D 92463 (Pages : 2) Name.......................................

Reg. No...................................

Ph.D. PRELIMINARY QUALIFYING EXAMINATION
JANUARY 2016

Paper II—Elective—Mathematics

METRIC SPACES AND FUZZY SET THEORY

Time : Three Hours Maximum : 70 Marks

Answer **either** *Part A* **or** *Part B of each question.*
Each question carries 17½ marks.

Unit I

1. (A) (a) Define a metric space. If $X = R^2$ and for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$, verify whether (X, d_1) and (X, d_2) are metric space where $d_1 = | x_1 - y_1 | + | x_2 - y_2 |$ and $d_2 = \max(| x_1 - y_1 |, | x_2 - y_2 |)$.

(b) Show that the function $f : R \to (-1, -1)$ defined by $f(x) - \frac{x}{1+|x|} \; \forall \, x \in R$ is a homeomorphism. Also show that this function is uniformly continuous on R.

Or

(B) (a) State and prove Cantor's intersection theorem.

(b) Define : (i) Separable metric space ; and (ii) Second countable metric space. Prove that a metric space (X, d) is separable if and only if it is second countable.

Unit II

2. (A) (a) Prove that a subset A of the usual Metric space R is connected if and only if it is an interval.

(b) Prove that a metric space (X, d) is disconnected if and only if there exists a continuous mapping from (X, d) on to the discrete two element space (X_0, d_0).

Or

(B) State and prove Banach Contraction Theorem.

Turn over

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Unit III

3. (A) Define a fuzzy number. Give an example of a fuzzy number. If M (1/2/3) and N (2/3/4) are two triangular fuzzy numbers, find $M * N$ where $* \in \{ +, -, \text{Max}, \text{Min} \}$.

Or

(B) (a) Define a fuzzy set. Give one example. If A and B are any two fuzzy sets defined by the membership functions A (x) and B (x), check whether the following operations satisfy the De Morgan's laws :

$(A \cup B)(x) = \min(1, A(x) + B(x)), (A \cap B)(x) = \max(0, A(x) + B(x) - 1)$ and

$\bar{A}(x) = 1 - A(x)$. Justify your answer.

(b) Define a fuzzy relation and a fuzzy equivalence relation with one example each. Describe fuzzy logic mathematically.

Unit IV

4. (A) (a) Describe two applications of fuzzy logic.

(b) Explain the term fuzzy logic rule base.

Or

(B) (a) Define static fuzzy system and explain its modelling.

(b) Explain the terms SISO and MIMO.

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D 92462 **(Pages : 2)** **Name..................................**

Reg. No..............................

Ph.D. PRELIMINARY QUALIFYING EXAMINATION, JANUARY 2016

Mathematics

Paper II—THEORY OF FUZZY SETS AND GRAPHS

Time : Three Hours Maximum : 70 Marks

Answer **either** *part A* **or** *part B of each question.*
Each question carries 17½ *marks.*

Unit I

1. A (a) Define fuzzy set and give a summary of general features of fuzzy membership functions.

(b) Write short notes on Interval Arithmetic techniques.

Or

B If ρ and w are fuzzy relations on a fuzzy subset of μ of S. Then show that :

(a) If ρ and w are symmetric, then $\rho \circ \bar{w}$ is symmetric if and only if $\rho \circ w = w \circ \rho$.

(b) If ρ is symmetric, then So is every power of ρ.

(c) If ρ is symmetric, then ρ^t is a symmetric relation on μ^t for all $t \in [0, 1]$.

Unit II

2. A (a) In any graph G show that the number of vertices of odd degree is even.

(b) Prove that in a connected graph G with atleast 3 vertices, any two longest paths have a vertex in common.

(c) If G is a line graph, show that $K_{1,3}$ is a forbidden subgraph of G.

Or

B (a) Show that the line graph of the star $K_{1,n}$ is the compute graph K_n.

(b) Prove or disprove :

(i) For any two graphs, $G_1 \times G_2 \cong G_2 \times G_1$.

(ii) Lexicographic product $G_1[G_2] \cong G_2[G_1]$.

(iii) Any triangulated graph is perfect.

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Unit III

3. A (a) Define a fuzzy tree. Show that (μ,ρ) is a fuzzy cycle if and only if (μ,ρ) is not a fuzzy tree.

(b) Prove or disprove :

(i) If (u,v) is a bridge then $\rho^{\infty}(u,v)=\rho(u,v)$.

