C 33853 (Pages : 2) Name..................................

Reg. No................................

Ph.D. COURSEWORK EXAMINATION, DECEMBER 2017

Mathematics

Paper II—TOPOLOGY AND FUZZY SET THEORY

Time : Three Hours Maximum : 70 Marks

Answer **all** *questions.*
Each question carries 17½ *marks.*

UNIT I

1. A (a) Let D be a dense subset of a space X and let $Y \subset X$. Show by an example that $D \cap Y$ need not be dense in Y.

 (b) Let X be a space and $A \subset X$. Prove that int (A) is the union of all open sets contained in A. It is also the largest open subset of X contained in A.

 (c) For a subset A of X, prove that $\bar{A} = A \cup A'$.

Or

 B (a) Let (X, τ) be a topological space and $A \subset X$. Prove that A is a compact subset of X if and only if the subspace $(A, \tau/A)$ is compact.

 (b) State and prove Lebesgue covering Lemma.

 (c) Let X_1 and X_2 be connected topological spaces and $X = X_1 \times X_2$ with the product topology. Prove that X is connected.

UNIT II

2. A (a) Prove that every completely regular space is regular.

 (b) For a topological space X, prove that, if X is regular then for any $x \in X$ and any open set G containing x, there exists an open set H containing x such that $\bar{H} \subset G$.

 (c) Prove that regularity is a hereditary property.

Or

 B (a) Prove that every map from a comact space into a T_2 space is closed and the range of such a map is a quotient space of the domain.

 (b) Prove that a subset of X is a box if and only if it is the intersection of a family of walls.

 (c) Prove that if a product is non-empty, then each co-ordinate space is embeddable in it.

Turn over

UNIT III

3. A (a) Define support of a fuzzy set $\tilde{A}$ and give an example.

(b) Determine all a-level sets of the fuzzy set,

$\tilde{A} = \{(3,1),(4,.2),(5,.3),(6,.4),(7,.6),(8,.8),(10,1),(12,.8),(14,.6)\}$.

(c) Define the algebraic product of two fuzzy sets $\tilde{A}$ and $\tilde{B}$.

If $\tilde{A} = \{(3,.5),(5,1),(7,.6),\}$ and $\tilde{B} = \{(3,1),(5,.6)\}$ find the algebraic product $\tilde{A} \cdot \tilde{B}$.

Or

B (a) Define convex fuzzy set and give an example.

(b) Determine the intersection and union of the complements of the fuzzy sets,

$\tilde{A} = \{(2,.4),(3,.6),(4,.8),(5,1),(6,.8),(7,.6),(8,.4)\}$, $\tilde{B} = \{(2,.4),(4,.8),(5,1),(7,.6)\}$.

(c) Define the bounded sum of two fuzzy sets $\tilde{A}$ and $\tilde{B}$.

If $\tilde{A} = \{(4,.5),(6,1),(8,.6)\}$ and $\tilde{B} = \{(4,1),(6,.6)\}$ determine the bounded sum $\tilde{A} \oplus \tilde{B}$.

UNIT IV

4. A (a) Define the entropy of a fuzzy set $\tilde{A} = \{x, \mu_{\tilde{A}}(x)\}$ and give an example.

(b) Define the union and intersection of two fuzzy sets of type 2.

(c) Distinguish between fuzzy measure and measure of fuzziness. Illustrate with an example.

Or

B (a) Define fuzzy measure ?

(b) Let $X = \{0, 1, 2, ... 10\}$ with possibility $\prod(\{x\})$ given by :

X	...	0	1	2	3	4	5	6	7	8	9	10
$\prod(\{x\})$	...	0	0	.1	.9	.3	.2	1	.9	.8	.9	.5

Compute possibility $\prod(A)$ if $A = (1, 4, 6, 9)$.

(c) Define fuzzy number $\tilde{M}$ and give an example.

(4 × 17½ = 70 marks)

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Ph.D. COURSEWORK EXAMINATION, DECEMBER 2017

Mathematics

Paper II—GRAPH THEORY

Time : Three Hours | Maximum : 70 Marks

Answer **either** *Part A* **or** *Part B of each question.*
Each question carries 17½ *marks.*

I. (A) (a) Prove that graph isomorphism is an equivalence relation.

(b) How many non-isomorphic graphs are there on 4 vertices. Draw all non-isomorphic graphs on 4 vertices.

(c) Write a short note on Peterson graph.

Or

(B) (a) Prove that every closed odd walk contains an odd cycle.

(b) Prove that an edge of a graph is a cut edge if and only if it belongs to a cycle.

(c) State and prove Turan's theorem.

II. (A) State and prove Hall's matching condition.

Or

(B) (a) Prove that if G has no isolated vertices then $\alpha'(G) + \beta'(G) = n(G)$.

