D 93626 (Pages : 2) Name..

Reg. No.....................................

FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

Statistics

STA 1C 05—SAMPLING THEORY

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (i) State the principle of statistical regularity and indicate its importance.

(ii) What are the advantages of sampling methods ?

(iii) What are the advantages of PPS sampling ?

(iv) Explain Lahiri's method.

(v) What are regression estimators ?

(vi) Obtain the expression for the bias of ratio estimators in the case of simple random sampling.

(vii) What is systematic random sampling ?

(viii) What are the advantages of cluster sampling ?

(4 × 4 = 16 marks)

Section B

Answer **all** *questions.*
Each question carries 16 *marks.*

II. A) a) Show that in case of simple random sampling the probability that a specified unit of the population being selected in any given draw is equal to the probability of its being selected at the first draw.

b) Show that in simple random sampling s^2 is an unbiased estimator of S^2. Also obtain an unbiased estimator of the variance of the $\bar{y}$ in simple random sampling without replacement.

(6 + 10 = 16 marks)

Or

Turn over

B) a) Discuss the methods for the estimation of sample size in simple random sampling.

b) Explain optimum allocation.

(8 + 8 = 16 marks)

III. A) a) Distinguish between simple random sampling and PPS sampling ?

b) Explain Des Raj's ordered estimator and show that it is unbiased. Obtain its sampling variance.

(6 + 10 = 16 marks)

Or

B) a) Distinguish between ordered and unordered estimators in PPS sampling.

b) Define the Horwitz-Thompson estimator for population total in case of PPS sampling. Obtain the expression for its variance.

(5 + 11 = 16 marks)

IV. A) a) Compare the ratio estimator with mean per unit.

b) What are unbiased ratio type estimators ? Define Hartley and Ross estimator and show that it is unbiased. Obtain its sampling variance under simple random sampling without replacement.

(6 + 10 = 16 marks)

Or

B) a) Obtain an expression for the approximate bias of the regression estimator and also obtain the large sample variance of the regression estimator.

b) When the bias of the ratio estimator will be small ?

(12 + 4 = 16 marks)

V. A) a) Explain multi-stage sampling. What are the situations in which we can use it ?

b) Explain unequal cluster sampling and obtain the unbiased estimator of the mean. Also derive its variance.

(6 + 10 = 16 marks)

Or

B) a) Explain multi-phase sampling and mention its advantages. How it differs from multistage sampling ?

b) Explain various sources of non-sampling errors.

(8 + 8 = 16 marks)

D 93625 (Pages : 2) Name.....................................

Reg. No.................................

FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

Statistics

STA 1C 04—DISTRIBUTION THEORY

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (i) Define discrete uniform distribution and derive its mean and variance.

(ii) Write a short note on hypergeometric distribution.

(iii) Derive the M.G.F. of gamma distribution.

(iv) Define beta distribution of first kind and second kind. Also state their properties.

(v) Write a short note on bivariate normal distribution.

(vi) Let (X, Y) be a random vector of continuous type with p.d.f. *f*. Obtain the p.d.f. of : (a) $U = X - Y$ and (b) $V = \frac{X}{Y}$.

(vii) Define F distribution and state its important properties.

(viii) Define Chi-square distribution and list its applications.

(4 × 4 = 16 marks)

Section B

Answer **either** *part -A* **or** *part - B of* **all** *questions.*
Each question carries 16 *marks.*

II. A) a) Define Logarithmic distribution and derive its M.G.F.

b) Write a short note on hypergeometric and its association with other distributions.

(8 + 8 = 16 marks)

Or

B) a) Derive the r^{th} order moment of discrete uniform distribution.

b) Explain the multinomial distribution and its marginal distribution.

(8 + 8 = 16 marks)

Turn over

III. A) a) Derive the r^{th} order moment of Pareto distribution.

b) Explain the role and significance of transformed distribution with illustration.

(8 + 8 = 16 marks)

Or

B) a) Write the statistical properties of bivariate normal distribution.

b) Derive the characteristic function of generalized Laplace distribution.

(8 + 8 = 16 marks)

IV. A. a) Let (X, Y) be a bivariate normal random vector with parameters $\mu_1, \mu_2, \sigma_1, \sigma_2$ and ρ. Let $U_1 = \sqrt{X^2 + Y^2}$ and $U_2 = \frac{X}{Y}$. Find the joint density of (U_1, U_2) and find the marginal density of U_1 and U_2.

b) Derive the p.d.f. of the median and mid-range of order statistics from a random sample of size *n*.

(8 + 8 = 16 marks)

Or

B. a) Let X_1, X_2, X_3 be i.i.d. random variables with common density function

$$f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}.$$

Find the p.d.f. of $Y = X_1 + X_2 + X_3$.

b) Let $Y_1 < Y_2 < Y_3 < Y_4$ denote the order statistics of a random sample from exponential distribution with parameter λ = I. Compute the probability of an event $Y_4 \ge 3$.

(8 + 8 = 16 marks)

V. A. a) Obtain the mean and variance of *t* distribution.

b) Show that non-central Chi-square satisfies additive property.

(8 + 8 = 16 marks)

Or

B. a) Derive the p.d.f. of non-central F distribution.

b) Write in detail about the interrelation between F and χ^2 distribution.

(8 + 8 = 16 marks)

[4 × 16 = 64 marks]

D 93624 **(Pages : 3)** Name.....................................

Reg. No.................................

FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

Statistics

STA 1C 03—PROBABILITY THEORY—I

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions ; each question carries* 4 *marks.*

I. (i) Define indicator random variable and mention any two properties.

(ii) What is induced probability space ? Explain with an illustration.

(iii) Show that the expected value of a bounded variate X always exists.

(iv) State Holder's inequality and mention its relation with Schwarz' inequality.

(v) State Cauchy's criterion of convergence.

(vi) Define convergence almost surely and mutual convergence.

(vii) Define tail events. State Kolmogorov's 0-1 law.

(viii) Define independence of events. State Borel-Cantelli lemma.