(ii) If $G=(\mu,\rho)$ is a fuzzy tree then G is not compute.

(iii) A compute fuzzy graph cannot be a bridge.

Or

B (a) If (μ,ρ) is a partial fuzzy subgraph G then show that :

(i) If (L,γ) is an partial fuzzy subgraph of I(*s*).

(ii) $(\mu,\rho)\simeq(L,\gamma)$.

(b) Show that (μ,ρ) is a fuzzy line graph if and only if $(\text{supp}(\mu),\text{supp}(\rho))$ is a line graph and $\forall(u,v)\in\text{supp}(\rho),\rho(u,v)=\mu(u)\wedge\mu(v)$.

Unit IV

4. A State and prove Fulkerson and gross characterization for fuzzy graphs.

Or

B (a) Define a weakly connected fuzzy graph.

(b) Show that every acyclic fuzzy graph is acyclic by *t*-cuts.

(c) If G is a compute fuzzy tree then show that G is a weak fuzzy tree.

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D 92461 **(Pages : 3)** **Name.......................................**

Reg. No....................................

Ph.D. PRELIMINARY QUALIFYING EXAMINATION, JANUARY 2016

Mathematics

Paper II—THEORY OF GRAPHS AND GROUPS

Time : Three Hours Maximum : 70 Marks

Answer **either** *part A* **or** *part B of each question.*
Each question carries 17½ marks.

Unit I

I. (A) (a) Define a graph and give an example of a graph with six vertices and atleast eight edges, without using any diagram.

(1 + 3 marks)

(b) Define lexicographic product of graphs and show that this operation is not symmetric.

(1 + 3 marks)

(c) Prove that the automorphism group of a graph and its complement are same.

(3 marks)

(d) Prove that a connected simple graph G is 3-edge connected if and only if, every edge of G is the intersection of the edge set of two cycles in G.

(3 marks)

(e) If $\alpha'(G)$ and $\beta'(G)$ are the candinalities of maximum matching and minimum edge covering of a graph G with n vertices and without isolated vertices, then prove that $\alpha'(G) + \beta'(G) = n$.

(3½ marks)

Or

(B) (a) Define self-complementary graphs and obtain a necessary condition for a positive integer to be the order of a self-complementary graph.

(1 + 3 marks)

(b) Derive an expression for the maximum number of edges in a simple graph with n vertices and k components.

(1 + 3 marks)

(c) State and prove any one characterisation of a cut edge. (3 marks)

(d) Prove that any set of k vertices in a k-connected $(k \geq 2)$ graph G lie on a cycle.

(3 marks)

(e) Prove that any $(n - 1)$ regular simple graph on $2n$ vertices has a l-factor.

(3½ marks)

Turn over

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Unit II

II. (A) (a) Determine the chromatic number of the Petersen graph. (3 marks)

(b) Define critical graph and prove that any critical graph is connected. (1 + 3 marks)

(c) Prove that there exists a triangle-free k-chromatic graph for every integer $k \geq 1$. (6½ marks)

(d) Define the chromatic polynomial $f(G, \lambda)$ of a graph G and show that :

$f(G, \lambda) = f(G - e, \lambda) - f(G \cdot e, \lambda)$ for any edge e of G.

(4 marks)

Or

(B) (a) Determine the chromatic number of the wheel W_n. (3 marks)

(b) Define a vertex cut and prove that no vertex cut in a critical graph G is a clique. (1 + 3 marks)

(c) Define edge chromatic index of a graph G and determine the edge chromatic index of K_n. (1 + 5 marks)

(d) Define triangle free graphs and prove that a simple graph G having n vertices is triangle-free if, and only if, $f(G, \lambda) = \lambda(\lambda - 1)^{n-1}$.

(1 + 3½ marks)

Unit III

III. (A) (a) If N is a normal subgroup of a group G, show that the map $\gamma : G \to G/N$ defined by $\gamma(a) = aN$ for $a \in G$ is a homomorphism.

(3½ marks)

(b) Prove that a homomorphism of a cyclic group is completely determined by the value of the homomorphism on a generator of the group.

(4 marks)

(c) Define the commutator subgroup G' of a group G and prove that G / G' is abelian and also prove that G/N is abelian if, and only if $G' \leq N$.

