# ENHANCING MATHEMATICS LEARNING THROUGH EVIDENCE BASED INSTRUCTION FOCUSING ON LANGUAGE OF MATHEMATICS IN ELEMENTARY SCHOOLS OF KERALA 

By
SARABI M.K.

Supervised By

## Dr. ABDUL GAFOOR K.

## DEPARTMENT OF EDUCATION UNIVERSITY OF CALICUT <br> 2019

## Dr. Abdul Gafoor K.

Professor
Department of Education
University of Calicut
Calicut university PO
Pin- 673635
gfr.abdul@yahoo.co.in

## Certificate

This is to certify that the thesis entitled "ENHANCING MATHEMATICS LEARNING THROUGH EVIDENCE BASED INSTRUCTION FOCUSING ON LANGUAGE OF MATHEMATICS IN ELEMENTARY SCHOOLS OF KERALA" is an authentic record of research work carried out by SARABI M.K., for the degree of doctor of philosophy in education, University of Calicut, under my supervision and guidance and that no part thereof has been presented before for any other degree, diploma or associateship in any other university.

Place: Calicut University
Date:

Prof. (Dr.) Abdul Gafoor K.
(Supervising teacher)

Dr. Abdul Gafoor K.<br>Professor<br>Department of Education<br>University of Calicut<br>Calicut university PO<br>Pin- 673635<br>gfr.abdul@yahoo.co.in

## Certificate

This is to certify that the thesis entitled "ENHANCING MATHEMATICS
LEARNING THROUGH EVIDENCE BASED INSTRUCTION FOCUSING ON LANGUAGE OF MATHEMATICS IN ELEMENTARY SCHOOLS OF

KERALA" is an authentic record of research work carried out by SARABI M.K., for the degree of doctor of philosophy in education, University of Calicut, under my supervision and guidance and that no part thereof has been presented before for any other degree, diploma or associateship in any other university.

The thesis is not revised as there was no suggestion from the adjudicators for the same. It is further certified that the soft copy of the thesis submitted is the same as the printed copy submitted herewith.

Place: Calicut University
Date: $17-07-2020$


Prof. (Dr.) Abdul Gafoor K.
(Supervising teacher)
Dr. K. ABDUL GAFOOR
Professor
Department of Education
University of Calicut
Calicut University P.O-673635

## DECLARATION

I, SARABI M.K., do hereby declare that this thesis entitled as "ENHANCING MATHEMATICS LEARNING THROUGH EVIDENCE BASED INSTRUCTION FOCUSING ON LANGUAGE OF MATHEMATICS IN ELEMENTARY SCHOOLS OF KERALA" is a genuine record of the research work done by me under the supervision of Dr. Abdul Gafoor K. Professor, Department of Education, University of Calicut, and that no part of this thesis has been presented earlier for the award of any degree, diploma or associateship in any university.

Place: Calicut University
SARABI M. K. Date:

## Acknowledgement

This thesis has been brought to its final form with the support and encouragement from several individuals and institutions. I would like to express my sincere gratitude to all of them who contributed in many ways to the successful completion of my Ph.D. work.

First and foremost, I would like to express my heartfelt gratefulness and respect to my supervising teacher $\operatorname{Dr}$. Abdul Gafoor K, Professor, Department of Education, University of Calicut, for his immense knowledge, patience and persistent motivation throughout the study. His intense dedication towards research and critical analysis have afways motivated me in research work. Sir has always made himself available to clarify my doubts despite his hectic schedule. Sir was atways around at times I thought that it is impossible to continue and that made the completion of thesis possible. It was a wonderful opportunity to do my doctoral research under his masterfy guidance and expertise. Words cannot describe the obligation that I have towards him. I could not imagine having a better supervisor for my Ph.D. study. My heart and mind are overflowing with respect and gratitude to Gafoor sir.

I would like to express my gratefulness to all teachers who taught me so far. I feel honored to share my heartfelt thanks to all my teachers of the Department of Education Prof. (Dr.) C. Naseema, Prof. (Dr.) P. Vsfa, Prof. (Dr.) P. K. Aruna, Prof. (Dr.) K. P. Meera, Prof. (Dr.) C. M. Bindhu, Dr. Baiju K. Nath, Dr. T. Vasumathi, and Dr. A. Hameed for providing contusive and supportive environment for research. I would like to express my heartfelt gratitude to all the Heads of the Department of Education during my research period, for supporting me. I would also like thank Research Admission Committee for giving valuable suggestions.

I extend my sincere gratitude to Prof. (Dr.) K.P. Suresh, Department of Education, Central University of Kerala for his valuable suggestions during my PQE viva. Also, I express my sincere gratitude to Prof. (Dr.) Anil Kumar K., Regional Institute of Education, Mysuru for his vafuable suggestions during my SRF viva. I extend my honest gratitude to the department librarian and office staff of department of education. I would like to express my gratitude to the heads and teachers of schools, where data collection was done, for their cooperation during my data collection. I am thankfulto all my dear students who participated in the study wholeheartedly.

From the 6ottom of my heart, I thank (Dr. Mumthas $\mathcal{N}$.S., for valuable suggestions, and I express my love and care to $\mathcal{N u w e e n}$ Gafoor and Ishaan Gafoor, for hosting me and sharing their valuable time for me.

I would like to express my heartfelt gratitude to Prof. (Dr.) K. Manikandan, Department of psychology, University of Calicut for the positive words and kindness shown towards me.

I express my love and gratitude to Abidha for helping me in data entry and proof reading of the whole thesis. I am thankful to Minichechi for translating my research tools. I express my gratitude to Sihab, $\mathcal{N a s e e 6 k a , ~ \mathcal { N a m s h e e d , ~ S u m a ~ a n d ~ S a n a m ~ f o r ~ t h e i r ~ }}$ help at the time of submission of the thesis. I would like to express my gratitude to Remya, Fashida, Asha, Nasim, FAskarka, Suneer, Mahful, Balu and Deepak,for helping me to collect data. I would like to thank all my fellow researchers for their help and support. I would like to share my love to all my friends who have studied $\mathcal{M}$. Phil. and $\mathcal{M} . E d$. courses in the department during the tenure of my research work.

