C 3014–A

(Pages : 6)

Name.......................................

Reg. No....................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE [SPECIAL] EXAMINATION MARCH 2021

Mathematics

MAT 6B 13 (E03)—C PROGRAMMING FOR MATHEMATICAL COMPUTING

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

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MAT 6B 13 (E03)—C PROGRAMMING FOR MATHEMATICAL COMPUTING

(Multiple Choice Questions for SDE Candidates)

1. # Define is a :

   (A) Statement.
   (B) Preprocessor compiler directive.
   (C) Function statement.
   (D) None.

2. Which one among the following statements is true ?

   (A) Every C program ends with an END word.
   (B) Main ( ) is where the program stops its execution.
   (C) A line in a program may have more than one statement.
   (D) All variables in C must be declared for their types before they are used in the program.

3. The modulus operator % can be used :

   (A) Only for floating point data.
   (B) Integer data.
   (C) Exponential functions.
   (D) None.

4. What will be the output of the program :

```
#include <stdio.h>
int main( )
{
    int y =12 ;
    const int x = y ;
    printf( "%d\n", x);
    return 0;
}
```

   (A) 12.
   (B) Garbage value.
   (C) Error.
   (D) None.

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5. What will be printed when the sample code below is printed ?

```
int x =0;
for( x=1; x<4; x++);
printf("x= %d\n", x);
```

(A) 0. (B) 1.

(C) 3. (D) 4.

6. An integer constant in *c* must have :

(A) At least one digit. (B) Commas and blanks.

(C) Decimal points. (D) None.

7.
```
int i =3;
switch =4;
{
default:
    ;
case 2:
i+= 4;
if (i = = 8)
    {
        i++;
if (i = = 9) break ;
        i +=1;
        }
    i- = 3 ;
        break;
    case 8
        i+= 4
        break;
            }
        printf( "i = %d\n",i)'
```

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The output of the code is" :

(A) i = 4. (B) i = 6.

(C) i = 9. (D) i = 8.

8. When applied to a variable, what does the unary operator yield ?

(A) The variable's address. (B) The variable's right value.

(C) The variable's binary form. (D) The variables Value.

9. Text enclosed in a pair of quotation marks is a data type :

(A) Integer. (B) Long.

(C) String. (D) Variable.

10. A memory location with some data that will not change is a :

(A) Constant . (B) Variable.

(C) String. (D) Integer.

11. The keyword used to declare a variable is :

(A) Const. (B) Var.

(C) String. (D) Dim.

12. To declare more than one variable on the same line, separate the variables with :

(A) Commas. (B) Colons.

(C) Pipes. (D) Semicolons.

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13. What is accomplished by the assignment statement "Hourlyplan = txtPay.Text" ?

(A) The value in the variable Hourly Plan is copied into the txtPay.Text text box.

(B) The value in the variable Hourly Plan is compared to the value in txtPay .

(C) The value entered in the txtPay text box is copied into the variable Hourly plan.

(D) The value entered in the txtPay text box is compared to the value in the variable \ Hourly plan.

14. An expression can be a :

(A) Constant.

(B) Variable.

(C) Combination of constants, variables, and arithmetic operations that result in a value.

(D) All of the above.

15. A string literal must be enclosed in :

(A) Quotation marks (").

(B) Single quotes (').

(C) Pound signs (#).

(D) Exclamation points(!).

16. A memory location with data that will change is recognized as :

(A) Variable.

(B) Constant.

(C) Keyword.

(D) None.

17. The format specification for printing an integer number with minimum field width w is :

(A) % wf.

(B) % ld.

(C) % wd.

(D) % wf.

18. The operator ? : is :

(A) A multi-way decision operator.

(B) Two-way conditional operator.

(C) Loop operator.

(D) None.

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19. The while statement is an :

(A) Exit controlled loop statement.

(B) Entry controlled loop statement.

(C) Both (A) and (B).

(D) None.

20. Which of the following statement is correct for For loop ?

(A) Entry controlled loop.

(B) Nesting is allowed.

(C) The starting value of the control variable must be less than its ending value.

(D) Both A and B above.

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C 3014 (Pages : 3) Name..................................

Reg. No..................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE [SPECIAL] EXAMINATION MARCH 2021

Mathematics

MAT 6B 13 (E03)—C PROGRAMMING FOR MATHEMATICAL COMPUTING

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Which symbol is used to denote a pre-processor statement ?
2. What is an executable object code ?
3. What are the different categories of operators in C ?
4. What are the logical operators ? Give meaning of each.
5. Write the syntax of conditional operators.
6. Distinguish between = and = = operators in C.
7. What is the new line character in C ?
8. What is the header file containing getch() function.
9. Write the syntax of the **while** statement in C.
10. What is the use of clrscr () in C ?
11. What is debugging a program ?
12. Name the inventors of the programming language C.

(12 × 1 = 12 marks)

Section B

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

13. What are the steps involved in executing a C program ?
14. What are the character categories in C ?
15. Write a brief note on keywords.

Turn over

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16. Give any two branching statements with syntax.

17. Write the syntax and draw the flow chart of the do..... while statement.

18. Write a short note on header files.

19. Give any two drawback of switch statement.

20. Write a short note on continue statement.

21. Specify escape sequences and their purpose in C.

22. What are the effects of the formats %d, %f, %1f and %g in printf function.

23. What is meant by function prototype ?

24. Specify any two string handling functions and their actions.

(8 × 3 = 24 marks)

Section C

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

25. Explain in detail any three input statements with suitable example.

26. Discus about differenr types of errors in C.

27. Write a program to fin*the roots of a quadratic equation with a provision for inputting coefficients of the equation.

28. Compare break and goto statements.

29. Write a program to print the following out put using for loop :

```
        1
      2   2
    3   3   3
  4   4   4   4
5   5   5   5   5
```

30. What is an array ? Explain the need for array variable.

31. Explain multiple branching statements in C.

32. Explain user defined functions in C.

33. Write a note on reading strings from terminal with suitable example.

(5 × 6 = 30 marks)

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Section D

Answer any **one** *question.*
The question carries 14 *marks.*

34. Explain, in detail with syntax, flow chart, and examples, different looping statement in C.

35. a) Explain the handling of multidimensional arrays with syntax and suitable example.

    b) Briefly explain different categories of functions.

36. a) Explain the working of any two string handling functions with syntax.

    b) Write a program to copy one string to another and count the number of characters copied, (without using string handling functions).

(1 × 14 = 14 marks)

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C 3013 (Pages : 4) Name..

Reg. No....................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE [SPECIAL] EXAMINATION MARCH 2021

Mathematics

MAT 6B 13 (E02)—LINEAR PROGRAMMING

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

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1. What is meant by a Polyhedral Convex set ?
2. State graphical solution algorithm for an LPP.
3. Write down the Mathematical form of a general LPP.
4. What are the slack and surplus variables ?
5. Define a basic solution of a system of m linearly independent equations with n unknowns.
6. State Minimax theorem.
7. Write down the dual of the following LPP :

   Maximize $z = 100x_1 + 15x_2$
   subject to the constraints :
   $$x_1 + 4x_2 \le 20$$
   $$4x_1 + x_2 \le 35$$
   $$x_1, x_2 \ge 0.$$
8. What is the transportation problem ?
9. Define non-degenerate basic feasible solution.
10. What is an unbalanced transportation problem ? How can you convert it into a balanced transportation problem ?
11. Write down the mathematical formulation of an assignment problem.
12. What is the number of basic variables in a balanced transportation problem with m origins and n destinations.

