C 4404 (Pages : 2) Name..................................

Reg. No..................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2021

Statistics

STA 2C 03—REGRESSION ANALYSIS AND TIME SERIES

Time : Two Hours Maximum : 60 Marks

Use of calculator and Statistical table are permitted.

Section A (Short Answer Type Questions)

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. Define regression analysis.
2. What type of linear relation is between x and y when the co-efficient of correlation between them is (i) -1 ; and (ii) $+1$.
3. Define qualitative variable.
4. What is the meaning of zero correlation between x and y ?
5. Define rank correlation.
6. Write modified formula of co-efficient of rank correlation when tied rank appears.
7. The regression line x on y is $2x - 3y + 5 = 0$. Identify the regression co-efficient x on y.
8. The line of regression x on y is $x - 0.3y - 35 = 0$. Identify the value of x when $y = 6$.
9. Define non-linear regression.
10. What is the principle of least squares ?
11. State any *two* uses of time series ?
12. What is the graphical method of measuring trend in a time series ?

(8 × 3 = 24 marks)

Turn over

Section B (Short Essay/Paragraph Type Questions)

Answer at least **five** *questions.*
Each question carries 5 marks.
All questions can be attended.
Overall Ceiling 25.

13. Explain how the scatter diagram helps in correlation and regression analysis.

14. Calculate Pearson's co-efficient of correlation using the following data :

x	1	2	3	4	5
y	3	5	12	15	20

15. Explain the two types of regression lines. What are their uses ?

16. Define regression co-efficients. Prove that the geometric mean of regression co-efficient gives the absolute value of the co-efficient of correlation.

17. The lines of regression x on y and y on x are $3x - 2y + 5 = 0$ and $5x - 4y + 4 = 0$. Find the means of x and y.

18. Explain the method of fitting of regression equation of the form $y = ab^x$ using the data $(x_1, y_1), (x_2, y_2), \ldots\ldots, (x_n, y_n)$.

19. Explain the method of semi-average to measure trend in a time series.

(5 × 5 = 25 marks)

Section C (Essay Type Questions)

Answer any **one** *question.*
The question carries 11 *marks.*

20. Fit a straight line of the form $y = ax + b$ using the following data :

x	2	4	6	8	10	12
y	9	12	18	30	34	42

21. Define time series. Explain various components of time series with suitable examples.

(1 × 11 = 11 marks)

C 4403 (Pages : 3) Name..

Reg. No......................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION, APRIL 2021

Statistics

STA 2C 02—REGRESSION ANALYSIS AND PROBABILITY THEORY

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. Define rank correlation coefficient.
2. What are the limits for Pearson's coefficient of correlation ?
3. If $b_{XY} = -0.90$ and $b_{YX} = -0.40$, find the value of the correlation coefficient.
4. The two lines of regression are $x + 2y - 5 = 0$ and $2x + 3y = 8$. Find the means of x and y.
5. The coefficient of correlation between X and Y is 0.60. Their covariance is 4.80 and variance of X is 9. Find the standard deviation of Y.
6. If $r_{12} = r_{23} = r_{13} = r$, find the value of $r_{12.3}$.
7. Define discrete and continuous sample space.
8. Give axiomatic definition of probability.
9. State multiplication theorem of probability for three events.
10. If A and B are two independent event, then $P(A \cap B)$ = ————.
11. For two mutually exclusive events A and B, given that $P(A \cup B) = \frac{3}{8}$ and $P(B) = \frac{1}{4}$. Find P(A).
12. State any two properties of distribution function of a discrete random variable.

(8 × 3 = 24 marks)

21. (i) The probability that a student passes Statistics test is $\frac{2}{3}$, the probability that he passes both Statistics and Mathematics test is $\frac{14}{45}$. The probability that he passes at least one test is $\frac{4}{5}$. What is the probability that he passes the Mathematics test ?

(ii) A problem in Statistics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved, if all of them try independently ?

(1 × 11 = 11 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended
Overall Ceiling 25.

13. Define correlation and explain different types of correlation.

14. Explain the concept of regression and write down the equations of lines of regression. When do these lines coincide ?

15. Define independent and mutually exclusive events. Can two events be independent and mutually exclusive simultaneously ? Support your answer with an example.