(5 marks)

(d) Compute the factor group $(\mathbb{Z}_4 \times \mathbb{Z}_6) / \langle (0, 1) \rangle$. (5 marks)

Or

(B) (a) Prove that a homomorphism ϕ of a group G is a one-to-one function if, and only if, the kernel of ϕ is $\{e\}$, where e is the identity element in G.

(4 marks)

(b) Let G be a group and $\phi : \mathbb{Z} \times \mathbb{Z} \to G$ is a homomorphism with $\phi(1, 0) = h$ and $\phi(0, 1) = k$, find $\phi(m, n)$.

(5 marks)

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(c) Prove that a factor group of a cyclic group is cyclic. (3½ marks)

(d) If G is the internal direct product of H and K, then prove that G/H is isomorphic to K in a natural way.

(5 marks)

Unit IV

IV. (A) (a) Show that a function from a finite set S to itself is one-to-one if, and only if, it is onto.

(3 marks)

(b) Define alternating group and show that the alternating group A_n is a subgroup of the group S_n of all permutations of a set of n elements.

(1 + 3 marks)

(c) Determine the number of different 11-bead necklaces that can be made using two colours.

(4½ marks)

(d) Prove that $Cay\left(\{(0,1),(0,1)\}:\mathbb{Z}_m \oplus \mathbb{Z}_n\right)$ has a Hamiltonian circuit when n divides m.

(6 marks)

Or

(B) (a) Do the set of all odd permutations of a set of n elements form a group ? Why ?

(3 marks)

(b) Find a non-cyclic subgroup of order 4 in S_4. (4 marks)

(c) Determine the number of ways of colouring a cube with three colours. (4½ marks)

(d) Let G be a finite Abelian group and S be any non-empty generating set for G. Prove that Cay (S : G) has a Hamiltonian path.

(6 marks)

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D 92457 **(Pages : 3)** **Name.....................................**

Reg. No...................................

Ph.D. PRELIMINARY QUALIFYING EXAMINATION, JANUARY 2016

Mathematics

Paper II—TOPOLOGY, FUZZY SET THEORY AND LATTICE THEORY

Time : Three Hours Maximum : 70 Marks

Answer **either** *part (A)* **or** *part (B) of each question.*

Unit I (Topology)

I. (A) (i) If $\{T_\alpha\}$ is a family of topologies on a set X, show that $\cap T_\alpha$ is a topology on X. Is $\cup T_\alpha$ a topology on X ? Justify.

(ii) Prove or Disprove :

(a) Product of two Lindelöf space is Lindelöf.

(b) Every closed subspace of a compact space is compact.

(iii) Show that every regular space with countable basis is normal.

(B) (i) Let $Y \subset X$ and X is an ordered set with the order topology. Is the order topology of Y obtained by restricting the order relation in X to Y is the same as the topology that Y inherits as a subspace of X ? Justify.

(ii) (a) State and prove sequence lemma.

(b) Give example of a Hausdorff space with countable basis which is not metrizabe.

(iii) (a) In finite complement topology on $\mathbb{R}$, to what point or points does the sequence $\left\{X_n = \frac{1}{n}\right\}$ converge.

(b) State Urysohu lemma, Urysohu metrization theorem and intermediate value theorem.

[3 × 10 = 30 marks]

Turn over

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Unit II (Fuzzy Set Theory)

II. (A) (i) Define basic set theoretic operations for fuzzy sets and provide suitable examples.

(7 marks)

(ii) Define t-norms and t-conorms and give two examples for each. (6 marks)

(iii) Let $X = \{0, 1, 2, \ldots\ldots\ldots\ldots 10\}$ with possibility $\pi(\{x\})$ given by :

x	0	1	2	3	4	5	6	7	8	9	10
$\pi(\{x\})$	0	0	0	0	0	0.1	0.5	0.8	1	0.8	0.5

Compute possibility $\pi(A)$ for $A = \{1, 5, 8, 9\}$.

(7 marks)

VI. (B) (i) Determine α-level sets and strong α-level sets for the following fuzzy sets :

$A = \{(3, 1), (4, .2), (5, .3), (6, .4), (7, .6), (8, .8), (10, 1), (12, .18), (14, .6)\}$.

