D 94077 (Pages : 4) Name.......................................

Reg. No...................................

FIRST SEMESTER (CBCSS) DEGREE EXAMINATION, NOVEMBER 2020

Mathematics

MEC 1C 01—MATHEMATICAL ECONOMICS

(2019 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 15	Maximum : 15 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 15.
2. The candidate should check that the question paper supplied to him/her contains all the 15 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MEC 1C 01—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

1. Which of the following are NOT true ?

   (A) $f(x) = ax^n$ implies $f'(x) = anx^{n-1}$.

   (B) $f(x) = 4x^5 - 3x^2$ implies $f'(x) = 20x^{-5} - 6x^{-2}$.

   (C) $f(x) = 4x + 3/x^2$ implies $f'(x) = 4 - 6/x^3$.

   (D) $f(x) = \ln(x)$ implies $f'(x) = x^{-1}$.

2. Which of the following statements are (in general) true ?

   (A) Marginal cost (MC) is minimised where MC = Average Variable Cost (AVC).

   (B) Total Cost (ATC) is minimised where MC = ATC.

   (C) Average Variable Cost (A VC) is minimised where MC = AVC.

   (D) Total revenue is maximised where MC = Marginal Revenue (MR).

3. The law which studies the direct relationship between price and quantity supplied of a commodity is :

   (A) Law of demand.

   (B) Law of variable proportion.

   (C) Law of supply.

   (D) None of the above.

4. In case of perfectly inelastic supply the supply curve will be :

   (A) Rising.
   (B) Vertical.
   (C) Horizontal.
   (D) Falling.

5. When a percentage in price results in equal change in quantity supplied, it is called :

   (A) Elastic supply.
   (B) Perfectly inelastic.
   (C) Elasticity of supply
   (D) Unitary elastic supply.

6. Suppose the supply for product A is perfectly elastic. If the demand for this product increases :

   (A) The equilibrium price and quantity will increase.

   (B) The equilibrium price and quantity will decrease.

   (C) The equilibrium quantity will increase but the price will not change.

   (D) The equilibrium price will increase but the quantity will not change.

7. If the co-efficient of income elasticity of demand is higher than 1 and the revenue increases, the share of expenditures for commodity X in total expenditure :

   (A) Will increase. (B) Will decrease.

   (C) Will remain constant. (D) Can not be determined.

8. If the demand curve for product A moves to the right, and the price of product B decreases, it can be concluded that :

   (A) A and B are substitute goods.

   (B) A and B are complementary goods.

   (C) A is an inferior good, and B is a superior good.

   (D) Both goods A and B are inferior.

9. If a price increase of 50 % results in an increase in the quantity supplyed of an economic good from 10 to 20 pieces, calculate the co-efficient of price elasticity of supply :

   (A) 1/4. (B) 1/2.

   (C) 1. (D) 2.

10. Which of the following statements is false :

    (A) Perfect competition involves many sellers of standardized products.

    (B) Monopolistic competition involves many sellers of homogeneous products.

    (C) The oligopoly involves several producers of standardized or differentiated products.

    (D) Monopoly involves a single product for which there are no close substitutes.

11. If the price elasticity of demand for wine is estimated to be -.6, then a 20 % increase in price of wine will lead to —————— in quantity demanded of wine at that price.

    (A) 12 % increase. (B) 12 % decrease.

    (C) 19.6 % increase. (D) 20.6 % decrease.

12. If the cross price elasticity of demand for two product is negative, then the two products are ————————.

(A) Complementary to each other.
(B) Perfectly substitute for each other.
(C) Completely competitive.
(D) Unrelated.

13. When the price of complementary products falls, the demand of the other product will :

(A) Fall.
(B) Increases.
(C) Remain stable.
(D) Drops by 25 %.

14. Goods which are perfect substitute of each other will have elasticity of substitution :

(A) Unity.
(B) Less than 1.
(C) More than 1.
(D) Infinite.

15. In question No. 201 if at 15,000, the dealer is prepared to supply on 1250 sets of TV the elasticity of supply is :

(A) 1.
(B) 2.
(C) 0.75.
(D) 1.4.

D 94077 (Pages : 2 + 4 = 6) Name.......................................

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FIRST SEMESTER (CBCSS) DEGREE EXAMINATION, NOVEMBER 2020

Mathematics

MEC 1C 01—MATHEMATICAL ECONOMICS

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. What is a Utility function ?
2. Define Long run costs.
3. Define Law of supply.
4. Give the maning of MRS_{YX}.
5. What is an Indifference Curve ?
6. Explain Cross elasticity of demand.
7. If $TC = 8Q^2 + 10Q + 15,$ what will be the MC ?
8. Define Consumer equilibrium.
9. What is meant by Investment multiplier ?
10. Explain Optimization.
11. Explain Arc elasticity method.
12. What is meant by Shift in demand ?

(8 × 3 = 24 marks)

Turn over

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Derive the relationship between MC and AC

14. What is income elasticity of demand ? The demand for a commodity is given by $Q = 5000 + 4Y$, where Q is the demand and Y is the income. What will be the income elasticity of demand when the income of the consumer is Rs. 7000 ?

15. What is a demand curve ? Why does a demand curve slope downward ?

16. Explain long run average cost curve. Why it is known as a planning curve ?

17. State and explain the law of diminishing marginal utility. Show how law of demand can be derived from it.

18. What is a budget line ? Derive the slope of a budget line.

19. What is marginal productivity ? Using the production function $Q = AL^a K^b$, show that, if the factors of production are paid according to their marginal product, the total product will be exhausted. (Q = total product = labor, K = capital)

(5 × 5 = 25 marks)

Section C

Answer any **one** *questions.*
The question carries 11 *marks.*

20. Explain cardinal utility analysis of demand. Derive consumer equilibrium using cardinal utility method.

21. Explain the significance of Lagrange multiplier and maximize the function $X_1X_2 + 2X_1$ Subject to the constraint $4X_1 + 2X_2 = 60$.

(1 × 11 = 11 marks)

D 94076 (Pages : 4) Name..

Reg. No......................................

FIRST SEMESTER (C.B.C.S.S.) DEGREE EXAMINATION, NOVEMBER 2020

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2020 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(Multiple Choice Questions for SDE Candidates)

1. 'What is the value of x after this statement, assuming initial value of x is 5? 'If x equals to one then $x = x + 2$ else $x = 0$' :

   (A) 1. (B) 3.

   (C) 0. (D) 2.

2. The compound statement A $\rightarrow$ (A $\rightarrow$ B) is false, then the truth values of A, B are respectively.

   (A) T, T. (B) F, T.

   (C) T, F. (D) F, F.

3. Let P (x) denote the statement "$x > 7$". Which of these have truth value true ?

   (A) P (0) (B) P (4).

   (C) P (6). (D) P (9).

4. When to proof P $\rightarrow$ Q true, we proof P false, that type of proof is known as :

   (A) Direct proof. (B) Contrapositive proofs.

   (C) Vacuous proof. (D) Mathematical Induction.

5. In the principle of mathematical induction, which of the following steps is manda- tory ?

   (A) Induction hypothesis. (B) Inductive reference.

   (C) Induction set assumption. (D) Minimal set representation.

6. By induction hypothesis, the series $1^2 + 2^2 + 3^2 + \ldots\ldots. + p^2$ can be proved equivalent to :

   (A) $p^2 + 27$. (B) $\frac{p*(p+1)*(2p+1)}{6}$.

   (C) $\frac{p*(p+1)}{4}$. (D) $p + p^2$.

7. For any positive integer m, . . . is divisible by 4.

   (A) $5m^2 + 2$. (B) $3m + 1$.

   (C) $m^2 + 3$. (D) $m^3 + 3m$.

8. Let gcd $(a, b) = d$. If c divides a and c divides b, then :

(A) $c \leq d$.
(B) $c \geq d$.
(C) $c = 1$.
(D) Cannot determine c with the given information.

9. If $a \mid b$ and $a \mid c$, then :

(A) $b \mid c$.
(B) $c \mid a$.
(C) $a \mid (b + c)$.
(D) $b \mid a$.

10. If a and b are non zero integers with $a \mid b$, then gcd (a, b) equals :

(A) $|a|$.
(B) b.
(C) ab.
(D) a.

11. The product of any three consecutive integers is divisible by :

(A) 36.
(B) 9.
(C) 6.
(D) 8.

12. If a is an odd integer then gcd $(3a, 3a + 2)$ equals :

(A) 3.
(B) 5.
(C) 1.
(D) 2.

13. $(1001111)_2 =$ ——————.

(A) 79.
(B) 89.
(C) 69.
(D) 99.

14. The linear Diophantine equation $ax + by = c$ has a solution if and only if :

(A) gcd $(a, c) \mid b$.
(B) gcd $(a, b) \mid c$.
(C) gcd $(c, b) \mid a$.
(D) $c \mid$ gcd (a, b).

15. A composite number n for which $a^n \equiv a \pmod{n}$ is called :

(A) A Pseudoprime.
(B) A Prime.
(C) A pseudoprime to the base a.
(D) An absolute pseudoprime.