16. On each of 30 items, two measurements X and Y are made and following data were obtained :

$$\sum X = 120, \sum Y = 90, \sum X^2 = 600, \sum Y^2 = 300, \sum XY = 330.$$

Calculate the product moment correlation coefficient and the slope of regression line of Y on X.

17. The ranks of 16 students in Mathematics and Physics are as follows. The numbers within the brackets denote the rank of students in Mathematics and Physics in order.

(1, 1), (2, 10), (3, 3), (4, 4), (5, 5), (6, 7), (7, 2), (8, 6), (9, 8), (10, 11), (11, 15), (12, 9), (13, 14), (14, 12), (15, 16), (16, 13). Calculate the rank correlation coefficient.

18. From a group eight children, five boys and three girls, three children are selected at random. Calculate the probabilities that the selected group contains (i) no girl, (ii) at least one girl and (iii) more girls than boys.

19 A random variable X has the following probability mass function :

x	:	0	1	2	3	4	5	6	7	8
$f(x)$	:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

Find the value of a.

Find $P(X \le 3)$ and $P(0 \le X \le 5)$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. The equations of two lines of regression are $3x + 12y - 10 = 0$ and $3y + 9x - 46 = 0$.

    (i) Identify the regression lines.

    (ii) Obtain the value of x, when $y = 10$ and the value of y, when $x = 3$.

    (iii) If variance of X is 9, find the standard deviation of Y.

C 4402–A (Pages : 4) Name....................................

Reg. No..................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2021

Statistics

STA 2C 02—PROBABILITY THEORY

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 15	Maximum : 15 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 15.
2. The candidate should check that the question paper supplied to him/her contains all the 15 questions in serial order.
3. Each question is provided with choices (a), (b), (c) and (d) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

Section C (Essay Type Questions)

Answer any **one** *question.*
The question carries 11 *marks.*

20. (a) If A and B are two independent events prove that A^c and B^c are also independent.

    (b) Define the mutual independence of three events A, B and C. Also illustrate that the pairwise independence of A, B and C need not imply their mutual independence.

21. (a) Cauchy-Schwartz Inequality for two random variables X and Y.

    (b) Using this inequality prove $-1 \le r_{XY} \le +1$, where r_{XY} is the coefficient of correlation between X and Y.

(1 × 11 = 11 marks)

STA 2C 02—PROBABILITY THEORY

(Multiple Choice Questions for SDE Candidates)

1. Mangoes numbered 1 through 18 are placed in a bag for delivery. Two mangoes are drawn out of the bag without replacement. Find the probability such that all the mangoes have even numbers on them ?

   (A) 43.7 %. (B) 34 %.

   (C) 6.8 %. (D) 9.3 %.

2. Two *t*-shirts are drawn at random in succession without replacement from a drawer containing 5 red *t*-shirts and 8 white t-shirts. Find the probabilities of all the possible outcomes :

   (A) 1. (B) 13.

   (C) 40. (D) 346.

3. A random variable X can take only two values, 2 and 4 i.e., P (2) – 0.45 and P (4) = 0.97. What is the Expected value of X ?

   (A) 3.8. (B) 2.9.

   (C) 4.78. (D) 5.32.

4. A man is dealt 4 spade cards from an ordinary deck of 52 cards. He is given 2 more cards. Let x be the probability that they both are the same suit. Which is true ?

   (A) $0.2 < x \leq 0.3$. (B) $0 < x \leq 0.1$.

   (C) $0.1 < x \leq 0.2$. (D) $0.3 < x \leq 0.4$.

5. Let A and B be events with P (A) = 6/15, P (B) = 8/15, and $P\left((A \cup B)^C\right)$ = 3/15. What is $P(A \cap B)$?

   (A) 1/15. (B) 2/15.

   (C) 3/15. (D) 4/15.

6. A pair of fair dice is tossed. Find the probability that the greatest common divisor of the two numbers is one.

(A) 12/36. (B) 15/36.

(C) 17/36. (D) 23/36.

7. Given E (X) = 5 and E (Y) = – 2, then E (X – Y) is :

(A) 3. (B) 5.

(C) 7. (D) – 2.

8. If C is a constant (non-random variable), then E (C) is :

(A) 0. (B) 1.

(C) $cf(c)$. (D) c.

9. A probability density function be represented by :

(A) Table. (B) Graph.

(C) Mathematical equation. (D) Both (b) and (c).

10. A fair die is rolled. Probability of getting even face or face more than 4 is :

(A) 1/3. (B) 2/3.

