1

PAIR PRODUCTION IN NON ABELIAN GAUGE FIELDS

by

SETHUMADHAVAN PUZHAKKAL

Thesis submitted to the
University of Calicut
in partial fulfillment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY
under Faculty of Science

Department of Physics
University of Calicut
AUGUST 2007

\

PAIR PRODUCTION IN NON ABELIAN GAUGE FIELDS

by

SETHUMADHAVAN PUZHAKKAL

Thesis submitted to the
University of Calicut
in partial fulfillment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY
under Faculty of Science

Department of Physics
University of Calicut
AUGUST 2007

2

Dedicated to my wife and children

\

PAIR PRODUCTION IN NON ABELIAN GAUGE FIELDS

by

SETHUMADHAVAN PUZHAKKAL

Thesis submitted to the
University of Calicut
in partial fulfillment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY
under Faculty of Science

Department of Physics
University of Calicut
AUGUST 2007

CERTIFICATE

Certified that the work presented in this thesis is a bonafide work done by Mr.Sethumadhavan Puzhakkal, under my guidance in the Department of Physics, University of Calicut and that this work has not been included in any other thesis submitted previously for the award of any degree.

V.M.Bannur

Supervising Guide

University of Calicut

AUGUST 2007

PAIR PRODUCTION IN NON ABELIAN GAUGE FIELDS

by

SETHUMADHAVAN PUZHAKKAL

Thesis submitted to the
University of Calicut
in partial fulfillment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY
under Faculty of Science

Department of Physics
University of Calicut
AUGUST 2007

DECLARATION

I declare that the work presented in this thesis is based on the original work done by me under the guidance of Dr. V.M.Bannur, Department of Physics, University of Calicut and has not been included in any other thesis submitted previously for the award of any degree either to this university or to any other university/institution.

Sethumadhavan Puzhakkal

University of Calicut

AUGUST 2007

PAIR PRODUCTION IN NON ABELIAN GAUGE FIELDS

by

SETHUMADHAVAN PUZHAKKAL

Thesis submitted to the
University of Calicut
in partial fulfillment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY
under Faculty of Science

Department of Physics
University of Calicut
AUGUST 2007

ACKNOWLEDGEMENTS

I am indebted to several people around me for the successful completion of this thesis, in its present form. No doubt, Dr.V.M.Bannur, my guide throughout, who offered me scholarly and analytical suggestions and insights, is the first and foremost among them to be thanked and acknowledged. In addition to scientific and technical aspects, his moral encouragement was a chief motivator for me to move ahead. His educative suggestions, opening up vistas of new possibilities, has humbled me as a teacher, and enriched me as a student. I can only acknowledge him with reverence and humility.

During the progress of this effort, almost everyday moved ahead, with encouragement from Professor Dr. B.R.S.Babu, and as such, I should recall his efforts with warmth, and gratitude. So must I, Dr.K.M.Varier, HOD of Physics, for the facilities and help he extended with pleasure. And in this, how can I forget to express my thanks to all other faculties of the department, whose cooperation never diminished from the best possible.

Mr.K.M.Udayanandan, without whose association and collaboration this thesis could not obtain the current form, I should not forget to acknowledge. Words simply refuse to expand to express the myriad set of activities he extended himself for the success of this effort. Sessions for motivation, inspiration, encouragement, moral support, deep rational debates to the core of things.

I must also recall with joy the sense of dedication to my wife Mini.K and children Shikhil and Nikhil exhibited while I followed this call with single minded devotion, often withdrawing my normal support. I thank them specially and acknowledge their sacrifices. That leave me to express my sincere gratitude to my co-worker Dr.A.K.Anila who gave me the wherewithal to finish this work in time.

My thanks for typing and text layout and related Latex conversion goes to Kumaran Moosad, and his colleagues, Pankajam and Abduraoof, Centre for Advanced Software Engineering, Calicut.

Sethumadhavan Puzhakkal

1

PAIR PRODUCTION IN NON ABELIAN GAUGE FIELDS

by

SETHUMADHAVAN PUZHAKKAL

Thesis submitted to the
University of Calicut
in partial fulfillment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY
under Faculty of Science

Department of Physics
University of Calicut

AUGUST 2007

ABSTRACT

This thesis work titled pair production in non-abelian gauge fields is divided into six chapters. Chapter 1 deals with the review of gauge theory for fundamental interactions.

Our aim is to calculate quark-antiquark pair production probability, which eventually leads to the formation of QGP, it is necessary to understand the entire history and development of the formation of QGP. This work is elucidated in chapter 2.

It has been our aim to study the pair production probability in the color flux tube model by taking the effect of non-abelian interaction. By assuming two different model solutions for the color potential, $q\bar{q}$ pair production probability is calculated separately in chapter 3 and chapter 4.

We have considered the picture of the meson where quarks, are connected by flux tubes in the last two chapters. In another model of mesons the quarks and gluons move in potential. We assumed that quarks and gluons move in cornell potential and calculated the flux tube density by fitting the EOS with the Lattice gauge theory results. This work is carried out in chapter 5.

The initial stages of QGP immediately after $q\bar{q}$ production quarks and antiquarks may be moving in cornell potential. Hence the study of their trajectory may be important. This calculation is performed in the last chapter.

1

PAIR PRODUCTION IN NON ABELIAN GAUGE FIELDS

by

SETHUMADHAVAN PUZHAKKAL

Thesis submitted to the
University of Calicut
in partial fulfillment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY
under Faculty of Science

Department of Physics
University of Calicut
AUGUST 2007

6

Contents

PAIR PRODUCTION IN NON ABELIAN GAUGE FIELDS

by

SETHUMADHAVAN PUZHAKKAL

Thesis submitted to the
University of Calicut
in partial fulfillment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY
under Faculty of Science

Department of Physics
University of Calicut
AUGUST 2007

12

List of Figures

1

PAIR PRODUCTION IN NON ABELIAN GAUGE FIELDS

by

SETHUMADHAVAN PUZHAKKAL

Thesis submitted to the
University of Calicut
in partial fulfillment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY
under Faculty of Science

Department of Physics
University of Calicut

AUGUST 2007

\13

List of Tables

PAIR PRODUCTION IN NON ABELIAN GAUGE FIELDS

by

SETHUMADHAVAN PUZHAKKAL

Thesis submitted to the
University of Calicut
in partial fulfillment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY
under Faculty of Science

Department of Physics
University of Calicut
AUGUST 2007

Chapter 1

Review of Gauge Theory for Fundamental Interactions

1.1 Introduction

In this chapter, the gauge theory of fundamental interaction is discussed in detail. Symmetry plays an important role in Physics, because correct identification of various symmetries in nature can reveal the secret of nature. The existence of a symmetry implies that some variable is unmeasurable and the corresponding conjugate variable is conserved. Each symmetry is followed by some invariance and that implies some conservation laws. For this idea we owe much to Amalie Emmy Noether (1882-1935). It was she who first showed that, if there is a symmetry, a conservation law cannot be far behind. For example the translational symmetry means we cannot determine the absolute position in space and the total momentum is conserved. Similarly rotational symmetry leads to conservation of angular momentum, Time symmetry leads to conservation of energy, space-time symmetry leads to energy-momentum tensor, phase symmetry leads to conservation of charge etc. Generalizing this concept that physical measurements are relative, Hermann Weyl proposed that absolute magnitude or norm of the physical vector is also not absolute but should depend on space-time points. A new connection would then be necessary in order to relate the length of vectors at different points. This idea became known as scale or gauge invariance

The idea of gauge invariance was first proposed by German Physicist Hermann Weyl (1885-1955), drawing inspiration from Einsteins general theory of relativity. The fundamental concept in both special and the general theories of relativity is that there is no absolute frame of reference, and the description of motion are relative.

With the development of Quantum Mechanics(QM) the Weyls original theory was given a new meaning by realizing that the phase of a wave function could be a new local variable. Thus the gauge transformation was interpreted as a change in the phase of the wave function. There are two types of gauge symmetries in Quantum Mechanics. One is the global gauge symmetry and the other is the local gauge symmetry. Global gauge symmetry leads to global invariance and the local gauge symmetry leads to generate a force field. For example global gauge symmetry of the Dirac-Lagrangian leads to conservation of charge and electro magnetic force is due to local gauge invariance. Symmetry thus plays a vital role in dictating dynamics.

1.2 The Gauge Theory of the Electromagnetic Field

In Maxwells electromagnetic theory there is a certain phenomenon known as gauge invariance. In physical terms it means the following. Normally one discusses electromagnetic fields in terms of the electric field and the magnetic field. However Maxwell formulated his theory in terms of the scalar potential Φ and vector potential A. Maxwell also showed how the E and B fields are related to Φ and A. It was then discovered that there is a flexibility in the choice of A. ie. Even though A is chosen in different ways, the physically measurable E and B turn out to be the same. This flexibility in the choice of A is referred to as gauge freedom, and making a particular choice is called fixing the gauge. When there is a freedom it implies that some thing remains invariant, which in turn means that something is conserved. In this case there is gauge freedom which leads to gauge invariance. What is the corresponding conserved quantity. Till Hermann Weyls exploration, no one knew, in fact no one even knew that such questions could be asked and such connections discovered. All that became clear, thanks to Noethers work, with which Weyl was familiar.

1.3 Global and Local Gauge Transformation

In physics, gauge theories are a class of physical theories based on the idea that symmetry transformations can be performed locally as well as globally. Most powerful theories are described by Lagranagian which are invariant under certain symmetry transformation groups. When they are invariant under a transformation identically performed at every space time point they are said to have a global symmetry. Gauge theory extends this idea by requiring that the Lagrangian must possess local symmetries as well. It should be possible to perform these symmetry transformations in a particular region of space time without affecting what happens in another region.

1.4 Global Gauge Transformation of Dirac Field

Consider the Dirac Lagrangian L for an electron. Let ψ be the wave function. Suppose we modify the wave function by adding a phase factor to it as follows.

$$\psi_x \rightarrow \psi'_x = e^{i\theta}\psi - - - - - - (1)$$

Now corresponding to the rotation by angle θ in phase space there is a formal transformation $U(\theta)$ in wave function space, which acts on ψ . Thus

$$\psi'(x) = U(\theta)\psi(x)$$

which on comparison with $Eq.(1)$ reveals that

$$U(\theta) = e^{i\theta}$$

Now what is the implication of $Eq.(1)$? We know that ψ is a function of spatial coordinates. $Eq.(1)$ implies that the phase of ψ is being changed by the same angle at all points of space. In other words, θ is not a function of x,such a uniform global change of phase is referred to as a global gauge transformation. It turns out that the Dirac Lagrangian is invariant under a global gauge transformation. Let us consider the free Lagrangian density of Dirac fied

$$L = i\bar{\psi}(x)r_\mu\partial_\mu\psi(x) - \bar{\psi}(x)m\psi(x)$$

Under global transformations

$$\psi(x) \rightarrow \psi'(x) = e^{i\theta}\psi(x)$$

and

$$\bar{\psi}(x) \rightarrow \bar{\psi}' = e^{-i\theta}\bar{\psi}(x)$$

$$L' = i\bar{\psi}'(x)r_\mu\partial_\mu\psi'(x) - \bar{\psi}'(x)m\psi'(x)$$

$$L' = ie^{-i\theta}\bar{\psi}(x)r_\mu\partial_\mu e^{i\theta}\psi(x) - e^{-i\theta}\bar{\psi}(x)me^{i\theta}\psi(x)$$

$$= i\bar{\psi}(x)r_\mu\partial_\mu\psi(x) - \bar{\psi}(x)m\psi(x)$$

$$L' = L$$

The set of such possible transformations form the abelian group $U(1)$. Thus we can say that L is invariant under global gauge group $U(1)$. Abelian just records the property that the group multiplication is commutative. ie.

$$U(\theta_1)U(\theta_2) = U(\theta_2)U(\theta_1)$$

$$e^{i\theta_1}e^{i\theta_2} = e^{i\theta_2}e^{i\theta_1}$$

Now the question is what is the corresponding conserved quantity. Noethers theorem tells us that there is a conserved current given by

$$j^\mu = \bar{\psi}r_\mu\psi$$

satisfying

$$\partial_\mu j^\mu = 0 (\mu = 0, 1, 2, 3)$$

or

$$\partial_0 j^0 + \nabla . J = 0$$

and yielding

$$\frac{d}{dt}\int j^0 d^3x = -\int \nabla . J d^3x$$

$$\frac{d}{dt}\int j^0 d^3x = -\int J.ds = 0$$

Where

$$\int j^0 d^3x = Q$$

is called charge of the field in general and is the generation of the gauge transformation. If Q is the electric charge operator, we call the transformation as the gauge transformation in the charge space. These transformations form a group of continuous transformations with one parameter only and so are termed as $U(1)$. In fact all additive quantum numbers such as the Lepton numbers, the Baryon numbers and the Strangeness etc, will generate gauge transformation corresponding to the group $U(1)$.

1.5 Local Gauge Transformations of Dirac Field

Suppose we decide to make the phase change different at different space points. i.e. we have

$$\psi(x) \rightarrow \psi'(x) = e^{i\theta(x)}\psi(x) - - - - - - - - (2)$$

The transformation above is called a local gauge transformation. Does this leave the Lagrangian (L)invariant?. We will see it now. No, it does not. The Dirac Lagrangian is

$$L = i\bar{\psi} r_\mu \partial_\mu - \bar{\psi} m \psi$$

under the local gauge transformation ($Eq.(2)$) Lagrangian becomes

$$L' = i\bar{\psi}' r_\mu \partial_\mu \psi' - \bar{\psi}' m \psi'$$

$$L' = ie^{-i\theta} \bar{\psi} r_\mu \partial_\mu e^{i\theta} \psi - e^{-i\theta} \bar{\psi} m e^{i\theta} \psi$$

$$L' = ie^{-i\theta} \bar{\psi} r_\mu \partial_\mu e^{i\theta} \psi - \bar{\psi} m \psi$$

But

$$\partial_\mu e^{i\theta} \psi = e^{i\theta} \partial_\mu \psi + ie^{i\theta} \partial_\mu \theta \psi$$

$$\therefore L' = ie^{-i\theta} \bar{\psi} r_\mu (e^{i\theta} \partial_\mu \psi + ie^{i\theta} \partial_\mu \theta \psi) - \bar{\psi} m \psi$$

$$L' = i\bar{\psi} r_\mu \partial_\mu \psi - \bar{\psi} r_\mu \partial_\mu \theta \psi - \bar{\psi} m \psi$$

$$L' = i\bar{\psi} r_\mu \partial_\mu \psi - \bar{\psi} m \psi - \bar{\psi} r_\mu \partial_\mu \theta \psi$$

$$L' = L - \bar{\psi} r_\mu \partial_\mu \theta \psi$$

$$L' \neq L$$

i.e. the Lagrangian is not invariant under local gauge transformations. Then why we bother? There is a reason. Suppose, if we are able to find a suitable L_1such that $L + L_1$ remains invariant under local gauge transformations then something interesting happens. i.e to restore the invariance, a new field say $A\mu$ is introduced to cancel this unwanted extra term. These newly introduced fields are called the gauge fields. These gauge fields are added to the derivative terms to define a new derivative such that it transforms as

$$D_\mu\psi \rightarrow D'_\mu\psi' = e^{i\theta}D_\mu\psi$$

Thus defining

$$D_\mu = \partial_\mu - ieA_\mu$$

$$D'_\mu = \partial_\mu - ieA'_\mu$$

Where $A_\mu \rightarrow A'_\mu$ so as to cancel the extra term which breaks the invariance. Thus the new Lagrangian which is invariant under local gauge transformation can be written as

$$L = i\bar{\psi}r_\mu D_\mu\psi - m\bar{\psi}\psi$$

and

$$L' = i\bar{\psi}'r_\mu D'_\mu\psi' - m\bar{\psi}'\psi'$$

Substituting for

$$\bar{\psi}', \psi' and D'_\mu$$

, we get

$$L' = ie^{-i\theta}\bar{\psi}r_\mu(\partial_\mu - ieA'_\mu)e^{i\theta}\psi - m\bar{\psi}\psi$$

$$L' = ie^{-i\theta}\bar{\psi}r_\mu\partial_\mu e^{i\theta}\psi + e^{-i\theta}\bar{\psi}r_\mu eA'_\mu e^{i\theta}\psi - m\bar{\psi}\psi$$

$$L' = ie^{-i\theta}\bar{\psi}r_\mu(e^{i\theta}\partial_\mu\psi + ie^{i\theta}(\partial_\mu\theta)\psi) + e^{-i\theta}\bar{\psi}r_\mu eA'_\mu e^{i\theta}\psi - m\bar{\psi}\psi$$

$$L' = i\bar{\psi}r_\mu\partial_\mu\psi - \bar{\psi}r_\mu(\partial_\mu\theta)\psi + \bar{\psi}r_\mu eA'_\mu\psi - m\bar{\psi}\psi$$

$$L' = i\bar{\psi}(r_\mu\partial_\mu - ir_\mu eA'_\mu)\psi - m\bar{\psi}\psi - \bar{\psi}r_\mu(\partial_\mu\theta)\psi$$

$$L' = i\bar{\psi}r_\mu(\partial_\mu - ieA'_\mu)\psi - m\bar{\psi}\psi - \bar{\psi}r_\mu\partial_\mu(\theta)\psi$$

But

$$L = i\bar{\psi}r_\mu(\partial_\mu - ieA_\mu)\psi - m\bar{\psi}\psi$$

For invariance

$$L' = L$$

Equating the two we get

$$\bar{\psi}r_\mu eA'_\mu\psi - \bar{\psi}r_\mu(\partial_\mu\theta)\psi = \bar{\psi}r_\mu eA_\mu\psi$$

From this we get

$$A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\theta$$

Hence the local gauge invariant Lagrangian is obtained by replacing the ordinary derivative by another derivative called the covariant derivative. The new gauge field $A\mu$ is interacting with Dirac field exactly the same way as the photon field interacting with matter. i.e the additional term in the new Lagrangian in this case is nothing but the interaction of the matter field with an external electromagnetic field. Thus the mere requirement of local phase invariance has generated an interaction term between the matter field and the gauge field. This is the essence of the gauge principle for generating the dynamical theories.

1.6 Global Gauge Transformation of Klein - Gordon Field

We consider a complex scalar field $\Phi(x)$. The Lagrangian density of a complex Klein-Gordon field in the absence of electromagnetic coupling is

$$L = \partial_\mu \Phi^* \partial^\mu \Phi - m^2 \Phi^* \Phi - - - - - - - - (1)$$

Lagrangian density is invariant under the transformation

$$\Phi(x) \rightarrow e^{i\theta}\Phi(x), \Phi^*(x) \rightarrow e^{-i\theta}\Phi^*(x)$$

and

$$\partial^\mu \Phi(x) \rightarrow e^{i\theta}\partial^\mu \Phi(x), \partial_\mu \Phi^*(x) \rightarrow e^{-i\theta}\partial_\mu \Phi^*(x)$$

Since θ does not depend upon space time point, this transformation is called global gauge transformation and the theory is said to be global gauge invariant under the group $U(1)$. Noethers theorem tells us that there is a conserved current j^μ is given by

$$j^\mu = \Phi^* \partial^\mu \Phi - (\partial^\mu \Phi^*)\Phi$$

Satisfying the continuity equation

$$\partial_\mu j^\mu = 0$$

or

$$\partial_0 j^0 + \nabla . J = 0$$

And yielding

$$\frac{d}{dt}\int j^0 d^3x = -\int \nabla.J d^3x = \int J.ds = 0$$

Where

$$\int j^0 d^3x = Q$$

is called a charge of the field in general and is the generator of the gauge transformation. If Q is the electric charge operator, we call the transformation as the gauge transformation in the charge space. These transformations form a group of continuous transformations with one parameter only and so are termed $U(1)$ as already explained.