16. The composite numbers n that are pseudoprime to every base a are called :

(A) Pseudoprime. (B) Prime.

(C) Pseudoprime to the base a. (D) Absolute pseudoprime.

17. If $p, q_1, q_2, \ldots, q_n$ are all primes and $p \mid q_1 q_2 \ldots q_n$, then :

(A) $p = q_k$ for some k. (B) $p = 2$.

(C) $q_k = 2$ for some k. (D) $p \mid q_k$ for some k.

18. If a is a solution of $P(x) \equiv 0 \pmod{n}$ and $a \equiv b \pmod{n}$, then :

(A) ab is also a solution. (B) $a + b$ is also a solution.

(C) $a - b$ is also a solution. (D) b is also a solution.

19. If a is an odd integer, then $a^2 \equiv --- \pmod{8}$.

(A) 1. (B) 2.

(C) 3. (D) 4.

20. The solution of $25x \equiv 15 \pmod{29}$ is :

(A) $x \equiv 18 \pmod{29}$. (B) $x \equiv 29 \pmod{29}$.

(C) $x \equiv 18 \pmod{19}$. (D) $x \equiv 17 \pmod{19}$.

D 94076 (Pages : 2 + 4 = 6) Name...

Reg. No....................................

FIRST SEMESTER (C.B.C.S.S.) DEGREE EXAMINATION, NOVEMBER 2020

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2020 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Find the converse and the contra positive of the implication"If today is Thursday, then I have a "test today."

2. What is the truth value of $\exists x\, P(x)$ where P (x) is the statement "$x^2 > 10$" and the universe of discourse consists of the positive integers not exceeding 4 ?

3. State which rule of inference is the basis of the following arguments: "It is below freezing now. Therefore, it is either below freezing or raining now".

4. Prove the theorem "The integer n is odd if and only if n^2 is odd".

5. Prove that there is a prime number greater than 3.

6. State Well-Ordering Property of $\mathbb{N}$.

7. Prove that there is no polynomial $f(n)$ with integral coefficients that will produce primes for all integers n.

8. If $d = (a, b)$ and d' is any common divisor of a and b, then prove that $d' \mid d$.

9. State Dirichlet's Theorem.

10. Find least number which when divided by 9 gives the remainder 8, when divided by 8 gives the remainder 7, when divided by 7 gives the remainder 6, ..., when divided by 3 gives the remainder 2, when divided by 2 gives the remainder 1.

11. Determine whether the linear Diophantine equations :

    $6x + 8y + 12z = 10$ and $6x + 12y + 15z = 10$ are solvable.

12. Let m and n be positive integers such that $m \mid n$. Then prove that $2^{m-1} \mid 2^{n-1}$.

Turn over

13. Define $\tau(n)$ and $\sigma(n)$. Find $\tau(12)$ and $\sigma(12)$.

14. Prove that $\phi(n) = n - 1$ if and only if n is prime.

15. Solve the linear congruence $35x \equiv 47 \pmod{24}$.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Write the truth table for the implication $p \to q$.

17. Verify that $p \wedge T = p$ and $p \vee F \equiv p$.

18. State and prove the Pigeonhole principle.

19. Find the number of positive integers ≤ 2076 and divisible by neither 4 nor 5.

20. State and prove Fermat's Little Theorem.

21. Find all solutions of the congruence $35x \equiv 47 (\text{mod} 24)$.

22. Show that a positive integer a is self-invertible modulo p if and only if $a \equiv \pm 1 (\text{mod}\, p)$.

23. Show that $n^7 - n$ is divisible by 42.

(5 × 6 = 30 marks)

Section C (Essay Questions)

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Show that $\sqrt[3]{3}$ is irrational.

(b) Express the statement

"Every student in this class has studied calculus" as a universal quantification.

25. (a) Explain the Two Queens Puzzle.

(b) Let a and b be any positive integers. Then prove that the number of positive integers $\leq a$ and divisible by b is $\lfloor a/b \rfloor$.

26. (a) Using Euclidean Algorithm calculate gcd (12378, 3054). Also represent the greatest common divisor as a linear combination of 12378 and 3054.

(b) State Fundamental Theorem of Arithmetic and find the canonical decomposition of 2520.

27. (a) State and prove Wilson's Theorem.

(b) Prove that $5^{2n+2} - 24n - 25$ is divisible by 576.

(2 × 10 = 20 marks)

D 13606-A (Pages : 4) Name.......................................

Reg. No...................................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MEC 1C 01—MATHEMATICAL ECONOMICS

(2019—2020 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 15	Maximum : 15 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 15.
2. The candidate should check that the question paper supplied to him/her contains all the 15 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MEC 1C 01—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

1. Which of the following are NOT true ?

   (A) $f(x) = ax^n$ implies $f'(x) = anx^{n-1}$.

   (B) $f(x) = 4x^5 - 3x^2$ implies $f'(x) = 20x^{-5} - 6x^{-2}$.

   (C) $f(x) = 4x + 3/x^2$ implies $f'(x) = 4 - 6/x^3$.

   (D) $f(x) = \ln(x)$ implies $f'(x) = x^{-1}$.

2. The law which studies the direct relationship between price and quantity supplied of a commodity is :

   (A) Law of demand. (B) Law of variable proportion.

   (C) Law of supply. (D) None of the above.

3. In case of perfectly inelastic supply the supply curve will be :

   (A) Rising. (B) Vertical.

   (C) Horizontal. (D) Falling.

4. When a percentage in price results in equal change in quantity supplied, it is called ?

   (A) Elastic supply. (B) Perfectly inelastic.

   (C) Elasticity of supply. (D) Unitary elastic supply.

5. If the co-efficient of income elasticity of demand is higher than 1 and the revenue increases, the share of expenditures for commodity X in total expenditure :

   (A) Will increase. (B) Will decrease.

   (C) Will remain constant. (D) Can not be determined.

6. Calculate the average fixed cost (AFC), for a level of production Q = 20, knowing that the total cost function is : $TC = 200 + 3Q + 2Q^2$.

   (A) 1060. (B) 200.

   (C) 20. (D) 10.

7. On the market with perfect competition :

   (A) The firm is a "price-taker," meaning, it takes over the market price.

   (B) The firm is a "price-maker", meaning, it determines the market price.

   (C) The companies' products are differentiated.

   (D) Input barriers are minimal, and exit barriers are maximal.

8. If the price elasticity of demand for wine is estimated to be -.6, then a 20 % increase in price of wine will lead to ———————— in quantity demanded of wine at that price.

   (A) 12 % increase. (B) 12 % decrease.

   (C) 19.6 % increase. (D) 20.6 % decrease.

9. If the price of coffee falls by 8 % and the demand for Tea declines by 2 %. The cross price elasticity of demand for Tea is :

   (A) 0.45. (B) 0.25.

   (C) + 0.44. (D) – 0.30.

10. When the price of complementary products increases, the demand of the other product will ?

   (A) Falls. (B) Increases.

   (C) Remains same. (D) Increases by 25 %.

11. An individual is spending his entire income on two items A and B equally. If income elasticity of A is 4 what is income elasticity of B :

   (A) 4. (B) 2.

   (C) 3. (D) 1.

12. Cross elasticity of a nearly perfect substitute products will be :

   (A) Infinite. (B) Zero.

   (C) > 1. (D) < 1.

13. Cross elasticity of complementary products will be :

   (A) Infinite. (B) Zero.

   (C) > 1. (D) < 0.

14. If the disposal income of a household decreases by 10 % and the demand for X commodity remains same. The income elasticity of X is :

(A) 0. (B) 0.5.

(C) 0.5 (D) 2.5.

15. Which of these would lead to increase in quantity supplied at a given price :

(A) Increase in VAT. (B) Increase in excise duty.

(C) Increase in import duty. (D) Reduction in levies.

D 13606 (Pages : 2) Name.......................................

Reg. No...................................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MEC 1C 01—MATHEMATICAL ECONOMICS

(2019—2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
2 marks each.
Maximum marks : 20.

1. What is disposable income ?
2. Write any two important determinants of demand.
3. Define elasticity of supply.
4. Point out the difference between a short run cost function and a long run cost function.
5. Write the relation between AR, MR and elasticity of demand.
6. Define average and marginal cost function.
7. Define utility function.
8. What is an indifference curve ?
9. Write any two criticisms against utility approach.
10. What is the major difference between the average concept and marginal concept in economics ?
11. Define (i) Increasing function ; and (ii) Decreasing function.
12. If $z = x^2 + 5xy$ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

Section B

Answer any number of questions.
5 marks each.
Maximum marks : 30.

13. Explain arc price elasticity and point price elasticity.

14. With the help of a diagram explain the concept of supply.

15. Find out the output at which the average cost function is minimum from the total cost function $TC = 2q^2 + 5q + 18$.

