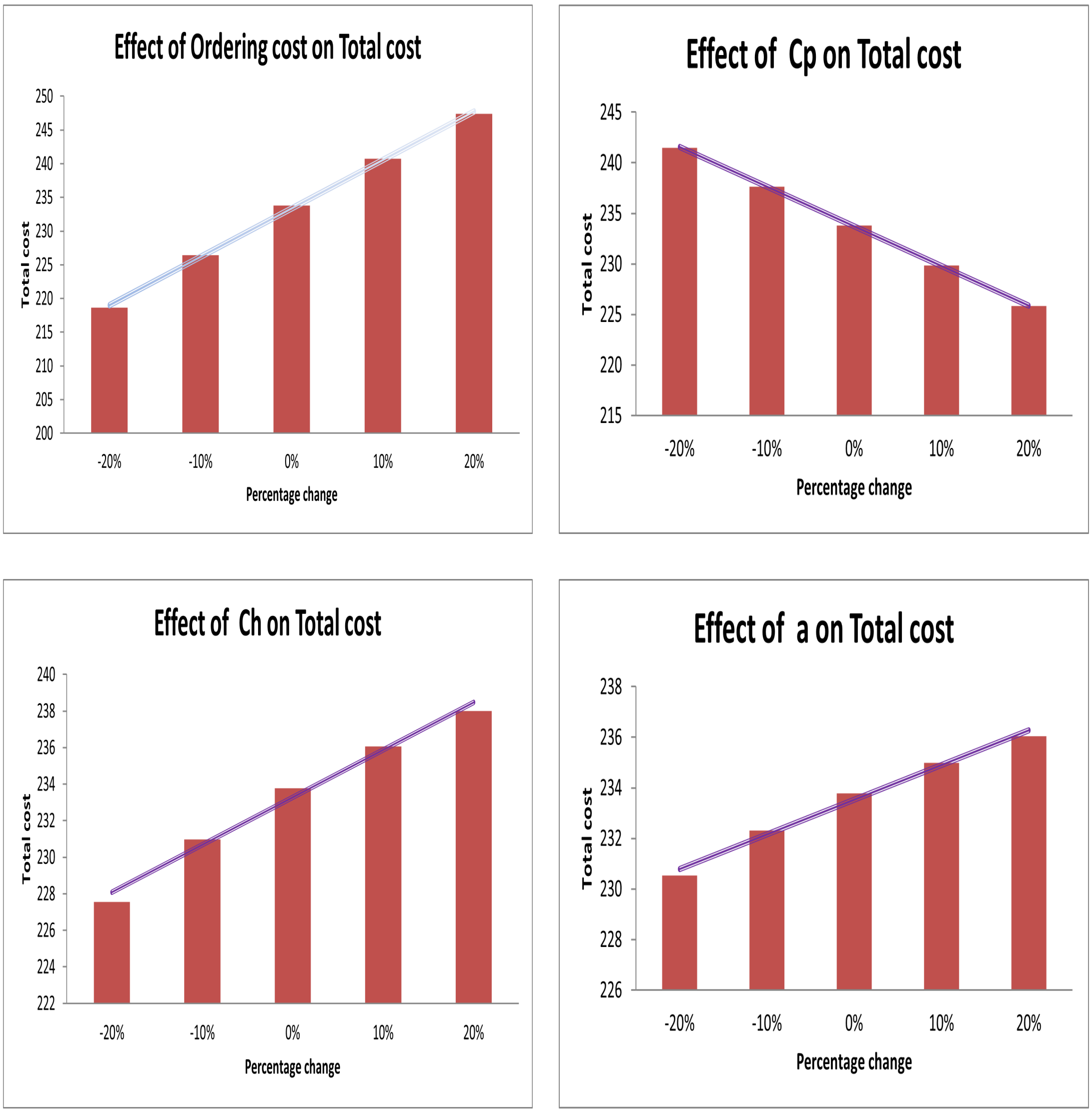

Figure 4.2: Graphical representation to show the variation of total cost with respect to different parameters.

4.8 Conclusion

The current chapter is mainly focused on addressing deteriorating and ameliorating inventory items to connect the inventory model to real life practices. Demand for

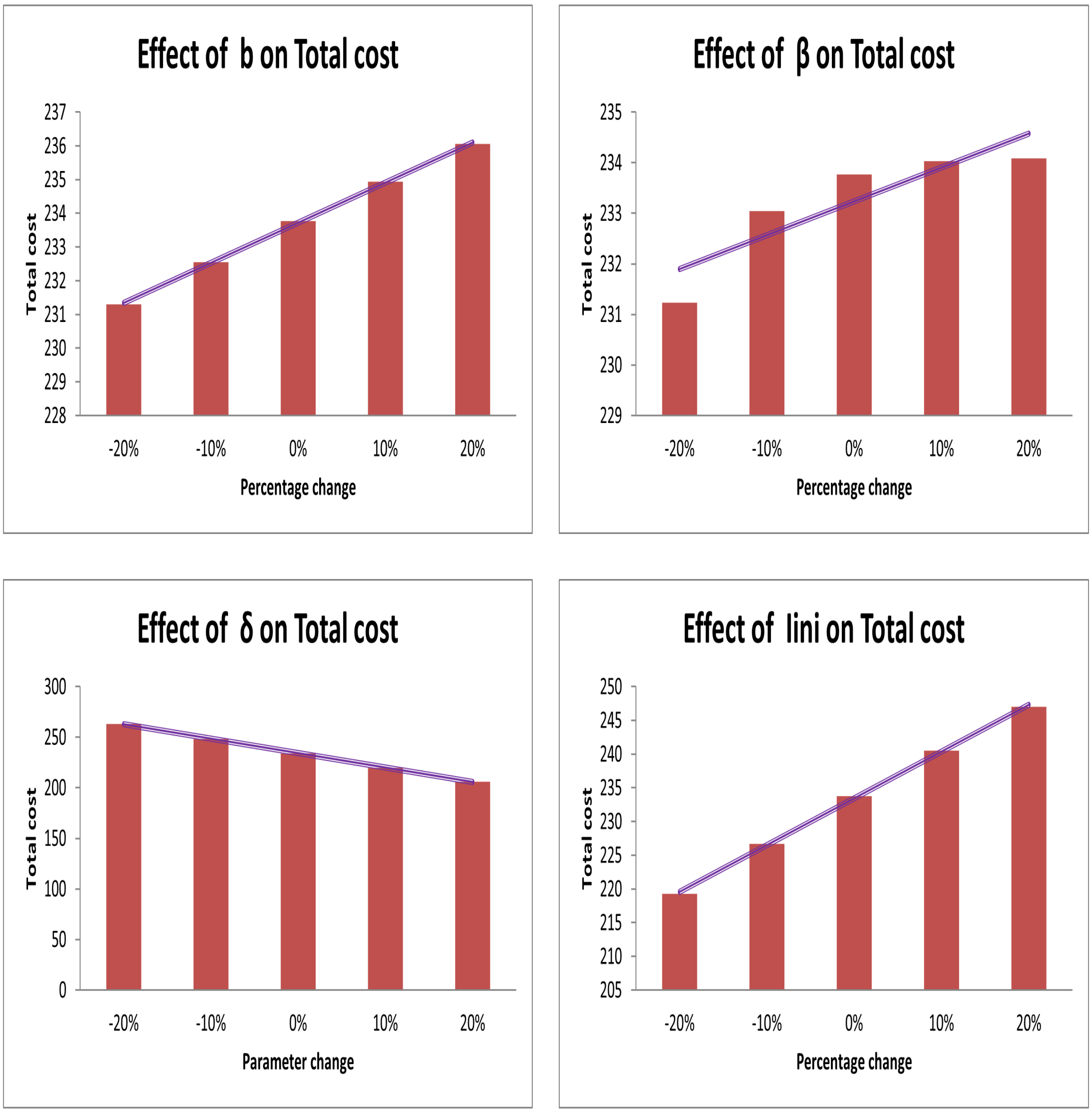

Figure 4.3: Graphical representation to show the variation of total cost with respect to different parameters.

