**D 93598** (Pages : 4) Name..................................

Reg. No...............................

# FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

Mathematics

MAT 1C 05—NUMBER THEORY

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

## Part A

*Answer* **all** *questions.*
*Each question carries 2 marks.*

1. Define completely multiplicative function and give an example of a multiplicative function which is not completely multiplicative.

2. If P is an odd positive integer, prove that $(-1/P) = (-1)^{(P-1)/2}$.

3. For all $x \geq 1$, prove that :

$$\sum_{n \leq x} \mu(n)\left[\frac{x}{n}\right] = 1.$$

4. For $n \geq 1$, prove that the $n^{th}$ prime $p_n$ satisfies the inequality :

$$\frac{1}{6} n \log n < p_n.$$

5. Describe briefly about frequency analysis in cryptography.

6. What is meant by 'hash functions' in cryptography ?

(6 × 2 = 12 marks)

## Part B

*Answer any* **five** *questions.*
*Each question carries 4 marks.*

7. If $n \geq 1$, prove that :

$$\phi(n) = \sum_{d/n} \mu(d) \cdot \frac{n}{d}.$$

Turn over

8. If $d(n)$ denotes the number of positive divisors of an integer $n$, then prove that :

$$\prod_{t/n} t = n^{d(n)/2}.$$

9. Determine whether 2/9 is a quadratic residue or non-residue mod 383.

10. For $x \geq 1$, prove that :

$$\sum_{n \leq x} \frac{1}{n^s} = \frac{x^{1-s}}{1-s} + G(s) + O\left(x^{-s}\right) \text{ if } \begin{matrix} s > 0 \\ s \neq 1 \end{matrix}.$$

11. Define the Chebyshev's functions $\Psi(x)$ and $I(x)$ and obtain a relation connecting them.

12. Prove that $\lim\limits_{x \to \infty} \left( \frac{M(x)}{x} - \frac{H(x)}{x \log x} \right) = 0.$

13. Find the inverse of the matrix $\begin{pmatrix} 1 & 3 \\ 4 & 3 \end{pmatrix}$ mod 29.

14. Describe briefly about RSA cryptosystem.

(5 × 4 = 20 marks)

## Part C

*Answer* **either** A **or** B *of each questions.*
*Each question carries* 16 *marks.*

15. (A) (a) If $f$ and $g$ are multiplicative functions, prove that their Dirichlet product $f * g$ is also a multiplicative function.

(b) If $f$ is an arithmetical function with $f(1) \neq 0$, prove that there is a unique arithmetical function $f^{-1}$ with :

$$f * f^{-1} = f^{-1} * f = I.$$

where I is the identify function.

(B) (a) For every odd prime $p$, prove that :

$$(2/p) = (-1)^{(p^2-1)/8}.$$

(b) If $p$ and $q$ are distinct odd primes, then prove that :

$$(p/q)(q/p) = (-1)^{(p-1)(q-1)/4}.$$

16. (A) (a) State and prove Euler's summation formula.

(b) For $x \geq 2$, prove that :

$$\sum_{p \leq x} \left[\frac{x}{p}\right] \log p = x \log x + O(x)$$

where the sum is extended over all primes $\leq x$.

(B) (a) If $0 < a < b$, prove that there exists an $x_0$ such that :

$$\pi(ax) < \pi(bx) \text{ if } x \geq x_0.$$

(b) Prove that the prime number theorem implies $\lim_{x \to \infty} \frac{M(x)}{x} = 0$.

17. (A) (a) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(Z/NZ)$ and let $D = ad - bc$. Prove that the following are equivalent.

(i) g.c.d. (D, N) = 1 ;

(ii) A has an inverse matrix ;

(iii) If $x$ and $y$ are not both 0 in $(Z/NZ)$, then $A\begin{pmatrix} x \\ y \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$;

(iv) A gives a 1 – to –1 correspondence of $(Z/NZ)^2$ with itself.

(b) The message "!IWGVIEX! ZRADRYD", which was sent using a linear enciphering transformation of digraph vectors in a 29-letter alphabet, in which A – Z have numerical equivalents 0 – 25, blank = 26, ? = 27, ! = 28 was intercepted. It is known that the last five letters of the plain-text are the sender's signature "MARIA" :

(i) Find the deciphering matrix and read the message.

(ii) Find the enciphering matrix and send the following reply in code : "DAMN FOG ! JO".

(B) (a) Briefly describe about digraph transformations.

(b) Suppose that both plaintexts and ciphertexts consist of trigraph message units, but while plain texts are written in the 27-letter alphabet (Consisting of A – Z and blank = 26), cipher texts are written in the 28-letter alphabet obtained by adding the symbol "/" (with numerical equivalent 27) to the 27 letter alphabet. It is required that each user A choose $n_A$ between $27^3$ and $28^3$ so that a plaintext trigraph in the 27-letter and then $C = P^{e_A} \bmod n_A$ corresponds to a ciphertext trigraph in the 28-letter alphabet. If the deciphering key is $K_D = (n, d) = (21583, 20787)$, decipher the message "YSNAUOZHXXH" (one blank at the end).

(3 × 16 = 48 marks)

**D 93597** **(Pages : 3)** **Name.......................................**

**Reg. No..................................**

# FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

Mathematics

MAT 1C 04—DISCRETE MATHEMATICS

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

## Part A

*Answer* **all** *questions.*
*Each question carries* 2 *marks.*

1. Prove that the number of vertices of odd degree in any graph is even.
2. Define a Hamiltonian graph. Give an example of a Hamiltonian graph with 6 vertices.
3. Prove *or* disprove : Intersection of two chains is a chain.
4. Describe the Hasse diagram of the partially ordered set $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ with partial order, 'divides'.
5. Define a finite automaton. Illustrate with an example.
6. Define a grammar G and find the associated L (G).

(6 × 2 = 12 marks)

## Part B

*Answer any* **five** *questions.*
*Each question carries* 4 *marks.*

7. Prove that a connected graph G with at least two vertices contains at least two vertices that are not cut vertices.
8. If G is Hamiltonian, then prove that for every non-empty proper subset S of V, $|\omega(G - S)| \leq |S|$.
9. If the degree of each vertex of a graph G is an even positive integer, prove that G is an edge-disjoint union of cycles.

**Turn over**

10. Let $(X \leq)$ be a poset. Prove that X can be expressed as the union of maximal chains.

11. Write in disjunctive normal forms :

(i) $x_1' x_2 (x_1' + x_2 + x_1 x_3)$ ; and (ii) $(xy + x'y + x'y')' (x + y)$.

12. (a) If $\Sigma = \{a, b\}$ and $L = \{aa, bb\}$, using set notation describe $\bar{L}$.

(b) Prove that $(L_1 L_2)^R = L_2^R L_1^R$.

13. Show that $L = \{a^n : a \geq 4\}$ is regular.

14. Find a dfa that recognize the set of all strings on $\Sigma = \{a, b\}$ starting with the prefix *ba*.

(5 × 4 = 20 marks)

## Part C

*Answer* **either** *A* **or** *B of each of the following* **three** *questions.*
*Each question carries* 16 *marks.*

15. A (a) Prove that the following statements are equivalent.

(i) G has exactly one cycle ;

(ii) G is connected and $n = m$ ;

(iii) For some edge $e$ of G, $G - e$ is a tree ; and

(iv) G is connected and the set of edges of G that are not cut edges forms a cycle.

(b) For a simple graph G with $n$ vertices, $n \geq 2$, which is complete, prove that $k(G) = n - 1$.

B. (a) Prove that $K_5$ and $K_{3,3}$ are non-planar.

(b) Describe the common features of $K_5$ and $K_{3,3}$.

16. A. Let X be a finite set and $\leq$ be a partial order on X. Define a binary relation R on X by $xRy$ if and only if $y$ covers $x$. Prove that $\leq$ is the smallest order relation on X containing R.

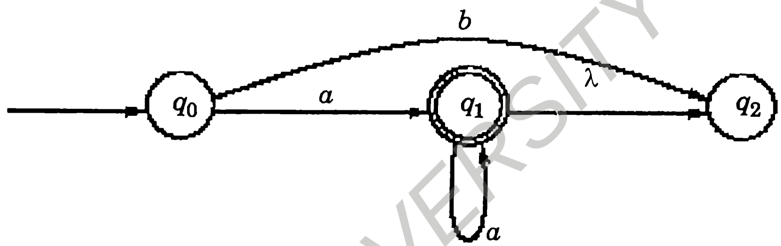
B. (a) Given integers $0 \leq r_1 < r_2 < \ldots < r_k \leq n$, prove that there exists one and only one symmetric function of $n$ Boolean variables whose characteristic numbers are $r_1, r_2, \ldots, r_k$.

(b) Prove that the set of all Boolean functions of $n$ Boolean variables $x_1, x_2, \ldots, x_n$ is a subalgebra of the Boolean algebra of all Boolean functions of these variables.

17. A. (a) For every nfa with arbitrary number of final states, prove that there exists an equivalent nfa with only one final state.

(b) Show that if L is regular, then $L^R$ is regular.

B. Convert the nfa given by the following transition graph into an equivalent dfa.




(3 × 16 = 48 marks)

**D 93596** (Pages : 4) Name..................................

Reg. No...............................

# FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

Mathematics

MAT 1C 03—REAL ANALYSIS—I

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

## Part A

*Answer* **all** *questions.*
*Each question carries 2 marks.*

1. Let Y be an open subset of a metric space X. If a subset E of Y is open relative to Y, then prove that E is open in X.

2. If E is an infinite subset of a compact set K, then prove that E has a limit point in K.

3. Prove that composition of continuous maps is continuous.

4. Let $f$ be a bijective continuous map from a metric space X onto a metric space Y. Is $f^{-1}$ continuous ? Justify your answer.

5. Suppose $f \geq 0$, is continuous on $[a, b]$ and $\int_a^b f(x)\,dx = 0$. Prove that $f(x) = 0$ for all $x \in [a, b]$.

6. Let $f_1, f_2$ be bounded real functions and $\alpha$ be monotonic increasing real function on $[a, b]$. If $f_1, f_2$ are Riemann-Stieltjes integrable with respect to $\alpha$ on $[a, b]$, then prove that $f_1 + f_2$ is Riemann-Stieltjes integrable with respect to $\alpha$ on $[a, b]$.

7. If $\{f_n\}$ and $\{g_n\}$ converge uniformly on a set E, then prove that $\{f_n + g_n\}$ converge uniformly on E.

8. Let $\{f_n\}$ be a uniformly bounded sequence of continuous functions on a compact set E. Does $\{f_n\}$ has a convergent subsequence ? Justify your answer.

(8 × 2 = 16 marks)

## Part B

*Answer any* **four** *questions.*
*Each question carries 4 marks.*

9. Let X be a metric space and $E \subset X$. If $p$ is a limit point of E, then prove that every neighborhood of $p$ contains infinitely many points of E.

10. Prove that closed subsets of compact sets are compact.

11. Prove that continuous image of a connected metric space is connected.

12. Let $f$ be differentiable in $(a, b)$. If $f'(x) \geq 0$ for all $x \in (a, b)$, then prove that $f$ is monotonically increasing.

13. Let $f$ be a bounded function and $\alpha$ be a monotonic increasing function on $[a, b]$. If $P^*$ is a refinement of P, then prove that :

$$U(P^*, f, \alpha) \leq U(P, f, \alpha).$$

14. Let $\mathcal{C}(X)$ denote the set of all complex valued, continuous, bounded functions defined on a metric space X. Prove that $\mathcal{C}(X)$ is a complete metric space with respect to the metric.

$$d(f, g) = \sup_{x \in X} |f(x) - g(x)|.$$

(4 × 4 = 16 marks)

## Part C

*Answer A* **or** *B of the following questions.*
*Each question carries* 12 *marks.*

### Unit I

15. (A) (a) Let $\{F_\alpha\}$ be a collection of closed subsets of a metric space X. Prove $\bigcap_\alpha F_\alpha$ is closed.

(b) Prove that every bounded infinite subset of $\mathbb{R}^k$ has a limit point.

(B) (a) If $\{I_n\}$ is a sequence of intervals in $\mathbb{R}^1$ such that $I_n \supset I_{n+1}$, $n = 1, 2, 3, \ldots$, then prove that $\bigcap_{n=1}^{\infty} I_n$ is not empty.

(b) Let P be a non-empty perfect set in $\mathbb{R}^k$. Prove that P is uncountable.

## Unit II

16. (A) (a) Let $f$ be a continuous real function on a compact metric space X. Prove that $f$ attains its maximum and its minimum on X.

(b) Let $f$ be monotonic on an open interval $(a, b)$. Prove that the set of points of $(a, b)$ at which $f$ is discontinuities is at most countable.

(B) (a) Prove that continuous image of a compact metric space is compact.

(b) Let $f$ and $g$ be continuous real functions on $[a, b]$ which are differentiable in $(a, b)$. Prove that there is a point $x \in (a, b)$ at which :

$$[f(b) - f(a)]\, g'(x) = [g(b) - g(a)]\, f'(x).$$

## Unit III

17. (A) (a) Let $f$ be a bounded, monotonic real function and $\alpha$ be a continuous, monotonic increasing real function on $[a, b]$. Prove that $f$ is Riemann-Stieltjes integrable with respect to $\alpha$ on $[a, b]$.

(b) For $1 < s < \infty$, define :

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}.$$

Prove that $\zeta(s) = \dfrac{s}{s-1} - s \displaystyle\int_1^{\infty} \frac{x - [x]}{x^{s+1}}\, dx.$

(B) (a) Let $f$ be Riemman integrable on $[a, b]$. For $a \le x \le b$, let

$$F(x) = \int_a^x f(t)\, dt.$$

Prove that F is continuous on $[a, b]$.

(b) Let $\gamma$ be a curve on $[a, b\}$. If $\gamma'$ is continuous on $[a, b]$, then prove that $\gamma$ is rectifiable and

$$\Lambda(\gamma) = \int_a^b |\gamma'(t)|\, dt.$$

Turn over

## UNIT IV

18. (A) (a) Let $\{f_n\}$ be a sequence of functions differentiable on $[a, b]$ and $\{f_n(x_0)\}$ converges for some point $x_0$ on $[a, b]$. If $\{f'_n\}$ converges uniformly on $[a, b]$, then prove that $\{f_n\}$ converges uniformly on $[a, b]$ to a function $f$ and

$$f'(x) = \lim_{x \to \infty} f'_n(x)$$

for all $x \in [a, b]$.

(b) If $\{f_n\}$ is a pointwise bounded sequence of complex functions on a countable set E, then prove that $\{f_n\}$ has a subsequence $\{f_{n_k}\}$ such that $\{f_{n_k}(x)\}$ converges for every $x \in E$.

(B) (a) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

(b) Let K be compact and $f_n \in \mathcal{C}(K)$ for $n = 1, 2, 3, ....$ If $\{f_n\}$ is pointwise bounded and equicontinuous on K, then prove that $\{f_n\}$ contains a uniformly convergent subsequence.

(4 × 12 = 48 marks)

**D 93595** (Pages : 4) Name.....................................

Reg. No.................................

# FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

Mathematics

MAT 1C 02—LINEAR ALGEBRA

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

## Part A

*Answer* **all** *questions.*
*Each question carries 2 marks.*

1. Show that if V is a finite-dimensional vector space, then any *two* bases of V have the same number of elements.
2. Define linearly dependent subset of a vector space V and prove that if two vectors are linearly dependent, one of them is a scalar multiple of the other.
3. Let $\beta = (\alpha_1, \alpha_2, \alpha_3)$ be the basis of $C^3$ defined by $\alpha_1 = (1, 0, -1), \alpha_2 = (1, 1, 1), \alpha_3 = (2, 2, 0)$. Find the dual basis of $\boldsymbol{\beta}$.
4. Let F be a field and let $f$ be the linear functional on $F^2$ defined by $f(x_1, x_2) = ax_1 + bx_2$. If T is the linear operator defined by $T(x_1, x_2) = (x_1 - x_2, x_1 + x_2)$, then find $T^t f$, where $T^t$ denotes the transpose of T.
5. Show that similar matrices have the same characteristic polynomial.
6. Let P be the linear operator on $R^2$ defined by $P(x_1, x_2) = (x_1, 0)$. What is the minimal polynomial for P.
7. Let V be a real vector space and E be a projection on V. Prove that $(I + E)$ is invertible and find $(I + E)^{-1}$.
8. Prove that an orthogonal set of non-zero vectors is linearly independent.

(8 × 2 = 16 marks)

**Turn over**

## Part B

*Answer any* **four** *questions.*
*Each question carries* 4 *marks.*

9. Is the vector $(3,-1,0,-1)$ in the subspace of $R^4$ spanned by the vectors $(2,-1,3,2)$, $(-1,1,1,-3)$ and $(1,1,9,-5)$? Justify your answer.

10. Let V and W be vector spaces over the field F and let U be an isomorphism from V onto W. Prove that $T \mapsto UTU^{-1}$ is an isomorphism of $L(V, V)$ onto $L(W, W)$.

11. Let V be a finite-dimensional vector space over the field F and let W be a subspace of V. Show that $\dim W + \dim W^{\circ} = \dim V$.

12. Let W be an invariant subspace for a linear operator T on a vector space V. Prove that :
    - (i) The characteristic polynomial for $T_W$ divides the characteristic polynomial for T.
    - (ii) The minimal polynomial for $T_W$ divides the minimal polynomial for T.

13. Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & -2 & 2 \\ 2 & -3 & 2 \end{bmatrix}$. Is A similar over the field of real numbers to a triangular matrix ? Justify your answer.

14. Let W be a finite dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W. Show that the mapping $\beta \mapsto \beta - E\beta$ is the orthogonal projection of V onto $W^{\perp}$.

(4 × 4 = 16 marks)

## Part C

*Answer* **either** *A* **or** *B of each of the following questions.*
*Each question carries* 12 *marks.*

15. A (a) Show that if $W_1$ and $W_2$ are finite-dimensional subspaces of a vector space V, then $W_1 + W_2$ is finite dimensional and $\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$.

(b) Let V be the vector space of all 2 × 2 matrices over R. Let $W_1$ be the subspace of the matrices of the form $\begin{bmatrix} x & -x \\ y & z \end{bmatrix}$ and $W_2$ be the subspace of the matrices of the form $\begin{bmatrix} a & b \\ -a & c \end{bmatrix}$. Find the dimensions of $W_1$, $W_2$, $W_1 + W_2$ and $W_1 \cap W_2$.

B (a) Let V be an $n$-dimensional vector space over the field F and Let $\beta$ and $\beta'$ be two ordered bases of V. Show that there is a unique $n \times n$ invertible matrix P with entries in F such that :

(i) $[\alpha]_{\beta} = P[\alpha]_{\beta'}$ ; and (ii) $[\alpha]_{\beta'} = P^{-1}[\alpha]_{\beta}$

for every vector $\boldsymbol{\alpha}$ in V.