16. Find the marginal and average functions of the total function given by $3Q^2 + 7Q + 12$ at $Q = 3$ and $Q = 5$.

17. Distinguish between cardinal and ordinal utility analysis.

18. Write the properties of indifference curve.

19. $f(x, y) = x^y$, verify that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.

Section C

Answer any **one** *question.*
The question carries 10 *marks.*

20. Define a demand function and explain all the variables in the demand function.

21. A firm producing two goods x and y has the profit function $\pi = 64x - 2x^2 + 4xy - 4y^2 + 32y - 14$. Find out the profit maximizing level of output for each of the two goods and test to be sure the profits are maximized.

(1 × 10 = 10 marks)

D 13605 (Pages : 3) Name...

Reg. No.....................................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MTS 1C 01—MATHEMATICS—I

(2019—2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 20 marks.

1. Find the derivative of $f(x) = x^2 - x$ at $x = 2$.

2. Find $\lim_{x \to 0} \frac{\sqrt{x^2 + 100} - 10}{x^2}$.

3. Find the tangent line to the curve $y = \sqrt{x}$ at $x = 4$.

4. Find the derivative of $y = \left(x^2 + 1\right)\left(x^3 + 3\right)$.

5. Give the parameterization of the circle $x^2 + y^2 = 1$.

6. Find $\lim_{y \to 1} \sec\left(y \sec^2 y - \tan^2 y - 1\right)$.

7. Suppose that $F'(x) = x$ for all x and that $F(3) = 2$. What is $F(x)$?

8. Suppose that f is differentiable on the whole real line and that $f'(x)$ is constant. Prove that f is linear.

9. Prove that for the curve $y = c \sin \frac{x}{a}$, every point at which it meets the x-axis is a point of inflection.

Turn over

10. Find the maximum and minimum points and values for the function $f(x) = \left(x^2 - 8x + 12\right)^4$ on the interval $[-10, 10]$.

11. Find $\sum_{k=1}^{7} \left(3 - k^2\right)$.

12. Find $\int_0^1 \frac{\left(3x^2 + x^4\right)}{\left(1 + x^2\right)^2} dx$.

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum 30 marks.

13. If $\sqrt{5 - 2x^2} \le f(x) \le \sqrt{5 - x^2}$ for $-1 \le x \le 1$, find $\lim_{x \to 0} f(x)$.

14. Find the linearization of $f(x) = \sqrt{x+1} + \sin x$ at $x = 0$. How is it related to the individual linearizations for $\sqrt{x+1}$ and $\sin x$?

15. An oil slick has area $y = 30x^3 + 100x$ square meters x minutes after a tanker explosion. Find the average rate of change in area with respect to time during the period from $x = 2$ to $x = 3$ from $x = 2$ to $x = 2.1$. What is the instantaneous rate of change of area with respect to time at $x = 2$?

16. Use implicit differentiation to find dy/dx if $6y^2 + \cos y = x^2$.

17. Prove that the curve $y = \frac{x}{1 + x^2}$ has three points of inflection and they are collinear.

18. Evaluate $\lim_{x \to \infty} \frac{x^n}{e^x}$, where n is natural number.

19. Find the area of the region enclosed by the curves $x + y^2 = 3$ and $4x + y^2 = 0$.

Section C

Answer any **one** *question.*
The question carries 10 *marks.*
Maximum 10 *marks.*

20. (a) State and prove the quotient rule of differentiation for positive integers.

(b) Prove that $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, (n \neq -1).$.

(c) A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder.The second plane crosses the first plane at 45° angle at the center of the cylinder. Find the volume of the wedge.

21. (a) On what interval is $f(x) = x^3 - 2x + 6$ increasing or decreasing ?

(b) Find the asymptotes of the graph of $f(x) = -\frac{8}{x^2 - 4}$.

(c) Find the equation of the line tangent to the parametric curve given by the equations $x = (1 + t^3)^4 + t^2, y = t^5 + t^2 + 2$ at $t = 1$.

(1 × 10 = 10 marks)

D 13604-A (Pages : 4) Name.......................................

Reg. No....................................

FIRST SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2021

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2020 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 20 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(Multiple Choice Questions for SDE Candidates)

1. The truth value of given statement is 4 + 3 = 7 or 5 is not prime :

   (A) False. (B) True.

2. $(A \vee F) \vee (A \vee T)$ is always :

   (A) True. (B) False.

3. The number of primes is :

   (A) Finite. (B) Infinite.

   (C) Uncountable.

4. Difference of two distinct prime numbers is :

   (A) Odd and prime. (B) Even and composite.

   (C) None of the mentioned.

5. For $n \geq 1$, there are at least ———— primes less than 2^2.

   (A) n. (B) $n - 1$.

   (C) $n + 1$.

6. If $a \equiv b \mod n$, then :

   (A) $n \mid a$ and $n \mid b$. (B) $n \mid b$ only.

   (C) $n \mid (a - b)$.

7. $a \equiv b \bmod n$ and $b \equiv c \bmod n$, then :

   (A) $a \equiv c \bmod n$. (B) $a = b$.

   (C) $a = c$.

8. If $a \equiv b \bmod n$, then :

   (A) $a - b = n$. (B) $a - b = kn$, for some integer k.

   (C) $a + b = kn$, for some integer k.

9. If $a \equiv b \bmod n$, and $m \mid n$, then :

(A) $a \equiv b$ mod.

(B) $a6 \equiv b \bmod m$.

(C) $a \equiv b \bmod (n / m)$.

10. If $a \equiv b \bmod n$ and $\gcd(a, n) = d$, then $gcd\ (b, n)$ is :

(A) a.

(B) d.

(C) nd.

11. A positive integer is divisible by 9 if and only if the sum of the digits in its deci-mal representation is divisible by :

(A) 3.

(B) 81.

(C) 9.

12. If $gcd\ (a, n) = 1$, then the congruence $ax \equiv b \bmod n$ has :

(A) Infinitely many solutions modulo n.

(B) Unique solution modulo n.

(C) More than one solution modulo n.

13. The system of linear congruences $ax + by \equiv r \pmod{n}$ and $cx + dy \equiv s \pmod{n}$ has a unique solution modulo n whenever :

(A) $gcd\ (ad - bc, n) = 1$.

(B) $gcd\ (ad, bc) = 1$.

(C) $gcd\ (ad, bc) = n$.

14. If p is a prime, then for any integer a :

(A) $a^p \equiv a \bmod p$.

(B) $a^{p-1} \equiv 1 \bmod p$.

(C) $a^{p-1} \equiv -1 \bmod p$.

15. $\sigma(12)$ is :

(A) 28.

(B) 27.

(C) 16.

16. $\sigma(n) = n + 1$ if and only if :

(A) n is an odd number. (B) n is an even number.

(C) n is a prime number.

17. The functions τ and σ are both multiplicative functions. The statement is :

(A) False. (B) True.

(C) Partially true.

18. Which of the following statement is true ?

(A) The functions τ and σ are both multiplicative functions.

(B) The Euler's Phi-function is injective.

(C) The Euler's Phi-function is not Multiplicative.

19. Given integers a, b, c, $gcd\ (a, bc) = 1$ if and only if :

(A) $gcd\ (a, b) = 1$ and $gcd\ (a, c) = 1$. (B) $gcd\ (a, b) = 1$ and $gcd\ (b, c) = 1$.

(C) $gcd\ (a, b) = 1$ and $gcd\ (a, c) = b$.

20. $\varphi\left(2^3\right)$ is :

(A) 2. (B) 3.

(C) 7.

D 13604 (Pages : 3) Name.......................................

Reg. No....................................

FIRST SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2021

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2020 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

(Answer any number of questions.
Each carries 2 marks.
Maximum marks that can be earned from this section is 25)

1. What is a logical statement or a proposition ?
2. Define a vacuously true statement and give an example.
3. If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, show that $ac \equiv bd \pmod{n}$.
4. Prove or disprove : Every composite number n has a prime factor $\leq \lfloor \sqrt{n} \rfloor$.
5. If $(a, b) = d$, show that a/d and b/d are relatively prime.
6. State Dirichlet's Theorem.
7. If n is a positive integer such that $(n-1)! \equiv -1 \pmod{n}$, show that n is a prime.
8. State Euler's theorem.
9. State the Division algorithm for integers.
10. What are linear diophantine equations in two variables x and y.
11. State the necessary and sufficient condition(s) for a linear congruence $ax \equiv b \pmod{m}$ to have a unique solution.
12. Use the divisibility criterion to check whether 10000234 divisible by eight or not.
13. Express the g.c.d (100,13) as a linear combination of 100 and 13.

14. Compute the value of $\varphi(100)$.

15. Explain how canonical decomposition is useful in finding the least common multiple of two positive integers.

Section B

(Answer any number questions from this section.
Each question carries 5 marks.
Maximum that can be earned from this section is 35)

16. Evaluate the boolean expression $\sim\left[(x \leq y) \wedge (y > z)\right]$ when $x = 3, y = 4$ and $z = 5$.

17. Construct the truth table of $(p \to q) \wedge (q \to p)$.

18. Prove that there is no positive integer between 0 and 1.

19. There are n guests at a party. Each person shakes hands with everybody else exactly once. Define recursively the number of handshakes $h(n)$ made.