($12 \times 1 = 12$ marks)

Turn over

Section B

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

13. A paper mill produces two grades of paper namely X and Y . Owing to raw material restrictions, it cannot produce more than 400 tons of grade X and 300 tons of grade Y in a week. There are 160 production hours in a week. It requires 0.2 and 0.4 hours to produce a ton of products X and Y, respectively with corresponding profit of Rs. 200 and Rs. 500 per ton. Formulate the above problem as an LPP to maximize the profit and find the optimum product mix.

14. Show that the set $S = \{(x_1, x_2) : 3x_1^2 + 2x_2^2 \leq 6\}$ is convex.

15. Write the characteristics of the standard form of Linear Programming Problem.

16. Write down the following LPP in standard form :

    Maximize $z = 3x_1 + 2x_2 + 5x_3$
    subject to the constraints :

    $$2x_1 - 3x_2 \leq 3$$
    $$x_1 + 2x_2 + 3x_3 \geq 5$$
    $$3x_1 + 2x_3 \leq 2$$
    $$x_1, x_2, x_3 \geq 0.$$

17. State the simplex algorithm for the computation of an optimum solution of LPP.

18. Explain two phase simplex method.

19. Discuss the relationship between primal and its dual.

20. Prove that every loop has an even number of cells.

21. Write all the steps for Vogel's Approximation methods of solving a transportation problem.

22. Explain degeneracy in a transportation problem.

23. In an assignment problem, if we add or subtract a constant to every element of a row or column of the cost matrix, then prove that an assignment plan which minimizes the total cost for the new matrix, also minimizes the total cost for the original cost matrix.

24. Describe a method of drawing minimum number of lines in the context of assignment problem.

(8 × 3 = 24 marks)

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Section C

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

25. Let A be an $m \times n$ matrix, and b an m- vector then show that $\{x \in R^n : Ax \leq b\}$ is a convex set.

26. Solve the following LPP by graphical method :

Minimize $Z = 20x_1 + 10x_2$

subject to the constraints :

$$x_1 + 2x_2 \leq 40$$
$$3x_1 + x_2 \geq 30$$
$$4x_1 + 3x_2 \geq 60$$
$$x_1, x_2 \geq 0.$$

27. Find the degenerate basic feasible solutions of the system :

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$
$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$
$$x_i \geq 0, i = 1, 2, 3, 4.$$

28. State and prove fundamental theorem of linear programming.

29. Use simplex method to solve the following LPP :

Maximize $z = 3x_1 + 2x_2$

subject to the constraints :

$$x_1 + x_2 \leq 4$$
$$x_1 - x_2 \leq 2$$
$$x_1, x_2 \geq 0.$$

30. Use penalty method to

Maximize $z = 3x_1 + 2x_2 + 3x_3$

subject to the constraints :

$$2x_1 + x_2 + x_3 \leq 2$$
$$3x_1 + 4x_2 + 2x_3 \geq 8$$
$$x_1, x_2, x_3 \geq 0.$$

31. Solve the following assignment problem in order to minimize the total cost. The cost matrix given below gives the assignment cost when different operators are assigned to various machines :

	Operators				
Machines	I	II	III	IV	V
A	30	25	33	35	36
B	23	29	38	23	26
C	30	27	22	22	22
D	25	31	29	27	32
E	27	29	30	24	32

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Turn over

32. Prove that the necessary and sufficient condition for the existence of a feasible solution to the transportation problem is that total demand is equal to the total supply.

33. Determine an initial feasible solution to the following transportation problem using the north-west corner rule :

	D_1	D_2	D_3	D_4	Supply
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
Required	6	10	15	4	35

(5 × 6 = 30 marks)

Section D

Answer at least **one** *question.*
The question carries 14 *marks.*

34. Let $A \subseteq \mathbb{R}^n$ be any set. Prove that $\langle A \rangle$, the convex hull of A, is the set of all finite convex combinations of vectors in A.

35. Solve the LPP by simplex method :

Minimize $z = x_1 - 3x_2 + 2x_3$
subject to the constraints :

$$3x_1 - x_2 + 2x_3 \le 7$$
$$-2x_1 + 4x_2 \le 12$$
$$-4x_1 + 3x_2 + 8x_3 \le 10$$
$$x_1, x_2, x_3 \ge 0.$$

36. Find the optimal solution of the following transportation problem whose cost matrix is given as under :

	D_1	D_2	D_3	D_4	Supply
O_1	2	2	2	1	3
O_2	10	8	5	4	7
O_3	7	6	6	8	5
Demand	4	3	4	4	15

(1 × 14 = 14 marks)

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C 3012-A (Pages : 4) Name.......................................

Reg. No...................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE (SPECIAL) EXAMINATION MARCH 2021

Mathematics

MAT 6B 13 (E01)—GRAPH THEORY

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

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INSTRUCTIONS TO THE CANDIDATE

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2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
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MAT 6B 13 (E01)—GRAPH THEORY

(Multiple Choice Questions for SDE Candidates)

1. Let T = (V, E) be a tree such that $|V| \geq 2$. Then T has :

   (A) At least two pendant vertices. (B) At least three 2pendant vertices.

   (C) At least four pendant vertices. (D) No pendant vertices.

2. Let T be a tree. Then which of the following statements are true :

   (A) There exists two paths between every pair of vertices.

   (B) T is minimally connected.

   (C) T has n vertices and n - 2 edges.

   (D) T has n vertices and n edges.

3. Let G = (V, E) be a connected graph and let $v \in V$. Then v is a central point if :

   (A) $e(v) > r(G)$. (B) $e(v) < r(G)$.

   (C) $e(v) = r(G)$. (D) $e(v) \geq r(G)$.

4. Let G = (V, E) be a graph such that $|E| = 8$ and $\deg(v) = 2$ for all $v \in V$. Then $|V|$ = ————.

   (A) 6. (B) 2.

   (C) 8. (D) 10.

5. Let G_1 be a (p_1, q_1) graph and G_2 be a (p_2, q_2) graph. Then $G_1 + G_2$ is :

   (A) $(p_1 + p_2, q_1 + q_2)$ graph. (B) $(p_1 q_1, p_2 q_2)$ graph.

   (C) $(p_1 + p_2, q_1 + q_2 + p_1 p_2)$ graph. (D) $(p_1 - p_2, q_1 - q_2)$ graph.

6. For what values of n is the graph K_n Eulerian ?

   (A) n is a even number. (B) n is an odd number.

   (C) n is a composite number. (D) None of these.

7. $K_{m,n}$ is Eulerian if and only if :

   (A) m and n are even. (B) m and n are odd.

   (C) m is even and n is odd. (D) m is odd and n is even.

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8. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. Let f be an isomorphism from G_1 to G_2. Let $v \in V_1$. Then which of the following statements are true ?

(A) $\deg(v) = \deg(f(v))$.
(B) $\deg(v) > \deg(f(v))$.
(C) $\deg(v) < \deg(f(v))$.
(D) $\deg(v) \neq \deg(f(v))$.

9. The number of perfect matchings in the complete bipartite graph $K_{m,n}$ is :

(A) n.
(B) $n\,!$.
(C) $(n-1)\,!$.
(D) n^2.