1.7 Local Gauge transformation of Klein-Gordon Field

Here we would like to see what would happen if we allow the parameter θ to depend upon the space time coordinates. Thus

$$\Phi(x) \rightarrow e^{i\theta(x)}\Phi(x), \Phi^*(x) \rightarrow e^{-i\theta(x)}\Phi^*(x)$$

The important point to note here is that the Lagrangian density Eq.(1) is not invariant under this transformation because the derivative acts on phase also and so transformation becomes

$$\partial^\mu\Phi(x) \rightarrow \partial^\mu e^{i\theta}\Phi(x) = e^{i\theta}\partial^\mu\Phi(x) + i\theta(x)e^{i\theta}(\partial^\mu\theta(x))\Phi(x) - - - - - - - - -(2)$$

We have

$$L = \partial_\mu \Phi^* \partial^\mu \Phi - m^2 \Phi^* \Phi$$

Under the transformation

$$L \to L' = \partial_\mu e^{-i\theta} \Phi^* \partial^\mu e^{i\theta} \Phi - m^2 e^{-i\theta} \Phi^* e^{i\theta} \Phi$$

$$L' = \partial_\mu (e^{-i\theta} \Phi^*) \partial^\mu (e^{i\theta} \Phi) - m^2 \Phi^* \Phi$$

using $Eq.(2)$ we get

$$L' = [e^{-i\theta} \partial_\mu \Phi^* - i\theta e^{-i\theta} \partial_\mu(\theta) \Phi^*][e^{i\theta} \partial^\mu \Phi(x) + i\theta e^{i\theta} \partial^\mu(\theta) \Phi] - m^2 \Phi^* \Phi$$

$$L' = \partial_\mu \Phi^* \partial^\mu \Phi(x) + \theta^2 \partial_\mu(\theta) \partial^\mu(\theta) \Phi^* \Phi + i\theta \partial_\mu \Phi^* \partial^\mu(\theta) \Phi - i\theta \partial_\mu(\theta) \Phi^* \partial^\mu \Phi(x) - m^2 \Phi^* \Phi$$

It shows that

$$L \neq L^1$$

Now we define

$$\partial^\mu \to \partial^\mu + iqA^\mu, A^\mu \to A^\mu - \frac{1}{q} \partial^\mu(\theta)$$

Now ∂^μ is replaced by $\partial^\mu + iqA^\mu$, the Lagrangian density becomes

$$L = (\partial_\mu - iqA_\mu) \Phi^* (\partial^\mu + iqA^\mu) \Phi - m^2 \Phi^* \Phi$$

$$L = \partial_\mu \Phi^* \partial^\mu \Phi - iqA_\mu [\Phi^* \partial^\mu \Phi - (\partial^\mu \Phi^*) \Phi] + q^2 A_\mu A^\mu \Phi^* \Phi - m^2 \Phi^* \Phi$$

This Lagrangian is invariant under local gauge transformation and it contains the gauge field A_μ. Finally we add the kinetic energy of the electro magnetic field to the above Lagrangian density. This term also must be gauge invariant. If we define

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

We see that $F^{\mu\nu}$ is invariant under the transformation

$$A^{\mu}(x) \to A^{\mu}(x) - \frac{1}{q}\partial^{\mu}\theta(x)$$

we are thus lead to the final Lagrangian

$$L = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (\partial_{\mu} - iqA_{\mu})\Phi^{*}(\partial^{\mu} + iqA^{\mu})\Phi - m^{2}\Phi^{*}\Phi$$

We have achieved the invariance under the local gauge transformation by introducing a vector field A_{μ}. If this vector field A_{μ} is the electromagnetic vector potential and $F_{\mu\nu}$ is the electromagnetic field tensor, the above Lagrangian then describes a charged Klein -Gordon field interacting with the electromagnetic field. Let us note that L does not contain a mass term like$\frac{1}{2}M^{2}A^{\mu}A_{\mu}$, since such a term would violate the invariance under the transformation. In other words gauge invariance forces the vector bosons to be massless. This is correct for the photon which is massless.

1.8 Geometrical Description of Gauge Theory

The geometrical picture of gauge theory provides a common platform for discussing electromagnetic, strong nuclear, weak nuclear and gravity. It depends only on very general properties of gauge principle. Any particle or system which carries an internal quantum number like charge, isospin, color etc is considered to have a direction in its internal symmetry space. In this picture a particle is identified with its space time coordinates and the orientation in its internal space. Thus as the particle moves through space time, it traces out a path in its internal space above the space time trajectory. In order to compare this internal space direction at different space time points we need to define a connection. This connection must be capable of relating all possible directions and orientations in the internal space to each other. The set of all such rotations form a symmetry group, and the group transformations lead to a connection which will be identified with a gauge potential field. A general form of a local symmetry transformation from an arbitrary group can be written as

$$U\psi = e^{-ig\sum_k \theta^k(x)T^k}\psi$$

Here g is a general coupling constant. For example $g = e$, the electric charge for the electromagnetic $U(1)$ the gauge group,T^k are the generators of the internal symmetry group and satisfying commutation relations.

$$[T^i, T^j] = iC^{ijk}T^k$$

Where C^{ijk} are called the structure constants of the Lie group defined by the commutation relations. Let a particles wave function be split into external and internal parts corresponding to the space time coordinate and the internal space coordinate as

$$\psi(x) = \sum_\alpha \psi_\alpha(x)u_\alpha$$

Here u_α form a set of basis vectors in the internal space, $\psi_\alpha(x)$ is a component of $\psi(x)$ in the basis of u_α and they transform like

$$\psi_\beta(x) = U_{\beta\alpha}\psi_\alpha(x)$$

When this particle moves from x to $x + dx$ through an external potential field, $\psi(x)$ changes by

$$d\psi = \psi(x + dx) - \psi(x)$$

$$= \sum_\alpha (\partial_\mu \psi_\alpha) dx^\mu u_\alpha + \psi_\alpha du_\alpha$$

The du_α can be calculated from an infinitesimal internal rotation associated with an external displacement dx. We have

$$U(dx) = e^{-iq\sum_k d\theta^k T^k}$$

Where $d\theta^k = \partial_\mu \theta^k dx^\mu$ which rotates the internal basis by an infinitesimal amount du_α

$$\therefore \quad U(dx)u_\alpha = u_\alpha - du_\alpha$$

$$= [\delta_{\alpha\beta} - iq\sum_k \partial_\mu \theta^k dx^\mu T^k_{\alpha\beta}]u_\beta$$

Thus

$$du_\alpha = iq\sum_k \partial_\mu \theta^k dx^\mu T^k_{\alpha\beta} u_\beta$$

Now defining a new connection

$$(A_\mu)_{\alpha\beta} = \sum_k \partial_\mu \theta^k T^k_{\alpha\beta}$$

We get

$$d\psi = \sum_{\alpha,\beta}[\partial_\mu \psi_\alpha \delta_{\alpha\beta} - iq(A_\mu)_{\alpha\beta}\psi\alpha]dx^\mu u_\beta$$

Thus defining

$$D_\mu \psi_\alpha = \sum_\beta [\partial_\mu \delta_{\alpha\beta} - iq(A_\mu)_{\alpha\beta}]\psi_\beta$$

This new derivative is called the gauge covariant derivative, which describes the changes in both external and internal parts of $\psi(x)$. For electromagnetic $U(1)$ gauge group the internal space is one dimensional,so we have

$$D_\mu \psi = (\partial_\mu - iqA_\mu)\psi$$

and A_μ transforms as

$$A'_\mu = UA_\mu U^{-1} - \frac{i}{q}(\partial_\mu U)U^{-1}$$

1.9 Field tensor and its Abelian and Non-Abelian nature

The field tensor $F_{\mu\nu}$ can be derived geometrically using Stokes theorem. For example in electrodynamics the line integral over the potential A; $\oint A.dx$ can be interpreted geometrically as the net change in the internal directions of a test particle which has been moved around a closed path. This expression is therefore is a phase change of the particles wave functions. Consider a test particle moving with a successive displacements dx and dy around a closed path. The net change in its internal direction in a two different path can be calculated.

The guage transformation for the displacement $x \rightarrow x + dx$ along path(1) can be written as

$$U_x(dx) = 1 - iqA_\mu(x)dx^\mu$$

Then along path(2) ie. $x + dx \rightarrow x + dx + dy$

$$U_{x+dx}(dy) = 1 - iqA_\mu(x + dx)dy^\nu$$

Using Taylor expansion and keeping only lower order terms

$$U_{x+dx}(dy)U_x(dx) = 1 - iqA_\mu(x)dx^\mu - iqA_\nu(x)dx^\nu - q^2A_\nu(x)A_\mu(x)dy^\nu dx^\mu - iq\partial\mu A_\nu(x)dy^\nu dx^\mu$$

Similarly for the $path(3)$ and $path(4)$, we get

$$U_{x+dy}(dx)U_x(dy) = 1 - iqA_\nu(x)dx^\nu - iqA_\mu(x)dx^\mu - q^2A_\mu(x)A_\nu(x)dx^\mu dy^\nu - iq\partial_\nu A_\mu(x)dx^\mu dy^\nu$$

Thus the net change in its internal orientation is

$$U(dy)U(dx) - U(dx)U(dy) = -iq\{\partial\mu A_\nu - \partial_\nu A_\mu + iq[A_\mu, A_\nu]\}dx^\mu dy^\nu$$

Where A_μ and A_ν do not commute because they are different combinations of the internal group generators T^k. Thus the derivatives $\partial_\mu A_\nu$ and $\partial_\nu A_\mu$ are also not equal in general. Thus the gauge transformation for the different paths does not produce the same phase. Now comparing with the Stokes Theorem, dxdy is the surface area enclosed by the path, then one identifies the non - abelian version of the field tensor defined as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + iq[A_\mu, A_\nu]$$

1.10 Abelian Gauge Fields

When the gauge group generators commute each other, gauge field is called an abelian gauge. Mathematically speaking, for an abelian gauge field the last term in the above equation vanishes. Then the field tensor becomes

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

For example quantum electrodynamics is an abelian gauge theory. Then the components of this gauge field tensor can be identified as the usual electric and magnetic fields in the Maxwells electromagnetic theory. The time components are the electric fields and the space components are the magnetic fields.

1.11 Non- Abelian Gauge Fields

Following the gauge theory of electromagnetic, In 1954 C.N Yang and R.Mills constructed a field theory for the strong interactions which is exactly gauge invariant. They postulated the local gauge group as $SU(2)$ Isotopic spin group. A new isotopic spin potential was therefore postulated by them in analogy with electro magnetic potential. However the greater complexity of the $SU(2)$ compared to $U(1)$ makes the Yang Mill potential quite different from that of electromagnetic. The most general form of the Yang Mills potential is a linear combination of the generators of the $SU(2)$ group similar to the angular momentum operators. Thus

$$A_\mu = \sum_i A^i_\mu L_i$$

This explicitly displays the dual act of Yang - Mills potential as both the field in space time and an operator in the isotopic spin space. The potential has three charge components corresponding to the three independent angular momentum components. In this description a neutron is transformed into a proton by absorbing a unit of isospin from the Yang-Mills gauge field A_μ. This shows that the Yang-Mills gauge field A_μ must itself carry an electric charge unlike the electromagnetic potential. However the gauge invariance demands them to be massless. Thus the short range nuclear force could not be explained by this. But Yang-Mill theory established a foundation for the modern gauge theory and provides a new insight into the newly discovered internal quantum number to determine the fundamental form of the interactions. As a result of the subsequent developments in particle physics especially the introduction of quarks and its color degrees of freedom, the Yang - Mill theory was revived to describe the $SU(3)$ color dynamics, In this $SU(3)$ local gauge symmetry the gauge potential $A\mu$ carries eight charges corresponding to the three colors for quarks and this field is represented as

$$A_\mu = A^\ell_\mu T^\ell$$

And the field tensor derived earlier becomes

$$F_{\mu\nu} = F^\ell_{\mu\nu} T^\ell$$

Where T^ℓ are the generators of $SU(3)$ group and are related to the Gell-Mann λ- matrices as $T^\ell = \frac{1}{2}\lambda^\ell$ and they are given as

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

and they satisfy the commutation relation

$$[T^\ell, T^m] = i f^{\ell mn} T^n$$

Then the color field tensor becomes

$$G^a_{\mu\nu} = \partial\mu A^a_\nu - \partial\nu A^a_\mu - g f^{amn} A^m_\mu A^n_\nu$$

Where f^{amn} is the fine structure constant.From the expression for $G^a_{\mu\nu}$, it is evident that these color gauge fields interact themselves. Thus this theory becomes a non linear field theory unlike electromagnetic $U(1)$. Because of the non linear interactions of Yang-Mills potentials among themselves the quantum chromodynamics become almost impossible to solve exactly. Also the dynamics of these fields possess peculiar properties compared to the electromagnetic case(see the comparison table). Like in QED the gauge potential corresponds to the photon, here in QCD these non-linear gauge potential corresponds to color photons called gluons. The colored quarks interact through the exchange of these colored gluons. The interactions of these color gluons among themselves are the ones which cause the theory to become nonlinear unlike QED.

1.12 Distinction between QED and QCD

QED	QCD
1)It is a quantum field theory describing the electron and its associated electromagnetic field	1)It is also a quantum field theory describing the quarks and the associated color field
2)The gauge field is photon	2)The gauge field is gluons
3)There is only one photon	3)There are eight gluons
4)Photon carries no charge	4)Gluon carries color charge
5)Photons do not interact themselves	5)Gluon interacts themselves
6)The charged particles and the photons exist in free states in nature	6)Quarks and gluons do not exist as free states in nature- permanently confined objects.
7)It is an abelian gauge theory	7)It is a non-abelian gauge theory.
8)It is $U(1)$ gauge theory	8)It is $SU(3)$ color gauge theory
9)The coulomb potential exists to large separation $V(r) = q/r$	9)The gluon fragmentation and recombination contribute to generate non coulombic potential $V(r) = ar$ or br^2
10)QED Vacuum is perfect diamagnetic	10) QCD Vacuum is perfect dielectric.

1.13 References

[1]Amitabba Lahiri ,Palash-B.pal , A First book of Quantum Field Theory Narosa Publishing House (2001)

[2]Bannur .V.M.Ph.D Thesis B.C.C.N T86/121/161 (1988)

[3]Bjorken, J.D. Phys.Rev D 27 , 140 (1983)

[4]Bogolioubov P.L,Ann.Inst.Henripoincare Quarks Confinement and Quark Model Sec.2 ,163(1967)

[5]C.S.Warke and R.S.Bhalerao, Pramana J.Phys.38,1 ,37 ,Jan(1992)

[6]CarlosAragaodeCarvalhoandS.GoulartRosaJr,J.Phys.A.Math.Gen.13,3233(1980)

[7]G.Venkataraman The Big and the Small Universities Press, India

[8]Gang Chen,Phys.Letts A.326,55(2004)

[9]Gell-Mann.M and Neeman.Y The EightFold WayBenjamin,NewYork (1964)

[10]Gell-Mann M and Pais A Phys.Rev. 97 ,1387 (1995)

[11]H. Satz, Nature 324, 13 (1986)

[12]H.Haugerud and F.Ravndal,arxiv.cond.matt/9509041,Vol1,7,sept(1995)

[13]H.Haugerud,T.Haugset,arxiv.cond.matt/9605100Vol1,16May(1996)

[14]Helge kragh,Quantum Generations,University Press (2001)

[15]Ihsan Yilmaz I, J.Mod.Phys. D,24(2005)

[16]Itzykson and ZuberQuantumFieldTheoryMcGrawHillPublications(1988)

[17]J.Dunning- Davies, J.Phys. A.Math.Gen.14, 3013(1981)

[18]Jackson J.D,Classical electro dynamics,Wiley, Newyork (1974)

[19]John A.Eisele ,Modern Quantum mechanics with Applications to Elementary Particle Physics(1968)

[20]Jose A. Cuesta, Phys. Rev.E,65, 031406(2002)

[21]Lee .T.D Particle Physics and Introduction to Field Theory Harwood Academic Publishers (1981)

[22]M.P Khanna Introduction to Particle Physics Prentice Hall of India ,NewDelhi(2002)

[23]Peter A.carruthers Introduction to Unitary Symmetry Inter Science Publishers , NewYork (1966) Los Alamos Science ,No 11 Summer/ Fall , published by LosAlamos National laboratory , New Mexico (1984)

[24]Prashant Shukla, An Introduction to Relativistic heavy ion collisions and Quark-Gluon

[25]Plasma,Lecture notes for 46th batch of physics trainee scientific Officers of BARC,Bombay

[26]Ramond.P.Field Theory A modern PrimerBenjamin Cummings.(1981)

[27]Sakurai J.J Advanced Quantum Mechanics Addison ,Wesley (1967)

[28]Schwinger Selected Papers of Quantum Electro Dynamics Dover ,New York(1953)

[29]Silvan.S.Schweber QED and the men who made it Dyson, Feynman ,Schwinger and Tomonaga , University Press (1999)

[30]Stanislaw Mrowczy nski ,Acta- Physica Polonica.B29 ,3711 (1998)

[31]Tarasov LThis Amazingly Symmetrical World Mir Publishers (1986)

****

1

PAIR PRODUCTION IN NON ABELIAN GAUGE FIELDS

by

SETHUMADHAVAN PUZHAKKAL

Thesis submitted to the
University of Calicut
in partial fulfillment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY
under Faculty of Science

Department of Physics
University of Calicut
AUGUST 2007

Chapter 2

Quark Gluon Plasma

2.1 Introduction

A millionth of a second after the Big Bang, the universe was an incredibly dense plasma, so hot that no nuclei nor even nuclear particles could exist. The plasma consisted of quarks, the particles that compose nucleons and some other elementary particles, and gluons(the massless particles that carry the force between quarks). Gluons are the particles that quarks exchange as they interact or in the language of modern physics, gluons mediate the strong force between quarks. Since quarks make up protons and neutrons, this leads to the force that holds protons and neutrons together in a nucleus.The term plasma is adopted in analogy with electrodynamics plasma(where plasma is defined as a medium containing many charged particles governed by electromagnentic forces.It is a quasi neutral gas of charged particles which possesses collective behavior.The plasma medium is described macroscopically by its temperature and density and changes in the plasma are calculated by using conservation laws such as conservation of energy,momentum and mass). A plasma is a matter in which charges are screened due to the presence of other mobile charges.In QGP the color charges of the quarks and gluons are screened.

2.2 Quarks and Gluons

The last forty three years have completely changed our understanding of the strong interactions, which in nuclear physics are often called the nuclear forces, and of the particles undergoing them, for which one uses the generic name of hadrons. They consist of baryons like proton, neutron, hyperons and their resonances, their antiparticles the antibaryons, and mesons like pion, kaon and their resonances. The key to the new understanding is the representation of all hadrons as composites of basic fermions called quarks(q) and antiquarks ($\bar{q}$), with Quantum Chromo Dynamics(QCD) as the non-abelian gauge field theory describing the strong interaction between $q's$, and $\bar{q}'s$, the gluons being the quanta of the QCD gauge fields. Experimentally one knows six types (flavors) of quarks, labeled u and d (up and down) for the constituents of nucleons, s for strange, c for charm and b for bottom quarks and t for top quark. In addition to the partons, matter as we know it today also consists of leptons. All leptons are fermions. The electrically charged ones are the electron e^-, its antiparticle the positron e^+, the positive and negative muons and tau leptons, while the neutral ones are neutrinos associated one to one with the charged leptons. The leptons do not undergo the strong interaction, i.e., are not directly coupled to the gluons. All basic fermions (leptons and quarks) undergo with various couplings. The electroweak interactions, described by another non-abelian gauge field theory called Electro Weak Theory (EWT) with the photons and weak bosons W^+,W^- and Z^* as quanta of the gauge fields. The photons, very familiar to all physicists, ofcourse mediate the well known electromagnetic interactions, whereas the weak ones are mediated by the very heavy W and Z bosons. The description of matter in terms of quarks and leptons, QCD and EWT with their gauge field quanta, is usually called the Standard Model of particle physics.

Non-abelian gauge field theories, first invented in 1954 by C.N.Yang and R.L.Mills, are generalizations of Quantum Electro Dynamics (QED), itself the relativistic, quantum version of Maxwells theory of electromagnetism and QCD is non-abelian in the sense that, contrary to QED, their gauge groups are non-abelian.