($4 \times 4 = 16$ marks)

Section B

Answer **either** *part-A* **or** *part-B of* **all** *questions ; each question carries* 16 *marks.*

II. A. (a) Show that the probability function defined on all intervals of the form $(a, b] \subseteq \mathcal{R}$ defines an extension uniquely to the minimal field containing all the intervals. (10 marks)

(b) A function $F(x, y)$ of two variates X, Y is defined by $F(x, y) = 1$, if $x + y \geq 0$; $F(x, y) = 1$, if $x + y < 0$. Examine whether $F(x, y)$ can be a distribution function of some two-dimensional random variable. (6 marks)

Or

Turn over

B (a) Distinguish probability space and induced probability space. (6 marks)

(b) State and prove Jordan decomposition theorem. (10 marks)

III. A (a) Define Gamma distribution. Obtain its moment generating function and hence find $E(X)$.

(8 marks)

(b) Show that $E\left(\frac{|X|^r}{1+|X|^r}\right)-\frac{a^r}{1+a^r} \leq P\{|X| \geq a\} \leq \frac{1+a^r}{a^r} E\left(\frac{|X|^r}{1+|X|^r}\right)$. (8 marks)

Or

B (a) If $E(X^r)$ exists, then show that $E(X^t)$ need not exist if $t > r$. (6 marks)

(b) State and prove C_r – inequality. (10 marks)

IV. A (a) Define convergence in probability. Show that

$$X_n \xrightarrow{P} 0 \text{ iff } E\left(\frac{|X_n|}{1+|X_n|}\right) \to 0, \text{ as } n \to \infty.$$ (10 marks)

(b) Let $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$. Show that $X_n Y_n \xrightarrow{P} XY$. (6 marks)

Or

B (a) Define convergence in distribution. Show that $X_n \xrightarrow{P} c$ implies that $F_n(x) \to 0$ for $x < c$, $F_n(x) \to 1$ for $x \geq c$, and conversely.

(10 marks)

(b) Define convergence in mean square. Verify a sequence of variates $\{X_n\}$ defined with $P(X_n = 0) = 1 - (1/n^2)$, $P(X_n = n) = 1/n^2$, $n = 1, 2, 3, \ldots$ is convergent in mean square.

(6 marks)

V. A (a) What is lattice distribution ? State the inversion formula for lattice distribution.

(8 marks)

(b) If $X \sim N(\mu, \sigma^2)$; $Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$, show that $X = \mu + \sigma Z$ has its characteristic function given by $\varphi_X(t) = \exp\left(it\mu - \left(t^2\sigma^2\right)/2\right)$.

(8 marks)

Or

B (a) State and prove Borel 0-1 criterion.

(10 marks)

(b) Find the distribution for which characteristic function is $\varphi(t) = e^{-|t|}, -\infty < t < \infty$.

(6 marks)

[4 × 16 = 64 marks]

D 93623 (Pages : 2) Name.....................................

Reg. No.................................

FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

Statistics

STA 1C 02—MATHEMATICAL METHODS FOR STATISTICS-II

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (i) Distinguish between Sigma field and Borel sigma field.

(ii) What is a measure ? Define finite measure and sigma-finite measure.

(iii) What is Borel measurable function ? How does it differ from Lebesgue measurable function ?

(iv) Define measurable space. State the conditions for a function defined on a measurable space to be simple.

(v) State Radon-Nikodym theorem. What is its significance ?

(vi) Define signed measures and absolute continuity of one measure with respect to another.

(vii) Define complex n-space, and vector addition and vector multiplication on complex n-space.

(viii) State the condition for linear dependence of vectors in a vector space. Give an example.

(4 × 4 = 16 marks)

Section B

Answer **either** *part-A* **or** *part-B of* **all** *questions.*
Each question carries 16 *marks.*

II. A) (a) Define monotone class of sets. Show that sigma field is a monotone class and a monotone field is a sigma field.

(b) Let $\{A_n, n = 1, 2, ...\}$ be a finite, disjoint class of sets in P, each contained in a given set A_0, such that $A_0 \subset \bigcup_{i=1}^{\infty} A_i$, where P is the class of all bounded, left closed and right opened intervals, prove that $\mu(A_0) \leq \sum_{i=1}^{\infty} \mu(A_i)$.

(8 + 8 = 16 marks)

Or

Turn over

B) (a) Let μ be a finite, non-negative and additive set function on a sigma field. If μ is either continuous from below at every set in the sigma field or continuous from above at 0, show that μ is a measure.

(b) Define outer measure. State any two properties on outer measures.

(8 + 8 = 16 marks)

III. A) (a) Define fundamental in measure and convergence in measure of a sequence of measurable functions. State any two properties relating to convergence of a sequence of measurable functions.

(b) State and prove Fatou's lemma.

(8 + 8 = 16 marks)

Or

B) (a) Let $\{f_n\}$ be a sequence of measurable functions which converges in measure to f and to g. Show that $\{f_n\}$ is fundamental in measure and $f = g$ almost everywhere.

(b) Let f be measurable, g be integrable and $|f| \le |g|$ a.e, Prove that f is integrable.

(8 + 8 = 16 marks)

IV. A) (a) Let μ be a signed measure and v be a finite signed measure, such that $v << \mu$. Prove that for every $\varepsilon > 0$, there is a $\delta > 0$, such that $|v|(A) < \varepsilon$ for every measurable set A, for which $|\mu|(A) < \delta$.

(b) Define product space and state Fubini's theorem.

(10 + 6 = 16 marks)

Or

B) (a) What are double and iterated integrals ?

(b) Show that every section of a measurable set is a measurable set.

(6 + 10 = 16 marks)

V. A) (a) Define matrix representation of a linear operator T : V —> V relative to a basis.

(b) Define subspace of a vector space. Show that the intersection of any number of subspaces of a vector space V is a subspace of V.

(6 + 10 = 16 marks)

Or

B) (a) Define linear span. Show that the vectors (1, 1, 1), (1, 2, 3) and (1, 5, 8) span R^3.

(b) Let $F : R^2 \to R^2$ be the linear operator defined by $F(x, y) = (2x + 3y, 4x - 5y)$. Find the matrix representation of F relative to the basis $S = \{u_1, u_2\} = \{(1, -2), (2, -5)\}$.