I express my heartfelt gratitude to my parents Katheem $\mathcal{M}$. K. and Safiya T. K., and gratefulfy remember the sacrifices they have made, their endless patience and the freedom given to me. I cannot leave this page without mentioning my younger brother Sabith for the various sought of support he has given in every kind of a tough situation. I would also express thanks to my elder brother Zayed and my in-law for 6eing supportive in my research work. I would like to express my innermost love to Hadi, my nephew, a real 6lessing in my life who puts color in my world with his innocent love.

I thankfully remember former office assistant Sajnechi, my dear friend for the Love and support she has extended. I also thankfully remember former section officer Sudheer sir of department of education for the support he has offered.

I extend my sincere gratitude to my dear friend Ramettan, Infratech, for his techinical support in preparing this report and for the care he has extended.

I express my wholehearted thanks to all who have helped and cooperated with me for the fulfilment of my dream.

SARABI M. K.

## CONTENTS

## LIST OF TABLES

## LIST OF FIGURES

## LIST OF APPENDICES


IV ANALYSIS ..... 239-322
Language related difficulties in mathematics learning ..... 240
Students' perception of difficulties in mathematical tasks ..... 240
Difficulty in mathematics sourcing from nature of mathematics ..... 245
Achievement in components of language of mathematics ..... 249
Summary of language related difficulties in learning mathematics ..... 253
among elementary school students of Kerala
Effects of language integrated mathematics instruction ..... 255
Main effects of language integrated mathematics instruction on mathematics learning outcomes ..... 255
Interaction of language integrated mathematics instruction with ..... 302
control variables on mathematics learning outcomes
Summary of effect of language integrated mathematics instruction on ..... 320 mathematics learning among elementary school students in Kerala
V SUMMARY, FINDINGS, AND SUGGESTIONS ..... 323-384
Restatement of the problem ..... 323
Variables of the study ..... 324
Hypotheses of the study ..... 325
Methodology in brief ..... 327
Major findings of the study ..... 332
Tenability of the hypotheses ..... 352
Discussion of findings ..... 355
Limitations of the study ..... 369
Conclusion ..... 371
Educational implications ..... 374
Suggestions for further research ..... 382
REFERENCES ..... 385-408
APPENDICES

## LIST OF TABLES

| Table <br> No. | Title | Page <br> No. |
| :---: | :---: | :---: |
| 1 | List of schools from where students were drawn into sample for standardization of tools | 176 |
| 2 | Factor loading and communalities from principal component analysis of perceived nature of elementary school mathematics that makes it difficult to learn | 181 |
| 3 | Summary of the factors derived in exploratory factor analyses of the perceived task difficulties in the five areas of elementary school mathematics | 182 |
| 4 | Component and subcategory wise items in three sets of tests of difficulties in language of mathematics | 188 |
| 5 | Dimension wise and factor wise distribution of items of scale of attitude towards mathematics | 192 |
| 6 | Dimension wise factor loading of each items of scale of attitude towards mathematics | 194 |
| 7 | Illustrative items from five dimensions of scale of attitude towards mathematics | 195 |
| 8 | Coefficients of reliability of scale of attitude towards mathematics and its dimensions | 196 |
| 9 | Illustrative items from the two dimensions of scale of self-efficacy in mathematics | 198 |
| 10 | Coefficients of reliability and validity of scale of self-efficacy in mathematics and its dimensions | 200 |
| 11 | Dimension wise factor loading of each items of scale of selfefficacy in mathematics | 201 |
| 12 | Blueprint for test of previous achievement in mathematics | 202 |
| 13 | Illustrative items from test of previous achievement in mathematics by the cognitive objectives | 203 |
| 14 | Blueprint for tests of achievement in mathematics units | 208 |
| 15 | Illustrative items for each cognitive domain objective from the tests of achievement in mathematics units | 209 |
| 16 | Coefficients of reliability and validity of the tests of achievement in mathematics units | 212 |


| Table <br> No. | Title | Page <br> No. |
| :---: | :---: | :---: |
| 17 | Illustrative items from scales of self-efficacy in the five select mathematics units of standard VII | 214 |
| 18 | Number of items and range of scores in the draft and final forms of scales of self-efficacy in the five select mathematics units of standard VII with corresponding appendix numbers of their Malayalam and English versions | 215 |
| 19 | Coefficients of reliability and validity of scales of self-efficacy in mathematics units | 216 |
| 20 | Distribution of items in the test of verbal comprehension in Malayalam by the components and the type of items used | 218 |
| 21 | Techniques used in language integrated mathematics instruction | 221 |
| 22 | Allocation of lessons and strategies in language integrated mathematics instruction in each of the five select units of standard VII mathematics | 222 |
| 23 | Details of part 1 survey sample used to identify student perception of difficulties in mathematical tasks and their reasons for difficulty | 224 |
| 24 | List of Schools from which Data was Collected for Part II Survey | 225 |
| 25 | Distribution of experimental sample by the levels of verbal comprehension in Malayalam, non-verbal intelligence and previous achievement in mathematics | 226 |
| 26 | Test of significance of difference between the mean scores of verbal comprehension in Malayalam of the control and experimental groups | 227 |
| 27 | Test of significance of difference between the mean scores of nonverbal intelligence of the control and experimental groups | 229 |
| 28 | Mann-Whitney test of significance of difference between median scores of previous achievement in mathematics of the control and experimental groups | 230 |
| 29 | Percentage of students perceiving difficulty in tasks involved in factors of number concept | 241 |
| 30 | Percentage of students perceiving difficulty in tasks involved in mathematical symbols and notations | 241 |
| 31 | Percentage of students perceiving difficulty in tasks involved in factors of mathematical operations | 242 |


| Table <br> No. | Title | Page <br> No. |
| :---: | :--- | :--- | :--- |
| 32 | Percentage of students perceiving difficulty in tasks involved in <br> mathematical abstractions | 243 |
| 33 | Percentage of students perceiving difficulty in tasks involved in <br> factors of problem solving | 243 |
| 34 | Percentage of students perceiving difficulty sourcing from nature of <br> mathematics | 245 |
| 35 | Correlation of task difficulties in mathematics with nature of <br> mathematics and its factors | 247 |
| 36 | Test of significance of difference between means of achievement in <br> mathematics after language integrated mathematics instruction and | 257 |
|  | practice in solving mathematics problems (control) |  |
|  | Test of significance of difference between means of achievement in <br> algebra after language integrated mathematics instruction and <br> practice in solving mathematics problems (control) | 259 |
| 38 | Test of significance of difference between means of achievement in <br> arithmetic after language integrated mathematics instruction and <br> practice in solving mathematics problems (control) | 261 |
| 39 | Test of significance of difference between means of achievement in <br> geometry after language integrated mathematics instruction and <br> practice in solving mathematics problems (control) | 264 |
| 40 | Test of significance of difference between means of achievement in <br> geometry after language integrated mathematics instruction and <br> practice in solving mathematics problems (control) | 267 |