As every physicist knows, the extraordinary thing about quarks and gluons is that they are never seen as free particles. This is another peculiarity of QCD. In QCD the so-called color charge operators which express the couplings between the quarks and gluons (including the direct couplings between gluons imposed by the non-abelian nature of the gauge group) are assumed to be completely hidden. This property, called confinement and so far not yet derived mathematically from the field equations, implies that all observable particles with strong interactions, i.e. all hadrons and nuclei, have zero color charges. They must be colorless composites of the basic color-charge carriers, i.e. the quarks and gluons, the strong interactions between them being analogous to the Van der Waals forces between the electrically neutral atoms and molecules of ordinary physics. Indeed, all known hadrons are composites of this type: qqq for baryons, $\bar{q}\bar{q}\bar{q}$ for antibaryons and $q\bar{q}$ for mesons. Colorless composites involving gluons are also possible, the simplest ones being gluon-gluon glueballs, but they have never been seen experimentally.

2.3 Quark-Gluon plasma (QGP)

While the problems around the possible production and detection of blobs of QGP in nuclear collisions are still mostly unsolved, years of work in lattice QCD (i.e. the Monte Carlo computer treatment of QCD in a lattice approximation of space and imaginary time) have accumulated impressive theoretical evidence for the prediction that, at sufficiently high temperatures($T > 200MeV = 2.3 \times 10^{12}K$), hadronic matter in bulk melts into an equilibrium plasma of deconfined quarks and gluons, with the simultaneous disappearance of the quark vacuum condensates. No way is known to create macroscopic quantities of QGP in the laboratory, but the hot big bang theory of the expanding universe implies that at early times all hadronic matter must havc been in the form of QGP. The problem of how this plasma of colored QCD partons made its transition to confined hadronic matter (eventually only nucleons) has recently attracted much attention among cosmologists, and also possibility of having some, perhaps abnormal, forms of cold QGP inside neutron stars has occupied astrophyscists. These interesting considerations, and more generally the undisputable fact that very hot and dense hadronic matter in bulk plays and important role in astrophysics and cosmology, provide additional reasons for various experimental programme on ultra-relativistic nuclear collisions.

While there is little doubt that the early universe, at an age of perhaps 10 to 100 microseconds after the big bang, went through the transition of QGP to confined hadronic matter, many scenarios are possible for the detailed course of events in this transition. As an example of current speculations in this field, we mention the work of J.H Applegate et al. Who discuss the possibility that the hadronic phase transition may have created inhomogeneities in the space distribution of nucleons, with possible consequences up to the time of early nucleosynthesis (T around 10^9K, age of the universe around one minute). The homogeneities would have appeared during the co-existence period of regions of plasma and of confined hadrons. While they would then have disappeared rapidly for the neutrons by diffusion, they would have survived much longer for the protons which, due to their electric charge, have very small mobility in the electric plasma prevailing at those times. It turns out that such effects may have important consequences for the dark matter problem which has become so central in modern astrophysics.

2.4 Experimental Techniques of Observing QGP

While quarks are basic to the understanding of hadron spectroscopy, for which they were invented in 1964 by M.Gell-Mann and G.Zweig, it is in the field of high energy collisions that they manifest most directly their great explanatory and predictive power. We briefly review the various types of reactions which have been studied systematically so far.

2.5 Electron-Positron Annihilation

The cleanest case is electron-positron annihilation into hadrons at centre of mass (CM) energies E_{CM} above a few GeV. The most striking feature here is that, although the e^+e^- annihilate first in a virtual photon γ^*, a state of maximum spin 1, most of the final states consist of two narrow hadron jets J, J1 oriented back to back in the CM frame, with only a few low-energy particles moving in directions transverse to these jets. This jet structure is naturally explained by QCD. The partons which carry electric charge are quarks(q) and antiquarks($\bar{q}$), the gluons(g) being electrically neutral. Hence the virtual photon materializes into a q and a $\bar{q}$ wich fly apart with great energy in opposite directions. Since they carry color charges, a QCD field (more precisely a color-electric field) builds up between them. In a first phase, this field transforms into many q, $\bar{q}$ and g by formation of a parton shower (the basic branching processes of QCD are $q \rightarrow qg$, $\bar{q} \rightarrow \bar{q}g$, $g \rightarrow gg$ and ggg, $g \rightarrow q\bar{q}$). Then comes the hadronization phase in which partons of compensating color charges form the outgoing hadrons.

The low spin of the virtual photon(0 or 1) implies that the angle between the jets J, J1 and the incident e^+e^-has a broad distribution close to $1 + \cos^2\theta$. This is brilliantly confirmed by experiment and contrasts with the narrow jet structure of single events. The latter is characterized by the fact that the outgoing hadrons have low values, of order $300MeV/c$, for their transverse momenta (i.e. the components of their momenta perpendicular to the average jet direction). That at large E_{CM} the initially created q and $\bar{q}$ give rise to such narrow jets is linked with the relative weakness of QCD couplings for hard processes, i.e. for processes involving large momentum transfers. This is the celebrated property of asymptotic freedom of QCD, which makes renormalized perturbation calculus applicable to the parton shower formation. In contrast, the hadronization cannot yet be calculated systematically because the momentum transfers involved are low (of order $1GeV/c$ or smaller) and for these so-called soft processes the QCD,couplings are too large for perturbative methods to apply. For the moment one describes them with plausible non-perturbative algorithms which were developed over the years in a close interplay of theory and experiment.

Fortunately, many features of e^+e^- annihilation at high energy do not depend on the details of the soft processes. This is true not only for the jet structure described above, but also for the total cross section which can be calculated perturbatively and depends directly on the number of color states of the quarks(three per flavour) and their fractional electric charges($-1/3$ for d, s and b quarks, $2/3$ for u, c and t quarks). These fundamental quantities are thereby tested experimentally in a very direct way.

Occasionally, e^+e^- annihilations occur for which, early in the parton shower, one or more very energetic gluons are emitted at large angle to the initial q and $\bar{q}$. Such hard gluons then produce additional hadronic jets with calculable angular and energy distributions. It is in this way that convincing evidence has been found for gluons.

2.6 Deep Inelastic Lepton Scattering (DILS)

Next in complication is the DILS reaction where a lepton(It could be a muon or a neutrino) exchanges a virtual photon with a nucleon(proton) under such conditions that the four-momentum transfer squared Q^2 is much larger than $1GeV^2$ (hence the name deep inelastic scattering). The quantity Q^2 is defined by

$$Q^2 = (P - P')^2 - (P_0 - P_0')^2$$

Where P, P' are the momentum vectors of the lepton before and after the scattering, and P_0, P_0' the corresponding energies (note that Q^2 is always positive from energy-momentum conservation). The virtual photon is absorbed by a q or $\bar{q}$ in the nucleon and deflects it violently (hard process!), while the remaining partons in the nucleon tend to fly on, leading to the formation of two hadronic jets J, J1 for reasons similar to those explained for e^+e^- annihilation. In the present case, the jets are back to back in the rest frame of the total hadronic system created by the photon exchange.

The photon is absorbed by one of the three valence quarks which compose the nucleon , the remaining two flying on (diquark). It also frequently happens that the photon is absorbed by one of the additional sea q and $\bar{q}$, the sea being the cloud of virtual partons which are present in the nucleon wave function as a consequence of their coupling with the valence quarks (the nucleon sea is analogous to the cloud of virtual photons and e^+e^- pairs attached to an electron in QED, but it is quantitatively more important because the QCD couplings are stronger). Also in this case of DILS on a sea parton one has formation of two hadronic jets by color separation.

The great interest of DILS is that it probes directly the internal quark structure of the nucleon. One measures the cross section as a function of Q^2 and the variable.

$$x = Q^2/2m_p(P_0 - P_0')_{\ell ab}$$

Where m_p is the proton mass and $(...)_{\ell ab}$ refers to energies in the rest frame of the initial nucleon. Energy-momentum conservation implies that x is restricted to values between 0 and 1. The very simple QED formulae for single photon exchange show that DILS is controlled by structure functions $f(x, Q^2)$ which describe the distribution in x of (anti) quarks in the nucleon at a space-time resolution of order Q^{-1} which is much smaller than $(1GeV)^{-1} = 0.2fm$. The variable x has a direct physical interpretation if one considers the nucleon in a frame where it has ultra-relativistic velocity (large momentum frame); x is then the fraction of the nucleon momentum carried by the (anti)quark which absorbs the virtual photon. There are several structure functions corresponding to various spin configurations, but the dominant one, usually called $f_2(x, Q^2)$, simply expresses the probability distribution in x of all q and $\bar{q}$ in the nucleon.

The x-dependence of the structure functions cannot be calculated perturbatively from QCD because it reflects the nucleon wave function in terms of quarks and gluons, which is dominated by small momentum components. Thanks to DILS it can now be measured very well. Perhaps the most remarkable result of these measurements is that, in a large momentum frame, all q and $\bar{q}$ in a nucleon carry only about half of the nucleon momentum, which implies that the other half must be carried by the gluons in the nucleon sea.

The Q^2 dependence of the structure functions is not only measurable, it can also be calculated at large Q^2 from perturbative QCD. Although it is small, it has provided some of the most successful tests of QCD.

2.7 High mass Dilepton Production in Hadronic Collisions (Drell-Yan Process)

The Drell-Yan processes first proposed by S.Drell and T.M. Yan, brings us one elegant step beyond DILS. This time, instead of using an incident lepton to knock a quark out of a nucleon, one takes a purely hadronic reaction and concentrates on the very rare collisions producing a dilepton, i.e. a pair of leptons of opposite electric charges, usually muons. If the effective mass of the dilepton(i.e. its total energy in its own rest frame) is much larger than 1 GeV, the production mechanism is a hard process; a quark of one incident hadron and an antiquark of the other (the pion) annihilate into a virtual photon which then materializes into the dilepton. The process is obviously controlled by the structure functions(SF) of the two incident hadrons. Since the nucleon SF is known from DILS, the π_p Drell-Yan process leads to a experimental determination of the pion SF. The validity of the argument can be tested by doing the same experiment for pp collisions where both SF are known.

While the results agree qualitatively with expectations, discrepancies by about a factor 2 (called the K-factor by the specialists) appear in the comparison with perturbative QCD calculations. This shows that further effects are at work, which is not surprising since the incident hadrons can briefly interact before the $q\bar{q}$ annihilation and the momentum transfers involved are not as large as in the best DILS experiments. The study of dilepton production has therefore led to a fruitful exploration of what is called semi-hard effects, i.e. QCD effects which are at the borderline of applicability of perturbation calculus and require the use of ad hoc approximations.

The Drell-Yan process is again such that nuclear environment effects can be studied by using nuclear targets.

2.8 Purely Hadronic Reactions

We now discuss hadron-hadron collisions at high E_{CM}. The most common type is two incident protons. It produces two hadronic jets J1, J2 flying out in directions very close to those of the incident hadrons, all outgoing particles having low P_T, where P_T is defined as the momentum component perpendicular to the incident direction.

Curiously, these low P_T collisions, which have by far the largest cross sections and for which the largest amount of experimental information is available, turn out to be the most difficult ones to interpret. Theoretical work here is much more descriptive and less deductive than for hard processes. It extracts empirical regularities from the data and constructs plausibile dynamical models able to reproduce them. By lack of consensus among model builders the situation is rather confusing. However, all the other evidence in favour of QCD (in hard collisions and hadron spectroscopy) is so strong that the problem of low P_T collisions is now principally the problem of interpreting them in QCD terms.

It is therefore significant that the two most elaborate and successful models now available (Dual Parton Model and Lund Fritiof Model,) are entirely formulated in terms of QCD partons. While they differ in many ways, they have in common a basic feature which is well supported by the data and is likely to correspond to reality; one valence quark of each hadron seems to be held back in an early phase of the collision while the remaining diquarks tend to fly through. The resulting color fields then materialize into partons which finally go through the hadronization phase. While these steps are qualitatively analogous to those we have encountered in hard processes, the situation here is more complex and much more uncertain because non-perturbative QCD is now playing a central role. On the other hand, this implies that low P_T collisions are likely to provide an important input for the search of reliable non-perturbative methods for hadron production, i.e. for the confinement mechanism which is clearly the most important unsolved problem in QCD.

This is not to say that perturbative QCD plays no role in purely hadronic reactions. On the contrary, one of the earliest and most elegant QCD predictions concerns them and has been brilliantly confirmed by experiment. It deals with the so-called high P_T collisions, i.e. those very rare high energy collisions of hadrons which produce particles with transverse momenta much larger than 1GeV. They turn out to be of the 4 jet type. In the E_{CM} range above 50 GeV, they are successfully explained by perturbative QCD. In this example one valence quark q_1 of proton p_1 makes a high momentum transfer, large angle collision(hard process) with one valence quark q_2 of proton p_2,. In addition to the jets J1, J2 generated by the remaining diquarks, the QCD fields created sideways by q_1 and q_2 give rise to two further jets J1,J2 which fly off with large P_T. The distribution of J1 and J2 in angle and energy can be calculated perturbatively from the Feynman diagrams by making use of the structure functions of the incident hadrons as measured in DILS. The high P_T collisions observed at the CERN proton-antiproton collider ($E_{CM} = 630 GeV$) give impressive agreement with theory. The calculations show that at these high E_{CM} the hard process occurs not only between valence quarks by more frequently between sea partons, dominantly gluons. A high energy hadron collider is therefore and excellent producer of jets initiated by gluons.

Finally it should be noted that the separation between hard and soft collisions of hadrons is not sharp. There is a very important intermediate regime of semi-hard processes, still poorly known. As one reaches CM energies of several hundred GeV, more and more collisions show minijets i.e. hadronic jets of a few GeV emitted in transverse directions, the study of which will undoubtedly be useful for unraveling semihard QCD physics.

2.9 Ultra-Relativistic Nuclear Physics

We now get back to nuclear physics to discuss those very high energy processes where quarks must play an important role, simply because they are known to be essential for our understanding of the same processes on free nucleons. The problem here will be, is the quark behavior modified by the nuclear matter environment, or is it the same as in the case of free nucleons?

2.10 Deep Inelastic Lepton Scattering on Nuclei. (The EMC effect)

The first modification of the quark behavior of nucleons in nuclei was found in 1982 by the Eropean Muon Collaboration (EMC) using the CERN muon beam at laboratory energies 120 to 280 GeV, and comparing iron with deuterium targets. Contrary to previous expectations, the ratio

$$R(x) = \frac{f_{Fe}(x)}{f_D(x)}$$

of the structure functions f_2 for the two nuclei (normalized to a single nucleon) was found to be substantially different. Similar results were found soon afterwards with electrons at SLAC. This so-called EMC effect attracted immediately much theoretical attention, but it became apparent that different mechanisms are at work in various ranges of the variable x. Little is known so far on the Q^2 dependence, which we left out in the definition of R; the trends reported below are averaged over Q^2.This ambitious programme may go a long way in unraveling how the quark structure of nucleons is distorted inside nuclear matter.

2.11 Drell-Yan Process in Nuclei

The Drell-Yan process on nuclei, one sees immediately that two nuclear effects must be expected. Firstly, the structure function of the nucleon in the nuclear target is modified as in the EMC effect. Secondly, the incident pion must penetrate nuclear matter before its antiquark annihilates a quark of the target to produce the virtual photon which then materializes in the muon pair. Both effects have been detected recently by the NA10 Collaboration in a negative pion beam at CERN (laboratory energy of pions 140 and 286 GeV), in a comparison of deuterium and tungsten targets. The tungsten structure function is found to be modified similarly to the EMC effect. One observes in addition a broadening of the transverse momentum distribution of the muon pair, which is attributed to soft interactions of the incident pion in the target nucleus before the dimuon is produced. As in DILS, although the effects are small, systematic studies are possible by varying the target and the kinematical variables of the dimuon.

2.12 Ultra-Relativistic Nuclear Collisions

In this section we deal with collisions of high energy hadron-nucleus and nucleus-nucleus collisions, i.e. the modifications of the processes considered earlier when one or both of the colliding object is a nucleus. Since the 1960s, a number of hadron-nucleus experiments have been performed. For low P_T collisions, the observed increase in the multiplicity of outgoing hadrons can be readily understood in terms of Glaubers multiple collision theory in nuclei, based on the eikonal approximation. The most interesting feature resulting from this analysis is that there is very little cascading, i.e., that hadrons produced in a collision on one nucleon in the nucleus very rarely create additional particles by colliding on a second nucleon. The only exception is a small amount of cascading for hadrons produced with energies which are low in the rest frame of the nucleus.

The near absence of cascading is easily understood in terms of relativistic time dilation. Soft hadron production takes a time of order $1fm/c$ in the rest frame of the produced particle(corresponding to momentum transfers of order $200MeV/c$). When the produced particle is relativistic with respect to the nucleus rest frame, time dilation implies that it forms mostly outside the nucleus, and cascading is then not possible. It would be of obvious interest to make this argument more quantitative. Very little has been done so far in this direction, partly due to the incompleteness of the experimental information, but also because of the lack of knowledge concerning the elementary collisions themselves. Now that good models for low P_T hadron-hadron collisions become available (Dual Parton Model, Lund Fritiof Model, as mentioned earlier), more refined work should be possible if very accurate and complete data would be provided.

Since the early 1980s, however, the interest has massively shifted towards the study of ultra-relativistic nucleus-nucleus collisions, mainly in the hope of creating very dense and hot blobs of hadronic matter. In such blobs, it is conceivable that the QCD partons($q, \bar{q}$) and gluons could exist for times up to $10-20fm/c$ as a disordered fluid, i.e. a quark-gluon plasma with all their color and spin degrees of freedom liberated, before the expansion and cooling of the blob forces the partons back into confinement as constituents of final-state hadrons. There are also theoretical reasons to believe that the quark vacuum condensates mentioned earlier disappear in the QGP state.

After several years of purely theoretical speculations, contact with reality began late in 1986 with the first heavy ion experiments at Brookhaven national laboratorie's(BNL) Relativistic heavy ion collider (^{16}O and ^{28}Si beams, laboratory energy up to 15 GeV per nucleon) and at CERN's large hadron collider(LHC)(^{16}O and ^{32}S beams, up to 200 GeV per nucleon). First results, often still preliminary, were presented at the Quark Matter 87 Conference. Not surprisingly, they indicate that common collisions, despite multiplicity and transverse energy distributions extending to spectacularly large values, are essentially explainable by combining Glauber theory with good low P_T nucleon-nucleon models(the transverse energy is the sum of $[m_i^2 + p_{Ti}^2]^{1/2}$ for all observed particles). In fact, small discrepancies will probably be taken care of by improvements of the nucleon-nucleon models, which still contain quite some flexibility. Conversely, the model calculations for normal collisions will hopefully provide guidance on how to select special events likely to have produced particularly hot and dense blobs of hadronic matter. This may give a more realistic basis for rediscussing the difficult problem of defining reliable signals for QGP formation, a question which up to now had to be treated on highly uncertain theoretical grounds.For hard processes, the most interesting result so far concerns the production of muon pairs (dimuons) in the CERN oxygen beam (200 GeV per nucleon) on a uranium target. The NA38 experiment measured the ratio between dimuons from the decay of the J/Ψ. They find that this ratio decreases substantially when the transeverse energy E_T of the underlying event is large (E_T is measured on neutrals only for instrumental reasons). One finds a reduction of the ratio by about a factor 2/3 when comparing $E_T > 50GeV$ to $E_T < 28GeV$. In addition, the J/Ψ suppression is strongest at low transverse momentum P_T of the dimuon.

It is remarkable that this behavior had been predicted under the assumption that the J/Ψ is produced in a blob of QGP. The main argument underlying the prediction is that the color force between the c and the $\bar{c}$

quarks is screened in the plasma (the partons in the QGP carry color charges and therefore produce the QCD analogue of Debye screening in an ordinary electric plasma). This weakens and may suppress the $c\bar{c}$ binding needed for J/Ψ formation, especially in high E_T collisions which are more likely to contain a short-lived blob of QGP, and preferably at low P_T since the $c\bar{c}$ then stays longer in the blob.

It is nevertheless too early to conclude that the quark-gluon plasma has been seen, becauseJ/Ψ suppression may be due to other causes. What is already clear, however, is that this type of experiment reveals remarkable effects of considerable size, and much more detailed work will undoubtedly be done in the years to come.

