D 91551 (Pages : 3) Name..................................

Reg. No...............................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

M.Sc. Statistics

STA 3E 02—OPERATIONS RESEARCH—I

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (i) Define feasible region of a LPP and show that the feasible region is a convex set.

(ii) Describe the phenomenon of cycling in the simplex method.

(iii) State the significance of duality. Prove that the dual of the dual is the primal.

(iv) What are balanced and unbalanced transportation problems ?

(v) Define Assignment problem. Can it be considered as a particular case of a transportation problem ? If so, give reasons.

(vi) State the general form of integer programming problem. Distinguish between pure and mixed integer programming problem.

(vii) State minimax theorem. Discuss the role of it in game theory.

(viii) What are the major limitations and applications of game theory ?

(4 × 4 = 16 marks)

Section B

Answer **either** *Part* (A) **or** *Part* (B) *of all questions.*
Each question carries 16 *marks.*

II. (A) (a) Discuss the main features of simplex method used for solving the optimization problems.

(b) Distinguish between two-phase method and Big-M method. Discuss the different possibilities arising at the terminal stage of Big-M method.

(8 + 8 = 16 marks)

Or

Turn over

(B) (a) Show that if either the primal or the dual problem has a finite optimum solution, then the other problem has a finite optimum solution and the extremes of the two linear functions are equal.

(b) Explain the dual-simplex method for the solution of LPP.

(8 + 8 = 16 marks)

III. (A) (a) Explain Vogel's method of finding an initial basic feasible solution for a transportation problem. How do you improve upon this solution ?

(b) Describe how an assignment problem can be treated as a transportation problem. Show that the optimal solution to the assignment problem remains the same if a constant is added or subtracted to any row or column of the cost matrix.

(8 + 8 = 16 marks)

Or

(B) (a) Find the optimum solution of the following transportation problem :

		Destination				*Supply*
		W_1	W_2	W_3	W_4	
Source	1	21	16	25	13	11
	2	17	18	14	23	13
	3	32	27	18	41	19
	Demand	6	10	12	15	

(b) Describe the method of processing jobs through two machines.

(10 + 6 = 16 marks)

IV. (A) (a) How is a mixed integer program different from a pure integer program ? Explain the merits and demerits of "rounding off" a continuous optimal solution to a LPP to obtain an integer solution.

(b) Describe branch and bound method for solving an integer programming problem.

(8 + 8 = 16 marks)

Or

(B) (a) Solve the following integer linear programming problem optimally :

Maximize $Z = 10x_1 + 8x_2$

subject to

$$2x_1 + 4x_2 \le 25$$

$$4x_1 + 6x_2 \le 27$$

$$x_1, x_2 \ge 0 \text{ and integers.}$$

(16 marks)

V. (A) (a) How do you solve a game when (i) Saddle point exists, and (ii) Saddle point does not exist ?

(b) Show how a two-person zero-sum game problem can be reduced to a linear programming problem.

(8 + 8 = 16 marks)

Or

(B) (a) Explain the theory of dominance in the solution of rectangular games. Illustrate with examples.

(b) Explain the algebraic method for solving a rectangular game.

(8 + 8 = 16 marks)

[4 × 16 = 64 marks]

D 91550 (Pages : 3) Name.....................................

Reg. No.................................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

M.Sc. Statistics

STA 3E 01—TIME SERIES ANALYSIS

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 marks.

1. (a) What are the main steps involved in the exploratory analysis of an observed time series ?

   (b) How do you fit a polynomial trend to a time series ?

   (c) Obtain the ACF of a stationary AR (1) process.

   (d) Let $\{a_t\}$ be a white noise sequence. Check if the time series defined by the model by $X_t = 0.4 X_{t-1} + 0.5X_{t-2} + a_t$ is stationary.

   (e) Derive a forecast formula for a stationary AR (2) process.

   (f) Obtain the *Yule–Walker* estimate of the parameter of an invertible MA (1) process.

   (g) Obtain the spectral density of an M A (2) process.

   (h) Define an ARCH (2) process $\{Y_t\}$ and obtain the ACF of $\{Y_t^2\}$.

(4 × 4 = 16 marks)

Section B

Answer **either** *part* (A) **or** *part* (B) *of all questions.*
Each question carries 16 *marks.*

2. (A) (a) Show that $\{Z_t\}$ defined by $Z_t = A \cdot \cos(\theta t) + B \cdot \sin(\theta t)$, $t = 1, 2, \ldots$, is weakly stationary when A and B are uncorrelated *Uniform* (0, 1) random variables, and θ is a positive constant. Is $\{Z_t\}$ strictly stationary ?

   (b) Explain the double exponential smoothing method of forecasting. Describe the Holt's method of smoothing.

(9 + 7 = 16 marks)

Or

Turn over

(B) (a) Obtain the autocorrelation function of a moving average process of order q.

(b) Derive the random shock form of an ARIMA (1, 1, 1) model.