(b) State and prove Tutte's 1-factor theorem.

III. (A) State and prove Perfect Graph Theorem.

Or

(B) Show that every triangulated graph is perfect.

IV. (A) In an undirected graph G, prove that the following conditions are equivalent.

(i) G is a split graph.

(ii) G and its complement are triangulated.

(iii) G contains no induced subgraph isomorphic to $2K_2$, C_4 or C_5.

Or

(B) (a) Show that if G is a permutation graph then so is the complement of G.

(b) Let π be a permutation of the numbers $\{1, 2, \ldots, n\}$. Prove that the canonical coloring of $G[\pi]$, as produced by canonical sorting strategy algorithm is a minimum coloring.

(4 × 17½ = 70 marks)

C 33851 (Pages : 2) Name..................................

Reg. No..............................

Ph.D. COURSEWORK EXAMINATION, DECEMBER 2017

Mathematics

Paper II—ADVANCED FLUID DYNAMICS

Time : Three Hours Maximum : 70 Marks

Answer **either** *Part A* **or** *Part B of each question.*
Each question carries 17½ *marks.*

Unit I

1. (A) (a) Define the coefficient of viscosity of a fluid. Derive the Newton's law of viscosity.

   (b) Show that the most general motion of a fluid element consists of translation, rotation and deformation.

Or

   (B) (a) Explain what is coefficient of thermal conductivity and write down the dimension of coefficient of thermal conductivity.

   (b) Derive the Navier- Stoke's equations for the motion of a viscous fluid.

Unit II

2. (A) What is dynamical similarity ? What are the conditions under which the fluid motions are dynamically similar. Explain.

Or

   (B) Differentiate between Reynolds number, Froude number and Mach number. What are their physical importance in the dynamics of viscous compressible fluids ?

Unit III

3. (A) (a) What are the important non-dimensional coefficients in the dynamics of viscous fluids ?

   (b) Explain the phenomenon of boundary layer separation. What are the factors determining the motion in the boundary layer ?

Or

   (B) (a) Explain steady incompressible flow with constant fluid properties.

   (b) Explain the process of transpiration cooling.

Turn over

Unit IV

[redacted]. (A) (a) What are nano fluids ? What are the advantages of nano fluids over conventional fluids ?

(b) What are the key factors in increasing the heat transfer coefficient in nano fluids ? Explain.

Or

(B) (a) Explain a method used for measuring the thermal conductivity of Nano fluids.

(b) Brief the concepts of Density of nano fluids and specific heat of nano fluids.

($4 \times 17\frac{1}{2} = 70$ marks)

C 33770 (Pages : 3) Name..............................

Reg. No..............................

Ph.D. COURSEWORK EXAMINATION, DECEMBER 2017

Mathematics

Paper I—RESEARCH METHODOLOGY

Time : Three Hours Maximum : 70 Marks

Answer **all** *questions.*
Each question carries 17½ *marks.*

UNIT I

1. A. (a) Explain the meaning of the LATEX command.

\document class [12pt, a4 paper] {article}.

(b) Write down a LATEX source code to produce the following symbols : $\equiv, \geq, \ni, \infty$.

(c) Write down a LATEX source code to produce the following : $\lim_{x \to 0} \frac{\sin x}{x} = 1$.

(d) Write down a LATEX source code to produce the following : $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

(e) Write down a LATEX source code to produce the following : $\frac{dy}{dx} = \sin x$.

Or

B. (a) Write down a LATEX source code to produce the following table :

SI.No	Name	Test	Assignment	Total
1	Pranav	20	10	30
2	Gokul	19	9	18
3	Diya	18	8	26
4	Neethu	17	7	24

(b) Write down a LATEX source code to produce $x = \frac{y + \frac{z}{2}}{x^2 + 1}$.

(c) Write down LATEX source code to produce the following :

$$\begin{aligned} F(X) &= 1, x \geq 0 \\ &= 0, x < 0 \end{aligned}$$

Turn over

Unit II

2\. A. (a) Write down LATEX source code to produce the following :

$$\begin{array}{ccc} a+b+c & uv & 5 \\ ab & u-v & 10 \\ a & u+v & 500 \end{array}$$

(b) Explain LATEX source code to produce enumerated lists with an example ?

(c) Write down LATEX source code to produce roman, italic, boldface and slanted type faces.

Or

B. (a) Write down LATEX source code to produce the following :

Month	Jan
	Feb
	March
	April
Year	2013
	2014
	2015
	2016

(b) Write down LATEX source code to get following environments with examples ?

1. Flushleft and flushright.
2. Description.
3. Displaymath.
4. Eqnarray.

(c) Write a short note on different page styles in LATEX.