(8 + 8 = 16 marks)

[4 × 16 = 64 marks]

D 93622 (Pages : 3) Name..................................

Reg. No..............................

FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

Statistics

STA IC 01—MATHEMATICAL METHODS FOR STATISTICS—I

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (i) If f is a bounded function and α be a non-decreasing function on $[a, b]$, show that the lower RS integral does not exceed the upper RS integral.

(ii) Define beta and gamma functions. Show that beta function is symmetrical in its constants.

(iii) Show that a monotonically increasing sequence which is not bounded above diverges.

(iv) What is an alternating series ? State the conditions for testing the convergence of an alternating series.

(v) Examine the equality of the partial derivatives f_{xy} and f_{yx} of $f(x, y) = x^3 + e^{xy^2}$.

(vi) State Taylor's theorem on partial derivatives.

(vii) State the conditions to be satisfied by the generalized inverse of a matrix.

(viii) Explain the standard form of a system of linear equations.

(4 × 4 = 16 marks)

Section B

Answer **either** *Part A* **or** *Part B of all questions.*
Each question carries 16 *marks.*

II. (A) (a) State and prove second mean value theorem.

(b) If f is RS integrable on $[a, b]$ with respect to a monotonically non-decreasing function α on $[a, b]$ and if $|f(x)| \le K$, find the upper bound for $\left|\int_a^b f(x)\, d\alpha(x)\right|$.

(8 + 8 = 16 marks)

Or

Turn over

(B) (a) Let S (P, f, α) be the Riemann - Stieltjes sum, where P is the partition of $[a, b]$, f is the bounded function on $[a, b]$ and α is the monotonic non-decreasing function on $[a, b]$.

Prove that $\lim \lim_{\|P\| \to 0} S(P, f, \alpha)$ exists and is equal to $\int_a^b f(x)\, d\alpha(x)$.

(b) Examine the convergence of $\int_0^2 \frac{1}{2x - x^2}\, dx$.

(10 + 6 = 16 marks)

III. (A) (a) Define sequence. Show that a sequence cannot converge to more than one limit.

(b) Prove that a series of positive terms either converges or diverges but never oscillates.

(8 + 8 = 16 marks)

Or

(B) (a) State and prove Cauchy's general principle of convergence for series.

(b) Justify the following statement citing two illustrations : A bounded sequence need not be convergent.

(8 + 8 = 16 marks)

IV. (A) (a) What are partial derivatives of a function of two variables ? State partial derivatives of higher order.

(b) Show that $f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ is continuous at the origin.

(8 + 8 = 16 marks)

Or

(B) (a) Define limit, continuity and differentiability of a function $f(x, y)$ at (x_0, y_0).

(b) Define extreme values of a function. Show that $f(x, y) = x^4 + x^2 y + y^2$ has a minimum at (0, 0).

(8 + 8 = 16 marks)

V. (A) (a) Define characteristic polynomial and minimal polynomial. State any *two* properties connecting both the polynomials.

(b) If $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$, verify whether it is diagonalizable. If yes, find the matrix P such that $P^{-1}AP$ is diagonal.

(8 + 8 = 16 marks)

Or

(B) (a) Show that rank of the generalized inverse of a matrix A equals the rank of A.

(b) Find the characteristic polynomial of $A = \begin{bmatrix} 3 & -1 & 1 \\ 7 & -5 & 1 \\ 6 & -6 & 2 \end{bmatrix}$. Find the algebraic and geometric multiplicities of one of the eigenvalues of the matrix.

(8 + 8 = 16 marks)

[4 × 16 = 64 marks]

D 93620 **(Pages : 3)** **Name.....................................**

Reg. No..................................

FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

Statistics

STA 1C 04—DISTRIBUTION THEORY

(2010 Admission onwards)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 marks.

I. 1 If X_1 and X_2 are independent geometric random variables, show that min (X_1, X_2) is geometric.

2. Derive the probability generating function of multinomial distribution and hence obtain the mean and variance.

3. Derive the mean and variance of hypergeometric random variable.

4. If X_1, X_2 are independent exponential random variables with mean 1, find the distribution of $X_1 - X_2$.

5. Define Pareto distribution. Find its mean and variance.

6. If the joint distribution of X and Y is given by

$$F(x, y) = 1 - e^{-x} - e^{-y} + e^{-(x+y)}; x > 0, y > 0,$$

find the marginal and conditional densities. Are X and Y independent ?

7. Derive the formula for the joint distribution of two order statistics.

8. Distinguish between central and non-central Chi-square distributions.

(4 × 4 = 16 marks)

Turn over

Section B

Answer **either** *Part A* **or** *Part B of* **all** *questions.*
Each question carries 16 *marks.*

II. A (i) Show that binomial distribution as a special limiting distribution of hypergeometric distribution. Define negative binomial distribution. Why is it so called ?

(ii) Define geometric distribution and derive its mean and variance. State and prove the lack of memory property of geometric distribution.

Or

B (i) If X and Y are independent Poisson variates, show that the conditional distribution of X given X + Y is binomial.

(ii) Prove that Poisson distribution is a limiting case of the negative binomial distribution.

III. A (i) If X and Y are independent standard norma! variates, derive the distribution of $Z = X/Y$.

(ii) If X and Y are independent exponential variates with common mean 1, derive the distribution of $X/X + Y$.

Or

B (i) Obtain the characteristic function of standard Laplace distribution.

(ii) If X and Y are independent $U(0,1)$ random variables find the p.d.f. of

(1) $U = X + Y$; (2) $V = X - Y$; and (3) $W = |X - Y|$.

IV. A (i) If $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ are order statistics of a random sample from

$f(x) = \theta e^{-\theta x}$; $0 < x < \infty$

find (i) $E\left(X_{(r:n)}\right)$; and (ii) $V\left(X_{(r:n)}\right)$.

(ii) Define bivariate normal distribution. If (X, Y) are bivariate normally distributed then derive the conditional distribution of Y given $X = x$.