| Table <br> No. | Title |  |  |
| :---: | :--- | :--- | :--- | | Page |
| :---: |
| No. |


| Table <br> No. | Title | Page <br> No. |  |
| :---: | :--- | :--- | :--- |
| 55 | Result of 2 x 2 ANOVA of gain in self-efficacy in mathematics by <br> treatment and verbal comprehension in Malayalam | 310 |  |
| 56 | Comparison of mean gain scores of self-efficacy in mathematics <br> among students with low or high verbal comprehension in <br> Malayalam after language integrated mathematics instruction and <br> practice in solving mathematics problems (control) | 310 |  |
| 57 | Result of 2 x 2 ANOVA of gain in self-efficacy in mathematics by <br> treatment and non-verbal intelligence | 313 |  |
| 58 | Comparison of mean gain scores of attitude towards mathematics <br> among students with low or high verbal comprehension in | 316 |  |
| 59 | Malayalam after the language integrated mathematics instruction <br> and the practice in solving mathematics problems (control) | Comparison of mean gain scores of attitude towards mathematics <br> among students with low and high non-verbal intelligence after the | 318 |
| 60 | language integrated mathematics instruction and the practice in <br> solving mathematics problems (control) |  |  |

## LIST OF FIGURES

| Figure No. | Title | Page <br> No. |
| :---: | :---: | :---: |
| 1 | A strategy for instruction that focus on the language of mathematics | 97 |
| 2 | Outline of the study | 168 |
| 3 | Components of language of mathematics | 185 |
| 4 | Scheme of analysis of the phase I survey study | 232 |
| 5 | Scheme of analysis of the phase III experimental study | 235 |
| 6 | Percentage of standard VIII students perceiving difficulty in mathematical tasks | 244 |
| 7 | Percentage of standard VIII students perceiving difficulty sourcing from nature of mathematics content and teaching-learning | 246 |
| 8 | Achievement in components of language of mathematics among standard VIII students | 249 |
| 9 | Achievement in subcategories of components of language of mathematics among standard VIII students | 251 |
| 10 | Ogives of the scores of achievement in mathematics after language integrated mathematics instruction and practice in solving mathematics problems | 258 |
| 11 | Ogives of the scores of achievements in algebra after language integrated mathematics instruction and practice in solving mathematics problems | 260 |
| 12 | Ogives of the scores of achievement in arithmetic after language integrated mathematics instruction and practice in solving mathematics problems | 262 |
| 13 | Ogives of the scores of achievements in geometry after language integrated mathematics instruction and practice in solving mathematics problems | 265 |
| 14 | Ogives of the gain scores of self-efficacy in mathematics after language integrated mathematics instruction and practice in solving mathematics problems | 268 |
| 15 | Ogives of the gain scores of self-efficacy in learning mathematics after language integrated mathematics instruction and practice in solving mathematics problems | 272 |
| 16 | Ogives of the gain scores of self-efficacy in solving mathematics problems after language integrated mathematics instruction and practice in solving mathematics problems | 275 |


| Figure | Title | Page <br> No. |
| :---: | :--- | :--- |
| 17 | Ogives of the scores of self-efficacy in algebra after language integrated <br> mathematics instruction and practice in solving mathematics problems | 278 |
| 18 | Ogives of the scores of self-efficacy in arithmetic after language <br> integrated mathematics instruction and practice in solving mathematics <br> problems | 280 |
| Ogives of the scores of self-efficacy in geometry after language |  |  |
| 19 |  |  |
| integrated mathematics instruction and practice in solving mathematics |  |  |
| problems |  |  |$\quad 283$


| Figure <br> No. | Title | Page <br> No. |
| :---: | :--- | :--- |
| 29 | Line graph with error bars of self-efficacy in mathematics of students <br> with high and low verbal comprehension in Malayalam in the control <br> (practice in solving mathematics problems) and experimental (language | 312 |
| integrated mathematics instruction) groups |  |  |$\quad$| Line graph with error bars of self-efficacy in mathematics of students |
| :--- |
| with high and low non-verbal intelligence in the control (practice in |
| solving mathematics problems) and experimental (language integrated | 314

## LIST OF APPENDICES

| Appendix | Title | Page No. |
| :---: | :---: | :---: |
| A1 | Factor loadings and communalities based on principal component analysis of difficulties in tasks related to 1) number concept, 2) mathematical operations and 3 ) problem solving for elementary school students | A1-1 |
| A2 | Questionnaire on students' difficulties in learning-Malayalam | A2-1 to 5 |
| A3 | Questionnaire on students' difficulties in learning- English | A3-1 to 5 |
| B | Glossary of terms and symbols in elementary school mathematics | B- 1 to 12 |
| C 1 | Test of difficulties in language of mathematics-set A | C1-1 to 6 |
| C 2 | Scoring key for test of difficulties in language of mathematics-set A | C2-1 |
| D1 | Test of difficulties in language of mathematics-set B | D1 to 6 |
| D2 | Scoring key for test of difficulties in language of mathematics-set B | D2-1 |
| E 1 | Test of difficulties in language of mathematics-set C | E1 to 6 |
| E 2 | Scoring key for test of difficulties in language of mathematics-set C | E2-1 |
| F1 | Data and results of item analysis and the items selected to the final version of scale of attitude towards mathematics | F1-1 |
| F2 | Scale of attitude towards mathematics (draft) - Malayalam | F2-1 to 3 |
| F3 | Scale of attitude towards mathematics (draft) - English | F3-1 to 3 |
| F4 | Scale of attitude towards mathematics (final) - Malayalam | F4-1 to 2 |
| F5 | Scale of attitude towards mathematics (final) - English | F5-1 to 2 |
| F6 | Response sheet of scale of attitude towards mathematics (final)Malayalam | F6-1 |
| F7 | Response sheet of scale of attitude towards mathematics (final)- English | F7-1 |
| G1 | Data and results of item analysis on scale of self-efficacy in mathematics | G1-1 |
| G2 | Scale of self-efficacy in mathematics (draft)- Malayalam | G2-1 to 3 |
| G3 | Scale of self-efficacy in mathematics (draft)- English | G3-1 to 3 |
| G4 | Scale of self-efficacy in mathematics (final)- Malayalam | G4-1 to 2 |
| G5 | Scale of self-efficacy in mathematics (final)- English | G5-1 to 2 |
| H1 | Data and results of item analysis of test of previous achievement in mathematics | H1-1 |