(8 + 8 = 16 marks)

3. (A) (a) Obtain the stationary region of an AR (2) process in terms of the autoregressive parameters.

(b) Show that an ARMA (1, 1) process can be represented as an auto-regressive process of infinite order.

(8 + 8 = 16 marks)

Or

(B) (a) Let $Z_t - \mu = \phi\left(Z_{t-1} - \mu\right) + a_t$, $t = 1, 2, ..., N$, where $|\phi| < 1$ and $\{a_t\}$ is a sequence of i.i.d. N $(0, \sigma^2)$ random variables. Obtain the maximum likelihood estimators of μ, ϕ and σ^2. State the asymptotic properties of the resulting MLEs.

(b) Obtain an explicit form of an l – *step* ahead MMSE forecast for a stationary AR (2) process based on a realization up to a time point n.

(10 + 6 = 16 marks)

4. (A) (a) Derive a formula for computing a *one – step* ahead forecast for an ARIMA (0, 1, 1) model. Show that the final formula reduces to a simple exponential smoother.

(b) What is the role of residual analysis in time series model diagnosis ?

(c) How do you choose order of an ARMA model for a given data ?

(8 + 4 + 4 = 16 marks)

Or

(B) (a) Find mean, variance and ACF of $\{X_t\}$ defined by $X_t = \theta + X_{t-1} + at$, where $X_0 = 0$, θ is a constant and $\{a_t\}$ is a white noise. Is $\{X_t\}$ weakly stationary ? If not, how do you get a stationary version ? Justify your answer.

(b) Define an MA (q) process and obtain the ACF of an invertible MA (q) process.

(8 + 8 = 16 marks)

5. (A) (a) What do you mean by Fourier analysis of time series ? Explain with an example.

(b) Define : (i) A sample spectrum ; and (ii) Periodogram.

(c) Establish the relationship between a sample spectrum and the periodogram.

(4 + 4 + 8 = 16 marks)

Or

(B) (a) Define ARCH (p) model and state conditions for its stationarity.

(b) Obtain the mean, variance and kurtosis of the marginal distribution of a stationary GARCH (1, 1) process. Compare the kurtosis with that of normal distribution.

(4 + 12 = 16 marks)

[4 × 16 = 64 marks]

D 91549 **(Pages : 3)** **Name.......................................**

Reg. No...................................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

M.Sc. Statistics

STA 3C 13—STOCHASTIC PROCESSES

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A

Answer any **four** *questions.*
Each question carries 4 marks.

1. (a) Explain different types stochastic processes based on nature of state space and index set. Give one example for each.

   (b) State and prove the Chapman-Kolmogorov Equations.

   (c) Define Poisson process. Obtain its relation with uniform distribution.

   (d) Define Renewal reward process and Regenerative process.

   (e) Obtain the integral equation satisfied by the renewal function of a renewal process.

   (f) Explain a gamblers ruin problem.

   (g) Explain the characteristics of a Queueing system.

   (h) Define weak stationary process. Give an example.

(4 × 4 = 16 marks)

Part B

Answer **either** *part A* **or** *part B.*
Each question carries 16 *marks.*

II. A (a) Explain the concept of communication between the states of a Markov chain.

Prove that communication is a class property.

(b) Define transient state and recurrent state. Show that all the states of an irreducible finite' chain are positive recurrent.

Turn over

B (a) Explain the concept of steady state distribution. If P is the TPM of an ergodic chain and π is its stationary distribution show that $\pi = \pi P$.

(b) Define stationary distribution of a Markov chain. Classify the states of the Markov chain whose TPM is given below :

$$\begin{bmatrix} 1/3 & 0 & 2/3 & 0 \\ 0 & 1/4 & 3/4 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 1/3 & 1/3 & 1/3 \end{bmatrix}.$$

III. A (a) Show that a stochastic process $\{N(t)\}$ is a Poisson process iff its interarrival time distribution is Poisson.

(b) Let $\{N(t)\}$ is a Poisson process with parameter λ. Suppose that each occurrence of the events has a constant probability p of being recorded independently. If $\{M(t)\}$ is the number of events being recorded in an interval of length t. Then show that $\{M(t)\}$ is a Poisson process with parameter λp.

B (a) Derive the steady state distribution of the Poisson process.

(b) Describe a one dimensional and two-dimensional random walk. Show that states of one dimensional and two-dimensional symmetric random walk are recurrent.

IV. A (a) Derive Kolmogrov Equations for continuous time Markov Process.

(b) Define a birth and death processes. Derive the forward Kolmogorov differential equation satisfied by the process.

B (a) Define Galton Watson branching processes. Obtain the recurrence relation between the PGFs of a branching process.

(b) Show that probability of extinction of GW branching process is the smallest positive root of the equation $s = P(s)$, where $P(s)$ is the PGF of the offspring distribution.