inventory items is another criterion and it is taken as linearly dependent on time. The shortages which are partially backlogged with a loss depending on the customers waiting time is another assumption. Time dependent deterioration and amelioration with rate follow two parameter Weibull distribution making the model more applicable in different fields. Addressing of this research gap in the related works of literature makes a unique contribution to the present paper. The real life scenarios where growing up items are considered is the main field in the study. Initial items kept in the system growing up concerning amelioration and small deterioration rate. These items remain in stock until demand increases. Early reaching the time until maximum inventory will retain the optimal cost to be minimum. The different parameters in the inventory model were checked for whether its change directly depended on the optimum cost. When major parameters change from -20% to a maximum of 20%, a notable change is seen in the optimum cost. Large increasing variation is seen in optimum cost concerning the initial ordering cost, holding cost, demand parameters, ameliorating parameter and initial inventory level. So to make the cost function a minimum take appropriate actions in decreasing these parameters. The purchasing cost and backlogging parameter show direct variation concerning the optimum cost. Hence the sensitivity analysis provides the parameters that decision makers need to pay attention to minimize the optimum cost.

Chapter 5

AN INTEGRATED SUPPLY CHAIN MODEL WITH PRODUCT OF THE DEMAND INFLUENCED BY ITS PRICE, FRESHNESS AND ADVERTISEMENT STRATEGY

5.1 Introduction

The freshness of the product influences the buying behavior of customers. The Use of food items including fruits, vegetables, dairy products and meat is related to optimal quality and safety standards. The maximum lifetime of such products is vital in their management. Customers are more health conscious in the case of perishable products, so maximum care in the minimum time is an essential criterion in buying behavior. Awareness regarding food safety and health concerns influences the demand for and sale of fresh products. Hence, it is necessary to meet the maximum demand before

Some contents of this chapter are based on Praveen and Manoharan (2024).

expiration. The focus on selling price is thus connected to product demand. The management must carefully consider the selling price following the freshness of the product. As the price increases there is a high chance of demand decreasing. A product's selling price and quality are the most important determinants in their purchasing decision. In the course of increasing sales, implementing an effective advertisement strategy is essential. Advertisement policies create awareness and generate demand for fresh products in the market. Effective advertisement will lead to positive consequences that drive its demand. Advertisement policy helps to communicate with the consumers regarding the product to address their needs and wants. The purchase medium, such as in-store advertising, media advertising, and print advertising can implement different advertising policies. Another important factor is the consideration of carbon tax policies to reduce the effect of carbon emissions, which is vital for sustainable development. As product management and development increase, the corresponding eco-friendly nature of the environment also has to be maintained. Regulation methods to control carbon emissions are thus a key area in the supply chain processes.

The study of the supply chain model based on integrating the individual mindsets of the manufacturer and the retailer has been an exciting area for many supply chain researchers. Identifying problem areas in the system, making suggestions to the business for further steps and obtaining the optimum objective of improving inventory management are the key objectives of these processes. The integrated policy makes retailers and manufactures relationships with each other and with the business flexible and convenient. Market demands vary with the business and consideration of market demand according to the field of study is essential in evaluating supply chain policy. Several concerns regarding a customer's purchase moment include the product's price and the product's freshness. Advertisement strategy influences buying behavior at the moment and during the purchase discussion. This chapter investigates a single-manufacturer single-retailer supply chain model where the market demand depends on selling and product freshness under the consideration of carbon emissions.

Researchers developed various integrated models that considered different conditions to guide business. Palanivel and Uthayakumar (2015) investigated EOQ model with the impact of inflation on pricing and inventory strategies, where demand is related to advertisement and price. A supply chain system with seller to its buyer with deteriorating products of a maximum lifetime is explored by Wang et al. (2014). Zhang et al. (2015) developed the one-manufacturer-one-retailer supply chain model for items with controllable deterioration rate incorporate with price dependent demand. They included preservation technology to reduce deterioration in the model. Various carbon tax policies introduced in the government sector to meet the sustainable development of specific firms were discussed by Kuo et al. (2016). Manna et al. (2017) investigated an imperfect production system for deteriorating items with advertisement rate follows an exponential form over time. Shah and Vaghela (2017) developed a decaying inventory model considering the demand interrelated with time and advertising frequency. Later on, Giri et al. (2017) evolved a two-echelon supply chain model with a single-vendor and a single-buyer. The single product considered in the study is deteriorating in nature and buyer is adopting the preservation technology investment. Sekar and Uthayakumar (2017) provided a multi-production run inventory model for deteriorating items with scrap, failure rework and penalty cost. Saha and Chakrabarti (2018) investigated a supply chain model for perishable products considering the trade credit policy that incorporates inventory levels and advertisement-driven demand. Shen et al. (2019) proposed an inventory model with carbon tax policy and preservation technologies to maximize profit. Gautam et al. (2020) evolved a supplier-retailer inventory problem to deal with deterioration under price-sensitive demand. Giri et al. (2020) considered a two-level supply chain consist of a single-vendor and single- buyer model with the vendor produces a single product in lot. Lu et al. (2020) developed a model with optimum equilibrium and the importance of adding carbon policy measures. Rout et al. (2020) introduced the supply chain model with regulating carbon emission polices. Yadav et al. (2021) developed a two manufacture one retailer model by considering the effect of carbon emissions. Sebatjane and Adetunji (2021) formulated a supply chain model for growing items where demand is dependent on the inventory level and expiration date of the product. Choudhury et al. (2021) presented an integrated single vendor-buyer production inventory model for deteriorating items that incorporate quantities and quality losses. They considered demand as a function of the selling price and the freshness condition of the product. Choudhury et al. (2022) developed a multi layer inventory supply chain with deterioration and pollution where demand follows a function of selling price and freshness.

The relevant literature found that setting a selling price and freshness index with advertisement strategies is a crucial aspect of the supply chain model. Sustainable development and carbon emission have to be considered in the study of supply chain processes. To the best of our knowledge, no works have been found that consider the combination of these aspects in an integrated supply chain process. Hence this chapter has developed a manufacturer-retailer supply chain model based on these facts and proposed an appropriate model for solving this. Numerical examples were used to validate the model. The variation of different parameters was discussed with a sensitivity analysis. On these lines different sections of this chapter are presented.