$B = \left\{(x, \mu_B(x)) ; \mu_B(x) = \left[1 + (x - 10)^2\right]^{-1}\right\}$ for $\alpha = .3, .5$ and $.8$.

(7 marks)

(ii) Distinguish between Fuzzy measure and measures of fuzziness. Illustrate with suitable examples.

(7 marks)

(iii) Determine the intersection of $B = \{(2, .4), (3, .6), (4, .8), (5, 1), (6, .8), (7, .6), (8, .4)\}$ and $C = \{(2, .4), (4, .8), (5, 1), (7, .6)\}$.

Using (a) Hamacher operator with $\gamma = 0.25$.

(b) Yager operator with $p = 10$.

(6 marks)

[20 marks]

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Unit III (Lattice Theory)

V (A) (i) (a) Prove or Disprove :

Every chain is a lattice.

(b) Give an example of a poset ϕ in which inf ϕ doesnot exist.

(ii) In a modular lattice M with 0, show that an n element set $\{a_1, a_2, ...a_n\} \subseteq M - \{0\}$ is independent iff $(a_1 \vee a_2 \vee .... \vee a_i) \wedge a_{i+1} = 0 \forall i = 1, 2, ... n.$

(B) (i) State Zorn's lemma. Also prove that if L is a distributive lattice, I is an ideal and D is the dual of L with $I \cap D = \phi$. Then $\exists$ a prime ideal P of L such that $P \geq I$ and $P \cap D = \phi$.

(ii) Show that every complete Boolean Algebra R-generated by a chain C with 0 is finite.

[2 × 10 = 20 marks]

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D 92456 **(Pages : 3)** **Name................................**

Reg. No..............................

Ph.D. PRELIMINARY QUALIFYING EXAMINATION, JANUARY 2016

Mathematics

DOMINATION IN GRAPHS

Time : Three Hours Maximum : 70 Marks

Answer **either** *(A)* **or** *(B) of each question.*
All questions carry equal marks.

I. (A) (a) Define and illustrate with one example each the terms : (i) domination number ; (ii) Connected domination number ; and (iii) independent domination number. Compute all these parameters for the Peterson graph.

(4½ + 1½ = 6 marks)

(b) Define the term 'wounded spider'. Prove that for any tree T, $r(T) = n - \Delta(T)$ if and only if T is a wounded spider. Illustrate with an example.

(1 + 3 + 1½ = 5 ½ marks)

(c) Discuss any three bounds for $r(G)$ with examples. (6 marks)

Or

(B) (a) Obtain a necessary and sufficient condition that a dominating set is a minimal dominating set. Illustrate.

(4 + 1 = 5 marks)

(b) Explain the application of domination number in social networking theory. Prove that if a r-set S of a connected graph G of order $n \geq 2$ is a status of G, then S is an independent dominating set of cardinality too. Illustrate with an example.

(3 + 4 + 1 = 8 marks)

(c) Compute r, r_c, r_t for $G = P_3 \times P_4$.

(3 × 1½ = 4½ marks)

II. (A) (a) If a graph G has $r(G) \geq 2$, prove that $m \leq \left[\frac{1}{2}(n - r(G))(n - r(G) + 2\right]$. Illustrate with an example.

(5 + 1½ = 6½ marks)

Turn over

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(b) Prove or disprove : Every maximal independent set in a graph G is a minimal dominating set of G.

(5 marks)

(c) Define the term 'irredundance number'. Prove that every minimal dominating set in a graph G is a maximal irredundant set of G.

(6 marks)

Or

(B) (a) For any graph G, prove that $n - m \le r\,(G) \le n + 1 - \sqrt{2m+1}$. Discuss the case when equality holds in the lower bound.

(6 marks)

(b) Prove that if a graph G has no isolates, $r\,(G) \le \beta_2\,(G)$. Discuss the case when $G = k_3 \circ k_1$.

(6 marks)

(c) Explain with examples, the terms 'hereditary' and 'super hereditary'. Prove that for any graph G and hereditary property p, $\alpha\,(p) + \beta\,(p) = n$.

(2½ + 3 = 5½ marks)

III. (A) (a) Explain the terms, fractional domination number and fractional irredundant number of a graph. Illustrate with examples.

(5 marks)

(b) State true *or* false. Justify :

(i) No tree is in CVR. (3 marks)

(ii) If a graph $G \in CVR$, then i (G) = (V/G).