V. A (a) Define weak stationary process. Consider the stochastic process $\{X(t), t > 0\}$, where $X(t) = A\cos(\omega t) + B\sin(\omega t)$, where A and B are uncorrelated random variables such that $E(A) = E(B) = 0$ and $V(A) = V(B) = 1$. Show that $\{X(t), t > 0\}$ is a weak stationary process.

(b) Define renewal processes. State and prove renewal theorem.

B (a) Describe the characteristics of a Queueing system. Obtain the steady system size distribution of an M/M/1 Queueing system.

(b) Define Brownian motion process. Explain how Brownian motion process can be obtained from a random walk through a limiting process.

(4 × 16 = 64 marks)

D 91548 (Pages : 2) Name.......................................

Reg. No...................................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

M.Sc. Statistics

STA 3C 12—MULTIVARIATE ANALYSIS

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (a) Differentiate between simple and partial correlations.

(b) If $X \sim Np(\mu, \Sigma)$, derive the characteristic function of X.

(c) State and prove the additive property of multivariate normal distribution.

(d) State multivariate central limit theorem.

(e) Is Hotelling's T^2 is a generalization of the square of students-t statistic ? Justify the answer.

(f) Explain the concept of principal components.

(g) What are canonical variates and canonical correlations ?

(h) Explain spericity test.

(4 × 4 = 16 marks)

Part B

Answer **either** *part A* **or** *part B.*
Each question carries 16 *marks.*

II. A) (a) Define multivariate normal distribution. If X Np(μ, Σ), find the distribution of Y = CX, where C is a non-singular matrix of order *p*.

(b) If $X \sim Np(\mu, \Sigma)$, then show that $Q = (X - \mu)\Sigma^{-1}(X - \mu) \sim \chi_{(p)}$. Explain the hypothesis test concerning the mean vector μ of $Np(\mu, \Sigma)$ where Σ is known.

Turn over

B) (a) Let $X \sim Np(\mu, \Sigma)$, obtain the MLE's of μ and Σ.

(b) Define one sample Hotelling's T^2 statistics. Show that Hotelling's T^2 statistic is invariant under non-singular transformation.

III. A) (a) Derive the characteristic function of Wishart distribution.

(b) Prove that Wishart distribution is a generalization of $\sigma^2\chi^2$ distribution.

B) (a) Define Wishart distribution. State and prove any three properties of Wishart distribution.

(b) Describe multivariate Fisher-Behren problem.

IV. A) (a) Establish the relation between principal components and the eigen vectors of the variance covariance matrix.

(b) Derive the distribution of the sample correlation coefficient.

B) (a) Let $X_1, X_2, \ldots, X_n$ i.i.d. random variables such that $X_i \sim Np(\mu, \Sigma)$, $i = 1, 2, \ldots, n$. Prove that the mean vector X and SP matrix A are independent.

(b) Derive the distribution of Hotelling T^2 Statistic.

V. A) (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from $Np(\mu, \Sigma)$. Obtain the distribution of the sample generalized variance.

(b) Explain Bayes classification rule.

B) (a) Derive the linear discriminant function for classifying an observation between two multivariance normal distributions.

(b) Derive the test criterion to test the hypothesis that mean vectors of two multivariate normal populations are equal when they have same unknown covariance matrix.

($4 \times 16 = 64$ marks)

D 91547 (Pages : 3) Name..................................

Reg. No..............................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

M.Sc. Statistics

STA 3C 11—STATISTICAL INFERENCE—II

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 marks.

I. (a) Define most powerful test.

(b) Does the Laplace family of pdf's $f_\theta(x) = \frac{1}{2}e^{-|x-\theta|}, -\infty < x < \infty,\ \theta \in R$ posses MLR property.

(c) Define UMP Unbiased test.

(d) Define locality most powerful test.

(e) Define ASN function. Explain its uses.

(f) State fundamental identity of SPRT.

(g) Distinguish between parametric and non parametric tests.

(h) Define Kendall's tau.

(4 × 4 = 16 marks)

Section B

Answer **either** *Part A* **or** *Part B.*
Each question carries 16 *marks.*

II. (A) (a) A sample of size one is taken from population with distribution P (λ). To test $H_0 : \lambda = 1$ against $H_1 : \lambda = 2$ consider the non-randomized test :

$$\phi(x) = \begin{cases} 1 \text{ if } x > 3 \\ 0 \text{ if } x \geq 3 \end{cases}$$

Find the probabilities of type I and type II errors. If it is required to achieve a size equal to 0.05 how will you modify the test.

(b) State and prove Generalized Neyman- pearson lemma.

Turn over

(B) (a) Define MLR Property. Prove that $U(0, \theta)$ has MLR in $X_{(n)}$.

(b) Define UMP test. Obtain the UMP test for testing $H_0 : M \leq M_0$ against $H_1 : M > M_0$ based on a single observation from hypergeometric distribution with p.m.f.

$$f(x, M) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{x}}, x = 0, 1, 2, \ldots, M.$$

III. (A) (a) Define likelihood ratio test. Obtain the, asymptotic distribution of the likelihood ratio test statistic.

(b) Obtain the likelihood ratio test for testing the equality of means of two normal populations with equal variances.

