C 61700

P.G. ENTRANCE EXAMINATION, MAY 2019

MATHEMATICS

Time : Two Hours — Maximum : 200 Marks

Instructions to Candidates

Question Paper : The Question paper contains **50 Multiple Choice Questions**. Among the four options of each question given as A, B, C and D, only one will be the most appropriate answer. Mark the bubble containing the letter corresponding to the most appropriate answer in the **OMR** answer sheet using ball point pen (Blue or Black).

Negative Marking : Total four marks will be given for each correct answer. One mark will be deducted for each wrong answer.

Symbols Used : Throughout this question paper $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ and $\mathbb{C}$ denote the set of natural numbers, the set of integers, the set of rational numbers, the set of real numbers and the set of complex numbers respectively.

Caution : Mathematical Tables and Calculators are **NOT** permitted in the Examination Hall.

C 61700 **(Pages : 8)**

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1. The value of $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$ is equal to :

   (A) $\pi\sqrt{2}$. (B) $\sqrt{2}$.

   (C) $\sqrt{\pi}$. (D) None of the above.

2. The Euler method for solving the differential equation $y' = f(x, y), y(x_0) = y_0$ is given by the recursive formula :

   (A) $y_{n+1} = y_n + hf(x_{n+1}, y_{n+1})$. (B) $y_{n+1} = y_n + f(x_n, y_n)$.

   (C) $y_{n+1} = y_n + hf(x_n, y_n)$. (D) $y_{n+1} = y_{n+1} + hf(x_n, y_n)$.

3. Which of the following will be an integrating factor of the differential equation $ty' + (t+2)y = t^3$?

   (A) te^t. (B) e^t.

   (C) t^2e^t. (D) e^{2t}.

4. Let $f(x)$ be a polynomial of degree four having extreme values at $x = 1$ and $x = 2$. If $\lim_{x \to 0}\left[1 + \frac{f(x)}{x^2}\right] = 3$, then $f(2)$ is equal to :

   (A) 0. (B) – 8.

   (C) – 4. (D) 4.

5. The value of the integral $\int_0^9 \left[\sqrt{x} + 2\right] dx$, where [ ] is the greatest integer function is :

   (A) 23. (B) 22.

   (C) 31. (D) 27.

6. The curve represented by the equation $4x^2 + 16y^2 - 24x - 32y - 12 = 0$ is :

(A) Parabola. (B) A pair of lines.

(C) A hyperbola. (D) An ellipse.

7. The initial value problem $|y'| + |y| = 0, y(0) = 0$ has :

(A) No solution. (B) Only one solution.

(C) Two solutions. (D) Infinitely many solutions.

8. The Fourier series of the function $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, & 0 < x < \pi \end{cases}$, $f(x) = f(x + 2\pi)$ contains :

(A) Only sine terms. (B) Constant term and cosine terms.

(C) Both cosine and sine terms. (D) Only cosine terms.

9. The value of the integral $\int_0^\infty \frac{x^7(1 - x^{12})}{(1 + x)^{28}} dx$ is :

(A) 1. (B) 7.

(C) 0. (D) 27.

10. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function satisfying $f(x) = 5\int_0^x f(t)\,dt + 2$ for all $x \in \mathbb{R}$.

Then $f(1)$ equals to :

(A) $5e$. (B) 5.

(C) e^5. (D) e.

11. The function $g(x) = \int_z^{x + 2\pi} \sin t\,dt$ for all $x \in \mathbb{R}$ is :

(A) Not differentiable. (B) Neither continuous nor differentiable.

(C) A continuous function. (D) continuous but not differentiable.

12. Newton's iterative method formula for computing $\sqrt{28}$ is :

(A) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{28}{x_n}\right)$. (B) $x_{n+1} = \frac{1}{2}\left(x_n - \frac{28}{x_n}\right)$.

(C) $x_{n+1} = \left(x_n + \frac{28}{x_n}\right)$. (D) None of these.

13. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=x^3$. Then which of the following is false :

(A) f is one-one. (B) f is onto.

(C) f is bijective. (D) None of the above.

14. Let A be the set of all sequences of positive integers. Then which of the following is correct :

(A) A is denumerable. (B) A is countable.

(C) A is uncountable. (D) None of the above.

15. Let A and B be infinite sets. Then which of the following is true ?

(A) $A \cap B$ is infinite. (B) $A \cup B$ is infinite.

(C) Both (A) and (B) are true. (D) None of the above.

16. Let $f: A \to B$ and $g: B \to A$ be such that $g \circ f$ is the identity function. Then which of the following is true ?

(A) g is one to one. (B) g is onto.

(C) f is onto. (D) None of the above.

17. Let X be any set and let $f: X \to X$. Then which of the following is true ?

(A) If f is one to one, then f is onto. (B) If f is onto, then f is one to one.

(C) Both (A) and (B) are true. (D) None of the above.