| Appendix | Title | Page <br> No. |
| :---: | :---: | :---: |
| H2 | Test of previous achievement in mathematics (draft)- Malayalam | H2-1 to 4 |
| H3 | Test of previous achievement in mathematics (draft)- English | H3-1 to 4 |
| H4 | Scoring key for test of previous achievement in mathematics (draft) | H4-1 |
| H5 | Test of previous achievement in mathematics (final)- Malayalam | H5-1 to 3 |
| H6 | Test of previous achievement in mathematics (final)- English | H6-1 to 3 |
| H7 | Scoring key for test of previous achievement in mathematics (final) | H7-1 |
| H8 | Response sheet for test of previous achievement in mathematics (final) | H8-1 |
| I 1 | Data and results of item analysis of tests of achievement in mathematics units | I 1-1 |
| I 2 | Test of achievement in parallel lines (draft)-Malayalam | I 2-1 to 3 |
| 13 | Test of achievement in parallel lines (draft)-English | I 3-1 to 3 |
| I4 | Test of achievement in parallel lines (final)-Malayalam | I 4-1 to 3 |
| I 5 | Test of achievement in parallel lines (final)-English | I 5-1 to 3 |
| 16 | Response sheet for test of achievement in parallel lines (final) | I 6-1 |
| I7 | Test of achievement in unchanging relations-Malayalam | I 7-1 to 2 |
| 18 | Test of achievement in unchanging relations-English | I 8-1 to 2 |
| 19 | Response sheet for test of achievement in unchanging relations (draft) | I 9-1 |
| I 10 | Test of achievement in repeated multiplications (draft)- Malayalam | I 10-1 to 2 |
| I 11 | Test of achievement in repeated multiplications (draft)- English | I 11-1 to 2 |
| I 12 | Test of achievement in repeated multiplications (final)- Malayalam | I 12-1 to 2 |
| I 13 | Test of achievement in repeated multiplications (final)- English | I 13-1 to 2 |
| I 14 | Response sheet for test of achievement in repeated multiplications (final) | I 14-1 |
| I 15 | Achievement test in area of triangle-Malayalam | I 15-1 to 2 |
| I 16 | Achievement test in area of triangle-English | I 16-1 to 2 |
| I 17 | Response sheet of achievement test in area of triangle | I 17-1 |
| I 18 | Test of achievement in square and square root-Malayalam | I 18-1 to 3 |
| I 19 | Test of achievement in square and square root-English | I 19-1 to 2 |
| I 20 | Response sheet for test of achievement in square and square root | I 20-1 |
| I 21 | Scoring key for test of achievement in mathematics units | I 21-1 |


| Appendix | Title | Page <br> No. |
| :---: | :---: | :---: |
| J1 | Data and results of item analysis on scale of self-efficacy in units of mathematics | J1-1 |
| J2 | Scale of self-efficacy in parallel lines (draft)- Malayalam | J2-1 to 2 |
| J3 | Scale of self-efficacy in parallel lines (draft)- English | J3-1 to 2 |
| J4 | Scale of self-efficacy in parallel lines (final)- Malayalam | J4-1 to 2 |
| J5 | Scale of self-efficacy in parallel lines (final)- English | J5-1 to 2 |
| J6 | Scale of self-efficacy in unchanging relations (draft)- Malayalam | J6-1 to 2 |
| J7 | Scale of self-efficacy in unchanging relations (draft)- English | J7-1 to 2 |
| J8 | Scale of self-efficacy in unchanging relations (final)- Malayalam | J8-1 to 2 |
| J9 | Scale of self-efficacy in unchanging relations (final)- English | J9-1 to 2 |
| J10 | Scale of self-efficacy in repeated multiplications (draft)- Malayalam | J10-1 to 2 |
| J11 | Scale of self-efficacy in repeated multiplications (draft)- English | J11-1 to 2 |
| J12 | Scale of self-efficacy in repeated multiplications (final)- Malayalam | J12-1 to 2 |
| J13 | Scale of self-efficacy in repeated multiplications (final)- English | J13-1 to 2 |
| J14 | Scale of self-efficacy in area of triangle- Malayalam | J14-1 to 2 |
| J15 | Scale of self-efficacy in area of triangle- English | J15-1 |
| J16 | Scale of self-efficacy in square and square root- Malayalam | J16-1 to 2 |
| J17 | Scale of self-efficacy in square and square root- English | J17-1 |
| K1 | Data and results of item analysis of test of verbal comprehension in Malayalam | K1-1 |
| K2 | Test of verbal comprehension in Malayalam (draft) | K2-1 to 9 |
| K3 | Scoring key for test of verbal comprehension in Malayalam (draft) | K3-1 |
| K4 | Test of verbal comprehension in Malayalam (final) | K4-1 to 6 |
| K5 | Scoring key for test of verbal comprehension in Malayalam (final) | K5-1 |
| K6 | Response sheet of test of verbal comprehension in Malayalam (final) | K6-1 |
| L | Model work book for language integrated mathematics instructions | L-1 to 9 |
| M | Model work book for practice in solving mathematics problems | M1 to 9 |
| N | Distribution of verbal comprehension in Malayalam | N-1 |
| O | Distribution of non-verbal intelligence | O-1 |
| P | Distribution of previous achievement in mathematics | P-1 |
| Q | Distribution of achievement in mathematics after intervention | Q-1 |