10. If G is a plane (p, q) graph with r faces and k components then $p - q + r - k =$ ————.

(A) 1.
(B) 2.
(C) 3.
(D) 4.

11. If a (p_1, q_1) graph and (p_2, q_2) graph are homoeomorphic, then $p_1 - p_2 + q_2 - q_1 =$ ————.

(A) 1.
(B) 0.
(C) 2.
(D) 3.

12. If a (p, q) graph is self dual, then $2p - q =$ ————.

(A) 1.
(B) 0.
(C) 2.
(D) 4.

13. The origin and terminus of a longest path in a tree has degree :

(A) 0.
(B) 1.
(C) 2.
(D) 3.

14. Every block of a tree is :

(A) k_2.
(B) k_3.
(C) k_4.
(D) k_5.

15. Which of the following statements are true ?

(A) Every Hamiltonian graph is 0-connected.
(B) Every Hamiltonian graph is 1-connected.
(C) Every Hamiltonian graph is 2-connected.
(D) Every Hamiltonian graph is 3-connected.

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C 3012 (Pages : 3) Name..................................

Reg. No...............................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE (SPECIAL) EXAMINATION MARCH 2021

Mathematics

MAT 6B 13 (E01)—GRAPH THEORY

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Edges with same pair of vertices are called ————.
2. Give an example of a simple graph with 4 vertices and 5 edges.
3. Define complete bipartite graph.
4. A graph without edges is called ————.
5. Define Vertex degree.
6. Define k-regular graph.
7. Define Forest
8. Define Trail.
9. Define Hamilton cycle.
10. Define connected graph.
11. Define component of a graph.
12. Define face of a plane graph.

(12 × 1 = 12 marks)

Section B

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

13. Draw the graph G = (V, E) where V = $\{a, b, c, d, e\}$, and E = $\{(a, b), (b, c), (c, c), (c, d), (b, d)\ (d, e), (b, e), (b, e)\}$.
14. Define Graph isomorphism.

Turn over

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15. Define subgraphs and spanning subgraphs.

16. Define complement of a graph. Give an example.

17. Define bipartite graph, and complete bipartite graph.

18. Let u and v be distinct vertices of a tree T. Then show that there is precisely one path from u to v.

19. State a neeessary andsufficient condition for a graph to be Euler.

20. What is the smallest integer n such that the complete graph K_n has at least 500 edges.

21. Is Petersen graph Euler ? Why ?

22. State Whitney's theorem.

23. Define and illustrate Jordan curve.

24. Define critical planar graph. Give an example.

(8 × 3 = 24 marks)

Section C

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

25. In any graph G, show that there is an even number of odd vertices.

26. Define and illustrate with example : (1) Vertex deleted subgraph ; (2) Edge deleted subgraph and (3) underlying simple graph.

27. In Petersen graph give a trail which is not a path and a walk which is not a trail.

28. Let G be a k-regular graph, where k is an odd number. Prove that the number of edges in G is a multiple of k.

29. Draw the graph whose adjacency matrix is given by $\begin{bmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 2 & 1 \\ 1 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$.

30. Define incidence matrix of a graph and illustrate it with an example.

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31. A connected graph G is Euler if and only if it is a union of edge disjoint cycles.

32. Let u and v be distinct vertices of a tree T. Then show that there is precisely one path from u to v.

33. Prove that any tree with at least two vertices is a bipartite graph.

(5 × 6 = 30 marks)

Section D

Answer any **one** *question.*
Each question carries 14 *marks.*

34. Let T be a loop less graph. Then show that T is a tree if and only if for any distinct vertices u and v of T, there is precisely one path from u to v.

35. (a) Given any *two* vertices u and v of a graph G, show that every $u - v$ walk contains a $u - v$ path.

    (b) Define join of two graphs. Also show that $K_{m,n} = \overline{K_m} + \overline{K_n}$.

36. (a) Define Hamilton path and Hamiltonian cycle. Also give an example of a graph which is n not Hamiltonian but having a Hamiltonian path.

    (b) Let G be a simple graph with n vertices and let u and v be non-adjacent vertices in G such that $d(u) + d(v) \geq n$. Then show that G is Hamiltonian if and only if G + uv is Hamiltonian.

(1 × 14 = 14 marks)

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C 3011- A (Pages : 4) Name..................................

Reg. No..................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE [SPECIAL] EXAMINATION MARCH 2021

Mathematics

MAT 6B 12—NUMBER THEORY AND LINEAR ALGEBRA

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

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INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
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MAT 6B 12—NUMBER THEORY AND LINEAR ALGEBRA

(Multiple Choice Questions for SDE Candidates)

1. A subset W of a vector space V is a subspace if :

   (A) The sum of elements of W belongs to W.

   (B) The sum and scalar multiples of elements of W belongs to W.

   (C) The scalar multiples of elements of W belongs to W.

   (D) The sum of elements of V belongs to W.

2. The intersection of any set of subspaces of a vector space V is :

   (A) Not a subspace of V. (B) A subspace of V.

   (C) Need not be a subspace of V. (D) A proper subspace of V.

3. If B_1 and B_2 are any *two* finite bases of a vector space V then :

   (A) Number of elements in B_1 > number of elements in B_2.

   (B) Number of elements in B_1 < number of elements B_2.

   (C) Number of elements in B_1 = number of elements in B_2.

   (D) Number of elements in B_1 ≠ number of elements in B_2.

4. If the map $f : V \to W$ is linear. Then the kernel or null space of f is :

   (A) $f^{\to}(W)$. (B) $f^{\to}(\{0_V\})$.

   (C) $f^{\leftarrow}(\{0_W\})$. (D) $f^{\leftarrow}(W)$.

5. Suppose a and b are integers with $a \neq 0$, then $a \mid b$ if :

   (A) $a = bc$, c is some integer. (B) $b = ac$, c is some integer.

   (C) $c = ab$, c is some integer. (D) $ab = 1$.

6. gcd $(-5, 5) =$ ————————.

   (A) 3. (B) 1.

   (C) 5. (D) – 5.

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7. Let a and b be integers, not both zero. Then a and b are relatively prime if and only if there exists integers x and y such that :

(A) $1 = ax + by$.

(B) $2 = ax + by$.

(C) $ab = ax + by$.

(D) $a - b = ax + by$.

8. The number of primes is :

(A) Finite.

(B) Infinite.

(C) Uncountable.

(D) 1729.

9. If $\gcd(a, b) = d$, then $\gcd(a/d, b/d) =$ ————————.

(A) 1.

(B) b.

(C) a.

(D) d.

10. If a is an odd integer then $\gcd(3a, 3a + 2) =$ ————————.

(A) 3.

(B) 5.

(C) 1.

(D) 2.

11. Every positive integer $n > 1$ can be expressed as a product of :

(A) Composite numbers.

(B) Prime numbers.

(C) Even numbers.

(D) Odd numbers.

12. If p is a prime and $p|ab$, then :

(A) $p|a$ only.

(B) $p|b$ only.

(C) $p|a$ or $p|b$.

(D) $p|a$ and $p|b$.

13. *The Sieve of Eratosthenes* is used for finding :

(A) All primes below a given integer.

(B) All even numbers below a given integer.

(C) All odd numbers below a given integer.

