C 83152

Name.......................................

Reg. No...................................

FOURTH SEMESTER M.Sc. (CCSS) DEGREE EXAMINATION, JUNE 2015

Mathematics

MAT 4E 08—GRAPH THEORY

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 8 marks.

1. Construct a graph with $k = 3$, $\lambda = 4$ and $\delta = 5$.
2. Show that connected k-regular bipartite graph is 2-connected.
3. Define k-critical graphs. Prove that a graph is 3-critical if and only if it is an odd cycle.
4. Find the chromatic polynomial of a cycle of length n.

(4 × 8 = 32 marks)

Part B

Answer **either** *A* **or** *B of each question.*
Each question carries 24 marks.

5. (A) (a) Prove that in a 2-connected graph G, any two longest cycles have at least two vertices in common.

(b) If C is any cycle of a simple block G with at least three vertices, then prove that there exists a sequence of non-separable subgraphs $C = B_0, B_1, \ldots, B_r = G$ such that B_{i+1} is an edge-disjoint union of B_i and a path P_i, where the only vertices common to B_i and P_i are the end vertices of P_i, $0 \le i \le r-1$.

(B) (a) Prove that in any network N, the value of any flow f is less than or equal to the capacity of any cut K.

(b) Determine the parameters α, α', β and β' for the Petersen graph P.

6. (A) (a) Let G be a graph with n vertices and independence number α. Prove that $\frac{n}{\alpha} \le \chi \le n - \alpha + 1$.

(b) Prove that in a critical graph G, no vertex cut is a clique.

(c) Let G be a graph and H be a subgraph of G. Prove that $\chi(H) \le \chi(G)$ and $\psi(H) \le \psi(G)$, where $\chi(G)$ is the chromatic number and $\psi(G)$ is the pseudo chromatic number of G.

(B) (a) Prove that $\chi'(K_n) = \begin{cases} n-1 & \text{if } n \text{ is even} \\ n & \text{if } n \text{ is odd,} \end{cases}$ where K_n is the complete graph on n vertices and χ' is the edge chromatic number.

(b) Prove that a simple graph G on n vertices is a tree if and only if its chromatic polynomial is $\lambda(\lambda - 1)^{n-1}$.

(2 × 24 = 48 marks)

C 83151 (Pages : 2) Name..................................

Reg. No..................................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2015

(CCSS)

Mathematics

MAT 4E 07—ALGEBRAIC TOPOLOGY

Time : Three Hours Maximum : 80 Marks

Part A

Answer any **all** *questions.*
Each question carries 4 *marks.*

1. How many faced does an n-simplex have ? Justify your answer.
2. Define an oriented n-simplex and an oriented geometric complex.
3. Compute the o-dimensional homology group, $H_0(K)$, for the complex K consisting of all proper faces of a 2-simplex $\langle a_0 a_1 a_2 \rangle$ with orientation induced by the ordering $a_0 < a_1 < a_2$.
4. Prove that there are only five regular simple polyhedra.
5. Prove that every simplicial mapping $\phi : |K| \to |L|$ is continuous.
6. Prove that the mesh of a complex K of positive dimension is the maximum length of its 1-simplexes.
7. Prove that "equivalence of loops" is an equivalence relation on the set of loops in a space X with base point x_0.
8. Prove that the fundamental group of the punctured plane $\mathbb{R}^2\{p\}$ is isomorphic to the group $\mathbb{Z}$ of integers under addition.

(8 × 4 = 32 marks)

Part B

Answer **either** *A* **or** B *of each question.*
Each question carries 16 *marks.*

9. A (a) Prove that a simplex σ is the smallest convex set which contains all vertices of σ.

(b) Let K be an oriented complex, σ^p an oriented p-simplex of K and σ^{p-2} a (p-2) face of σ^p. Prove that

$$\Sigma\left[\sigma^p, \sigma^{p-1}\right]\left[\sigma^{p-1}, \sigma^{p-2}\right] = 0,\ \sigma^{p-1} \in K.$$

Turn over

B (a) Prove that homology grouops of a complex K are independent of the choice of orientation for its simplexes.

(b) Let K be a complex with r combinatorial components. Prove that $H_0(K)$ is isomorphic to the direct sum of r copies of the group Z of integers.