| Appendix | Title | Page <br> No. |
| :---: | :---: | :---: |
| R | Distribution of achievement in algebra after intervention | R-1 |
| S | Distribution of achievement in arithmetic after intervention | S-1 |
| T | Distribution of achievement in geometry after intervention | T-1 |
| U | Distribution of pretest scores of self-efficacy in mathematics | U-1 |
| V | Statistical constants of the distribution of posttest scores of attitude towards mathematics, self-efficacy in mathematics and dimensions of attitude and self-efficacy | V-1 |
| W | Distribution of gain scores of self-efficacy in mathematics | W-1 |
| X | Distribution of pretest scores of self-efficacy in learning mathematics | X-1 |
| Y | Distribution of gain scores of self-efficacy in learning mathematics | Y-1 |
| Z | Distribution of pretest scores of self-efficacy in solving mathematics problems | Z-1 |
| AA | Distribution of gain scores of self-efficacy in solving mathematics problems | AA-1 |
| AB | Distribution of self-efficacy in algebra after intervention | AB-1 |
| AC | Distribution of self-efficacy in arithmetic after intervention | AC-1 |
| AD | Distribution of self-efficacy in geometry after intervention | AD-1 |
| AE | Distribution of pretest scores of attitude towards mathematics | AE-1 |
| AF | Distribution of gain scores of attitude towards mathematics | AF-1 |
| AG | Distribution of pretest scores of like towards mathematics | AG-1 |
| AH | Distribution of gain scores of like towards mathematics | AH-1 |
| AI | Distribution of pretest scores of engagement with mathematics | AI-1 |
| AJ | Distribution of gain scores of engagement with mathematics | AJ-1 |
| AK | Distribution of pretest scores of self-belief in mathematics | AK-1 |
| AL | Distribution of gain scores of self-belief in mathematics | AL-1 |
| AM | Distribution of pretest scores of active learning of mathematics | AM-1 |
| AN | Distribution of gain scores of active learning of mathematics | AN-1 |
| AO | Distribution of pretest scores of enjoyment of mathematics | AO-1 |
| AP | Distribution of gain scores of enjoyment of mathematics | AP-1 |


#### Abstract

The context of this study is the increasing recognition of the importance of language of mathematics in its instruction as a prerequisite for understanding and applying mathematics effectively. Rationale of the study bases on the presumption that universal language of mathematics to a certain extent explains the extra challenges to students in learning mathematics than in many other school subjects. It is further argued that in spite of the universal language of mathematics, the language of 'doing mathematics within the classroom' is far from universal, because mathematical communication is not culture free and is affected by linguistic peculiarities and pedagogic conventions and practices of different cultures and languages of instruction. Hence this study makes an attempt to develop and test a mathematics instruction that take adequate care on attainment in language specific to mathematics in ways that enhance student's achievement of mathematical concepts and problem solving skills as well as the related attitudes and self-efficacy beliefs. With a multi method design, it started with content analysis of school mathematics textbooks and tests in Malayalam medium and then surveyed standard VIII students' perception of difficulties in mathematics and their achievement in language of mathematics. An instruction that integrates the language aspects of mathematics on which students have difficulties, as informed by evidences from the test of language of mathematics and report of their difficulties in mathematics in general and language of its content and teaching learning process in particular at the end of primary schooling, was developed. Effect of this language integrated mathematics instruction,through Malayalam,on the outcome variables were studied with pre-test post-test control group design in groups that were matched on previous achievement in mathematics, verbal comprehension in Malayalam and non-verbal intelligence.Guided practice in solving mathematics problems in the unit end exercises of the select five units was the control. The experimental study sample was comparable to population in case of attitude towards mathematics and self-efficacy in mathematics as well and hence the findings from the experiment are generalizable to the population of standard 7 students in elementary schools of Kerala and beyond. The results indicated that, for elementary school students, content and language of mathematics are perceived as much difficult as problem solving in mathematics. Such difficulties source equally or even more from surface structure than deep structures of its language- relating to density of difficult concepts, prevalence of symbols, notations, and unfamiliar words that make their learning strenuous, rote and uninteresting, among others. Task difficulty in mathematics correlated more with nature of content of mathematics, than with its teaching learning. Language integrated mathematics instruction had medium effect on achievement in arithmetic and small effect on self-efficacy in arithmetic. But on all other variables viz., achievements in mathematics in total or in algebra or geometry, there is a gain of 25 percentile rank points after language integrated mathematics instruction, indicating large effects.It strengthened self-efficacy beliefs both in learning mathematics and in solving mathematics problems much more than that after practice in solving mathematics problems; and in algebra or in geometry equivalent to a gain of 30 or more percentile ranks, over and above practice in solving mathematics problems. Language integrated mathematics instruction, if used instead of guided practice in mathematics problem solving, enhances attitude towards mathematics and its dimensions viz., like towards mathematics, engagement with mathematics, self-belief in mathematics, active learning in mathematics and enjoyment of mathematics to a large extent. Effect of language integrated mathematics instruction is more among the students in the lower quartile, and comparatively less among the students in the upper quartile, of achievement and self-efficacy in mathematics. It is concluded that mathematics teachinglearning in schools among other things has to value precise and unambiguous use of language and hence has to integrate these in a deliberate, conscious and yet holistic approach. If textbooks, teachers, classroom environments and teaching-learning processes give required focus on the language of mathematics in a learner appropriate way, it will considerably reduce the feeling of difficulty in mathematics and further enhance cognitive and affective outcomes of mathematics learning.


## Chapter I

## INTRODUCTION

- Need and Significance of the Study
- Statement of the Problem
- Definition of Key Terms
- Variables of the Study
- Objectives of the Study
- Hypotheses of the Study
- Methodology
- Scope and Delimitation of the Study

Mathematics is related to every facet of life and hence is a significant aspect of human learning and knowledge. Despite this recognition and corresponding weightage to its instruction, school mathematics continues to be difficult for the students. The reasons for difficulty in mathematics may vary from curriculum to curriculum and topic to topic. Kerala Curriculum Framework (2007) observes difficulty in mathematics learning sourcing from imbibing basic tenets and unpalatable theories of mathematics, difficulties with methods of forming ideas, repetitive nature of exercises to gain proficiency in calculations, disparity between mathematics in daily life and school mathematics, introduction of symbols and figures, and also from the over emphasize given to the established methods of calculation. The basic nature of mathematics itself, including its language and the skills involved makes it difficult to learn. The characteristics of mathematics like abstractness, accuracy, brevity, symbols and notations and cumulativeness are considered as possible factors causing difficulty in learning it.