(b) Show that if A is an $m \times n$ matrix with entries in the field F, then row rank (A) = column rank (A).

16. A (a) Let T be a linear transformation from V into W. Show that T is non-singular iff T carries each linearly independent subset of V onto a linearly independent subset of W.

(b) Let V be a finite-dimensional vector space over the field F. For each vector $\boldsymbol{\alpha}$ in V define $L_{\alpha}(f) = f(\alpha), f \in V^*$. Show that the mapping $\alpha \mapsto L_{\alpha}$ is an isomorphism of V onto V**.

B (a) Let W be the subspace of $R^5$ which is spanned by the vectors $\alpha_1 = (2, -2, 3, 4, -1), \alpha_2 = (0, 0, -1, -2, 3)\ \alpha_3 = (-1, 1, 2, 5, 2), \alpha_4 = (1, -1, 2, 3, 0)$. Find a basis for W°.

(b) Let $W_1$ and $W_2$ be subspaces of a finite dimensional vector space V. Show that

$(W_1 \cap W_2)^\circ = W_1{}^\circ + W_2{}^\circ$

17. A Let T be the linear operator on $R^3$ which is represented in the standard ordered basis by the matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. Prove that T is diagonalizable by exhibiting a basis for $R^3$, each vector of which is a characteristic vector of T.

B Let T be a linear operator on a finite dimensional vector space V. Show that if $f$ is the characteristic polynomial for T, then $f(T) = 0$.

18. A (a) Let $W_1, W_2, \ldots, W_k$ be subspaces of a finite-dimensional vector space V such that $V = W_1 \oplus \ldots \oplus W_k$. Show that there exists $k$ linear operators $E_1, E_2, \ldots, E_k$ on V such that :

(i) each $E_i$ is a projection ;

(ii) $E_i E_j = 0$ if $i \neq j$ ;

(iii) $I = E_1 + E_2 + \ldots + E_k$ ; and

(iv) The range of $E_i$ is $W_i$.

(b) Let V be a finite-dimensional vector space and let $W_1$ be any subspace of V. Prove that there is a subspace $W_2$ of V such that $V = W_1 \oplus W_2$.

B Let W be the subspace of $R^2$ with the standard inner product. Let E be the orthogonal projection of $R^2$ onto W. Find :

(a) A formula for $E(x_1, x_2)$ ;

(b) The matrix of E in the standard ordered basis ;

(c) $W^{\perp}$; and

(d) An orthonormal basis in which E is represented by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

(4 × 12 = 48 marks)

**D 93594** (Pages : 4) Name..................................

Reg. No..............................

# FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

Mathematics

MAT 1C 01—ALGEBRA—I

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

## Part A

*Answer* **all** *questions.*
*Each question carries* 2 *marks.*

1. Show that (1, 2) is a generator of the cyclic group $\mathbb{Z}_2 \times \mathbb{Z}_3$.
2. Describe an isometry of the plane that fixes only one point.
3. Let $G = S_3$ act on the set {1, 2, 3} by $\sigma x = \sigma(x)$ for $\sigma \in S_3$ and $x \in X$. Find the isotropy group $G_x$ for $x = 1$.
4. Let D be an integral domain and $F = D \times D / \sim$ be the field of quotients of D where the notations are the usual ones. Show that for $a \neq b$, $(a, 1)$ and $(b, 1)$ are not $\sim$–related.
5. Verify whether $x^3 + x^2 + x + 1$ is irreducible in $\mathbb{Z}_2[x]$.
6. Verify whether the ideal generated by 4 is maximal in $\mathbb{Z}_{10}$.
7. Find $irr(\alpha, \mathbb{Q})$ where $\alpha = \sqrt{2} + \sqrt{3}$.
8. Verify whether $\mathbb{Q}(\pi)$ is an algebraic extension of $\mathbb{Q}$.

(8 × 2 = 16 marks)

## Part B

*Answer any* **four** *questions.*
*Each question carries* 4 *marks.*

9. Find all generators of the cyclic group $\mathbb{Z}_3 \times \mathbb{Z}_4$.
10. Let $S_n$ be the symmetric group on $n$ symbols and $A_n$ be the alternating group. Show that $S_n/A_n$ is isomorphic to $\mathbb{Z}_2$.

11. Let X be a G-set and $x \in X$. Show that $G_x = (g \in G : gx = x)$ is a subgroup of G.

12. Describe the field of quotients of the integral domain $Z_2$.

13. Prove that $x^5 + 6x^3 + 9x + 3$ is irreducible over the rationals.

14. Show that doubling the cube by straight edge and compass is an impossible construction.

(4 × 4 = 16 marks)

## Part C

*Answer* **either** *part A* **or** *part B of each of the* **four** *questions.*
*Each question carries* 12 *marks.*

15. (A) (a) Let $m$ and $n$ be relatively prime. Show that $\mathbb{Z}_m \times \mathbb{Z}_n$. is isomorphic to $\mathbb{Z}_{mn}$.

(b) Show that the group $\mathbb{Z} \times \mathbb{Z}_2$ is not cyclic.

(c) Show that the direct product $G_1 \times G_2$ of groups $G_1$ and $G_2$ contains a subgroup isomorphic to $G_1$ and a subgroup isomorphic to $G_2$.

(B) Let H be a subgroup of a group G. Show that the following are equivalent.

(a) $ghg^{-1} \in H$ for all $g \in G$ and $h \in H$.

(b) $gHg^{-1} = H$ for all $g \in G$.

(c) $gH = Hg$ for all $g \in G$.

16. (A) (a) Let X be a G-set. For each $g \in G$ let $\sigma_g : X \to X$ be defined by $\sigma_g(x) = gx$ for all $x \in X$. Show that :

(i) $\sigma_g$ is a permutation of X.

(ii) $\phi : G \to S_X$ defined by $g \mapsto \sigma_g$ is a homomorphism where $S_X$ is the group of all permutations of X.

(b) Give an example of a G-set where $G = \mathbb{Z}_4$.

(B) (a) Let G be a finite group and X be a finite G-set. Let $r$ be the number of orbits in X. For $g \in G$ let $X_g = \{x \in X : gx = x\}$. Show that

(i) $\sum_{g \in G} |X_g| = \sum_{x \in X} |G_x|$ where $G_x$ is the isotropy group of $x$.

(ii) $r|G| = \sum_{g \in G} |X_g|$.

(b) Give a non trivial action of $\mathbb{Z}_3$ on $X = \{1, 2, 3\}$ and describe $X_g$ for each $g \in \mathbb{Z}_3$.

17. (A) (a) State and prove division algorithm in the polynomial ring F $[x]$ where F is a field.

(b) Show that $a \in F$ is a zero of $f(x) \in F[x]$ if and only if $(x - a)$ is a factor of $f(x)$.

(B) (a) Let R be a ring and N be an ideal of R. Show that $\eta : R \to R/N$ defined by $a \mapsto a + N$ for $a \in R$ is a ring homomorphism.

(b) Let $\phi : R \to R'$ be a ring homomorphism. Show that :

(i) $\text{Ker}\phi$ is an ideal of R.

(ii) $R/\text{Ker}\phi$ is isomorphic to $\phi$ (R).

18. Let E be an extension of a field F and $\alpha \in E$.

(a) Prove that if the evaluation homomorphism $\phi_\alpha : F[x] \to E$ is one to one then $\alpha$ is not algebraic over F.

(b) Show that if $\alpha$ is algebraic over F then there exists a polynomial $p(x) \in F[x]$ satisfying the following.

(i) $p(\alpha) = 0$.

(ii) If $f(x) \in F[x]$ and $f(\alpha) = 0$ then $p(x)$ divides $f(x)$.

(B) (a) Let E be a finite extension of degree $n$ of a finite field F of $q$ elements. Show that the number of elements of E is $q^n$.

(b) Prove that a finite field of $p^n$ elements exists for each prime $p$ and each natural number $n$.

($4 \times 12 = 48$ marks)

**D 93427** (Pages : 3) Name.......................................

Reg. No...................................

# FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2020

(CBCSS)

Mathematics

MTH 1C 05—NUMBER THEORY

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

*General Instructions.*

## Part A

*Answer* **all** *questions.*
*Each question has weightage* 1.

1. Find all integers $n$ such that :

$$\phi(n) = n/2.$$

2. If $n \geq 1$, prove that :

$$\wedge(n) = \sum_{d/n} \mu(d) \cdot \log \frac{n}{d}.$$

3. Prove that $[2x] - 2[x]$ is either 0 or 1.

4. For $x > 0$, prove that :

$$0 \leq \frac{\psi(x)}{x} - \frac{\mathcal{J}(x)}{x} \leq \frac{(\log x)^2}{2\sqrt{x}\log 2}.$$

5. For all $x \geq 1$, prove that :

$$\sum_{p \leq x} \frac{\log p}{p} = \log x + O(1).$$

6. Evaluate the Legendre' symbol $(73 | 383)$.
7. Prove that Legendre's symbol $(n | p)$ is a completely multiplicative function $n$ of $n$.
8. Briefly describe about digraph transformation.

(8 × 1 = 8 weightage)

**Turn over**

## Part B

*Answer any* **six** *questions by choosing two questions from each unit.*
*Each question carries a weightage of 2.*

### Unit I

9. For $n \geq 1$, prove that :

$$\phi(n) = n \prod_{p/n} \left(1 - \frac{1}{p}\right).$$

10. State and prove the selberg identity.

11. If $x \geq 1$, prove that :

$$\sum_{n > x} \frac{1}{n^s} = O\left(x^{1-s}\right) \text{ if } s > 1.$$

### Unit II

12. Prove that the relation :

$$\lim_{x \to \infty} \frac{\pi(x) \log x}{x} = 1$$

implies the relation :

$$\lim_{x \to \infty} \frac{\mathcal{J}(x)}{x} = 1.$$

13. Let $\{a(n)\}$ be a non-negative sequence such that :

$$\sum_{n \leq x} a(n) \left[\frac{x}{n}\right] = x \log x + O(x) \text{ for all } x \geq 1 \cdot$$

For $x \geq 1$, prove that :

$$\sum_{n \leq x} \frac{a(n)}{\cdot n} = \log x + O(1).$$

14. State and prove the Abel's identity.

**D 93426** (Pages : 4) Name.......................................

Reg. No..................................

# FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2020

(CBCSS)

Mathematics

MTH 1C 04—DISCRETE MATHEMATICS

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

## Part A

*Answer* **all** *questions in Part A.*
*Each question carries a weightage of* 1.

1. State a necessary and sufficient condition for two simple graphs G and H to be isomorphic.
2. Prove or disprove : No loop can belong to an edge cut.
3. Is it true that every eulerian graph is connected. Justify.
4. If G is a connected plane graph, show that a cut edge of G belong to exactly one face.
5. Define maximal element and maximum element. Illustrate with an example.
6. If $(X, +, ., ')$ is a Boolean algebra, prove that : (i) $x + x = x$ and (ii) $x \cdot x = x$ for $x \in X$.
7. Prove that $(uv)^R = v^R u^R$ for all $u, v \in \sum^+$.
8. Define a finite automaton. Illustrate with an example.

(8 × 1 = 8 weightage)

## Part B

*Answer* **six** *questions in Part B choosing* **two** *from each unit.*
*Each question carries a weightage of* 2.

### Unit I

9. (a) Show that the number of edges of a simple graph with $n$ vertices and $w$ components is :

$$\frac{(n-w)(n-w+1)}{2}.$$

(b) Is it true ; if G is a simple graph with $\delta \geq \frac{n-2}{2}$, then G is connected. Justify.

10. (a) Prove that a simple graph is a tree if and only if any two distinct vertices are connected by a unique path.

(b) Show that the number of edges of a tree on a vertices is $n-1$.

11. (a) If G is a plane graph and $f$ is a face of G then prove that there exists a plane embedding of G in which $f$ is the exterior face.

(b) Prove or disprove : All planar embedding of a given planar graph have the same number of faces.

## Unit II

12. (a) Let X be a set and $\leq$ be a binary relation on X which is reflexive and transitive. Define R on X by $x$ R$y$ if $x \leq y$ and $y \leq x$. Verify whether R is an equivalence relation.

(b) Prove or disprove : A longest, chain in a partially ordered set $(X \leq)$ is maximal, but the converse need not hold.

13. If $(X, +, ., ')$ is a Boolean algebra,, prow that, for all $x, y, z \in X$.

(a) $x+(y+z)=(x+Y)+z$ and (b) $(x')' = x$.

14. (a) What is the disjunctive normal form of a Boolean function.

(b) Write the Boolean function $f(x_1, x_2, x_3) = (x_1 + x_2')x_3' + x_2x_1'(x_2 + x_1'x_3)$ in its disjunctive normal form.

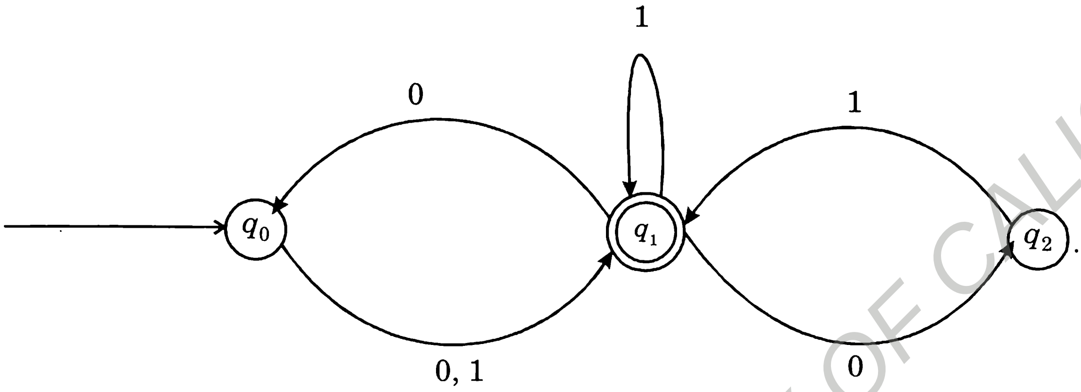
## Unit III

15. (a) Find *dfa* that accepts the set of all strings with exactly one $a$ where $\sum = \{a, b\}$.

(b) Find an *nfa* that accepts $\{a\}^*$ and is such that if in its transition graph a single edge is removed without any other changes, the resulting automaton accepts $\{a\}$.

16. Find a dfa that accepts the set of all strings on $\sum = \{a, b\}$ staring with the Prefix $ab$ ·'

17. Which of the given strings are accepted by the following automaton : (i) 01001 ; (ii) 10010 ; (iii) 000.




(6 × 2 = 12 weightage)

## Part C

*Answer any* **two** *questions in Part C.*

*Each question carries a weightage of* 5.

18. (a) Let T be a graph with $n$ vertices, then prove the following are equivalent :

    (i) T is a tree.

    (ii) T has no cycles and has $n - 1$ edges.

    (iii) There exists a unique path between any two vertices in T.

    (b) Find all non-isomorphic trees with 5 vertices.

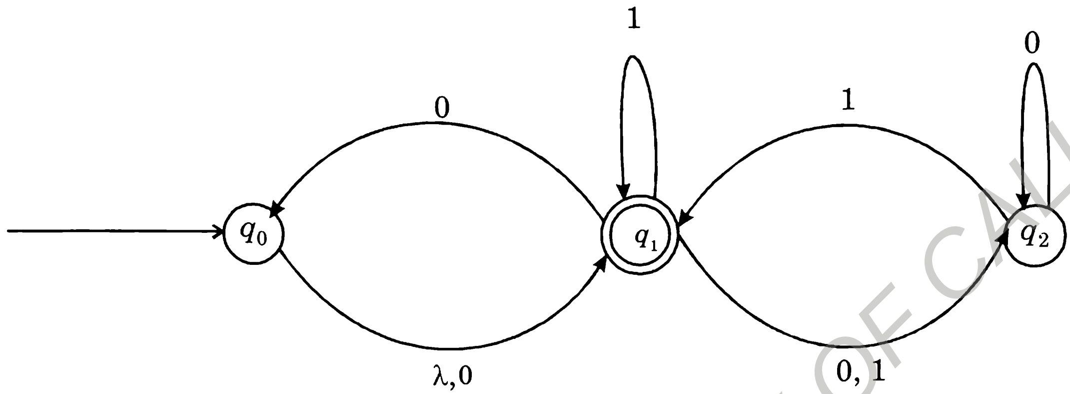
19. (a) Prove that $K_5$ is non-planar.

    (b) For any simple planar graph G, prove that $\delta(G) \leq 5$.

20. (a) Show that every Boolean function on $n$ variables $x_1, x_2, \ldots, x_n$ can be uniquely expressed as the sum of terms of the form $x_1^{\epsilon_1} x_2^{\epsilon_1} \ldots x_n^{\epsilon_n}$ where each $x_i^{\epsilon_i}$ is either $x_i$ or $x_i'$.

    (b) Show that every finite Boolean algebra is isomorphic to a power set Boolean algebra.

21. Convert the *nfa* given by the following diagram into an equivalent *dfa*.




(2 × 5 = 10 weightage)

**D 93425** (Pages : 3) Name.......................................

Reg. No..................................

# FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2020

(CBCSS)

Mathematics

MTH 1C 03—REAL ANALYSIS I

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

## General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

## Part A (Short Answer Questions)

*Answer* **all** *the questions.*
*Each question carries* 1 *weightage.*

1. Construct a bounded set of real numbers with exactly three limit points.
2. Let Y be an open subset of a metric space. If a subset E of Y is open relative to Y, then prove that E is open in X.
3. Let $f$ be a continuous mapping of a metric space X into a metric space Y. If $E \subset X$, then prove that $f(\overline{E}) \subset \overline{f(E)}$.
4. Give an example of a differentiable function $f$ on $\mathbb{R}$ such that $f'$ is not continuous at 0.
5. Let $f_1, f_2$ be bounded functions and $\alpha$ be a monotonic increasing function on $[a, b]$. If $f_1$ and $f_2$ are Riemann-Stieltjes integrable with respect to $\alpha$ on $[a, b]$, then prove that $f_1 + f_2$ is Riemann-Stieltjes integrable with respect to $\alpha$ on $[a, b]$.
6. Let $f$ be a bounded function and $\alpha$ be a monotonic increasing function on $[a, b]$ such that $|f|$ is Riemann-Stieltjes integrable with respect to $\alpha$. Is $f$ Riemann-Stieltjes integrable with respect to $\alpha$? Justify your answer.

7. Let $\gamma$ be a curve in the complex plane, defined on $[0,2\pi]$ by $\gamma(t) = e^{2\pi i t \sin\frac{1}{t}}$. Prove that $\gamma$ is not rectifiable.