(C) 1/2. (D) 5/6.

11. If A and B are two not-independent events, then the probability that both A and B will happen together is :

(A) $P(A \cup B) = P(A)P(B/A)$. (B) $P(A \cup B) = P(A)P(B)$.

(C) $P(A \cup B) = P(A) + P(B)$. (D) $P(A \cup B) = P(A)$.

12. A random variable is said to be ——— if its range set is either finite or countably infinite.

(A) Continuous. (B) Discrete.

(C) Both (a) and (b). (D) None of these.

13. If X is the number of heads obtained in tossing 3 coins, E (X) is :

 (A) 1. (B) 2.

 (C) 1.5. (D) None of these.

14. The distribution of a random variable X where X takes values 0 and 1 with probabilities p and q respectively such that $p + q = 1$ is called :

 (A) Bernoulli Distribution. (B) Binomial distribution.

 (C) Poisson distribution. (D) None of these.

15. If X and Y are two random variables, then E (X + Y) is

 (A) E (X) · E (Y). (B) E (X) + E (Y).

 (C) E (X) + E (Y) – E (X, Y). (D) E (X) + E (Y) – E (X) · E (Y).

C 4402 (Pages : 3) Name..................................

Reg. No..................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2021

Statistics

STA 2C 02—PROBABILITY THEORY

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical table are permitted

Section A (Short Answer Type Questions)

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. Define (a) Random experiment ; (b) Event.
2. If the events A = { 1, 2, 3, 4, 5} and B = {4, 6, 7} are exhaustive events, identify the events : (i) $A \cap B^c$; (ii) $(A \cup B)^c$.
3. If P (A) = 0.4, P (B) = 0.5 and $P(A \cup B) = 0.7$. Find P (A/B^c)
4. State multiplication theorem on probability for two events A and B. If P (A/B) = P (A) = P (B) = 0.4. Find $P(A \cup B)$.
5. Define probability density function and state any *two* of its properties.
6. Obtain the distribution function of X, with p.d.f. $f(x) = 3x^2$, for $0 < x < 1$.
7. Find the value of k, if $f(x) = \left(\frac{k}{2}\right)^x$, for $x = 1, 2, 3...$ is the probability mass function of X.
8. If E (X) = 2, E (X^2) = 8, find V (3X – 2).
9. Obtain the mean and variance of a random variable X with m.g.f. $M_X(t) = (1 - t)^{-1}$, $t < 1$.
10. Define characteristic function of a random variable and state its advantage over m.g.f.

Turn over

11. Find c, if $f(x, y) = c(x + 2y)$, for $x = 1,2$; $- y = 0, 1$ is the joint p.m.f. of (X, Y).

12. Define independence of two random variables X and Y.

(8 × 3 = 24 marks)

Section B (Short Essay/Paragraph Type Questions)

Answer at least **five** *questions.*

Each question carries 5 marks.

All questions can be attended.

Overall Ceiling 25.

13. Mentioning the underlying assumptions clearly, state axiomatic definition of probability.

Using this definition establish $0 \leq P(A) \leq 1$ for an event A.

14. A box contains 3 blue and 2 red balls. Another box contains 2 blue and 3 green balls. One of the identical boxes is selected and two balls were drawn without replacement. It is found that the two balls are blue. What is the probability that only green balls to remain in the selected box ?

15. The p.m.f. of X, $f(x) = \frac{2x^2 - 1}{k}$, for $x = 1, 2, 3, 4$ and $f(x) = 0$ *elsewhere* (i) Find k ; (ii) Write the distribution function $F(x)$.

16. Given the p.d.f. of X as $f(x) = 1$, for $0 < x < 1$. Find the p.d.f. of $Y = -2 \log_e X$.

17. In a game three balls are drawn from a box containing 5 white and 7 black balls. 10 points are given for each white ball drawn and 5 points are given for each black ball drawn. Calculated the expected points per game for a long run of the game.

18. For X with p.d.f. $f(x) = kx(2 - x)$, for $0 < x < 1$; $f(x) = 0$, elsewhere. Obtain (a) k ; (b) Mean and variance of X.

19. For two random variables X and Y, prove that (i) $V(X - Y) = V(X) + V(Y) - 2\,Cov(X, Y)$; (ii) $Cov(X - a, Y - b) = Cov(X, Y)$, where a and b are two constants.