(B) (a) Define UMP test. Find a UMP size α test for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$ based on a sample of n observations from the population with p.m.f.

$$f_\theta(x) = \theta^x (1-\theta)^{1-x}, x = 0, 1; 0 < \theta < 1.$$

(b) Briefly explain Union-intersection and Intersection -Union tests.

IV. (A) (a) Explain Wilcoxon signed rank test.

(b) Explain Mann-Whitney-Wilcoxon test.

(B) (a) Explain the Chi-square test for homogeneity.

(b) Define median test. Derive its null distribution.

V. (A) (a) Define Sequential probability ratio test and derive the boundary values of it.

(b) Obtain the SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1, \theta_0 < \theta_1$ based on observations from $B(n, \theta)$ at strength (α, β).

(B) (a) Prove that the Sequential probability ratio test terminates with probability one.

(b) Obtain the OC function corresponding to the SPRT for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1, \mu_0 < \mu_1$ based on observations from $N(\mu, \sigma^2)$ at strength (α, β), where σ^2 is known.

(4 × 16 = 64 marks)

D 91546 (Pages : 3) Name.......................................

Reg. No..................................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

M.Sc. Statistics

STA 3E 02—OPERATIONS RESEARCH—I

(2013 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer only **four** *questions.*
Each question carries 4 *marks.*

I. 1. Define primal problem and dual problem. Describe the general rules for writing the dual of a linear programming problem.

2. Explain two-phase method for solving a given linear programming problem.

3. Write a short note on parametric programming.

4. How the problem of degeneracy arises in a transportation problem. ? Explain how one overcomes it.

5. What is symmetric game ? Show that the value of a symmetric game is zero.

6. Distinguish pure and mixed integer programming problems. Give examples.

7. Define the following (i) Pay-off matrix ; (ii) Saddle point ; and (iii) Rectangular game.

8. Explain the principal assumptions made while dealing with sequencing problems.

(4 × 4 = 16 marks)

Section B

Answer **either** *part (A)* **or** *part (B).*
Each question carries 16 *marks.*

II. (A) (a) Use simplex method to solve the following Linear Programming Problem :
Maximize $Z = 4x_1 + 10x_2$
subejct to the constraints

$$2x_1 + x_2 \le 50,$$
$$2x_1 + 5x_2 \le 100,$$
$$2x_1 + 3x_2 \le 90,$$
$$x_1 \ge 0 \text{ and } x_2 \ge 0.$$

Turn over

(b) Show that the set of feasible solutions of an L.P.P. is a convex set. If $f(X)$ is minimum at more than one of the vertices of the feasible solution space, then it is minimum at all those points which are the convex linear combinations of these vertices.

(B) (a) Explain the simplex procedure for solving a linear programming problem.

(b) Show that the dual of the dual is the primal. Also prove that the optimum value of $f(X)$ of the primal, if it exists, is equal to the optimum value of $\varphi(Y)$ of the dual.

III. (A) (a) Explain any *one* method for finding the initial solution of transportation problem. Also explain the method of obtaining the optimal solution of a transportation problem.

(b) In an assignment problem, if we add or subtract a constant to every element of any row (or column) of the cost matrix, then prove that an assignment that minimizes the total cost on one matrix also minimizes the total cost on the other matrix.

(B) (a) What is an assignment problem ? Give an algorithm to solve an assignment problem.

(b) Find the sequence that minimizes the total time required in performing the following jobs on 3 machines in the order ABC. Processing times (in hours) are given in the following table :

Job	1	2	3	4	5
Machine A	8	10	6	7	11
Machine B	5	6	2	3	4
Machine C	4	9	8	6	5

IV. (A) Solve the following all integer programming problem using the branch and bound method :

Minimize $Z = 3x_1 + 2.5x_2$
subject to the constraints

$$x_1 + 2x_2 \geq 20,$$
$$3x_1 + 2x_2 \geq 50 \text{ and}$$
x_1, x_2 are non-negative integers.

(B) Explain cutting plane method for solving an integer programming problem. Also explain application of zero-one programming.

V. (A) (a) Explain the relation between a linear programming problem and a two person zero sum game.

(b) Let $\left(a_{ij}\right)$ be the $m \times n$ pay-off matrix for a two-person zero-sum game. If $\underline{v}$ denotes the maximin value and $\overline{v}$ the minimax value of the game, then prove that $\overline{v} \geq \underline{v}$.

(B) (a) Explain how to find the solution of a two person zero sum game. Give an example.

(b) State and prove the fundamental theorem of game theory.

(4 × 16 = 64 marks)

D 91544 **(Pages : 2)** **Name.....................................**

Reg. No.................................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

M.Sc. Statistics

STA 3C 13—STOCHASTIC PROCESS

(2013 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (a) State Kolmogorov consistency theorem. What is its significance ?