10. A (a) State and prove that Euler-Poincare theorem.

(b) Let K be a pseudomanifold with α_0 vertices, α_1 1-simplexes and α_2 2-simplexes. Prove that (i) $\alpha_1 = 3(\alpha_0 - \chi(K))$ and (ii) $\alpha_0 \geq \frac{1}{2}\left(7 + \sqrt{49 - 24\chi(K)}\right)$ where $\chi(K)$ is the Euler characteristic of K.

B (a) Let $|K|$ and $|L|$ be polyhedra with triangulations K and L respectively and $f: |K| \to |L|$ a continuous function such that K is star related to L relative to f. Prove that f has a simplicial approximation $g: |K| \to |L|$.

(b) If two continuous maps $f, g: S^n \to S^n$ are homotopic, prove that they have the same degree.

11. A (a) Suppose that loops $\alpha, \alpha^1, \beta, \beta^1$ in a space X have base point x_0 and satisfy the relations $\alpha \sim_{x_0} \alpha'$ and $\beta \sim_{x_0} \beta'$. Show that $\alpha * \beta \sim_{x_0} \alpha' * \beta'$.

(b) If a space X is path connected and x_0 and x_1 are points in X, prove that the fundamental groups $\pi_1(X, x_0)$ and $\pi_1(X, x_1)$ are isomorphic.

B (a) Prove that two loops α and β in S' with base point 1 are equivalent if and only if they have the same degree.

(b) Let X be a space for which there is an open cover $\{V_i\}$ of X such that $\cap V_i \neq \phi$, each V_i is simply connected and for $i \neq j$, $V_i \cap V_j$ is path connected. Prove that X is simply connected.

(3 × 16 = 48 marks)

C 83150 (Pages : 3) Name..............................

Reg. No..............................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2015

(CCSS)

Mathematics

MAT 4E 06—FUNCTIONAL ANALYSIS—II

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 4 *marks.*

1. Show that any two comparable complete norms on a linear space are equivalent.
2. Let Y be a subspace of a normed space X. For $x' \in X'$, let $F(x') = x'/y$. Show that F is a surjective linear map from X' to Y' such that $\|F(x')\| \leq \|x'\|$ for all $x' \varepsilon X'$.
3. Let M = diag $(k_1, k_2, \ldots)$ be a diagonal matrix and X be the sequence space l^2. Show that if $k_n \to 0$ as $n \to \infty$, then M defines a compact operator on X.
4. Show that among all the normed spaces $L^p([0, 1])$, $1 \leq p \leq \infty$, only the space $L^2([0, 1])$ is a Hilbert space.
5. Let $\{u_\alpha\}$ be an orthonormal set in an inner product space X and $x \varepsilon X$. Let $E_x = \{u_\alpha : \langle x, u_\alpha \rangle \neq 0\}$. Show that E_x is countable.
6. Let H be a Hilbert space and A ε BL (H). Show that A is injective if and only if R (A*) is dense in H.
7. Let He be a Hilbert space and A, B ε BL (H) with A self-adjoint. Show that AB = 0 if and only if $R(A) \perp R(B)$.
8. Let H be a finite dimensional Hilbert space over K and A ε BL (H). Show that if there is an orthonormal basis for H consisting of eigenvectors of A, then A is a normal operator on H.

(8 × 4 = 32 marks)

Part B

Answer **either** *A* **or** B *of each question.*
Each question carries 16 *marks.*

9. A (i) Let X be a Banach space, A ε BL (X) and $\|A^p\| < 1$ for some positive integer p. Show that the bounded operator I-A is invertible.

Turn over

(ii) Let X be the sequence space l^2 and $A : X \to X$ be defined by $A(x) = \left(0, x(1), \frac{x(2)}{2}, \frac{x(3)}{3}, \ldots\right)$ for all $x = (x(1), x(2), \ldots) \varepsilon X$. Determine $\sigma_e(A), \sigma_a(A)$ and $\sigma(A)$.

B Let $1 \le p \le \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. Show that the dual of K^n with the norm $\| \|_p$ is linearly isometric to K^n with the norm $\| \|_q$.

10. A (i) Let X and Y be normed spaces and $F : X \to Y$ be linear. Show that F is a compact map if and only if for every bounded sequence (x_n) in X, $(F(x_n))$ has a subsequence which converges in Y.