2.13 A New State of Matter (QGP)

The year 1994 marked the beginning of the CERN lead beam programme. A beam of 33 TeV (or 160 GeV per nucleon) lead ions from the SPS now extends the CERN relativistic heavy ion programme, started in the mid eighties, to the heaviest naturally occurring nuclei. A run with lead beam of 40 GeV per nucleon in fall of 1999 complemented the program towards lower energies. Seven large experiments participate in the lead beam program, measuring many different aspects of lead-lead and lead-gold collision events: NA44, NA45/CERES, NA49, NA50, NA52/NEWMASS, WA97/NA57, and WA98. Some of these experiments use multipurpose detectors to measure simultaneously and correlate several of the more abundant observables. Others are dedicated experiments to detect rare signatures with high statistics. This coordinated effort using several complementing experiments has proven very successful. The present document summarizes the most important results from this program at the dawn of the RHIC era: soon the relativistic heavy ion collider at BNL will allow to study gold-gold collisions at 10 times higher collision energies.

Physicists have long thought that a new state of matter could be reached if the short range repulsive forces between nucleons could be overcome and if squeezed nucleons would merge into one another. Present theoretical ideas provide a more precise picture for this new state of matter: it should be a quark-gluon plasma (QGP), in which quarks and gluons, the fundamental constituents of matter, are no longer confined within the dimensions of the nucleon, but free to move around over a volume in which a high enough temperature and/or density prevails. This plasma also exhibits the so-called "chiral symmetry" which in normal nuclear matter is spontaneously broken, resulting in effective quark masses which are much larger than the actual masses. For the transition temperature to this new state, lattice QCD calculations give values between 140 and 180 MeV, corresponding to an energy density in the neighborhood of $1GeV/fm^3$, or seven times that of nuclear matter. Temperatures and energy densities above these values existed in the early universe during the first few microseconds after the Big Bang.

It has been expected that in high energy collisions between heavy nuclei suffciently high energy densities could be reached such that this new state of matter would be formed. Quarks and gluons would then freely roam within the volume of the fireball created by the collision. The individual quark and gluon energies would be typical of a system at very high temperature (above 200 MeV) even if the system should not have enough time to fully thermalize. Positive identification of the quark-gluon plasma state in relativistic heavy ion collisions is, however, extremely difficult. If created, the QGP state would have only a very transient existence. Due to color confinement, a well-known property of strong interactions at low energies, single quarks and gluons cannot escape from the collision , they must always combine to color-neutral hadrons before being able to travel to the detector. This process is called "hadronization". Thus, regardless of whether or not QGP is formed in the initial stage, the collision fireball later turns into a system of hadrons. In a head-on lead-lead collision at the SPS about 2500 particles are created (NA49) of which more than 99.9% are hadrons. Evidence for or against formation of an initial state of deconfined quarks and gluons at the SPS thus must be extracted from a careful and quantitative analysis of the observed final state.

A common assessment of the collected data leads us to conclude that we now have compelling evidence that a new state of matter has indeed been created, at energy densities which had never been reached over appreciable volumes in laboratory experiments before and which exceed by more than a factor 20 that of normal nuclear matter. The new state of matter found in heavy ion collisions at the SPS features many of the characteristics of the theoretically predicted quark-gluon plasma.

The evidence for this new state of matter is based on a multitude of different observations. Many hadronic observables show a strong nonlinear dependence on the number of nucleons which participate in the collision. Models based on hadronic interaction mechanisms have consistently failed to simultaneously explain the wealth of accumulated data. On the other hand, the data exhibit many of the predicted signatures for a quark-gluon plasma. Even if a full characterization of the initial collision stage is presently not yet possible, the data provide strong evidence that it consists of deconfined quarks and gluons.

We emphasize that the evidence collected so far is "indirect" since it stems from the measurement of particles which have undergone significant reinteractions between the early collision stages and their final observation. Still, they retain enough memory of the initial quark-gluon state to provide evidence for its formation, like the grin of the Cheshire Cat in Alice in Wonderland which remains even after the cat has disappeared. It is expected that the present "proof by circumstantial evidence" for the existence of a quark-gluon plasma in high energy heavy ion collisions will be further substantiated by more direct measurements (e.g. electromagnetic signals which are emitted directly from the quarks in the QGP) which will become possible at the much higher collision energies and fireball temperatures provided by RHIC at Brookhaven and later the LHC at CERN.

In the following the most important experimental findings and their interpretation are described in more detail:

Hadrons are strongly interacting particles. In nuclear collisions, after being first created, they undergo many secondary interactions before escaping from the collision "fireball". When they are finally set free, the fireball volume has expanded by about a factor 30-50; this information can be extracted from two-particle correlations between identical hadrons by a method called "Bose-Einstein interferometry" (NA44, NA49, WA98). At this point, the relative abundances and momentum distributions of the hadrons still contain important memories of the dense early collision stage which can be extracted by a comprehensive analysis of the hadronic final state. More than 20 different hadron species, including a few small anti-nuclei (anti-deuteron, anti-helium), have been measured by the seven experiments (NA44, NA45, NA49, NA50, NA52, WA97, WA98). A combined analysis of their momentum distributions and two-particle correlations shows that, at the point where they stop interacting and "freeze out", the fireball is in a state of tremendous explosion, with expansion velocities exceeding half the speed of light, and very close to local thermal equilibrium at a temperature of about $100 - 120$ MeV. This characteristic feature gave rise to the name "Little Bang". The observed explosion calls for strong pressure in the earlier collision stages. Recently measured anisotropies in the angular distribution of the momenta perpendicular to the beam direction (NA49, NA45, WA98) indicate that the pressure was built up quickly, pointing to intense rescattering in the early collision stages.

An earlier glimpse of the expanding system is provided by a measurement of correlated electron-positron pairs, also called dileptons (NA45). These data show that in sulphur-gold and lead-gold collisions the expected peak from the ρ vector meson (a particle which can decay into dileptons even before freeze-out) is completely smeared out. Simultaneously, NA45 finds in lead-gold collisions an excess of dileptons in the mass region between 250 and 700 MeV, by about a factor 3 above expectations from hadron decays scaled from proton-nucleon to lead-gold collisions. Theory explains this by a broadening of the ρ's spectral function, resulting from scattering among pions and nucleons in a very dense hadronic fireball, just below the critical energy density for quark-gluon plasma formation. The ρ meson mixes with its partner under chiral symmetry transformations, signalling the onset of chiral symmetry restoration as matter becomes denser and denser.

The theoretical analysis of the measured hadron abundances (NA44, NA45, NA49, NA50, NA52, WA97, WA98) shows that they re ect a state of "chemical equilibrium" at a temperature of about 170 MeV. This points to an even earlier stage of the collision. In fact, such temperatures (corresponding to an energy density of about $1GeV/fm^3$) are the highest allowed ones before, according to lattice QCD, hadrons should dissolve into quarks and gluons. The observations are explained by assuming that at this temperature the hadrons were formed by a statistical hadronization process from a pre-existing quark-gluon system. Theoretical studies showed that at CERN energies subsequent interactions among the hadrons, while causing pressure and driving the expansion and cooling of the fireball, are very ineffective in changing the abundance ratios. This is why, after accounting for the decay of unstable resonances, the finally measured hadron yields rejects rather accurately the conditions at the quark-hadron transition.

A particularly striking aspect of this apparent "chemical equilibrium" at the quark-hadron transition temperature is the observed enhancement, relative to proton-induced collisions, of hadrons containing strange quarks. Globally, when normalized to the number of participating nucleons, this enhancement corresponds to a factor 2 (NA49), but hadrons containing more than one strange quark are enhanced much more strongly (WA97, NA49, NA50), up to a factor 15 for the Ω hyperon and its antiparticle (WA97)! Lead-lead collisions are thus qualitatively diferent from a superposition of independent nucleon-nucleon collisions. That the relative enhancement is found to increase with the strange quark content of the produced hadrons contradicts predictions from hadronic rescattering models where secondary production of multi-strange (anti)baryons is hindered by high mass thresholds and low cross sections. Since the hadron abundances appear to be frozen in at the point of hadron formation, this enhancement signals a new and faster strangeness-producing process before or during hadronization, involving intense rescattering among quarks and gluons. This effect was predicted about 20 years ago as a quark-gluon plasma signature, resulting from a combination of large gluon densities and a small strange quark mass in this color deconfined, chirally symmetric state. Experimentally it is found not only in lead-lead collisions, but even in central sulphur-nucleus collisions, with target nuclei ranging from sulphur to lead (NA35, WA85, WA94). This is consistent with estimates of initial energy densities above the critical value of $1 GeV/fm^3$ even in those collisions.

Evidence for the formation of a transient quark-gluon phase without color confinement is further provided by the observed suppression of the charmonium states J/ψ, c $\bar{c}$, and ψ' (NA50). These particles contain charmed quarks and antiquarks (c and $\bar{c}$) which are so heavy that they can only be produced at the very beginning when the constituents of the colliding nuclei still have their full energy. As one varies the size of the colliding nuclei and the centrality of the collision one finds, after subtracting the expected absorption effects from final state interactions between the c) pair and the nucleons of the interpenetrating nuclei, a succession of suppression patterns: The most weakly bound state, ψ', is suppressed already in sulphur-uranium collisions (NA38), the intermediate $c\bar{c}$ seems to disappear quite suddenly in semicentral lead-lead collisions, and in the most central lead-lead collisions an additional reduction of the J/ψ yield indicates that now also the strongly bound J/ψ ground state itself is significantly suppressed (NA50). The observation of c anti c suppression is indirect, via its 30-40% contribution to the measured J/Ψ yield which is expected from scaling proton-proton measurements. Charmonium suppression was predicted 15 years ago as a consequence of color screening in a quark-gluon plasma which should keep the charmed quark-antiquark pairs from binding to each other. According to this prediction, suppressing the J/ψ requires temperatures which are about 30% above the color deconfinement temperature, or energy densities of about $3GeV/fm^3$. This agrees with estimates of the initial energy densities reached in central lead-lead collisions, based on calorimetry or on a back-extrapolation from the freeze-out stage to the timei before expansion started. It was tried to reproduce the data by assuming that the charmonia are destroyed solely by final state interactions with surrounding hadrons; none of these attempts can account for the shape of the centrality dependence of the observed suppression. On the other hand, the interpretation of this pattern in terms of color screening by deconfined quarks and gluons leads to the prediction of a similar suppression pattern at RHIC in much smaller nuclei; this prediction will soon be tested.

In spite of its many facets the resulting picture is simple: the two colliding nuclei deposit energy into the reaction zone which materializes in the form of quarks and gluons which strongly interact with each other. This early, very dense state (energy density about $3 - 4GeV/fm^3$, mean particle momenta corresponding to $T^a = 240MeV$) suppresses the formation of charmonia, enhances strangeness and begins to drive the expansion of the fireball. Subsequently, the "plasma" cools down and becomes more dilute. At an energy density of $1GeV/fm^3$ ($T^a = 170MeV$) the quarks and gluons hadronize and the final hadron abundances are fixed. At an energy density of order $50MeV/fm^3$ ($T = 100\{120MeV\}$) the hadrons stop interacting and the fireball freezes out. At this point it expands with more than half the light velocity.

This does not happen only in a few "special" collision events, but essentially in every lead-lead collision: characteristic observables, like the average transverse momentum of produced particles or the kaon/pion ratio, show only the statistically expected uctuations in a thermalized ensemble, around average values which are the same in all collisions (NA49). Since the kaon/pion ratio is essentially fixed at the point of hadronization, this indicates the absence of long-range correlations like those expected in a fully-developed thermodynamic phase transition. A better theoretical understanding of the phase-transition dynamics might emerge from these observations. The short-range character suggests similarities with the transition found in high-T_c superconductivity.

"Direct" observation of the quark-gluon plasma may be possible via electromagnetic radiation emitted by the quarks during the hot initial stage. Searches for this radiation were performed at the SPS (WA98, NA45, NA50) but are difficult due to high backgrounds from other sources. For sulphur-gold collisions WA80 and NA45 established that not more than 5% of the observed photons are emitted directly. For lead-lead collisions WA98 have reported indications for a significant direct photon contribution. Preliminary data from NA45 are consistent with this finding, but so far not statistically significant. NA50 has seen an excess by about a factor 2 in the dimuon spectrum in the mass region between the f and J/ψ vector mesons. The predicted electromagnetic radiation rates at the above mentioned temperatures are marginal for detection. While under these conditions it is a great experimental achievement to have obtained positive evidence for a signal, its connection with the predicted "thermal plasma radiation" is not yet firmly established.

This is expected to change at the higher collision energies provided by RHIC and LHC. The much higher initial temperatures (up to nearly 1000 MeV for lead-lead collisions at the LHC have been predicted) and longer plasma lifetimes should facilitate the direct observation of the plasma radiation and lead to the production of additional heavy charm quarks by gluon-gluon scattering in the QGP phase. The much higher initial energy densities which can be reached at RHIC and LHC give us more time until the quarks and gluons rehadronize, thus allowing for a quantitative characterization of the quark-gluon plasma and detailed studies of its early tharmalization processes and dynamical evolution. Finally, the higher collision energies allow for the production of jets with large transverse momenta, whose leading quarks can be used as "hard penetrating probes" within the quark-gluon plasma. At RHIC a set of four large detectors, with complementary goals and capabilities, ensures that all experimental aspects of ultrarelativistic heavy ion collisions are optimally covered. The ability of the collider to simultaneously accelerate and collide nuclei of different sizes and energies promises a complete understanding of systematic trends as one proceeds from proton-proton via proton-nucleus to gold-gold collisions. As in solid state physics, where the knowledge of the basic interaction Lagrangian (QED) does not permit to reliably predict many bulk properties and where the detailed understanding of the latter is usually driven by experiment, we expect that such a systematic experimental study of strongly interacting matter will eventually lead to a quantitative understanding of "bulk QCD". We are looking forward to these far-reaching opportunities provided by RHIC and LHC.

NA44 Collaboration:

1. H. Beker et al., "MT -dependence of boson interferometry in heavy ion collisions at the CERN SPS", Physical Review Letters 74 (1995) 3340-3343

2. I.G. Bearden et al., "Collective expansion in high-energy heavy ion collisions", Physical Review Letters 78 (1997) 2080-2083

3. I.G. Bearden et al., "Strange meson enhancement in Pb-Pb collisions", Physics Letters B 471 (1999) 6-12

NA45/CERES Collaboration:

1. G. Agakichiev et al., "Low-mass e+ e-pair production in 158 A GeV Pb-Au collisions at the CERN SPS, its dependence on multiplicity and transverse momentum", Physics Letters B 422 (1998) 405-412
2. B. Lenkeit et al., "New results on low-mass lepton pair production in Pb-Au collisions at 158 GeV/c per nucleon", Nuclear Physics A 654 (1999) 627c-630c
3. B. Lenkeit et al., "Recent results from Pb-Au collisions at 158 GeV/c per nucleon obtained with the CERES spectrometer", Nuclear Physics A 661 (1999) 23c-32c

NA49 Collaboration:

1. T. Alber et al., "Transverse energy production in 208Pb+Pb collisions at 158 GeV per nucleon", Physical Review Letters 75 (1995) 3814-3817
2. H. Appelshuser et al., "Hadronic expansion dynamics in central Pb+Pb collisions at 158 GeV per nucleon", European Physical Journal C 2 (1998) 661-670
3. F. Sikler et al., "Hadron production in nuclear collisions from the NA49 experiment at 158 GeV/c A", Nuclear Physics A 661 (1999) 45c-54c

NA50 Collaboration:

1. M.C. Abreu et al., "Anomalous J/y suppression in Pb-Pb interactions at 158 GeV/c per nucleon", Physics Letters B 410 (1997) 337-343

2. M.C. Abreu et al., "Observation of a threshold effect in the anomalous J/y suppression", Physics Letters B 450 (1999) 456-466

3. M.C. Abreu et al., "Evidence for deconfinement of quarks and gluons from the J/y suppression pattern measured in Pb-Pb collisions at the CERN SPS", CERN-EP-2000-013, submitted to Physics Letters B

NA52/NEWMASS Collaboration:

1. R. Klingenberg et al., "Strangelet search and antinuclei production studies in Pb+Pb collisions", Nuclear Physics A 610 (1996) 306c-316c

2. G. Ambrosini et al., "Baryon and antibaryon production in Pb-Pbcollisions at 158 A GeV/c", Physics Letters B 417 (1998) 202-210

3. G. Ambrosini et al., "Impact parameter dependence of K, and production in fixed target Pb + Pb collisions at 158 GeV per nucleon", New Journal of Physics 1 (1999) 22.1-22.23

WA97/NA57 Collaborations:

1. E. Andersen et al., "Strangeness enhancement at mid-rapidity in Pb-Pb collisions at 158 A GeV/c", Physics Letters B 449 (1999) 401-406

2. F. Antinori et al., "Production of strange and multistrange hadrons in nucleus-nucleus collisions at the SPS", Nuclear Physics A 661 (1999) 130c-139c

3. F. Antinori et al., "Transverse mass spectra of strange and multistrange particles in Pb-Pb collisions at 158 A GeV/c", CERN-EP-2000-001, submitted to European Physical Journal C

WA98 Collaboration:

1. R. Albrecht et al., "Limits on the production of direct photons in 200 A GeV32 S+Au collisions", Physical Review Letters 76 (1996) 3506-3509

2. M.M. Aggarwal et al., "Centrality dependence of neutral pion production in 158 A GeV 208Pb+ 208Pb collisions, Physical Review Letters 81 (1998) 4087-4091; 84 (2000) 578-579(E)

3. M.M. Aggarwal et al., "Freeze-out parameters in central 158 A GeV 208Pb+ 208Pb collisions", Physical Review Letters 83 (1999) 926-930

2.14 Pair Production in QED

Pair production refers to the creation of an elementary particle and its antiparticle, usually from a photon (or another neutral boson). This is allowed, provided there is enough energy available to create the pair at least the total rest mass energy of the two particles and that the situation allows both energy and momentum to be conserved (though not necessarily on shell). All other conserved quantum numbers (angular momentum, electric charge) of the produced particles must sum to zero, thus the created particles shall have opposite values of each (for instance, if one particle has strangeness +1 then another one must have strangeness −1). In nuclear physics, this occurs when a high-energy photon interacts with an atomic nucleus, allowing it to produce an electron and a positron without violating conservation of momentum. Since the momentum of the initial photon must be absorbed by something, pair production cannot occur in empty space out of a single photon; the nucleus is needed to conserve both momentum and energy. Pair production is the chief method by which energy from gamma rays is observed in condensed matter. The photon needs only to have a total energy of twice the rest mass(m_e) of an electron (1.022 MeV) for this to occur as described above; if it is much more energetic, heavier particles may also be produced. These interactions were first observed in Patrick Blackett's counter-controlled bubble chamber, leading to the 1948 Nobel Prize in Physics. In semiclassical general relativity, pair production is also invoked to explain the Hawking radiation effect. According to quantum mechanics, at short scales short-lived particle-pairs are constantly appearing and disappearing ; in a region of strong gravitational tidal forces, the two particles in a pair may sometimes be wrenched apart before they have a chance to mutually annihilate.

When this happens in the region around a black hole, one particle may escape, with its antiparticle being captured by the hole. Pair production is also the hypothesized mechanism behind the Pair instability supernova type of stellar explosions, where pair production suddenly lowers pressure inside a supergiant star, leading to a partial implosion, and then explosive thermonuclear burning. Supernova SN 2006gy is hypothesized to have been a pair production type supernova.

As I already said the phenomenon of producing electron-positron pair from vacuum in the presence of an external source is called pair production. The external source may be a static or varying electric field.

Pair creation does not occur in a pure magnetic field. This is clear since a constant field cannot transfer energy to a charged particle. It is the acceleration due to the electric field that enables particles to leak through the potential barrier. Hence the physically relevant case can be taken to be the one of a pure electric field. In 1954 J. Schwinger computed the pair production probability in a pure static field. It turns out that the estimates based on his calculation are totally adequate. In 1970s some experimentalists working on intense optical lasers have raised the question of testing nonlinear effects of vacuum quantum electrodynamics. In connection with this E. Brezin and C. Itzykson theoretically investigated the possibility of observing the creation of pairs of charged particles (electrons and positrons) in oscillating electric fields. One might think of the collective effects of millions ($2mc^2$) of photons concentrated in a small volume and materializing their energy. In their calculation they showed that why Schwingers estimation lead to the inobservability of the phenomenon of pair production. They proved that to observe the phenomenon the only possibility would be to increase the available maximal field by four orders of magnitude. While these rules out the observation of the absorptive aspects of non-linearities, it might be interesting in the future to look for dispersive effects.