(3 marks)

(c) Prove that S is a kernel of a digraph D, then S is both maximal independant and minimal absorbant. Illustrate with an example.

(5 + 1.5 = 6.5 marks)

Or

(B) (a) Prove : If D is a symmetric digraph, then D has a kernel and $S \le V$ is a kernel iff S is a maximal independent set.

(5 marks)

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(b) Obtain a characterization of UVR graphs. Show that $K_{r,8}$; $3 \le r \le 8$ are in UVR.

(5 + 1½ = 6½ marks)

(c) Prove that a graph $G \in CVR$ has order $n = (\Delta(G) + 1)((r(G) - 1) + 1)$ then G is regular. Illustrate.

(4 + 2 = 6 marks)

IV. (A) (a) If G is a connected graph with $n \ge 3$ vertices, prove that $r_t(G) \le \frac{2n}{3}$. Illustrate for the Petersen graph.

(5 + 2 = 7 marks)

(b) If G is a connected graph of $n \ge 3$, prove that $r_c(G) = n - E_T(G) \le n - 2$.

(5 marks)

(c) For any graph G, prove $r_2(G) \le \beta_2(G)$. Give an example when equality occurs.

(5½ marks)

Or

(B) (a) Explain the notation $r_k(G)$. Prove that $r_k(G) >, \frac{kn}{\Delta(G) + k}$.

(4 marks)

(b) If a graph G does not contain the star $K_{1,k+1}, k \ge 2$ as an induced subgraph, then $i(G) \le (k-1) r(G) - (k-2)$.

(6 marks)

(c) Prove that for a connected graph G,

$n/(\Delta(G) + 1) \le r_c(G) \le 2m - n.$

Discuss when the equality occurs.

(5 + 2½ = 7½ marks)

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D 92391 (Pages : 3) Name..

Reg. No......................................

Ph.D. PRELIMINARY QUALIFYING EXAMINATION, JANUARY 2016

Paper I—RESEARCH METHODOLOGY

Time : Three Hours Maximum : 70 Marks

Answer **all** *questions.*
Each question carries 17½ *marks.*

Unit I

1. A (a) Explain the meaning of the LaTeX command.

\documentclass[12pt,twocolumn]{article}.

(b) Write down a LaTeX source code to produce the following symbols :

‡, §, ©, Æ, Ø.

(c) Write down a LaTeX source code to produce the following :

$x \equiv y \pmod{c+d}$ *and* $\gcd(m,n) = c \bmod d$.

(d) Write down a LaTeX source code to produce the following :

$A \xrightarrow{\varphi} B \xleftarrow{\gamma} C$.

(e) Write down a LaTeX source code to produce the following :

$$\overbrace{3+\underbrace{2+\ldots+2}_{23}+3}^{25} = 2\times 23+6 = 52$$

Or

B (a) Write down a LaTeX source code to produce the following :

$\log xy = \log x + \log y$ and $\lim\limits_{n\to\infty} x_n = 0$.

Turn over

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(b) Write down a LaTeX source code to produce the following :

$$\Delta p\left(\frac{\partial \mathrm{R}}{\partial \mathrm{p}}\right)+\Delta q\left(\frac{\partial \mathrm{R}}{\partial \mathrm{q}}\right)=-\mathrm{R}(p,q).$$

(c) Write down a LaTeX source code to produce the following :

Sl. no.	Items	Rate	Quantity	Amount
1	Rice	40	10	400
2	Wheat	26	5	130
3	Coconut oil	125	4	500
4	Gingelly oil	110	3	330
			Total	1360

Unit II

2. A (a) Write down a LaTeX source code to produce the following :

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ a & b & c \\ \gamma & \beta & \delta \end{bmatrix}.$$

(b) Describe new theorem commands on LaTeX to produce theorem like objects named Lemma, Theorem, Proposition and Corollary.

(c) Explain the working of BIBTeX program.

Or

B (a) Describe basic commands for producing contents and list of tables.

(b) Explain the following commands with sufficient examples :

\cite, \nocite, \ref, \pageref, \label.

(c) Describe commands for producing Index. Give an example of an input containing items for index and specify the commands to generate the index.

Unit III

3. A (a) Prove that any two integral systems are isomorphic.

(b) If $A \preceq B$ and $B \preceq A$, then prove that $A \sim B$.