(5 × 5 = 25 marks)

Section C (Essay Type Questions)

Answer any **one** *question.*
The question carries 11 *marks.*

20. (a) If A and B are two independent events prove that A^c and B^c are also independent.

    (b) Define the mutual independence of three events A, B and C. Also illustrate that the pairwise independence of A, B and C need not imply their mutual independence.

21. (a) Cauchy-Schwartz Inequality for two random variables X and Y.

    (b) Using this inequality prove $-1 \le r_{XY} \le +1$, where r_{XY} is the coefficient of correlation between X and Y.

(1 × 11 = 11 marks)

C 4401 (Pages : 3) Name..................................

Reg. No...............................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2021

Statistics

STA 2B 02—BIVARIATE RANDOM VARIABLE AND PROBABILITY DISTRIBUTIONS

Time : Two Hours and a Half Maximum : 80 Marks

Use of Calculator and Statistical table are permitted.

Section A (Short Answer Type Questions)

Answer at least ten *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 30.

1. What is the expected profit for a seller per item, if he is selling a large number of this item per day with various profits Rs. 2, 3 or 4 according to the buyer with a probability 0.2, 0.35 and 0.45 respectively ?

2. For any random variable X, show that V (X) is always non-negative.

3. Define characteristic function of a random variable X.

4. Define the conditional probability mass function $X/Y = y$, for two discrete random variables X and Y.

5. Find c if the joint p.d.f of (X, Y) is given by $f(x, y) = c, 0 < x < 1, 0 < y < 1$.

6. List any *two* properties of joint p.m.f. $f(x, y)$ of two discrete random variables X and Y.

7. For two independent random variables X and Y, show that V (X – Y) = V (X) + V (Y).

8. For random variables X, show that Cov (X, X) = V (X).

9. Define a Bernoulli random variable.

10. Identify the parameters of X following binomial distribution with mean 12 and variance 3.

11. Obtain the m.g.f. of X following geometric distribution with parameter p.

Turn over

12. If X is the number shown when an unbiased die is thrown. Name the probability distribution of X. Write the p.m.f. of X and find E (X).

13. If the variance of X following Poisson distribution is 5, find P (X = 5).

14. Define negative binomial distribution.

15. State Bernoulli's law of large numbers.

(10 × 3 = 30 marks)

Section B (Short Essay/Paragraph Type Questions)

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. If X and Y are two independent random variables, prove that Cov (X, Y) = 0. Establish that the converse need not true.

17. Explain two methods of finding raw moments of X, when moment generating function of X exists.

18. If $\phi_X(t)$ is the characteristic function of X, prove that $|\phi_X(t)| \leq 1$.

19. Find k, if $f(x, y) = \frac{x+y}{k}$; $x = 0, 1, 2$; $y = 1, 2$ is a joint p.m.f. of (X, Y). Verify whether X and Y are independent.

20. If $f(x, y) = \begin{cases} e^{-(x+y)}; & x > 0 ; y > 0 \\ 0, & \text{elsewhere} \end{cases}$ is the joint p.d.f. of (X, Y), obtain the conditional p.d.fs of X given Y and Y given X.

21. State and prove the additive property of binomial distribution.

22. Define hyper geometric distribution. Find the mean of X following this distribution with parameters N, M and n where M is not exceeding N.

23. If P (X = 2) = P (X = 3), where X follows Poisson distribution, find the m.g.f. of X.

(5 × 6 = 30 marks)

Section C (Essay Type Questions)

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. State and prove Cauchy-Schwartz inequality. Using this, prove that $0 \le r_{X,Y} \le 1$, where $r_{X,Y}$ is the co-efficient of correlation between X and Y.

25. Given the joint p.d.f. of two random variables X and Y as ;

$$f(x, y) = \begin{cases} c(2x + y), & \text{for } 2 < x < 6 ; 0 < y < 5 \\ 0, & \text{otherwise} \end{cases}$$. Find (a) c ; (b) P (3 < X < 4, Y > 2) ; (c) P (X + Y > 4);

and (iv) $f_{X/Y}(x/y)$.

26. State the limiting conditions and prove that under the stated conditions binomial distribution approaches to Poisson distribution.

27. State and prove Chebycheve's inequality. Also find a lower bound to P (2 < X < 6) where the mean and variance of X are 4 and 3 respectively.

(2 × 10 = 20 marks)

C 4187 (Pages : 4) Name.......................................