(b) Define a random walk with two barriers.

(c) Establish the additive property of Poisson process.

(d) Define a compound Poisson processes. Give one example.

(e) Define renewal reward process.

(f) Briefly explain insurers ruin problem.

(g) Briefly explain the important characteristics of a queue.

(h) Explain Gaussian process and discuss its stationarity.

($4 \times 4 = 16$ marks)

Section B

Answer **either** *part (a)* **or** *part (b) of all questions.*
Each question carries 16 *marks.*

II. (a) (i) With suitable examples describe different classifications of a stochastic process.

(ii) State and prove a necessary and sufficient condition for the persistence of the state of a Markov chain.

Or

(b) (i) When do you say that two states of a Markov chain communicate ? Show that communication is an equivalence relation. Also show that communicating states have same period.

(ii) Define branching process. Obtain its mean when the initial size is 1.

Turn over

III. (a) (i) Derive the distribution of inter arrival times and waiting times for the occurrence of events in a Poisson process.

(ii) Explain the decomposition property of a Poisson process.

Or

(b) (i) Define a non homogenous Poisson process and derive its pgf.

(ii) Derive the Kolmogorov differential equations to be satisfied by a birth and death process.

IV. (a) (i) Define a renewal process. Derive expressions for renewal function and renewal density.

(ii) State and prove the basic renewal theorem.

Or

(b) (i) Show that the renewal function M (t) satisfies the renewal equation $M = F + F * M$.

(ii) Explain the concepts of residual, current and total life times in renewal theory and inspection paradox.

V. (a) (i) What is Little's formula ? What is its use ? Describe the M/G/1 queueing system.

(ii) Obtain the steady state probability distribution of an M/M/c queue. Also obtain the mean queue length.

Or

(b) (i) Define Brownian Bridge. If $\{X(t), t \geq 0\}$ is a Brownian motion process, show that $\{Z(t), 0 \leq t \leq 1\}$ is a Brownian Bridge process when $Z(t) = X(t) - t\,X(1)$.

(ii) How will you define a stationary stochastic process ? Show that the two forms of stationarity are equivalent for a Gaussian process.

(4 × 16 = 64 marks)

D 91543 (Pages : 3) Name.....................................

Reg. No.................................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

Statistics

STA 3C 11—STATISTICAL INFERENCE—II

(2013 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 marks.

I. (i) A sample of size 1 is taken from an Poisson distribution with mean θ. Consider the test function

$$\phi(x) = \begin{cases} 1 \text{ if } x > 1 \\ 0 \text{ if } x \leq 1 \end{cases}$$

for test the hypothesis $H_0 : \theta = 1$ against $H_1 : \theta = 2$. Find the probability of type 1 error, probability of type 2 error and power of the test.

(ii) Define MLR property. Show that $U(0, \theta)$ posses MLR property.

(iii) Derive the likelihood ratio test for $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda \neq \lambda_0$, based on a sample of size n from an exponential distribution with mean $1/\lambda$.

(iv) Define an unbiased test and UMPU test.

(v) Describe Kolmogorov-Smirnov one sample test procedure.

(vi) Explain Chi-square test of homogeneity.

(vii) Describe sequential unbiased estimation.

(viii) State the fundamental identity of SPRT. What is its use ?

(4 × 4 = 16 marks)

Turn over

Section B

Answer **either** *(A)* **or** *(B) of all questions.*
Each question carries 16 *marks.*

II. A. (a) State and prove Neyman-Pearson lemma.

(b) A random sample of size n is taken from a normal population with mean θ and variance unity. Is there exist a UMP level α test for test the hypothesis $H_0 : \theta \leq 2$ against $H_1 : \theta > 2$? If so what is it ?

Or

B. (a) Let X be a uniform random variable over $(0, \theta)$. Obtain a UMP test of size α for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$ based on a random sample of size n on X.

(b) Prove that a UMP test of one-sided alternatives always exists for a family of distributions possessing monotone likelihood ratio property.

III. A. (a) Derive the likelihood ratio test for testing the equality of means of two normal populations with common unknown variances.

(b) Obtain a UMP unbiased size α test for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$ on a random sample of size n from $N(\mu, 1)$.

Or

B. (a) Explain likelihood ratio test procedure for testing composite hypotheses.

(b) Based on a random sample of size n from Cauchy distribution $C(\theta, 1)$, derive LMP test for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$.

IV. A. (a) Show that Kolmogorov-Smirnov test statistic is distribution free.

(b) Describe a non-parametric test for testing whether the third quartile of a population is a pre-assigned value.

Or

B. (a) Define Mann-Whitney and Wilcoxon statistics. Establish a linear relationship between them. Obtain the mean and variance of Mann-Whitney Wilcoxon statistic under H_0.

(b) Explain the sign test. Discuss its uses.