5.2 Motivation, research questions and gaps

The optimization of a firm's profit is related to price, expiration, freshness and advertisement strategy with the carbon emissions aspects. This chapter highlights these facts to address the impact on sustainable business needs. It should be emphasized that no work has considered a combination of price, freshness and advertisement strategy when developing models in supply chain processes. The effect of deterioration related to shelf time plays a vital role in the management model. As motivated by the literature, we developed the research objectives for this chapter, summarised below.

- To study the impact of expiration and freshness index on the profit function of manufacturer retailer model.
- To investigate the effect of advertisement policies on buying decision.
- To find out the selling price of each good item from the retailer's side.
- To compare the optimum profit with and without the effect of carbon tax policies.

The theme of the chapter is to fill the research gap in the manufacturer retailer model by introducing the freshness index, selling price and advertisement policy and their combinations under the policies of carbon tax to reduce emissions. In this chapter, we developed the model and illustrated how this model fills this gap.

5.3 Problem Description

In supply chain study, while considering fresh products, expiration and freshness are vital terms as these affect the buying behavior. The proper management can result in a significant reduction in operating costs of the inventory system. This model is concerned with a two-echelon supply chain inventory model with freshness and price dependent demand. The quantity of carbon emissions and reduction cost are vital in the supply chain processes so is included in the model. The proposed model incorporates several key elements. Single manufacturer single retailer transaction of a single product to maximize their profit. At the retail level, the demand function is affected by the product's selling price and freshness level. The effect of advertisement frequency on the profit function is compared. Implementation of carbon tax policies results are presented. Within the framework of these conditions, we frame an integrated supply chain model as a profit maximization problem. For the model's proper functioning, we considered the following notations and assumptions throughout this chapter.

5.3.1 Notations

Objective Function

Symbol	**Description**
TPR	Average total profit per unit time of the retailer
TPM	Average total profit per unit time of the manufacturer

Decision Variables

Symbol	Description
A	Advertisement frequency (positive integer)
p	Unit selling price of retailer
T_r	Ordering cycle length of retailer

Cost Parameters

Symbol	Description
C_r	Replenishment cost per order of retailer
C_m	Fixed starting cost of manufacturer
G	Expenditure per advertisement
C_p	Production cost of manufacturer
w	Supplier's wholesale price
C_{hr}	Holding cost per unit time of retailer
C_{hm}	Holding cost per unit item per unit time of manufacturer
C_c	Unit carbon emission cost
C_d	Deterioration cost per unit per unit time

Inventory Parameters

Symbol	Description
a	Market size ($a > 0$)
b	Price sensitivity in demand ($b > 0$)
γ	Advertising elasticity ($0 \leq \gamma < 1$)
L	Products limited shelf life
$I_r(t)$	Retailer's inventory level at time t
$I_m(t)$	Manufacture's inventory level at time t
T_m	Production starting time for the manufacturer
θ	A constant deterioration rate of manufacturer, ($0 \leq \theta \leq 1$)
e_r	Carbon emission related to warehousing per unit
e_m	Carbon emission related to production per unit
P	A constant production rate of manufacturer
Q	Order size (units)

5.3.2 Assumptions

The developed integrated manufacturer-retailer supply chain model formulated upon the following assumptions:

1. A two-echelon supply chain with a manufacturer produces some kind of deteriorating items and then delivers them to a retailer. Finally, the retailer sells the products to the customers.

2. Deterioration occurs within the retailers inventory cycle is considered as the quantity loss rate of items during the product's limited shelf life (denoted as L). Deterioration rate is mathematically expressed as follows:
$\theta(t) = \frac{1}{1+L-t}, 0 \leq t \leq T \leq L.$

3. Consumer purchasing decisions are influenced by printed expiration dates. In essence, a consumer's likelihood to purchase an item decreases as the item approaches its expiration date. The freshness index at time t is linearly decreasing from 1 at the beginning to 0 at the maximum lifetime (L). The freshness index ($F(t)$) is given by:$F(t) = \frac{L-t}{L}, 0 \leq t \leq T \leq L.$

4. The demand rate is determined by the item's selling price, freshness index and advertising strategies. Sales are influenced by the selling price and customers always want to buy the best product at an affordable price. Therefore, it is assumed that the demand for a particular commodity is inversely proportional to its selling price. The freshness of the item also impacts customer buying behavior. Initially, when the product is fresh, there is no significant effect on demand. However, as the product loses its freshness over time, demand decreases. Additionally, advertising policies are pivotal in cultivating product recognition and popularity. Incorporating these presumptions, we may infer that demand is contingent upon a relationship between advertising frequency, product freshness, and pricing, as expressed below: $D(A, t, p) = A^{\gamma}(a - bp)\frac{L-t}{L}$, where $\gamma \in (0, 1)$, $(a - bp) > 0$ and $0 \leq t \leq T \leq L$

5. Shortages are not allowed.

5.4 Mathematical formulation

The supply chain process proposed in this chapter deals with the assumptions given previously. The flow of items from manufacturer to retailer and from retailer to customer is illustrated in this model. Consider the inventory situation where the production system at the manufacturer begins the process after a time delay. The retailer sells a single type of fresh item in the imperfectly competitive market. At this end, the inventory level depletes due to the combined effect of both demand and deterioration. The representation of the inventory flow for the manufacturer and retailer is presented in figure 5.1 .

The retailer's model

The schematic diagram of the retailers model shows the change in inventory level over the replenishment cycle. Initially retailer receives order quantity Q of fresh items from the manufacturer at $t = 0$. The effect of carbon emission occurs during this period due to keeping the items fresh for extended periods. The firm has to pay costs under the sustainability policies. Here, we consider that both the manufacturer and retailer have to pay a carbon tax for the respective emissions that take place on their side. The retailer's inventory level $I_r(t)$ declines during the period $[0, T_r]$ due to the combined effect of consumer demand and deterioration of the item. Hence, the rate of change of inventory level at any instant t is governed by the differential equation

$$\frac{dI_r(t)}{dt} + \frac{1}{1+L-t} I_r(t) = -A^{\gamma}(a - bp)\frac{L-t}{L}, t \in [0, T_r]. \tag{5.4.1}$$

subject to the boundary condition $I_r(T_r) = 0$, the retailers inventory level is

$$I_r(t) = \frac{A^{\gamma}(a-bp)}{L}\left[(1+L-t)(T_r - t) + (1+L-t)\ln\left(\frac{1+L-T_r}{1+L-t}\right)\right], \quad 0 \le t \le T_r. \tag{5.4.2}$$

using $I_r(0) = Q$, the retailer ordering quantity Q is

$$Q = \frac{A^{\gamma}(a-bp)(1+L)}{L}\left[T_r + \ln\left(\frac{1+L-T_r}{1+L}\right)\right]. \tag{5.4.3}$$