Inorder to reach the aforementioned conclusion, it is necessary to estimate how much the production rate depends on the frequency of the field. This appears as a challenging exercise since it will allow us to describe the transition between two extreme domains. All the problems relating to pair production have been completely solved theoretically and experimentally verified by the scientific community by using high intensity X-ray lasers. Nowadays we face another challenging problem i.e. pair production in QCD, Physics community is behind this now.

2.15 Pair Production in QCD

The process by which quark - anti quark pairs are created in hadron-hadron or nucleus-nucleus collision is called pair production in QCD. Actually it is a topic of great interest in high energy physics and nuclear physics. More over it has been a subject of extensive research since the early days of strong interactions. In ultra relativistic heavy ion collisions, it is believed that a large amount of energy is deposited in a small space time region. It is this energy stored in the form of color electric field energy which is thought to be responsible for the production of quark- anti quark ($q\bar{q}$) pairs. The corresponding production rate can be calculated using different mechanisms such as Schwingers proper time method, Balian-Bloch multiple reflection expansion of Greens function method, generalization of the complex multiple reflection technique of knoll and Schaffer method etc.Why this study is important,since creation of large number of quark- antiquark pairs together with the gluons ,produce through interactions ,eventually leads to the formation of QGP.Their theoretical and experimental confirmation is very crucial to the validity of QCD. In our thesis we adopt a realistic non-abelian model for chromoelectric field which is the exact solution of field tensor equation governing QCD and for calculating the pair production amplitude we use the complex multiple reflection technique given by Biswas and Guha. The details regarding pair production is written on chapter 3.

2.16 Solution of Yang-Mills Field Equation

The Yang-Mills field equation corresponding to the groups $SU(2)$ without external sources is given by

$$\partial_\mu G_a^{\mu\nu} + g\varepsilon_{abc} A_{\mu b} G_c^{\mu\nu} = 0$$

Where $G_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g\varepsilon_{abc} A_b^\mu A_c^\nu$ is the Field tensor with μ,ν runs over 0,1,2,3 and a runs over 1,2 and 3. For a given a there are 16 $G^{\mu\nu}$, hence the total number of $G^{\mu\nu}$ are 48.Using the temporal gauge $A^0 = 0$ and assume space homogeneity (space derivatives vanish),we obtain only 3 non-vanishing field equations. When $a = 1$, $\mu = 1$ we get

$$\partial_0^2 A_1^1 + g^2 A_1^1[(A_2^2)^2 + (A_3^3)^2] = 0$$

When $a = 2$, $\nu = 2$ we get

$$\partial_0^2 A_2^2 + g^2 A_2^2[(A_1^1)^2 + (A_3^3)^2] = 0$$

and When $a = 3$,$\nu = 3$ we get

$$\partial_0^2 A_3^3 + g^2 A_3^3[(A_1^1)^2 + (A_2^2)^2] = 0$$

Finally we assume that all color directions are same i.e. $A_1^1 = A_2^2 = A_3^3 = A$ Now our equations become an elegant single non linear second order differential equation.

$$\frac{\partial^2 A}{\partial t^2} + 2g^2 A^2 = 0$$

The solution of this equation turns out to be a Jacobian elliptical function i.e.

$$A = \left(\frac{2g^2}{3}\right)^{\frac{1}{4}} \mu \quad cn[\left(\frac{8g^2}{3}\right)^{\frac{1}{4}} \mu(t), \frac{1}{\sqrt{2}}]$$

where $cn(x, k)$ is the Jacobian elliptic cosine function of argument x and modulus k and μ^4 is the energy density. In the Jacobian elliptic function the parameter k whose value lies between 0 and 1. When $k = 0$ it will be a cosine function which is oscillatory. This oscillatory model solution has already been used to calculate the pair creation probability in non-abelian guage field by several authors. When $k = 1$, Jacobian elliptical function becomes hyperbolic secant function which is used as the model solution for calculating pair creation probability in chapter 4.

2.17 Color Coupled Klein-Gordon Equation

Here we develop Klein-Gordon equation in QCD in the external color potential. For this we choose our signature as -2. Energy momentum four vector $p^{\mu} = (\frac{E}{c}, p)$ and $p_{\mu} = (\frac{E}{c}, -p)$

$$p^{\mu}p_{\mu} = \frac{E^2}{c^2} - p^2 = m^2c^2$$

Using this, Klein- Gordon equation becomes (taking $c = 1$)

$$(E^2 - p^2 - m^2)\phi = 0$$

or

$$(p_0^2 - p^2 - m^2)\phi = 0 \tag{1}$$

Where

$$E^2 = -\bar{h}^2\frac{\partial^2}{\partial t^2} = -\partial_0^2 = p_0^2$$

and

$$p^2 = \frac{\partial^2}{\partial x^2} = -\nabla^2$$

(taking $\bar{h} = 1$) In the presence of external potential, we get

$$\left((p_0 - eA_0)^2 - (p - eA)^2 - m^2\right)\phi = 0$$

$$\left(\partial_0^2 - \nabla^2 - e(p.A + A.p) + e^2A^2 + m^2\right)\phi = 0$$

$$\left(\partial_0^2 - \nabla^2 + 2ieA_{\alpha}\partial_3 + e^2A_{\alpha}^2 + m^2\right)\phi = 0$$

In Yang-Mills theory, this equation becomes

$$(\partial_0^2 - \nabla^2 + 2ig\tau_\alpha A_\alpha + g^2 A_\alpha^2 + m^2)\begin{pmatrix}\phi_+ \\ \phi_-\end{pmatrix} = 0 \qquad (2)$$

Where g is the coupling constant and $\tau_\alpha(\alpha = 1,2,3)$ are the Pauli spin matrices with $\tau_1 = \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}$, $\tau_2 = \begin{pmatrix}0 & -i \\ i & 0\end{pmatrix}$ and $\tau_3 = \begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix}$. Pauli spin matrices are introduced to respect $SU(2)$ group. Equation 2 is coupled in ϕ_+ , ϕ_-. To decouple this, first of all we have to substitute for A_α, the color potential. This is obtained by solving the field equation (see previous section) which governs Quantum Chromo Dynamics. For example

$$A_\alpha = \sqrt{E_\alpha} tanh \sqrt{E_\alpha} t$$

Equation 2 becomes

$$\left(\partial_0^2 - \nabla^2 + 2igtanh^2\sqrt{E_0}t\begin{pmatrix}\sqrt{E_3} & \sqrt{E_1} - i\sqrt{E_2} \\ \sqrt{E_1} + i\sqrt{E_2} & -\sqrt{E_3}\end{pmatrix}\partial_3 + g^2 E_\alpha tanh^2\sqrt{E_\alpha}t + m^2\right)\begin{pmatrix}\phi_+ \\ \phi_-\end{pmatrix} = 0 \qquad (3)$$

Diagonalising the matrix by a unitary transformation after finding their eigen values and eigen functions, we get decoupled equations

$$\left(\partial_0^2 + p_\perp^2 + m^2 + (p_3 + g\sqrt{E_0}tanh\sqrt{E_0}t)^2\right)\psi_+ = 0$$

and

$$\left(\partial_0^2 + p_\perp^2 + m^2 + (p_3 - g\sqrt{E_0}tanh\sqrt{E_0}t)^2\right)\psi_- = 0$$

Put

$$p_{\perp}^2 + m^2 + (p_3 + g\sqrt{E_0} tanh\sqrt{E_0} t)^2 = \omega^2(t)$$

, we get

$$\left(\partial_0^2 + \omega^2(t)\right)\psi_+ = 0$$

This is the Klein-Gordon for one dimension,we solve this by using W.K.B method.

2.18 References

[1]Abramowitz M and Segun I A Hand Book of Mathematical Functions (1968)

[2]Alfred Actor, Rev. Mod Phys. 51 , 3, July 1979

[3]Alfred George Greenhill, The Applications of Elliptic functions Dover publications, Inc. Newyork

[4]Arthur Cayley,An elementary treatise on elliptic functions- Dover publications.Inc.Newyork

[5]Avijith.K.Ganguly, Predhiman K. Kaw and Jitendra C. Parikh, Phys. Rev. D48, 2983(1983)

[6]B.Banerjee,R.S.Bhalerao and V. Ravishankar,Phys.Lett.B224,16(1989)

[7]B.Petersson,Phys.Rev.Lett, 75,4169(1995)

[8]B.R.Duffy, E.J.Parkes,Phys.Letts A 214 ,271(1996)

[9]B.S. Balakrishna, Phys. Rev. Lett. 60,16(1988)

[10]A.Bailas and W. Czyz, Ibid, 30, 237 (1884), B. Banerjee, R.S. Bhalerao and V . Ravisankar, Phys. Lett. B 224, 16 (1989)

[11]Bannur.V.M,Barve,Celenza,MisraandShakin,Phys.Rev.D34,3530(1986)

[12]Bijan.S.S ,Phys. Rev. C, 65,054904(2002)

[13]Cecile Martin and D. Vautherin,Phys. Rev. D38,11,3593 (1998)

[14]E.J.Parkes, B.R.Duffy, Phys.Letts A 229,217(1997)

[15]E.J.Parkes, B.R.Duffy,P.CAbbott, Phys.Letts A 295,280(2002)

[16]E.J.Parkes, J.Phys.A.Maths.Gen.27, L497(1994)

[17]E.J.Parkes,Z.Zhu,B.R.Duffy,H.C.Huang,Phys.Letts A 248 ,219(1998)

[18]Engui Fan, Jian zhang, Phys.Letts A 305, 383(2002)

[19]F.Bowman,Introduction to elliptic functions,John wileyand sons.Inc.New York

[20]HarrisHancock,EllipticIntegrals- Dover publications.Inc.Newyork

[21]Ren-Xin Xu.Chin,J.Astron.astro phys 0, 200(x)

[22]Shau- Jin Chang,Phys.Rev.D29,2(1984)

[23]ShibajiBanerjee,Sanjay.K.Gosh J.Phys.G.Nucl.Part.Phys26,L1(2000)

[24]V.M.Bannur,Pramana J.Phys.51,3,4,405 (1998)

[25]V.M.Bannur,Pramana J.Phys.59,4,671(2002)

[26]W.Malfliet ,Am.J.Phys.60,7 July (1992)

[27]Wolf gang ketterle and N.J.Van Druten,Phys.Rev.A,54,1(1996)

[28]Y.Kluger,J.M.Eisenbergand B.Svetitsky,Phys.Rev.Lett. 67,18,2427(1991)

[29]Y.Kluger,J.M.Eisenberg and B.Svetitsky,Phys.Rev.D45,12(1992)

[30]Yan-Ze Peng, Phys.Letts A 314 ,401(2003)

****

1

PAIR PRODUCTION IN NON ABELIAN GAUGE FIELDS

by

SETHUMADHAVAN PUZHAKKAL

Thesis submitted to the
University of Calicut
in partial fulfillment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY
under Faculty of Science

Department of Physics
University of Calicut
AUGUST 2007

Chapter 3

Pair production in Nonabelian Gauge Fields-Model 1

3.1 Introduction

The quark-antiquark creation probability in hadron-hadron ornucleus-nucleus collisions has been a topic of great interest in high-energy physics. Moreover, it has been the subject of extensive research since the early days of strong interactions. In ultra relativistic heavy-ion collisions (URHIC), it is believed that a large amount of energy is deposited in a small space-time region. It is this energy stored in the form of colour electric field energy that is thought to be responsible for the production of quark-antiquark ($q\bar{q}$) pairs and eventually leads to the formation of quark-gluon plasma (QGP). The gluon-induced pair creation, in the presence of a strong colour electric field mechanism is again discussed here with a new non-Abelian model solution, which reveals a surprising result. The advantages of this model solution are several. One is that it is an actual non-Abelian model compared to models that reflect only the non-Abelian nature as previously used by several authors [1 − 3]. The second is that calculation is purely analytical, without resorting to any approximation. Thirdly, we obtained a consistent result using real physics that is better compared with previous results by other authors [2 − 5].

3.2 Basic concept of the problem

When two nucleons collide at high energy, they almost just pass through each other. In the process, nucleons are excited, but in addition they leave a flux tube of deposited energy in the region of space they pass through. This energy is rapidly transformed into hadrons, the secondary particles usually produced in such collisions. From analysis of proton-proton collision data, it is known that the deposition of energy at a higher rate is approximately $\frac{1}{3}$ GeV fm^{-3}. It does not increase much more with a further increase in collision energy, which only makes the flux tube longer.

If, instead of nucleons, two heavy nuclei collide (for simplicity, both with mass number A), a multiple superposition of the phenomenon just described is obtained, so that a given region of the flux tube now receives a much greater deposition of energy [6]. Simple geometric arguments reveal an energy density of at least $\epsilon N A^{\frac{1}{3}}$ GeV fm^{-3}. This means that an energetic collision of two ^{238}U nuclei provides an average deposition of approximately 6 GeV fm^{-3}, which is well above the level needed for plasma formation [7].

Many authors have considered $q\bar{q}$ pair production in the above flux tube model [4,5,8-14]. The basic idea is that when two nuclei collide, they pass each other and become colour-charged, and are thus connected by colour flux tubes. In these flux tubes, a strong colour electric field is set up, which makes the vacuum unstable to pair production via the Schwinger mechanism [1]. As a result, the colour field energy in the flux tube is transformed into the energy of $q\bar{q}$ pairs. Thus, in the collision process, a large number of $q\bar{q}$ pairs, together with gluons produced through interactions, ultimately leads to the formation of a QGP. The self-interaction of gluons in the flux tube is likely to polarise the medium between the two receding nuclei. This leads to a characteristic normal mode of oscillations. Therefore, the colour field in the flux tube should be time-dependent (colour particles are coupled to each other via the gauge fields). These collective oscillations are non-Abelian. Hence, we assume a non-Abelian colour field model in which the characteristic frequency of collective oscillation depends on the amplitude of the oscillation.

During a nucleus–nucleus collision, a QGP is formed at the centre of the tube, which is of parallel plates of colour sources. Here we take an approximation whereby in the region of QGP formation, i.e. at the centre point between the receding nuclei, there is no colour source, only colour fields. This approximation is justifiable, because when compared to the size of the QGP formation region, the receding nuclei are farther away at approximately infinity. Hence, the colour source effect can be neglected. The fields left behind at the centre of QGP formation by the receding nuclei are nothing but the solution of the Yang-Mills equation in vacuum. Furthermore, it is suggested that the colour field should be time-dependent owing to the non-Abelian interactions of gluons.

3.3 Model solution

According to our present level of understanding, all fundamental interactions except for gravity are described by non-Abelian gauge theories. The gauge bosons of various non-Abelian symmetries mediate different interactions. All known fundamental fermion fields are divided into two broad classes. One class, called leptons, such as the electron, muon and neutron, do not have any strong interactions. In gauge theory terms, we can state that these are singlets under strong interaction gauge groups. The other class, called quarks, can be described by SU(3) symmetry, and therefore have strong interactions. Under global SU(3) symmetry, the three colours of the quarks transform like a triplet. If we try to gauge this colour symmetry, we obtain a theory called quantum chromodynamics (QCD). This contains eight gauge fields that are called gluons. To understand features due to non-Abelian effects, we have to solve the Yang-Mills equation. For simplicity, we consider the time-dependent vacuum solution of the SU(2) Yang-Mills equation, which satisfies:

$$\partial_\mu G_a^{\mu\nu} + g \in_{abc} A_{\mu b} G_c^{\mu\nu} = 0,$$

where the field tensor is

$$G_a^{\mu\nu} = \partial^\mu A_a^\mu - \partial^\nu A_a^\mu + g \in_{abc} A_b^\mu A_c^\nu,$$

and a, b, c are colour indices that take values 1,2,3 and Lorentz indices $\mu, \nu =$

$0, 1, 2, 3$, with metric $(1, -1, -1, -1)$. $\in_{abc}$ is an anti-symmetric Levicivita tensor.

Taking the temporal gauge $A^0 = 0$, the space homogeneity and the colour potential A_a are assumed to be the same in all colour directions, and the Yang-Mills equation is solved to obtain the colour potential $A(t)$. The general solution of this is a Jacobian elliptical function. Here we assume a model solution for the color potential

$$\vec{A}(t) = -\sqrt{2}\frac{E_\alpha}{\omega_0} \tan h\omega_0 t \qquad \text{where } \omega_0 = \sqrt{2gE}$$

$$\vec{\epsilon} = -\frac{\partial \vec{A}}{\partial t} = \sqrt{2}E_0 \sec h^2 \omega_0 t. \tag{3.1}$$

This form of electric field is quite physical,where the electric field dies off smoothly as function of time .It is due to the nonlinearity of the gauge theory.Further the qualitative feature of the equation(3.1) are obvious and easy to understand. The colour field has an impulse profile. The amplitude and frequency of the plasma oscillation increases with the increase in the strength of the external impulse field .The impulse profile is roughly symmetric about t=0 , where the field assumes its maximum value$\sqrt{2}E_o$. It produces charged particles and accelerates them, producing a positive current and an associated field that continues to oppose the external field until the net field vanishes.At that time the particle production ceases and current reaches a maximum value.However the external field is dying away,since its life time is small the part of the electric field due to the particle's own motion quickly finds itself too strong. The excess of field strength begins to produce particles. It accelerates these in the opposite direction to the particles generating the existing current whilst simultaneously decelerating the particles in that current that continues until the particle current vanishes, at which point the net field has acquired its largest value, particle production continues and with that a negative net current appears and grows.Moreover it shows that the colour field in the flux tube is time-dependent (colour particles are coupled to each other via gauge fields) and the characteristic frequency of oscillation depends on the amplitude of the oscillation, which is one of the hallmarks of non-linear as opposed to linear oscillators.

Here we wish to calculate the quark–anti-quark pair production amplitude from mesons (scalars)in QGP for which we have to solve the Klein-Gordan[K.G] equation which is the governing equation of scalar particles. For this, we follow the method given by Biswas and Guha [9]. The basic principle in their calculation is evaluation of the action integral $s(t_1, t_2)$. This is evaluated by solving the colour-coupled Klein-Gordan equation in the external colour potential.This equation is directly obtained from K.G equation in the presence of external field in quantum electrodynamics by replacing the coupling constant 'e' by the coupling constant 'g' in quantum chromodynamics and introducing the Pauli spin matrices to respect SU(2).The SU(2) colour coupled K.G equation in the external colour potential is evaluated to be (τ_α with $\alpha = 1, 2, 3$ are Pauli spin matrices):

$$[\partial_0^2 - \nabla^2 + 2ig\tau_\alpha A_\alpha \partial_3 + g^2 A_\alpha^2 + m^2] \begin{bmatrix} \phi_+ \\ \phi_- \end{bmatrix} = 0 \qquad (3.2)$$

with

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and $A_\alpha = -\sqrt{2}\frac{E_\alpha}{\omega_0} \tan h\omega_0 t$ with $E^2 = \sum_{\alpha=0}^{3} E_\alpha^2$

Owing to the non-Abelian effect in Eq. (3.2), it may be seen that ϕ_+, and ϕ_- are coupled. Thus, to solve the above equation, it has to be decoupled. It may be shown that the above equation can be decoupled by a unitary transformation in colour space defined by

$$U^+ = \begin{bmatrix} \frac{\sqrt{E^2-E_3^2}}{E-E_3} & \frac{E_1-iE_2}{\sqrt{E^2-E_3^2}} \\ \frac{\sqrt{E^2-E_3^2}}{E+E_3} & -\frac{E_1-iE_2}{\sqrt{E^2-E_3^2}} \end{bmatrix}, \text{ where } E^2 = E_1^2 + E_2^2 + E_3^2.$$

The (column vector) wave function in turn transforms into

$$U^+ \begin{pmatrix} \phi_+ \\ \phi_- \end{pmatrix} = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix} \quad .$$

The decoupled equations are evaluated as:

$$\left[\partial_0^2 - \nabla^2 - 2\sqrt{2}\frac{ig}{\omega_0} E \tanh \omega_0 t \partial_3 + \frac{2g^2E^2}{\omega_0^2} \tanh^2 \omega_0 t + m^2\right] \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix} = 0, \tag{3.3}$$

using $-\nabla^2 = p_1^2 + p_2^2 + p_3^2$ and $\partial_3 = ip_3$.