(c) Prove that the natural ordering and the cardinal ordering agree on the set $\mathbb{N}$ of natural numbers.

Or

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B (a) Prove that any set of ordinals is well-ordered.

(b) Using Zorn's lemma, prove that every set is well ordered.

(c) Prove that an infinite set includes a subset of cardinal number $\aleph_0$.

(d) Prove that the axiom of choice is equivalent to the assertion that any two cardinal numbers are comparable.

Unit IV

4. A (a) Suppose that statements P, Q, R and S are assigned the truth values T, F, F and T, respectively. Find the truth value of each of the following statements :

(i) $(P \vee Q) \vee R$.

(ii) $R \rightarrow (S \wedge P)$.

(iii) $S \leftrightarrow P \rightarrow (\neg P \vee S)$.

(iv) $R \wedge S \rightarrow (P \rightarrow \neg Q \vee S)$.

(b) Explain the following :

(i) Consistent theory.

(ii) Formal complete theory.

(c) Let A and B be formulas. Prove that $\models A \leftrightarrow B$ if and only if A is equivalent to B.

Or

B (a) Explain the following terms using truth tables :

(i) Negation.

(ii) Conjunction.

(iii) Disjunction.

(iv) Conditional.

(b) Let $A = \neg((\neg P \vee Q) \wedge (Q \wedge (R \vee \neg P)))$ be a formula and let A_d be its denial. Prove that $\models \neg A \leftrightarrow A_d$.

(c) Let A (x) be a formula which is free for y. Prove that :

(i) $\models (x)A(x) \rightarrow A(y)$.

(ii) $\models A(y) \rightarrow (\exists x)A(x)$.

(4 × 17½ = 70 marks)

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D 13976 (Pages : 2) Name..................................

Reg. No..............................

Ph.D. PRELIMINARY QUALIFYING EXAMINATION, DECEMBER 2016

Mathematics

Paper II (Elective) – THEORY OF GRAPHS AND GROUPS

Time : Three Hours Maximum : 70 Marks

Answer **either** *Part A* **or** *Part B of each question.*
Each question carries 17½ *marks.*

Unit I

I. (A) (a) Define 'Line graph'. Is every graph a line graph?
Justify your answer. Prove : The line graph of a simple graph.
G is a path if and only if G is a path.

(7½ marks)

(b) Define 'Tournament'. Draw all tournaments on 4 vertices. Prove that every tournament contains a directed hamiltonian path.

(6 marks)

(c) Show that the connectivity and edge connectivity of a simple cubic graph G are equal. Illustrate with an example.

(4 marks)

Or

(B) (a) Prove : A graph G with atleast three vertices is 2-connected if and only if any two veertices of G are connected by atleast two internally disjoint paths.

(5 marks)

(b) (i) Prove or disprove : For a simple graph G, both G and G^c cannot be connected.

(ii) Define automorphism group of a graph G. Does the exist non-isomorphic graph with the same autmorphism graph G. Justify.

(3 + 4½ = 7½ marks)

(c) Give an example of a cubic graph having no.1 factor. Justify. Is it true that every tree has atmost one perfect matching. Justify.

(5 marks)

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Unit II

II. (A) (a) Prove that for positive integer k, there is a triangle-free graph with chromatic number k.

(5 marks)

(b) Prove that $X^1(K_n) = \begin{cases} n-1 & \text{if } n \text{ is even} \\ n & \text{if } n \text{ is odd} \end{cases}$

(7½ marks)

Turn over

(c) Obtain a necessary and sufficient condition that a graph G on *n* vertices is a tree using chromatic polynomials.

(5 marks)

Or

(B) (a) Find the edge chromatic number of Petersen graph.
Prove : If G is a loopless bipartite graph X^1 (G) = Δ (G)

(7½ marks)

(b) Define a '*k*-critical graph'. Prove that every critical graph is a block.

(6 marks)

(c) Find a graph G where chromatic polynomial is $\lambda^5 - 6\lambda^4 + 11\lambda^3 - 6\lambda^2$.

(4 marks)

Unit III

III. (A) (a) Prove that every group is ismorphic to a group of permutations. Compute the left regular representation of Z_4.