Or

B (i) Find the distribution of sample range based on random sample from U (0, 1).

(ii) Let if $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ be order statistics from a population with absolutely continuous distribution function F. Derive the p.d.f. of $X_{(r)}$ and joint p.d.f. of $X_{(r)}$ and $X_{(s)}$.

V. A (i) If $X \sim N(\mu, 1)$ show that $Y = X^2$ has non-central Chi-square distribution. Also show that the square of a non-central *t*-statistic is a non-central F statistic.

(ii) If $F_n(x)$ denotes the empirical distribution function based on a sample of size n from a continuous population, derive the distribution of $Y = n F_n(x)$ and $\text{Var}(Y)$.

Or

B (i) Given a sample from $f(x) = 1/\theta, 0 < x < \theta$; derive the distribution of the sample range.

(ii) Define bivariate normal distribution and show that its marginal distributions are univariate normal distributions.

(4 × 16 = 64 marks)

D 93617 (Pages : 3) Name.......................................

Reg. No..................................

FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2020

(CCSS)

Statistics

STA IC 01—MATHEMATICAL METHODS FOR STATISTICS—I

(2010 Admission onwards)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (a) State first and second mean value theorems for Riemann-Stieltjes integral.

(b) Define functions of bounded variation on interval A. Give one example.

(c) Prove that the sequence $\{f_n\}$, where $f_n(x) = \frac{x}{1+nx^2}$ converges uniformly on any closed interval I.

(d) Define uniform convergence of sequence of functions. State Cauchy condition for uniform convergence of a sequence of functions.

(e) Show that the function $f(x, y) = \begin{cases} \frac{x^2 y}{x^4 + y^2}, & x^2 + y^2 \neq 0 \\ 0, & x = y = 0 \end{cases}$.

Posses first order partial derivative everywhere, including the origin, but the function is discontinuous at the origin.

(f) Prove that $(y - x - b)^4 + (x - a)^4$ has minima at $(a, a + b)$.

(g) Define Hermitian matrix. What can you say about the Eigen values of a Hermitian matrix ?

(h) What do you mean by positive definite matrix ? Give one example.

(4 × 4 = 16 marks)

Turn over

Section B

Answer the question from **either** *Section A* **or** *B of* **all** *questions.*
Each question carries 16 *marks.*

II. A (a) If $f \in R(\alpha)$ on $[a, b]$ and α has a continuous derivative α' on $[a, b]$, show that $\int_a^b f(x)\alpha'(x)\,dx$ exists and $\int_a^b f(x)\,d\alpha(x) = \int_a^b f(x)\alpha'(x)\,dx$.

(b) Show that the integral $\int_0^{\frac{\pi}{2}} \log \sin x \, dx$ is convergent and hence evaluate it.

(8 + 8 = 16 marks)

Or

B (a) Assume that α is a function of bounded variation on $[a, b]$. Let $V(x)$ denote the total variation of α on $[a, x]$ if $a < x \le b$ and $V(a) = 0$. Let f be bounded on $[a, b]$. If $f \in R(\alpha)$ on $[a, b]$, then show that $f \in R(V)$ on $[a, b]$.

(b) State and prove Euler's summation formula.

(8 + 8 = 16 marks)

III. A (a) Define uniform convergence of a series of functions. Discuss the uniform convergence of $\sum_{n=1}^{\infty} \frac{1}{n} \sin nx$.

(b) State and prove a set of sufficient conditions for uniform convergence of a series.

(6 + 10 = 16 marks)

Or

B (a) If $\{f_n(x)\}$ is a sequence of continuous functions on $E \subset R$ and if f_n converges to f on E, prove that f is continuous on E.

(b) State and prove Weierstrass M-test.

(9 + 7 = 16 marks)

IV. A (a) Explain how do you definite limit and continuity of multivariate functions. Illustrate it through an examples.

(b) Show that the length of the axes of the section of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ by the plane $lx+my+nz=0$ are the roots of the quadratic in r^2, $\frac{l^2 a^2}{r^2-a^2}+\frac{m^2 b^2}{r^2-b^2}+\frac{n^2 c^2}{r^2-c^2}=0$.

(6 + 10 = 16 marks)

Or

B (a) Investigate the maxima and minima of the function $21x-12x^2-2y^2+x^3+xy^2$.

(b) Establish a necessary condition for the existence of an extrema of a bivariate function. What are the sufficient conditions for the same ?

(8 + 8 = 16 marks)

V. A (a) Show that $H=\begin{bmatrix} A & B \\ 0 & C \end{bmatrix}$ is idempotent if and only if A and C are idempotent.

(b) Show that $m \times n$ matrix of real numbers has a generalized inverse.

(8 + 8 = 16 marks)

B (a) State and prove a necessary and sufficient condition on A so that the quadratic form $X'AX$ is positive definite.

(b) What is meant by spectral decomposition of a matrix ? Give an application of such decomposition.

(10 + 6 = 16 marks)

D 93447 (Pages : 3) Name.....................................

Reg. No.................................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2020

(CBCSS)

Statistics

MST 1C 04—PROBABILITY THEORY

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Each question carries 2 weightage.

1. Prove that the intersection of arbitrary number of σ-fields is also a σ-field.
2. Show that distribution function can have atmost of countable number of discontinuity points.
3. Define a characteristic function. Will it always exists ? Justify your answer.
4. Let $X_n \sim \chi^2_{(n)}, n = 1, 2, \ldots\ldots$. Find the limiting distribution of $\frac{X_n}{n^2}$.
5. State Kolmogorov three series theorem.
6. Let $X_1, X_2, \ldots.$ be a sequence of independent random variables with p.m.f.

   $P[X_k = 1] = \frac{1 - 2^{-k}}{2} = P[X_k = -1]$ and $P\left[X_k = 2^k\right] = 2^{-(k+1)} = P\left[X_k = -2^k\right]$. Examine whether SLLN holds.
7. State Lindeberg-Feller Central limit theorem.