Eq. (3.4) can be written in the form:

$$\left[\partial_0^2 + w(t)\right] \Psi_+ = 0 \text{ and } \left[\partial_0^2 - w(t)\right]\Psi_- = 0 \tag{3.4}$$

with

$$w(t) = \left[w^2 + \left(p_3 + \frac{\sqrt{2}gE}{w_0}\tanh w_0 t\right)^2\right]^{1/2}$$

, where

$$w^2 = p_1^2 + p_2^2 + m^2 = p_\perp^2 + m^2$$

This is nothing but the Klein-Gordan (KG) equation for one dimension. Now, we try to solve the decoupled equations using the WKB method. The validity of the approximation can easily be checked [2], i.e., $\frac{d}{dt}\left(\frac{1}{w(t)}\right) \ll 1$ or $\left|\frac{\dot{w}(t)}{w^2(t)}\right| \ll 1$.

To solve our KG equation, we follow the method using the WKB approximation in complex time, given by Biswas and Guha [9].

3.4 WKB approximation in complex time

In standard WKB approximation with real time t, a wave function is written as

$$\psi = \frac{c_1}{[w(t)]^{1/2}} e^{\int iw(t)dt} + \frac{c_2}{[w(t)]^{1/2}} e^{-\int iw(t)dt}$$

In complex time t, we define

$$\int_{t_1}^{t} w(t)dt = s(t, t_1) = \int_{t_1}^{t} [w^2 + V(t)]^{1/2} dt,$$

where $V(t) = (p_3 + \sqrt{2}\frac{gE}{w_0} \tanh w_0 t)^2$

The turning points are determined from:

$w(t) = 0$ i.e. $[w^2 + V(t)]^{1/2} = 0$.

The boundary conditions are chosen such that,

$$\psi = e^{is(t,t_1)} \sim e^{\int_{t_1}^{t} w(t)dt} \text{ when } t \rightarrow -\infty$$

$$\psi = e^{is(t,t_1)} + be^{-is(t,t_1)},$$

i.e.

$$\psi = e^{i\int w(t)dt} + be^{-i\int w(t)dt} \text{ when } t \rightarrow \infty.$$

Here b is called the reflection coefficient. Consider the pair production as a consequence of reflection in time; then the reflection coefficient is identified as the pair production amplitude [9].

The reflection coefficient is calculated as

$$b = -\frac{ie^{2is(t_1,t_0)}}{1 + e^{2is(t_1,t_2)}} \tag{3.5}$$

From this, the reflection amplitude $|b|^2$ can be calculated, which is our pair creation amplitude according to the complex time WKB approximation.

3.5 Evaluation of $s(t_1, t_2)$, $s(t_1, t_0)$ and the pair creation amplitude

$$s(t_1, t_2) = \int_{t_1}^{t_2} \left[w^2 + \left(p_3 + \frac{\sqrt{2}gE}{w_0} \tan h w_0 t \right)^2 \right]^{1/2} dt, \qquad (3.6)$$

where $t_1 = \frac{1}{w_0} \tan h^{-1} \frac{w_0}{\sqrt{2gE}} (iw - p_3)$ and $t_2 = \frac{-1}{w_0} \tan h^{-1} \frac{w_0}{\sqrt{2gE}} (iw + p_3)$.

To solve Eq. (3.6), put $p_3 + \frac{\sqrt{2}gE}{w_0} \tanh w_0 t = iw \sin\theta$

$$s(t_1, t_2) = \frac{iw^2}{\sqrt{2}gE} \int_{-\pi/2}^{+\pi/2} \frac{\cos^2\theta d\theta}{\left[1 - \frac{w_0^2}{2g^2E^2}(iw\sin\theta - p_3)^2\right]}.$$

This can easily be integrated to give

$$s(t_1, t_2) = -\frac{\pi i w_0}{2\sqrt{gE}} + \frac{\pi i}{2\sqrt{gE}} \left[\sqrt{w^2 + \left(\frac{w_0}{2} + p_3\right)^2} \right] + \left[\sqrt{w^2 + \left(\frac{w_0}{2} - p_3\right)^2} \right] \qquad (3.7)$$

In the Schwinger limit ($w_0 \to 0$), $S(t_1, t_2)$ becomes

$$s(t_1, t_2) = \pi i \sqrt{\frac{p^2 + m^2}{2gE}}. \qquad (3.8)$$

This is so because if we let $w_0 = 0$ in our model solution, we have the static field, that is, the expected Schwinger result.This is justifiable because when ω_0 goes to zero, the coupling constant 'g' goes to zero. When this happens the self interaction of gluon fields and colour coupling between quarks vanish, thus the non-abelian effect goes off there by SU(2) becomes U(1) which is abelian.

Similarly, $s(t_1, t_0)$ can be evaluated as:

$$s(t_1, t_0) = -\frac{\pi i w_0}{4\sqrt{gE}} + \frac{\pi i}{4\sqrt{gE}} \left[\sqrt{w^2 + \left(\frac{w_0}{2} + p_3\right)^2}\right] + \left[\sqrt{w^2 + \left(\frac{w_0}{2} - p_3\right)^2}\right]. \tag{3.9}$$

In the Schwinger limit $w_0 \to 0$, $s(t_1, t_0)$ becomes:

$$s(t_1, t_0) = \frac{\pi i}{2}\sqrt{\frac{p^2 + m^2}{2gE}}. \tag{3.10}$$

Inserting the values for $s(t_1, t_2)$ and $s(t_1, t_0)$ in Eq. (3.5), we obtain:

$$|b| = \frac{e^{\frac{\pi w_0}{2\sqrt{gE}}} \times e^{\frac{-\pi}{\sqrt{2gE}}\left[\sqrt{w^2+(\frac{w_0}{2}+p_3)^2}\right]+\left[\sqrt{w^2+(\frac{w_0}{2}-p_3)^2}\right]}}{1 + e^{\frac{\pi w_0}{\sqrt{gE}}} \times e^{\frac{-\pi}{\sqrt{2gE}}\left[\sqrt{w^2+(\frac{w_0}{2}+p_3)^2}\right]+\left[\sqrt{w^2+(\frac{w_0}{2}-p_3)^2}\right]}}. \tag{3.11}$$

The pair creation probability is:

$$|b|^2 = \frac{e^{\frac{\pi w_0}{\sqrt{gE}}} \times e^{\frac{-2\pi}{\sqrt{2gE}}\left[\sqrt{w^2+(\frac{w_0}{2}+p_3)^2}\right]+\left[\sqrt{w^2+(\frac{w_0}{2}-p_3)^2}\right]}}{\left|1 + e^{\frac{\pi w_0}{\sqrt{gE}}} \times e^{\frac{-\pi}{\sqrt{2gE}}\left[\sqrt{w^2+(\frac{w_0}{2}+p_3)^2}\right]+\left[\sqrt{w^2+(\frac{w_0}{2}-p_3)^2}\right]}\right|^2}. \tag{3.12}$$

To estimate the pair creation probability, we let $p_3 = 0$, with the integration range of order of magnitude $2\sqrt{2}\frac{gE}{w_0}$,

$$b^2 = \frac{e^{\frac{\pi w_0}{2\sqrt{gE}}} \times e^{\frac{-2\pi}{\sqrt{2gE}}\left[\sqrt{p^2+m^2+\frac{w_0^2}{4}}\right]}}{\left|1 + e^{\frac{\pi w_0}{\sqrt{gE}}} \times e^{\frac{-\pi}{\sqrt{2gE}}\left[\sqrt{p^2+m^2+\frac{w_0^2}{4}}\right]}\right|^2} \tag{3.13}$$

This shows that the pair creation probability depends crucially on the collective oscillation frequency w_0, a result that is expected. Inserting $w_0 = 0$ in Eq. (3.13), we obtain:

$$|b|^2 = \frac{e^{-2\pi\sqrt{\frac{p^2+m^2}{2gE}}}}{\left|1 + e^{-\pi\sqrt{\frac{p^2+m^2}{2gE}}}\right|^2}. \tag{3.14}$$

This shows that when $w_0 \to 0$ (static limit) independently of gE, we recover the well-known Schwinger result in a better form [1].

From Eq. (3.13) it is evident that when $\frac{db^2}{d(gE)} = 0$, we obtain $gE \cong m$ and $\frac{d^2(b)^2}{d^2(gE)}$ is negative. This shows that the pair creation probability is maximum for $gE \cong m$. Thus, when the quark–anti-quark mass is comparable with the field strength (gE), the pair creation probability dominates.

From this, we can infer that for the non-Abelian problem of interest, we first have to evaluate the ranges of values for w_0.

3.6 Discussion on the range of values for gE and ω_0

For the validity of WKB approximation ,we have $\left|\frac{\dot{w}(t)}{w^2(t)}\right| \ll 1$.A simple calculation shows that $\left|\frac{\dot{w}(t)}{w^2(t)}\right| \cong \frac{gE}{\omega^2} = \frac{gE}{p_\perp^2+m^2}$ i.e. $\frac{gE}{p_\perp^2+m^2} \ll 1$. For gE, we take the estimate discussed by Pavel and Brink [5]. Thus, for p–p collision, $gE \approx 0.2$ GeV^2, for ^{32}s on ^{32}s, gE ¡ 0.6 GeV^2 and for U–U collisions, $gE \leq 1.2$ GeV^2. As gE ranges from 0.2 to 1.5 GeV^{2}and m for quarks varies from 0.3 to 177 GeV, the condition for the validity of WKB approximation is satisfied .Now we estimate the pair creation probability for our model in the range 0.63–1.732 GeV. It is also evident that the pair creation probability depends crucially on the magnitude of w_0; when $w_0 \to 0$, we recovered the Schwinger limit. From this we conclude that the well-known Schwinger estimate underestimates the pair creation probability by several orders of magnitude, as pointed out by other authors [2,3]. Thus, here we calculate the pair creation probabilities w and w_s for two cases, one (w) for the dynamic case (w_0 depends on gE), and the other (w_s) for the static case $w_0 \to 0$, independently of gE. w and w_s are calculated for a range of mass values: $m_u = m_d = 0.31$ GeV, $m_s = 0.505$ GeV, $m_c = 1.5$ GeV, $m_b = 5$ GeV, $m_t = 177$ GeV, i.e. m varies from 0.3 to 177 GeV. For the calculations we use constituent quark masses rather than current quark masses. On physical grounds we believe that the constituent masses ought to be used , since the colour flux-tube model incorporates the effect of confinement of the colour electric field and of the physical vacuum around it. To materialise pairs from a vacuum, the particles ought to move a distance equal to the Compton wavelength $\frac{\hbar}{mc}$ under the influence of gE. For the light (u, d quarks) the use of current masses ($m = 10$ MeV) would give $\frac{\hbar}{mc} \sim 20$ fm and such length scales appear unreasonable for both p–p and A–A collisions. We therefore argue that pair creation via the flux tube model would be physically meaningful if $\frac{\hbar}{mc} \leq 1$ fm or $m > 200$ MeV.

3.7 Conclusion

From the pair creation probability values in Table 1, we can infer the following;

1. The pair creation probability calculated by the Schwinger mechanism (static limit) shows that the probability of production $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $c\bar{c}$ and $b\bar{b}$ increases with increasing field strength. This cannot be justified because the oscillating field, the potential barrier through which the pair tunnels, moves up and down with time. When there is no barrier, the particle may tunnel through with a tunnelling time $\tau_1 = -\frac{m}{gE}$, where m is the mass of the particle and gE is the strength of the field. If τ_1 is much less than the period of oscillation of the field $\tau_1 \sim \frac{1}{w_0}$, then tunnelling would take place during the half-period when there is no barrier and we essentially have a static situation (Schwinger limit). On the other hand, if $\tau_1 \gg \tau_f$, the pair cannot tunnel before the time interval of the oscillating electric field. In this limit, the pair creation rate depends on the frequency w_0. This is exactly what we obtained from our results.

2. Comparison of W and W_s shows that estimation of the pair creation probability by the Schwinger mechanism is an underestimate by several

orders of magnitude, which confirms the observations made by other authors [2,3].

3. Table 1 shows that according to our model the pair creation probability neither increases nor decreases with field strength (gE), but dominates when the quark-anti-quark mass is comparable with the strength of the field. In case of $u\bar{u}$ quark-anti quark production ($m = 0.31$ GeV) the pair creation probability is maximum when $gE \approx 0.3$ GeV^2. This shows that p-p collision is relevant for the creation of $u\bar{u}$. In the case of $s\bar{s}$ quark–anti-quark production ($m = 0.5$ GeV), the pair creation probability is maximum when $gE \approx 0.5$ GeV^2. This shows that collision such as between ^{32}s and ^{32}s is relevant for $s\bar{s}$ creation. In the case of charm quark-antiquark production, U-U is relevant.

4. To summarise, we described an important non-Abelian effect on the pair creation probability in collisions in high-energy physics. Once again we established that the colour field should be time-dependent due to non-Abelian interactions and should oscillate with a collective frequency w_0, which should depend on the field strength. It is evident that the collective frequency ranges from 0.63 to 1.732 GeV. Our model has several advantages over other models [2-5]. First, we assumed a non-Abelian colour field model, which is an exact solution of the Yang-Mills equation. Second, our calculation is completely analytical and no assumption is made to simplify it. Third, there is no need for any of the cumbersome calculations that other authors adopted [1-4]. Evaluation of the action integral $s(t_1, t_2)$, $s(t_1, t_0)$ is simple, albeit lengthy. Our result does not violate any of the former results in this field; moreover, it recovered the well-known Schwinger result. Finally, we obtained a very

consistent result. If our model is good and the pair creation probability is truly given by Eq. (3.13), it should be possible to test it in heavy ion collision experiments.

3.8 References

[1]J.Schwinger, Phys. Rev. 82, 664 (1951)

[2]E.Brezin, C. Itzykson, Phys. Rev. D 2, 1191 (1970)

[3]A.K.Ganguly, P.K. Kaw, J.C. Parikh, Phys. Rev. D 48, 2983 (1983)

[4]C.Martin, D. Vautherin, Phys. Rev. D 38, 3593 (1988)

[5]H.P. Pavel, D.M. Brink, Z. Phys. C 51, 119 (1991)

[6]J.D. Bjorken, Phys. Rev. D 27, 140 (1983)

[7]H. Satz, Nature 324, 13 (1986)

[8]A. Actor, Rev. Mod. Phys. 51, 3 (1979)

[9]S. Biswas, J. Guha, Pramana J. Phys.40, 467 (1993)

[10]F.E.Low,Phys.Rev.D12,163(1975)

[11]A. Casher, H. Neuberger, S. Nussinov, Phys. Rev D 20, 179 (1979)

[12]T.S. Biro, H.B. Nielsen, J. Knoll, Nucl. Phys. B 245, 449 (1984)

[13]N.K. Glendenning, T. Matsui, Phys. Rev. D 28, 2890 (1983)

[14]A.Bailas, W. Czyz, B. Banerjee, Phys. Lett. B 224, 16 (1989).

****

Table 3.1: **Pair creation probability** w **AND** w_s **for different values of** m **and field strength**

m (GeV)	gE (GeV^2)	w_s	w
0.3	0.2	0.1500	0.2494
Up quark or down quark	0.3	0.1820	0.2503
	0.5	0.2018	0.2165
	0.8	0.2183	0.2007
	1.0	0.2242	0.1949
	1.3	0.2298	0.1894
	1.5	0.2324	0.1875
0.5	0.2	0.0711	0.1545
Strange quark	0.3	0.1020	0.1864
	0.5	0.1426	0.2499
	0.8	0.1739	0.2389
	1.0	0.1864	0.2298
	1.3	1.9890	0.2190
	1.5	0.2049	0.2136
1.5	0.2	5.79×10^{-4}	2.06×10^{-5}
Charm quark	0.3	6.42×10^{-4}	3.12×10^{-5}
	0.5	8.84×10^{-4}	4.05×10^{-3}
	0.8	0.0230	0.0246
	1.0	0.0333	0.0499
	1.3	0.0489	0.0896
	1.5	0.0580	0.1174
5.0	0.2	1.63×10^{-11}	2.09×10^{-19}
Bottom quark	0.3	1.79×10^{-9}	4.32×10^{-15}
	0.5	1.52×10^{-7}	1.67×10^{-12}
	0.8	4.07×10^{-6}	1.14×10^{-9}
	1.0	1.51×10^{-5}	1.52×10^{-8}
	1.3	5.89×10^{-5}	8.57×10^{-7}
	1.5	1.16×10^{-4}	3.37×10^{-5}
	5.0	-	7.69×10^{-3}

\

PAIR PRODUCTION IN NON ABELIAN GAUGE FIELDS

by

SETHUMADHAVAN PUZHAKKAL

Thesis submitted to the
University of Calicut
in partial fulfillment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY
under Faculty of Science

Department of Physics
University of Calicut
AUGUST 2007

Chapter 4

Pair Production in Nonabelian Gauge Fields-A Resonance Phenomenon -Model 2

In this chapter we calculate the $(q\bar{q})$ pair production probability in the colour flux tube model by taking the effect of non-Abelian interactions in the theory. By solving $SU(2)$ Yang Mills equation we obtained an exact expression for electric field which is found to be time dependent and oscillating with a characteristic frequency ω_0 which depends on the amplitude of the field strength. By using WKB approximation in complex time we calculated the pair creation probability and it is seen that when ever the strength of the field (gE) is comparable to the quark-aniquark energy (p^2+m^2) the corresponding pair creation probability is maximum, and for the static field $\omega_0 \rightarrow 0$, we recovered the famous Schwinger result.

4.1 Introduction

What happens in Ultra Relativistic Heavy Ion Collisions(URHIC)? The question has attracted a lot of attention supported by extensive research from the early days of strong interactions. In high energy Physics, the quark antiquark creation probability in hadron-hadron or nucleus-nucleus collisions had received great attention. It is believed that a large amount of energy is deposited in a small space-time region in all such collisions, stored in the form of color electric field energy, it is thought to be responsible for the production of quark antiquark($q\bar{q}$) pairs which eventually results in quark-gluon plasma(QGP). Here we discuss, once more, the gluons induced pair creation , in the presence of strong color electric field, using a new non-abelian solution. This approach we find, produce surprising results.

What are the advantages of this approach? The basic one is that the solution is non-abelian actually and shines over several others from authors[1-3] which are only models. The second advantage is that we do not rely or resort to any approximation, simply as we depend, on pure calculations. Final advantage is that it leads to a result consistent, with real physics, and when compared with former results from authors[2-5], our results are better.

4.2 Visualization of the Problem

When two high energy nucleons collide, they almost pass through each other, exciting themselves. But in addition in the space they pass through they leave behind a flux tube of deposited energy with great rapidity. This energy get rapidly transformed into hadrons. The depositions of energy at higher rate is about $\frac{1}{3}GeV fm^{-3}$, as can be ascertained from proton-proton collision data. This increase can occur for energy deposition even if collision energy increase further, but it elongates the flux tube further.

However, a given region of flux tube can receive a much larger deposition of energy[6], when a multiple superposition of two heavy nuclei collides occur, to the tune of at least $\varepsilon \sim NA^{1/3}GeV fm^{-3}$, as can be deduced from simple geometrical arguments. This means that an energetic collision of two ^{238}U nuclei an average deposition of about $6GeV fm^{-3}$, well above, what is required for plasma formation[7].

Pair production in the flux tube model[4,5,8-14], had been considered by many authors. In flux tubes, the strong electric color field that is set up, makes the vacuum unstable to pair production via Schwinger Mechanism[1]. The color field in the flux tube is transformed into the energy of pairs. In the collision, a large number of pairs and in addition gluons get realized, leading ultimately to the formation of a quark-gluon plasma(QGP). The self interaction of the gluons in the flux tube is likely to polarize the medium between two receding nuclei. This leads to characteristic normal mode of oscillations.