20. Prove that no prime of the form $4n + 3$ can be expressed as the sum of two squares.

21. Compute the remainder when 3^{247} is divided by 25.

22. Solve the linear congruence $35x \equiv 47 \pmod{24}$.

23. Let b be an integer ≥ 2. Suppose $b + 1$ integers are randomly selected. Prove that the difference of two of them is divisible by b.

Section C

(Answer any **two** *questions from this section.*
Each question carries 10 *marks.*
Maximum that can be earned from this section is 20*)*

24. (a) Construct truth table for $p \to q \leftrightarrow \sim p \wedge q$

(b) Prove by the method of contradiction : The square of an odd integer is odd. Moreover, rewrite the proposition symbolically with UD = set of all integers.

25. (a) State and prove the weak version of principle of mathematical induction.

(b) Define the least common multiple $[a, b]$ of two positive integers and show that $[a, b] = \frac{ab}{(a, b)}$.

26. (a) Find the remainder when 24^{1947} is divided by 17.

(b) State and prove the Fermat's little theorem.

27. State and prove the Fundamental theorem of arithmetic.

D 13603-A (Pages : 4) Name.....................................

Reg. No...................................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2019 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(Multiple Choice Questions for SDE Candidates)

1. What is the value of x after this statement, assuming initial value of x is 5 ? 'If x equals to one then $x = x + 2$ else $x = 0$.

   (A) 1.
   (B) 3.
   (C) 0.
   (D) 2.

2. The compound statement $A \to (A \to B)$ is false, then the truth values of A, B are respectively :

   (A) T, T.
   (B) F, T.
   (C) T, F.
   (D) F, F.

3. Let P (x) denote the statement "$x > 7$". Which of these have truth value true ?

   (A) P (0).
   (B) P (4).
   (C) P (6).
   (D) P (9).

4. When to proof $P \to Q$ true, we proof P false, that type of proof is known as :

   (A) Direct proof.
   (B) Contrapositive proofs.
   (C) Vacuous proof.
   (D) Mathematical Induction.

5. In the principle of mathematical induction, which of the following steps is manda-tory :

   (A) Induction hypothesis.
   (B) Inductive reference.
   (C) Induction set assumption.
   (D) Minimal set representation.

6. For any positive integer m,... is divisible by 4.

   (A) $5m^2 + 2$.
   (B) $3m + 1$.
   (C) $m^2 + 3$.
   (D) $m^3 + 3m$.

7. Suppose that P (n) is a propositional function. Determine for which positive integers n the statement P (n) must be true if : P (1) is true ; for all positive integers n, if P (n) is true then P (n + 2) is true :

   (A) P (3).
   (B) P (2).
   (C) P (4).
   (D) P (6).

8. Let gcd $(a, b) = d$. If c divides a and c divides b, then :

(A) $c \le d$.
(B) $c \ge d$.
(C) $c = 1$.
(D) Cannot determine c with the given information.

9. If a and b are non-zero integers with $a \mid b$, then gcd (a, b) equals :

(A) $|a|$.
(B) b.
(C) ab.
(D) a.

10. The product of any three consecutive integers is divisible by :

(A) 36.
(B) 9.
(C) 6.
(D) 8.

11. If a is an odd integer then gcd$(3a, 3a + 2)$ equals :

(A) 3.
(B) 5.
(C) 1.
(D) 2.

12. If a, b are two distinct prime number than highest common factor of a, b is :

(A) 2.
(B) 0.
(C) 1.
(D) ab.

13. $(1001111)_2 = \ldots$

(A) 79.
(B) 89.
(C) 69.
(D) 99.

14. What is the one's complement of the number 1010110 ?

(A) 1111111.
(B) 0101001.
(C) 1100110.
(D) None of the mentioned.

15. An integer n is called a pseudoprime if :

(A) $n \mid 2^n - 2$.
(B) n is composite and $n \mid 2^n - 2$.
(C) n is prime and $n \mid 2^n - 2$.
(D) n is composite and $n \mid 2n - 1$.

16. A composite number n for which $a^n \equiv a \pmod{n}$ is called :

(A) A pseudoprime. (B) A prime.

(C) A pseudoprime to the base a. (D) An absolute pseudoprime

17. The composite numbers n that are pseudoprime to every base a are called :

(A) Pseudoprime. (B) Prime.

(C) Pseudoprime to the base a. (D) Absolute pseudoprimes.

18. If $p, q_1, q_2, \ldots, q_n$ are all primes and $p \mid q_1 q_2, \ldots q_n$, then :

(A) $p = q_k$ for some k. (B) $p = 2$.

(C) $q_k = 2$ for some k. (D) $p \mid q_k$ for some k.

19. If a is a solution of $P(x) \equiv 0 \pmod{n}$ and $a \equiv b \pmod{n}$, then :

(A) ab is also a solution. (B) $a + b$ is also a solution.

(C) $a - b$ is also a solution. (D) b is also a solution.

20. The Sieve of Eratosthenes is used for finding :

(A) All primes below a given integer.

(B) All even numbers below a given integer.

(C) All odd numbers below a given integer.

(D) All composite numbers below a given integer.

D 13603 (Pages : 3) Name.....................................

Reg. No.................................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 25 marks.

1. What is meant by a *proposition* ? Is "$3 + 4 = 9$" a proposition ? Why ?
2. What is meant by *negation* of a proposition ? Write the negation of the proposition "Ravi is taller than Joseph".
3. What is meant by the *contraposition* of an implication ? Give the contrapositive implication of "If $a > b$, then $b < c$."
4. Prove that every nonempty set of nonnegative integers has a least element.
5. Express $(231)_{10}$ in base 3.
6. Determine whether 1601 is a prime number.
7. Express $(28, 12)$ as a linear combination of 28 and 12.
8. Define the LCM of two numbers a, b. What is the LCM of 22 and 33 ?
9. Define the relation a congruent to b modulo n. Is 23 congruent to 4 modulo 5? Why?
10. Define complete set of residues modulo m. Give an example for such a set modulo 7 starting at 15.
11. Find the remainder when 16^{53} is divided by 7.
12. State Fermat's Little theorem and Wilson's theorem.
13. Find the remainder when 245^{1040} is divided by 18.

Turn over

14. Define *multiplicative* functions. Show that $f(n) = n^2$ is multiplicative.

15. Define the function σ. Evaluate $\sigma(12), \sigma(18)$.

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum 35 marks.

16. Prove directly that the product of any two odd integers is an odd integer and that the product of any two even integers is an even integer.

17. Let b be an integer $b \geq 2$. Suppose $b+1$ integers are randomly selected. Prove that the difference of atleast two of them is divisible by b.

18. Prove that every integer $n \geq 2$ has a prime factor.

19. Define Fermat numbers. Show that $641 | f_5$.

20. Find the general solution to the LDE $1492x + 1066y = -4$.

21. Let a and b be positive integers. Prove that $[a, b] \times (a, b) = ab$.

22. Prove the Wilson's theorem : If p is a prime, then $(p-1)! \equiv -1 \pmod p$. Without using it, verify that $10! \equiv -1 \pmod{11}$.

23. Prove that if f is a multiplicative function, then $F(n) = \sum_{d|n} f(d)$ is also multiplicative.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*
Maximum 20 *marks.*

24. (a) Prove by contradiction : There is no largest prime number.

(b) Prove by using contrapositive : "If the square of an integer is odd,then the integer is odd".

(c) There is a male barber in a certain town. He shaves all those men and only those men who do not shave themselves. Explain why the question "does the barber shave himself ?" results in a paradox.

25. (a) State the division algorithm and prove it.

(b) Find the quotient q and remainder r when 305 is divided by 16.

(c) State and prove the pigeohole principle.

26. If a cock is worth five coins, a hen three coins, and three chicks together one coin, how many cocks, hens, and chicks, totaling 100, can be bought for 100 coins ?

27. (a) Let p be a prime and a any integer such that $p \nmid a$. Prove that the least residues of the integers $a, 2a, 3a, \ldots\ldots, (p-1)a$ modulo p are a permutation of the integers $1, 2, 3, \ldots\ldots, (p-1)$.

(b) Let m be a positive integer and a any integer with $(a, m) = 1$. Prove that $a^{\phi(m)} \equiv 1 \pmod{m}$.

D 12647-A (Pages : 4) Name.....................................

Reg. No..................................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2021

Mathematics

MEC 1C 01—MATHEMATICAL ECONOMICS

(2021 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 15**	**Maximum : 15 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 15.
2. The candidate should check that the question paper supplied to him/her contains all the 15 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MEC 1C 01—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

1. $y = 100/x + 4x$ has :
   - (A) A maximum point where $x = 5$.
   - (B) A minimum point where $x = 5$.
   - (C) A maximum point where $x = 0$.
   - (D) A minimum point where $x = -5$.