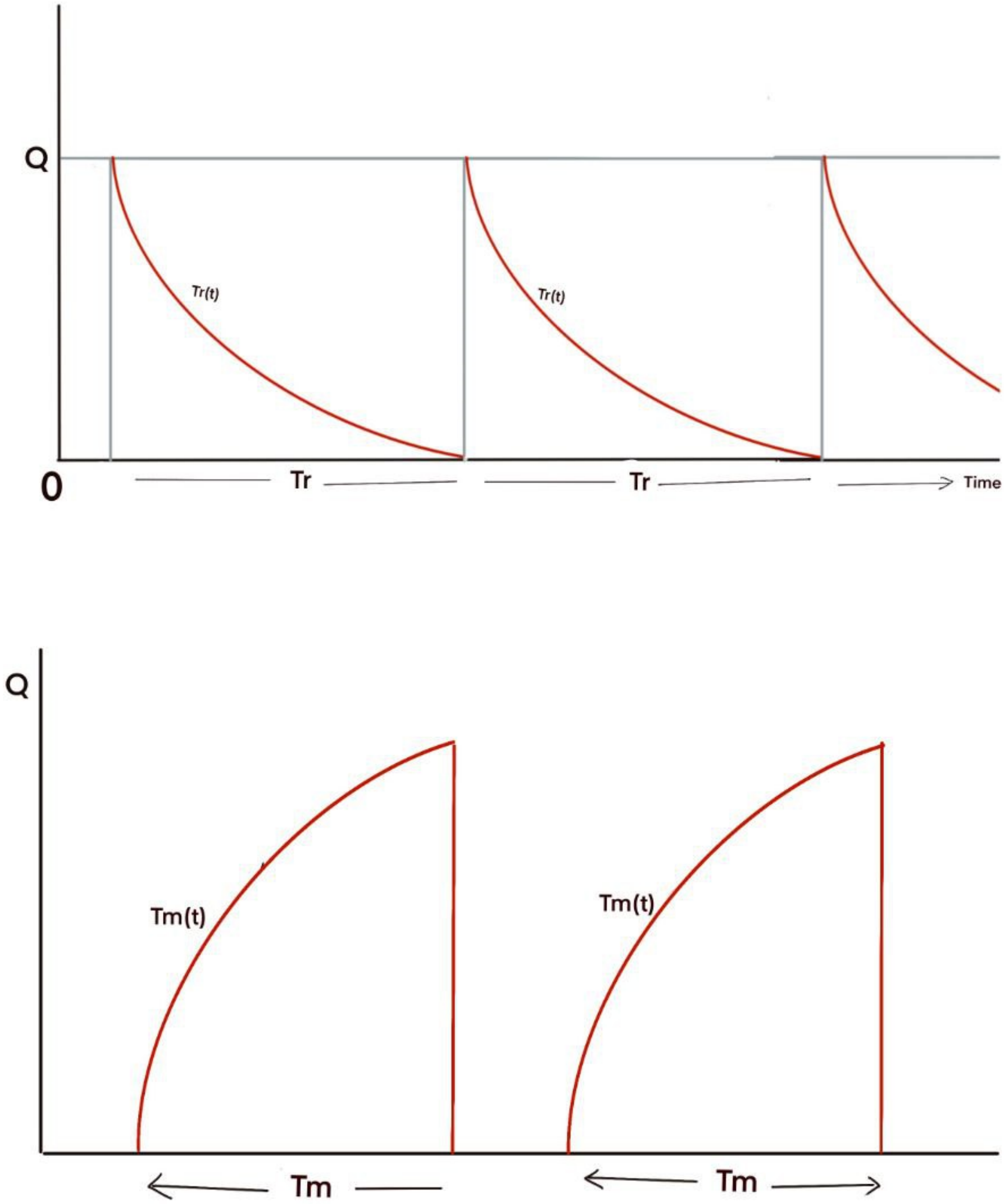

Figure 5.1: Retailer's and Manufactures inventory behaviour

The retailer's profit per unit time is

$$TPR = \text{Sales revenue}\,(SR_b) - \text{Ordering cost}\,(OC_b) - \text{Holding cost}\,(HC_b)$$
$$- \text{Cost of carbon emission}\,(CC_b) - \text{Purchase cost}\,(PC_b)$$
$$- \text{Advertisement cost}\,(AC_b).$$

The components associated with retailers total profit function are given below:

1. The total sales revenue of the retailer during the cycle is

$$SR_b = p\int_0^{T_r} D(A,t,p)\,dt$$

$$SR_b = pA^{\gamma}(a-bp)\left[T_r - \frac{T_r^2}{2L}\right]. \tag{5.4.4}$$

2. The retailer's ordering cost per cycle is

$$OC_b = C_r. \tag{5.4.5}$$

3. The cost of advertisement is

$$AC_b = G \times A. \tag{5.4.6}$$

4. The retailer's purchase cost per cycle is $PC = wQ$

$$PC_b = w\frac{A^{\gamma}(a-bp)(1+L)}{L}\left[T_r + \ln\left(\frac{1+L-T_r}{1+L}\right)\right]. \tag{5.4.7}$$

5. The retailer's holding and carbon emission cost per cycle is given by

$$HC_b + CC_b = (C_{hr} + C_c e_r)\int_0^{T_r} I_r(t)\,dt$$

$$\begin{aligned}HC_b + CC_b = (C_{hr} + C_c e_r)\frac{A^{\gamma}(a-bp)}{L}\Bigg[&\frac{(1+L)^2}{2}\ln\left(\frac{1+L-T_r}{1+L}\right)\\ &-\frac{T_r^3}{6} - \frac{T_r^2}{4} + \frac{(1+L)T_r^2}{2} + \frac{(1+L)T_r}{2}\Bigg].\end{aligned} \tag{5.4.8}$$

Hence, the retailer's profit per unit time in any replenishment cycle is obtained by combining the components of the profit function found above as

$$\begin{aligned} TPR = \frac{1}{T_r}\Bigg[& pA^{\gamma}(a-bp)\left(T_r - \frac{T_r^2}{2L}\right) - C_r - G\times A \\ & - (C_{hr}+C_c e_r)\frac{A^{\gamma}(a-bp)}{L}\Bigg(\frac{(1+L)^2}{2}\ln\left(\frac{1+L-T_r}{1+L}\right) - \frac{T_r^3}{6} - \frac{T_r^2}{4} \\ & + \frac{(1+L)T_r^2}{2} + \frac{(1+L)T_r}{2}\Bigg) \\ & - w\frac{A^{\gamma}(a-bp)(1+L)}{L}\left(T_r + \ln\left(\frac{1+L-T_r}{1+L}\right)\right)\Bigg]. \end{aligned} \tag{5.4.9}$$

The retailer's profit function can be written in a flexible format by re arranging terms as

$$\begin{aligned} TPR = \frac{1}{T_r}\Bigg[& A^{\gamma}(a-bp)\left(w_1 T_r^3 - \frac{pT_r^2}{2L} - w_2 T_r^2\right) \\ & + A^{\gamma}(a-bp)\left(pT_r - w_3 T_r\right) - C_r - GA \\ & - A^{\gamma}(a-bp)w_4\ln\left(\frac{1+L-T_r}{1+L}\right)\Bigg]. \end{aligned} \tag{5.4.10}$$

where the variables are defined as follows:
$w_1 = \frac{(C_{hr}+C_c e_r)}{6L}$; $w_2 = \frac{(C_{hr}+C_c e_r)(1+2L)}{4L}$; $w_3 = \frac{(1+L)}{L}(w + \frac{(C_{hr}+C_c e_r)}{2})$; and $w_4 = w\frac{(1+L)}{L} + (C_{hr}+C_c e_r)\frac{(1+L)^2}{2L}$.