Reg. No..................................

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION APRIL 2021

Statistics

SG 2C 02—REGRESSION ANALYSIS, TIME SERIES AND INDEX NUMBERS

Time : Three Hours Maximum : 80 Marks

Section A (One Word Questions)

Answer **all** *questions.*
Each question carries 1 mark.

1. ———————— is the GM of Laspeyer's and Paasche's index numbers.

2. If $\rho(x, y) = -1$, there is ————————.

   a) Negative correlation.
   b) Perfect negative correlation.
   c) Non correlation.
   d) None of the above.

3. Regression equation of x on y is ————————.

4. ———————— index number no consideration is given to the quantity.

   a) Unweighted index number.
   b) Paasche's index number.
   c) Weighted index number.
   d) Fisher's index number.

5. Spearman's rank correlation co-efficient is ————————.

6. Simplest method of correlation analysis is ————————.

   a) Karl Pearson's correlation method.
   b) Scatter diagram.
   c) Spearman's rank correlation method.
   d) None of the above.

7. ———————— index number which satisfies both time reversal and factor reversal test.

8. The regression co-efficient $b_{xy} = 0.2$ and the correlation coefficient between x and y 0.4. Then regression co-efficient $b_{yx} =$ ————————.

   a) 0.2.
   b) 0.6.
   c) 0.8.
   d) 0.4.

Turn over

9. Correlation between two variables means they are ——————.

10. Accurate method of trend analysis :

    a) Graphical method.
    b) Semi average method.
    c) Moving average method.
    d) Principle of least square method.

(10 × 1 = 10 marks)

Section B (One Sentence Questions)

Answer **all** *questions.*
Each question carries 2 marks.

11. What is positive correlation ?

12. Define index number.

13. What is multiplicative model ?

14. Compare Laspeyer's and Paasche's method.

15. Define Karl Pearson's co-efficient of correlation.

16. What are the components of time series ?

17. Define time reversal test.

(7 × 2 = 14 marks)

Section C (Paragraph Questions)

Answer any **three** *questions.*
Each question carries 4 marks.

18. Explain scatter diagram.

19. Use the economic data to construct unweighted index number :

Articles		A	B	C	D	E	F
Price	2003	15	17	14	10	11	13
	2006	12	13	14	9	12	10

20. Distinguish between correlation and regression.

21. The two regression lines in a bivariate study is as follows $3x + 12y = 19$ and $3y + 9x = 46$. Obtain $\bar{x}, \bar{y}$ and $r(x, y)$.

22. Explain the models of time series.

(3 × 4 = 12 marks)

30. Calculate correlation co-efficient for the following data :

X	65	66	67	67	68	69
Y	67	68	65	68	72	72

31. Explain the construction of cost of living index numbers.

32. Explain the moving average method. Also give its merits and Demerits.

(2 × 10 = 20 marks)

Section D (Short Essay Questions)

Answer any **four** *questions.*
Each question carries 6 *marks.*

23. Explain the importance of index numbers.

24. Obtain the best fitted parabola for the following data :

X	0	1	2	3	4
Y	1	1.8	3.3	2.5	6.3

25. Find the unweighted index number for the year 1980 based on the year 1970

Commodity		A	B	C	D	E
Price	1970	6	2	4	10	8
	1980	10	2	6	12	12

26. Discuss briefly the methods of correlation analysis.

27. Fit a trend line and estimate the production in the year 1990 for the following data :

Year	1982	1984	1986	1988	1990	1992	1994
Production (in units)	77	81	88	94	94	96	98

28. Illustrate how moving average is calculated for even periods.

(4 × 6 = 24 marks)

Section E (Essay question)

Answer any **two** *questions.*
Each question carries 10 *marks.*

29. Calculate price index number for 2008 with 2004 as base from the following data :

1) Marshall-Edge worth method.

2) Fisher's method.

Commodity	2004		2008	
	Price	Quantity	Price	Quantity
A	4	2	18	6
B	3	5	2	2
C	8	7	24	4

Turn over

C 4185-A (Pages : 4) Name.......................................

Reg. No...................................

SECOND SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, APRIL 2021

Statistics

STS 2C 02—PROBABILITY DISTRIBUTIONS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

STS 2C 02—PROBABILITY DISTRIBUTIONS

(Multiple Choice Questions for SDE Candidates)

1. Let x denote the number of heads obtained when a fair coin is tossed thrice ; then P (x = 1) is :

(A) $\frac{3}{8}$.
(B) $\frac{6}{8}$.
(C) $\frac{2}{8}$.
(D) None of these.

