

D 33616

(Pages : 3)

Name.....

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, FEBRUARY 2013

(CCSS)

Mathematics

MAT 1C 04—DISCRETE MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A

Answer all questions.

Each question carries 4 marks.

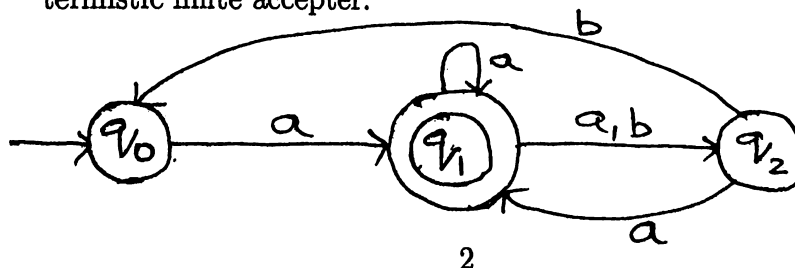
- I.
1. Let  $G$  be a graph with  $n$  vertices and  $m$  edges. Assume that each vertex of  $G$  is of degree either  $k$  or  $k + 1$ . Show that the number of vertices of degree  $k$  in  $G$  is  $(k + 1)n - 2m$ .
  2. Let  $G$  be a simple graph. Prove that  $\Gamma(G) = \Gamma(G^c)$ , where  $\Gamma(G)$  denote the automorphism group of  $G$ .
  3. Let  $D$  be a digraph with no directed cycles. Prove that there exists a vertex whose indegree is 0.
  4. Prove that a tree with atleast two vertices contains atleast two pendant vertices(vertices of degree 1).
  5. Define Eulerian graph and give an example of it. Is every complete graph  $K_n$  with  $n \geq 2$  is Eulerian? Justify your answer.
  6. Show that the complement of a simple planar graph with 11 vertices is nonplanar.
  7. Let  $A = \{a, b\}$  and let  $L = L = \{a(ab)^n : n = 0, 1, 2, \dots\}$ . Find a grammar that generates  $L$ .
  8. Define nondeterministic finite accepter(nfa). Find an nfa with 3 states that accepts the language  $\{ab, aba, ab^2\}$ .
  9. Is  $L = \{(a^2)^n : n \geq 1\}$  regular? Justify your answer.
  10. Let  $(X, \leq)$  be a partially ordered set and let  $x \in X$ . Let  $A = \{z \in X : x < z\}$ . Prove that an element  $y \in X$  covers  $x$  if and only if  $y$  is a minimal element of  $A$ .
  11. Let  $(X, +, \cdot, ')$  be a Boolean algebra. Prove that  $x + x = x$  for all  $x \in X$ .
  12. Obtain the disjunctive normal form of the function  $x'_1x_2(x'_1 + x_2 + x_1x_2)$ .

Turn over

### Part B

Answer A or B of each questions.  
Each question carries 8 marks.

- II. A. (a) Prove that a graph is bipartite if and only if it contains no odd cycles.  
(b) Compute  $K_2^c \vee K_3^c$ .
- B. (a) Prove that the set of all automorphisms of a simple graph  $G$  is a group with respect to the composition of functions.  
(b) Let  $G$  be a connected graph. Prove that an edge  $e = xy$  of a graph  $G$  is a cut edge if and only if  $e$  doesnot belong to any cycle of  $G$ .
- III. A. (a) Prove that every connected graph with  $n$  vertices and  $n - 1$  edges is a tree.  
(b) Let  $G$  be a graph and let  $\delta(G) \geq 2$ . Prove that  $G$  contains a cycle.
- B. (a) Prove that the following are equivalent for a conneted graph  $G$ .  
(i)  $G$  is Eulerian.  
(ii) The degree of each vertex of  $G$  is an even positive integer.  
(iii)  $G$  is an edge-disjoint union of cycles  
(b) Let  $G$  be a simple planar graph with atleast three vertices. Prove that  $m \leq 3n - 6$ , where  $m$  and  $n$  are respectively the number of edges and number of vertices in  $G$ .
- IV. A. (a) Define equivalent grammars. Give an example of equivalent grammar.  
(b) Let  $M = (Q, \Sigma, \delta, q_0, F)$  be a deterministic finite accepter accepting the language  $L$ . Find the language accepted by the finite accepter  $M' = (Q, \Sigma, \delta', q_0, Q - F)$ .
- B. (a) Show that the language  $L = \{vww : v, w \in \{a, b\}^*, |v| = 2\}$  is regular.  
(b) Construct a determistic finite accepter equivalent to the following nonde-  
termistic finite accepter.