8. If the sequences $\{f_n\}$ and $\{g_n\}$ converge uniformly on a set E, then prove that then sequence $\{f_n + g_n\}$ converge uniformly on E.

(8 × 1 = 8 weightage)

## Part B

*Answer any* **two** *questions of each unit.*
*Each question has weightage 2.*

### Unit I

9. Let A be the set of all sequences whose elements are the digits 0 and 1. Prove that A is countable.

10. Prove that a closed subset of a compact space is compact.

11. Let X be a connected metric space, Y be a metric space and let $f : X \to Y$ be a surjective continuous map. Prove that Y is connected.

### Unit II

12. Let $f$ be a real function defined on $[a, b]$ and let $f$ be differentiable on $(a, b)$. If $f'(x) \geq 0$ for all $x \in (a,b)$ then prove that $f$ is monotonically increasing.

13. If $f$ is differentiable on $[a, b]$, then prove that $f'$ cannot have any simple discontinuities on $[a, b]$.

14. If $f$ is Riemann-Stieltjes integrable with respect to $\alpha$ on $[a, b]$ and if $a < c < b$, then prove that $f$ is Riemann-Stieltjes integrable with respect $\alpha$ on $[a, c]$ and on $[c, b)$ and

$$\int_a^c f\, d\alpha + \int_c^b f\, d\alpha = \int_a^b f\, d\alpha.$$

### Unit III

15. Prove that the series $\sum_{n=1}^{\infty} (-1)^n \frac{x^2 + n}{n^2}$ converges uniformly in every bounded interval.

16. Let $\{f_n\}$ be a sequence of integrable functions and let $f$ be an integrable function such that $f_n \to f$. Is it true that $\int f\, dx = \lim \int f_n\, dx$? Justify your answer.

17. Let $\mathscr{C}$ (X) denote the set of all complex valued, continuous, bounded functions defined on a metric space X. Prove that $\mathscr{C}$ (X) is a complete metric space with respect to the metric :

$$d(f,g) = \sup_{x \in X} | f(x) - g(x) |.$$

(6 × 2 = 12 weightage)

## Part C

*Answer any* **two** *from the following four questions* (18–21).
*Each question has weightage* 5.

18. (a) Prove that a subset E of a metric space is open if and only if its complement $E^c$ is closed.

    (b) Prove that monotonic functions have no discontinuities of the second kind.

19. (a) Let $f$ be a bounded function and a be a monotonic increasing function on $[a, b]$. If $P_1$ is a refinement of P, then prove that :

    $$L(P, f, \alpha) \leq L(P_1, f, \alpha)$$

    (b) Let $f$ be a bounded function and $\alpha$ be a monotonic increasing function on $[a, b]$. If $f$ is continuous on $[a, b]$, then prove that $f$ is Riemann-Stieltjes integrable with respect to $\alpha$ on $[a, b]$.

20. (a) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

    (b) Let $\{f_n\}$ be a sequence of functions, differentiable on $[a, b]$ and such that $\{f_n(x_0)\}$ converges for some point $x_0$ on $[a, b]$. If $\{f_n'\}$ converges uniformly on $[a, b]$, then prove that $\{f_n\}$ converges uniformly on $[a, b]$, to a function $f$, and

    $$f'(x) = \lim_{n \to \infty} f_n'(x) \text{ for all } x \in [a, b].$$

21. (a) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

    (b) Let K be a compact metric space and let $f_n \in C(K)$ for $n = 1, 2, 3, \ldots$ and $\{f_n\}$ converges uniformly on K. Prove that $\{f_n\}$ is equicontinuous on K.

(2 × 5 = 10 weightage)

**D 93424** (Pages : 3) Name.......................................

Reg. No..................................

# FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2020

(CBCSS)

Mathematics

MTH 1C 02—LINEAR ALGEBRA

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

## General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

## Part A

*Answer* **all** *the questions.*
*Each question carries* 1 *weightage.*

1. Show that if V is a finite-dimensional vector space, then any two bases of V have the same number of elements.

2. Verify that the function T from $R^2$ into $R^2$ defined by $T(x_1, x_2) = (x_1 - x_2, 0)$ is a linear transformation.

3. Let T be the linear operator on $C^2$ defined by $T(x_1, x_2) = (x_1, 0)$ and let $B = \{(1,0),(0,1)\}$ and $B' = \{(1, i),(-i, 2)\}$ be two ordered bases for $C^2$. What is the matrix of T relative to the pair $B, B'$ ?

4. Show that if S is any subset of a finite-dimensional vector space V, then $(S^o)^o$ is the subspace spanned by S.

5. Let T be the linear operator on $R^2$, the matrix of which in the standard ordered basis is $A = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$. prove that the only subspaces of $R^2$ invariant under T are $R^2$ and the zero space.

**Turn over**

6. Prove that if E is the projection on R along N, then (I – E) is the projection on N along R.

7. Define inner product on a vector space and show that if $(\alpha \mid \beta) = 0$ for all $\beta$ in an inner product space V, then $\alpha = 0$.

8. Consider C, with the standard inner product. Find an orthonormal basis for the subspace spanned by $\beta_1 = (1, 0, i)$ and $\beta_2 = (2, 1, 1+i)$.

(8 × 1 = 8 weightage)

## Part B

*Answer any* **two** *questions from each of the following units.*
*Each question carries 2 weightage.*

### Unit I

9. Show that a non-empty subset W of a vector space V is a subspace of V iff for each pair of vectors $\alpha, \beta$ in W and each scalar $c$ in the scalar field F the vector $c\alpha + \beta$ is in W.

10. Find the co-ordinate matrix of the vector (1, 0, 1) in the basis of $c^3$ consisting of the vectors $(2i, 1, 0), (2, -1, 1), (0, 1+i, 1-i)$ in that order.

11. Let T be the linear operator on $R^3$ defined by $T(x_1, x_2, x_3), (3x_1, x, -x_2, 2x_1 + x_2 + x_3)$. Is T invertible ? If so, find a rule for $T^{-1}$.

### Unit II

12. Let V be a finite-dimensional vector space over the field F, and let $B = \{\alpha_1, \ldots, \alpha_n\}$ be a basis for V. Show that there is a unique dual basis $B^* = \{f_1, \ldots, f_n\}$ for the dual space $V^*$.

13. Let V and W be vector spaces over the field F, and let T be a linear transformation from V into W. Show that the null space of $T^t$ is the annihilator of the range of T. Show further that if V and W are finite-dimensional then rank $(T^t)$ = rank (T).

14. Let T be a diagonalizable linear operator on an $n$-dimensional vector space V, and let W be an invariant subspace under T. Prove that the restriction operator $T_w$ is diagonalizable.

### Unit III

15. Let V be a finite-dimensional vector space and let $w_1$ be any subspace of V. Show that there is a subspace $w_2$ of V such that $V = w_1 \oplus w_2$.

16. Let E be a projection of V and let T be a linear operator on V. Prove that the range of E is invariant under T iff ETE = TE.

17. Show that an orthogonal set of non-zero vectors is linearly independent in an inner product space.

(6 × 2 = 12 weightage)

## Part C

*Answer any* **two** *questions.*
*Each question carries 5 weightage.*

18. If $w_1$ and $w_2$ are finite-dimensional subspaces of a vector space V, then show that :

    (i) $w_1 + w_2$ is a finite-dimensional subspace of V.

    (ii) $\dim w_1 + \dim w_2 = \dim (w_1 \cap w_2) + \dim (w_1 + w_2)$

19. Let W be the subspace of $R^5$ which is spanned by the vectors $\alpha_1 = (1,2,1,0,0)$, $\alpha_2 = (0,1,3,3,1)$, $\alpha_3 = (1,4,6,4,1)$. Find a basis for $w^0$.

20. Let T be a linear operator on a finite dimensional vector space V. Prove that if $f$ is the characteristic polynomial for T, then $f(T) = 0$.

21. Let W be a finite-dimensional subspace of an inner product space V and let E be the projection of V on W. Show that :

    (i) E is an idempotent linear transformation of V onto W.

    (ii) $W^{\perp}$ is the null space of E.

    (iii) $V = W \oplus W^{\perp}$.

(2 × 5 = 10 weightage)

**D 93423** (Pages : 2) Name.........................................

Reg. No....................................

## FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2020

(CBCSS)

Mathematics

MTH 1C 01—ALGEBRA I

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

### Part A

*Answer* **all** *questions.*
*Each question has weightage* 1.

1. Verify whether $\phi(x,y)=(x+1,y)$ is an isometry of the plane.
2. Find a generator of the cyclic group $\mathbb{Z}_2 \times \mathbb{Z}_3$.
3. Describe all abelian groups of order 20 upto isomorphism.
4. Find all subgroups of the quotient group $\mathbb{Z}/6\mathbb{Z}$.
5. Let G be a group of order 18. Find the number of 3-Sylow subgroups of G.
6. Give all elements of the group given by the presentation $\left(a : a^5 = 1\right)$.
7. Find the inverse of $2i+j+k$ in the ring of quaternions.
8. Let $\phi : \mathbb{Z}\times \rightarrow \mathbb{Z}\times\mathbb{Z}$ be the homomorphism defined by $(x,y)\mapsto(x,0)$. Find $\ker\phi$.

(8 × 1 = 8 weightage)

### Part B

*Answer any* **six** *questions, choosing two from each unit.*
*Earth question has weightage* 2.

Unit 1

9. Verify whether $\mathbb{Z}_5 \times \mathbb{Z}_6$ is a cyclic group.
10. Show that $\mathbb{Z}/5\mathbb{Z}$ is isomorphic to $\mathbb{Z}_5$.
11. Verify whether $\mathbb{Z}_6$ is simple.

Unit 2

12. Show that (1 2 3) and (1 3 2) are conjugates in the symmetric group $S_3$.

13. Verify whether the following series of groups are isomorphic :

$0 < (5) < \mathbb{Z}_{15}$ and $0 < (3) < \mathbb{Z}_{15}$.

14. Show that every group of order 15 is cyclic.

Unit 3

15. Show that the ring of all endomorphisms of the group $\mathbb{Z}$ of integers is commutative.

16. Let $\phi_2 : \mathbb{Q}[x] \to \mathbb{Q}$ be the evaluation homomorphism with $\phi_2(x) = 2$. Find the kernal of $\phi_2$.

17. Verify whether $x^3 + 3x + 2 \in \mathbb{Z}_5[x]$ is irreducible.

(6 × 2 = 12 weightage)

## Part C

*Answer any* **two** *questions.*
*Each question has weightage 5.*

18. (a) Let G be a group and H be a normal subgroup of G.
    i. Show that the coset multiplication $(aH)(bH) = (ab)H$ is well defined.
    ii. Verify that G / H is a group.

    (b) Describe the factor group $\mathbb{Z}_{20}/H$ where H is the subgroup generated by 5.

19. (a) Let H, K be groups and G = H × K. Show that :
    i. $\bar{H} = \{(h, e) : h \in H\}$ is a normal subgroup of G where is the identity of K.
    ii. $G / \bar{H}$ is isomorphic to K.

    (b) Show that factor groups of cyclic groups are cyclic.

20. Let G be a group and N be a normal subgroup of G and H be any subgroup of G. Show that :
    (a) H N = N H.
    (b) H N is a subgroup of G.
    (c) $H \cap N$ is a normal subgroup of G.
    (d) H N / N is isomorphic to $H / (H \cap N)$.

21. (a) Let F be a field and $f(x) \in F[x]$. Show that $a \in F$ is a zero of $f(x)$ if and only if $(x - a)$ is a factor of $f(x)$.

    (b) Prove that a polynomial of degree 3 in F [x] is irreducible if and only if it has no zero in F.

(2 × 5 = 10 weightage)

**D 93245** (Pages : 2) Name..........................................

Reg. No.....................................

# FIRST SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION NOVEMBER 2020

(CUCSS)

Mathematics

MT IC 05—DISCRETE MATHEMATICS

(2016 Admissions)

Time : Three Hours Maximum : 36 Weightage

## Part A (Short Answer Questions)

*Answer* **all** *questions.*
*Each question has weightage* 1.

1. Compute the number of edges in the complete tripartite graph $K_{l,m,n}, l, m, n \in \mathbb{N}$.
2. In any simple graph, prove that the number of vertices of odd degree is even.
3. Define (i) $\kappa(G)$, connectivity and (ii) $\lambda(G)$, line connectivity of a graph G.
4. With standard notation, draw a graph G with $\kappa(G) = 1, \lambda(G) = 2$ and $\delta(G) = 3$.
5. In a tree, prove that every edge is a cut edge.
6. Prove that graph $K_5$ is nonplanar.
7. Define a total order. Give an example of a partial order which is not a total order.
8. Let P be the set of all real valued functions defined on a non-empty set X and $\leq$ be defined on P by $f \leq g$ if $f(x) \leq g(x)$ for every $x \in X$. Under what conditions $(P, \leq)$ is a chain.
9. Prove that an equivalence relation defined on a non-empty set X partitions the set.
10. Define a Boolean algebra.
11. Define a Boolean function on $n$ variables.Give an example of a Boolean function of 5 variables.
12. Explain the concatenation of two strings with an example.
13. Design a *dfa* which accepts string 1 only.
14. If $\sum = \{0,1\}$, design an *nfa* to accept set of strings either ending with two consecutive ones or two consecutive zeros.

(14 × 1 = 14 weightage)

**Turn over**

## Part B (Paragraph Type)

*Answer any* **seven** *questions from the following ten questions.*
*Each question has weightage* 2.

15. Prove that two graphs G and H are isomorphic if and only if $\bar{G} \cong \bar{H}$.

16. Prove that a tree of order $n$ has $n - 1$ edges, $n \in \mathbb{N}$.

17. With usual notation, prove that $\kappa(G) \leq \lambda(G)$, in any simple graph G.

18. Prove that every maximal outerplanar graph G with $p$ points has $2p - 3$ edges.

19. Prove that the Petersen graph is nonplanar.

20. Define a chain in a poset. Prove that the intersection of two chains is a chain.

21. Let $(X, +, ., ')$ be a Boolean algebra. Prove that $x + x \cdot y = x$ for all $x, y \in X$.

22. Prepare the table for values of the function $f(x, y, z) = (x + y)z'$.

23. Find a *dfa* that accepts all strings on $\sum(a, b)$ starting with the prefix $aa$.

24. Find a *dfa* for the language $L = \{ab^4wb^2 : w \in \{a, b\}^*$.

(7 × 2 = 14 weightage)

## Part C (Essay Type)

*Answer any* **two** *questions from the following four questions.*
*Each question has weightage* 4.

25. Define line graph of a graph. If $G_1$ and $G_2$ are isomorphic simple graphs, then prove that $L(G_1)$ and $L(G_2)$ are isomorphic.

26. Prove that a nontrivial connected graph is Eulerian if and only if all its vertices are of even degree.

27. Prove that every Boolean algebra is isomorphic to a power set Boolean algebra.

28. Find grammars for $\sum(a, b)$ that generate the sets of all strings with exactly one $a$.

(2 × 4 = 8 weightage)

**D 93244** (Pages : 3) Name.......................................

Reg. No..................................

# FIRST SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION NOVEMBER 2020

(CUCSS)

Mathematics

MT 1C 04—NUMBER THEORY

(2016 Admissions)

Time : Three Hours Maximum : 36 Weightage

## Part A

*Answer* **all** *questions.*
*Each question carries a weightage of* 1.

1. If $(m, n) = 1$, show that $(\varphi(m), \varphi(n)) = 1$.

2. If $I(n) = \left[\frac{1}{n}\right]$ show that $(f * I)(n) = f(n)$ for any arithmetic function $f$ and $n \in \mathbb{N}$.

3. State the generalized Möbius inversion formula. When will this give the Möbius inversion formula ?

4. Define the big O notation and explain the meaning of the notation $f(x) \sim g(x)$ as $x \to \infty$.

5. State and prove the Legendere's identity.

6. Express $\upsilon(x)$ in terms of $\pi(x)$ and $\pi(x)$ in terms of $\upsilon(x)$.

7. State the prime number theorem. Also give its equivalent forms in terms of $\psi(x)$ and $\upsilon(x)$.

8. Using prime number theorem, prove that every interval $[a, b]$ with $0 < a < b$ contains a real number of the form $p/q$ where $p, q$ are primes.

9. State Shapiro's theorem.

10. Define quadratic residue and non-residue mod $p$ for an odd prime $p$.

11. What is the value of $(2|25)$ ?

12. State the quadratic reciprocity law. Verify it for primes 3 and 5.

13. Define enciphering and deciphering transformations.

14. How do classical and public key cryptosystems differ ?

(14 × 1 = 14 weightage)

## Part B

*Answer any* **seven** *questions.*
*Each question carries a weightage of* 2.

15. For $n \geq 1$, prove that $\sum_{d|n} \mu(d) = \left[\frac{1}{n}\right]$.

16. Prove that the Dirichlet product $f * g$ of two multiplicative functions is again multiplicative.

17. For $n \geq 1$, prove that $\sum_{d|n} \lambda(d) = \begin{cases} 1 & \text{if } n \text{ is a square} \\ 0 & \text{otherwise} \end{cases}$.

18. State and prove the Euler summation formula.

19. For $n \geq 1$, prove that the $n^{\text{th}}$ prime $p_n$ satisfies the inequalities

$$\frac{1}{6} n \log n < p_n < 12\left(n \log n + n \log \frac{12}{e}\right).$$

20. If $p_n$ is the $n^{\text{th}}$ prime, prove that if $\lim_{x \to \infty} \frac{\pi(x) \log \pi(x)}{x} = 1$ then $\lim_{x \to \infty} \frac{\pi(x) \log x}{x} = 1$.

21. Prove that $\lim_{x \to \infty} \left(\frac{M(x)}{x} - \frac{H(x)}{x \log x}\right) = 0$.

22. Determine those odd primes $p$ for which 3 is a quadratic residue and those for which it is a non-residue.

23. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(\mathbb{Z}/N\mathbb{Z})$ and set $D = ad - bc$. Prove that if $x$ and $y$ are not both 0 in $\mathbb{Z}/N\mathbb{Z}$, then $A\begin{pmatrix} x \\ y \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \gcd(D, N) = 1$.

24. How does RSA cryptosystem work ?

(7 × 2 = 14 weightage)

## Part C

*Answer any* **two** *questions.*
*Each question carries a weightage of* 4.

25. Prove that the set of all arithmetic functions $f$ with $f(1) \neq 0$ forms an abelian group with respect to Dirichlet convolution.

26. Prove the following : There is a constant A such that $\sum_{p \leq x} \frac{1}{p} = \log \log x + A + O\left(\frac{1}{\log x}\right)$ for all $x \geq 2$.