Importance of language in curriculum and in teaching-learning is obvious. Language is the medium through which students gain access to the curriculum and through which they display-and are assessed for-what they have learned. Language cannot be separated from what is taught and learned in school (Lucas, Villegas \& Freedson-Gonzalez, 2008). Over and above the relevance of language in general, special challenges that the academic language poses for learners in schools also are well recognized. Learners use language for purposes different from those used in routine conversations (Lucas, Villegas \& Freedson-Gonzalez, 2008).

Though mathematics is often seen as language free - its subject matter being symbols and complex abstract relationships among numbers, categories, geometric forms, variables and the like which require further interpretation, perhaps more than any other subject - teaching and learning mathematics depends on language. Thus, precise mathematical language is critical to student understanding (Reynolds, 2010). This is especially so for elementary school students who are still developing their proficiency in the language of the classroom (Barwell, 2008).

Compared to any other subject area, mathematics, and its language causes extra challenges. Children learning mathematics are in double disadvantage compared to learning of languages, social sciences and natural sciences. Mathematics is usually taught without explicitly introducing its language. Learners are often left to discover the language of mathematics unassisted. This makes learning mathematics unnecessarily difficult and time consuming for many. This is one of the relevance and need for the language of mathematics being explicitly introduced to learners. As with any other school subject, facility with language is a prerequisite for understanding and applying mathematics effectively (Baber, 2011).

It is often accepted that learning of mathematics calls for higher cognitive investment in learning concepts and principles, processes and related skills, and specific language for communicating what is being learnt. This enhanced cognitive load in learning mathematics divides the cognitive energy in mastering mathematical concepts, processes and related skills, and specific language for communication, resulting in comparatively low attainment. Students and teachers usually compensate the deficit in cognitive energy by focusing in mathematical processes and skills which seems to be unique to mathematics as a
subject. However, cumulatively the low level of attainment in language specific to mathematics, and to some extent its concepts and principles, creates bottlenecks in learning even the mathematical processes and skills. This makes mathematics the most difficult subject being taught at school for the majority of students. Failure to attain cognitive outcomes of learning mathematics results in reduced self-efficacy and fear of mathematics. One solution to this problem in mathematics instruction now being advocated is to take adequate care of attainment in language, specific to mathematics in ways that enhance students' achievement of mathematical concepts, problem solving skills and the attitude towards the subject. One question this research ponders on is, what challenges students feel in learning mathematics in schools owing to the Malayalam medium through which it is taught in schools of Kerala.

Moreover, there is growing recognition of language of mathematics as an important element of instruction (Thompson \& Rubenstein, 2000) at school level. Ramanujam, Subramanian and Suchdev (2006) indicates mathematical communication as an important element of mathematics process among other processes like formal problem solving, use of heuristics, estimation and approximation, optimization, reasoning and proof, use of patterns, visualization, representation, and making connections. Precise and unambiguous use of language of mathematics and its communication be taught to the learners in order to appreciate the significance of such conventions and their use (NCERT, 2005).

Mathematical proficiency means the ability to understand, judge, do, and use mathematics. Such abilities are required both in settings where one deals directly with mathematics as well as other contexts in and out of school where mathematics plays a role (Niss, 2003). Many of these competences are related to

## 4 EVIDENCE BASED INSTRUCTION OF LANGUAGE OF MATHEMATICS

mathematical symbols and formalism and communicating with and about mathematics. Niss (2003) lists skills in handling mathematical symbols and formalism such as a) decoding and interpreting symbolic and formal mathematical language, and understanding its relations to natural language, b) understanding the nature and rules of formal mathematical systems (both syntax and semantics) c) translating from natural language to formal/symbolic language and d) handling and manipulating statements and expressions containing symbols and formulae. Other such skills include, skills related to mathematical communication such as a) understanding others' written, visual or oral 'texts', in a variety of linguistic registers, about matters having a mathematical content; and b) expressing oneself, at different levels of theoretical and technical precision, in oral, visual or written form, about such matters.

Though mathematical language is universal and shared by all those doing mathematics, the language of 'doing mathematics within the classroom' is far from being universal (Gorgorio \& Planas, 2001; Novotna \& Moraova, 2005). The assumption that students learning mathematics will automatically acquire and "absorb" the discourse and be able to communicate mathematical ideas (Tharpe, 2017) is increasingly under challenge. Previous researches show that proficiency in the language of mathematics and mathematics performance are related. Chard (2003) suggests that understanding the language of mathematics gives students the skills they need to think about, talk about, and assimilate new mathematics concepts as they are introduced, and vocabulary knowledge provides young learners with a mathematics foundation. Studies on the effect of various types of vocabulary or communication strategy suggest that focused academic language intervention would be beneficial to mathematics learners. So, this study is a step to identify students' language related difficulties in learning
mathematics and to develop an instructional strategy focusing on the language of mathematics which will help to improve their performance in cognitive and affective learning outcomes of mathematics.

## Need and Significance of the Study

Mathematics is taught without explicitly introducing its own language and the learner is left to discover the language unassisted (Baber, 2011) which makes difficulty in meaningful understanding of mathematical concepts. Students may excel in mathematics computation, but their ability to apply their skills will suffer if they do not understand the mathematics vocabulary used in instructions and story problems (Bruun, Diaz \& Dykes, 2015). In children's mathematical learning, cognitive aspects includes various elements of its language (Walls as cited in Sparrow \& Hurst, 2010).

Learning outcomes at the elementary stage decided by NCERT (2017) advocates that the learners are expected to realize and use mathematics as an important tool that they can talk about, use and explore. Also it enumerates two language related outcomes in curricular expectations of mathematics learningdeveloping language and symbolic notations at primary stage and learning to provide reasoning and convincing arguments to justify one's own conclusions in mathematical context at upper primary stage. For deeper understanding of mathematics, effective communication is crucial (Sammons, 2018). Current curricula of mathematics focus much on computation than on communication because it does not give enough opportunity to the students for oral or written communication in mathematics (Huggins \& Maiste, 1999).