2. Which of the following statements are (in general) true ?
   - (A) Marginal Cost (MC) is minimised where MC = Average Variable Cost (AVC).
   - (B) Total Cost (ATC) is minimised where MC = ATC.
   - (C) Average Variable Cost (AVC) is minimised where MC = AVC.
   - (D) Total revenue is maximised where MC = Marginal Revenue (MR).

3. The law which studies the direct relationship between price and quantity supplied of a commodity is :
   - (A) Law of demand.
   - (B) Law of variable proportion.
   - (C) Law of supply.
   - (D) None of the above.

4. When a percentage in price results in equal change in quantity supplied, it is called ?
   - (A) Elastic supply.
   - (B) Perfectly inelastic.
   - (C) Elasticity of supply.
   - (D) Unitary elastic supply.

5. Few sellers is the feature of :
   - (A) Monopoly.
   - (B) Oligopoly.
   - (C) Perfect competition.
   - (D) Monopolistic competition.

6. Supply curve of a perfectly competitive firm is :
   - (A) Vertical.
   - (B) Upward sloping.
   - (C) Horizontal.
   - (D) Downward sloping.

7. Suppose the supply for product A is perfectly elastic. If the demand for this product increases:

(A) The equilibrium price and quantity will increase.

(B) The equilibrium price and quantity will decrease.

(C) The equilibrium quantity will increase but the price will not change.

(D) The equilibrium price wili increase but the quantity will not change.

8. If the demand curve for product A moves to the right, and the price of product B decreases, it can be concluded that :

(A) (A) and (B) are substitute goods.

(B) (A) and (B) are complementary goods.

(C) (A) is an inferior good, and (B) is a superior good.

(D) Both goods (A) and (B) are inferior.

9. If a price increase of 50% results in an increase in the quantity supplyed of an economic good from 10 to 20 pieces, calculate the co-efficient of price elasticity of supply.

(A) 1/4. (B) 1/2.

(C) 1. (D) 2.

10. An economic agent contracts a loan of 15.000 lei, which he will repay in three equal annual installments. What will be the total interest paid, knowing that the annual interest rate is 12% per year ?

(A) 3.600 lei. (B) 1.800 lei.

(C) 5.400 lei. (D) 1.500 lei.

11. Calculate the average fixed cost (AFC), for a level of production Q = 20, knowing that the total cost function is: $TC = 200 + 3Q + 2Q^2$.

(A) 1060. (B) 200.

(C) 20. (D) 10.

12. On the market with perfect competition :

(A) The firm is a "price-taker", meaning, it takes over the market price.

(B) The firm is a "price-maker", meaning, it determines the market price.

(C) The companies' products are differentiated.

(D) Input barriers are minimal, and exit barriers are maximal.

13. Which of the following statements about monopoly is true :

(A) There are several companies producing a specific product.

(B) There is only one producing company, but the product has close substitutes.

(C) There are no competitors on the relevant market.

(D) Input barriers are low.

14. Which of the following can be considered as the basic features of public goods ?

(A) Are state-owned;

(B) Are characterized by non-excludability and non-rivalry;

(C) Are characterized by excludability and rivalry;

(D) May be positive or negative.

15. Which of the following solutions are not part of the ways of internalizing externalities ?

(A) The imposition of fines on the producer of negative externalities.

(B) The introduction of taxes and duties that bring private costs to the level of social costs.

(C) Closure of companies producing positive or negative externalities.

(D) The association of the negative externality manufacturer with the receptor of such an effect.

D 12647 (Pages : 2) Name..

Reg. No....................................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION, NOVEMBER 2021

Mathematics

MEC 1C 01—MATHEMATICAL ECONOMICS

(2021 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer atleast **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall ceiling 24.

1. What is a demand curve ?
2. Define market demand curve.
3. What is meant by market equilibrium ?
4. Define cost elasticity.
5. Define average and marginal revenue.
6. Define elasticity of average cost.
7. What is a cardinal utility function ?
8. Define an indifference map.
9. Write any two properties of indifference curve.
10. Define total revenue, marginal revenue and average revenue.
11. Define : (i) A convex function ; (ii) A concave function.
12. If $f(x, y) = 3x^2y + x^3y^2$ find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(8 × 3 = 24 marks)

Turn over

Section B

Answer atleast **five** *questions.*
Each question carries 5 marks.
All questions can be attended.
Overall ceiling 25.

13. With the help of suitable diagrams explain demand schedule and demand function.

14. Briefly explain cross price elasticity.

15. Explain the various assumptions on the problem of cost production.

16. Distinguish between short run and long run cost functions.

17. What are the criticism against utility approach ?

18. Find the slope of the average cost curve in terms of average cost and marginal cost.

19. Suppose the price p and quantity q of a commodity are related by the equation $q = 30 - 4p - p^2$. Find elasticity of demand at $p = 2$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
Each question carries 11 marks.

20. (a) Explain the important determinants of price elasticity.

    (b) Prove that the elasticity of demand at different points on the same demand curve is different.

21. Use Lagrange multiplier method to optimize $z = 4x^2 - 2xy + 6y^2$ subject to the constraint $x + y = 72$. Also estimate the effect on the value of the objective function from 1-unit change in the constant of the constraint.

(1 × 11 = 11 marks)

D 12646 (Pages : 3) Name.......................................

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FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MTS 1C 01—MATHEMATICS—I

(2021 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. Calculate the slope of the tangent line to the graph of $f(x) = x^2 + 1$ when $x = -1$.

2. Find $\lim\limits_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - x}$.

3. Find the derivative of $y = \sqrt{x}$ for $x > 0$.

4. Find $\dfrac{d}{dx}\left[\cos\left(\sqrt{1 + \cos x}\right)\right]$.

5. Find the linearization of $f(x) = \cos x$ at $x = \pi/2$.

6. Show that there is a number c such that $c^3 - c^2 = 10$.

7. Find $\lim\limits_{t \to 0} \cos\left(\dfrac{x}{\sqrt{19 - 3\sec 2t}}\right)$.

8. Suppose that f is differentiable on the whole real line and that $f'(x)$ is constant. Prove that f is linear.

Turn over

9. Find the critical points of $f(x) = 3x^4 - 8x^3 + 6x^2 - 1$.

10. Find the inflection points of $f(x) = x^2 + (1/x)$.

11. Using limits of Riemann sums, establish the equation $\int_a^b c\,dx = c(b-a)$, where c is a constant.

12. Find $\int_0^2 \left(\frac{t^2}{4} - 7t + 5\right) dt$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Find $\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$.

14. Show that the line $y = mx + b$ is its own tangent at any point $(x, mx + b)$ on the line.

15. Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 1 ft/s. How fast is the area of the spill increasing when the radius of the spill is 20 ft ?

16. Use implicit differentiation to find d^2y/dx^2 if $5x^3 - 7y^2 = 10$.

17. Find the maximum and minimum points and values for the function $f(x) = (x^2 - 8x + 12)^4$ on the interval $[-10, 10]$.

18. Use l'Hôpital's Rule to find $\lim_{x \to 0} \frac{\sin x - x}{x^3}$.

19. Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x, -1 \le x \le 2$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. (a) Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

(b) Find $\frac{dy}{dx}$ if $y = \int_1^{x^2} \cos t \, dt$.

21. (a) Find the absolute extrema of $h(x) = x^{2/3}$ on $[-2, 3]$.

(b) Find the volume of the solid generated by the revolution about the x-axis of the loop of the curve $y^2 = x^2 \frac{3a - x}{a + x}$.

(c) Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$.

(1 × 11 = 11 marks)

D 12645-A (Pages : 4) Name.......................................

Reg. No....................................

FIRST SEMESTER (CBCSS–UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2021 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(Multiple Choice Questions for SDE Candidates)

1. What is the value of x after this statement, assuming initial value of x is 5 ? 'If x equals to one then $x = x + 2$ else $x = 0$'.

   (A) 1. (B) 3.

   (C) 0. (D) 2.

2. The compound statement $A \rightarrow (A \rightarrow B)$ is false, then the truth values of A, B are respectively :

   (A) T, T. (B) F, T.

   (C) T, F. (D) F, F.

3. $(A \vee F) \vee (A \vee T)$ is always :

   (A) True. (B) False.

4. Let P (x) denote the statement "$x > 7$". Which of these have truth value true ?

   (A) P (0). (B) P (4).

   (C) P (6). (D) P (9).

5. When to proof $P \rightarrow Q$ true, we proof P false, that type of proof is known as :

   (A) Direct proof (B) Contrapositive proofs.

   (C) Vacuous proof. (D) Mathematical Induction.

6. In the principle of mathematical induction, which of the following steps is manda-tory ?

   (A) Induction hypothesis. (B) Inductive reference.

   (C) Induction set assumption. (D) Minimal set representation.

7. For any positive integer m,... is divisible by 4.

   (A) $5m^2 + 2$. (B) $3m + 11$.

   (C) $m^2 + 3$. (D) $m^3 + 3m$.

8. Let gcd $(a, b) = d$. If c divides a and c divides b, then :

   (A) $c \leq d$. (B) $c \geq d$.