27. State and prove the Gauss' lemma.

28. Suppose we know that our adversary is using an enciphering matrix A in the 26-letter alphabet. If we intercept the ciphertext "WKNCCHSSJH" and identify first word to be "GIVE", find the deciphering matrix and read the complete message.

(2 × 4 = 8 weightage)

**D 93243** (Pages : 3) Name.....................................

Reg. No..................................

# FIRST SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION NOVEMBER 2020

(CUCSS)

Mathematics

MT 1C 03—REAL ANALYSIS–I

(2016 Admissions)

Time : Three Hours Maximum : 36 Weightage

## Part A (Short Answer Questions)

*Answer* **all** *questions.*
*Each question has weightage* 1.

1. If $f$ is a continuous function from a metric space X into $\mathbb{R}$, show that the set $Z(f) = \{x \in X / f(x) = 0\}$ is closed in X.

2. Give an example of a continuous function on $(0,1]$ which is not uniformly continuous. Justify your answer.

3. Show that a finite subset of a metric space has no limit points.

4. Show that the Cantor set is perfect.

5. Give an example of a bounded metric space which is not compact. Justify your answer.

6. If $f : [a,b] \to \mathbb{R}$ is a function which has a local maximum at a point $x \in (a,b)$ and $f'(x)$ exists, show that $f'(x) = 0$.

7. If $f$ is a bounded real function and $\alpha$ is a monotonically increasing real function on $[a,b]$, show that $\int_{-a}^{b} f\, d(\alpha) \leq \int_{a}^{-b} f\, d(\alpha)$.

8. If $f \geq 0, f$ is continuous on $[a,b]$ and $\int_a^b f(x)\, dx = 0$, prove that $f(x) = 0$ for all $x \in [a,b]$.

9. Show that the function given by $f(x) = \begin{cases} x^2 \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is differentiable at every $x \in \mathbb{R}$.

10. Show that for every continuous function $f : [0,1] \to [0,1]$ there is a point $x \in [0,1]$ such that $f(x) = x$.

**Turn over**

11. Evaluate the sum of functions $\sum_{n=0}^{\infty}\frac{x^2}{\left(1+z^2\right)^n}$ on $\mathbb{R}$.

12. Verify whether the sequence of functions $\{f_n\}$ given by $f_n(x)=\frac{x^2}{x^2+(1-nx)^2}$, $n=1,2,3,....$ has a uniformly convergent subsequence.

13. Verify whether the sequence $\{f_n'\}$ converge to $f'$, where $f_n(x)=\frac{\sin nx}{\sqrt{n}}$ and $f(x)=\lim_{n\to\infty}f_n(x)$.

14. Prove that every uniformly convergent sequence of bounded real functions on $\mathbb{R}$ is uniformly bounded.

(14 × 1 = 14 weightage)

## Part B

*Answer any* **seven** *questions.*
*Each question has weightage* 2.

15. Prove that a ball in $\mathbb{R}^k$ is convex.

16. Construct a bounded set of real numbers having exactly three limit points and justify your answer.

17. If $f$ is monotonic, $\alpha$ is continuous and monotonically increasing on $[a,b]$, prove that $f\in R(\alpha)$ on $[a,b]$.

18. Show by an example that the L' Hospital's rule may fail to be true for complex valued functions.

19. If $f$ is differentiable on $[a,b]$ and $\lambda$ is a number such that $f'(a)<\lambda<f'(b)$, prove that there exists a point $x\in(a,b)$ such that $f'(x)=\lambda$.

20. Define a rectifiable curve on $[a,b]$. Verify whether the curve $f(t)=\cos t+i\sin t$ is rectifiable on $[0,\pi]$.

21. State and prove the Cauchy criterion for uniform convergence of sequence of real functions on a subset E of a metric space.

22. Prove that a sequence $\{f_n\}$ converges to $f$ with respect to the supremum metric of C(X) if and only if $f_n\to f$ uniformly on X.

23. Prove that the set C ([0, 1]) of all continuous complex valued functions on [0, 1] is a complete metric space in the supremum metric.

24. Let $f:[a,b]\to\mathbb{R}$ be a differentiable function. Show that its derivative $f'$ cannot have any simple discontinuities on [a, b}.

(7 × 2 = 14 weightage)

## Part C

*Answer any* **two** *questions.*
*Each question has weightage 4.*

25. Prove that every continuous function on a compact metric space into a metric space is uniformly continuous.

26. Prove that a subset $E \subset \mathbb{R}$ is connected if and only if it has the following property : if $x, y \in E$ and $x < z < y$, then $z \in E$.

27. Let $f \in R$ on $[a,b]$. Prove that the function $F(x) = \int_a^x f(t)\,dt$, for $a \le x \le b$, is continuous on $[a,b]$.

28. If $\{f_n\}$ is a pointwise bounded sequence of complex valued functions on a countable set E of a metric space, prove that $\{f_n\}$ has a subsequence that converges pointwise on E.

(2 × 4 = 8 weightage)

**D 93242** (Pages : 4) Name..................................

Reg. No..................................

# FIRST SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION NOVEMBER 2020

(CUCSS)

Mathematics

MT 1C 02—LINEAR ALGEBRA

(2016 Admissions)

Time : Three Hours Maximum : 36 Weightage



## Part A

*Answer **all** the questions.*
*Each question carries weightage 1.*

1. Let V be a vector space over a field K. Show that for $0 \in K$ and any vector $u \in V, 0u = 0$.

2. Consider $V = \mathbb{R}^3$ as a vector space over $\mathbb{R}$. Show that W is not a subspace of V, where $W = \left\{(a, b, c) : a^2 + b^2 + c^2 \leq 1\right\}$.

3. For which value of $k$ will the vector $u = (1, k, 5)$ in $\mathbb{R}^3$ be a linear combination of the vectors $v = (1, -3, 2)$ and $w = (2, -1, 1)$?

4. Show that the mapping $F : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $F(x, y) = (x + 1, 2y, x + y)$ is not linear.

5. Let $\phi$ be the linear functional on $\mathbb{R}^2$ defined by $\phi(x, y) = x - 2y$. For the linear operator T on $\mathbb{R}^2$, defined by $T(x, y) = (y, x + y)$, find $\left(T^t(\phi)\right)(x, y)$, where $T^t$ stands for transpose of T.

6. Let $\lambda$ be an eigenvalue of an operator $T : V \to V$. Show that $V_\lambda$, the eigenspace of $\lambda$ is a subspace of V.

7. Let $A = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$. Find all eigenvalues and the corresponding eigenvectors of A viewed as a matrix over the complex field $\mathbb{C}$.

**Turn over**

8. Show that the vectors

$e_1 = (1, 0, 0, \ldots, 0), e_2 = (0, 1, 0, \ldots., 0), e_3 = (0, 0, 1, \ldots., 0), \ldots., e_n = (0, 0, 0, \ldots., 1)$ is a basis for $\mathbb{R}^n$.

9. For the matrix $A = \begin{bmatrix} 2 & -3 \\ 7 & -4 \end{bmatrix}$, find a polynomial having the matrix A as a root.

10. Is $\mathbb{R}^2$ a subspace of $\mathbb{R}^3$ ? Justify your answer.

11. Is there a linear transformation that maps $(1, 0)$ to $(5, 3, 4)$ and maps $(3, 0)$ to $(1, 3, 2)$ ?

12. Let $\mathbb{R}^3$ be equipped with the standard inner product. What is the orthogonal projection of the vector $x = (-10, 2, 8)$ onto the vector $y = (3, 12, -1)$ ?

13. Find the norm of $(3, 4) \in \mathbb{R}^2$ with respect to the usual inner product.

14. Let $\alpha = (1, 2), \beta = (-1, 1)$. If $\gamma$ is a vector such that $\langle \alpha, \gamma \rangle = -1$ and $\langle \beta, \gamma \rangle = 3$, find $\gamma$.

(14 × 1 = 14 weightage)

## Part B

*Answer any* **seven** *questions.*
*Each question carries weightage* 2.

15. Let U and W be subspaces of $\mathbb{R}^3$ defined by $U = \{(a, b, c) : a = b = c\}$ and $W = \{(0, b, c)\}$. Show that $\mathbb{R}^3 = U \oplus W$.

16. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear mapping defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Find a basis and the dimension of the image U of T.

17. Let V be a finite dimensional vector space over the field F, and let W be a subspace of V. If $W^0$ denotes the annihilator of W, then prove that $\dim W + \dim W^0 = \dim V$.

18. Let T be the linear operator on $\mathbb{R}^2$, the matrix of which in the standard ordered basis is $A = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$.

Prove that the only subspaces of $\mathbb{R}^2$ invariant under T are $\mathbb{R}^2$ and the zero subspace.

19. In the space $C[0, \pi]$, consisting of all continuous functions on the interval $[0, \pi]$, with the inner product $\langle f, g \rangle = \int_0^\pi f(t) g(t) dt$, consider the sequence of functions $\{w_k\}$ defined by $\{w_k\} = \cos(kt), k = 0, 1, 2, \ldots\ldots$ Is this an orthogonal sequence ? Justify your answer.

20. Find a unit vector orthogonal to $v_1 = (1, 1, 2)$ and $v_2 = (0, 1, 3)$ in $\mathbb{R}^3$.

21. Let W be the subspace of $\mathbb{R}^5$ spanned by $u = (1, 2, 3, -1, 2)$ and $v = (2, 4, 7, 2, -1)$. Find a basis for the orthogonal complement $W^\perp$ of W.

22. State and prove the Cauchy-Schwarz inequality.

23. Let $v_1, v_2, \ldots\ldots, v_n$ be non-zero eigenvectors of an operator $T : V \to V$ belonging to distinct eigenvalues $\lambda_1, \lambda_2, \ldots\ldots, \lambda_n$. Prove that $v_1, v_2, \ldots\ldots, v_n$ are linearly independent.

24. Verify that the following is an inner product in

$\mathbb{R}^2 : \langle u, v \rangle = x_1 y_1 - 2x_1 y_2 - 2x_2 y_1 + 5x_2 y_2$, where $u = (x_1, x_2), v = (y_1, y_2)$.

(7 × 2 = 14 weightage)

## Part C

*Answer any* **two** *questions*
*Each question carries weightage* 4.

25. Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. If V is finite dimensional show that $\text{rank}(T) + \text{nullity}(T) = \dim V$.

26. Let V and W be vector spaces over the field F, and let T be a linear transformation from V into W. The null space of $T^t$ (transpose of T) is the annihilator of the range of T. If V and W are finite dimensional, then prove that :

    (i) $\text{rank}(T^t) = \text{rank}(T)$

    (ii) the range of $(T^t)$ is the annihilator of the null space of T.

27. Apply Gram-Schmidt process to the vectors $w_1 = (1, 0, 3)$ $w_2 = (2, 2, 0)$ and $w_3 = (3, 1, 2)$ to compute an orthonormal basis for $\mathbb{R}^3$.

28. Suppose $V = W_1 \oplus W_2 \oplus ..... \oplus W_r$. The projection of V into its subspace $W_k$ is the mapping $E : V \to V$ defined by $E(v) = W_k$, where $v = w_1 + w_2 + .... + w_r, w_i \in W_i$, Show that (i) E is linear ; and (ii) $E^2 = E$.

(2 × 4 = 8 weightage)

**D 93241** (Pages : 3) Name.......................................

Reg. No..................................

# FIRST SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION NOVEMBER 2020

(CUCSS)

Mathematics

MT 1C 01—ALGEBRA—I

(2016 Admissions)

Time : Three Hours Maximum : 36 Weightage

## Part A

*Answer* **all** *questions.*
*Each question carries weightage* 1.

1. Verify whether $\phi(x, y) = (x+2, y+3)$ is an isometry of the Euclidean plane $\mathbb{R}^2$.

2. Give two non-isomorphic groups of order 10.

3. Find the order of $(3, 10, 9)$ in the group $\mathbb{Z}_4 \times \mathbb{Z}_{12} \times \mathbb{Z}_{15}$.

4. Show that if G is non-abelian, then the factor $G/Z(G)$ is not cyclic.

5. Find the ascending central series of $S_3$.

6. Verify whether the series $\{0\} < 10\mathbb{Z} < \mathbb{Z}$ and $\{0\} < 25\mathbb{Z} < \mathbb{Z}$ are isomorphic.

7. Give an example to show that a factor ring of an integral domain may be a field.

8. Derive class equation.

9. For a prime $p$, prove that every group G of order $p^2$ is abelian.

10. Prove that a non-zero polynomial $f(x) \in F[x]$ of degree $n$ can have almost $n$ zeros in a field F.

11. For $n \in \mathbb{N}$, prove that $\mathbb{Z}/n\mathbb{Z}$ is isomorphic to $\mathbb{Z}_n$.

12. Prove that no group of order 36 is simple.

13. Let H and K be normal subgroups of a group G. Give an example showing that we may have H isomorphic to K but $G/H$ is not isomorphic to $G/K$.

14. Prove that $f(x) = x^4 - 2x^2 + 8x + 1$ is irreducible over $\mathbb{Q}$.

(14 × 1 = 14 weightage)

## Part B

*Answer any* **seven** *questions.*
*Each question has weightage* 2.

15. Find all abelian groups, upto isomorphism, of order 360.

16. Let H be a normal subgroup of G. Prove that the cosets of H form a group $G/H$ under the binary operation $(aH)(bH) := (ab)H$.

17. Find isomorphic refinements of the series $\{0\} < 8\mathbb{Z} < 4\mathbb{Z} < \mathbb{Z}$ and $\{0\} < 9\mathbb{Z} < \mathbb{Z}$.

18. Let $p$ be a prime. Let G be a finite group and let $p$ divides G. Prove that G has a subgroup of order $p$.

19. What is meant by a free group ?

20. Show that the presentation $(a, b : a^3 = 1, b^2 = 1, ba = a^2 b)$ gives a group isomorphic to $S_3$.

21. Let F be a field and let $f(x) \in F[x]$. Suppose $f(x)$ is of degree 2 or 3. Prove that $f(x)$ is irreducible over F if and only if it lias a zero in F.

22. Let F be the ring of all functions mapping $\mathbb{R}$ into $\mathbb{R}$, and let C be the subring of F consisting of all the constant functions in F. Is C an ideal in F ? why ?

23. Let G be a group. If H is any subgroup of G and N is a normal subgroup of G. Prove that HN is a subgroup of G.

24. Let H and K be normal subgroups of a group G with $K \leq H$. Show that $G/H$ is isomorphic to $(G/K)/(H/K)$.

(7 × 2 = 14 weightage)

## Part C

*Answer any* **two** *questions.*
*Each question has weightage 4.*

25. Prove that every finite group of isometries of the plane is isomorphic to either $\mathbb{Z}_n$ or to a dihedral group $D_n$ for some positive integer $n$.

26. Let X be a G-set. For $x, y \in X$ let $x \sim y$ if $gx = y$ for some $g \in G$. Show that :

    (a) $\sim$ is an equivalence relation on X.

    (b) If $[x]$ is the equivalence class containing $x$ then $\left|[x]\right| = (G : G_x)$ where $G_x = \{g \in G : gx = x\}$.

27. State and prove first Sylow theorem.

28. (a) State Eisenstein Criterion for irreducibilty of polynomials in $\mathbb{Q}[x]$.

    (b) Prove that the polynomial $\phi_p(x) = \dfrac{x^p - 1}{x - 1}$ is irreducible over $\mathbb{Q}$.

(2 × 4 = 8 weightage)

**C 84011** (Pages : 3) Name.......................................

Reg. No..................................

# M.Sc. (PREVIOUS) DEGREE [CBCSS] EXAMINATION, APRIL/MAY 2020

(PVT/SDE)

M.Sc. Mathematics–First Semester

MTH IC 05—NUMBER THEORY

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

## Part B

### Section A

*Answer* **all** *questions.*
*Each question has weightage* 1.

1. Prove that $\phi(mn) = \phi(m).\phi(n)(d/\phi(d))$,

   where $d = (m,n)$ and $\phi$ is the Euler totient function.

2. If *f* is multiplicative, prove that

   $$\sum_{d/n} \mu(d)\, f(d) = \prod_{p/n} (1 - f(p)).$$

3. Prove that $[x] + \left[x + \frac{1}{2}\right] = [2x]$.

4. For $x \geq 2$, prove that $I(x) = \pi(x)\log x - \int_2^x \frac{\pi(t)}{t}\, dt$.

5. If $0 < a < b$, prove that there exists an $x_0$ such that $\pi(ax) < \pi(bx)$ if $x \geq x_0$.

6. If P is an odd positive integer, prove that $(-1/p) = (-1)^{(p-1)/2}$.

7. Briefly describe about digraph transformations.

8. What is meant by hash functions in public key cryptography ?

(8 × 1 = 8 weightage)

Turn over

## Section B

*Answer* **six** *questions by choosing* **two** *questions from each unit.*
*Each question has weightage 2.*

### Unit I

9. If $n \geq 1$, prove that $\phi(n) = \sum_{d/n} \mu(d) \cdot \frac{n}{d}$.

10. Define the derivative $f'$ of an arithmetical function $f$. If $f$ and $g$ are arithmetical functions, then prove that $(f * g)' = f' * g + f * g'$.

11. For $x \geq 2$, prove that $\sum_{p \leq x} \left[\frac{x}{p}\right] \log p = x \log x + O(x)$, where the sum is extended over all primes $\leq x$.

### Unit II

12. Prove that the relation $\lim_{x \to \infty} \frac{I(x)}{x} = 1$

implies $\lim_{x \to \infty} \frac{\pi(x) \log x}{x} = 1$.

13. Let $\{a(n)\}$ be a non-negative sequence such that $\sum_{n \leq x} a(n) \left[\frac{x}{n}\right] = x \log x + O(x)$ for all $x \geq 1$. Prove that there is a constant $B > 0$ such that $\sum_{n \leq x} a(n) \leq Bx$ for all $x \geq 1$.

14. For $n \geq 1$, prove that the $n^{th}$ prime $p_n$ satisfy the inequality $p_n < 12\left(n \log n + n \log \frac{12}{e}\right)$.

### Unit III

15. Let $p$ be an odd prime. Prove that for all integers $n$,

$(n/p) \equiv n^{(p-1)/2} \pmod{p}$.

16. In a long string of cipher text which was encrypted by means of an affine map on single-letter message units in the 26-letter alphabet, it is observed that the most frequently occurring letters are "Y" and "V", in that order. Assuming that those cipher text message units are the encryption of "E" and "T" respectively, read the message "QAOOYQQEVHEQV".

17. Briefly describe about RSA cryptosystem.

(6 × 2 = 12 weightage)

## Section C

*Answer any two questions.*
*Each question has weightage 5.*

18. Show that the set of all arithmetical functions $f$ with $f(1) \neq 0$ forms an abelian group with respect to the Dirichlet product.