(ii) Let X and Y be Banach spaces and $F \varepsilon BL(X, Y)$ show that $F \varepsilon CL(X, Y)$ iff $F' \varepsilon CL(Y', X')$.

B (i) Let X be a normed space and $A \varepsilon CL(X)$. Show that every non-zero spectral value of A is an eigenvalue of A.

(ii) Let X be a normed space and $A \varepsilon CL(X)$. Show that every eigenspace of A corresponding to a non-zero eigenvalue of A is finite dimensional.

11. A (i) Let $\{x_1, x_2, \ldots\}$ be a linearly independent subset of an inner product space X. Define

$y_1 = x_1, u_1 = \frac{y_1}{\|y_1\|}$ and for $n = 2, 3, \ldots,$

$$y_n = x_n - \langle x_n, u_1 \rangle u_1 - \ldots - \langle x_n, u_{n-1} \rangle u_{n-1}, u_n = \frac{y_n}{\|y_n\|}.$$

Show that $\{u_1, u_2, \ldots\}$ is an orthonormal set in X and for $n = 1, 2, \ldots,$ span $\{u_1, u_2, \ldots\} = \text{span}\{x_1, x_2, \ldots\}$.

(ii) Show that $\left\{u_n(t) = \frac{e^{int}}{\sqrt{2\pi}} : n = 0, \pm 1, \pm 2, \ldots\right\}$ is an orthonormal basis for the Hilbert space $H = L^2([-\pi, \pi])$.

B (i) State and prove projection theorem.

(ii) Let H be a Hilbert space, G a subspace of H and $g \varepsilon G'$. Show that there is a unique $f \varepsilon H'$ such that $f / G = g$ and $\|f\| = \|g\|$.

12. A (i) Let H be a Hilbert space and $A \varepsilon BL(H)$. Show that A is unitary if and only if $\|A(x)\| = \|x\|$ for all $x \varepsilon H$ and A is surjective.

(ii) Let H be a non-zero Hilbert space and A ε BL (H) be self-adjoint. Show that

$$\{m_A, M_A\} \subset \sigma_a(A) = \sigma(A) \subset [m_A, M_A].$$

B (i) Let H be a Hilbert space and A ε BL (H) be compact. Show that A* is compact.

(ii) Let A be a compact self-adjoint operator on a non-zero Hilbert space H. Show that there is some $x_0 \varepsilon H$ with $\|x_0\| = 1$ such that

$$|\langle A(x_0), x_0 \rangle| = \sup\{|\langle A(x), x \rangle| : x \varepsilon H, \|x\| = 1\}.$$

(3 × 16 = 48 marks)

C 62487

Name.................................

Reg. No.............................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2014

Mathematics

MAT 4E 08—GRAPH THEORY

Time : One Hour and a Half — Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 8 *marks.*

1. Let H be a subgraph of a graph G. Is k (H) $\le k$ (G) ? Justify your answer.
2. Show that the Cylindrical edge connectivity of the Petersen graph is 2.
3. If a graph G is k-critical, then prove that δ (G) $\ge k - 1$.
4. Find a graph G whose chromatic polynomial is $\lambda^5 - 6\lambda^4 + 11\lambda^3 - 6\lambda^2$.

(4 × 8 = 32 marks)

Part B

Answer (A) **or** (B) *of each question.*
Each question carries 24 *marks*

5. (A) (i) Prove that the connectivity and edge connectivity of a simple cubic graph G are equal.
   (ii) Prove that a connected simple graph G is 3-edge connected if and only if every edge of G is the intersection of the edge sets of two cycles of G.

   (B) (i) Prove that a graph G with at least three vertices is 2-connected if and only if any two vertices of G lie on a common cycle.
   (ii) Prove that in a network N with source s and sink t, the maximum value of a flow from s to t is equal to the minimum value of the capacities of all the cuts in N.

6. (A) (i) Prove that in a critical graph G, no vertex cut is a clique.
   (ii) If G is a loopless bipartite graph, then prove that $\chi'(G) = \Delta(G)$.

   (B) Prove that for every positive integer k, there exists a triangle-free graph with chromatic number k.

(2 × 24 = 48 marks)

C 62486 (Pages : 2) Name.....................................