Now the problem is to analyze the retailer's profit function 5.4.10 and determine the optimal decision variables p, T_r and A. The relevant optimization problem can be expressed as follows:

Problem 1:

Maximize $TPR(p, T_r, A)$

subject to $w \le p, A > 0, T_r > 0.$

The manufacture's model

Initially, the retailer orders Q unit of fresh products to the manufacturer. Thereafter, the manufacturer begins manufacturing at time point $t = T_r - T_m$ according to the orders received from the retailer. The carbon emission during the production period is taken into account. The manufacturer's inventory level increases due to production rate and constant deterioration rate. Hence, the rate of change of inventory level at any instant t is governed by the differential equation

$$\frac{dI_m(t)}{dt} = P - \theta I_m(t), t \in [T_r - T_m, T_r]. \tag{5.4.11}$$

subject to the boundary condition $I_m(T_r - T_m) = 0$ and $I_m(T_r) = Q$, the manufacture's components can be found as below: Production time length is

$$T_m = \frac{1}{\theta} \ln \frac{P}{P - Q\theta}. \tag{5.4.12}$$

Inventory level for the manufacturer is

$$I_m(t) = \frac{P}{\theta}\left[(1 - e^{\theta(T_r - T_m - t)}\right], \quad T_r - T_m \leq t \leq T_r. \tag{5.4.13}$$

The manufacturer's profit per unit time is

$$\begin{aligned} TPM = {} & \text{Sales revenue}\,(SR_m) - \text{Production cost}\,(PDC_m) - \text{Holding cost}\,(HC_m) \\ & - \text{Cost of carbon emission}\,(CC_m) \\ & - \text{Deterioration cost}\,(DC_m) - \text{Setup cost}\,(AS_m). \end{aligned}$$

$$\begin{aligned} TPM = \frac{1}{T_r} & \Big[wQ - (C_p + C_c e_m)PT_m - C_d\,(PT_m - Q) \\ & - C_m - (C_{hm} + C_c e_m) \int_{T_r - T_m}^{T_r} I_m(t)\,dt\Big]. \end{aligned}$$

$$\begin{aligned} TPM = \frac{1}{T_r} & \Big[wQ - (C_p + C_c e_m)PT_m - C_d\,(PT_m - Q) - C_m \\ & - \frac{(C_{hm} + C_c e_m)P}{\theta^2}\left(\theta T_m + e^{-\theta T_m} - 1\right)\Big]. \end{aligned} \tag{5.4.14}$$

The optimum value of T_r^* provide the maximum profit value of the manufacturer.

5.5 Optimal solutions and theoretical results

The retailer's profit function expressed in equation 5.4.10 is checked to find the optimal values of decision variables p, T_r and A that maximizes the profit function $TPR(p, T_r, A)$. For the optimum problem it is necessary to demonstrate the concavity of the profit function with respect to p, T_r and A. To find the optimal values of the respective variables, the other two variables are considered to be fixed, and then the first derivative and second derivative of the profit function are taken concerning one of the decision variables. For any fixed T_r, A, and w, the profit function $TPR(p, T_r, A)$ is concave in p. The optimal solution for p can be derived from setting $\frac{\partial TPR(p,T_r,A)}{\partial p} = 0$. The partial derivatives of the profit function $TPR(p, T_r, A)$ expressed in equation 5.4.10 with respect to the retailer's selling price p is provided here. The first-order partial derivative is:

$$\begin{aligned}\frac{\partial TPR}{\partial p} = \frac{1}{T_r}\Bigg[&-A^\gamma b\left(w_1 T_r^3 - \frac{T_r^2}{2L} - w_2 T_r^2 + T_r - w_3\right)\\ &+A^\gamma(a-bp)\left(-\frac{T_r^2}{2L} + T_r\right) + A^\gamma b w_4 \ln\left(\frac{1+L-T_r}{1+L}\right)\Bigg].\end{aligned} \tag{5.5.1}$$

Setting $\frac{\partial TPR}{\partial p} = 0$, we obtain the critical value of p. The second-order partial derivative is given by $\frac{\partial^2 TPR}{\partial p^2} = A^\gamma b\left(\frac{T_r^2}{2L} - T_r\right)$. Since $\frac{\partial^2 TPR}{\partial p^2} < 0$ for $T_r < 2L$, the profit function TPR is concave in p. Therefore, the critical value derived from the first derivative corresponds to the maximum profit. Hence, for any given values $A > 0$ and $T_r > 0$, the profit function $TPR(p, T_r, A)$ is concave with respect to p and there exists a unique p that maximizes the retailers profit.

Optimum value of the advertisement frequency A^* can easily find out after checking the first and second derivative of profit function $TPR(p, T_r, A)$ with respect to A. When business coordination is profitable, it can be seen that $\frac{\partial^2 TPR}{\partial p^2} < 0$. Since A is an integer variable the optimum value A^* can be find by putting value giving approach.

For any fixed p, A, and w, the profit function $TPR(p, T_r, A)$ is concave in T_r. The optimal solution for T_r can be derived from setting $\frac{\partial TPR(p,T_r,A)}{\partial T_r} = 0$. Arrange the profit function $TPR(p, T_r, A)$ from equation 5.4.10 such that it is divided into ratio of two functions of T_r. Let the two functions are $G(T_r)= T_r$ and

$$\begin{aligned} F(T_r) = A^{\gamma}(a-bp)\left(w_1T_r^3 - \frac{pT_r^2}{2L} - w_2T_r^2 + pT_r - w_3T_r\right) \\ - C_r - GA - A^{\gamma}(a-bp)w_4 \ln\left(\frac{1+L-T_r}{1+L}\right). \end{aligned} \tag{5.5.2}$$

Based on the fact that any real-valued function in fractional form $W(T) = \frac{F(T)}{G(T)}$ is pseudo-concave, if $F(T)$ is a differentiable, positive, and concave function, but $G(T)$ is a differentiable, positive, and convex function the retailer profit function $TPR(p, T_r, A)$ is concave in T_r.

5.6 Numerical examples

In this section numerical examples are conducted by considering different examples where all parameters are taken in proper units.

Example 1: Assume the parameters as a=110, b=1.1, L=4, γ=0.1, C_r=520, C_{hr}=1.2, G=40, w=32, C_p=4, P= 500, C_d= 1.2, C_m= 700, C_{hm}= 1.1, θ=0.08, C_c= 1.2, e_m=0.8 and e_w=0.06. Then, by solving equation for retailer's profit function and manufacturers profit function we obtained the optimum values of p, A^* and T_r as $p^* = 69.52$, A^*=4 and T^*=1.470, which satisfy the conditions and using the values of p^*, A^* and T^* in equations 5.4.10 and 5.4.14 provide the optimum relevant profit of the retailer as TPR^*=495.47 and profit of the manufacturer is TPM^*=521.600.