(4 × 2 = 8 weightage)

Turn over

Part B

Answer any **four** *questions.*
Each question carries 3 *weightage.*

8. Define limit of a sequence of sets. Examine whether the following sequence of sets is convergent.

$$A_{2n} = \left(0, \frac{1}{2n}\right), A_{2n+1} = \left[-1, \frac{1}{2n+1}\right].$$

9. State and prove Basic inequality.

10. Characteristic function is real iff the distribution function is symmetric about zero.

11. Prove that $X_n \xrightarrow{a.s.} X \Rightarrow X_n \xrightarrow{P} X$. Check whether the converse is true.

12. If $\{F_n(x)\}$ is a sequence of distribution functions, then there exist a subsequence of $\{F_n(x)\}$ which converges weakly.

13. State and prove Khintchine's WLLN.

14. Let $x_1, x_2, \ldots\ldots, x_n$ be i.i.d. N (0,1) RVs. Define

$$U_n = \left(\frac{x_1}{x_2} + \frac{x_3}{x_4} + \ldots + \frac{x_{2n-1}}{x_{2n}}\right)$$

$$V_n = x_1^2 + x_2^2 + \ldots + x_n^2 \text{ and } Z_n = \frac{U_n}{V_n}.$$

Find limiting distribution of Z_n.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

15. Define convergence in probability. Prove that if $X_n \xrightarrow{P} X$, then there exist a sub-sequence $\{X_{nk}\}$ of $\{X_n\}$ which converges a.s to X.

[illegible]

D 93446 (Pages : 2) Name.......................................

Reg. No..................................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2020

(CBCSS)

Statistics

MST 1C 03—DISTRIBUTION THEORY

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Each question carries 2 weightage.

1. For the p.m.f. $P[X=j]=\frac{a_j\theta^j}{f(\theta)}$, $j=0,1,2,3,\ldots,\theta>0$, where $a_j\geq 0$ and $f(\theta)=\sum_{j=0}^{\infty}a_j\theta^j$. Find the pgf of X.
2. Let X and Y be independent random variables with pmf's $b(m, p)$ and $b(n, p)$ respectively. Show that X given X + Y is hypergeometric.
3. Let X and Y be iid N(0,1) random variables. Show that X + Y and X – Y are independent.
4. If $X_1, X_2, \ldots, X_n$ are iid random variables following the Weibull distribution. Find the distribution of $X_{(1)} = \text{Min}\,(X_1, X_2, \ldots, X_n)$.
5. If X is lognormal obtain the distribution of $\frac{1}{X}$.
6. Define exponential family of distributions. Identify two distributions belongs to this family.
7. Define noncentral Chi-square distribution. When will this reduce to central Chi-square ?

(4 × 2 = 8 weightage)

Turn over

Part B

Answer any **four** *questions.*
Each question carries 3 *weightage.*

8. Define negative binomial distribution, Establish the reproductive property of it.

9. Obtain the poisson distribution as a limiting case of the negative binomial distribution.

10. Let X_1, X_2, X_3 be iid random variables with common exponential distribution $f(x) = e^{-x}, x > 0$. Let

    $Y_1 = X_1 + X_2 + X_3$, $Y_2 = \dfrac{X_1 + X_2}{X_1 + X_2 + X_3}$ and $Y_3 = \dfrac{X_1}{X_1 + X_2}$. Show that Y_1, Y_2 and Y_3 are independent.

11. If X and Y are independent Gamma variates with parameters α and β. Find the distribution of

    $U = X + Y$ and $V = \dfrac{X}{X + Y}$.

12. Define Cauchy distribution and obtain its Characteristic function.

13. Define Pareto distribution. If Y is a random variable following Pareto distribution, find the distribution of $X = \log Y$.

14. Derive the distribution of the range of a random sample of size n from U[0, 1] distribution.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

15. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from Normal distribution with mean μ and variance σ^2. Show that sample mean and sample variance are independently distributed.

16. Define non-central t distribution. Derive the probability density function of it.

17. Let X_1, X_2 be iid U[0, 1] random variables. Let $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$, find the p.d.f's of Y_1 and Y_2.

18. Define order statistics. Find the p.d.f's of $X_{(1)}$ and $X_{(n)}$ of a random sample of size n from standard logistic distribution.

(2 × 5 = 10 weightage)

D 93445 (Pages : 2) Name..................................

Reg. No..............................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2020

(CBCSS)

Statistics

MST 1C 02—ANALYTICAL TOOLS FOR STATISTICS—II

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer any **four** *questions*
Weightage 2.

1. Explain linear dependency and independency of vectors over a field.
2. Define basis and dimension of a vector space with suitable example.
3. Prove that the inverse of amatrix exist iff it is non singular
4. Define Hermition and skew Hermition matrices. Give examples.
5. Define algebraic and geometric multiplicities.
6. State rank -nullity theorem.
7. Examine the definiteness of the quadratic form $2x^2 + 3y^2 + 4xy$.

(4 × 2 = 8 weightage)

Part B

Answer any **four** *questions*
Weightage 3.

8. Explain linear dependency and independency of vectors over a field. Examine whether the following vectors are linearly independent. {(1, 1, 1), (1, 3, 2), (2, 1, 1)}
9. Define sub space. Show that, the intersection of any number of subspaces of a vector space V is also a subspace of V.
10. What do you mean by algebraic and geometric multiplicity of an eigen value . Establish how they are related
11. Show that a set of orthogonal vectors are linearly independent.
12. Prove that the characteristics roots of a Hermition matrix are real.

Turn over

13. Define Moore-Penrose the g-inverse. Show that it is unique.

14. Define g- inverse. Compute the g-inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 1 \\ 0 & 2 & 0 & 5 \end{bmatrix}$.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Weightage-5.

15. (i) If t W_1 and W_2 be subspaces of a vector space V, then prove that $W_1 + W_2$ is a subspace of V. And $W_1 + W_2$ is the smallest subspace of V containing W_1 and W_2.

(ii) Let W be the sub space of R^4 spanned by vectors

$\{(1, -2, 5, -3), (2, 3, -1, 4), (3, 8, -3, -5)\}$ Find the basis and dimension of W.