Reg. No..................................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2014

(CCSS)

Mathematics

MAT 4E 07—ALGEBRAIC TOPOLOGY

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 4 *marks.*

1. Define a triangulable space and give an example.
2. Show that the barycentric co-ordinates of each point in a simplex are unique.
3. Prove that, for the boundary operator ∂, the composition $\partial\partial : C_p(K) \to C_{p-2}(K)$ is the trivial homomorphism, where K is an oriented complex and $p \geq 2$.
4. Prove that there are only five regular simple polyhedra.
5. State and prove the Brouwer fixed point theorem.
6. For $m \neq n$, prove that S^m and S^n are not homeomorphic.
7. Let X be a topological space and $x_0 \in X$. Define the fundamental group of X at x_0.
8. Prove that every contractible space is simply connected.

(8 × 4 = 32 marks)

Part B

Answer **either** *A* **or** *B of each question.*
Each question carries 16 *marks.*

9. A (a) Prove that a set $\{a_0, a_1, a_2, \ldots, a_k\}$ of points in R^n is geometrically independent iff the set of vectors $\{a_1 - a_0, a_2 - a_0, \ldots, a_k - a_0\}$ is linearly independent.

(b) Let K be an oriented complex, σ^p an oriented p-simplex of K and σ^{p-2}, a $(p-2)$, face of σ^p. Prove that :

$$\sum\left[\sigma^p, \sigma^{p-1}\right]\left[\sigma^{p-1}, \sigma^{p-2}\right] = 0, \sigma^{p-1} \in K.$$

Turn over

B (a) Prove that the homology groups of a complex are independent of the choice of orientation for its simplexes.

(b) Compute all homology groups of the complex K where K is the closure of the 2-simplex $< a_0 \, a_1 \, a_2 >$ with orientation induced by $a_0 < a_1 < a_2$.

10. A (a) State and prove Euler-Poincare theorem.

(b) Prove that an n-pseudomanifold K is orientable if and only if the n^{th} homology group H_n (K) is not the trivial group.

B (a) For any complex K, prove that $\lim_{s \to \infty}$ mesh $K^{(s)} = 0$.

(b) Define a vector field on S^n. Show that there is a vector field on S^n, $n \geq 1$ if and only if n is odd.

11. A (a) Show that for each homotopy class $[\alpha]$ in $\pi_1(X, x_0)$, the inverse of $[\alpha]$ with respect to the multiplication ' 0 ' and the identity element [c] is the class $[\bar{\alpha}]$ where $\bar{\alpha}(t) = \alpha(1-t), t \in I$.

(b) Let X be a space and let α, β, γ and δ be loops with base point x_0 in X. Exhibit a homotopy which shows that $(\alpha * \beta) * (\gamma * \delta) \sim x_0 (\alpha * (\beta * \gamma) * \delta)$.

B (a) Define the degree of a loop in S'. Prove that two loops α and β in S' with base point I are equivalent iff they have the same degree.

(b) If A is a deformation retract of a space X and x_0 is a point of A, prove that $\pi_1(X, x_0)$ is isomorphic to $\pi_1(A, x_0)$.

(3 × 16 = 48 marks)

C 43791

(Pages : 2)

Name..

Reg. No......................................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JULY 2013

(CCSS)

Mathematics

MAT 4E 08—GRAPH THEORY

(2009 Admissions)

Time : One Hour and a Half

Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 8 *marks.*

1. Let G be a graph, L(G) be its line graph and let $k(G) = k$. Is $k(L(G)) = k$? Justify your answer.

2. Let $b(v)$ denote the number of blocks of a simple connected graph G to which a vertex v belongs. Prove that the number of blocks $b(G)$ of G is given by :

$$b(G) = 1 + \sum_{v \in V(G)} (b(v) - 1).$$

3. Prove that in a critical graph G, no vertex cut is a clique.

4. Prove that the chromatic polynomial of a wheel with n vertices is $\lambda(\lambda-2)^n + (-1)^n \lambda(\lambda-2)$.

(4 × 8 = 32 marks)

Part B

Answer A **or** *B of each question.*
Each question carries 24 *marks.*

5. A. (i) Prove that a graph G with atleast three vertices is 2-connected if and only if any two vertices of G are connected by atleast two internally disjoint paths.