Mathematics is an artificially constructed formal language, although technically it is not a natural human language; however natural everyday

## 6 EVIDENCE BASED INSTRUCTION OF LANGUAGE OF MATHEMATICS

language is used to teach the formal language of mathematics (Meiers \& Trevitt, 2010). Students go into the classroom with natural everyday language obtained through interactions at home and within the community influenced by social and cultural factors outside the school, instead of mathematics language (Simpson \& Cole, 2015). Problems allied to linguistic features of mathematics may source from 1) less proficiency in medium of instruction (Adams \& Cohen, 2010; Adetula, 1990; Howie, 2003; Latu, 2005; Vilenius-Tuohimaaa, Aunola \& Nurmi, 2008; Vukovic \& Lesaux, 2013), 2) learning mathematics in a language other than mother tongue (bilingual students) (Abedi \& Lord, 2001; Nordin, 2005; Yushau \& Bokhari, 2003); and 3) the academic vocabulary/ language of mathematics (Ilany \& Margolin, 2010; Mbugua, 2012; Powell, Driverb, Robertsc \& Fallc, 2017).

The failure to attain cognitive outcomes of learning mathematics results in affective reactions in learners, making them feel negativity, anxiety, and fear of mathematics. In order to make mathematics attractive to students one of the major steps thus required is helping students master the language of mathematics. But, teachers of mathematics usually focus on the mathematical competencies like having to learn the process of mathematics and its operations. One question this study probes among others is, does an instruction that explicitly supports learner understanding of language used in mathematics teaching and learning will add to affective reactions of students such as selfbeliefs in mathematics and like towards and engagement with mathematics.

Since the language of mathematics is precise and concise, it is essential for students to learn the key terms and words used in mathematics (Cuevas, 1991). This academic language is the formalized language of school
mathematics essential for communicating, defining and forming concepts, and constructing mathematics knowledge (Gottlieb \& Ernst-Slavit, 2014; Kersaint, Thompson, \& Petkova, 2009; Kim, 2015; Lim, Stallings \& Kim, 2015). Academic language in school, poses special challenges as learners use this language for purposes different from those used in daily life (Lucas, Villegas \& Freedson-Gonzalez, 2008). Acquiring the appropriate language is the key to making mathematics intelligible as symbolism and logic both influence learning and problem solving in mathematics. However, the means and processes through which mathematics instruction can integrate language objectives is less than clear even where the medium of instruction is in international languages like English, not to speak of the uncertainties and indefiniteness in languages like Malayalam, despite it being the medium of instruction for millions of students in schools of Kerala. This study is specially to equip the researcher and other stakeholders to understand and appreciate the difficulties emerging from deep and surface structure of mathematics language as used in Malayalam medium schools of Kerala where this study is situated.

Mathematics language do not develop naturally as a child develops a natural language and it need to be learned through conscious practice (Dekeyser, 2007; NCTM, 2000) by engaging learners in authentic, real-life functional use of the language (Moschkovich, 2012). The important role of language-rich activity in the classroom, through "conversation", "discussion" or "discourse" is increasingly being recognized owing to reasons including developments in classroom practice, professional discourse, and policy (Morgan, Craig, Schuette \& Wagner, 2014) and increasing attention to language which surely reflects upon the "social turn" (Lerman, 2000) in mathematics education as well. This gaining importance of the social

## 8 EVIDENCE BASED INSTRUCTION OF LANGUAGE OF MATHEMATICS

environment within which mathematics education takes place inevitably leads to raised significance of language and other forms of communication in learning environment. While the central role of language in the learning, teaching, and doing of mathematics is increasingly being recognized, an agreement about the roles language play in various teaching learning contexts within mathematics education and their impact on various outcomes are still lacking (Morgan, Craig, Schuette \& Wagner, 2014). This research is one attempt to test the effectiveness of an evidenced based means of integrating language in mathematics instruction.

Language has a key role in the mathematics learning as fluency in it provides access to the world of mathematics (Esty, 1992). However, language is only the start of good communication (Tharpe, 2017). NCTM (1991) identified communication, with discourse as a key component, as one of the six standards for teaching mathematics. Good communication in the classroom requires good, appropriate, and concise language used by the teacher, required of students, and fostered by teacher designed opportunities for learning and practice (Tharpe, 2017). But this needs to be supported through research and development of means by which language, in different cultural contexts be integrated in classrooms.

There has been a longstanding interest in the issues involved in teaching and learning mathematics in different languages. Though these issues can be universal to school level mathematics education, the answers to them need not be as universal like the issues themselves. This lack of universality arises because mathematical communication is not culture free (Novotna \& Moraova, 2005), and is affected by linguistic peculiarities and pedagogic conventions and
practices of different cultures. Even as political struggles over choice of language of instruction continue, research needs to add to our understanding of how characteristics of specific languages may affect the nature of the mathematics that is done using the language as well as how they may affect student learning (Craig \& Morgan, 2015).

In anticipation of these developments, the position paper of the national focus group on the teaching of mathematics (NCERT, 2006) suggested a decade earlier that the main goal which should be given the first preference is universal inclusion. For the principle of inclusion to be meaningful even the language used in textbook must be sensitive to language uses of all children especially at primary level. For a vast majority of Indian children, the language of mathematics learned in schools is far removed from their everyday speech and hence forbidding. It also notes that school mathematics should takes place in a situation where children see mathematics as something to talk about, to communicate, to discuss among them, to work together on. A special mention was made on problems created by the language used in textbooks, especially at the elementary level. As implied above, in carrying out such a linguistic analysis, teachers must identify the key vocabulary and subject-specific terminology that students need to understand. They must also review written texts (textbooks, worksheets, study guides) associated with the mathematics learning. This study attempted to analyze mathematics textbooks in Malayalam to develop a test of language of mathematics such that students' hard spots in mathematics at this level being communicated through Malayalam could be identified.

Learners will need additional cognitive demands in solving mathematical problems due to the language used to present those problems. A cognitive load
explanation of the difficulty in learning mathematics advocated by Sweller (1994) suggests that schema acquisition and automation are the primary mechanism of learning. Mastery over subject depends on these two processes. Schema is the basic unit of learning. Function of learning is to store automated schemas in long-term memory. In most cases, the learner will have the mathematical schema; but only when the process of automation starts to work one can use it. As working memory is very limited, automated schemas in long term memory will be helpful in reducing working memory load. Language acquisition should be automated, such that working memory can be used for solving mathematical problems and not for reading and comprehending the language of the problem.