   (C) $c = 1$. (D) Cannot determine c with the given information.

9. If a and b are relatively prime then :

(A) $a \mid b$.
(B) $b \mid a$.
(C) $\gcd(a, b) = 1$.
(D) $\text{lcm}(a, b) = 1$.

10. The product of any three consecutive integers is divisible by :

(A) 36.
(B) 9.
(C) 6.
(D) 8.

11. If a is an odd integer then $\gcd(3a, 3a + 2)$ equals :

(A) 3.
(B) 5.
(C) 1.
(D) 2.

12. If a, b are integers such that $a > b$ then $\text{lcm}(a, b)$ lies in :

(A) $a > \text{lcm}(a, b) > b$.
(B) $a > b > \text{lcm}(a, b)$.
(C) $lcm(a, b) \geq a > b$.
(D) None of the mentioned.

13. $(1001111)_2$ =---

(A) 79.
(B) 89.
(C) 69.
(D) 99.

14. The linear Diophantine equation $ax + by = c$ has a solution if and only if :

(A) $\gcd(a, c) \mid b$.
(B) $\gcd(a, b) \mid c$.
(C) $\gcd(c, b) \mid a$.
(D) $c \mid \gcd(a, b)$.

15. A composite number n for which $a^n \equiv a(\text{mod}n)$ is called :

(A) A pseudoprime.
(B) A prime.
(C) A pseudoprime to the base a.
(D) An absolute pseudoprime.

16. The composite numbers n that are pseudoprime to every base a are called :

(A) A pseudoprime.
(B) A prime.
(C) A pseudoprime to the base a.
(D) An absolute pseudoprime.

17. If a is a solution of $P(x) \equiv 0 \pmod{n}$ and $a \equiv b \pmod{n}$, then :

(A) ab is also a solution.
(B) $a + b$ is also a solution.
(C) $a - b$ is also a solution.
(D) b is also a solution.

18. *f* a is an odd integer then $a^2 - 1$ is :

(A) A multiple of 7. (B) A multiple of 8.

(C) A multiple of 11. (D) A multiple of 9.

19. If a is an odd integer, then $a^2 \equiv --- \pmod 8$:

(A) 1. (B) 2.

(C) 3. (D) 4.

20. The solution of $25x \equiv 15 \pmod{29}$ is :

(A) $x \equiv 18 \pmod{29}$. (B) $x \equiv 29 \pmod{29}$.

(C) $x \equiv 18 \pmod{19}$. (D) $x \equiv 17 \pmod{19}$.

D 12645 (Pages : 3) Name....................................

Reg. No..................................

FIRST SEMESTER (CBCSS–UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2021 Admissions)

Time : Two Hour and a Half Maximum : 80 Marks

Section A

Answer atleast **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall ceiling 30.

1. Verify that $p \vee p \equiv p$ and $p \wedge p \equiv p$.

2. Let P(x)denote the statement "$x > 3$." What is the truth value of the quantification $\exists x \, P(x)$, where the universe of discourse is the set of real numbers ?

3. State the barber paradox presented by Bertrand Russell in 1918.

4. Prove that if n is a positive integer, then n is odd if and only if $5n + 6$ is odd.

5. Prove the following formula for the sum of the terms in a "geometric progression" :

$$1 + r + r^2 + \ldots + r^n = \frac{1 - r^{n+1}}{1 - r}.$$

6. Let a and b positive integers such that $a \mid b$ and $b \mid a$. Then prove that $a = b$.

7. Briefly explain Mahavira's puzzle.

8. Find the number of positive integers ≤ 2076 and divisible by neither 4 nor 5.

9. Prove that every composite number n has a prime factor $\leq \lfloor \sqrt{n} \rfloor$.

10. Show that any two consecutive Fibonacci numbers are relatively prime.

Turn over

11. Let a and b be integers, not both zero. Then prove that a and b are relatively prime if and only if there exist integers α and β such that $1 = \alpha a + \beta b$.

12. Prove that if $a|$ and $b|c$, and $(a, b) = 1$, then $ab|c$.

13. Prove that every integer $n \geq 2$ has a prime factor.

14. Let f_n denote the n^{th} Fermat number. Then prove that $f_n = f_{n-1}^2 - 2f_{n-1} + 2$, where $n \geq 1$.

15. Express gcd (28, 12) as a linear combination of 28 and 12.

(10 × 3 = 30 marks)

Section B

Answer atleast **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall ceiling 30.

16. Show that the propositions $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

17. Show that theassertion "All primes are odd" is false.

18. Let b be an integer ≥ 2. Suppose $b + 1$ integers are randomly selected. Prove that the difference of two of them is divisible by b.

19. If p is a prime and $p | a_1 a_2 \ldots a_n$, then prove that $p | a_i$ at for some i, where $1 \leq i \leq n$.

20. Show that $11 \times 14n + 1$ is a composite number.

21. There are infinitely many primes of the form $4n + 3$.

22. Show that $2^{11213} - 1$ is not divisible by 11.

23. Prove that if $n \geq 1$ and gcd $(a, n) = 1$,then $a^{\phi(n)} \equiv 1 \pmod{n}$.

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) State six standard methods for proving theorems and briefly explain any two of them with the help of examples.

    (b) Using the laws of logic simplify the Boolean Expression $(p \wedge \neg q) \vee q \vee (\neg p \wedge q)$.

25. (a) Prove that there is no polynomial $f(n)$ with integral coefficients that will produce primes for all integers n.

    (b) State the prime number theorem and find six consecutive integers that are composites.

26. (a) State and prove Fundamental Theorem of Arithmetic.

    (b) Find the largest power of 3 that divides 207!

27. (a) Let p be a prime and a any integer such that $p \mid a$. Then show that the least residues of the integers $a, 2a, 3a, \ldots, (p-1)a$ modulo p are a permutation of the integers

    $1, 2, 3, \ldots, (p-1)$.

    (b) Find the remainder when 24^{1947} is divided by 17.

(2 × 10 = 20 marks)

D 12511-A **(Pages : 4)** **Name..**

Reg. No......................................

FIRST SEMESTER (CUCBCSS-UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

ME 1C 01—MATHEMATICAL ECONOMICS

(2016—2018 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 20 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

ME 1C 01—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

1. The demand for essential good is :

   (A) Elastic. (B) Inelastic.

   (C) Relatively elastic. (D) Relatively inelastic.

2. The ratio of the percentage change in sales of one product to the percentage change in price of another product is called :

   (A) Arc price elasticity. (B) Point price elasticity.

   (C) Cross price elasticity. (D) Income elasticity.

3. Perfectly elastic supply curve is :

   (A) Parallel to X axis.

   (B) Parallel to Y axis.

   (C) Sloping curve passing through the origin.

   (D) None of the these.

4. The variables in the demand function which are related to price are :

   (A) Own price of the product. (B) Price of compliment.

   (C) Price of substitutes. (D) All the above.

5. When the purchases of goods increase with rising levels of income, such goods are called :

   (A) Inferior goods. (B) Normal goods.

   (C) Giffen goods. (D) Laxurious goods.

6. In a demand curve price is measured along the :

   (A) Vertical axis. (B) Horizontal axis.

   (C) Both (A) and (B). (D) None.

7. A shift of the whole demand curve to the left indicates :

   (A) An increase in demand. (B) Decrease in demand.

   (C) Demand is constant. (D) None.

8. The average responsiveness of the dependent variable to changes in the independent variable over some interval is measured by :

   (A) Point elasticity. (B) Arc elasticity.

   (C) Supply elasticity. (D) None.

9. When the cross price elasticity $e_c = 0$, the goods are ?

   (A) Substitutes. (B) Complements.

   (C) Normal. (D) Independent.

10. For a less elastic supply curve, η_p^s is :

   (A) Less than 1. (B) More than 1.

   (C) Equal to 1. (D) Zero.

11. The total of the quantities demanded by all consumers in an economy at each price is called :

   (A) Market demand curve. (B) Market supply curve.

   (C) Market equilibrium. (D) None of these.

12. When $|e_p| < 1$, the demand is :

   (A) Elastic. (B) Inelastic.

   (C) Unitarily elastic. (D) None.

13. A given percentage change in price results in an equal percentage change in sales, indicates :

   (A) Unitary price elasticity. (B) Inelastic price elasticity.

   (C) Elastic price elasticity. (D) None.

14. Profit is equal to total revenue minus :

   (A) Explicit costs. (B) Implicit costs.

   (C) Implicit costs and explicit costs. (D) Wages and rents.

15. The elasticity of demand η_d in terms of AR and MR is :

   (A) $\frac{AR - MR}{AR}$. (B) $\frac{AR - MR}{MR}$.

   (C) $\frac{MR}{AR - MR}$. (D) $\frac{AR}{AR - MR}$.

16. A distinction between cost of production and expenses of production is made by :

(A) Engel. (B) Marshall.

(C) Keynes. (D) None of these.