V. A. (a) Obtain the SPRT of strength (α, β) for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1 (> \lambda_0)$, where λ is the parameter of the Poisson distribution.

(b) If (α, β) is the strength of the SPRT with boundary points (A, B), which terminates with probability one, then show that $A \geq \frac{\beta}{1-\alpha}$ and $B \leq \frac{1-\beta}{\alpha}$.

Or

B. (a) Show that the SPRT eventually terminates with probability one.

(b) Let $X_n, n \geq 1$ be i.i.d. observations from a Bernoulli distribution $B(1, \theta)$, obtain the SPRT of strength (α, β) for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (> \theta_0)$.

(4 × 16 = 64 marks)

D 91076 (Pages : 2) Name.......................................

Reg. No...................................

THIRD SEMESTER M.A./M.Sc./M.Com. DEGREE (REGULAR) EXAMINATION NOVEMBER 2020

(CBCSS)

Statistics

MST 3E 13—BIOSTATISTICS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Section A

Answer at least **three** *questions.*
Each question carries 2 *marks.*
All questions can be attended.
Overall Ceiling 6.

1. Give an example of a statistical problem in biomedical research.
2. What is the importance of Raleigh distribution in survival analysis ?
3. What is a censored data ?
4. What do you mean by random censoring ?
5. Give an index for measuring the probability of death under competing risks.
6. What do you mean by genetic drift ?
7. What is the importance of randomized dose-response studies ?

(3 × 2 = 6 weightage)

Section B

Answer at least **three** *questions.*
Each question carries 4 *marks.*
All questions can be attended.
Overall Ceiling 12.

8. Write down the principles of biostatistical design in medical studies.
9. Explain log-normal distribution and its applications in survival analysis.
10. Distinguish between type I and type II random censoring.
11. Explain Kaplan - Meier methods.

Turn over

12. Describe ML method.

13. Describe Hardy-Weinberg equilibrium.

14. Explain the planning and design of a randomized clinical trial.

(3 × 4 = 12 weightage)

Section C

Answer at least **two** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 12.

15. Describe the parametric methods for comparing two survival distributions.

16. Obtain the MLE for the mean of the exponential distribution under type-I and type-II censoring.

17. Describe stochastic epidemic models.

18. Describe phase I-IV clinical trials.

(2 × 6 = 12 weightage)

D 91075 (Pages : 2) Name.................................

Reg. No.................................

THIRD SEMESTER M.A./M.Sc./M.Com. DEGREE (REGULAR) EXAMINATION NOVEMBER 2020

(CBCSS)

Statistics

MST 3E 10—STATISTICAL QUALITY CONTROL

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Section A

Answer at least **three** *questions.*
Each question carries 2 *weightage.*
All questions can be attended.
Overall Ceiling 6.

1. How will you measure the performance of different sampling plans ?
2. In what way sequential sampling plan differs from multiple sampling plan ?
3. What are sampling plans with double specification limits ?
4. How are control limits used to improve specification limits ?
5. A tool wear occurs due to k-types of defects. Suggest a method of controlling them using a single control chart.
6. What is meant by OC function of a control chart ?
7. What do you mean by process control in industrial statistics ?

(3 × 2 = 6 weightage)

Section B

Answer at least **three** *questions.*
Each question carries 4 *weightage.*
All questions can be attended.
Overall Ceiling 12.

8. Under what circumstances you would like to use an AOQL sampling plan ? Derive the AOQ function for the double sampling attributes plan.
9. Derive the sequential sampling plan for attributes when the probability of accepting lots with proportion defectives less than or equal to p_1 is greater than 1-α and the probability of accepting lot with proportion defectives greater than or equal to p_0 is less than β.

Turn over

10. Describe the various ways in which control charts may be modified to meet special situations. Explain how you can find out lack of control even if all points are within control limits.

11. Define ARL function for a control chart. For a p chart, central line = 0.04, LCL = 0.005, UCL = 0.075. If samples of size of 100 are taken, find the ARL function when the process is in control with respect to the above p chart and when process proportion defective has shifted to 0.06.

12. Explain how a control chart helps in controlling measurable characteristics in production processes.

13. What are rational subgroups and what are the considerations in selecting them ? Do they limit the ability of the control chart to detect assignable causes ? If so explain.

14. Distinguish between process control and product control.

(3 × 4 = 12 weightage)

Section C

Answer at least **two** *questions.*
Each question carries 6 *weightage.*
All questions can be attended.
Overall Ceiling 12.

15. Derive the expression for ATI and AOQ for single sampling plan with rectification scheme.

16. Discuss the relationship that may exist between specification limits and control limits and suggest suitable modification of the usual control charts in different situations.

17. (a) How do you obtain the natural tolerance limits of a production process ? How do you make use of this knowledge of process tolerance in reviewing specification tolerances ?

    (b) If x has exponential distribution $f(x) = e^{-x}$; $x > 0$ and 0 elsewhere, construct control chart such that probability of an observation falling above UCL is 0.05 and below LCL is 0.10.