19. Prove that the prime number theorem implies $\lim_{x\to\infty} \frac{M(x)}{x} = 0$.

20. Assume $n \not\equiv 0 \pmod p$ and consider the least positive residue mod $p$ of the following $(p-1)/2$ multiples of $n$ :

$$n, 2n, 3n, \ldots, \left(\frac{p-1}{2}\right)^{n}.$$

If $m$ denotes the number of these residues which exceed P/2, then prove that $(n|p) = (-1)^{m}$.

21. (a) State and prove Euler's summation formula.

(b) Find the inverse of $A = \begin{pmatrix} 2 & 3 \\ 7 & 8 \end{pmatrix} \in M_2(z/26z)$.

(2 × 5 = 10 weightage)

**C 84011** (Pages : 4 + 3 = 7) Name.....................................

Reg. No..................................

**M.Sc. (PREVIOUS) DEGREE [CBCSS] EXAMINATION, APRIL/MAY 2020**

(PVT/SDE)

M.Sc. Mathematics–First Semester

MTH 1C 05—NUMBER THEORY

(2019 Admissions)

**Part A**

| | DD | | MM | | YEAR | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Date of Examination :** | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | FN/AN |

**Time : 15 Minutes** **Total No. of Questions : 20**



## INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. Immediately after the commencement of the examination, the candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Write the Name, Register Number and the Date of Examination in the space provided.
4. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer book.
5. **Candidate should handover this Question paper to the invigilator after 15 minutes and before receiving the question paper for Part B Examination.**

MTH 1C 05—NUMBER THEORY

**Part A**

Multiple Choice Questions :

1. If $n > 1$, then $\sum_{d/n} \mu(d) =$

   (A) 0 or 1.
   (B) 0.
   (C) –1.
   (D) 1.

2. The divisor sum of the Euler totitent function is :

   (A) $n+1$.
   (B) 1.
   (C) 0.
   (D) $n$.

3. Which of the following statement is not true ?

   (A) Dirichlet multiplication is commutative.
   (B) Dirichlet multiplication is commutative and associative.
   (C) Dirichlet multiplication is commutative but not associative.
   (D) Dirichlet multiplication is associative.

4. Which of the following statement is true ?

   (A) An arithmetical function $f$ has an inverse if $f(1) = 0$.
   (B) An arithmetical function $f$ has an inverse if $f(1) \neq 0$.
   (C) An arithmetical function $f$ has an inverse if $f(0) = 0$.
   (D) Every arithmetical function has an inverse.

5. The value of A (9) is :

   (A) 1.
   (B) log 3.
   (C) 3.
   (D) 0.

6. Which of the following statement is true ?

   (A) Euler totitent function is multiplicative as well as completely multiplicative.
   (B) Euler totitent function is multiplicative but not completely multiplicative.
   (C) Euler totitent function is not multiplicative.
   (D) Euler totitent function is completely multiplicative.

7. The Dirichlet product of two completely multiplicative functions is completely multiplicative :

(A) False. (B) True.

8. The Dirichlet inverse of the Liouville's function $\lambda(n)$ is :

(A) $[\mu(n)]$. (B) $\varphi(n)$.

(C) $\mu(n)$. (D) $\lambda(n)$.

9. If $x \geq 2$, the asymptotic formula for $\log[x]!$ is :

(A) $\log x - x + O(\log x)$. (B) $x\log x - x + O(\log x)$.

(C) $x + O(\log x)$. (D) $x\log x + O(\log x)$.

10. The fractional part of $x$ is :

(A) $[x]$. (B) $x + [x]$.

(C) $x - [x]$. (D) $[x] - x$.

11. The prime number theorem is equivalent to :

(A) $\lim_{x\to\infty} \frac{\theta(x)}{x} = 1$. (B) $\lim_{x\to\infty} \frac{\theta(x)}{x} = 0$.

(C) $\lim_{x\to\infty} \frac{\theta(x)}{\psi(x)} = 1$. (D) $\lim_{x\to\infty} \frac{\theta(x)}{M(x)} = 1$.

12. The highest power of 10 that divides 1000! is :

(A) 249. (B) 219.

(C) 229. (D) 239.

13. Which of the following statement is equivalent to the relation $M(x) = o(x)$ as $x \to \infty$ :

(A) $\psi(x) \sim x$ as $x \to \infty$. (B) $\theta(x) \sim x$ as $x^2 \to \infty$.

(C) $\varphi(x) \sim x$ as $x \to \infty$. (D) $M(x) \sim x$ as $x \to \infty$.

14. The function $M(x)$ is defind as :

(A) $\sum_{n\le x}\lambda(n)$.
(B) $\sum_{n\le x}\mu(n)$.
(C) $\sum_{n\le x}\varphi(n)$.
(D) $\sum_{n\le x}A(n)$.

15. Let $p_n$ denote the $n^{th}$ prime. Then the statement $\lim_{x\to\infty}\frac{p_n}{n\log n}=1$ is logically equivalent to :

(A) $\lim_{x\to\infty}\frac{x\log\pi(x)}{x}=1$.
(B) $\lim_{x\to 1}\frac{x\log\pi(x)}{x}=1$.
(C) $\lim_{x\to\infty}\frac{x\log\pi(x)}{x}=0$.
(D) The prime number theorem.

16. Public key encryption is advantageous over Symmetric key Cryptography because of :

(A) Speed.
(B) Key exchange.
(C) Space.
(D) Key length.

17. A(n) ———— algorithm transforms ciphertext to plaintext.

(A) Encryption or decryption.
(B) Decryption.
(C) Encryption.
(D) Neither encryption nor decryption.

18. A combination of an encryption algorithm and a decryption algorithm is called a :

(A) Secret.
(B) Key.
(C) Cipher.
(D) None of the above.

19. Which of the following options is not correct according to the definition of the Hash Function ?

(A) They compress the input values.
(B) Hash Function are mathematical functions.
(C) The hash functions work on arbitrary length input but produces fixed length output.
(D) None of the above.

20. "The Hash Function takes an input of arbitrary length and converts it into a fixed length output." Which of the following names can we use of denoting the output of the hash function ?

(A) Message Digest.
(B) Hash Code.
(C) Hash value.
(D) All of the above.

**C 84010** (Pages : 4) Name.....................................

Reg. No..................................

# M.Sc. (PREVIOUS) DEGREE [CBCSS] EXAMINATION, APRIL/MAY 2020

(PVT/SDE)

M.Sc. Mathematics–First Semester

MTH 1C 04—DISCRETE MATHEMATICS

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

## Part B

### Section A

*Answer* **all** *questions in Section A.*
*Each question carries* 1 *weightage.*

1. Prove that a simple graph with $n$ vertices can have at most $\frac{n(n-1)}{2}$ edges.

2. Prove or disprove : There exists graphs in which every edge is a cut edge.

3. Show that every connected graph contains a spanning tree.

4. For a connected plane graph G, with usual notations, prove that $n - m + f = 2$.

5. Define a lattice. What is a lattice diagram ? Illustrate with an example.

6. If $(X, +, \cdot, ')$ is a Boolean algebra, prove the laws of absorption : (i) $x + x \cdot y = x$; and (ii) $x \cdot (x + y) = x$.

7. Find a grammar for the language $L = \{w : |w| \bmod 3 > 0\}$.

8. What is an extended transition function ? Illustrate with an example.

(8 × 1 = 8 weightage)

## Section B

*Answer* **six** *questions in Section B choosing* **two** *from each unit.*
*Each question carries 2 weightage.*

### Unit I

9. (a) Prove that Aut $(G)$, the set of all automorphisms of a simple graph G is a group with respect to the composition of mappings.

   (b) Prove or disprove : $K_n = K_1 \vee K_{n-1}$.

10. (a) Prove that the deletion of edges of a minimum-edge cut of a connected graph G results in a disconnected graph with exactly two components.

    (b) If there is a unique path between any *two* vertices of a simple connected graph G, then prove that G is a tree.

11. (a) If G is a simple planar graph with at least 3 vertices, then prove that $m \leq 3n - 6$.

    (b) In a simple planar graph G, prove that $\delta(G) \leq 5$.

### Unit II

12. Let $(X, \leq)$ be a finite partially ordered set. Then prove :

    (a) Every chain in X is contained in a maximal chain ;

    (b) Every non-empty subset of X has a supremum, if X is a lattice.

13. If $(X, +, \cdot, ')$ is a Boolean algebra and $x, y, z \in X$ prove :

    (a) $(x \cdot y) \cdot z = x \cdot (y \cdot z)$; and

    (b) $x \cdot 1 = x$.

14. (a) What is conjunctive normal form of a Boolean function ?

    (b) Write the Boolean function $f(x_1, x_2, x_3) = x_1 x_2 + x_2' x_3$ in its DNF.

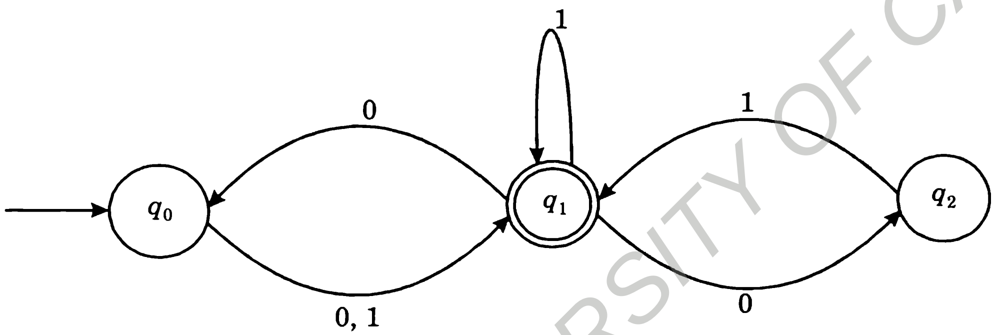
## Unit III

15. For $\sum = \{a, b\}$, find dfa that accept the set of all strings with no more than three $a$'s.

16. (a) Describe the rules defining the language $L(r)$ associated with the regular expression $r$.

(b) Give three major differences between dfa and nfa.

17. Which of the given strings are accepted by the following automaton (i) 00 ; (ii) 01001 ; and (iii) 10010.




(6 × 2 = 12 weightage)

## Section C

*Answer any **two** questions in Section C.*
*Each question carries 5 weightage.*

18. (a) For any non-trivial connected graph G, prove the following statements are equivalent :

(i) G is eulerian,

(ii) the degree of each vertex of G is an even positive integer,

(iii) G is an edge-disjoint union of cycles.

(b) Prove that an edge $e = xy$ of a connected graph G is a cut edge of G if and only if $e$ belongs to no cycle of G.

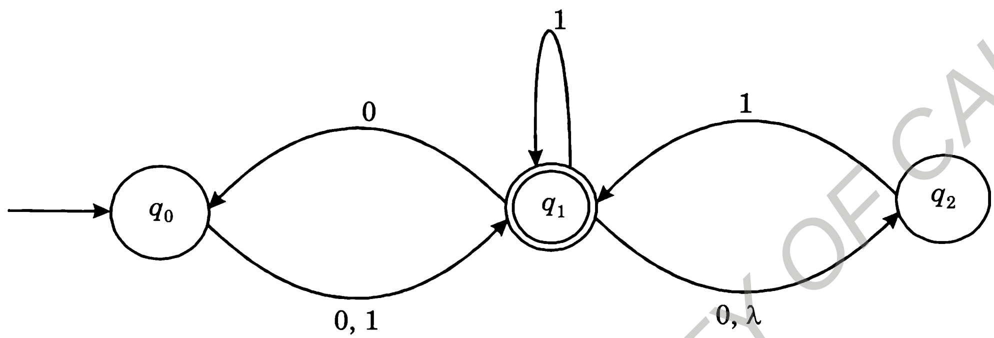
19. (a) Prove that $K_{3,3}$ is non-planar.

(b) Show that the graph is planar if and only if it is embeddable on a sphere.

20. (a) Show that every finite Boolean algebra is isomorphic to a power set Boolean algebra.

(b) If $(X, +, \cdot, ')$ is a finite Boolean algebra, prove that every non-zero element of X contains at least one atom.

21. Convert the nfa given by the following transition diagram into an equivalent dfa.




(2 × 5 = 10 weightage)

**C 84010** (Pages : 4 + 4 = 8) **Name.......................................**

**Reg. No...................................**

# M.Sc. (PREVIOUS) DEGREE [CBCSS] EXAMINATION, APRIL/MAY 2020

(PVT/SDE)

M.Sc. Mathematics–First Semester

MTH 1C 04—DISCRETE MATHEMATICS

(2019 Admissions)

**Part A**

| | DD | | MM | | YEAR | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Date of Examination :** | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | FN/AN |

**Time : 15 Minutes** **Total No. of Questions : 20**

## INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. Immediately after the commencement of the examination, the candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Write the Name, Register Number and the Date of Examination in the space provided.
4. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer book.
5. **Candidate should handover this Question paper to the invigilator after 15 minutes and before receiving the question paper for Part B Examination.**

## MTH 1C 04—DISCRETE MATHEMATICS

### Part A

Multiple Choice Questions :

1. What is the maximum number of edges in a bipartite graph having 12 vertices :
   - (A) 24.
   - (B) 38.
   - (C) 36.
   - (D) 32.
2. Total number of regular graphs of degree 2 on 12 vertices :
   - (A) 6.
   - (B) 7.
   - (C) 8.
   - (D) 9.
3. A graph is self complementary if it is isomorphic to it's complement. For all self complementary graphs on $n$ vertices, $n$ is :
   - (A) A multiple of 4.
   - (B) Even.
   - (C) Odd.
   - (D) Congruent to 0 mod 4, or 1 mod 4.
4. For a given graph G having $v$ vertices and $e$ edges which is connected and has no cycles, which of the following statements is true ?
   - (A) $v = e$.
   - (B) $v = e + 1$.
   - (C) $v + 1 = e$.
   - (D) $v = e - 1$.
5. A connected undirected graph containing $n$ vertices and $n - 1$ edges ————————.
   - (A) Cannot have cycles.
   - (B) Must contain at least one cycle.
   - (C) Can contain at most two cycles.
   - (D) Must contain at least two cycles.
6. What is the radius of the Petersen graph ?
   - (A) 2.
   - (B) 3.
   - (C) 4.
   - (D) None of the above.
7. Let $G = C_n$. Then :
   - (A) There is path of length 5.
   - (B) There is a closed path of length 5.
   - (C) G is bipartite.
   - (D) None of the above.

8. Which of the following statement is/are TRUE ?

P : A cycle is a walk with end vertices are same.

Q : A cycle is a path with end vertices are same.

(A) P only.
(B) Q only.
(C) Both P and Q.
(D) Neither P and Q.

9. Which of the following properties does a simple graph not hold ?

(A) Must be connected.
(B) Must be unweighted.
(C) Must have no loops or multiple edges.
(D) Must have no edges.

10. The number of elements in the adjacency matrix of a graph having 7 vertices is ————————.

(A) 7.
(B) 14.
(C) 36.
(D) 49.

11. If G is the forest with 54 vertices and 17 connected components, G has ———————— total number of edges.

(A) 37.
(B) 71.
(C) 17.
(D) 54.

12. How many of the following statements are correct ?

1 All cyclic graphs are complete graphs.

2 All complete graphs are cyclic graphs.

3 All paths are bipartite.

4 All cyclic graphs are bipartite.

5 There are cyclic graphs which are complete.

(A) 1.
(B) 2.
(C) 3.
(D) 4.

13. Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is a connected graph, then the number of bounded faces in any embedding of G on the plane is equal to :

(A) 4.
(B) 5.
(C) 6.
(D) 7.

14. Radius of a graph, denoted by rad (G) is defined by ———————— ?

(A) $\text{Max}\{e(v): v \in V(G)\}$.

(B) $\text{Min}\{e(v): v \in V(G)\}$.

(C) $\text{Max}\{d(v,v): v,u \in V(G)\}$.

(D) $\text{Min}\{d(u,v): v,u \in V(G)\}$.

15. If a graph is planar, then it is embeddable on a :

(A) Circle.

(B) Square.

(C) Sphere.

(D) Triangle.

16. A grammar G is defined as a quadruple, $G = (V,T,S,P)$, where P is a finite set of :

(A) Paths.

(B) Productions.

(C) Power sets.

(D) Partitions.

17. Two finite accepters, $M_1$ and $M_2$, are said to be equivalent if :

(A) $L(M_1) = L(M_2)$.

(B) $L = L(M_D)$.

(C) $L(M_1) = G_D$.

(D) $F_{M1} = L(M_2)$.

18. If S is a strict partial order on a set X, then which of the following is a partial order :

(A) $S \cap \Delta X$.

(B) $\Delta X \cup X$.

(C) $\Delta X \cap X$.

(D) $\Delta X \cup S$.

19. Let $(X, +, .)$ be a Boolean algebra. Then for all $x, y, z \in X$ such that $(x+y)' = x' \cdot y'$ is called :

(A) Law of Absorption.

(B) Law of Idempotency.

(C) Law of Complementation.

(D) De Morgan's Law.

20. Let $(X, \leq)$ be a lattice with minimum element 0 and maximal element 1. Then $y$ is a complement of $x$, if :

(A) $x \vee y = 0, x \wedge y = 0$.

(B) $x \vee y = 0, x \wedge y = 1$.

(C) $x \vee y = 1, x \wedge y = 0$.

(D) $x \vee y = 1, x \wedge y = 1$.

**C 84009** (Pages : 3) Name.....................................

Reg. No..................................

# M.Sc. (PREVIOUS) DEGREE [CBCSS] EXAMINATION, APRIL/MAY 2020

(PVT/SDE)

M.Sc. Mathematics–First Semester

MTH 1C 03—REAL ANALYSIS–I

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

## Part B

### Section A (Short Answer Questions)

*Answer* **all** *questions.*
*Each question has* 1 *weightage.*

1) Prove that balls in $\mathbb{R}^k$ are convex.

2) Is arbitrary intersection of opens sets in a metric space open ? Justify your answer.

3) Let $[x]$ denote the largest integer contained in $x$. What discontinuities does the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = [x]$ have ?

4) Give an example of a continuous real function defined on [– 5, 5] which is not differentiable at 3.

5) Show by an example that the mean value for real valued functions need not hold for complex valued functions.

6) Let $f$ be a bounded real function and $\alpha$ be a monotonic increasing real function on $[a,b]$. If $f$ is Riemann-Stieltjes integrable with respect to $\alpha$ on $[a,b]$, then prove that $|f|$ is Riemann-Stieltjes integrable with respect to $\alpha$ on $[a,b]$.

7) Let $\gamma$ be a curve in the complex plane, defined on $[0,2\pi]$ by $\gamma(t) = e^{it}$.