(D) All composite numbers below a given integer.

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14. The system of linear congruences $ax + by \equiv r \pmod{n}, cx + dy \equiv s \pmod{n}$ has a unique solution modulo n whenever :

(A) $\gcd(a - c, n) = 1$.

(B) $\gcd(a - d, n) = 1$.

(C) $\gcd(ad - bc, n) = 2$.

(D) $\gcd(ad - bc, n) = 1$.

15. The solution of $25x \equiv 15 \pmod{29}$ is :

(A) $x \equiv 18 \pmod{29}$.

(B) $x \equiv 29 \pmod{29}$.

(C) $x \equiv 18 \pmod{19}$.

(D) $x \equiv 17 \pmod{19}$.

16. A number theoretic function f is said to be multiplicative if whenever $\gcd(m, n) = 1$, then :

(A) $f(mn) = f(m) + f(n)$.

(B) $f(mn) = f(m) f(n)$.

(C) $f(mn) = f(m) - f(n)$.

(D) $f(mn) = f(m)/f(n)$.

17. The domain of number theoretic function is the :

(A) Set of prime numbers.

(B) Set of negative integers.

(C) Set of rational numbers.

(D) Set of positive integers.

18. If n and r are positive integers with $1 \le r < n$, then the binomial co-efficient $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ is :

(A) An integer.

(B) A prime.

(C) Irrational.

(D) An even integer.

19. If n is a prime, then $\phi(n) =$ ——————

(A) n.

(B) $n - 1$.

(C) $n + 1$.

(D) $n - 2$.

20. For $n > 2, \phi(n)$ is :

(A) An odd integer.

(B) An even integer.

(C) Irrational.

(D) Prime.

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C 3011 (Pages : 3) Name..............................

Reg. No..............................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE [SPECIAL] EXAMINATION MARCH 2021

Mathematics

MAT 6B 12—NUMBER THEORY AND LINEAR ALGEBRA

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Define relatively prime integers. Give an example.
2. State the relationship between gcd and lcm of two positive integers.
3. Give an example for a Diophantine equation in two variables.
4. Write the canonical form of 720.
5. State Wilson's theorem.
6. Define an arithmetic multiplicative function.
7. Find $\phi(25)$.
8. Define subspace spanned by a non-empty subset of a vector space.
9. Give the canonical basis of $\mathbb{R}^n$.
10. Find the kernel of the differentiation map $D : \mathbb{R}_n[x] \to \mathbb{R}_n[x]$.
11. Define nullity of a linear map.
12. State the Dimension theorem.

(12 × 1 = 12 marks)

Section B

Answer at least **eight** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 48.

13. Show that the square of any odd positive integer is of the form $8k+1$.
14. Prove that if $d|n$, then $(2^d - 1) | (2^n - 1)$.
15. Find gcd (143, 227).
16. Prove that there are infinite number of primes.

Turn over

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17. Let a and b be arbitrary integers. Prove that $a \equiv b \pmod{n}$ if and only if a and b leave the same non-negative remainder when divided by n.

18. Find the units digit of 3^{100}.

19. Show that $28! + 233$ is divisible by 31.

20. Prove that $\phi(n) = n - 1$ if and only if n is prime.

21. Find the highest power of 5 dividing 1000!.

22. Verify whether the set of all $n \times n$ symmetric matrices over a field F is a subspace of the space of all $n \times n$ matrices over F.

23. Prove that if S is a subset of a vector space V, then $\langle S \rangle = \text{span } S$.

24. Show that the set of vectors {(1, 0, –1), (1, 2, 1), (0, –3, 2)} is a basis of $\mathbb{R}^3$.

25. Show that the mapping $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x, y) = (x + y, x - y)$ is linear.

26. Find the rank and nullity of the linear map $f : \mathbb{R}^3 \to \mathbb{R}$ given by $f(x, y, z) = x$.

(8 × 6 = 48 marks)

Section C

Answer at least **five** *questions.*
Each question carries 9 *marks.*
All questions can be attended.
Overall Ceiling 45.

27. Let a and b be integers, not both zero. Prove that a and b are relatively prime if and only if there exist integers x and y such that $1 = ax + by$. Deduce that if $a \mid bc$ with $\gcd(a, b) = 1$, then $a \mid c$.

28. Prove the relationship between gcd and lcm of two positive integers.

29. Solve the system of congruences $x \equiv 1 \pmod{3}, x \equiv 2 \pmod{5}, x \equiv 3 \pmod{7}$.

30. State Fermat's Theorem. Is its converse true ? Justify your answer.

31. Prove that τ and σ are multiplicative functions.

32. Define a vector space. Verify whether the set $\mathbb{R}^2$ a vector space under the operations

$$(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, 0) \text{ and } c(x_1, x_2) = (cx_1, 0).$$

33. Show that every linearly independent subset of a finite dimensional vector space can be extended to form a basis of the vector space.

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34. Show that the linear mapping $f:\mathbb{R}^3 \to \mathbb{R}^3$ given by $f(x,y,z)=(x+y+z,2x-y-z,x+2y-z)$ is both surjective and injective.

35. If the set {(1, 1, 0), (1, 0, 1), (0, 1, 1)} is a basis of $\mathbb{R}^3$ and if $f:\mathbb{R}^3 \to \mathbb{R}^2$ is linear such that $f(1, 1, 0) = (1, 2)$, $f(1, 0, 1) = (0, 0)$, $f(0, 1, 1) = (2,1)$, determine f completely.

(5 × 9 = 45 marks)

Section D

Answer any **one** *question.*
The question carries 15 *marks.*

36. State and prove Division algorithm. Illustrate with an example.

37. Prove that the quadratic congruence $x^2+1\equiv 0 \pmod{p}$, where p is an odd prime, has a solution if and only if $p\equiv 1 \pmod{4}$.

38. Let V and W be vector spaces over a field F and $f: V \to W$ be a linear map. Prove that :

(a) Range (f) is a subspace of W.

(b) If V is spanned by a finite subset S of V, then Range (f) is spanned by f (S).

(c) If X is a finite dimensional subspace of V such that $X \cap \ker f = \{0\}$ and if B = $\{v_1, v_2, v_3, \ldots, v_n\}$ is a basis X, then f (B) is a basis of f (X).

(1 × 15 = 15 marks)

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C 3010-A

(Pages : 4)

Name.......................................

Reg. No...................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE [SPECIAL] EXAMINATION MARCH 2021

Mathematics

MAT 6B 11—NUMERICAL METHODS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B) and (C) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

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MAT 6B 11—NUMERICAL METHODS

(Multiple Choice Questions for SDE Candidates)

1. Approximate value = True value + ——————.

   (A) Expected value. (B) Error.

   (C) Absolute value.

2. —————— is a root of the equation $x^2 + 4x + 4 = 0$.

   (A) 2. (B) -2.

   (C) 3.

3. A real root of the equation $x^2 - 2x - 3 = 0$ lies in ——————.

   (A) $[2,4]$. (B) $[5,6]$.

   (C) $[1,2]$.

4. $\mathrm{E} f(x) =$ ——————.

   (A) $f(x+h)$. (B) $f(x-h)$.

   (C) $f(x)$.

5. $\Delta =$ ——————.

   (A) $\mathrm{E} - 1$. (B) E.

   (C) $\mathrm{E} + 1$.