Example 2: All the data from example 1 are retained except we took A= 3 and A =5, then process is repeated to find the optimal values of total profit functions TPR^* and TPM^* and the results are compared in table and presented in figures. The figures shows the concave nature of the retailer's profit function with respect to the variables p, T_r. .

The effect of carbon tax policies concerning the three distinct values of advertise-

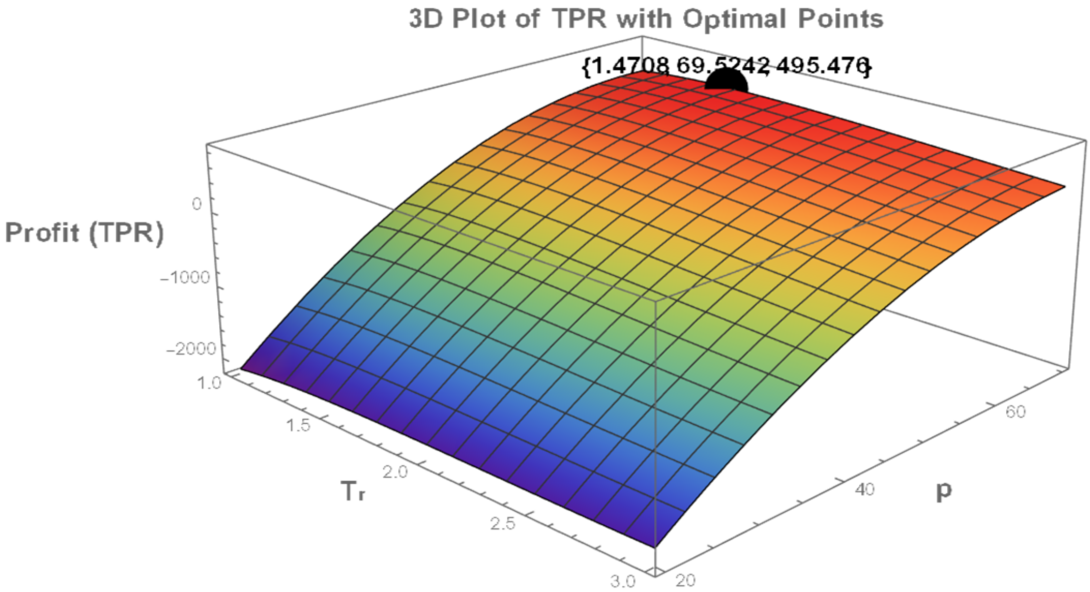

Figure 5.2: Total average profit according to various choices of parameters with p, T_r and TPR along the x-axis,the y-axis and the z-axis respectively for A=4.

Table 5.5: Values with and without Carbon Tax Policy

	A=3		A=4		A=5	
	UCT	WCT	UCT	WCT	UCT	WCT
T_r	1.44	1.47	1.47	1.49	1.49	1.52
p	69.45	69.16	69.52	69.22	69.6	69.29
TPR	495.63	517.99	495.47	518.76	490.05	514.17
TPM	488.57	533.32	521.96	567.63	549.65	596.02

Note: UCT = Under Carbon Tax, WCT = Without Carbon Tax

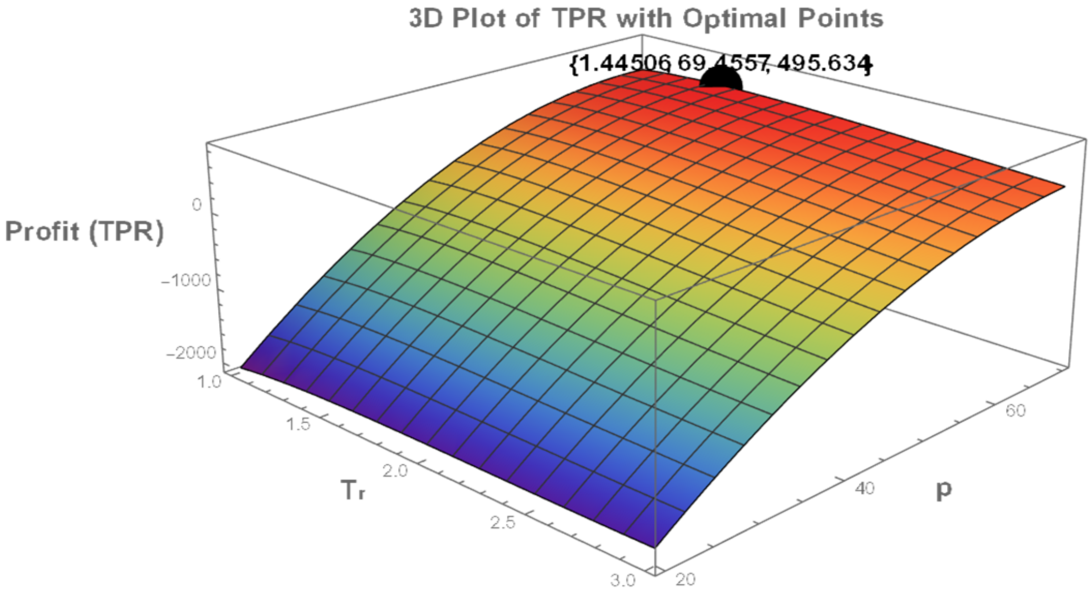

Figure 5.3: Total average profit according to various choices of parameters with p, T_r and TPR along the x-axis,the y-axis and the z-axis respectively for A= 3.

ment frequency, A is compared in the table 5.5. Under each value of A, the profit functions TPR and TPM are higher when there is no carbon tax policy. The difference is that payment reduction is possible due to the lack of policy implementation. However, the slight change in profit indicates that implementing carbon tax policies is more sustainable for the environment without much affecting the profit. When advertisement frequency increases, there is a trend of increase in the profit functions. Hence our proposed model with the effect of carbon emission is prevalent in the sustainable supply chain processes.

5.7 Sensitivity analysis

The selected parameters under the supply chain processes were checked for sensitivity analysis in order to get an idea of how the changes in these parameters effect in total profit function of retailer's and manufactures. Total profit is also checked for the combined changes due to the variation in the parameters. The response of the total profit are investigated after changing the values of selected parameters by changing of -20%, -10%, 0%, +10%, and +20%. From the numerical examples we specifically focus on the example with A=4 for comparison. The results are shown in Table 5.6