4.3 Solution of SU(2) Yang -Mills equation

According to our present level of understanding all fundamental interactions except gravity are described by non-Abelian gauge theories. The gauge bosons of various non-Abelian symmetries mediate different interactions. All known fundamental fermions fields are divided into two broad classes. One class, called Leptons, such as the electrons, muon and neutrinos, do not have any strong interactions. In gauge theory parlance we can say that these are singlets under the strong gauge interaction gauge group. The other class, called, called quarks, can be described by $SU(3)$ symmetry and therefore have strong interactions. Under a global $SU(3)$ symmetry the three colors of quarks transform like a triplet. If try to gauge this color symmetry, we obtained a theory called Quantum chromo dynamics (QCD). This will contain eight gauge fields $G^a_{\mu\nu}$, which are called gluons. To understand the features due to non-abelian effect one has to solve the Yang Mills equation. For simplicity here we consider the time dependent vacuum solution of $SU(2)$ Yang-Mills equation, which satisfies

$$\partial_\mu G^{\mu\nu}_a + g\varepsilon_{abc} A_{\mu b} G^{\mu\nu}_c = 0$$

Where the field tensor

$$G^{\mu\nu}_a = \partial^\mu A^\nu_a - \partial^\nu A^\mu_a + g\varepsilon_{abc} A^\mu_b A^\nu_c$$

a,b,c are color indices which take values 1,2,3 and Lorentz indices μ,ν=0,1,2,3 with metric $(1, -1, -1, -1)$. ε_{abc} is an anti symmetric Levicivita tensor. Taking the temporal gauge $A^0 = 0$, space homogeneity $(\nabla\Phi = 0)$, the Yang-Mills equation becomes

$$A_a + g^2 A_a (A^2 - A^2_a) = 0$$

Where $A^2 = A_1^2 + A_2^2 + A_3^2$ We assume that all three colors vary in time in the same way:

$$A_1(t) = A_2(t) = A_3(t) = A(t)$$

Then the Yang-mills equation reduces to

$$A(t) + 2g^2 A^3(t) = 0$$

It is a non-linear equation in one degree of freedom and admitting an analytical solution as the Jacobian elliptic cosine function. When the modulus of the cos function is taken to be one, solution becomes hyperbolic secant function. The color potential is evaluated to be

$$A(t) = E_0/\omega_0 sech\omega_0 t$$

where

$$\omega_0 = (8g^2/3)^{1/4}\sqrt{E_0}/2$$

Therefore

$$E = -\frac{\partial A}{\partial t} = E_0 sech\omega_0 t \tan h\omega_0 t$$

It shows that the color field in the flux tube is time dependant (color particles are coupled to each other via the gauge fields) and the characteristic frequency of oscillation depends on the amplitude of oscillation. It is actually one of the hallmarks of non-linear oscillators opposed to linear ones is that their frequency verses with the amplitude. Here we wish to calculate the quark- anti quark pair production amplitude in quark- gluon plasma. For this, we follow the method given by S. Biswas and S. Guha [9]. The basic ingredient in their calculation is the evaluation of action integral $S([t_1, t_2])$. This will be evaluated by solving the color coupled Klein Gordan equation in the external color potential. For the color $SU(2)$ group the equations to be solved are (τ_α, with $\alpha = 1, 2, 3$ are paulispin matrices)

$$[\partial_0^2 - \nabla^2 + 2ig\tau_\alpha A_\alpha \partial_3 + g^2 A_\alpha^2 + m^2] \begin{bmatrix} \phi_+ \\ \phi_- \end{bmatrix} = 0 \tag{4.1}$$

with

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

It is due to non-abelian effect in the equation 2, it may be seen that ϕ_+ and ϕ_- are coupled. So, to solve the above equation, it has to be decoupled. It may be shown that the above equation can be decoupled by a unitary transformation in color space defined by

$U^+ = \begin{bmatrix} \frac{\sqrt{E^2-E_3^2}}{E-E_3} & \frac{E_1-iE_2}{\sqrt{E^2-E_3^2}} \\ \frac{\sqrt{E^2-E_3^2}}{E+E_3} & -\frac{E_1-iE_2}{\sqrt{E^2-E_3^2}} \end{bmatrix}$, where $E^2 = E_1^2 + E_2^2 + E_3^2$. The (column vector) wave function in turn transforms into

$U^+ \begin{pmatrix} \phi_+ \\ \phi_- \end{pmatrix} = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix}$ The decoupled equations are evaluated to be

$$[\partial_0^2 + \omega(t)]\, \psi_+ = 0 \text{ and } [\partial_0^2 - \omega(t)]\, \psi_- = 0 \tag{4.2}$$

With $\omega(t) = [\omega^2 + (p_3 + \frac{\sqrt{2}gE}{\omega_0} tanh\omega_0 t)^2]^{1/2}$, where $\omega^2 = p_1^2 + p2^2 + m^2 = p_\perp^2 + m^2$ This is nothing but our Kelin-Gordan(K.G) equation for one dimension.

Now, we try to solve the decoupled equations using the W.K.B method.

The validity of the approximation can be easily checked.[2]

i.e., $\frac{d}{dt}(\frac{1}{\omega(t)}) \ll$ or $\frac{\omega(t)}{\omega^2(t)} \ll 1$

So as to solve our K.G. equation, we follow the method, W.K.B approximation in complex time, given by S.Biswas and J.Guha [9].

4.4 Evaluation of reflection coefficient using W.K.B. approximation in complex time.

In standard W.K.B approximation with a real time t, wave function is written as

$$\psi = \frac{c_1}{[\omega(t)]^{1/2}} e^{\int i\omega(t)dt} + \frac{c_2}{[\omega(t)]^{1/2}} e^{-\int i\omega(t)dt}$$

In complex time t, we define

$$\int_{t_1}^{t} \omega(t)dt = s(t, t_1) = \int_{t_1}^{t} [\omega^2 + V(t)]^{1/2} dt$$

Where $V(t) = (p_3 - \frac{gE}{\omega_0} sech\omega_0 t)^2$ The turning points are determined from$\omega(t) = 0$ i.e, $[\omega^2 + V(t)]^{1/2} = 0$ The boundary conditions are chosen such that,

$$\psi \sim e^{is(t,t_1)} \sim e^{\int_{t_1}^{t} i\omega(t)dt}$$

when $t \to -\infty$

$$\psi \sim e^{is(t,t_1)} + be^{-is(t,t_1)}$$

i.e, $\psi \sim e^{i\int \omega(t)dt} + be^{-i\int \omega(t)dt}$ When $t \to \infty$

Here b is called the reflection coefficient. Consider the pair production as a consequence of reflection in time; the reflection coefficient will be identified as pair production amplitude [9].

The reflection coefficient is calculated to be.

$$b = -\frac{ie^{2is(t_1,t_0)}}{1 + e^{2is(t_1,t_2}} \tag{4.3}$$

From this, the reflection amplitude b^2can be calculated which is our pair creation amplitude according to the complex time W.K.B approximation

4.5 Calculation of action integral and the pair creation amplitude

$$s(t_1, t_2) = \int_{t2}^{t1} [\omega^2 + (p_3 + \frac{\sqrt{2}gE}{\omega_0} tanh\omega_0 t)^2]^{1/2} dt \tag{4.4}$$

where $t_1 = \frac{1}{\omega_0} tanh^{-1} \frac{\omega_0}{\sqrt{2gE}} (i\omega - p_3)$ and $t_2 = -\frac{1}{\omega_0} tanh^{-1} \frac{\omega_0}{\sqrt{2gE}} (i\omega + p_3)$

To solve equation4.4, put $(p_3 + \frac{\sqrt{2gE}}{\omega_0} tanh\omega_0 t) = i\omega \sin\theta$

$\therefore s(t_1, t_2) = \frac{i\omega^2}{\sqrt{2gE}} \int_{-\pi/2}^{+\pi/2} \frac{\cos^2\theta d\theta}{[(p_3 - i\omega\sin\theta)\sqrt{1 - \frac{\omega_0^2(i\omega\sin\theta - p_3)^2}{2g^2E^2}}]}$

This can be easily integrated to give

$$s(t_1, t_2) = \frac{i\pi}{2} + \frac{i\pi}{2\sqrt{2gE}} [\sqrt{\omega^2 + (\frac{\omega_0}{2} + p_3)^2} + \sqrt{\omega^2 + (\frac{\omega_0}{2} - p_3)^2}] \tag{4.5}$$

Similarly $s(t_1, t_0)$ can be evaluated to be

$$s(t_1, t_0) = \frac{i\pi}{4} + \frac{i\pi}{4\sqrt{2gE}}[\sqrt{\omega^2 + (\frac{\omega_0}{2} + p_3)^2} + \sqrt{\omega^2 + (\frac{\omega_0}{2} - p_3)^2}] \tag{4.6}$$

Putting the values of $s(t_1, t_2)$and $s(t_1, t_0)$ in equation 4.3, we get

$$b = \frac{e^{\frac{\pi}{2}} \times e^{\frac{-\pi}{\sqrt{2gE}}[\sqrt{\omega^2 + (\frac{\omega_0}{2} + p_3)^2} + \sqrt{\omega^2 + (\frac{\omega_0}{2} - p_3)^2}]}}{1 + e^{-\pi} \times e^{\frac{-\pi}{\sqrt{2gE}}[\sqrt{\omega^2 + (\frac{\omega_0}{2} + p_3)^2} + \sqrt{\omega^2 + (\frac{\omega_0}{2} - p_3)^2}]}} \tag{4.7}$$

$\therefore$ The pair creation probability

$$b^2 = \frac{e^{-\pi} \times e^{\frac{-2\pi}{\sqrt{2gE}}[\sqrt{\omega^2 + (\frac{\omega_0}{2} + p_3)^2} + \sqrt{\omega^2 + (\frac{\omega_0}{2} - p_3)^2}]}}{[1 + e^{-\pi} \times e^{\frac{-\pi}{\sqrt{2gE}}[\sqrt{\omega^2 + (\frac{\omega_0}{2} + p_3)^2} + \sqrt{\omega^2 + (\frac{\omega_0}{2} - p_3)^2}]}]^2} \tag{4.8}$$

To estimate the pair creation probability we set $p_3 = 0$, the range of integration being of order of magnitude $2\frac{2gE}{\omega_0}$ as suggested by classical equation of motion.

$\therefore$

$$b^2 = \frac{e^{-\pi} \times e^{\frac{-4\pi}{\sqrt{2gE}}[\sqrt{p^2+m^2+\frac{\omega_0^2}{4}}]}}{[1 + e^{-\pi} \times e^{\frac{-4\pi}{\sqrt{2gE}}[\sqrt{p^2+m^2+\frac{\omega_0^2}{4}}]}]^2} \tag{4.9}$$

From equation 4.9, it can be seen that, when $\frac{db^2}{d(gE)} = 0$, we get

$gE = p^2+m^2$ and $\frac{d^2(b)^2}{d^2(gE)}$ is found to be negative. It shows that the pair creation becomes maximum when $gE = p^2 + m^2$. i.e. when ever the quark anti quark energy is comparable with the field strength the pair creation probability dominates. Thus we can say that it is a resonance phenomenon. Further equation 4.9 shows that the pair creation probability depends crucially upon the collective oscillation frequency ω_0 a result which is expected. Put $\omega_0 = 0$ in equation 4.9, we get

$$\therefore \quad b^2 = \frac{e^{-\pi} \times e^{\frac{-4\pi}{\sqrt{2gE}}[\sqrt{p^2+m^2}]}}{[1 + e^{-\pi} \times e^{\frac{-4\pi}{\sqrt{2gE}}[\sqrt{p^2+m^2}]}]^2}$$

4.6 Result

To conclude, we described an important non-abelian effect on the pair creation probability in collisions in high energy physics. Once again it is established that the colour field should be time dependent due to non-abelian interactions and should oscillate with a collective frequency ω_0 which should depend on field strength. Our model has several advantages over other models[2-5]. We assumed a non-abelian colour field model which is an exact solution of Yang-Mills equation, the second is that our calculation is completely analytic and no assumption is made to simplify it, thirdly we need not go for any cumbersome calculations as the authors adopted [1-4]. Evaluation of the action integral $s(t_1, t_2)$, $s(t_1, t_0)$ is simple even though lengthy. Our results do not violate any of the former results in this field. Finally, we obtained a very consistent result. If our model is good and the pair creation probability is truly given by our equation 4.9, it should be possible to test it in heavy ion collision experiments.We hope that the same resonance effect will hold well even if we calculate the pair production probability using the

Dirac equation.

4.7 References

[1]J.Schwinger, Phys. Rev. 82, 664 (1951)

[2]E.Brezin, C. Itzykson, Phys. Rev. D 2, 1191 (1970)

[3]A.K.Ganguly, P.K. Kaw, J.C. Parikh, Phys. Rev. D 48, 2983 (1983)

[4]C.Martin, D. Vautherin, Phys. Rev. D 38, 3593 (1988)

[5]H.P. Pavel, D.M. Brink, Z. Phys. C 51, 119 (1991)

[6]J.D. Bjorken, Phys. Rev. D 27, 140 (1983)

[7]H. Satz, Nature 324, 13 (1986)

[8]A. Actor, Rev. Mod. Phys. 51, 3 (1979)

[9]S. Biswas, J. Guha, Pramana J. Phys.40, 467 (1993)

[10]F.E.Low,Phys.Rev.D12,163(1975)

[11]A. Casher, H. Neuberger, S. Nussinov, Phys. Rev D 20, 179 (1979)

[12]T.S. Biro, H.B. Nielsen, J. Knoll, Nucl. Phys. B 245, 449 (1984)

[13]N.K. Glendenning, T. Matsui, Phys. Rev. D 28, 2890 (1983)

[14]A.Bailas, W. Czyz, B. Banerjee, Phys. Lett. B 224, 16 (1989).

****

PAIR PRODUCTION IN NON ABELIAN GAUGE FIELDS

by

SETHUMADHAVAN PUZHAKKAL

Thesis submitted to the
University of Calicut
in partial fulfillment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY
under Faculty of Science

Department of Physics
University of Calicut

AUGUST 2007

Chapter 5

Flux Tube Density using Cornell Potential

5.1 Introduction

In Chapter 3 and chapter 4 we have considered the picture of the meson where quarks are connected by a flux tube. In another model of mesons where there is constituent gluon approach , the quarks and gluons move in potential.The most phenomenological model of confinement may be the flux tube model.This model is consistent with the experimental observation of jet phenomena observed during hadron formation in e^+e^- which has already been discussed in chapter 2.Consider a meson made up of quark and anti-quark,which are connected by a flux tube of chromoelectric field in analogy to the formation of flux tubes in superconductors.In earlier days a constant field in the flux tube was assumed and the energy for quarks separated a distance r was used to be

$$V(r) = \frac{E^2 Ar}{8\pi} = \sigma r$$

where A is the cross section area of the tube,r is the separation between the quarks and σ is the string constant or flux tube density.Other Early potentials used are MIT bag model,RHO potential model etc .The potential models have been highly successful to describe the bound states of quarks.We in this section consider mesons as quarks and gluons confined in a Cornell potential(coulomb+linear) and calculated the string constant which is the flux tube density. Many models like bag model[1] ,quasiparticle model[2,3],strongly interacting QGP model[4], strongly coupled quark gluon plasma(SCQGP) model[5],Cornell potential model[6] etc were used to explain this complicated matter formed. All the models were not fully accepted by the physics community even though some models were succcesful to a great extent[5] . The Cornell potential model has been earlier used to compare the lattice results of the gluon plasma[6].Bijan.S.S [7] has earlier obtained an equation of state [EOS] for hadron production analytically using the same potential.

5.2 Cornell Potential model

The particle -particle, particle- antiparticle or antiparticle-antiparticle potential in a plasma environment is generally defined as

$$U(r,T) = z_i z_j V(r_{ij})$$

where $z_i = +1$ or $z_i = -1$ when the ith particle corresponds to a particle or antiparticle respectively. The Cornell potential $V(r_{ij})$ does not depend on the nature of the particles and is defined here as

$$V(r_{ij}) = \left(\frac{3\alpha_s}{r_{ij}} - C\sigma r_{ij}\right) \tag{5.1}$$

for gluon plasma

$$V(r_{ij}) = \left(\frac{4}{3}\frac{\alpha_s}{r_{ij}} - \sigma r_{ij}\right) \tag{5.2}$$

for quark-antiquark plasma where α_s , σ and C are strong coupling constant , string tension and Casimir scaling respectively.

5.3 Evaluation of Pressure using Mayer's Cluster Expansion Technique

For any plasma the equation for pressure is given by [13,14]

$$\frac{P}{T} = \sum_{i=1}^{N} n_i + S - \sum_{i\geq 1}^{N} n_i \frac{\partial S}{\partial n_i} \tag{5.3}$$

in natural units .Here n_i = number density of particle/antiparticle and

$$S = \sum_{\nu\geq 2}^{N} \frac{1}{4\pi^2} \int_0^\infty \frac{l^2 dl\,(-\kappa^2 V_l)^\nu}{\nu} \tag{5.4}$$

where V_l = interaction potential in momentum space , ν = number of particle rings and κ = inverse Debye length . For gluons let the number density be represented by n_g.Hence the equation(5.3) modifies to

$$\frac{P}{T} = n_g + S - n_g \frac{\partial S}{\partial n_g} \tag{5.5}$$

Taking

$$\kappa^2 = \frac{n_g}{T}$$

equation(5.5) becomes

$$\frac{P}{T} = \kappa^2 T + S - \kappa^2 \frac{\partial S}{\partial \kappa^2} \tag{5.6}$$

From equation (5.4)

$$\frac{\partial S}{\partial \kappa^2} = \frac{\kappa^2}{4\pi^2} \int_0^\infty l^2 dl \frac{V_l^2}{1 + \kappa^2 V_l} \tag{5.7}$$

The Cornell potential given by equation (5.1) is in coordinate space and V_l is the Fourier transform of it and is obtained as

$$V_l = \frac{12\pi\alpha_s}{l^2 + \alpha^2} + \frac{8\pi C\sigma}{(l^2 + \alpha^2)^2} \tag{5.8}$$

where α is a convergence parameter which will be taken to be zero at the end of the calculation. Substituting this value of V_l in equation (5.7) and performing integration we get

$$\frac{\partial S}{\partial \kappa^2} = \frac{18\pi\alpha_s^2\kappa^2 + 3\alpha_s\kappa\sqrt{2\pi C\sigma} - C\sigma}{\sqrt{12\pi\alpha_s\kappa^2 + 4\kappa\sqrt{2\pi C\sigma}}} + c_1 \tag{5.9}$$

where c_1 is term independent of κ^2. Integrating over κ^2 we get

$$S = \frac{1}{12\pi}\left(\sqrt{12\pi\alpha_s\kappa^2 + 4\kappa\sqrt{2\pi C\sigma}}\right)\left(12\pi\alpha_s\kappa^2 - 2\sqrt{2\pi C\sigma\kappa}\right) + c_1\kappa^2 \quad (5.10)$$

Substituting (5.9) and (5.10) in equation (5.6), we get

$$\frac{P}{T} = \kappa^2 T - \left(\frac{6\pi\alpha_s^2\kappa^{\frac{7}{2}} + \alpha_s\sqrt{2\pi C\sigma}\kappa^{\frac{5}{2}} + \frac{1}{3}C\sigma\kappa^{\frac{3}{2}}}{\sqrt{12\pi\alpha_s\kappa + 4\sqrt{2\pi C\sigma}}}\right) \quad (5.11)$$

We have for gluon plasma

$$n_g \approx \frac{g_I}{\pi^2}T^3\sum_{l=1}^{\infty}\frac{1}{l^3}$$

where g_I =16 is the internal degrees of freedom for gluons.

$$n_g \approx g_I T^3\frac{\zeta(3)}{\pi^2} \quad (5.12)$$

where $\zeta(3)$ is the Riemann-zeta function Substituting equation (5.12) in equation (5.11) we get

$$\frac{P}{T^4} = \lambda - \left(\frac{6\pi\alpha_s^2\lambda^{\frac{7}{4}} + \alpha_s\sqrt{2\pi C\sigma}\lambda^{\frac{5}{4}}\frac{1}{T} + \frac{1}{3}C\sigma\lambda^{\frac{3}{4}}\frac{1}{T^2}}{\sqrt{12\pi\alpha_s\lambda^{\frac{1}{2}} + 4\sqrt{2\pi C\sigma}\frac{1}{T}}}\right) \quad (5.13)$$

where $\lambda = \frac{g_I\zeta(3)}{\pi^2}$

The Fourier transform of the Cornell potential given by equation (5.2) is

$$V_l = \frac{16\pi\alpha_s}{3(l^2+\alpha^2)} + \frac{8\pi\sigma}{(l^2+\alpha^2)^2} \quad (5.14)$$

For quark-antiquark plasma equation(5.3) becomes

$$\frac{P}{T} = n_q + n_{\bar{q}} + S - n_q\frac{\partial S}{\partial n_q} - n_{\bar{q}}\frac{\partial S}{\partial n_{\bar{q}}} \tag{5.15}$$

Taking

$$\kappa^2 = \frac{n_q + n_{\bar{q}}}{T}$$

equation(16) becomes

$$\frac{P}{T} = \kappa^2 T - S - \kappa^2\frac{\partial S}{\partial \kappa^2} \tag{5.16}$$

Substituting the value of V_l in equation (5.7) and evaluating S and resubstituting in equation (5.16), we get

$$\frac{P}{T} = \kappa^2 T - \frac{\frac{32}{9}\pi\alpha_s^2\kappa^{\frac{7}{2}} + \frac{4}{3}\alpha_s\sqrt{2\pi\sigma}\kappa^{\frac{5}{2}} + \sigma\kappa^{\frac{3}{2}}}{6\sqrt{\frac{4}{3}\pi\alpha_s\kappa + \sqrt{2\pi\sigma}}} \tag{5.17}$$

We have for quark anti-quark plasma

$$n_q + n_{\bar{q}} \approx \frac{12n_f}{\pi^2}T^3\sum_{l=1}^{\infty}\frac{(-1)^{l-1}}{l^3}$$

where n_f is the number of flavors of the quark anti-quark plasma .