6. Divided difference operator is ——————.

   (A) Bilinear. (B) Asymmetrical.

   (C) Linear.

7. Every square —————— matrix will have an inverse.

   (A) Non-singular. (B) Singular.

   (C) Upper Triangular.

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8. A is an $n \times n$ matrix. Then AA^{-1} = ______________.

(A) A.

(B) I.

(C) A^2.

9. In Simpson's 1/3, n should be ______________.

(A) Odd.

(B) Even.

(C) Decimal Number.

10. In each step in the Gauss elimination method, the co-efficient of the first unknown in the first equation is called ______________.

(A) Term.

(B) Pivotal co-efficient.

(C) Pivoting.

11. The Gauss elimination method fails if any one of the pivotal co-efficients becomes ______________.

(A) 1.

(B) Constant.

(C) Zero.

12. The characteristics polynomial of an $n \times n$ matrix A is ______________.

(A) $A - \lambda I$.

(B) $|A - \lambda I| = 0$.

(C) $|A - \lambda I|$.

13. $|A - \lambda I| = 0$ of an $n \times n$ matrix A is called ______________.

(A) Characteristic polynomial.

(B) Characteristic equation.

(C) Eigen Vector.

14. The ______________ of the characteristic equation $|A - \lambda I| = 0$ is called characteristic roots.

(A) Co-efficients.

(B) Roots.

(C) Degree.

15. If λ is an eigen value, then column vector X such that ______________ is called an eigen vector associated with the eigen value λ.

(A) $AX > \lambda X$.

(B) $AX < \lambda X$.

(C) $AX = \lambda X$.

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16. $y' = x^2 + y^2$, then $y''' =$ ————————.

(A) $2x + 2yy'$.

(B) $2 + 2yy'' + 2(y')^2$.

(C) $2 + 2yy''$.

17. $y^{(n)} = y_0 + \int_{x_0}^{x} f\left(x, y^{(n-1)}\right) dx, n = 1, 2, 3, \ldots$ is the iterative formula for ————————.

(A) Euler method.

(B) Picard's method.

(C) Runge-Kutta method.

18. $y_{n+1} = y_n + h\, f(x_n, y_n), n = 0, 1, \ldots$ is the iterative formula for ————————.

(A) Picard's method.

(B) Euler method.

(C) Runge-Kutta method.

19. $E^2 f(x) =$ ————————.

(A) $f(x)$.

(B) $f(x + 2h)$.

(C) $f(x - 2h)$.

20. $D^2 f(x) =$ ————————.

(A) $f(x)$.

(B) $f''(x)$.

(C) $f'(x)$.

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C 3010 (Pages : 5) Name.....................................

Reg. No.................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE [SPECIAL] EXAMINATION MARCH 2021

Mathematics

MAT 6B 11—NUMERICAL METHODS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. State the sufficient condition for the convergence of sequence of approximations $x_{n+1} = \phi(x_n)$ in iteration method.

2. Evaluate $\Delta^{10}\left(x^7\right)$.

3. Give Newton's divided difference formula.

4. Write the relation between forward and backward difference operators.

5. Form the table of differences of the function $f(x) = x^2$ for $x = 0, 1, 2, 3, 4, 5, 6$.

6. State Gauss' backward central difference formula.

7. Given a set of n-values of (x, y), what is the formula for computing $\left[\dfrac{d^2y}{dx^2}\right]_{x_0}$.

8. What do you mean by numerical integration ?

9. State general formula for numerical integration.

10. What is partial pivoting and complete pivoting ?

11. State Predictor formula.

12. Write Runge-Kutta formula of fourth order to solve $\dfrac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$.

(12 × 1 = 12 marks)

Turn over

CHMK LIBRARY UNIVRSITY OF CALICUT

Section B

Answer at least **eight** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 48.

13. Find by iteration method, a real root of $2x - \log_{10} x = 7$.

14. Find the second divided difference of $f(x) = \frac{1}{x}$, using the points x_0, x_1, x_2.

15. If D stands for the differential operator $\frac{d}{dx}$, prove that $D = \frac{1}{h}\left[\Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - .....\right]$.

16. Find the missing values in the following table :

x		45	50	55	60	65
y	:	3.0	–	2.0	–	– 2.4

17. The following table gives the population of a town during the last six censuses. Estimate, using Newton's interpolation formula, the increase in the population during the period 1946 to 1948 :

Year	:	1911	1921	1931	1941	1951	1961
Population (in thousands)		12	15	20	27	39	52

18. If $y_1 = 4, y_3 = 12, y_4 = 19$ and $y_x = 7$, find x.

19. The following data represents e^{-x}.

x	:	– 1	– 0.5	0	0.5	1
$f(x)$	:	2.7183	1.6487	1	0.6065	0.3679

Obtain approximate value of $f''(-1)$.

20. A curve is given by the points of the table given below :

x	:	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	:	23	19	14	11	12.5	16	19	20	20

Apply Simpson's rule to find the area bounded by the curve, the x-axis and the extreme ordinates.

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21. Evaluate $\int_1^3 \frac{1}{x}\, dx$ by Simpson's rule with 8 strips. Determine the error by direct integration.

22. Decompose the matrix $\begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ in the form LU.

23. Apply Gauss-Jordan method to solve the equations :

$x + y + z = 9 ; 2x - 3y + 4z = 13 ; 3x + 4y + 5z = 40.$

24. Solve the system of equations $10x + 2y + z = 9 ; 2x + 20y - 2z = -44 ; -2x + 3y + 10z = 22$ by Gauss-Seidel iteration method.

25. Given $\frac{dy}{dx} - 1 = xy$ and $y(0) = 1$, obtain the Taylor series for $y(x)$ and compute $y(0.1)$ correct to four decimal places.

26. Find the largest eigen value of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 4 \end{bmatrix}$.

(8 × 6 = 48 marks)

Section C

Answer at least **five** *questions.*
Each question carries 9 *marks.*
All questions can be attended.
Overall Ceiling 45.

27. Using Ramanujan's method find a real root of the equation $e^{-x} = x$.

28. Find an interval of unit length which contains the smallest negative root in magnitude of the equation $2x^3 + 3x^2 + 2x + 5 = 0$. Using the end points of this interval as initial approximation perfom four iterations of the Regula-Falsi method.

29. Using Newton's forward difference formula, find the sum $S_n = 1^3 + 2^3 + 3^3 + ... + n^3$.

Turn over

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30. Interpolate by means of Gauss's backward formula the sales of a concern for the year 1976 given that :

Year	:	1940	1950	1960	1970	1980	1990
Sales (in Lakhs)	:	17	20	27	32	36	38

31. A function $f(x)$ representing the following data values has a minimum in the interval (0.5, 0.8). Find this point of minimum and the minimum value :

x	:	0.5	0.6	0.7	0.8
$f(x)$		2.3256	1.4632	0.9842	1.3282

32. Estimate the annual rate of cloth sales of 1935 from the following data :

Year	:	1920	1925	1930	1940
Sales of Cloth (in lakhs of metres)		250	285	328	444

33. Apply Lagrange's method to find the value of x when $f(x) = 15$ from the given data :

x	:	5	6	9	11
$f(x)$		12	13	14	16

34. Find the inverse of the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 2 & 2 \end{bmatrix}$ using Gauss elimination method.