16. Describe the spectral decomposition of a real symmetric matrix.

17. Solve the system of equations using Gauss elimination method.

$2x - y + z = 5$

$7x + 2y - 5z = 20$

$x + y + z = 6$

18. (i) State and prove Cayley-Hamilton theorem.

(ii) Explain Gram -Schmidt orthogonalization process.

(2 × 5 = 10 weightage)

D 93444 (Pages : 2) Name.......................................

Reg. No..................................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2020

(CBCSS)

Statistics

MST 1C 01—ANALYTICAL TOOLS FOR STATISTICS—I

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer any **four** *questions.*
Weightage 2 *for each question.*

1. Define directional derivative.

2. Define the limit of a multivariate function.

3. Define an analytic function. Give an example.

4. Distinguish between essential and isolated singularity.

5. Define Laplace transform of a function.

6. State Poisson integral formula.

7. If $L\{F(t)\} = f(s)$, show that $L\left\{\int_0^1 F(u)\,du\right\} = \frac{1}{s} f(s)$.

(4 × 2 = 8 weightage)

Part B

Answer any **four** *questions.*
Weightage 3 *for each question.*

8. Examine whether the limit of the function $f(x, y) = \frac{x^3 y^3}{x^2 + y^2}$ exist at $(0, 0)$.

9. Examine the function $21x - 12x^2 - 2y^2 + x^3 + xy^2$ for maximum and minimum.

Turn over

10. Show that every analytic function satisfies Cauchy-Riemann equations.

11. State and prove Jordan's lemma.

12. Show that if $f(z)$ is an entire function, which is bounded for all values of z then it is a constant.

13. Describe different forms of Fourier integral formula.

14. Evaluate $\int_0^\infty \frac{x^2}{x^4+5x^2+6}\,dx$.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Weightage 5 *for each question.*

15. (i) Explain the Lagrange multiplier method. Maximize $36 - x^2 - y^2$ subject to $x + 7y = 25$.

    (ii) Explain Riemann integral of a multivariable function.

16. State and prove Laurent's lemma.

17. (i) State and prove the necessary and sufficient condition for a function to be analytic.

    (ii) Find an analytic function whose real part is $e^x \cos y$.

18. State and prove Cauchy residue theorem.

(2 × 5 = 10 weightage)

D 93264 **(Pages : 4)** **Name.......................................**

Reg. No...................................

FIRST SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION NOVEMBER 2020

(CUCSS)

Statistics

ST IC 05—DISTRIBUTION THEORY

(2013 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries a weightage 1.

1. Define moment generating function. Give an example of a random variable whose moment generating function does not exist.
2. Obtain the probability generating function of geometric distribution with success probability p.
3. Specify the conditions under which binomial distribution tends to Poisson distribution.
4. Let X and Y be independent random variables following standard normal distribution. Identify the distribution of $\frac{X}{Y}$.
5. Obtain the variance of standard Weibul distribution.
6. Let X be a random variable with distribution function $F_X(x)$. Show that $F_X(x)$ follows uniform distribution in the interval $[0, 1]$.
7. Define location-scale family with an example.
8. Define marginal and conditional distributions in bivariate case.
9. If $X_i, i = 1, 2, ..., n$ be n random variables follows a distribution with distribution function $F_X(x)$. Obtain the distribution function of $X_{(r)}$, where $X_{(r)}$ is the r^{th} order statistic.

Turn over

10. Define non-central χ^2- distribution with non-centrality parameter α.

11. Explain any *two* uses of Students t-distribution.

12. If X_1 and X_2 are independent Chi-square variate with degrees of freedom n_1 and n_2, then write the distribution of $\frac{X_1}{X_2}$.

(12 × 1 = 12 weightage)

Part B

Answer any **eight** *questions.*
Each question carries a weightage 2.

13. The mean and variance of a binomial random variable X with parameters n and p is 16 and 8. Find i) $P(X=0)$; ii) $P(X=1)$; and iii) $P(X \geq 2)$.

14. If X_1 and X_2 be two independent random variables having geometric distribution, then show that the conditional distribution of $(X_1/(X_1+X_2)=n)$ is uniform.

15. Obtain the mean of power series distribution.

16. Derive the characteristic function of standard cauchy distribution.

17. If X has a uniform distribution in the interval $[0.1]$, then find the distribution of $-2\log X$.

18. Prove or disprove Normal distribution belongs to Pearson family of distributions.

19. Explain mixture distribution. Derive Pareto distribution as a mixture of gamma distribution and exponential distribution.

20. If X and Y are independent. Show that they are un correlated. Is the converse always true ? Justify your answer.

21. If $X_1, X_2, X_3, \ldots, X_n$ are independent random variables, X_i $i=1,2,3,\ldots,n$ having an exponential distribution with parameters $\lambda_i, i=1,2,3,\ldots,n$, then obtain the distribution of $X_{(1)}$, where $X_{(1)} = \text{Min}(X_1, X_2, X_3, \ldots, X_n)$.

22. If X is a Chi-square variate with n degrees of freedom, then prove that for large n the random variable $\sqrt{2X}$ follows normal distribution with mean $\sqrt{2n}$ and variance 1.

23. Obtain the variance of a students-t distribution with degrees of freedom n.

24. If $X_1, X_2, X_3, \ldots, X_m\ X_{m+1}, X_{m+2}, \ldots, X_{m+n}$ are independent normal variate with mean zero and variance σ^2. Obtain the distribution of $\dfrac{\sum_{i=1}^{m} X_i^2}{\sum_{i=m+n}^{m+n} X_i^2}$.

(8 × 2 = 16 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of 4.

25. If $X_1, X_2, X_3, \ldots, X_k$ are k independent Poisson variates with parameters $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_k$ respectively. Obtain the conditional distribution of $(X_1 \cap X_2 \cap \ldots\ldots \cap X_k / X)$ where $X = (X_1 + X_2 + \ldots\ldots X_k)$ is fixed.

26. i) Define an exponetial distribution with parameter λ.

ii) If X has an exponential distribution with parameter λ, then for every constant $a \geq 0$, show that $P\{Y \leq x \mid X \geq a\} = P\{X \leq x\}$ for all x, where $Y = X - a$.