We know $\sum_{l=1}^{\infty}\frac{(-1)^{l-1}}{l^3} = \frac{3}{4}\zeta(3)$. Hence

$$\frac{P}{T^4} = \beta - \frac{\frac{32\pi}{9}\alpha_s^2\beta^{\frac{7}{4}} + \frac{4}{3}\alpha_s\sqrt{2\pi\sigma}\beta^{\frac{5}{4}}\frac{1}{T} + \sigma\beta^{\frac{3}{4}}\frac{1}{T^2}}{6\sqrt{\frac{4}{3}\pi\alpha_s\beta^{\frac{1}{2}} + \sqrt{2\pi\sigma}\frac{1}{T}}} \tag{5.18}$$

where $\beta = \frac{9}{\pi^2}n_f\zeta(3)$

5.4 Comparison with LGT

It is seen that EOS with Cornell potential model fits very well with the lattice data for the three systems namely gluon plasma, 2-flavor and 3- flavor QGP . In Fig 1 we plotted $\frac{P(T)}{T^4}$ verses $\frac{T}{T_c}$ for pure gauge, 2-flavor and 3- flavor QGP along with lattice results. For each system α_s and σ are adjusted so that we get a good fit with the lattice results. Surprisingly good fits are obtained for all systems with $\alpha_s = 0.09, .01 and .009$ and $\frac{\sigma}{T_c^2} = 8, 15$ and 15 for gluon plasma ,2- flavor and 3-flavor respectively.

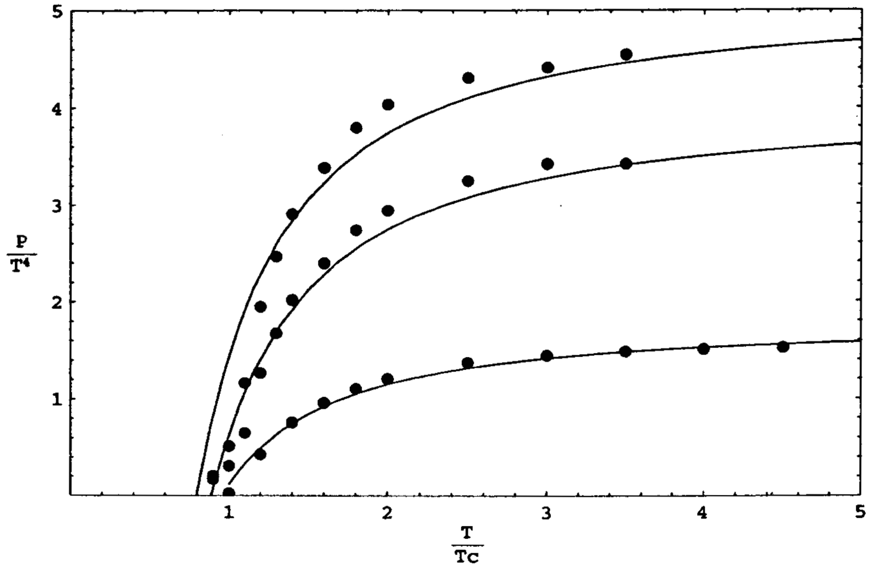

Figure 5.1: Plots of $\frac{P}{T^4}$ as a function of $\frac{T}{T_c}$ in Cornell potential and lattice results(dots) for pure gauge ,2-flavor and 3-flavor .Lower curve is for pure gauge, middle curve is for 2-flavor and upper curve is for 3-flavor

5.5 Conclusion

Using a phenomenological model like Cornell potential to treat QGP near and above T_c as " Coulomb + linear confinement potential" shows surprisingly good fit to the lattice results.Two system parameters were varied to get good fit to the lattice results of pressure . Earlier studies had shown that the string constant parameter is independent of the flavor of quark gluon plasma[15].Our studies confirm this result.It can be seen that the string constant $\frac{\sigma}{T_c^2} = 15$ for both 2-flavor and 3-flavor plasma with different values of α_s.Besides this the flux tube density for gluon plasma is smaller than the flux tube density for QGP.Above all the value obtained for flux tube density is well in agreement with what is predicted experimently.

5.6 References

[1]D.J.Rischke,M.I.Gorenstein,A.Schafer,H.Stocker Phys.Lett B**278**,19(1992)

[2]V.M.Bannur, Phys. Lett B, **647**,271-274(2007)

[3]V.M.Bannur The European Physical Journal C, **50**,629(2007)

[4]E.Shuryak,hep-ph/0405066(2004)

[5]V.M.Bannur, J.Phys.G:Nucl.Part.Phys. **32**,993-1002(2006)

[6]V.M.BannurPhys.Lett B. **362**,7-10(1995)

[7]Bijan.S.S ,Phys. Rev. C,**65**,054904(2002)

[8]F.Karsh,E.Laermann,A.Peikert,Phys.Lett.B **478**,447(2000)

[9]G.Boyd,J.Engels,F.Karsh,E.Laermann,C.Legeland,M.Lutgemeier, B.Petersson,Phys.Rev.Lett,**75**,4169(1995)

[10]G.Boyd,Nucl.Phys,B,**469**,419(1996)

[11]F.Karsh,Nucl.Phys.A**698**,199(2002)

[12]Laermann,O.Philipsen,Ann.Rev.Nucl.Part.Sci.**53**,163(2003)

[13]J.E.Mayer,J.Chem.Phys.**18**, 1426-1436(1950)

[14]R.Balescu,Statistical Mechanics of Charged Particles(Interscience ,New York,1963)

[15]E.Eichten,K.Gottfried,T.Kinoshita,K.D.Lane and T.M.Yan, Phys.Rev.D **17**,3090(1978).

****

1

PAIR PRODUCTION IN NON ABELIAN GAUGE FIELDS

by

SETHUMADHAVAN PUZHAKKAL

Thesis submitted to the
University of Calicut
in partial fulfillment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY
under Faculty of Science

Department of Physics
University of Calicut
AUGUST 2007

Chapter 6

Trajectory of quarks in Cornell Potential

6.1 Introduction

In the initial stages of QGP,immediatly after $q\bar{q}$ pair production,quarks and qluons may be moving in the Cornel potential and hence the study of their trajectory may be important.So in this chapter we obtained a physical picture of movements of quarks and anti-quarks confined in the Cornell potential. We obtained the trajectory of quarks in Cornell potential using Hamilton Jacobi method [4]. Hamilton-Jacobi equation[HJE] may be derived from Hamiltonian mechanics by treating S the action as the generating function for a canonical transformation of the classical Hamiltonian . The HJE is a single, first-order partial differential equation for the function S of the N generalized coordinates and the time t. The Hamiltonian of the quarks in

Cornell potential is given by

$$H = \frac{p^2}{2m} + \frac{4}{3}\frac{\alpha_s}{r} - \sigma r \tag{6.1}$$

This can be written as

$$H = \frac{p^2}{2m} + \frac{a}{r} + br \tag{6.2}$$

6.2 Hamilton-Jacobi Method

The Hamilton-Jacobi equation is given by

$$H + \frac{\partial S}{\partial t} = 0 \tag{6.3}$$

where S is the action which contains the complete information regarding the dynamics of the particle.To evaluate the action ,we start from the Hamilton-Jacobi equation .

$$\frac{p^2}{2m} + \frac{a}{r} + br + \frac{\partial S}{\partial t} = 0 \tag{6.4}$$

$$p^2 = p_r^2 + \frac{p_\theta^2}{r^2} = p_r^2 + \frac{l^2}{r^2} = \left(\frac{\partial S}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial S}{\partial \theta}\right)^2 \tag{6.5}$$

Hamiltonian represents the total energy E of the system

Then the action S is given by

$$S = -Et + l\theta + f(r)$$

$$S = -Et + l\theta + \int \left(\sqrt{2mE - \frac{l^2}{r^2} - \frac{2ma}{r} - 2mbr} \right) dr'$$

6.3 Trajectory of Quarks

The trajectory of quarks can be found out from the action.The equation of the trajectory is given by the condition $\frac{\partial S}{\partial l}$ = a constant

$$\frac{\partial S}{\partial l} = \theta - \int \frac{\frac{l}{r'^2}}{\left(\sqrt{2mE - \frac{l^2}{r'^2} - \frac{2ma}{r'} - 2mbr'} \right)} dr'$$

By proper choice of the pericentre of the trajectory $\frac{\partial S}{\partial l}$ is chosen to be equal to zero with this we get

$$\theta = \int \frac{\frac{l}{r'^2}}{\left(\sqrt{2mE - \frac{l^2}{r'^2} - \frac{2ma}{r'} - 2mbr'} \right)} dr'$$

put $r' = \frac{\rho c}{\omega_0}$,$l = \frac{\lambda mc^2}{\omega_0}$and $\varepsilon = \frac{E}{mc^2}$ we get

$$\theta = \int \frac{d\rho}{\rho \sqrt{\frac{2mEr^2}{\ell^2} - 1 - \frac{2mar}{\ell^2} - \frac{2mbr^3}{\ell^2}}}$$

Put $\frac{a\omega_0}{mc^3} = \alpha$ and $\frac{b}{mc\omega_0} = \beta$, we get

$$\theta = \int \frac{d\rho}{\rho \sqrt{\frac{2\varepsilon\rho^2}{\lambda^2} - 1 - \frac{2\rho\alpha}{\lambda^2} - \frac{2\beta\rho^3}{\lambda^2}}}$$

Making the change of variable $\rho = \frac{1}{x}$, we get

$$\theta = -\int \frac{dx}{x\sqrt{\frac{2\varepsilon}{\lambda^2 x^2} - 1 - \frac{2\alpha}{\lambda^2 x} - \frac{2\beta}{\lambda^2 x^3}}}$$

$$\theta = -\int \frac{dx}{\sqrt{\frac{2\varepsilon}{\lambda^2} - x^2 - \frac{2\alpha x}{\lambda^2} - \frac{2\beta}{\lambda^2 x}}}$$

$$\theta = -\int \frac{dx\sqrt{x}}{\sqrt{\frac{2\varepsilon x}{\lambda^2} - x^3 - \frac{2\alpha x^2}{\lambda^2} - \frac{2\beta}{\lambda^2}}} \tag{6.6}$$

In order to calculate this integral, the polynomial occurring in the denominator of the radicant will be decomposed in to factors. For this we will find its roots. They are given by the algebraic equation

$$x^3 + \frac{2\alpha x^2}{\lambda^2} - \frac{2\varepsilon x}{\lambda^2} + \frac{2\beta}{\lambda^2} = 0$$

$$x^3 + 3a_1x^2 + 3a_2x + a_3 = 0 \tag{6.7}$$

Written in normal form, having coefficients $a_0 = 1$, $a_1 = \frac{2\alpha}{3\lambda^2}$, $a_2 = -\frac{2\varepsilon}{3\lambda^2}$ and $a_3 = \frac{2\beta}{\lambda^2}$ To solve the cubic equation put $x = z - a_1$, ge get

$$z^3 + 3z(a_2 - a_1^2) + a_3 - 3a_1a_2 + 2a_1^3 = 0$$

$$z^3 + 3zH + G = 0 \tag{6.8}$$

Where the quadratic term is absent with the coefficients $H = a_2 - a_1^2$ and $G = a_3 - 3a_1a_2 + 2a_1^3$

Now put $z = u + v$, we get

$$z^3 - 3uvz - u^3 - v^3 = 0$$

Comparing this with eqn(6.8) we get

$$H = -uv \Rightarrow u^3v^3 = -H^3$$

and

$$G = -(u^3 + v^3)$$

If u^3 and v^3 are the roots, then

$$(t - u^3)(t - v^3) = 0$$

$$t^2 - (u^3 + v^3)t + u^3v^3 = 0$$

$$t^2 + Gt - H^3 = 0$$

$$t_{1,2} = \frac{-G \pm \sqrt{G^2 + 4H^3}}{2}$$

The nature of the roots of the eqn(6.8) depends on the values of the discriminant

$$\Delta = G^2 + 4H^3$$

Evaluating this, we get

$$\Delta = \frac{8}{27\lambda^8}\left(\frac{27\beta^2\lambda^4}{2} + \lambda^2\varepsilon^3 + 8\alpha^3\beta - 2\alpha^2\varepsilon^2 + 18\alpha\varepsilon\beta\right) \tag{6.9}$$

$$\lambda_0 = \sqrt{\frac{-\varepsilon^3}{\beta^2} + \frac{\varepsilon^6}{\beta^4} - 54[\frac{8\alpha^3}{\beta} - \frac{2\alpha^2\varepsilon^2}{\beta^2} + \frac{18\alpha\varepsilon}{\beta}]^{1/2}}$$

The positive root(the only meaningful solution) of the polynomial on the right hand side of eqn(6.9), the quantity Δ is negative, null or positive as $0 < \lambda < \lambda_0$, $\lambda = \lambda_0$ or $\lambda > \lambda_0$ respectively.

For $\Delta < 0$ eqn(6.7) has three simple roots,say x_1, x_2 and x_3. Then eqn(1) of the trajectory becomes

$$\theta = \int \sqrt{\frac{x}{(x_1 - x)(x_2 - x)(x_3 - x)}} dx$$

Then the implicit equation of the trajectory has the form

$$\frac{x_1}{\sqrt{(x_1 - x_3)x_2}} \Pi \left(arc\sin \sqrt{\frac{x_2(x_1 - \frac{1}{\rho^2})}{\frac{(x_1 - x_2)}{\rho^2}}}, \frac{x_1 - x_2}{x_2}, \sqrt{\frac{(x_1 - x_2)x_3}{(x_1 - x_3)x_2}} \right) = \theta \tag{6.10}$$

Where $\Pi(\psi, n, k)$ is the elliptic integral of the third kind

$$\Pi(\psi, n, k) = \int \frac{d\psi'}{(1 + n\sin^2\psi')\sqrt{1 - k^2\sin^2\psi'}}$$

6.4 Conclusion

From the above mathematical analysis we conclude the following things. Eq.(6.10) describes a finite motion within a ring bounded by circles of radii corresponding to the roots x_1 and x_2. During a complete rotation of the particle on trajectory, the pericenter suffers an angular shift. This trajectory is an open curve which is called a Rosette.This shows that the trajectory of quarks in Cornell potential is in a rosette shape.In the non-relativistic case, when $x_3 = 0$ the equation of the trajectory can be written as

$$\frac{X^2}{A^2} + \frac{Y^2}{B^2} = 1$$

which represents the equation of an ellipse with the centre at the field centre and of semi axis A and B.That is if consider quarks as non-relativistic particles and allow to move in Cornell potential they move in elliptical orbit.

6.5 References

[1]Goldsrein.H,Classical Mechanics,John Wiley Sons,Inc,NewYork1980

[2]Landau.L.D and Lifshits.E.M,Quantum Mechanics, Non Relativistic Theory, Pergamon Press, Oxford 1977

[3]Khanna M.L,Theory of Equations,JaiPrakashNathand Co,Meerut,1976

[4]Gradshteyn I.S and Ryzhik I.M, Table of Integrals, Series and Products, Academic Press, NewYork, London, 1965

[5]E.Eichten,K.Gottfried,T.Kinoshita,K.D.Lane and T.M.Yan, Phys.Rev.D 17,3090(1978)

[6]E.J.Eichten,Phy.Rev.D52,3,1726(1995).

****

1

PAIR PRODUCTION IN NON ABELIAN GAUGE FIELDS

by

SETHUMADHAVAN PUZHAKKAL

Thesis submitted to the
University of Calicut
in partial fulfillment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY
under Faculty of Science

Department of Physics
University of Calicut
AUGUST 2007

Chapter 7

Summary and Conclusion

Summarization and drawing valid conclusions based on the thesis work is dealt in this chapter.Based on non-abelian interactions,the pair production probability in the color flux tube model had been studied carefully.Initiating from pair production the study develops to the formation of QGP whose experimental confirmation is crucial to the validity of QCD.

In the first part of the thesis, comprising of chapter 3 and chapter 4 ,based on two different color potentials,the quark -antiquark pair production probability is calculated.With the first model calculation reveals that whenever the quark-antiquark mass is comparable to the field strength, the pair production probability is maximum.This result had been published in Pramana,Vol.69,No 2.August(2007).

The second model calculation shows that the pair production probability is maximum whenever the strength of the field is comparable to the quark-antiquark energy.This result has also been published in Acta Ciencia

Indica,Vol XXX111,P.No.4(2007)

In the second part (chapter 5) we have considered the quarks and antiquarks (connected by flux tubes)moving in Cornell potential.The flux tube density has calculated using this potential to compare with the lattice gauge theory results.The results corroborate each other.This work also got accepted by Physical Review C.

In part three (chapter 6),using Hamilton-Jacobi method, trajectory of quarks in Cornell potential is calculated.The trajectory is found to be a Rossette and in non-relativistic case,as expected, the trajectory turns out to be an ellipse.This result is under review from Pramana.

To put matters in full view, it may be recalled that chapter 1 and chapter 2 review the materials collected for studies.In connection with these two chapters ,a book on elementary particles titled "prabanchathinte janimridhikal" was published by Libi publications,calicut and a paper titled by "The gauge symmetry of work energy theorem" was published in Acta Ciencia Indica Vol XXX111,PNo.4(2007). While reviewing QGP another paper titled "Variation of Convective loss Cone Instability (LCI) in Magnetic Mirror"relating to fusion plasma is published in Ultrascientist of Physical Sciences, Vol 19, No.2, August(2007).

****

\

PAIR PRODUCTION IN NON ABELIAN GAUGE FIELDS

by

SETHUMADHAVAN PUZHAKKAL

Thesis submitted to the
University of Calicut
in partial fulfillment of the requirements
for the award of the degree of
DOCTOR OF PHILOSOPHY
under Faculty of Science

Department of Physics
University of Calicut
AUGUST 2007

Chapter 8

List of publications

1. A book on Elementary Particles titled by "Prabanjathinte Janimridhikal" Published by Libi Publications, Rly. Station Link Road, Calicut(2005)

2. $q\bar{q}$ pair production in non-abelian gauge fields- A resonance phenomena, Acta ciencia Indica, Vol, XXXIII, P No.1 (2007)

3. The Gauge Symmetry of Work-Energy Theorem - Acta ciencia Indica, Vol XXXIII, P No.4 (2007)

4. $q\bar{q}$ pair production in non-abelian gauge fields, Pramana, Vol 69,No.2,August(2007)

5. Variation of Convective loss Cone Instability (LCI) in Magnetic Mirror, Ultrascientist of Physical Sciences, Vol 19, No.2, August(2007)

6. EOS of QGP in Cornell - Potential, Physical Review C - Accepted

7. Trajectory of Quarks in RHO Potential - Pramana being reviewed.