18. Describe the construction and merits of EWMA charts.

(2 × 6 = 12 weightage)

D 91074 (Pages : 2) Name...

Reg. No..................................

THIRD SEMESTER M.A./M.Sc./M.Com. DEGREE (REGULAR) EXAMINATION, NOVEMBER 2020

(CBCSS)

Statistics

MST 3E 07—STATISTICAL DECISION THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Section A

Answer at least **three** *questions.*
Each question carries 2 *weightage.*
All questions can be attended.
Overall Ceiling 6.

1. Define an admissible decision rule.
2. What is mean by improper priors and conjugate priors ?
3. Explain Baye's risk principle.
4. What do you mean by a utility function ?
5. State separating hyper plane theorem.
6. What is an equaliser rule ?
7. Show that under absolute-error loss, the Baye's rule is the median of the posterior distribution.

(3 × 2 = 6 weightage)

Section B

Answer at least **three** *questions.*
Each question carries 4 *weightage.*
All questions can be attended.
Overall Ceiling 12.

8. Define the terms 'strategy', 'randomized strategy' and 'optimal strategy' with reference to game theory.
9. What is meant by randomized decision rule ? Define loss function and risk function of a randomized decision rule.

Turn over

10. Explain the direct method of solving statistical games.

11. Show that for Poisson family, the conjugate prior is gamma distribution.

12. Find the Jeffrey's improper prior for σ and posterior distribution in N (μ, σ^2) when μ known and σ unknown, based on a sample of size n.

13. What are the different techniques to determine the prior density ?

14. A sample of size n is taken from a Bernoulli distribution with parameter θ where θ has prior distribution Uniform (0, 1). Find posterior distribution of θ.

($3 \times 4 = 12$ weightage)

Section C

Answer at least **two** *questions.*
Each question carries 6 *weightage.*
All questions can be attended.
Overall Ceiling 12.

15. What is meant by non-informative priors ? Explain different methods of constructing a non-informative prior.

16. Define utility function. Explain axioms and the construction of utility function.

17. State and prove fundamental theorem of games.

18. (a) Explain Hierarchical Baye'sanalysis.

    (b) Show that Baye's rule admissible.

($2 \times 6 = 12$ weightage)

D 91073 (Pages : 2) Name.......................................

Reg. No..................................

THIRD SEMESTER M.A./M.Sc./M.Com. DEGREE (REGULAR) EXAMINATION, NOVEMBER 2020

(CBCSS)

Statistics

MST 3E 02—TIME SERIES ANALYSIS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Section A

Answer at least **three** *questions.*
Each question carries 2 *weightage.*
All questions can be attended.
Overall Ceiling 6.

1. What is meant by trend component of a time series ? How can we remove trend ?
2. Explain the relationship between time series and stochastic processes.
3. Define autocovariance function. What are the properties of autocovariance function of a stationary time series.
4. Define ARIMA (1, 1, 2) model.
5. Explain residual analysis of a time series data.
6. Define spectral density and periodogram.
7. Define ARCH time series model and discuss its properties.

($3 \times 2 = 6$ weightage)

Section B

Answer at least **three** *questions.*
Each question carries 4 *weightage.*
All questions can be attended.
Overall Ceiling 12.

8. Discuss various methods of trend estimation in a time series.
9. What is meant by saying that stochastic process is invertible ? Show that AR (p) process is invertible.

Turn over

10. Consider the process $Y_t = \varepsilon_t \cos ct + \varepsilon_{t-1} \sin ct, c \neq 0$. Determine the value of c for which the process is stationary.

11. Define ACF and PACF of stationary process. Explain the behavior of ACF and PACF of MA model.

12. State and prove Herglotz theorem.

13. Describe a test for stationarity of estimated noise sequence in a time series data.

14. Explain Portmanteau test.

(3 × 4 = 12 weightage)

Section C

Answer at least **two** *questions.*
Each question carries 6 *weightage.*
All questions can be attended.
Overall Ceiling 12.

15. Explain important components of a time series. Describe Holt-Winter smoothing procedure.

16. Obtain stationarity condition for autoregressive process of order p. Explain minimum mean square error prediction of time series.

17. Derive Yule-Walker equation for estimation of parameters of AR process. Suppose for an AR (2) model $Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \varepsilon_t, \varepsilon \sim WN(0, \sigma^2)$, the sample autocovariance based on 1500 samples are $r(0) = 5, r(1) = 0, r(2) = 2.5, r(3) = 0, r(4) = 1$. Obtain the estimates of ϕ_1, ϕ_2 and σ^2.

18. Derive the spectral density of ARMA (p, q) process given by $\Phi(B) Z_t = \Theta(B) \varepsilon_t$. Identify the stationary and invertible process Z_t having spectral density $S(f) = \dfrac{17 - 8 \cos 2\pi f}{13 - 12 \cos 2\pi f}$.