18. Which of the following is false ?

(A) An empty set is a subset of every set.

(B) An empty set is a subset of an empty set.

(C) Any subset of the empty set is an empty set.

(D) None of the above.

19. For statements p , q and r, we have that " p implies q" and " q implies r". Then which of the following is possible if q is false :

(A) p and r are both true. (B) p and r are both false.

(C) p is true and r is false. (D) None of the above.

20. What is the negation of the statement " there exists $x \in A$ such that for every $y \in A, y \le x$ " ?

(A) There exists $x \in A$ such that for every $y \in A, x \le y$.

(B) For every $x \in A$, there exists $y \in A, y \le x$ is false.

(C) For every $x \in A$, there exists $y \in A, y \le x$.

(D) None of the above.

21. Let (x_1, y_1) and (x_2, y_2) be two distinct vectors on a line l in $\mathbb{R}^2$. Then they are linearly independent if l is :

(A) $x = 0$. (B) $y = 0$.

(C) $x - y = 0$. (D) $y - x - 1 = 0$.

22. Let $A : \mathbb{R}^2 \to \mathbb{R}^2$ and $B : \mathbb{R}^2 \to \mathbb{R}^2$ be two linear operators on $\mathbb{R}^2$. Then rank of A + B is :

(A) equal to rank of A + rank of B.

(B) Less than or equal to rank of A + rank of B.

(C) Grater than or equal to rank of A + rank of B.

(D) None of the above.

23. Let V be a finite dimensional vector space. If v_1 and v_2 are two linearly independent vectors in V. Then the dimension of V is :

(A) 2. (B) At least two.

(C) At most two. (D) May be any integer.

24. Let V be a vector space dimension n over a field F and $A : V \to V$ be a linear operator. Then the characteristic polynomial of A has :

(A) n distinct roots. (B) n roots counting multiplicities.

(C) At most n roots. (D) At least n roots.

25. If V denotes a vector space over a field F, then which one of the following statements is not an axiom for the vector space V :

(A) For all $x, y \in V, x + y = y + x$. (B) For all $x, y, z \in V, (x + y) + z = x + (y + z)$.

(C) For all $x, y, z \in V, (xy)z = x(yz)$. (D) For all $x \in V$, and $k \in F\ (kx) \in V$.

26 Consider the functions given below :

(a) $\begin{cases} f(x, y) = \dfrac{xy}{x^2 + y^2} \text{ if } (x, y) \neq (0,0) \\ = 0 \text{ if } (x, y) = (0, 0) \end{cases}$. (b) $\begin{cases} f(x, y) = \dfrac{x^2 y}{x^4 + y^2} \text{ if } (x, y) \neq (0,0) \\ = 0 \text{ if } (x, y) = (0, 0) \end{cases}$.

(c) $\begin{cases} f(x, y) = \dfrac{\sin xy}{\sin x \sin y} \text{ if } (x, y) \neq (0,0) \\ = 1 \text{ if } (x, y) = (0, 0) \end{cases}$. (d) $f(x, y) = \sec x \tan y$.

(A) (a) and (b) are continuous at (0,0).

(B) (b) and (c) are continuous at (0,0).

(C) (c) and (d) are continuous at (0,0).

(D) (d) and (a) are continuous at (0,0).

27. The derivative of the function $f(x, y) = xy + y^2$ at the point (2, 5) is zero in the direction of :

(A) $\frac{5}{\sqrt{29}} i + \frac{4}{\sqrt{29}} j.$

(B) $\frac{-5}{13} i + \frac{2}{13} j.$

(C) $\frac{-12}{13} i + \frac{5}{13} j.$

(D) $\frac{12}{\sqrt{13}} i + \frac{5}{\sqrt{13}} j.$

28. Let $f(x, y) = x^2 + kxy + k^2y^2$. Find the value of k for which (0, 0) is a critical point of f :

(A) $k = 0.$

(B) $k = 1.$

(C) $k = 2.$

(D) For all values of k.

29. The value of $\iint xye^{x+y}\, dxdy$ is :

(A) $ye^y\left(xe^x - e^x\right).$

(B) $\left(ye^y - e^y\right)\left(xe^x - e^x\right).$

(C) $\left(ye^y - e^y\right)xe^x.$

(D) $\left(ye^y - e^y\right)\left(xe^x + e^x\right).$

30. The derivative of $f(x, y, z)$ at a point P is great in the direction of the vector $A = i + j - k$. In this direction the value of the derivative is $2\sqrt{3}$. Then the gradient vector of f at P is :

(A) $i + j - k.$

(B) $2i + 2j - 2k.$

(C) $\frac{1}{\sqrt{3}}(i + j - k).$

(D) $2\sqrt{3}\,(i + j - k).$

31. Which of the following is not true ?

(A) $e^{z_1 + z_2} = e^{z_1} e^{z_2}.$

(B) $e^{z_1 - z_2} = e^{z_1}/e^{z_2}.$

(C) $\left|e^z\right| = \left|e^{\operatorname{Im} z}\right|.$

(D) $\left|e^z\right| = \left|e^{\operatorname{Re} z}\right|.$

32. Let $f(z) = e^{\left(1/z^4\right)}$. Then $z = 0$ is :

(A) a pole of order 4.