35. Solve, by Euler's method, the equation $\frac{dy}{dx} = x + y, y(0) = 0$. Choose $h = 0.2$ and compute $y(0.4)$ and $y(0.6)$.

(5 × 9 = 45 marks)

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Section D

Answer any **one** *question.*
The question carries 15 marks.

36. (a) A rod is rotating in a plane about one of its ends. If the following table gives the angle θ radians which the rod ias turned for different values of time t seconds, find its angular velocity and angular acceleration when $t = 0.7$ seconds :

t seconds	:	0.0	0.2	0.4	0.6	0.8	1.0
θ radians	:	0.0	0.12	0.48	1.10	2.0	3.20

(b) Prove the following :

(i) $u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + \ldots \Delta^{n-1} u_{x-n} + \Delta^n u_{x-n-1}$.

(ii) $\Delta \nabla y_k = \nabla \Delta y_k = \delta^2 y_k$.

(iii) $\Delta \left(\frac{1}{y_k} \right) = - \Delta y_k / (y_k y_k + 1)$.

37. Find the LU decomposition of the matrix $\begin{bmatrix} 1 & 0 & 5 & 2 \\ -1 & 4 & 1 & 0 \\ 3 & 0 & 4 & 1 \\ -2 & 1 & 1 & 3 \end{bmatrix}$.

38. (a) Using Milne's method, obtain the solution of the differential equation $\frac{dy}{dx} = \frac{1}{x+y}, y(0) = 2$ at $x = 0.8$, given that $y(0.2) = 2.0933, y(0.4) = 2.1755, y(0.6) = 2.2493$.

(b) Determine the largest eigen value and corresponding eigen vector of the matrix $\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

(1 × 15 = 15 marks)

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C 3009-A **(Pages : 4)** **Name.......................................**

Reg. No...................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE (SPECIAL) EXAMINATION MARCH 2021

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 20 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

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MAT 6B 10—COMPLEX ANALYSIS

(Multiple Choice Questions for SDE Candidates)

1. Real part of $f(z) = \log z$ is :

(A) $\frac{1}{2}\log\left(x^2+y^2\right)$.

(B) $\log\left(x^2+y^2\right)$.

(C) $\log(x+iy)$.

(D) None of these.

2. If $f(z) = \left(x^2+ay^2-2xy\right)+i\left(bx^2-y^2+2xy\right)$ is analytic, then value of a and b is :

(A) $-1, 1$.

(B) $1, -1$.

(C) $1, 0$.

(D) None of these.

3. If S and T are domains in the complex plane, which of the following need NOT be true ?

(A) $S \cup T$ is a domain if $S \cap T \neq \phi$.

(B) $S \cup T$ is an open set.

(C) $S \cap T$ is an open set.

(D) $S \cap T$ is a domain.

4. If $f(z)$ is a real valued analytic function in a domain D, then :

(A) $f(z)$ is a constant.

(B) $f(z)$ is identically zero.

(C) $f(z)$ has modulus 1.

(D) None of these.

5. For real numbers x and y, $\sin(x+iy)$ equals :

(A) $\sin x \cosh y + i \cos x \sinh y$.

(B) $\cos x \cosh y - i \sin x \sinh y$.

(C) $\sin x \cosh y - i \cos x \sinh y$.

(D) $\cos x \cosh y + i \sin x \sinh y$.

6. An arc $z = z(t); a \leq t \leq b$ is simple if :

(A) $z(t)$ is continuous.

(B) $z(t)$ is a one to one function.

(C) $z(t)$ is such that $z(a) = z(b)$.

(D) None of these.

7. The only bounded entire functions are :

(A) Real valued functions.

(B) Harmonic functions.

(C) Constant functions.

(D) Exponential function.

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8. Suppose $f(z)$ is analytic inside and on unit circle. If $|f(z)| \le 1, \forall z$ with $|z| = 1$. Then an upper bound for $|f''(0)|$ is :

(A) $n!$.
(B) $\frac{1}{n!}$.
(C) n.
(D) None of these.

9. Value of the integral $\int_0^{\pi} e^{it}\, dt$ is :

(A) $2i$.
(B) 0.
(C) $2\pi i$.
(D) None of these.

10. If n is any non-zero integer, then $\int_0^{2\pi} e^{in\theta}\, d\theta$ equals :

(A) 0.
(B) 2π.
(C) 1.
(D) None of these.

11. A Maclaurin series is a Taylor series with centre :

(A) $z_0 = 1$.
(B) $z_0 = 0$.
(C) $z_0 = 2$.
(D) None of these.

12. The radius of convergence of the power series of the function $f(z) = \frac{1}{1-z}$ about $z = 1/4$ is :

(A) 1.
(B) $\frac{1}{4}$.
(C) $\frac{3}{4}$.
(D) 0.

13. A power series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$ always converges for :

(A) At least one point z.
(B) All complex numbers z.
(C) At all z which are either real or purely imaginary.
(D) At all z with $|z - z_0| < R$ for some $R > 0$.

14. The singular points of the function $f(z) = \frac{1}{4z - z^2}$ are :

(A) $z = 0$ and $z = -4$.
(B) $z = 0$ and $z = 4$.
(C) $z = 4$ and $z = -4$.
(D) $z = 2$ and $z = -2$.

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15. The constant term in the Laurent series expansion of $f(z) = \frac{e^z}{z^2}$ in the region $0 < |z| < \infty$ is :

(A) 0.

(B) $\frac{1}{2}$.

(C) 2.

(D) None of these.

16. Value of $\int_0^\infty \frac{\sin x}{x} dx$ is :

(A) $\frac{\pi}{2}$.

(B) $\frac{1}{2}$.

(C) ∞.

(D) 2π.

17. If $f(z)$ has a pole of order m at z_0, then at z_0, $\frac{1}{f(z)}$ has :

(A) A removable singularity.

(B) An essential singularity.

(C) A pole of order m.

(D) None of these.

18. For $f(z) = \frac{\tan z}{z}$, $z = 0$ is a :

(A) Essential singularity.

(B) Simple pole.

(C) Removable singularity.

(D) Double pole.

19. Singularities of a rational function are :

(A) Poles.

(B) Essential.

(C) Non-isolated.

(D) Removable.

20. The singularity of the function $\frac{\sin z}{z}$ at $z = 0$ is :

(A) Essential singularity.

(B) Simple Pole.

(C) Removable singularity .

(D) Double Pole.

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C 3009 (Pages : 4) Name..................................

Reg. No...............................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE (SPECIAL) EXAMINATION MARCH 2021

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Define Harmonic functions.

2. Is the function $f(z) = \bar{z}$ analytic on $\mathbb{C}$.

3. Define simply connected domain.

4. Write Cauchy-Riemann equations in polar co-ordinates.

5. If a function f fails to be analytic at a point z_0 but is analytic at some point in every neighborhood of z_0, then z_0 is called a ———— of f.

6. State Cauchy's integral formula.

7. What is the principal value of $(-i)^i$?

8. Write an upper bound of sin $z, z \in \mathbb{C}$.

9. Define *simple connected* domain.

10. State Maximum modulus principle.

11. Give an example of non-isolated singular point.

12. If $\lim_{z \to z_0} f(z) = \infty$, then z_0 is called ———— of f.

(12 × 1 = 12 marks)

Turn over

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Section B

Answer at least **eight** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 48.