Unit III

3\. A. (a) Show that for each natural number n, the finite cardinal n is the cardinal of the set of natural numbers which precede n in the natural ordering ?

(b) Prove that for every set $A, A < P(A)$.

(c) Show that a subset of a countable set is countable.

Or

B. (a) Show that if A is a well ordered set and f is an isomorphism of A into itself, then $a \leq f(a)$ for each a in A.

(b) Prove that the axiom of choice is equivalent to the assertion that any two cardinal numbers are comparable ?

(c) State Zorn's Lemma. Also write any two statements equivalent to the Zorn's lemma ?

UNIT IV

4. A. (a) Construct the truth table for the statement $(P \to Q) \leftrightarrow \neg P \vee Q$.

(b) When do we say that a theory is deductively complete, formally complete and negation complete ?

(c) Justify using laws of tautological conditionals and bi conditionals that $A \to \neg C$ is a consequence of $\neg A \vee B, C \to \neg B$

Or

B. (a) When are a set of axioms dependent and independent ?

(b) Verify whether $P \wedge (P \to Q) \to Q$ is a tautology.

(c) If truth values of P, Q, R and S are T, F, F and T respectively, find truth value of $P \vee Q \to (R \leftrightarrow \neg S)$.

(4 × 17½ = 70 marks)

C 25430 (Pages : 3) Name..

Reg. No......................................

Ph.D. COURSEWORK EXAMINATION, JULY 2017

Mathematics

Paper I—RESEARCH METHODOLOGY

Time : Three Hours Maximum : 70 Marks

Answer **all** *questions.*
Each question carries 17½ *marks.*

Unit 1

I. A (a) Explain the LATEX command :

\ document class [11 pt] {article}.

(b) Explain how to format the following :

1 Birkhäuser.

2 Njemač̄ka marka.

3 $(A \cup B)' \cap C$.

(c) Explain the meaning and scope of command :

\ use package

Or

B (a) Write the LATEX source code to produce the following table :

k	$Q_k(p)$
-1	Φ
0	1
1	$1+p$
2	$\frac{1}{3}\left(5p^2-2p\right)$

(b) Write down minimal structure of a LATEX document.

(c) Explain the following :

1 Low level page style interface.

2 fancyhdr.

3 truncate.

Unit 2

II. A (a) Write LATEX code to produce the following :

$$A = \begin{bmatrix} a_1^1 & a_2^1 & a_3^1 & \dots & a_n^1 \\ a_1^2 & a_2^2 & a_3^2 & \dots & a_n^2 \\ \vdots & & & & \\ a_1^m & a_2^m & a_3^m & \dots & a_n^m \end{bmatrix}.$$

(b) Write LATEX code to produce the following :

$$\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{6+\dots}}}}}$$

Or

B (a) Describe steps to create bibliography data base. Give examples.

(b) Write LATEX source code to produce the following :

Theorem 1 Every group of prime order is cyclic.

Corollary 5 For any finite group G, the order of subgroup H of G divides order of G.

Definition 7 (Zadeh 1965) A fuzzy set A in X (space of points) is characterized by its membership function $f_A : X \to [0, 1]$.

(c) Write LATEX source code to produce the following :

1 $\overline{(A \cup B)} = \bar{A} \cap \bar{B}.$

2 $\left(\prod_{i=1}^{n} Z_i\right)^c = \prod_{i=1}^{n} Z_i^c.$

Unit 3

III. A (a) Show that the relation $\leq$ well orders N :

(b) Prove or disprove :

1 $\lambda_0^1 + \lambda_0^1 = \lambda_0^1$.

2 $\lambda_0^{1\lambda_0^1} = \lambda_0^1$.

3 Any set of ordinals is well ordered.

(c) Prove that a chain is of order type w iff it is infinite and every proper segment is finite.

Or

B (a) Show that the set s (α) of all ordinals less than the ordinal α is a well ordered set of ordinal number α.

(b) Write notes on :

1 Axiom of choice.

2 Russel's paradox.

3 State and prove Tarski theorem.

Unit 4

IV. A (a) Construct truth table for the statement :

$$P \wedge Q \rightarrow (Q \wedge \neg Q \rightarrow R \wedge Q).$$

(b) Prove that $\models A \leftrightarrow B$ iff A eq B.

(c) Write the following formula as a truth function in outfix notation :

$$\neg P \rightarrow (Q \vee (R \wedge S)).$$

Or

B (a) With the help of truth tables, explain :

1 Conjunction.

2 Conditional.

3 Negation.

(b) Write notes on (1) formal compute theory ; (2) computeness and consistency of informal theories.

(c) Prove or disprove : Theory of groups is non-categorical.