   Prove that $\gamma$ is rectifiable.

8) Give an example of an equicontinuous family of functions.

(8 × 1 = 8 weightage)

## Section B

*Answer any* **two** *questions from each unit.*
*Each question has 2 weightage.*

### Unit I

9) Let $p$ be a limit point of a subset E of a metric space. Prove that every neighborhood of $p$ contains infinitely many points of E.

10) Prove that compact subsets of a metric space are closed.

11) Prove that continuous image of a connected set is connected.

### Unit II

12) Let $f$ be defined on $[a,b]$. If $f$ has a local maximum at a point $x \in (a,b)$ and if $f'(x)$ exists, then prove that $f'(x) = 0$.

13) If $f$ is differentiable on $[a,b]$, then prove that $f'$ cannot have any simple discontinuities on $[a,b]$.

14) For $1 < s < \infty$, define $\varsigma(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$.

Prove that $\varsigma(s) = \frac{s}{s-1} - s\int_1^{\infty} \frac{x-[x]}{x^{s+1}}\, dx$.

### Unit III

15) For $n$ = 1, 2, 3,... and $x$ real, let $f_n(x) = \frac{x}{1+nx^2}$.

Show that $\{f_n\}$ converges uniformly.

16) Prove that the series $\sum_{n=1}^{\infty} (-1)^n \frac{x^2+n}{n^2}$

does not converge absolutely for any value of $x$.

17) If K is compact, $f_n \in C(K)$ for $n$ = 1, 2, 3,... and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, then prove that $\{f_n\}$ is uniformly bounded on K.

(6 × 2 = 12 weightage)

## Section C

*Answer any* **two** *from the following four questions (18-21).*
*Each question has 5 weightage.*

18) (a) Let E be a non-empty set of real numbers which is bounded above. If $y = \sup E$, then prove that $y \in \overline{E}$.

(b) Let P be a non-empty perfect set in $R^k$. Prove that P is uncountable.

19) (a) Let X be a metric space with metric $d(x,y) = \begin{cases} 1 & \text{if } x \neq y \\ 0 & \text{if } x = y \end{cases}$

and Y be any metric space. Prove that every function from X to Y is continuous.

(b) Let $f$ be a continuous mapping of a compact metric space X into a metric space Y. Prove that $f$ is uniformly continuous.

20) (a) Let $f$ be a bounded real function and $\alpha$ be a monotonic increasing real function on $[a,b]$. If $f$ is Riemann-Stieltjes integrable with respect to $\alpha$ on $[a,b]$ if and only if for every $\in > 0$ there exists a partition P of $[a,b]$ such that $U(P,f,\alpha) - L(P,f,\alpha) < \in$.

(b) Let $f$ be a bounded real function and $\alpha_1$, $\alpha_2$ be monotonic increasing real functions on $[a,b]$. If $f$ is Riemann-Stieltjes integrable with respect to $\alpha_1$ and $\alpha_2$ on $[a,b]$, then prove that $f$ is Riemann-Stieltjes integrable with respect to $\alpha_1 + \alpha_2$ on $[a,b]$.

21) (a) If $\{f_n\}$ is a sequence of continuous functions on E and if $f_n \to f$ uniformly on E, then prove that $f$ is continuous on E.

(b) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

(2 × 5 = 10 weightage)

**C 84009** (Pages : 4 + 3 = 7) Name..............................

Reg. No..............................

**M.Sc. (PREVIOUS) DEGREE [CBCSS] EXAMINATION, APRIL/MAY 2020**

(PVT/SDE)

M.Sc. Mathematics–First Semester

MTH 1C 03—REAL ANALYSIS–I

(2019 Admissions)

**Part A**

| | DD | | MM | | YEAR | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Date of Examination :** | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | FN/AN |

**Time : 15 Minutes** **Total No. of Questions : 20**



## INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. Immediately after the commencement of the examination, the candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Write the Name, Register Number and the Date of Examination in the space provided.
4. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer book.
5. **Candidate should handover this Question paper to the invigilator after 15 minutes and before receiving the question paper for Part B Examination.**

# MTH 1C 03—REAL ANALYSIS–I

## Part A

Multiple Choice Questions :

1. Which one of the following is a countable subset of real numbers ?

   (A) (0, 1).
   (B) [0, 1].
   (C) $\mathbb{R} \setminus (0,1)$.
   (D) None of these.

2. Which one of the following is a matrix on $\mathbb{R}$ ?

   (A) $d(x,y) = |x+y|$.
   (B) $d(x,y) = |x^2 + y^2|$.
   (C) $d(x,y) = |x^2 - y^2|$.
   (D) $d(x,y) = |x-y|$.

3. Which one of the following is an open subset of real numbers ?

   (A) $\mathbb{Z}$.
   (B) $\mathbb{Q}$.
   (C) $\mathbb{R} \setminus \mathbb{Z}$.
   (D) $\mathbb{R} \setminus \mathbb{Q}$.

4. Which one of the following is not true in general ?

   (A) Union of open sets is open.
   (B) Union of closed sets is closed.
   (C) Intersection of closed sets is closed.
   (D) Intersection of open sets can be closed.

5. Which one of the following subsets of real numbers has no limit point in $\mathbb{R}$ ?

   (A) $\mathbb{Z}$.
   (B) $\mathbb{Q}$.
   (C) $\mathbb{R} \setminus \mathbb{Z}$.
   (D) $\mathbb{R} \setminus \mathbb{Q}$.

6. Which one of the following subsets of $\mathbb{R}^2$ is not convex ?

   (A) $\{(x,y) : x^2 + y^2 \geq 1\}$.
   (B) $\{(x,y) : x^2 + y^2 \leq 1\}$.
   (C) $\{(x,y) : x^2 + y^2 < 1\}$.
   (D) $\mathbb{R}^2$.

7. The subset [0, 1] of real numbers is :

   (A) Connected and compact.
   (B) Connected but not compact.
   (C) Compact but not connected.
   (D) Neither connected nor compact.

8. If $A = \mathbb{Z}, B = \mathbb{Q}, C = \mathbb{R} \setminus \mathbb{Z}$ and $D = \mathbb{R} \setminus \mathbb{Q}$, and $P_1$ is the property 'every points is a limit point', $P_2$ is the property 'contains all its limit points'; then which one of the following is correct ?

   (A) All sets A, B, C and D satisfy both $P_1$ and $P_2$.

   (B) All sets A, B, C and D satisfy $P_1$.

   (C) None of the sets A, B, C or D satisfy $P_1$.

   (D) B, C and D satisfy $P_1$ but do not satisfy $P_2$.

9. Among the subsets $X = \{(x,0,0) : x^2 < 1\}$, $Y = \{(x,y,0) : x^2 + y^2 < 1\}$ and $Z = \{(x,y,z) : x^2 + y^2 + z^2 < 1\}$ of $\mathbb{R}^3$, which are open in $\mathbb{R}^3$ ?

   (A) X, Y and Z.

   (B) Only X and Y.

   (C) Only Y and Z.

   (D) Only Z.

10. If $f(x) = [x], g(x) = x - [x]$ and $h(x) = x + [x]$, then :

   (A) All are continuous.

   (B) Only $f$ and $g$ are continuous.

   (C) Only $h$ is continuous.

   (D) None of them is continuous.

11. Which one of the following functions defined on $\mathbb{R}$ can be modified into a continuous function on $\mathbb{R}$ by suitably assigning the value of $f(0)$ ?

   (A) $f(x) = \begin{cases} \sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$.

   (B) $f(x) = \begin{cases} \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$.

   (C) $f(x) = \begin{cases} 2x, & \text{if } x > 0 \\ 1, & \text{if } x = 0 \\ 3x & \text{if } x < 0 \end{cases}$.

   (D) $f(x) = \begin{cases} x, & \text{if } x < 0 \\ -1, & \text{if } x = 0 \\ x+1 & \text{if } x > 0 \end{cases}$.

12. There exist monotonic functions defined on $\mathbb{R}$ which are discontinuous at :

   (A) Every point of $\mathbb{R}$.

   (B) Every point of $\mathbb{Q}$.

   (C) Every point of $(0,\infty)$.

   (D) Every point of (0, 1).

13. Which one of the following functions defined on $\mathbb{R}$ is not continuous on $\mathbb{R}$ ?

   (A) $f(x) = \begin{cases} \sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$.

   (B) $f(x) = \begin{cases} x\sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$.

   (C) $f(x) = \begin{cases} x^2\sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$.

   (D) None of these.

14. Which one of the following is always not true about a real continuous function $f$ defined on [0, 1] ?
    - (A) The range of $f$ can be an open interval.
    - (B) The range of $f$ must be an interval.
    - (C) The range of $f$ must be bounded.
    - (D) The range of $f$ must be closed.

15. The function $f:[0,2\pi)\to\mathbb{C}$ defind by $f(x)=\cos x+i\sin x$ is :
    - (A) A continuous bijection.
    - (B) Neither continuous nor a bijection.
    - (C) Continuous but not a bijection.
    - (D) A bijection but not continuous.

16. Which one of the following is true about the function $f(x)=1/x, x>0$?
    - (A) Continuous on $(0,\infty)$ but not uniformly continuous.
    - (B) Uniformly continuous on $(0,\infty)$.
    - (C) Not continuous on $(0,\infty)$.
    - (D) None of the above.

17. The function $f:\mathbb{R}^2\to\mathbb{R}$ defind by $f(x,y)=\dfrac{1}{x^2+y^2}$, for $(x,y)\neq(0,0)$ is :
    - (A) Continuous on $\mathbb{R}^2$ if $f(0,0)=0$.
    - (B) Continuous on $\mathbb{R}^2$ if $f(0,0)=r$, for any $r\neq 0$ in $\mathbb{R}$.
    - (C) Continuous on $\mathbb{R}^2$ for some real value of $f(0,0)$.
    - (D) Not continuous for any value of $f(0,0)$.

18. The derivative of $f(x)=x^2\sin\dfrac{1}{x}$, at $x\neq 0$ is :
    - (A) $2x\sin\dfrac{1}{x}-\cos\dfrac{1}{x}$.
    - (B) $2x\sin\dfrac{1}{x}+\cos\dfrac{1}{x}$.
    - (C) $-2x\sin\dfrac{1}{x}+\cos\dfrac{1}{x}$.
    - (D) $-2x\sin\dfrac{1}{x}-\cos\dfrac{1}{x}$.

19. If $\gamma:[0,\pi]\to\mathbb{R}^2$ is the curve $\gamma(t)=(\cos t,\sin t), t\in[0,\pi]$, then the length of $\gamma$ is :
    - (A) $2\pi$.
    - (B) $\pi$.
    - (C) $\pi/2$.
    - (D) $\pi/4$.

20. If $f_n(x)=x^n, n=1,2,\ldots. x\in[0,1]$, then the sequence $\{f_n\}$ converges to :
    - (A) $f(x)=x$.
    - (B) $f(x)=\begin{cases}0, & \text{if } x<1\\ 1, & \text{if } x=1\end{cases}$.
    - (C) $f(x)=0$.
    - (D) $f(x)=\begin{cases}1, & \text{if } x<1\\ 0, & \text{if } x=1\end{cases}$.

**C 84008** (Pages : 3) Name.......................................

Reg. No..................................

# M.Sc. (PREVIOUS) DEGREE [CBCSS] EXAMINATION, APRIL/MAY 2020

(PVT/SDE)

M.Sc. Mathematics–First Semester

MTH 1C 02—LINEAR ALGEBRA

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

## Part B

### Section A

*Answer* **all** *the questions.*
*Each question carries* 1 *weightage.*

1. Define linearly dependent subset of a vector space and show that if two vectors and linearly dependent, then one of them is a scalar multiple of the other.

2. Is there a linear transformation T from $R^3$ into $R^2$ such that

   $T(1,-1,1)=(1,0)$ and $T(1,1,1)=(0,1)$? Justify your answer.

3. Define trace function on the vector space of square matrices of order $n$ over a field F and show that it is a linear functional.

4. Let $f$ be the linear functional on $R^2$ defined by $f(x_1,x_2)=ax_1+bx_2$ and let T be the linear operator on $R^2$ defined by $T(x_1,x_2)=(-x_2,x_1)$. If $T^t$ is the transpose of T, find $T^t f$.

5. Show that similar matrices have the same characteristic polynomial.

6. Let T be the linear operator on $R^2$, the matrix of which in the standard order basis is $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$. Prove that the sub-space of $R^2$ spanned by the vector $(1,0)$ is invariant under T.

7. Let ( $l$ ) be the standard inner product on $R^2$. Let $\alpha=(1,2), \beta=(-1,1)$. If $\gamma$ is a vector such that $(\alpha|\gamma)=-1$ and $(\beta|\gamma)=3$, find $\gamma$.

**Turn over**

8. Let W be a finite-dimensional sub-space of an inner product space V, and let E be the orthogonal projection of V on W. Prove that $(E\alpha|\beta) = (\alpha|E\beta)$ for all $\alpha, \beta$ in V.

(8 × 1 = 8 weightage)

## Part B

*Answer any* **two** *questions from each of the following units.*
*Each question carries 2 weightage.*

### Unit I

9. Let V be a vector space over the field F. Show that the intersection of any collection of sub-spaces of V is a sub-space of V.

10. Show that the vectors $\alpha_1 = (1, 0, -1), \alpha_2 = (1, 2, 1), \alpha_3 = (0, -3, 2)$ form a basis for $R^3$. Express each of the standard basis vectors as linear combinations of $\alpha_1, \alpha_2$ and $\alpha_3$.

11. Let V and W be finite-dimensional vector spaces over the field F such that $\dim V = \dim W$. Let T be a linear transformation from V into W. Show that T is an invertible iff T is onto.

### Unit II

12. Let V be a finite-dimensional vector space over the field F, and let W be a sub-space of V. Show that $\dim W + \dim W^0 = \dim V$.

13. Let V be a finite-dimensional vector space over the field F. Show that there is an isomorphism of V into V**.

14. Let T be a linear operator on the $n$-dimensional vector space V such that T has $n$-distinct characteristic values. Prove that T is diagnolizable.

### Unit III

15. Let $E_1$ and $E_2$ be linear operators on a vector space V such that $E_1 + E_2 = I$. Show that $E_1E_2 = 0$ iff $E_i^2 = E_i$ for $i = 1, 2$.

16. Let W be a finite-dimensional sub-space of an inner product space V and let E be the orthogonal projection of V on W. Show that I-E is the orthogonal projection of V on $W^{\perp}$ with null space W.

17. Apply the Gram-Schmidt process to the vectors $\beta_1 = (3, 0, 4), \beta_2 = (-1, 0, 7), \beta_3 = (2, 9, 11)$, to obtain an orthonormal basis for $R^3$ with the standard inner product.

(6 × 2 = 12 weightage)

## Part C

*Answer any* **two** *questions.*
*Each question carries* 5 *weightage.*

18. Let V be an $n$-dimensional vector space over the field F, and let W be an $m$-dimensional vector space over F. Show that the space $L(V, W)$ is finite dimensional and has dimension $mn$.

19. (a) Let W be the sub-space of $R^5$ which is spanned by the vectors

    $\alpha_1 = (2, -2, 3, 4, -1), \alpha_2 = (-1, 1, 2, 5, 2), \alpha_3 = (0, 0, -1, -2, 3), \alpha_4 = (1, -1, 2, 3, 0)$. Find a basis for $W^0$.

    (b) Let $W_1$ and $W_2$ be sub-spaces of a finite dimensional vector space V. Prove that $(W_1 + W_2)^0 = W_1^0 \cap W_2^0$.

20. (a) Let V be a finite-dimensional vector space over the field F and let T be a linear operator on V. Show that T is diagonalizable iff the minimal polynomial for T has the form $p = (x - c_1) \dots (x - c_k)$ where $c_1, \dots, c_k$ are distinct elements of F.

    (b) Let T be a diagonalizable linear operator on the $n$-dimensional vector space V and let W be a sub-space which is invariant under T. Prove that the restriction operator $T_W$ is diagonalizable

21. Let W be the sub-space of $R^2$ spanned by the vector $(3, 4)$ using the standard inner product on $R^2$, let E be the orthogonal projection of $R^2$ onto W. Find :

    (a) A formula for $E(x_1, x_2)$.

    (b) The matrix of E in the standard ordered basis.

    (c) $W^{\perp}$.

    (d) An orthonormal basis in which E is represented by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

(2 × 5 = 10 weightage)

C 84008 (Pages : 4 + 3 = 7) Name.....................................

Reg. No..................................

# M.Sc. (PREVIOUS) DEGREE [CBCSS] EXAMINATION, APRIL/MAY 2020

(PVT/SDE)

M.Sc. Mathematics–First Semester

MTH 1C 02—LINEAR ALGEBRA

(2019 Admissions)

**Part A**

| | DD | | MM | | YEAR | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Date of Examination :** | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | FN/AN |

**Time : 15 Minutes** **Total No. of Questions : 20**



## INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. Immediately after the commencement of the examination, the candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Write the Name, Register Number and the Date of Examination in the space provided.
4. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer book.
5. **Candidate should handover this Question paper to the invigilator after 15 minutes and before receiving the question paper for Part B Examination.**

## MTH 1C 02—LINEAR ALGEBRA

### Part A

Multiple Choice Questions :

1. Which of the following is a subspace of the vector space $R^2$ over R ?

   (A) $\{(x, y)/y = 2x\}$.
   (B) $\{(x, y)/3x + 2y = 5\}$.
   (C) $\{(x, y)/7x + 2y = 5\}$.
   (D) $\{(x, y)/x^2 + y^2 = 1\}$.

2. Which of the following is not a basis of the vector space $R^2$ over R ?

   (A) $\{(2,3),(3,1)\}$.
   (B) $\{(1,3),(3,9)\}$.
   (C) $\{(2,5),(7,1)\}$.
   (D) $\{(5,1),(3,1)\}$.

3. Which of the following is not a linear combination of $u = (0, -1, 1)$ and $v = (2, 2, 1)$ ?

   (A) $(2,1,2)$.
   (B) $(2,0,3)$.
   (C) $(4,3,2)$.
   (D) $(0,5,7)$.

4. Suppose that $V_1 = (2,1,0,3)$, $V_2 = (3,-1,5,2)$ and $V_3 = (-1,0,2,1)$. Which of the following is not there in the span $\{V_1, V_2, V_3\}$ ?

   (A) $(2,3,-7,3)$.
   (B) $(1,1,1,1)$.
   (C) $(0,0,0,0)$.
   (D) $(4,6,-14,6)$.