- V. A. (a) Let  $k$  and  $n$  be positive integers with  $k \leq n!$ . Prove that there exist unique integers  $d_1, d_2, \dots, d_n$  such that  $0 \leq d_i \leq n - i$  for  $i = 1, 2, \dots, n$  and

$$k = 1 + \sum_{i=1}^n d_i (n - i)!.$$

- (b) Let  $(X, +, \cdot, ')$  be a Boolean algebra. For  $x, y \in X$ , define a relation  $\leq$  on  $X$  by  $x \leq y$  if  $x \cdot y' = 0$ . Prove that  $\leq$  is a partial order.
- B. (a) Let  $x_1, x_2, \dots, x_n$  be mutually independent Boolean variables. Prove that there are  $2^{2^n}$  Boolean functions on these  $n$  variables. Also prove that these  $2^{2^n}$  Boolean functions form a Boolean algebra.
- (b) Write the Boolean function  $(xy + x'y + x'y')(x + y)$  in conjunctive normal form.



Reg. No. : .....

Name : .....

M.Sc. (Previous) Degree Examination, August 2013

(IDE)

Branch : MATHEMATICS

Paper I : Linear Algebra

Time : 3 Hours

Max. Marks : 85

**Instructions :** 1) Answer **five** questions choosing **Part A** or **Part B** from **each** question.  
2) **All** questions carry **equal** marks.

I. A) Let  $W$  be the subspace of  $\mathbb{C}^3$  spanned by  $\alpha_1 = (1, 0, i)$  and  $\alpha_2 = (1+i, 1, -1)$ .

i) Show that  $\alpha_1$  and  $\alpha_2$  form a basis for  $W$ .

ii) Show that the vectors  $\beta_1 = (1, 1, 0)$  and  $\beta_2 = (1, i, 1+i)$  are in  $W$  and form another basis for  $W$ .

iii) What are the co-ordinates of  $\alpha_1$  and  $\alpha_2$  in the ordered basis  $\{\beta_1, \beta_2\}$  for  $W$ .

I. B) a) Let  $V$  be the set of real numbers. Regard  $V$  as a vector space over the field of rational numbers with the usual operations. Prove that this vector space is not finite dimensional.

b) Let  $V$  be the vector space over the complex numbers of all functions from  $\mathbb{R}$  into  $\mathbb{C}$ , i.e. the space of all complex-valued functions on the real line. Let  $f_1(x) = 1$ ,  $f_2(x) = e^{ix}$ ,  $f_3(x) = e^{-ix}$ .

i) Prove that  $f_1$ ,  $f_2$  and  $f_3$  are linearly independent.

ii) Let  $g_1(x) = 1$ ,  $g_2(x) = \cos x$ ,  $g_3(x) = \sin x$ . Find an invertible  $3 \times 3$  matrix  $p$

such that  $g_j = \sum_{i=1}^3 p_{ij} f_i \quad j = 1, 2, 3$ .