17. When marginal cost is greater than average cost, the total cost elasticity will be :

(A) Greater than 1. (B) Less than 1.

(C) Equal to 1. (D) None.

18. The equation $\eta_p = \frac{AR}{AR - MR}$ indicates that marginal revenue as a function of :

(A) Elasticity of demand. (B) Average revenue.

(C) Both (A) and (B) and None. (D) None.

19. The first order condition for utility maximization gives :

(A) $\frac{MU_1}{MU_2} = \frac{P_1}{P_2}$. (B) $\frac{MU_1}{P_2} = \frac{MU_1}{P_1}$.

(C) $\frac{MU_2}{MU_1} = \frac{P_1}{P_2}$. (D) None of the above.

20. The point at which the marginal utility first increases, reaches the maximum, then diminishes is called :

(A) Point of inflexion. (B) Minimum point.

(C) Saturation point. (D) None of these.

D 12511 (Pages : 4) Name.....................................

Reg. No.....................................

FIRST SEMESTER (CUCBCSS-UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

ME 1C 01—MATHEMATICAL ECONOMICS

(2016—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. The demand for essential good is :

   (a) Elastic. (b) Inelastic.

   (c) Relatively elastic. (d) Relatively inelastic.

2. When the demand curve shifts to right, there occurs :

   (a) An increase in demand.

   (b) Decrease in demand.

   (c) Demand is constant.

   (d) None.

3. When the cross price elasticity is positive, the products are ?

   (a) Substitutes. (b) Compliments.

   (c) Normal. (d) None of these.

4. Profit is equal to total revenue minus :

   (a) Explicit costs. (b) Implicit costs.

   (c) Implicit costs and explicit costs. (d) Wages and rents.

5. Total variable cost plus total fixed cost gives :

   (a) Total cost. (b) Average cost.

   (c) Marginal cost. (d) None of these.

6. The ratio of the proportionate change in average cost to the proportionate change in output is called elasticity of :

(a) Marginal cost. (b) Total cost.

(c) Average cost. (d) None.

7. The rate at which the consumer trades off one commodity for another is called :

(a) Marginal rate of technical substitution.

(b) Marginal rate of substitution.

(c) Equi-marginal utility.

(d) None of these.

8. The indifference curves for perfect substitutes are :

(a) Straight lines. (b) L-shaped.

(c) Curves. (d) Concave from above.

9. An attribute possessed by a commodity to satisfy a human want, to yield satisfaction to consumer is termed as :

(a) Utility. (b) Preference.

(c) Want. (d) None of these.

10. Let $50 + 10kk^2$ be a production function, where k represents capital. Then the marginal productivity when $k = 1$ is :

(a) 116. (b) 16.

(c) 58. (d) 8.

11. For the function $y = 4x_1x_2 + x_1^3 + 2x_2^2$, the partial derivative $\frac{\partial y}{\partial x_1}$ is :

(a) None of the following. (b) $4x_1 + 3x_1^2$.

(c) $4x_1 + 4x_2$. (d) $4x_2 + 3x_1^2$.

12. Behaviour of the function defined by $y = x^3 - 7x^2 + 6x - 2$ at $x = 4$ is :

(a) Increasing. (b) Stationary.

(c) Decreasing. (d) None.

Part B

Answer any **six** *questions in two or three sentences.*
Each question carries 3 *marks.*

13. What is 'Demand Curve' ?

14. Explain briefly the 'Law of Supply'

15. What is 'Elasticity' ? Write any two types of elasticity.

16. What is the difference between cost of production and expenses of production ?

17. What is the nature of short term cost functions ?

18. Write a short note on marginal rate of substitution.

19. What are the maxima and minima conditions of consumer's equilibrium.

20. Find all the two first order partial derivatives of $z = 3x^2y^3$.

21. Find the marginal revenue function, given the average revenue function AR = 10 – 0.5q.

(6 × 3 = 18 marks)

Part C

Answer any **six** *questions from the following.*
Each question carries 5 *marks.*

22. What are the properties of price elasticity of demand ?

23. Give the nature and property of a demand function for a normal good.

24. Distinguish between point elasticity and arc elasticity.

25. Suppose the price p and the quantity q of acommodity is related by the equation $q = 30 - 4p - q^2$. Find the elasticity of demand at $p = 2$.

26. Explain the concept of rate of commodity substitution.

27. Find the maximum profit : Given TR = 1400Q – 6Q^2, TC = 1500 + 80 Q.

28. Given $z = 8x^2 + 3y^2$, $x = 4ty = 5t$. Find $\frac{dz}{dt}$.

30. Find the critical points, given $z = 2y^3 - x^3 + 147x - 54y + 12$.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions from the following.*
Each question carries 10 *marks.*

31. Explain Demand function.

32. (a) Cost function is given by $\pi = a + bq + cq^2$. Prove that $\frac{d(AC)}{dq} = \frac{MC - AC}{q}$.

(b) Given TR = $1400q - 6q^2$ and TC = $1500 + 80q$. Calculate the maximum profit.

33. Explain briefly the concept of marginal rate of substitution.

34. Find the critical values for minimizing the costs of a firm producing two goods x and y when the total cost function is $c = 8x^2 - xy + 12y^2$ and the firm is bound by contract to produce a minimum combination of goods totaling 42, that is, subject to the constraint $x + y = 42$.

(2 × 10 = 20 marks)

D 12510 (Pages : 4) Name.......................................

Reg. No...................................

FIRST SEMESTER (CUCBCSS-UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MAT 1C 01—MATHEMATICS

(2016—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type Questions)

Answer **all** *questions (1-12).*
Each question carries 1 mark.

1. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\cos x}{\frac{\pi}{2} - x} = \ldots\ldots\ldots\ldots$

2. State sandwich theorem for limits.

3. What is a jump discontinuity ?

4. State Max-Min theorem for continuous functions.

5. Define point of inflection of a function $y = f(x)$.

6. What are the asymptotes of $y = \tan x$.

7. If $y = x^4 - 3\cos x + e^x$, $dy = \ldots\ldots$

8. Find the critical points of $f(x) = x^3 + 12x + 5$, in $[-3, 3]$.

9. When we say that a function $y = f(x)$ is concave up in $[a, b]$?

10. If f and g are two monic polynomials (leading coefficient is 1) of same degree, what is $\lim\limits_{x \to \infty} \dfrac{f(x)}{g(x)}$?

11. What is Riemann sum for a function f on the interval $[a, b]$.

12. If $f(x) > 0$, what is the area of the region bounded by the the graph of f, the x-axis and the ordinates $x = a$ and $x = b$.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions (13-24).*
Each question carries 2 mark.

13. Using formal definition of limit, show that $\lim_{x \to 1} (5x - 3) = 2$.

14. Using intermediate value theorem, show that there is a real number which is exactly one less than its cube.

15. Find left and right limits of the function f at $x = 2$, where $f(x) = \begin{cases} 3 - x & x \le 2 \\ \frac{x}{2} + 1, & x > 2. \end{cases}$

16. Let $f(x) = -x^3 + 12x + 5, x \in [-3, 3]$. Where does the function f assume extreme values and what are these values ?

17. Define removable discontinuity and give an example.

18. Verify Rolle's theorem for the function $f(x) = (x - 2)(x - 3)$ on the interval $[2, 3]$.

19. Find the horizontal/vertical asymptotes of the graph of $f(x) = \frac{x^3 - 1}{x^2 - 1}$.

20. Find the average of $y = 2x - x^2$ in $[0, 3]$.

21. Find the linearization of $f(x) = 2 - \int_2^{x+1} \frac{9}{1+t} dt$.

22. Find dy/dx if $y = \int_x^1 \sqrt{1 + t^2}\, dt$. Explain main steps in your calculation.

23. Find the area between $y = \sin x$, $x = -\pi/2, x = \pi/2$ and the x-axis.

24. Write down the main steps to find the volumes of solids by the method of slicing.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions (25-33).*
Each question carries 5 marks.

25. Define continuity and different types of discontinuity of a function $f(x)$ at a point a.

26. State Rolle's theorem and verify it for the function $f(x) = \frac{x^3}{3} - 3x + 2$ in the interval $[-3, 0]$.

27. State and prove L'Hospital's Rule (First form).

28. State Mean Value Theorem and verify for the function $y = 2x^3 - 3x^2$ in $[1, 2]$.

29. Evaluate $\lim_{x \to 0} \left[\frac{1}{x^2} - \cot^2 x\right]$.

30. Express the solution of the following initial value problem as an integral. $y' = \tan x, y(1) = 5$.

31. If f is a continuous function on $[a, b]$, show that :

$$(\min f) \cdot (b - a) \le \int_a^b f(x)\, dx \le (\max f) \cdot (b - a).$$

32. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

33. Find the volume, by slicing, of the solid which lies between planes perpendicular to the x-axis at $x = 0$ and $x = 4$. The cross sections perpendicular to the axis on the interval $[0, 4]$ are squares whose diagonals run from the parabola $y = -\sqrt{x}$ to the parabola $y = \sqrt{x}$.