5. What is the co-ordinate vector of $(0,1)$ relative to the basis containing $\alpha = (1,-1)$ and $\beta = (1,1)$ ?

   (A) $(2,1)$.
   (B) $\left(\frac{1}{2}, \frac{1}{2}\right)$.
   (C) $\left(\frac{1}{2}, -\frac{1}{2}\right)$.
   (D) $\left(\frac{1}{2}, 1\right)$.

6. What is the co-ordinate vector of $(1,1)$ relative to the basis containing $\alpha = (2,-4)$ and $\beta = (3,8)$ ?

   (A) $(2,1)$.
   (B) $\left(\frac{5}{28}, \frac{3}{14}\right)$.
   (C) $\left(\frac{5}{2}, -\frac{3}{8}\right)$.
   (D) $\left(\frac{4}{28}, \frac{1}{14}\right)$.

7. What is the co-ordinate vector of $(1,1)$ relative to the basis containing $\alpha=(1,1)$ and $\beta=(0,2)$ ?

(A) $\left(2,-\frac{1}{2}\right)$. (B) $\left(\frac{1}{2},\frac{1}{2}\right)$.

(C) $\left(\frac{1}{2},-\frac{1}{2}\right)$. (D) $\left(\frac{1}{2},1\right)$.

8. Let $T: R_2[x]\rightarrow R_4[x]$ defined by $T(p(x))=p(x^2)$. Then :

(A) T is a linear transformation with rank 5.

(B) T is a linear transformation with rank 3.

(C) T is a linear transformation with rank 2.

(D) T is not a linear transformation.

9. Let V be the vector space of ordered pairs of complex numbers over the field R, then dimension of V is :

(A) 1. (B) 2.

(C) 3. (D) 4.

10. Which of the following is not a vector space over R ?

(A) R over Q. (B) Q over R.

(C) R over R. (D) C over R.

11. Let A be a $5\times 5$ matrix with trace 15 and 2 and 3 are its eigen values of A each with multiplicity 2. Then determinant of A is :

(A) 150. (B) 180.

(C) 120. (D) 24.

12. Which of the following is correct ?

(A) R is a vector space over N. (B) R is a vector space over Z.

(C) R is a vector space over C. (D) None of the above.

13. Let V be a vector space of dimension 3 and let A and B be two disjoint subspaces having dimensions 2 and 1 respectively. Then :

(A) $V=A\cap B$. (B) $V=A\cup B$.

(C) $V=A+B$. (D) Nothing can be said.

14. Consider the vector space W over R spanned by the set :

$$\{(1,1,0,0),(1,1,0,0),(1,0,0,0),(0,1,0,0),(1,0,1,0),(1,1,1,0)\}$$

Then W is of dimension.

(A) 1. (B) 2.

(C) 3. (D) 4.

Turn over

15. Consider the vector space W over R spanned by the set

$\{(1, 1, 0, 0), (1, 0, 0, 0), (0, 1, 0, 0)\}$

Then W is of dimension.

(A) 1. (B) 2.

(C) 3. (D) 4.

16. Consider the vector space W over R spanned by the set

$\{(1, 1, 0), (0, 1, 0), (1, 0, 0)\}$.

Then W is of dimension.

(A) 1. (B) 2.

(C) 3. (D) 4.

17. Let $c$ be a characteristic value of T. Then what is the characteristic value of $f$ (T) :

(A) $f(c)$. (B) $c$.

(C) $c^2$. (D) None of these.

18. Let T be a linear operator on a finite dimensional vector space V. Let $c_1, ..., c_n$ be distinct characteristic values of T. Then which of the following is true if T is diagonalizable :

(A) Characteristic polynomial is $f = (x - c_1)(x - c_2)...(x - c_k)$.

(B) Characteristic polynomial is $f = (x - c_1)^{d_1} (x - c_2)^{d_2} ... (x - c_k)^{d_k}$.

(C) Characteristic polynomial is $f = (x - c_1) + (x - c_2) + ... + (x - c_k)$.

(D) None of these.

19. A unit vector which is orthogonal to the vector (2, 1, 6) of $R^3$ with respect to the standard inner product is :

(A) $(2, -2, -1)$. (B) $\sqrt{1/3}\,(2, -2, -1)$.

(C) $(1, -1, 1)$. (D) $\frac{1}{3}(2, -2, -1)$.

20. Consider the vector space C over R. Let T be a linear operator on C define by $T(z) = \bar{z}$. Then :

(A) T is one-one but not onto. (B) T is onto but not one-one.

(C) T is both one-one and onto. (D) T is neither one-one nor onto.

**C 84007** (Pages : 4 + 3 = 7) Name.....................................

Reg. No.................................

## M.Sc. (PREVIOUS) DEGREE [CBCSS] EXAMINATION, APRIL/MAY 2020

(PVT/SDE)

M.Sc. Mathematics–First Semester

MTH 1C 01—ALGEBRA–I

(2019 Admissions)

**Part A**

| | DD | | MM | | YEAR | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Date of Examination :** | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | FN/AN |

**Time : 15 Minutes** **Total No. of Questions : 20**



## INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. Immediately after the commencement of the examination, the candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Write the Name, Register Number and the Date of Examination in the space provided.
4. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer book.
5. **Candidate should handover this Question paper to the invigilator after 15 minutes and before receiving the question paper for Part B Examination.**

MTH 1C 01—ALGEBRA–I

**Part A**

Multiple Choice Questions :

1. The number of abelian groups of order 30 upto isomorphism is :

   (A) 0. (B) 1.

   (C) 2. (D) 3.

2. Which of the following is true :

   (A) Every abelian group of prime order is cyclic.

   (B) Every abelian group of prime power order is cyclic.

   (C) If $a$ divides the order of a group G, then G has a subgroup of order $a$.

   (D) None of these.

3. Let G be a non abelian group. Then the order of G could be :

   (A) 35. (B) 37.

   (C) 40. (D) 49.

4. Which of the following is true :

   (A) Every group of order 4 is abelian.

   (B) Every group of order 9 is abelian.

   (C) Every group of order 16 is abelian.

   (D) Every group of order 25 is abelian.

5. Let $\phi$ be a homomorphism from a group G to a group G' and H be a subgroup of G. Then which of the following is not true :

   (A) If H is cyclic, then $\phi(H)$ is cyclic subgroup of $G'$.

   (B) If H is abelian, then $\phi(H)$ is abelian subgroup of $G'$.

   (C) If H normal in G, then $\phi(H)$ is normal in $G'$.

   (D) If $H'$ is a normal subgroup of $G'$, then $\phi^{-1}(H')$ is a normal subgroup of G.

6. Let H be a normal subgroup of G. In which of the following G\H is not abelian.

   (A) $G = \mathbb{R}$ with addition and $H = \mathbb{Z}$. (B) G has order 100 and H has order 25.

   (C) G is not abelian and $H = \{e\}$. (D) None of these.

7. Which of the following is not a group action :

(A) $G = (R \setminus \{0\}..)$ and $X = \mathbb{C}$, for any $r \in R \setminus \{0\}, x \in \mathbb{C}$ define $r * x = rx$.

(B) $G = (R,+)$ and $X = \mathbb{C}$. For any $r \in R, x \in \mathbb{C}$, define $r * x = rx$.

(C) $G = (R,+)$ and $X = \mathbb{C}$. For any $r \in R, x \in \mathbb{C}$, define $r * x = r + x$.

(D) None of these.

8. Which of the following is not true :

(A) The intersection of normal subgroup G is normal.

(B) Let H be a normal subgroup of G and $m = (G : H)$. Then $a^m \in H$ for every $a \in G$.

(C) If H and N are subgroups of a group G and N is normal in G, then $H \cap N$ is normal in G.

(D) Let H be a subgroup of G such that $|G| = 2|H|$. Then H is normal in G.

9. Let G be a group of order 153. Which of the following is not true :

(A) The sylow 3– subgroup of G have order 9.

(B) There exist exactly one subgroup of order 17.

(C) The sylow 3-subgroup of G have order 3.

(D) The sylow 17- subgroup of G have order 17.

10. Let $\mathbb{Z}$ be the set of integers. Which of the following is an ideal ?

(A) The set of even integers.

(B) The set of all odd integers.

(C) The set of non negative integers.

(D) None of the above.

11. Let G be a group and $H \subset K \subset G$. Which of the following is true ?

(A) If K is normal in G, Then H is normal in G.

(B) If H is normal in G, Then H is normal in K.

(C) If H is normal in K, Then H is normal in G.

(D) None of these.

12. Which of the following is true ?

(A) Every group has many different presentations.

(B) Every group has two presentations that are not isomorphic.

(C) Every group has a finite presentation.

(D) Every group with a finite presentation is of finite order.

13. How many distinguishable ways can seven people be seated at a round table where there is no distinguishable "head" to the table :

(A) 7!. (B) 6!.

(C) 7. (D) 6.

14. Let H be the smallest subgroup of $S_n, n \geq 5$ containing all the three cycles. Then :

(A) $|H| = 2$. (B) $(G : H) = 2$.

(C) $H = S_n$. (D) H is abelian.

15. Let $H = \mathbb{Z}_2 \times \mathbb{Z}_6$ and $K = \mathbb{Z}_3 \times \mathbb{Z}_4$. Then :

(A) H is isomorphic to K since both are cyclic.

(B) H is isomorphic to K since 2 divides 6 and g.c.d(3, 4) = 1.

(C) H is not isomorphic to K since K is cyclic whereas H is not.

(D) H is not isomorphic to K since there is no homomorphism from H to $k$.

16. If G is a group and H is a subgroup of index 2 in G then which of the following statement is false :

(A) H is a normal subgroup of G. (B) H is not a normal subgroup of G.

(C) G is not simple. (D) None of these.

17. If $\pi_1$ is a projection homomorphism from $\mathbb{Z}_5 \times \mathbb{Z}_3$ onto $\mathbb{Z}_5$ with kernel $\{0\} \times \mathbb{Z}_3$ then $\mathbb{Z}_5 \times \mathbb{Z}_3 / \{0\} \times \mathbb{Z}_3$ is isomorphic to :

(A) $\mathbb{Z}_5$. (B) $\mathbb{Z}_2$.

(C) $\mathbb{Z}_2 \times \mathbb{Z}_2$. (D) $\{0\} \times \mathbb{Z}_3$.

18. Let F be a field. An element $a \in F$ is a zero of a polynomial $f(x)$ if and only if :

(A) $x$ is a factor of $f(x)$. (B) $a$ is a factor of $f(x)$.

(C) $(x - a)$ is factor of $f(x)$. (D) None of these.

19. Which of the following plane isometries can be elements of finite subgroup of the plane isometries :

(A) Translations. (B) Reflections.

(C) Glide reflection. (D) None of these.

20. Let X be a G-set $x \in X$, $Gx = \{g * x : g \in G\}$ and $G_x = \{g \in G : g * x = x\}$. If $|G| = 10$ and $|G_x| = 5$, then what is the order of $G_x$ ?

(A) 50. (B) 2.

(C) 15. (D) 1.

**C 84007** (Pages : 3) Name..........................................

Reg. No..........................................

# M.Sc. (PREVIOUS) DEGREE [CBCSS] EXAMINATION, APRIL/MAY 2020

(PVT/SDE)

M.Sc. Mathematics–First Semester

MTH 1C 01—ALGEBRA–I

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

## Part B

### Section A

*Answer* **all** *questions.*
*Each question has weightage* 1.

1. Verify whether $\phi(x,y)=(y,x)$ is an isometry of the plane.

2. Give a subgroup of order 4 in $\mathbb{Z}_2 \times \mathbb{Z}_6$.

3. Describe all abelian groups of order 100 upto isomorphism.

4. Let $\phi : \mathbb{Z}_6 \to \mathbb{Z}_{12}$ be a homomorphism such that $\phi(1)=4$. Find $\ker\phi$.

5. Let G be a group of order 36. Find the order of a 3-Sylow subgroup of G.

6. Give all elements of the group given by the presentation $\left(a,b : a^2 = b^2 = 1, ab = ba\right)$.

7. Let $x = i + 2j + k$ and $y = 2i + j + k$ be elements in the ring of quaternions. Find $xy$.

8. Let $\phi : F[x] \to F$ be defined by $a_0 + a_1x + .... + a_nx^n \mapsto a_0$. Find $\ker\phi$.

(8 × 1 = 8 weightage)

### Section B

*Answer any* **six** *questions choosing two from each unit.*
*Each question has weightage* 2.

#### Unit 1

9. Describe an isomorphism $\phi$ from $\mathbb{Z}_3 \times \mathbb{Z}_4$ to $\mathbb{Z}_{12}$ and verify that $\phi$ is an isomorphism.

10. Show that if G is a cyclic group and H is a subgroup of G then $G/H$ is cyclic.

**Turn over**

11. Show that every group of prime order is simple.

UNIT 2

12. Show that the group $\mathbb{Z}$ of integers has no composition series.
13. Show that if G is a group of order 20 then G is not simple.
14. Find all elements conjugate to (12) in the symmetric group $S_4$.

UNIT 3

15. Show that the ring of all endomorphisms of the group $\mathbb{Z} \times \mathbb{Z}$ is not commutative.
16. Let $\phi_i : \mathbb{C}[x] \to \mathbb{C}$ be the evaluation homomorphism with $\phi_i(x) = i$. Find an element in the kernal of $\phi_i$.
17. Let $\phi : R \to R'$ be a homomorphism of rings. Show that if $\ker\phi = 0$ then $\phi$ is one to one.

(6 × 2 = 12 weightage)

SECTION C

*Answer any* **two** *questions.*
*Each question has weightage* 5.

18. (a) Let G be a group and H be a normal subgroup of G. Show that
    i) $\gamma : G \to G/H$ defined by $g \mapsto gH$ is a homomorphism.
    ii) $\ker \gamma = H$.

    (b) Let H be a subgroup of the group $\mathbb{Z}$ of integers. Show that $\mathbb{Z}/H$ is cyclic.
19. (a) Let C be the commutator subgroup of a group G. Show that
    i) C is a normal subgroup of G.
    ii) G/C is abelian.
    iii) If N is a normal subgroup of G such that G/N is abelian then $C \subseteq N$.

    (b) Find the commutator subgroup of the symmetric group $S_3$.
20. (a) Let G be a group of order $p^n$ where $p$ is a prime and X be a finite G-set. Let $X_G = \{x \in X : gx = x \text{ for all } g \in G\}$. Show that $|X_G| \equiv |X| \pmod{p}$.

    (b) Show that any two $p$-Sylow subgroups of a group G are conjugates.

21. (a) Let $\mathbb{Q}$ be the field of rationals and $\mathbb{Z}$ be the ring of integers. Let $f(x) \in \mathbb{Z}[x]$. Show that $f(x)$ factors into a product of polynomials of degree $s$ and $t$ in $\mathbb{Q}[x]$ if and only if $f(x)$ has such a factorization in $\mathbb{Z}[x]$ with polynomials of degrees $s$ and $t$.

(b) Verify using Eisenstein criterion that the polynomial $x^5 + 6x^3 + 9x + 6$ is irreducible in $\mathbb{Q}[x]$.

(2 × 5 = 10 weightage)

**C 63426** (Pages : 3) Name..................................

Reg. No..............................

# FIRST SEMESTER M.Sc. (SSE) DEGREE EXAMINATION, MARCH 2019

Mathematics

Paper V—DISCRETE MATHEMATICS

(2003 Admissions)

Time : Three Hours Maximum : 80 Marks

## Part A

*Answer* **all** *questions.*
*Each question carries 4 marks.*

1. (a) Give an example of a complete lattice. Establish your claim.

   (b) If G is a $k$-regular graph, prove that $\delta = \frac{2|E|}{|V|} = \Delta$.

   (c) Prove that a graph is a tree if and only if it is minimally connected.

   (d) Find a *nfa* which accepts the set of all strings containing 1100 as a substring.

(4 × 4 = 16 marks)

## Part B

*Answer any* **four** *questions without omitting any unit.*
*Each question carries 16 marks.*

### Unit I

2. (a) Let (L, ≤) be a lattice. For $a, b, c \in L$, prove that :

   (i) $b \le c \Rightarrow a \cdot b \le a \cdot c$.

   (ii) $b \le c \Rightarrow a + b \le a + c$.

   (b) Prove that every chain is a distributive lattice.

3. (a) Express the Boolean function $f(x,y,z) = (x+y)(x+z) + y + z'$ in its disjunctive normal form.

   (b) If $x$ and $y$ are the elements of a Boolean algebra, prove that $x = y \Leftrightarrow xy' + x'y = 0$.

4. (a) Let $(X, +, \cdot, ')$ be a Boolean algebra. Prove that :

   (i) Every non-zero element of X contains at least one atom.

   (ii) Every two distinct atoms of X are mutually disjoint.

   (b) Draw the Hasse diagram of the set of all positive divisors of 30.

Turn over

## Unit II

5. (a) With usual notations, prove $\delta(G) \le \frac{2|E|}{|V|} \le \Delta(G)$ where G is a simple graph.

   (b) Prove that a vertex $v$ in a connected graph G is a cut vertex if and only if there exist vertices $u$ and $w$ distinct from $v$ such that every path connecting $u$ and $w$ contains the vertex $v$.

   (c) Prove that the edge connectivity of a connected graph G can not exceed the minimum degree of G.

6. (a) In a connected simple plane graph G with $|E| > 1$, prove that

   (i) $|E| \le 3|V| - 6$.

   (ii) There is a vertex $v$ in G such that $\deg v \le 5$.

   (b) If a connected planar simple graph is triangle free then prove that $|E| \le 2|V| - 4$.

   (c) Prove that a complete graph $K_n$ is planar if and only if $n \le 4$.

7. (a) Prove or disprove. A Hamiltonian cycle always provides a Hamiltonian path, but a Hamiltonian path may not lead to a Hamiltonian cycle.

   (b) Let T be a graph with $n$ vertices. Prove the following are equivalent :

   (i) T is a tree.

   (ii) T is connected and acyclic.

   (iii) T is connected and has $n - 1$ edges.

   (c) Prove that an undirected graph G has a spanning tree if and only if G is connected.

## Unit III

8. (a) Let $\Sigma = \{a, b\}$. Find a grammar that generates $L = \{a^n b^{n+1}; n \ge 0\}$.

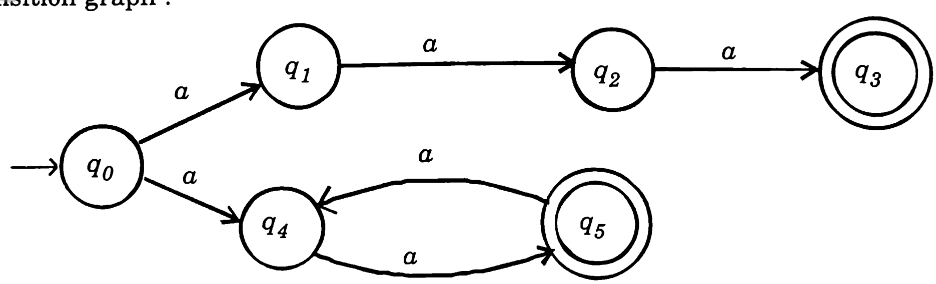
   (b) If $\Sigma = \{a, b\}$ and $L = \{aa, bb\}$, find $\overline{L}$.