13. Define entire functions. Give an example.

14. Briefly explain branch of logarithm. Also mention about *principal branch*, *branch cut* and *branch point*.

15. Show that Log $(1-i) = \frac{1}{2}\ln(2) - \frac{\pi}{4}i$.

16. Find the principal value of i^i.

17. Evaluate $\int_C (z+2)/z\,dz$, where C is the positively oriented semicircle $|z| = 2$ in the upper half plane.

18. State Cauchy's integral formula and its extension.

19. Show that the derivative of an analytic function is analytic.

20. Suppose that $z_n = x_n + iy_n \ (n = 1, 2, ...)$ and $S = X + iY$. Then show that $\sum_{n=1}^{\infty} z_n = S$ if and only if $\sum_{n=1}^{\infty} x_n = X$ and $\sum_{n=1}^{\infty} y_n = Y$.

21. Prove or disprove : If f is an analytic function on a connected domain D and C is a simple closed curve in D, then $\int_C f(z)\,dz = 0$.

22. Suppose $z_n = x_n + iy_n, n = 1, 2, 3, ..., x_n \to x$ and $y_n \to y$ then show that $z_n \to z$, where $z = x + iy$.

23. Assuming the Maclau in series of sin z, obtain the Maclau in series of sinh z.

24. Discuss about the residue of an analytic function at infinity.

25. Find the residue of $\frac{1}{z+z^2}$ at $z = 0$.

26. Show that the zeros of a non-zero analytic function are isolated.

(8 × 6 = 48 marks)

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Section C

Answer at least **five** *questions.*
Each question carries 9 *marks.*
All questions can be attended.
Overall Ceiling 45.

27. Derive Cauchy-Riemann equations.

28. Show that $\sin^{-1}(z) = -i \log\left[iz - \left(i - z^2\right)^{1/2}\right]$.

29. Let $f : \mathbb{C} \to \mathbb{C}$ be analytic. Define $g : \mathbb{C} \to \mathbb{C}$ by $g(z) = \overline{f(\overline{z})}$. Is g analytic ? Prove your claim.

30. Prove Liouville's theorem, and hence deduce the fundamental theorem of algebra.

31. State and prove the principle of domination of paths.

32. Suppose that $|f(z)| \leq |f(z_0)|$ at each point z in some neighborhood $|z - z_0| < \in$ in which f is analytic. Then show that $f(z)$ has the constant value $f(z_0)$ throughout that neighborhood.

33. $f z_1$ is a point inside the circle of convergence $|z - z_0| = R$ of a power series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$, then show that the series must be uniformly convergent in the closed disk $|z - z_0| \leq R_1$, where $R_1 = |z_1 - z_0|$.

34. Explain the role of residues in the evaluation of real improper integral.

35. Show that $\int_0^{2\pi} \frac{d\theta}{1 + a\sin\theta} = \frac{2\pi}{\sqrt{1 - a^2}}$.

(5 × 9 = 45 marks)

Section D

Answer any **one** *question.*
The question carries 15 *marks.*

36. (a) Define conjugate harmonic function. Show that $u(x, y) = 2x(1 - y)$ is harmonic and find its harmonic conjugate.

(b) If u is harmonic and v is harmonic conjugate to u, then show that $u + iv$ is analytic.

Turn over

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37. Define absolute convergence of a power series. Also show that if a power series $\sum_{n=1}^{\infty} a_n (z - z_0)^n$ converges when $z = z_1 \ (z_1 \neq z_0)$ then it is absolutely convergent at each point z in the open disk $|z - z_0| < |z_1 - z_0|$.

38. Evaluate $\int_0^{\infty} \frac{x^2}{x^6 + 1} \, dx$.

(1 × 15 = 15 marks)

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C 3008-A (Pages : 4) Name..................................

Reg. No..................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE [SPECIAL] EXAMINATION MARCH 2021

Mathematics

MAT 6B 09—REAL ANALYSIS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 20 Marks**

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INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B) and (C) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 6B 09—REAL ANALYSIS

(Multiple Choice Questions for SDE Candidates)

1. Let A be the open interval (–1, 1) then, the cluster points of A are :

   (A) 1 and –1 only.
   (B) All the points of (–1, 1).
   (C) All the points of [–1, 1].

2. Which of the following have no cluster points ?

   (A) $(3,4]$.
   (B) $[2,3]$.
   (C) $\{2,3,4\}$.

3. Let $f(x)=\begin{cases}\frac{|x|}{x}, & x\neq 0\\ 0, & x=0\end{cases}$. Then $\underset{x\to 0}{\text{Lt}}\, f(x)$ is :

   (A) 1.
   (B) –1.
   (C) None of these.

4. Given a, b satisfy $0<a<1,\ b>1$. Then which of the following sequences is convergent ?

   (A) $\frac{ab^n}{2^n}$.
   (B) $\frac{2^{3n}}{3^{2n}}$.
   (C) $\frac{n}{b^n}$.

5. Let $A\subseteq\mathbb{R}$ and $C\in\mathbb{R}$ such that for every $\delta>0$ there exist atleast one point $x\in A, x\neq c$ such that $|x-c|<\delta$. Then c is a ———— of A.

   (A) Isolated point.
   (B) Cluster point.
   (C) None of these.

6. When the graph of a function 'f' in $\mathbb{R}$ has a break such that the function value and the limit (does not exist) aren't the same, then 'f' is said to have a ———— discontinuity at that point.

   (A) Removable.
   (B) Jump.
   (C) None of these.

7. Consider $f(x)=6x^3-3x^2+2$. Then which of the following is true ?

   (A) 'f' has a zero in (0, 1).
   (B) 'f' has no zero.
   (C) 'f' has a zero in (–1, 1).

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8. Which of the following statements is true ?

(A) Every continuous function is uniformly continuous.

(B) Every uniformly continuous function is continuous.

(C) None of these.

9. Let $f:[a,b]\to\mathbb{R}$. If there exist a $L\in\mathbb{R}$ such that for every $\in>0$, there exist $\delta_{\in}>0$ such that for any tagged partition $\dot{p}$ of $[a,b]$ with $\|\dot{P}\|<\delta_{\in};|s(f;p)-L|<\in$, then '$f$' is :

(A) Riemann Integrable. (B) Lesbegue Integrable.

(C) None of these.

10. Let $g:[0,4]\to\mathbb{R}$ be defined by $g(x)=3$ for $0\le x\le 2$ and $g(x)=2$ for $2<x\le 4$ then $\int_0^4 g=$

(A) 5. (B) 7.

(C) 10.

11. Which of the following is true ?

(A) $f\in R[a,b]\Rightarrow f$ is bounded. (B) $f\in R[a,b]\Rightarrow f$ is continuous.

(C) $f\in R[a,b]\Rightarrow f$ is monotone.

12. Let $x\in[0,1]$ and 'f' be such that $f(x)=\begin{cases}1, & x \text{ is rational}\\ 0, & x \text{ is irrational.}\end{cases}$ Then :

(A) $f\in R[0,1]$. (B) $f\notin R[0,1]$.

(C) None of these.

13. Suppose $f:[a,b]\to\mathbb{R}$ and $f(x)=0$ except for finite number of points $c_1,c_2,c_3....c_n$ in $[a,b]$. Then :

(A) $f\notin R[a,b]$. (B) $\int_a^b f=0$.

(C) None of these.