(ii) Prove that a connected simple graph G is 3-edge connected if and only if every edge of G is the intersection of the edge sets of two cycles of G.

B. (i) Prove that in any network N, the value of any flow f is less than or equal to the capacity of any cut K.

(ii) Determine the values of the parameters α, α', β and β' for the Peterson graph.

Turn over

6. A. (i) Define critical graphs. Prove that a critical graph is connected.

(ii) If a connected graph G is neither an odd cycle nor a complete graph, then prove that $\chi(G) \leq \Delta(G)$.

B. (i) Let G be a loopless bipartite graph. Prove that $\chi'(G) = \chi(G)$.

(ii) Let G be a simple graph of order n and size m. Prove that :

1 $f(G;\lambda)$ is a monic polynomial of degree n in λ with integer coefficients and constant term zero.

2 Coefficient of $f(G;\lambda)$ are alternate in sign and the coefficient of λ^{n-1} is $-m$.

(2 × 24 = 48 marks)

C 43789 **(Pages : 3)**

Name.......................................

Reg. No...................................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JULY 2013

(CCSS)

Mathematics

MAT 4E 06—FUNCTIONAL ANALYSIS—II

(2009 Admissions)

Time : Three Hours

Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 4 marks.

1. Let X denote the sequence space l^1. Let $\| \|'$ be a complete norm on X such that if $\|x_n - x\|' \to 0$ then $x_n^{(j)} \to x(j)$ for every $j = 1, 2, \ldots$ show that $\| \|'$ is equivalent to the usual norm $\| \|_1$ on X.

2. Let X be a normed space and $A \varepsilon BL(X)$. Show that A is invertible iff A is bounded below and surjective.

3. Show that every eigen space of a compact operator on a normed space X corresponding to a non-zero eigenvalue of A is infinite dimensional.

4. Let $\langle , \rangle$ be an inner product on a linear space X. Show that for all $x, y \,\varepsilon\, X, |\langle x, y\rangle|^2 \leq \langle x, x\rangle \langle y, y\rangle$, where equality holds iff the set $\{x, y\}$ is linearly dependent.

5. Let $\{x_1, x_2, \ldots\}$ be an orthogonal set in an inner product space X and $k_1, k_2, \ldots$ be scalars having absolute value 1. Show that

$$\|k_1 x_1 + k_2 x_2 + \ldots + k_n x_n\| = \|x_1 + x_2 + \ldots + x_n\|.$$

6. Let X be an inner product space and $f \,\varepsilon\, X'$. Let $\{u_1, u_2, \ldots\}$ be an orthonormal set in X. Show that $\sum_n |f(u_n)|^2 \leq \|f\|^2$.

7. Let H be a Hilbert space and $A \,\varepsilon\, BL(H)$. Show that $\|A^*\| = \|A\|$ and $\|A^*A\| = \|A\|^2 = \|AA^*\|$.

8. Let $A \,\varepsilon\, BL(H)$ be self-adjoint. Show that $A^2 \geq 0$ and $A \leq \|A\| \cdot I$.

(8 × 4 = 32 marks)

Turn over

Part B

Answer A **or** *B of each question.*
Each question carries 12 *marks.*

9. A. (i) Let X be a normed space and $A \varepsilon BL(X)$ be of finite rank. Show that $\sigma_e(A) = \sigma_a(A) = \sigma(A)$.

(ii) Let X denote the sequence space l^2. Let $A : X \to X$ be defined by

$A(x) = \left(0, x(1), \frac{x(2)}{2}, \frac{x(3)}{3}, \ldots\right)$ for $x = (x(1), x(2), \ldots) \varepsilon X$. Determine $\sigma_e(A), \sigma_a(A)$ and $\sigma(A)$.

B. (i) Let $1 \leq p < \alpha$ and $\frac{1}{p} + \frac{1}{q} = 1$. For a fixed $y \varepsilon l^q$, let $f_y(x) = \sum_{j=1}^{\alpha} x(j)\, y(j), x \varepsilon l^p$. Show that $f_y \varepsilon (l^p)', \|f_y\| = \|y\|_q$ and the map $F : l^q \to (l^p)'$ defined by $F(y) = f_y$; $y \varepsilon l^q$ is a linear isometry from l^q onto $(l^p)'$.