- II. A) a) Prove that there is a linear transformation  $T$  from  $\mathbb{R}^3$  into  $\mathbb{R}^2$  such that  $T(1, -1, 1) = (1, 0)$  and  $T(1, 1, 1) = (0, 1)$  and find such a linear transformation.
- b) Let  $T$  be the linear operator on  $e^3$  for which  $T(1, 0, 0) = (1, 0, i)$ ,  $T(0, 1, 0) = (0, 1, 1)$ ,  $T(0, 0, 1) = (i, 1, 0)$ . Is  $T$  invertible? Justify your claim.
- II. B) a) Let  $V$  and  $W$  be finite-dimensional vector spaces over the field  $\mathbb{F}$ . Prove that  $V$  and  $W$  are isomorphic if and only if  $\dim V = \dim W$ .
- b) Let  $V, W$  and  $Z$  be finite-dimensional vector spaces over the field  $\mathbb{F}$ . Let  $T$  be a linear transformation from  $V$  into  $W$  and  $U$  a linear transformation from  $W$  into  $Z$ . If  $B, B'$  and  $B''$  are ordered bases for the spaces  $V, W$  and  $Z$  respectively, if  $A$  is the matrix of  $T$  relative to the pair  $B, B'$  and  $B$  is the matrix of  $U$  relative to the pair  $B', B''$  then prove that the matrix of composition  $UT$  relative to the pair  $B', B''$  is the product  $C = BA$ .
- III. A) a) Let  $V$  be a finite dimensional vector space over the field  $\mathbb{F}$  and  $W$  a subspace of  $V$ . Then prove that  $\dim W + \dim W^\circ = \dim V$ .
- b) Let  $V$  be a finite dimensional vector space over the field  $\mathbb{F}$ . Show that  $T \rightarrow T^*$  is an isomorphism of  $L(V, V)$  onto  $L(V^*, V^*)$ .
- III. B) a) Let  $\mathbb{K}$  be a commutative ring with identity and let  $n$  be a positive integer. Prove that there exists at least one determinant function on  $\mathbb{K}^{n \times n}$ .
- b) Let  $\mathbb{K}$  be a commutative ring with identity and let  $A$  and  $B$  be  $n \times n$  matrices over  $\mathbb{K}$ . Then prove that  $\det(A B) = (\det A) (\det B)$
- IV. A) a) Let  $U$  and  $T$  be two linear operator on a vector space  $V$ . Suppose that  $UT = TU$ . Then prove that both the range and null space of  $U$  are invariant under  $T$ .
- b) State primary decomposition theorem.
- c) Let  $V$  be a finite dimensional vector space over an algebraically closed field  $\mathbb{F}$ . Then prove that every linear operator  $T$  on  $V$  can be written as the sum of a diagonalizable operator  $D$  and a nilpotent operator  $N$ .



- IV. B) a) State and prove Cayley-Hamilton theorem for operators as a finite dimensional vector space.
- b) Find a projection  $E$  which projects  $\mathbb{R}^2$  onto the subspace spanned by  $(1, -1)$  along the subspace spanned by  $(1, 2)$ .
- V. A) a) Define the  $T$ -annihilator of a vector  $\alpha$  in the vector space  $V$  corresponding to the linear operator  $T$  on  $V$ .
- b) If  $U$  is a linear operator on the finite dimensional space  $W$  then prove that  $U$  has a cyclic vector if and only if there is some ordered basis for  $W$  in which  $U$  is represented by the companion matrix of the minimal polynomial for  $U$ .
- c) Let  $\mathbb{F}$  be a field and let  $B$  be an  $n \times n$  matrix over  $\mathbb{F}$ . Then prove that  $B$  is similar over the field  $\mathbb{F}$  to one and only one matrix which is in rational form.
- V. B) a) Let  $V$  be an  $n$ -dimensional vector space and let  $T$  be a linear operator on  $V$ . Suppose that  $T$  is diagonalizable.
- i) Show that  $T$  has  $n$ -characteristic values.
- ii) If  $T$  has  $n$  distinct characteristic values and if  $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$  is a basis of characteristic vectors for  $T$ . Show that  $\alpha = \alpha_1 + \dots + \alpha_n$  is a cyclic vector for  $T$ .
- b) State cyclic decomposition theorem.
-