(10 marks)

(b) Define the terms, 'Center' and 'Commutator subgroups'. Describe these for (i) Every Abelian group ; (ii) Every Abelian group.

(7½ marks)

Or

(B) (a) Let H be a subgroup of G. Prove : The left multiplication is well define by the equation (aH) (bH) = (ab) H if and only if aH = Ha for every a $\in$ G.

(7½ marks)

(b) State and prove fundamental homomorphism theorem.

(10 marks)

Unit IV

IV. (A) (a) State and prove Buruside's theorem.

(7½ marks)

(b) Define 'Cayley digraph of a group'. Illustrate with an example.

(5 marks)

(c) Describe the symmetrics of a square

(5 marks)

Or

(B) (a) Let G be a finite Abelian group. S be any generating set of G.
Prove : Cay (S, G) has a Hamiltonian path.

(7½ marks)

(b) Prove *or* disprove : Every permutation in S_n, n > 1 is a product of cycles.

(5 marks)

(c) Show that A_8 contains an element of only 15.

(5 marks)

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D 13914 (Pages : 2) Name.....................................

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Ph.D. PRELIMINARY QUALIFYING EXAMINATION, DECEMBER 2016

Mathematics

Paper I—RESEARCH METHODOLOGY

Time : Three Hours Maximum : 70 Marks

Answer **all** *questions. Each question carries 17.5 marks.*

Unit-1

I. A (a) Mention the various document classes in latex.

(b) Write a Minimal Structure for a latex article.

(c) Explain how to format the following four words.

(1) Häuhner-händler.

(2) naíef.

(3) débácle.

OR

B (a) Explain the command \documentclass [10pt,a4paper] {report}.

(b) Write the command for entering the text '40°C,

(c) Explain the meaning and scope of the command : \usepackage

Unit-2

II. A (a) Write the source code to produce the following table.

X	X^2
2	4
4	16
6	36

(b) Write the Latex Source Code to produce the following.

(c) How do you justify paragraph in LATEX.

OR

B (a) Describe the steps to create bibliography data base. Give three or four examples.

(b) Write the LATEX source code to produce the following :
Theorem 1 (Artin, 1927) Let a >1 be an integer that is not a square. Then, a is a primitive root modulo infinitely many prime numbers p.

(c) Give any two "description" lists in LATEX.

Turn over

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Unit-3

III A (a) Show that similarity is an equivalence relation on any collection of sets.

(b) Prove that $\alpha + 1 > \alpha$ for every ordinal α.

(c) Define Zorn's lemma and show that Zorn's lemma implies the 'Well ordering Theorem'.

OR

B (a) Explain Russel's paradox.

(b) Prove that any infinite set is similar to a proper subset of itself.

(c) State Hausdorff's maximal principle, Show that it implies Zorn's lemma.

IV A (a) Show with the help of truth tables that $P \leftrightarrow Q$ is the same as $(P \to Q) \wedge (Q \to P)$.

(b) Let x be any variable, B be any formula. Then prove that.

(i) If $B \to A(x)$, then $B \to (x) A(x)$.

(ii) If $A(x) \to B$, then $(\exists x) A(x) \to B$.

(c) Define the following :

(1) consistent theory.

(2) formal complete theory.

OR

B (a) Construct the truth table for the following statement. $P \to (P \to Q)$.

(b) Define a tautology, Show that $P \wedge (P \to Q) \to Q$ is a tautology.

(c) Let A (x) be a formula which is free for y. Prove that (x) A (x), then A (x).

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C 5393 (Pages : 2) Name..................................

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Ph.D. (PRELIMINARY QUALIFYING) EXAMINATION, JUNE 2016

Paper II—Elective

MODULAR FORMS

Time : Three Hours Maximum : 70 Marks

Answer **either** *A* **or** *B of each question.*
Each question carries equal marks.

I. A (a) Show that the group $SL_2(\mathbb{Z})$ acts on the upper half plane $\mathcal{H} = \{z \in \mathbb{C} : \mathrm{I}m\, z > 0\}$ by $\begin{pmatrix} a & b \\ c & d \end{pmatrix}.z = \frac{az+b}{cz+d}$.

(b) Show that $F = \left\{ z \in \mathcal{H} : -\frac{1}{2} \le \mathrm{Re}\, z \le \frac{1}{2}, |z| \ge 1 \right\}$ is a fundamental domain for the action of $SL_2(\mathbb{Z})$ on $\mathcal{H}$.