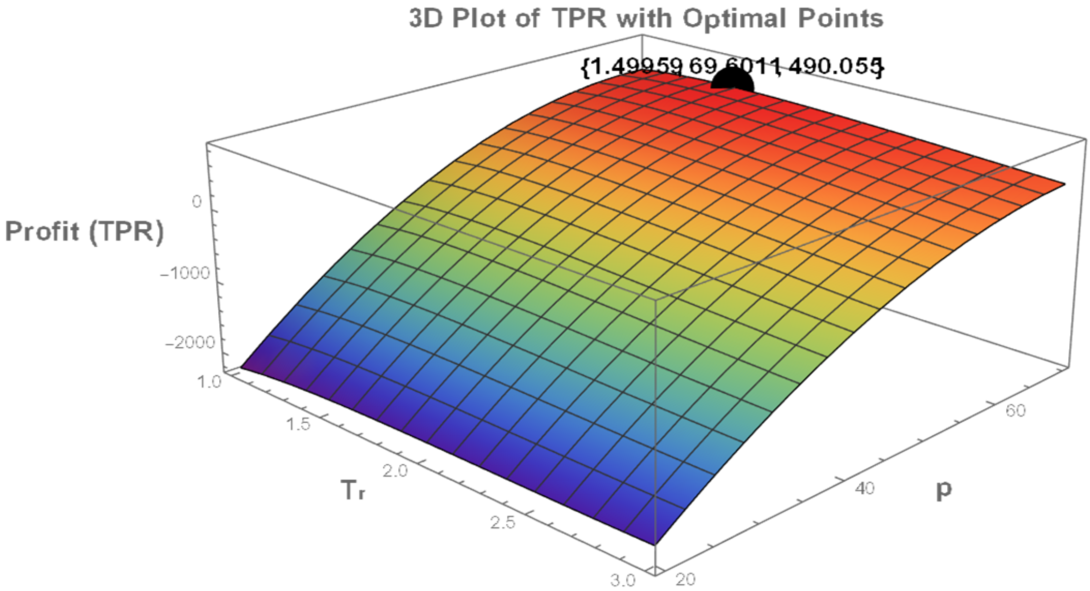

Figure 5.4: Total average profit according to various choices of parameters with p, T_r and TPR along the x-axis,the y-axis and the z-axis respectively for A= 5.

and Table 5.7. The analysis provide necessary conclusions regarding the effect of total profit due to the variation in the parameters.

Table 5.6: Table 1: Variation in p, T_r, TPR and TPM and $TPR+TPM$ with respect to different parameters

Parameter	Optimal T	Optimal p	Optimal TPR	Optimal TPM	TPR + TPM
	1.48336	69.3917	505.881	529.758	1035.64
	1.47704	69.4582	500.664	525.851	1026.52
w	1.4708	69.5242	495.476	521.964	1017.44
	1.46465	69.5898	490.316	518.097	1008.41
	1.45858	69.6548	485.185	514.251	999.436
	2.08314	61.2246	3.01128	261.527	264.538
	1.70692	65.1643	214.714	395.023	609.737
a	1.4708	69.5242	495.476	521.964	1017.44
	1.30106	74.0769	842.757	646.009	1488.77
	1.17065	78.7406	1255.38	768.434	2023.82
	2.08314	61.2246	3.01128	261.527	264.538
	1.70692	65.1643	214.714	395.023	609.737
b	1.4708	69.5242	495.476	521.964	1017.44
	1.30106	74.0769	842.757	646.009	1488.77
	1.17065	78.7406	1255.38	768.434	2023.82
	2.08314	61.2246	3.01128	261.527	264.538
	1.70692	65.1643	214.714	395.023	609.737
L	1.4708	69.5242	495.476	521.964	1017.44
	1.30106	74.0769	842.757	646.009	1488.77
	1.17065	78.7406	1255.38	768.434	2023.82
	1.49365	69.5852	469.382	499.154	968.536
	1.48218	69.5546	482.314	510.465	992.778
Γ	1.4708	69.5242	495.476	521.964	1017.44
	1.45954	69.4942	508.871	533.656	1042.53
	1.44837	69.4645	522.503	545.542	1068.05
	1.34227	69.1845	569.434	493.073	1062.51
	1.4075	69.3561	531.61	508.813	1040.42
C_r	1.4708	69.5242	495.476	521.964	1017.44
	1.53246	69.6894	460.845	533.	993.845
	1.5927	69.8521	427.565	542.28	969.845
	1.48336	69.3917	505.881	529.758	1035.64
	1.47704	69.4582	500.664	525.851	1026.52
$C_{hr} + C_c e_w$	1.4708	69.5242	495.476	521.964	1017.44
	1.46465	69.5898	490.316	518.097	1008.41
	1.45858	69.6548	485.185	514.251	999.436

Table 5.7: Table 2: Variation in p, T_r, TPR and TPM and $TPR+TPM$ with respect to different parameters

Parameter	Optimal T_r	Optimal p	Optimal TPR	Optimal TPM	$TPR+TPM$
P	1.53555	68.857	547.446	755.204	1302.65
	1.53555	68.857	547.446	723.264	1270.71
	1.53555	68.857	547.446	691.199	1238.64
	1.53555	68.857	547.446	659.043	1206.49
	1.53555	68.857	547.446	626.82	1174.27
C_m	1.53555	68.857	547.446	652.125	1199.57
	1.53555	68.857	547.446	606.539	1153.98
	1.53555	68.857	547.446	560.952	1108.40
	1.53555	68.857	547.446	515.366	1062.81
	1.53555	68.857	547.446	469.78	1017.23
C_{hm}	1.53555	68.857	547.446	561.428	1108.87
	1.53555	68.857	547.446	561.19	1108.64
	1.53555	68.857	547.446	560.952	1108.40
	1.53555	68.857	547.446	560.715	1108.16
	1.53555	68.857	547.446	560.477	1107.92
C_ce_m	1.53555	68.857	547.446	566.67	1114.12
	1.53555	68.857	547.446	563.811	1111.26
	1.53555	68.857	547.446	560.952	1108.40
	1.53555	68.857	547.446	558.094	1105.54
	1.53555	68.857	547.446	555.235	1102.68
C_p	1.53555	68.857	547.446	590.988	1138.43
	1.53555	68.857	547.446	575.97	1123.42
	1.53555	68.857	547.446	560.952	1108.40
	1.53555	68.857	547.446	545.934	1093.38
	1.53555	68.857	547.446	530.917	1078.36
θ	1.53588	68.8538	547.699	561.249	1108.95
	1.53571	68.8554	547.572	561.1	1108.67
	1.53555	68.857	547.446	560.952	1108.40
	1.53538	68.8587	547.319	560.804	1108.12
	1.53522	68.8603	547.192	560.656	1107.85
C_d	1.50655	69.151	524.673	543.845	1068.52
	1.50286	69.1891	521.712	541.618	1063.33
	1.49919	69.2269	518.76	539.397	1058.16
	1.49554	69.2646	515.817	537.183	1053.00
	1.49193	69.3022	512.884	534.976	1047.86

5.7.1 Discussions on sensitivity analysis

From the above table 5.6 and 5.7 we observe the following.

The sensitivity analysis of the parameters under study reveals that the variables change directly change the total profit functions of retailers and manufactures. The variables p and T_r, TPR and TPM directly effected by these variables as it is seen from the table. A change in the variable a increases the total profit function through total profit of manufacturer. In the case of b, it increases total profit, The variables L, γ and C_r also shows a change in total profit and total profit manufacturer.

5.8 Conclusion

A two echelon supply chain with manufacturer retailer policy is discussed in this chapter. We presented the model with product demand depend on its selling price, freshness index and advertisement strategy. application of deterioration in the process is another criteria for the model. As the fresh product items are maintained with at most care there is a chance of carbon emission due to the methods applied during both at retailer and manufacture level. We explore an inventory model for fresh products with the effect of carbon tax policies. Mathematical model is solved with respect to the carbon tax policies and compared the the profit values with and without effect of tax policy. From the results we observed that profit is higher when firm is working without policy but there is only a slight change in profit difference is noted. So the importance of reducing carbon emission will not affect by the profit difference. Hence for a sustainable growth of the business implementation of tax policy is must required. Besides these main objective of this chapter is to maximize the manufacturer's total profit with respect to the freshness of the product and price. In accordance with the manufactures profit we derived the cost and profit affect of retailer to get a comparison of the profit allocation.