2. Two dice are rolled. Let x be the maximum of the numbers that turn-up then P (x = 5) =

(A) $\frac{3}{36}$.
(B) $\frac{7}{36}$.
(C) $\frac{11}{36}$.
(D) None of these.

3. If x is the number of heads obtained in tossing 3 coins, E (x) =

(A) 1.
(B) 2.
(C) 1.5.
(D) None of these.

4. For a binomial distribution $P(x) = nC_x\, p^x \cdot q^{n-x}$, x = 0, 1, 2, . . . , n, which of the following is incorrect :

(A) $\mu_1^1 = np$.
(B) $\mu_2^1 = n(n-1)p^2 + np$.
(C) $\mu_2 = np$.
(D) $\mu_1 = 0$.

5. The measure of skewness β_1 is related to the central moments as :

(A) $\beta_1 = \frac{\mu_3}{\mu_2}$.
(B) $\beta_1 = \frac{\mu_2}{\mu_3}$.
(C) $\beta_1 = \frac{{\mu_3}^2}{\mu_2^3}$.
(D) $\beta_1 = \frac{\mu_2^3}{\mu_3^2}$.

6. The curve is said to be leptokurtic if :

(A) $\frac{\mu_4}{(\mu_2)^2} - 3 > 0$.
(B) $\frac{\mu_4}{(\mu_2)^2} - 3 < 0$.
(C) $\frac{\mu_4}{(\mu_2)^2} - 3 = 0$.
(D) None of these.

7. In binomial distribution, the variance σ^2 and mean μ are related by :

(A) $\sigma^2 = q\mu$.
(B) $\sigma^2 = \frac{\mu}{q}$.
(C) $q^2\sigma^2 = \mu$.
(D) None of these.

8. The range of the Bernoulli random variable is the set :

(A) $\mathbb{R}$.
(B) (0, 1).
(C) Integers.
(D) None of these.

9. The variance of the uniform (discrete) probability distribution is given by :

(A) $\sum\frac{(x-\mu)^2}{k^2}$.
(B) $\sum\frac{X_i}{k}$.
(C) $\sum x_i$.
(D) k.

10. Following is the probability distribution of a discrete random variable x. What is E $(2x + 3)$?

x	−3	−2	−1	0	1	2	3
P (x)	0.05	0.10	0.30	0	0.30	0.15	0.10

(A) 0.25.
(B) 3.25.
(C) 3.5.
(D) 0.50.

11. For the probability distribution given in problem (10), what is V $(2x + 3)$?

(A) 2.8875.
(B) 11.5500.
(C) 14.5600.
(D) None of these.

12. Let x be the number of heads obtained in four tosses of a fair coin. Find E (x) :

(A) $\frac{16}{16}$.
(B) $\frac{32}{16}$.
(C) $\frac{33}{16}$.
(D) None of these.

13. If $f(x,y) = \frac{1}{8}(6-x-y), 0 < x < 2 ; 2 < y < 4$. Then marginal probability density of x is :
$= 0$, otherwise

(A) $\frac{1}{4}(5-y)$.
(B) $\frac{1}{3}(4-x)$.
(C) $\frac{1}{4}(3-x)$.
(D) $\frac{1}{5}(4-y)$.

14. If $f(x,y) = 2, 0 < x < y < 1$ then marginal probability density function of y is :
$= 0$, otherwise,

(A) $2, 0 < x < y$. (B) $2y, 0 < y < 1$.

(C) 0. (D) 1.

15. Joint cumulative distribution function F (x, y) lies within the values :

(A) -1 and $+1$. (B) -1 and 0.

(C) $-\infty$ and 0. (D) 0 and 1.

16. If X and Y are two independent random variables, the cumulative distribution function F (x, y) is equal to :

(A) $F_1(x) . F_2(y)$. (B) P $(x \leq x, y \leq y)$.

(C) Both (A) and (B). (D) Neither (A) nor (B).

17. If X and Y have joint p.d.f. given by $f(x,y) = \frac{x+y}{21}, x = 1,2,3 \; i \; y = 1,2$. What is ρ_{xy} ?

(A) 0.027. (B) $-$ 0.027.

(C) $-$ 0.0346. (D) 0.0346.