B (a) Find a set of left (or right) coset representatives of $\Gamma = SL_2(\mathbb{Z})$ modulo the congruence subgroup $\Gamma(2)$.

(b) Draw a fundamental domain for $\Gamma(2)$ and find a set of $\Gamma(2)$-in equivalent cusp.

II. A (a) Define a modular form of weight k for $SL_2(\mathbb{Z})$ and prove that such functions form a linear space over $\mathbb{C}$.

(b) Derive the valence formula for a modular form of weight k for $SL_2(\mathbb{Z})$.

B (a) Suppose $f(z)$ is a modular form of weight k for $SL_2(\mathbb{Z})$ and $g(z)$ is a modular form of weight l for $SL_2(\mathbb{Z})$. Prove that the product $f(z)\,g(z)$ is a modular form of weight $k+l$ for $SL_2(\mathbb{Z})$.

(b) Prove that the space of modular forms of weight k for $SL_2(\mathbb{Z})$ is finite dimensional vector space by computing its dimension through valence formula.

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III. A (a) Define the Eisenstein series of weight k for $SL_2(\mathbb{Z})$ and derive the Fourier coefficients.

(b) Define the Peterson inner product. Prove that the Eisenstein series E_k of k is orthogonal to the space of cusp forms of weight k for $SL_2(\mathbb{Z})$.

B (a) Define Ramanujan Δ- function and show that it never vanishes on the upper half plane and has a simple zero at ∞.

(b) Prove that the Ramanujan tau function $\tau(n)$ satisfies

$$\tau(n) \equiv \sigma_{11}(n) \pmod{691}.$$

IV. A (a) Define n-th Poincaré series of weight k for $SL_2(\mathbb{Z})$. Derive Fourier coefficients.

(b) Prove that the Poincaré series span the space of cusp forms of weight k for $SL_2(\mathbb{Z})$ by deriving inner product of any cusp form with n-th Poincaré series.

B (a) Define n-th Hecke operator T_n and derive Fourier expansion of $T_n(f)$, where f is a modular form of weight for $SL_2(\mathbb{Z})$.

(b) Let $f = \sum_{n=1}^{\infty} a(n)\, q^n$ be a cusp form of weight k for $SL_2(\mathbb{Z})$ which is an eigen function for all Hecke operator T_n. Show that $a(1) \neq 0$.

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C 5391

Name.....................................

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Ph.D. (PRELIMINARY QUALIFYING) EXAMINATION, JUNE 2016

Paper II—Elective—TOPICS IN OPERATOR ALGEBRAS

Time : Three Hours | Maximum : 70 Marks

Answer **either** *A* **or** *B of each quuestion.*
Each question carries equal marks.

1. (A) (i) Define Gelfand transform on a Banach Algebra $\mathscr{B}$ and establish its fundamental properties.

   (ii) Prove that there is only one complex Banach algebra which is a division algebra (upto the appropriate isomorphism).

   (B) (i) If $\mathscr{B}$ is a Banach algebra and f is in $\mathscr{B}$, then prove tha $\sigma(f)$ is non empty.

   (ii) Prove that a complex multiplicative linear functional on a Banach Algebra has unit norm. Observe the role of completeness used in proving this result.

2. (A) State and prove Stone-Weierstrass theorem. More generally give a characterization of all unital closed self adjoint subalgebras of C(X).

   (B) Obtain the additional conditions (with necessary proofs) showing that the Gelfand transform on a Banach algebra which is not necessarily self adjoint is an isometric isomorphism.

3. (A) (i) Prove Spectral theorem for normal operators.

   (ii) Define positive operator on a Hilbert space. Use spectral theorem to obtain an alternate definition for it.

   (B) Give a general (operator free) proof showing that a self adjoint element in a C* algebra has a real spectrum. Also show that the spectrum of an element in C* algebra is independent of the ambient algebra.

4. (A) Prove that $L^{\infty}(\mu)$ is a maximal abelian W* algebra in $\mathscr{L}(L^2(\mu))$. Also observe that the weak operator topology and the w^* - topology coincide on $L^{\infty}(\mu)$.

   (B) Describe weak and strong operator topologies. Show that they are all distinct from the norm topology on the space of all bounded linear operators on a Hilbert space.

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