(2 × 6 = 12 weightage)

D 91070 (Pages : 2) Name.......................................

Reg. No..................................

THIRD SEMESTER M.A./M.Sc./M.Com. DEGREE (REGULAR) EXAMINATION, NOVEMBER 2020

(CBCSS)

Statistics

MST 3C 11—APPLIED REGRESSION ANALYSIS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Section A

Answer at least **three** *questions.*
Each question carries 2 weightage.
All questions can be attended.
Overall Ceiling 6.

1. If $\varepsilon \sim N\left(0, \sigma^2 I\right)$, what is the distribution of the OLS of β in the regression model $Y = X\beta + \varepsilon$.

2. What is meant by auto correlation ?

3. State Gauss Markov theorem.

4. Explain how residual plots are used to check the assumption of normality of the errors in linear model.

5. Write a note on local linear estimator.

6. What is a projection matrix ?

7. Explain forward selection step wise regression.

(3 × 2 = 6 weightage)

Section B

Answer at least **three** *questions.*
Each question carries 4 *weightage.*
All questions can be attended.
Overall Ceiling 12.

8. Explain the variance decomposition method of detecting multicollinearity and derive the expression for 'Variance Inflation Factor'.

9. Define heteroscedasticity. What are the consequences of heteroscedasticity in linear regression ?

10. Explain Nadaraya-Watson (Kernel regression) estimator ?

11. For the linear regression model $Y = \beta_0 + X\beta_1 + \varepsilon,\ \varepsilon \sim N\left(0, \sigma^2 I\right)$, obtain the distribution of residual sum of squares. Hence obtain an unbiased estimator for σ^2.

12. Explain hierarchical polynomial regression model ?

13. Discuss logistic regression model and explain the estimation procedure of parameters.

14. Derive the estimate of the parameters of a linear regression model using the method of least squares.

(3 × 4 = 12 weightage)

Section C

Answer at least **two** *questions.*
Each question carries 6 *weightage.*
All questions can be attended.
Overall Ceiling 12.

15. For the linear regression model $Y = X\beta + \varepsilon,\ \varepsilon \sim N\left(0, \sigma^2 I\right)$, obtain mle of β. Show that OLS estimator and $e'e$ are independent, where e is the residual.

16. Explain the four methods for scaling residuals bringing out the relationship between them.

17. Discuss multicollinearity. What are the consequences of multicollinearity ? Discuss how to detect multicollinearity in multiple linear regression.

18. What is the need for piecewise polynomial fitting ? Discuss the method of splines in this context.

(2 × 6 = 12 weightage)

D 90763 (Pages : 2) Name.....................................

Reg. No.................................

THIRD SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION NOVEMBER 2020

(CUCSS)

Statistics

STA 3C 11—STOCHASTIC PROCESSES

(2010 Admission onwards)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Weightage 1 *for each question.*

1. State the conditions for a stochastic process to be termed as a martingale process.
2. Give an illustration for a Markov chain.
3. Define random walk.
4. Define Poisson process.
5. What is meant by inter-arrival time ?
6. What is transition probability function ?
7. Define inspection paradox.
8. Give two examples for renewal process.
9. State elementary renewal theorem.
10. What is a Gaussian process ?
11. What is meant by a queue ?
12. Define weakly stationary process.

(12 × 1 = 12 weightage)

Part B

Answer any **eight** *questions.*
Weightage 2 *for each question.*

13. Let a Markov chain be defined with two states. Its transition probability matrix is given by $P = \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$. Find the stationary probabilities of the Markov chain.

Turn over

14. State the necessary and sufficient condition for a state to be recurrent or transient.

15. State the properties of generating functions in the context of a branching process.

16. Show that the Poisson process is a Markov process.

17. Distinguish discrete time and continuous time Markov processes.

18. List the properties of Poisson processes.

19. Write a short note on insurer ruin problem.

20. What is a renewal process ? Define the renewal function.

21. Let $\{N(t), t \geq 0\}$ be a renewal process. Derive the expression for the distribution of $N(t)$.

22. Describe queue discipline.

23. What is a stationary process ? Distinguish two types of stationary process.

24. State the properties of a Gaussian process.

(8 × 2 = 16 weightage)

Part C

Answer any **two** *questions.*
Weightage 4 for each question.

25. If $\{X_n, n = 0,1,2,.....\}$ be a branching process and q_n is the probability of extinction, show that q_n is non-decreasing and $\lim_{n\to\infty} q_n = \pi$, where $0 \leq \pi \leq 1$.

26. What is compound Poisson process ? Derive mean and variance of such a process.

27. Prove that $M(t) = F(t) + \int_0^t M(t-x)\, dF(x)$, where $M(t)$ is the renewal function.

28. Show that the interarrival time between two successive occurrences of a Poisson process having parameter λ has a negative exponential distribution with mean $1/\lambda$.

(2 × 4 = 8 weightage)