(6 × 5 = 30 marks)

Part D (Essay Questions)

Answer any **two** *questions (34-36).*
Each question carries 10 marks.

34. Trace the curve $(x^2 + y^2)\, x = a\,(x^2 - y^2),\, a > 0$.

35. State and prove Mean Value Theorem.

36. State and prove Fundamental Theorem of Calculus (Part 1).

(2 × 10 = 20 marks)

D 12509-A (Pages : 4) Name.....................................

Reg. No...................................

FIRST SEMESTER (CUCBCSS-UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MAT 1B 01—FOUNDATIONS OF MATHEMATICS

(2016—2018 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 1B 01—FOUNDATIONS OF MATHEMATICS

(Multiple Choice Questions for SDE Candidates)

1. If A and B are two sets such that $A \subseteq B$, then $A \cup B$ is :

   (A) A.
   (B) B.
   (C) $\varnothing$.
   (D) $A \cap B$.

2. For any two sets A and B, A – B = ————.

   (A) B – A.
   (B) $A \cap \bar{B}$.
   (C) $\bar{A} \cap B$.
   (D) $\bar{A} \cap \bar{B}$.

3. For any two sets A and B, A – B defined by :

   (A) $\{x : x \in A \text{ and } x \in B\}$.
   (B) $\{x : x \in A \text{ and } x \notin B\}$.
   (C) $\{x : x \notin A \text{ and } x \in B\}$.
   (D) $\{x : x \in A \text{ or } x \in B\}$.

4. The number of subsets of the set A = {x : x is a day of the week} is :

   (A) 7.
   (B) 2^6.
   (C) 2^7.
   (D) 14.

5. If $A \cap B = A$ and $A \cup B = A$, then :

   (A) $A \subseteq B$.
   (B) $B \subseteq A$.
   (C) A = B.
   (D) None of these.

6. A, B, C are three sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$ then :

   (A) A = B.
   (B) B = C.
   (C) A = C.
   (D) A = B = C.

7. If $(2x, x + y) = (8, 6)$ then y = ————.

   (A) 4.
   (B) 2.
   (C) – 2.
   (D) 5.

8. Let A = $\{a, b, c\}$ then the range of the relation R = $\{(a, b), (a, c), (b, c)\}$ defined on A is :

(A) $\{a, b\}$. (B) $\{c\}$.

(C) $\{a, b, c\}$. (D) $\{b, c\}$.

9. Let A = $\{1, 2, 3\}$ and R = $\{(1, 1), (2, 2), (1, 2), (2, 1), (2, 3)\}$ be a relation on A. Then R is :

(A) Reflexive. (B) Symmetric.

(C) Antisymmetric. (D) None of these.

10. If $f(x) = x^2 - 3x + 1$ and $f(2\alpha) = 2f(\alpha)$, then α = ————————.

(A) 3. (B) $\frac{1}{\sqrt{3}}$.

(C) $\frac{1}{\sqrt{2}}$ or $\frac{-1}{\sqrt{2}}$. (D) None of these.

11. Which of the following is not a Proposition ?

(A) Toronto is the Capital of India. (B) 1 + 1 = 2.

(C) $x + y = z$. (D) You pass the course.

12. $p \to q$ is false when :

(A) p is true and q is true. (B) p is true and q is false.

(C) p is false and q is true. (D) p is false and q is false.

13. Determine which of these conditional statements is false :

(A) If 1 + 1 = 2 then 2 + 2 = 5.

(B) If 1 + 1 = 3 then 2 + 2 = 4.

(C) If 1 + 1 = 3 then 2 + 2 = 5.

(D) If monkeys can fly, then 1 + 1 = 3.

14. Let Q (x, y) denote the statement "$x = y + z$". Then Q (3, 0) is :

(A) 3 = 0. (B) 0 = 3 + 3.

(C) 3 = 0 + 3. (D) 3 = 3 + 0.

15. Which of the following is not an expression for $p \to q$:

(A) If p, q.

(B) q when p.

(C) p follows from q.

(D) p implies q.

16. The bitwise AND of 01 and 11 is :

(A) 01.

(B) 11.

(C) 10.

(D) 00.

17. The Compound Propositions p and q are called logically equivalent if $p \leftrightarrow q$:

(A) A tautology.

(B) A contradiction.

(C) A contingency.

(D) None of these.

18. The tautology $(p \wedge (p \to q)) \to q$ is the basis of the true inference called ————.

(A) Law of detachment.

(B) Implication.

(C) Conjection.

(D) Resolution.

19. The solutions of the equation $x^2 + y^2 = z^2$, where x, y, z are integers are called :

(A) Pythagorean triples.

(B) Fermat triples.

(C) Perfect squares.

(D) Fermat squares.

20. An invalid argument form often used incorrectly as a rule of inference is :

(A) Proof.

(B) Conjecture.

(C) Theorem.

(D) Fallacy.

D 12509 (Pages : 3) Name.......................................

Reg. No.................................

FIRST SEMESTER (CUCBCSS-UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MAT 1B 01—FOUNDATIONS OF MATHEMATICS

(2016—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. Fill in the blanks : If A = {1, 2, 3, 4, 5, 6, 7} and B = {5, 6, 7, 8, 9} are any two sets, $A \oplus B =$ ————————.

2. Find n (A \B), if n(A) = 35 and n (A ∩ B) = 15.

3. Define an anti-symmetric relation.

4. Give an example of a ternary relation on $\mathbb{R}$, the set of real numbers.

5. Number of reflexive relations on a set of 3 elements is ————————.

6. Find the domain of the function $f(x) = \sqrt{25 - x^2}$.

7. Define, using indexed collection of sets, the union of arbitrary number of sets.

8. If $\sqrt{25 - x^2} \le f(x) \le \sqrt{25 + x^2}$, find $\lim_{x \to 0} f(x)$.

9. Solve for x and y if $(2x, x - y) = (6, 2)$.

10. The contrapositive of the statement " If I take an umbrella, then it will rain" is ————————.

11. What can you say about $\lim_{x \to 0^-} \frac{1}{x}$.

12. Fill in the blanks : The dual of the set equation $(A \cap U) \cup (B \cap A) = A$ is ————————.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any **nine** *out of twelve questions.*
Each question carries 2 marks.

13. Find the matrix of the relation R from A = {a, e, i, o, u} to B = {1, 100, 1000} given by R = {(a, 1), (a, 100), (e, 1000), (o, 1)}.

14. Evaluate the $\lim_{x \to 0} f(x)$, if $f(x) = \frac{|x| + x}{2x - |x+1|^2}$.

15. Construct a relation f on B = {a, b, c} which is neither symmetric and nor anti-symmetric.

16. Find the function obtained by shifting the graph of $f(x) = |x|$ right by 2 units.

17. Test whether the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = |x|$ is injective or not.

18. Define countable set and give an example.

19. Find all real values of x at which $f(x) = \cot x$ is discontinuous.

20. Find the power set of A = {–1, 0, 1}.

21. Show that $(p \wedge q) \to (p \vee q)$ is a tautology.

22. State the distributive law of disjunction over conjunction.

23. Let R = {(1, 1), (2, 1), (3, 2), (4, 3)}. Find the powers $R^2 = R \circ R$.

24. Show that the set inclusion relation is a partial order on the power set of a set S.

(9 × 2 = 18 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 5 marks.

25. Find f^{-1} by testing its existence when $f(x) = \frac{2x-3}{5x-7}$.

26. Find $g \circ f$ and $f \circ g$, if $f(x) = 2x + 1$ and $g(x) = x^2 - 2$.

27. Show that the interval [0, 1] is uncountable.

28. Discuss the continuity of the function $x \sin(1/x)$ at the origin.

29. If $A = \{[-n, n] : n \in \mathbb{Z}\}$, find $\bigcup_{n \in \mathbb{Z}} [-n, n]$ and $\bigcap_{n \in \mathbb{Z}} [-n, n]$.

30. Draw the graph of the function obtained by shifting the graph of $f(x) = x^2 + 1$ down by one unit.

31. Let $f : A \to B$ and $g : B \to C$ be functions. If $g \circ f$ is one-one, show that f is one-one.

32. Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

33. Translate into English the statement : $\forall x \forall y ((x > 0) \wedge (y < 0) \to (xy < 0))$, where the domain for both variables consists of all real numbers.

(6 × 5 = 30 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 10 *marks.*

34. (a) Show that the relation R on the set of integers given by aRb if $a - b$ is a multiple of 10.

(b) Find the composition $R \circ S$ and the corresponding matrix if R = {(1, 2), (1, 3), (4, 5)} and S = {(3, 1), (5, 4), (2, 1)}.

35. (a) Find the continuous extension of the function $g(x) = \frac{x^2 - 1}{x - 1}, x \neq 1$.

(b) Illustrate by examples the distinction between contrapositive and converse.

36. (a) Find $\lim_{x \to 16} \frac{x^4 - 16}{x - 2}$.

(b) Show that countable union of countable sets is countable.

(2 × 10 = 20 marks)