14. Which of the following is true ?

(A) Every convergent sequence is Cauchy.

(B) Every Cauchy sequence is convergent.

(C) Both (A) and (B).

Turn over

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15. Let $f:[a,b]\to\mathbb{R}$. Then which of the following is true ?

    (A) f is continuous on $[a,b]\Rightarrow f\in R[a,b]$.

    (B) $f\in R[a,b]\Rightarrow f$ is continuous on $[a,b]$.

    (C) $f\in R[a,b]\Rightarrow f$ is uniformly continuous on $[a,b]$.

16. Consider 'h' defined on [0, 1] by $h(x)=\begin{cases}x+1, & x \text{ is rational}\\ 0, & x \text{ is irrational.}\end{cases}$ Then which of these is true ?

    (A) $h\in R[0,1]$.

    (B) $h\notin R[0,1]$.

    (C) h is continuous on [0, 1].

17. Let H be defined on [0, 1] such that $H(x)=\begin{cases}k, & \text{for } x=\frac{1}{k}, k\in\mathbb{R}\\ 0, & \text{elsewhere}\end{cases}$. Then which of these is true ?

    (A) $H\in R[0,1]$.

    (B) $H\notin R[0,1]$.

    (C) H continuous on [0, 1].

18. If f is bounded on $[a,b]$ and if 'f' restricted to $[c,b]$, $c\in[a,b]$ is Riemann integrable, then which of the following does not hold true ?

    (A) $f\in R[a,b]$.

    (B) $f\notin R[a,b]$.

    (C) $\underset{c\to a^+}{\text{Lt}}\int_c^b f=\int_a^b f$.

19. If 'f' is continuous of every point of $[a,b]$ and F is any antiderivative of 'f' on $[a,b]$, then $\int_a^b f(x).\,dx=$

    (A) (b – a)(F (b) – F (a)).

    (B) F (b) – F (a).

    (C) None of these.

20. If $g(x)=\begin{cases}x, & |x|\ge 1\\ -x, & |x|<1\end{cases}$ and if $G(x)=\frac{1}{2}\left|x^2-1\right|$, then $\int_{-2}^{3} g(x).\,dx=$

    (A) $\frac{1}{2}$.

    (B) $\frac{5}{2}$.

    (C) $\frac{7}{2}$.

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C 3008 (Pages : 4) Name.......................................

Reg. No....................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE [SPECIAL] EXAMINATION MARCH 2021

Mathematics

MAT 6B 09—REAL ANALYSIS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. State Bolzano's Intermediate Value Theorem.
2. Define Uniformly continuous function.
3. Define Step function on an interval.
4. State the Composition theorem of Riemann integration.
5. Define Partition of a closed and bounded interval.
6. Define Indefinite integral of $f \in R[a, b]$, with base point a.
7. State the Squeeze theorem for Riemann integrability of a function.
8. State the Cauchy criterion for Riemann intagrability of a function.
9. For what value of r the series $\sum_{n=1}^{\infty} r^n \cos^2 nx$ uniformly convergent ?
10. Fill in the blanks : $\lim \left(\frac{\cos(nx+n)}{n}\right) =$ ——————.
11. Fill in the blanks : $\beta(1,1) =$ ——————.
12. Fill in the blanks : $\int_{-\infty}^{\infty} e^{-x^2}\, dx =$ ——————.

(12 × 1 = 12 marks)

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Section B

Answer at least **eight** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 48.

13. State the Boundedness Theorem. Show by an example that the boundedness theorem fails if the function is discontinuous in that interval.

14. State the Maximum Minimum Value Theorem. Show by an example that this theorem fails, if the interval is not closed.

15. Prove that the continuous image of a closed and bounded interval is a closed and bounded interval.

16. State the "non-uniform continuity criteria". Apply this result to test the uniform continuity of $f(x) = 1/x$ on $(0, \infty)$.

17. State the location of Roots Theorem. Use it to show that the equation $xe^x - 2 = 0$ has a root in $(0, 1)$.

18. If $f, g \in \mathrm{R}[a, b]$, then prove that $f \cdot g \in \mathrm{R}[a, b]$.

19. Evaluate $\int_{t=1}^{4} \frac{\sin \sqrt{t}}{\sqrt{t}} \, dt$.

20. If $f \in \mathrm{R}[a, b]$, then prove that $|f| \in \mathrm{R}[a, b]$.

21. Define Pointwise convergence of a sequence of functions. Test the convergence of (x^n).

22. If (f_n) and (g_n) are uniformly continuous functions in R, then show that $(f_n)(g_n)$ need not be uniformly continuous in R.

23. Distinguish between the absolute and uniform convergence of a series of functions ; also state the Cauchy criteria for uniform convergence of a series of functions.

24. Show that $\int_{x=a}^{b} \frac{1}{\sqrt{(b-x)^p}} \, dx$ converges, if $p < 1$; and diverges if $p \geq 1$.

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25. Express the integral $\int_{x=1}^{2} \frac{x^2}{\left(1-x^5\right)} dx$, in terms of Beta function.

26. Define Gamma function. Prove that $\Gamma(n) = (n-1)\,\Gamma(n-1)$.

(8 × 6 = 48 marks)

Section C

Answer at least **five** *questions.*
Each question carries 9 *marks.*
All questions can be attended.
Overall Ceiling 45.

27. State and prove the uniform continuity theorem.

28. (a) Define Lipschitz function.

(b) If $f : A \rightarrow R$ Lipschitz function on A, then prove that f is uniformly continuous on A.

29. If $f, g \in R[a, b]$, then prove that $f + g \in R[a, b]$ and $\int_a^b f + g = \int_a^b f + \int_a^b g$.

30. If $f : [a, b] \rightarrow R$ is continuous on $[a, b]$, then prove that $f \in R[a, b]$.

31. Test the uniform convergence of the series $\sum_{n=1}^{\infty} \frac{\sin\left(x^2 + nx\right)}{n\,(n+2)}$.

32. Discuss the convergence of $(f_n(x)) = \left(x^n (1-x)\right), x \in A = [0, 1]$.

33. (a) Distinguish between absolute and conditional convergence of an improper integral.

(b) Discuss the convergence of the Improper integral $\int_{x=\pi}^{\infty} \frac{\cos x}{x^2} dx$.

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34. Express the integral $\int_0^a x^{p-1}(a-x)^{q-1}\,dx\,;\,p,q>0$ in terms of Gamma function.

35. Prove that $\beta(m,n)=\dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

(5 × 9 = 45 marks)

Section D

Answer any **one** *question.*
The question carries 15 *marks.*

36. (a) If $f:A\to R$ is uniformly continuous on A and if (x_n) is a Cauchy sequence in A, then prove that $(f(x_n))$ is a Cauchy sequence in R.

(b) Evaluate $\int_0^\infty e^{-x^2}\,dx$, Gamma function.

37. (a) State and prove the Fundamental Theorem of calculus (2nd form).

(b) Discuss the uniform convergence $(f_n(x))=(x^n)$, $x\in A=[0,1]$, $x\in N$.

38. (a) State and prove the Cauchy criteria for uniform convergence of a sequence of functions.

(b) Prove that $\beta(m,n)=\int_{x=0}^{\infty}\dfrac{x^{m-1}}{(1+x)^{m+n}}\,dx$.

(1 × 15 = 15 marks)

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