18. Variance of an exponential distribution is :

(A) $\frac{1}{\theta}$. (B) $\frac{1}{\theta^2}$.

(C) $\theta \cdot e^{-\theta x}$. (D) None of these.

19. Which of the following holds true about a Cauchy Distribution ?

(A) Mean does exist. (B) Mean does not exist.

(C) Variance does exist. (D) None of these.

20. Which of the following of a Pareto Distribution exist ?

(A) Mean. (B) M.g.f.

(C) Both (A) and (B). (D) None of these.

C 4185 (Pages : 3) Name.......................................

Reg. No....................................

SECOND SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, APRIL 2021

Statistics

STS 2C 02—PROBABILITY DISTRIBUTIONS

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions in* **one** *word.*
Each question carries 1 *mark.*

Name the following :

1. The coefficient of $\frac{(it)^r}{r!}$ in the expansion of characteristic function.
2. The discrete distribution having memoryless property.
3. The distribution of $\frac{X_1}{X_2}$ where X_1 and X_2 are independent gamma variables with parameters n_1 and n_2 respectively.

Fill up the blanks :

4. If X and Y are two independent variables, the conditional distribution of X given $Y = y, f(x \mid y) =$ ————————.
5. If $X \sim B(n,p)$, the distribution of $y = n - X$ is ————————.
6. If $X \sim N(\mu,\sigma^2)$, the points of inflexion of normal curve are ————————.
7. The variance of the rectangular distribution $f(x) = \frac{1}{b-a}; a \le x \le b$ is equal to ————————.

Write true or false :

8. If X, Y and Z are three random variables, then $\text{cov}(X+Y,Z) = \text{cov}(X,Z) + \text{cov}(Y,Z)$.
9. For a geometric distribution mean is always less than the variance.
10. The existence of variances of the random variables is not necessary for applying weak law of large numbers.

(10 × 1 = 10 marks)

Turn over

Section B

Answer **all** *questions in one sentence each.*
Each question carries 2 marks.

11. Define mathematical expectation of a random variable.
12. What are the properties of moment generating function ?
13. Define conditional variance.
14. Define joint raw moments for the bivariate distribution.
15. Define geometric distribution.
16. If a random variable $X \sim N(40, 5^2)$, find $P(32 < X \leq 50)$.
17. Define convergence in probability.

(7 × 2 = 14 marks)

Section C

Answer any **three** *questions.*
Each question carries 4 marks.

18. State and prove the addition theorem of expectation.
19. What are the physical conditions for which binomial distribution is used ?
20. Show that in a Poisson distribution with unit mean, mean deviation about mean is $\frac{2}{e}$ times the standard deviation.
21. Define beta distributions of Type I and Type II. Give the relation between them.
22. State and prove Bernoulli's weak law of large numbers.

(3 × 4 = 12 marks)

Section D

Answer any **four** *questions.*
Each question carries 6 marks.

23. What is the expectation of the number of failures before the first success in an infinite series of independent trials with constant probability p of success in each trial ?
24. Two random variables X and Y have the following joint probability density function :

$$f(x,y) = \begin{cases} 2 - x - y, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the covariance between X and Y.

25. Find the m.g.f. of the random variables whose moments are (i) $\mu_r = (r+1)!2^r$ and (ii) $\mu_r = r!$

26. A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as Poisson variate with mean 1.5. Calculate the proportion of days on which (i) neither car is used and (ii) some demand is refused.

27. In a distribution exactly normal, 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of the distribution ?

28. Let X_i assume the values $+i$ and $-i$ with equal probabilities, show that law of large numbers cannot be applied to the independent variables $X_1, X_2, \ldots$

(4 × 6 = 24 marks)

Section E

Answer any **two** *questions.*
Each question carries 10 *marks.*

29. Prove that characteristic function is uniformly continuous.

30. Derive Poisson distribution as a limiting case of binomial distribution.

31. Explain the properties of normal distribution.

32. State and prove the Chebychev's inequality.

(2 × 10 = 20 marks)

C 4184 (Pages : 3) Name.......................................

Reg. No...................................

SECOND SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, APRIL 2021

Statistics

STS 2B 02—BIVARIATE RANDOM VARIABLE AND PROBABILITY DISTRIBUTIONS

Time : Three Hours Maximum : 80 Marks

Use of Calculator and Statistical tables are permitted.