27. Let the joint density of (X, Y) is

$$f(x, y) = \frac{e^{-(x+y)}\, x^3\, y^4}{\Gamma 4\, \Gamma 5} \quad x > 0, y > 0$$
$$= 0 \quad \text{otherwise.}$$

(i) Obtain the pdf of $U = \dfrac{X}{X + Y}$.

(ii) Find the expectation and variance of the random variable U.

28. Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample from a normal population with mean μ and variance σ^2. Then show that the sample mean $\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$ and sample variance $S^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n}$ are independently distributed.

(2 × 4 = 8 weightage)

D 93263 (Pages : 3) Name.......................................

Reg. No..................................

FIRST SEMESTER M.Sc. DEGREE [SUPPLEMENTARY] EXAMINATION NOVEMBER 2020

(CUCSS)

Statistics

ST 1C 04—REGRESSION AND LINEAR PROGRAMMING

(2013 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. When do you say that a linear parametric function is estimable ?
2. What are the assumptions of a simple linear regression models ?
3. Define orthogonal polynomials.
4. Define a generalized linear model.
5. Obtain the link function associated to a Poisson regression model.
6. Give the prediction problem on generalized linear model.
7. Define basic feasible solution of an LPP.
8. Define artificial variables.
9. What do you mean by degeneracy in Linear programming ?
10. State the fundamental theorem of duality.
11. Establish the difference between assignment problem and transportation problem.
12. Define the terms pure strategy and mixed strategy.

($12 \times 1 = 12$ weightage)

Part B

Answer any **eight** *questions.*
Each question carries 2 *weightage.*

13. Obtain the unbiased estimator of O^2 in a simple linear regression model.
14. Explain the methods for scaling residuals.
15. Describe the inference on polynomial regression models.

Turn over

16. Obtain the inference on logistics regression model.

17. Discuss the parameter estimation in GLM.

18. Explain how residual analysis performed in GLM.

19. Obtain all basic solution to the following system of linear equations :

$$x + 2y + z = 4$$
$$2x + y + 5z = 5.$$

20. Use simplex method to solve the following LPP :

Maximize $Z = 2x_1 - x_2 + x_3$

subject to $3x_1 + x_2 + x_3 \le 60$
$x_1 - x_2 + 2x_3 \le 10$
$x_1 + x_2 - x_3 \le 20$
$x_1, x_2, x_3 \ge 0$

21. Use Big- M method to solve the following LPP :

Maximize $Z = 6x_1 + 4x_2$

subject to $2x_1 + 3x_2 \le 30$
$3x_1 + 2x_2 \le 24$
$x_1 + x_2 \ge 3$
$x_1, x_2 \ge 0.$

22. Obtain the dual of the following primal problem :

Minimize $Z = x - 3x_2 - 2x_3$

subject to $3x_1 - x_2 + 2x_3 \le 7$
$2x_1 - 4x_2 \ge 12$
$-4x_1 + 3x_2 + 8x_3 = 10$
$x_1, x_2 \ge 0.$
$x_1, x_2 \ge 0$ and x_3 unrestricted in sign.

23. Describe travelling salesman problem :

24. Explain the method of finding optimal solution of a two -person zero - sum game.

(8 × 2 = 16 weightage)

Part C

Answer any **two** *questions.*
Each question carries 4 weightage.

25. State and prove Gauss Markove theorem.

26. Describe the procedure for estimating the parameters in a Poisson regression model.

27. Explain dual simplex algorithm.

28. Use Vogel's approximation method to obtain the initial feasible solution of the following transportation problem :

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

(4 × 2 = 8 weightage)

D 93262 **(Pages : 3)** **Name.......................................**

Reg. No...................................

FIRST SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION NOVEMBER 2020

(CUCSS)

Statistics

ST IC 03—ANALYTICAL TOOLS FOR STATISTICS—II

(2013 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *the questions.*
Weightage 1 *for each question.*

1. Give an example for a vector space.
2. Define basis of a vector space.
3. Define an inner product space.
4. Define idempotent matrix and give an example.
5. What do you meant by unitary matrix ?
6. Define rank of a matrix.
7. Prove that for a Hermitian matrix, the eigen values are all real.
8. What do you meant by minimal polynomial ? Find the minimal polynomial of an idempotent matrix.
9. Describe singular value decomposition of a matrix
10. What do you meant by reflexive g-inverse of a matrix ?
11. Define a quadratic form. How will you classify it ?
12. Define null space and nullity of a matrix.

($12 \times 1 = 12$ weightage)

Turn over

Part B

Answer any **eight** *questions.*
Weightage 2 for each question.

13. Suppose that the vectors x, y, z are linearly independent. Check the independence of the vectors $x+y, x-y$ and $x-2y+z$.

14. Show that sum of two subspaces is again a subspace.

15. If u and v are any *two* vectors in an inner product space, prove that $\| u+v \| \leq \| u \| + \| v \|$.

16. Let A be an $m \times n$ matrix and B be an $n \times p$ matrix. Show that $\rho(AB) \geq \rho(A) + \rho(B) - n$, where $\rho(A)$ denote the rank of A.

17. For a non singular matrix A, show that $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |A| \left| D - BA^{-1} C \right|$.

18. If A is a square matrix of order n having eigen values $\lambda_1, \lambda_2, \ldots, \lambda_n$. Show that (i) trace $A = \sum_{i=1}^{n} \lambda_i$; and (ii) $|A| = \prod_{i=1}^{n} \lambda_i$.

19. Distinguish between algebraic multiplicity and geometric multiplicity. Establish the relation between them.

20. Find the eigen values of an idempotent matrix

21. Explain spectral decomposition of a real symmetric matrix

22. Derive a necessary and sufficient condition for the linear system $Ax = b$ to be consistent.

23. Show that a generalized inverse always exists and is not unique.

24. Examine the nature of the quadratic form $x^2 - y^2 + z^2 + 6xy - 2yz$.

(8 × 2 = 16 weightage)

Part C

Answer any **two** *questions.*
Weightage 4 for each question.