(B) a zero of order 4.

(C) an essential singularity.

(D) a pole of infinite order.

33. The principal value of $(-i)^i$ is :

(A) $e^{\frac{\pi}{2}}$.
(B) $e^{\frac{-\pi}{2}}$.
(C) e^{π}.
(D) None of these.

34. Which of the following is not true ?

(A) $\overline{\cos(iz)} = \cos\left(\overline{iz}\right)$.
(B) $\overline{\cos z} = \cos\left(\bar{z}\right)$.
(C) $\overline{\sin(iz)} = \sin\left(\overline{iz}\right)$.
(D) $\overline{\sin z} = \sin\left(\bar{z}\right)$.

35. The value of the integral $\int_{|z|=1} e^{\left(1/z^2\right)}\, dz$ is :

(A) 0.
(B) 1.
(C) $e/2$.
(D) None of these.

36. Which of the following is not true ?

(A) The ring of integers is an integral domain.

(B) The ring of $\mathbb{Z}\left[\sqrt{2}\right] = \left\{a + b\sqrt{2} : a, b \in \mathbb{Z}\right\}$ is an integral domain.

(C) The ring $\mathbb{Z}_n$ of integers modulo n is an integral domain.

(D) The ring $\mathbb{Z}[x]$ of polynomials with integer coefficients is an integral domain.

37. Which of the following is a cyclic group ?

(A) $(\mathbb{Q}, +)$.
(B) $(\mathbb{Q} \setminus \{0\}, .)$.
(C) $(U_8, .)$ where $U_8 = \left\{z \in \mathbb{C} : z^8 = 1\right\}$.
(D) $(\mathbb{R}, +)$.

38. The order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 6 & 1 & 3 & 2 & 7 & 5 \end{pmatrix}$ is :

(A) 3.
(B) 4.
(C) 7.
(D) 12.

39. The number of automorphisms the group $\mathbb{Z}_{12}$ is :

(A) 1.

(B) 2.

(C) 4.

(D) 3.

40. The map $f:(\mathbb{R}^{+},.)\rightarrow(\mathbb{R},+)$ defined by $f(x)$ — $\ln x$ is :

(A) Not a homomorphism.

(B) An isomorphism.

(C) An onto but not a one to one homomorphism.

(D) A one to one but not a homomorphism.

41. Let X be a uniformly distributed random variable that takes value between 0 and 1. Then the value of E (X^3) is :

(A) $\frac{1}{4}$.

(B) $\frac{1}{8}$.

(C) $\frac{1}{16}$.

(D) $\frac{1}{2}$.

42. A convergent sequence of real numbers has :

(A) Exactly one limit.

(B) At most one limit.

(C) At least one limit.

(D) None of these.

43. The function $f:(0,\infty)\rightarrow\mathbb{R}$ defined by $f(x)=lnx$ is :

(A) One to one but not onto.

(B) Onto but not one one.

(C) Neither one one nor onto.

(D) One one and onto.

44. Which of the following is true for Riemann integrable function $f:[a,b]\rightarrow\mathbb{R}$?

(A) f is continuous.

(B) f is differentiable.

(C) f is bounded.

(D) f is monotone.

45. Let f be the function defined by $f(x)=\sum_{n=1}^{\infty}\left(\frac{x^n}{n}\right), -1<x<1.$ Then the derivative $f'(x)$ is :

(A) $\frac{1}{1+x}$.

(B) $1+x$.

(C) $\frac{1}{1-x}$.

(D) $\frac{x}{1-x}$.

46. Which of the following is not true for an LPP ?

(A) Objective function must be linear.

(B) All the decision variables must be non-negative.

(C) The number of decision variables must be equal to the number of constraints.

(D) All constraints must be linear relationships.

47. The reminder when the sum $111^{333} + 333^{111}$ is divided by 7 is :

(A) 1. (B) 3.

(C) 0. (D) 2.

48. The number of solutions of the congruence $6x \equiv 15 \pmod{21}$ is :

(A) 15. (B) 1.

(C) 3. (D) 1.

49. The last two digits of the decimal expansion of 3^{100} are :

(A) 23. (B) 27.

(C) 01. (D) 69.

50. Which of the following statement is false for the Euler's ϕ function ?

(A) For $n > 2, \phi(n)$ is an even integer.

(B) If n is an odd integer, then $\phi(2n) = \phi(n)$.

(C) If n is an odd integer, then $\phi(3n) = \phi(n)$.

(D) If n is an odd integer, then $\phi(2n) = 2\phi(n)$.