Section A (One Word Questions)

Answer **all** *questions.*
Each question carries 1 mark.

1. For the bivariate distribution function $(X, Y), F_{(X,Y)}(\infty,\infty) =$ ————.
2. If X and Y two independent random variables, E (X/Y) = ————.
3. For the bivariate distribution function $(X, Y), M_{X,Y}(t_1, t_2) =$ ————.
4. The first and second moments of X about 5 is 2 and 6, then V (X) = ————.
5. For two random variables X and Y, $V(aX - bY) =$ ————.
6. Cov $(X + a, Y + b) =$ ————.
7. Expectation of a Bernoulli random variable with parameter p is ————.
8. For a geometric random variable $X, P(X = x+1) = ---- \times P(X = x)$.
9. Random variable following gamma distribution ranges in between ————.
10. If X follow $N(0,1), P(X^2 < 1) =$ ————.

(10 × 1 = 10 marks)

Section B (One Sentence Questions)

Answer **all** *questions.*
Each question carries 2 marks.

11. Show that the 1st central moment about of a random variable X is zero.
12. Define independence of two random variables X and Y.
13. Show that the characteristic function $\phi_x(t)/t = 0$ is equal to 1.
14. Define the coefficient of kurtosis based on moments.

15. Find the values of the parameters of a binomial random variable with mean 4 and variance 3.

16. Find P (X = 0) for a Poisson random variable with mean 5.

17. If $X \sim N(\mu, \sigma)$, suggest the variable following log normal distribution.

(7 × 2 = 14 marks)

Section C (Paragraph Questions)

Answer any **three** *questions.*
Each question carries 4 *marks.*

18. State and prove multiplication theorem on expectation.

19. Find the m.g.f. of X with p.d.f $f(x) = \frac{1}{2}e^{-|x|}, -\infty < x < \infty$.

20. X and Y are independent random variables following Poisson distribution with parameters λ and μ respectively, where V (X) = 2 and V (Y) = 3. Find P (X + Y > 0).

21. If X follow $U[-a, a]$, find a such that $P[X > -1] = 0.7$.

22. Obtain the mean of X follow beta distribution of first kind with parameters p and q.

(3 × 4 = 12 marks)

Section D (Short Essay Questions)

Answer any **four** *questions.*
Each question carries 6 *marks.*

23. The joint p.m.f. of X and Y, $f(x,y) = k(2x+3y), x = 0, 1, 2 ; y = 1, 2, 3.$ Find (i) k ; (ii) conditional distribution of X given Y = 1.

24. For a random variable X, prove that $E(X) = E[E(X \mid Y)]$.

25. The bivariate m.g.f. of X and Y, with joint p.d.f. $f(x,y) = e^{-x-y}, x > 0 ; y > 0$. Verify whether X and Y are independent.

26. Obtain the mode of X following binomial distribution with parameters n and p.

27. If $X \sim P(\lambda)$; prove that $\mu_{r+1} = r\lambda\mu_{r-1} + \lambda\frac{d}{d\lambda}\mu$, where $\mu_{r-1}, \mu_r, \mu_{r+1}$ respectively be the $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ central moments of X.

28. 600 students are appearing a test independently with a probability of success 0.40. Using normal approximation, find the probability of the number of students passes :

(i) Between 260 and 280 inclusive.

(ii) Exactly 250.

(iii) Fewer than 230 and more than 250.

(4 × 6 = 24 marks)

Section E (Essay Questions)

Answer any **two** *questions.*
Each question carries 10 *marks.*

29. Let X and Y are two random variables with joint p.m.f. $f(x,y)=\begin{cases}2\,; 0<x<y<1\\ 0 \quad \text{elsewhere}\end{cases}$.

Find Correlation between X and Y.

30. If X and Y be independent random variables such that $P(X=r)=P(Y=r)=q^r p, r=0,1,2,\ldots$ where, p and q are positive numbers such that $p+q=1$. Find (i) the distribution of X + Y; (ii) The conditional distribution of X given X + Y.

31. Obtain the m.g.f. of X following (i) Exponential distribution with parameter λ. (ii) Gamma distribution with parameters λ and n. Hence prove that sum of n independent and identical exponential random variables follow gamma distribution.

32. Define standard normal distribution. Show that the quartile deviation of $X \sim N(\mu,\sigma)$ is 0.6745 σ.

(2 × 10 = 20 marks)