(ii) Show that if X is a finite dimensional normed space, then its dual X' has the same dimension as X.

10. A. (i) Let X and Y be normed spaces and $F : X \to Y$ be linear. Show that F is a compact map iff for every bounded sequence (x_n) in X, $(F(x_n))$ has a subsequence which converges in Y.

(ii) Let X and Y be normed spaces and $F \varepsilon BL(X, Y)$. Define the Transpose of F and show that if F is compact then the Transpose of F is also compact.

B. (i) Let X be a linear space, $A : X \to X$ linear and $A(x_n) = k_n x_n$ for some $0 \neq x_n \varepsilon X$ and $k_n \varepsilon k$ with $k_n \neq k_m$ whenever $n \neq m ; n = 1, 2, \ldots$. Show that $\{x_1, x_2, \ldots\}$ is linearly independent subset of X.

(ii) Show that every inner product space is a normed space.

11. A. (i) State and prove Bessel's inequality.

(ii) Let $\{u_\alpha\}$ be an orthonormal set in a Hilbert space H. Show that $\{u_\alpha\}$ is an orthonormal basis for H iff span $\{u_\alpha\}$ is dense in H.

B. (i) Let H be a Hilbert space, G be a subspace of H and $g \varepsilon G'$. Show that there is a unique $f \varepsilon H'$ such that $f / G = g$ and $\|f\| = \|g\|$.

(ii) Let H be a Hilbert space and $A \varepsilon BL(H)$. Show that there is a unique $B \varepsilon BL(H)$ such that for all $x, y \varepsilon H$, $\langle A(x), y \rangle = \langle x, B(y) \rangle$.

A. (i) Let H be a Hilbert space and $A \varepsilon BL(H)$. Show that A is unitary iff $\|A(x)\| = \|x\|$ for all $x \varepsilon H$ and A is surjective.

(ii) Let A be a self-adjoint operator on a finite dimensional Hilbert space H. Show that every root of the characteristic polynomial of A is real.

B. (i) Let $A \varepsilon BL(H)$ be compact. Show that A^* is compact.

(ii) Let $A \varepsilon BL(H)$. Show that A is compact iff A^*A is compact.

(4 × 12 = 48 marks)

C 27532-A (Pages : 2) Name..................................

Reg. No..................................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, MAY 2012

(CCSS)

Mathematics

MAT 4E 08—GRAPH THEORY

(2010 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 8 marks.

1 (a) Show that a connected k-regular bipartite graph is 2-connected.

(b) Determine $\lambda(K_n)$.

2 (a) Define maximal independent set and give an example of it.

(b) Let V be the vertex set of a graph G. Prove that a subset S of V is independent if and only if V–S is a covering of G.

3 Prove that for a graph G with n vertices and independence number α, $\frac{n}{\alpha} \le \chi \le n - \alpha + 1$.

4 Compute $f(C_4 ; \lambda)$.

(4 × 8 = 32 marks)

Part B

Answer A **or** *B of each question.*
Each question carries 24 marks.

5. A (a) Let G be a loopless graph. Prove that $\kappa(G) \le \lambda(G) \le \delta(G)$.

(b) Let x and y be a two vertices of a graph G. Prove that the maximum number of edge-disjoint (x, y)-paths in G is equal to the minimum number of edges of G whose deletion destroys all (x, y) paths in G.

Or

B (a) If C is any cycle of a simple block G with at lest three vertices, then prove that there exists a sequence of nonseparable subgraphs $C = B_0, B_1, \ldots, B_r = G$ such that B_{i+1} is an edge-disjoint union of B_i and a path P_i, where the only vertices common to B_i and P_i are end vertices of P_i $0 \le i \le r - 1$.

(b) Prove that for any graph G with $\delta > 0$, $\alpha' + \beta' = n$.

Turn over

6. A (a) Prove that $\chi(G) = 2$ if and only if G is a bipartite graph with at least one edge.

(b) Prove that Petersen graph P is the smallest snark and it is the unique snark on 10 vertices.

Or

B. Prove that for any simple graph $G, \Delta(G) \leq \chi'(G) \leq 1 + \Delta(G)$.

(2 × 24 = 48 marks)