25. If A and B are two subspaces of a finite dimensional vector space V, then show that $\dim(A+B)=\dim(A)+\dim(B)-\dim(A\cap B)$.

26. State and prove (i) Rank nullity theorem ; and (ii) Fundamental theorem on ranks.

27. State and prove Cayley-Hamilton theorem.

28. State and prove a necessary and sufficient condition for a quadratic form to be positive definite.

(2 × 4 = 8 weightage)

D 93261 **(Pages : 3)** **Name..**

Reg. No...

FIRST SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION NOVEMBER 2020

(CUCSS)

Statistics

ST 1C 02—ANALYTICAL TOOLS FOR STATISTICS–I

(2013 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *the questions.*
Weightage 1 *for each question.*

1. State the necessary condition for $f(x,y)$ to have an extreme value at $x = a, y = b$.
2. State existence theorem in case of function of two variables.
3. What is meant by line integral ?
4. State Poisson integral formula.
5. What are the sufficient conditions for a complex valued function $f(z)$ to be analytic ?
6. Give an example of a function has a pole of order 3 at $z = 1$ and having residue 10.
7. State fundamental theorem of algebra.
8. Compute the inverse Laplace transform of the function $\frac{1}{(s-1)^3}$.
9. Define Fourier infinite transform.
10. State the conditions of a function which can be represented as a Fourire series.
11. Examine the singularity of the function $f(z) = e^{\frac{1}{z}}$ at $z = 0$.
12. Define Riemann integrable functions over R^2.

(12 × 1 = 12 weightage)

Part B

Answer any **eight** *questions.*
Weightage 2 for each question.

13. Evaluate the integral $\iint_R (x^2 + 2y)\,dxdy$, where R = [0,1 ; 0, 2].

14. Find the maxima and minima of the function, $f(x,y) = x^3 + y^3 - 3x - 12y + 20$.

15. Explain Lagrange's method of multiplier to find stationary points of the function $f(x_1, x_2, ..., x_n)$.

16. State and prove Morera's theorem.

17. Evaluate $\frac{1}{2\pi i}\oint_C \frac{z^2}{z^2+4}\,dz$, where C is the square with vertices at $\pm 2, \pm 2 + 4i$.

18. Find the inverse Laplace transforms of : (i) $\frac{6s-4}{s^2-4s+20}$ (ii) $\frac{1}{(s-1)^4}$.

19. Find the Fourier series of the function sin $4x$ in the interval $[0, \pi]$.

20. Show that $f(z) = |z|^4$ is differentiable but not analytic at $z = 0$.

21. Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series valid for (a) $1 < |z| < 3$ (b) $0 < |z+1| < 3$.

22. State the Cauchy's residue theorem and determine the residues of $f(z) = \frac{z^2+4}{z^3+2z^2+2z}$ at the poles.

23. Find the Fourier series of the function $f(x) = x + x^2$ in the interval $[-1, 1]$.

24. State and prove Fourier integral theorem.

(8 × 2 = 16 weightage)

Part C

Answer any **two** *questions.*
Weightage 4 for each question.

25. Find the Fourier series of the function $f(x) = |x|$ for $-\pi \le x \le \pi$ and also show that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}.$$

26. State Taylor's expansion of $f(x,y)$ about $x = a$, $y = b$ and hence obtain Taylor's expansion of $x^2y + 3y - 2$ in powers of $(x-1)$ and $(y+2)$.

27. Solve the following differential equation by the Laplace transform method :

$x'' + y' + 3x = 15e^{-t}, y'' - 4x' + 3y = 15\sin 2t$ subject to

$x(0) = 35, x(0) = -48, y(0) = 27, y(0) = -55.$

28. Show that $\int_{0}^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}.$

(2 × 4 = 8 weightage)

D 93260 (Pages : 2) Name.......................................

Reg. No..................................

FIRST SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION NOVEMBER 2020

(CUCSS)

Statistics

ST 1C 01—MEASURE THEORY AND INTEGRATION

(2013 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *the questions.*
Weightage 1 *for each question.*

1. Define Reimann-Stieiltjes integral.
2. State mean value theorem.
3. Find the Lebesgue measure of the set of irrationals between 0 and 1.
4. Define the integral of a simple function.
5. State Radon-Nikodym theorem.
6. What do you understand by a sigma field ?
7. Define measure space.
8. State Holder inequality.
9. Give an example of an integral which depends on a parameter.
10. What do you mean by a normed linear space ?
11. State monotone class lemma.
12. State Caratheodory extension theorem.

(12 × 1 = 12 weightage)

Part B

Answer any **eight** *questions.*
Weightage 2 *for each question.*

13. Show that the sum, product and difference of two measurable functions are measurable.
14. Show that the following statements are equivalent :
    (i) f is a measurable function.
    (ii) $\forall \alpha, \{x : f(x) \leq \alpha\}$ is measurable.
    (iii) $\forall \alpha, \{x : f(x) > \alpha\}$ is measurable.

Turn over

15. Show that arbitrary intersection of sigma fields is a sigma field.
16. State Lebesgue dominated convergence theorem.
17. State and prove Minkowski's inequality.
18. State Fubini's theorem and point out its applications in Statistics.
19. Define Lebesgue-Stieltjes measure and show that Lebesgue measure is its particular case.
20. State Jordan decomposition theorem and explain its importance.
21. State and prove fundamental theorem of integral calculus.
22. Let R (α) denote the class of all R-S integrable functions on $[a, b]$. If $f, g \in R(\alpha)$ then show that

    $f+g \in R(\alpha)$ and $\int_a^b (f+g)\, d\alpha = \int_a^b f\, d\alpha + \int_a^b g\, d\alpha$.
23. State and prove a sufficient condition for the existence of Riemann-Stieltjes integral.
24. Establish the continuity property of measure.

(8 × 2 = 16 weightage)

Part C

Answer any **two** *questions.*
Weightage 4 *for each question.*

25. State and prove Weistrass theorem.
26. State and prove monotone convergence theorem.
27. Write short note on different modes of convergence. Illustrate each with an example.
28. State and prove Lebesgue decomposition theorem.

(2 × 4 = 8 weightage)