Chapter 6

SUMMARY AND FUTURE RESEARCH WORK.

6.1 Summary of the thesis

In this research work, some inventory models under various factors like time varying demand condition, different aspects of holding cost etc. have been studied. The models were developed for items that deteriorates, ameliorates over the planning horizon. The impact of inventory parameters are formulated and solved in different environments. The models illustrated with some innovative real-life related inventory problems in different directions.

In chapter 2 of the thesis, a deterministic EOQ model incorporating some practical features is discussed.The new model is developed with considering assumptions like demand function and holding costs being time-dependent, the inventory deteriorating at a variable rate over time, and assumptions like shortages being allowed and completely backlogged. The additive Weibull deterioration rate introduced in the model presented different phases of deterioration rate.

An inventory model for deteriorating items with an expiration period to evaluate the impact of both quantity and quality losses within a trade credit policy is discussed in the third chapter. The demand rate is depend on the selling price and the condition of inventory freshness. From the illustrated examples the effectiveness and validation of the optimality of the solution discussed. The analysis reveals that the higher profitability is obtained when total period of recycle order time and credit period granted by retailer is less than permissible delay period in payments offered by the supplier. Hence it involves period with no interest charges, as the retailer can cover the total purchasing cost by the time granted by the supplier. During this period retailer can receives sales revenue for all items at time. The chapter highlights the sensitivity and affirms the existence of a unique and optimal solution for the retailer.

To connect the combination of deterioration and amelioration a new model is proposed in the forth chapter of the thesis. An inventory model for deteriorating and ameliorating inventory items were discussed to connect real life practices. Early reaching the time until maximum inventory will retain the optimal cost to be minimum. From the illustrated examples the minimum total cost is obtained when the time to maximum inventory level reaches is minimum. The sensitivity analysis provided the impacts of parameters in the study. The optimum cost is effected most by variation in the initial ordering cost, holding cost, demand parameters, ameliorating parameter and initial inventory level. So appropriate actions in these parameters leads to reduction in cost function. The purchasing cost and backlogging parameter show direct variation concerning the optimum cost.

The effect of carbon tax policy to a supply chain process in the sustainable development era is illustrated in the fifth chapter of the thesis.A manufacturer retailer model is developed with the fact of maximizing the profit subject to the conditions of demand. From the examples provided in the chapter the profit difference with carbon emission costs were discussed. Advertisement frequency is directly depend on the profit of both retailer and manufacturer but high frequency will effect on reducing the revenue. So necessary balanced level have to be maintained.

6.2 Limitations and future research scopes

Since studying inventory control under various environments is a vast field, this research focuses on specific scenarios and models. In future studies, further exploration can be done by modifying or adding different real-life situations to the models. This could include considering more complex supply chain dynamics, incorporating additional variables like seasonal demand variations, or examining the impact of emerging technologies on inventory management. Additionally, future research could explore the effects of different types of assumptions such as economic fluctuations or natural disasters, on inventory control strategies. The model proposed in Chapter 2 dealt with incorporating production and non production periods and time dependent demand. This model can be extended by considering unequal cycle length in a finite time horizon, which is our limitation. The model can be illustrated with various demand options, pricing strategies, and implementation of quantity discount conditions for the end customers.

We discussed payment relaxation with the consideration of expiration date for Deteriorating items in chapter three. Implementation of preservation technology to reduce the quantity loss is a limitation to our study. The study does not explore offering discount facilities to overcome the issues related to quantity and quality loss during the inventory period. From the observations the model's scope could be extended by considering investments in preservation technology to mitigate deterioration losses and exploring various demand types such as inventory-dependent variables, single-vendor-and-multiple-buyers systems, and allowing shortages.

We dealt with deteriorating ameliorating items to develop the new model in chapter 4. A major limitation of the study is that it does not consider defective items if any during the amelioration and demand period. In light of the findings and limitations of this chapter, it is evident that there are promising opportunities to expand our research in various directions. Since this work deals with time dependent deterioration there is a chance of extending the work with other forms of deterioration. The amelioration rate can be considered as another form to develop the new system. Also, demand patterns can be changed to develop new models because the demand function is important in reducing the cost. Consideration of various cost functions associated with the study of growing up items will also be a quite worthwhile model to pursue in the future. The manufacturer-retailer process in the last chapter indicates demand depends on freshness, selling price, and advertisement frequency. There is high chance of getting modify this with demand as deal with functions of other significant variables such as stock level, preservation technologies, shelf space since it is fresh products and discounts. Other than this, multi-level supply chains with combinations of retailers may also developed.

Publications

1. Praveen V, P., Manoharan, M. (2020). Analysis of inventory control model for items having general deterioration rate. In: *Joshua V., Varadhan S., Vishnevsky V. (eds) Applied Probability and Stochastic Processes*, Infosys Science Foundation Series. Springer, Singapore. (pp. 293-306). https://doi.org/10.1007/978-981-15-5951-8_18

2. Praveen V, P., Manoharan, M. (2024) Inventory decisions for deteriorating and ameliorating items with partial backlogging and time varying demand. Accepted on 16 May 2024 and entered into the publication schedule of *International Journal of Operational Research*. Indexed in Scopus.

3. Praveen, V.P., Manoharan, M. (2025) An EOQ model for deteriorating items under two-level trade credit financing with expiration date. *Cent Eur J Oper Res* . https://doi.org/10.1007/s10100-025-00957-0. Indexed in Scopus.

4. Praveen, V.P., Manoharan, M. (2024) An integrated supply chain model with product of the demand influenced by its price, freshness and advertisement strategy. Communicated to *International Journal of Operations and Quantitative Management*. Indexed in Scopus.

Presentations

1. *Adoption of trade credit policy in optimization of inventory model with imperfect items.* National conference on Statistical insights and data science trends organized by the Department of Statistics (UG & PG), in collaboration with IQAC, SDM College (Autonomous), Ujire, and Karnataka state Statistics association (KSSA), held on 6th April 2024 at SDM PG Centre, Ujire.

2. *An EOQ model for deteriorating items under two-level trade credit financing with expiration date.* Seventh International Conference on Statistics for Twenty-first Century-2021 (ICSTC - 2021) organized by the International Statistics Fraternity(ISF) and Department of Statistics, University of Kerala, Trivandrum during 15 - 19 December, 2021.

3. *A novel inventory policy for imperfect items with stock dependent demand rate.* 10th World Congress in Probability and Statistics, jointly organized by the Bernoulli Society and IMS, from July 19 to 23, 2021.

4. *Analysis of inventory control model for items having general deterioration rate.* International Conference on Advances in Applied Probability and Stochastic Processes organized by Centre for Research in Mathematics, Department of Mathematics, CMS College Kottayam during 7-10 January 2019.

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