   (c) Define a deterministic finite automaton. What is the transition graph associated with a *dfa* ? Explain these terms with an example.

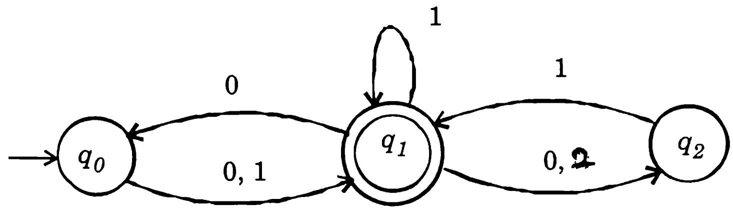
9. (a) Define the language accepted by a *dfa*. Illustrate it with an example.

   (b) Define a regular language. Give an example of a regular language.

   (c) Find a *dfa* that accepts the complement of the language defined by the *nfa* with the following transition graph :



10. (a) Convert the *nfa* given by the following transition graph into an equivalent *dfa* :




(b) Find a regular expression for the language $L = \{w : |w| \bmod 3 = 0\}$ on $\Sigma = \{a, b\}$.

(4 × 16 = 64 marks)

**C 63425** (Pages : 2) Name.................................

Reg. No.................................

# FIRST SEMESTER M.Sc. (SSE) DEGREE EXAMINATION, MARCH 2019

Mathematics

Paper IV—ORDINARY DIFFERENTIAL EQUATIONS AND CALCULUS OF VARIATION

(2003 Admissions)

Time : Three Hours Maximum : 80 Marks

## Part A

*Answer* **all** *the questions.*
*Each question carries* 4 *marks.*

I. (a) Find the general solution of the equation $(x^2-1)y''+(5x+4)y'+4y=0$ near its singular point $x=-1$.

(b) Prove that $\int x^p J_{p-1}(x)dx = x^p J_p(x)+C$.

(c) Describe the phase portrait of the system $\frac{dx}{dt}=x, \frac{dy}{dt}=0$.

(d) Find the stationary function of $\int_0^4 \left[xy'-(y')^2\right]dx$ which is determined by the boundary conditions $y(0)=0$ and $y(4)=3$.

(4 × 4 = 16 marks)

## Part B

*Answer any* **four** *questions without omitting any unit.*
*Each question carries* 16 *marks.*

### Unit I

II. (a) Find a power series solution of the equation $(1+x)y' = py, y(0)=1$, where $p$ is a constant.

(b) Find two independent Frobenius series solutions of the equation $2xy''+(x+1)y'+3y=0$.

III. (a) Find the general solution of the Chebyshev's equation $(1-x^2)y''-xy'+p^2y=0$, where $p$ is a constant.

(b) Show that the hypergeometric equation $x(1-x)y''+[c-(a+b+1)x]y'-aby=0$ has three regular singular points 0, 1 and $\infty$ with corresponding exponents 0 and $1-c$, 0 and $c-a-b$ and $a$ and $b$.

IV. (a) Obtain the hypergeometric series solution of the Legendre's equation $\left(1-x^{2}\right) y''-2 x y'+n(n+1) y=0$, where $n$ is a non-negative integer.

(b) Find the first three terms of the Legendre series of $f(x)=\begin{cases}0 & \text{if } -1 \leq x<0 \\ x & \text{if } \ 0 \leq x \leq 1\end{cases}$.

Unit II

V. (a) Show that $J_{-m}(x)=(-1)^{m} J_{m}(x)$ when $m$ is a non-negative integer.

(b) State and prove the orthogonality property for Bessel functions.

VI. (a) Find the general solution of the system :

$$\frac{dx}{dt}=x-2y, \frac{dy}{dt}=4x+5y.$$

(b) Determine the nature and stability properties of the critical point (0, 0) for the system :

$$\frac{dx}{dt}=4x-2y, \frac{dy}{dt}=5x+2y.$$

VII. (a) Let (0, 0) be a simple critical point of a non-linear system. Show that if the critical point (0, 0) of the related linear system of the above non-linear system is asymptotically stable, then the critical point (0, 0) of the non-linear system is also asymptotically stable.

(b) Show that (0, 0) is an asymptotically stable critical point of the system :

$$\frac{dx}{dt}=-y-x^{3}, \frac{dy}{dt}=x-y^{3}.$$

Unit III

VIII. (a) Let $u(x)$ be any non-trivial solution of $u''+q(x)u=0$, where $q(x)>0$ for all $x>0$. Show that if $\int_{1}^{\infty} q(x) dx=\infty$, then $u(x)$ has infinitely many zeros on the positive $x$-axis.

(b) Show that if $y(x)$ is non-trivial solution of $y''+q(x)y=0$, show that $y(x)$ has an infinite number of positive zeros if $q(x)>\frac{k}{x^{2}}$ for some $k>\frac{1}{4}$, and only a finite number if $q(x)<\frac{1}{4x^{2}}$.

IX. (a) Let $f(x, y)$ be a continuous function that satisfies a Lipschitz condition :

$$|f(x, y)-f(x, y_{2})| \leq K |y_{1}-y_{2}|$$

and a strip defined by $a \leq x \leq b$ and $-\infty<y<\infty$ show that if $(x_{0}, y_{0})$ is any point of the strip, then the initial value problem $y'=f(x, y), y(x_{0})=y_{0}$ has one and only one solution $y=y(x)$ on $a \leq x \leq b$.

(b) Consider the initial value problem $y'=x+y, y(0)=1$. Starting with $y_{0}^{(x)}=1$, apply Picard's method to calculate $y_{1}(x), y_{2}(x)$ and $y_{3}(x)$.

X. (a) Obtain Euler's differential equation.

(b) Show that the triangle with greatest area A for a given perimeter is equilateral.

(4 × 16 = 64 marks)

**C 63424** (Pages : 3) Name..................................

Reg. No..................................

# FIRST SEMESTER M.Sc. (SSE) DEGREE EXAMINATION, MARCH 2019

Mathematics

Paper III—REAL ANALYSIS—I

(2003 Admissions)

Time : Three Hours Maximum : 80 Marks

## Part A

*Answer* **all** *questions.*
*Each question carries* 4 *marks.*

I. (a) Define perfect sets. Give an example of a closed set which is not perfect.

(b) Prove that closed subsets of a compact set are closed.

(c) If $f$ is differentiable on $[a, b]$, then prove that $f'$ cannot have any simple discontinuities on $[a, b]$.

(d) Prove that the series $\sum_{n=1}^{\infty} (-1)^n \frac{x^2+n}{n^2}$ converges uniformly in every bounded interval.

(4 × 4 = 16 marks)

## Part B

*Answer any* **four** *questions without omitting any unit.*
*Each question carries* 16 *marks.*

### Unit I

II. (a) Prove that countable union of countable sets is countable.

(b) If $p$ is a limit point of a set E, then prove that every neighbourbood of $p$ contains infinitely many points.

III. (a) Let E be a set in $\mathbb{R}^k$. Prove that the following are equivalent :

(i) E is closed and bounded.

(ii) E is compact.

(iii) Every infinite subset of E has a limit point in E.

(b) Let E be a non-compact set in $\mathbb{R}^1$. Prove that there exists a continuous function on E which is not bounded.

IV. (a) Let $f$ be a continuous mapping of a metric space X into a metric space Y and let E be a connected subset of X. Prove that $f$(E) is a connected subset of Y.

(b) Prove that monotonic functions have no discontinuities of the second kind.

**Turn over**

## Unit II

V. (a) If $f$ and g are continuous real functions on $[a, b]$ which are differentiable in $(a, b)$, then prove that there is a point $x \in (a,b)$ such that

$$[f(b)-f(a)]g'(x)=[g(b)-g(a)]f'(x).$$

(b) Show by an example that the mean value theorem for real functions need not be true for complex valued functions.

VI. (a) Let $f$ be a bounded function and $\alpha$ be a monotonic increasing function on $[a, b]$. If $P_1$ is a refinement of P, then prove that

$$L(P,f,\alpha)\leq L(P_1,f,\alpha).$$

(b) Let $f$ be a bounded function and $\alpha$ be a monotonic increasing function on $[a, b]$. If $f$ is continuous on $[a, b]$, then prove that $f \in \mathcal{R}(\alpha)$ on $[a, b]$.

VII. (a) Let $f_1, f_2$ be bounded functions and $\alpha$ be a monotonic increasing function on $[a, b]$. If $f_1 \in \mathcal{R}(\alpha)$ and $f_2 \in \mathcal{R}(\alpha)$ on $[a, b]$, then prove that $f_1+f_2 \in \mathcal{R}(\alpha)$ on $[a, b]$ and

$$\int_a^b (f_1+f_2),\, d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha.$$

(b) Let $f$ be Riemann integrable on $[a, b]$. For $a \leq x \leq b$, let $F[x]=\int_a^x f(t)dt.$ If $f$ is continuous at a point $x_0$ of $[a, b]$, then prove that F is differentiable at $x_0$ and $F'(x_0)=f(x_0)$.

## Unit III

VIII. (a) Let $\gamma$ be a curve in the complex plane, defined on $[0,2\pi]$ by $\gamma(t)=e^{it}$. Show that $\gamma$ is rectifiable and find its length.

(b) Let $\alpha$ be monotonically increasing on $[a, b]$ and let $f_n \in \mathcal{R}(\alpha)$ on $[a, b]$ for $n = 1, 2, \ldots$. If $f_n \to f$ uniformly on $[a, b]$. Prove that $f \in \mathcal{R}(\alpha)$ on $[a, b]$ and $\int_a^b f d\alpha = \lim_{n\to\infty}\int_a^b f_n d\alpha.$

IX. (a) Let $\{f_n\}$ be a sequence of functions defined on E and let $|f_n(x)| \leq M_n$ for all $x \in E$, $n = 1, 2, \ldots$. If $\sum M_n$ converges, then prove that $\sum f_n$ converges uniformly on E.

(b) Let $\{f_n\}$ be a sequence of functions differentiable on $[a, b]$ and such that $\{f_n(x_0)\}$ converges for some point $x_0$ on $[a, b]$. If $\{f_n'\}$ converges uniformly on $[a, b]$, then prove that $\{f_n\}$ converges uniformly on $[a, b]$ to a function $f$ and $f'(x)=\lim_{n\to\infty} f_n'(x)$ for all $x \in [a,b]$.

X. (a) Let $\mathcal{C}$(X) denote the set of all complex valued, continuous, bounded functions defined on a metric space X. Prove that $\mathcal{C}$(X) is a complete metric space with respect to the metric :

$$d(f,g) = \sup_{x \in X} |f(x) - g(x)|.$$

(b) Let K be a compact set and let $f_n \in \mathcal{C}(K)$ for $n$ = 1, 2, .... If $\{f_n\}$ is pointwise bounded and equicontinuous on K, then prove that

(i) $\{f_n\}$ is uniformly bounded on K.

(ii) $\{f_n\}$ contains a uniformly convergent subsequence.

(4 × 16 = 64 marks)

**C 63423** (Pages : 3) Name..................................

Reg. No..................................

# FIRST SEMESTER M.Sc. (SSE) DEGREE EXAMINATION, MARCH 2019

Mathematics

Paper II—LINEAR ALGEBRA

(2003 Admissions)

Time : Three Hours Maximum : 80 Marks

## Part A

*Answer* **all** *questions.*
*Each question carries* 4 *marks.*

1. (a) Give two proper subspaces of $\mathbb{R}^3$ and verify whether their union is a subspace of $\mathbb{R}^3$.

   (b) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x, y) = (y, x)$. Is it a linear transformation ? If yes, describe it geometrically.

   (c) Let V be a finite dimensional vector space and $W_1$ be any subspace of V. Prove that there is a subspace $W_2$ of V such that $V = W_1 \oplus W_2$.

   (d) Find the characteristic values of $\begin{bmatrix} 0 & 0 & 0 \\ -4 & 6 & 0 \\ -1 & -2 & -2 \end{bmatrix}$. Is it diagnalizable ? Justify.

(4 × 4 = 16 marks)

## Part B

*Answer any* **four** *questions from this part without omitting any unit.*
*Each question carries* 16 *marks.*

### Unit I

2. (a) Prove that a non-empty subset W of a vector space V over the field F is a subspace of V if and only if $c\alpha + \beta \in W$, for each pair of vectors $\alpha, \beta \in W$ and for each $c \in F$.

   (b) Let V be a finite dimensional vector space over the field F and dim V = $n$. If $S = \{\alpha_1, \alpha_2, \ldots, \alpha_m\}$, $m < n$, is a set of linearly independent vectors in V, prove that S can be extended to form a basis for V over F.

3. Let V be the vector space of all complex valued functions on the real line. Let $f_1(x) = 1$, $f_2(x) = e^{ix}$, $f_3(x) = e^{-ix}$. Let $g_1(x) = 1$, $g_2(x) = \cos x$, $g_3(x) = \sin x$.

   (a) Prove that $\{f_1, f_2, f_3\}$ is a linearly independent subset of V.

   (b) Find an invertible 3 × 3 matrix P such that $g_j = \sum_{i=1}^{3} P_{ij} f_i$.

**Turn over**

4. (a) Let V be the vector space of the set of complex numbers over $\mathbb{R}$. Define $T : V \to M_2(\mathbb{R})$ by $T(x+iy) = \begin{bmatrix} x+7y & 5y \\ -10y & x-7y \end{bmatrix}$. Write the range of T and verify whether :

   (i) T is a one-one linear transformation.

   (ii) $T(z_1 z_2) = T(z_1)T(z_2)$.

   (b) Prove that two finite dimensional vector spaces over the same field are isomorphic if and only if they have the same dimension.

Unit II

5. (a) Let V be a finite dimensional vector space over the field F and $B = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ be a basis for V. Prove that there is a unique dual basis $B^* = \{f_1, f_2, \ldots, f_n\}$ for $V^*$ such that $f_i(\alpha_j) = \delta_{ij}$.

   Further prove that each $f \in V^*$ can be written as $f = \sum_{i=1}^{n} f(\alpha_i) f_i$ and each $\alpha \in V$ can be written as $\alpha = \sum_{i=1}^{n} f_i(\alpha)\alpha_i$.

   (b) Let B = {(1, 2), (2, 3). Is it a basis for $\mathbb{R}^2$ ? If so, find its dual basis.

6. (a) Let T be a linear operator on a finite dimensional vector space V. Let $c_1, c_2, \ldots, c_k$ be the distinct characteristic values of T. Let $W_i$ be the space of characteristic vectors with characteristic value $c_i$. Prove that $\dim(W_1 + W_2 + \ldots + W_k) = \dim W_1 + \dim W_2 + \ldots + \dim W_k$.

   (b) Let T be a linear operator on $\mathbb{R}^3$ which is represented in the standard basis by $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$.

   Find its characteristic vectors. Is it diagonalizable ? Justify.

7. Let T be a linear opeator on a finite dimensional vector space V.

   (a) Prove that the minimal polynomial for T divides the characteristic polynomial for T.

   (b) If W is invariant subspace for T, prove that for each $\alpha \in V : S(\alpha : W)$ is an ideal in the polynomial algebra F[$x$].

Unit III

8. (a) Let T be a linear operator on a finite dimensional vector space V and $V = W_1 \oplus W_2 \oplus \ldots \oplus W_k$ where each $W_i$ is invariant under T. Let $T_i = T \mid W_i$. Prove that $\det T = \det T_1 \cdot \det T_2 \ldots \det T_k$.

   (b) If V is a real vector space and E is an idempotent linear operator on V, prove that I + E is invertible. Also find $(I + E)^{-1}$.

9. Let V be a finite dimensional vector space, $\alpha \in V$ and T be a linear operator on V.

   (a) If the degree of the T-annihilator of $\alpha$ is $k$, find a basis for the cyclic subspace $Z(\alpha; T)$.

   (b) If U is the linear operator on $Z(\alpha; T)$ induced by T, prove that the minimal polynomial for U is the same as the T-annihilator of $\alpha$.

10. (a) Let V be an inner product space and W be a finite dimensional subspace of V and $\beta \in V$. If $\{\alpha_1, \alpha_2, \ldots \alpha_n\}$ is any orthonormal basis for W, find a best approximation to $\beta$ by vectors in W. Verify whether it is unique.

    (b) Apply the Gram-Schmidt process to the vectors $\beta_1 = (1, 0, 1)$, $\beta_2 = (1, 0, -1)$, $\beta_3 = (0, 3, 4)$ to obtain an orthonormal basis for $\mathbb{R}^3$ with the standard inner product.

(4 × 16 = 64 marks)

C 63422 (Pages : 2) Name..................................

Reg. No..................................

# FIRST SEMESTER M.Sc. (SSE) DEGREE EXAMINATION, MARCH 2019

Mathematics

Paper I—ALGEBRA—I

(2003 Admissions)

Time : Three Hours Maximum : 80 Marks

## Part A

*Answer* **all** *questions.*
*Each question carries* 4 *marks.*

I. (a) Consider the following mappings of the plane :

$$f(x,y) = (-y,x) \text{ and } g(x,y) = (x+4,2y).$$

Verify whether they are isometries of the plane.

(b) Find the order of (1, 1) in $\mathbb{Z}_4 \times \mathbb{Z}_6$.

(c) Let H be a subgroup of a group G and X be the set of all left cosets of H in G. Show that $a\text{H} \to ga\text{H}$ is an action of G on X.

(d) Describe the elements of the groupo represented as $\left(a,b : a^2 = b^2 = 1, ab = ba\right)$.

(4 × 4 = 16 marks)

## Part B

*Answer any* **four** *questions without omitting any unit.*
*Each question carries* 16 *marks.*

Unit I

II. (a) Prove that if $m$ and $n$ are relatively prime then $\mathbb{Z}_{mn}$ is isomorphic to $\mathbb{Z}_m \times \mathbb{Z}_n$.

(b) Describe all abelain groups of order 100 as direct product of cyclic groups.

III. (a) Let G be a group and H be a subgroup of G. Prove that the following are equivalent :—

(i) $ghg^{-1} \in \text{H}$ for all $g \in \text{G}$ and $h \in \text{H}$.

(ii) $g\text{H}g^{-1} = \text{H}$ for all $g \in \text{G}$.

(iii) $g\text{H} = \text{H}g$ for all $g \in \text{G}$.

(b) Show that if G is abelian then every subgroup of G is a normal subgroup.

Turn over