How can learners be supported to be fluent in language of mathematics such that it does not hinder their attention and energy from being focused on learning concepts and skills that are considered peculiarly assigned to mathematics learning in schools? What aspects and skills within language of mathematics constrain students at elementary level from learning mathematics skills? The researches on these sorts of questions are gaining attention, yet the answers emerging are still in beginning stage. Chow and Ekholm (2019) found that among young children, syntax is the strongest predictor of mathematics performance and vocabulary is not and hence vocabulary cannot be considered as an index of language ability among primary school students in the context of mathematics learning. However, general vocabulary was a stronger predictor for third-grade students with lower mathematics vocabulary scores (Powell, Driver, Roberts \& Fall, 2017). There are few studies in Indian context, though. Language ability in mathematics, reading ability in mathematics and performance in arithmetic skills are highly correlated (Rao, Ramaa \&

Gowramma, 2017). It was also found out that percentage of error is more in common words used both in general language and mathematics than in spatial terms.

Students' difficulties in translating mathematics word problems was a special area of interest (Cruz \& Lapinid, 2014). For example, Gooding (2009) identified reading and comprehension, reading all the information, distracting information, imagining the context, writing a number sentence, carrying out the calculation, and interpreting the answer in the context of the question all as language related difficulties in the context of word problems. Clearly, in test items, if unfamiliar language is used it contribute to the poor performance through difficulties in reading and comprehension especially so if they were less proficient in English. Teachers of these standard four students also perceive that mathematical language used in the test was difficult for the students to comprehend and students who had poor reading skills struggled to comprehend the questions (Sibanda, 2015). Students also points out lack of ability in understanding the content as well as instructional language as the reason for this difficulty (Nordin, 2005). Even as actual mathematics operation to be done is the same, when the text of the word problem had advanced vocabulary and complex syntactic structure, students' performance in mathematics problem solving was less, with even less performance if both the mathematics content and language are presented with complex nature (Barbu \& Beal, 2010). Understandably, linguistic modification of test items caused significant differences in mathematics performance as indicated in the slightly higher scores on the linguistically modified version. And the benefit of reduced linguistic complexity was more among students who were low or average in their mathematics performance (Abedi \& Lord, 2001). Items that gives focus on important

## 12 EVIDENCE BASED INSTRUCTION OF LANGUAGE OF MATHEMATICS

information, that are logically organized, and if they are free of unnecessary information can improve student performance in mathematics (Gillmor, Poggio \& Embretson, 2015). Also, skill on mathematics word problems was strongly related to skill in reading comprehension and there was no gender difference in word problem solving skills (Vilenius-Tuohimaa, Aunola \& Nurmi, 2008). Mathematics performance for the English language learners increased with English-reading proficiency, with the latter predicting not only student's mathematics test scores, but also their progress in the online mathematics tutorial, and mathematical self-concept (Beal, Adams \& Cohen, 2010).

Targeted language instruction for early language experiences did improve students' performance with influence on development in mathematics (Vukovic \& Lesaux, 2013a). Reviewed literature showed that around the globe, such language intervention to enhance mathematics learning, however, can take many forms and shades. There can be variation in their content- especially whether language of medium of instruction, or the mathematics vocabulary and their meaning are taught; and in their approaches - whether elements of reading and writing mathematics individually or in combination, or communication of language of mathematics as a whole or a more constructivist discourse approach.

Studies differ on their conclusions regarding the importance of various components of language of mathematics by grade level, the types of tasks involved and by the area of mathematics like algebra, arithmetic or geometry. To design interventions that addresses the needs of less performing students in mathematics, not only knowledge about student's ability to express, understand and learn mathematics is necessary (Vukovic \& Lesaux, 2013 ${ }_{\mathrm{b}}$ ) but also what language related difficulties they have in different areas of mathematics is to be
identified. For example, among first and second grade students, syntax is the strongest predictor of mathematics performance and vocabulary is not (Chow \& Ekholm, 2019), yet, in the third-grade students the relationship among general vocabulary, mathematics computation, and mathematics vocabulary were significantly stronger than in the fifth-grade students (Powell, Driver, Roberts \& Fall, 2017). Available evidence indicates that linguistic features had moderate effects on item difficulty at $4^{\text {th }}$ grade, dropping to small-to-medium effects at 10th grade (Shaftel, Belton-Kocher, Glasnapp \& Poggio, 2006). Contribution of non-verbal intelligence varies for different measures of mathematics performance (Tikhomirova, Voronina, Marakshina, Nikulchev, Ayrapetyan \& Malykh 2016). For instance, general verbal ability influences students' symbolic number skill, whereas phonological skills influence their arithmetic performance. Special features of algebraic language was a major reason of higher frequency of errors (75\%) made in translation from verbal to symbolic representation (Rodriguez-Domingo, Molina, Canadas \& Castro, 2012).

The above observations highlight the need for teachers to be reflective and critical users of classroom talk and understand their role in mathematical discourse in order to improve the quality of the learning opportunities presented to students in mathematics classroom (Brown \& Hirst, 2007). However, though teachers are aware about the linguistic challenges in mathematics instruction and the influence of language on students' performance in mathematics (Naidoo, 2015) they are not adequately equipped with their responsibilities in developing mathematical communication skills among the students (Kabael \& Baran, 2016). This is true for early career teachers as well, which make them inconsistently focus on how to teach mathematical vocabulary, anticipations for students’
precise use of mathematical terminology, and the use of multiple languages during classroom instruction (Turner, Roth McDuffie, Sugimoto, Aguirre, Bartell, Drake \& Witters, 2019).

Most of the studies and strategies reviewed indicate that researchers are preoccupied with surface structures of language like vocabulary, morphology of terms and their meaning, than the more holistic and deeper structures of language in their attempts to disentangle problems emerging out of language of mathematics. Accordingly, instruction that include the components such as terms may bring in positive outcomes. For example, symbols and syntactic structure in teaching improved student performance in mathematics (Mbugua, 2012). Mathematical vocabulary through daily activities improved students test scores as well self-efficacy beliefs in learning mathematics (Larson, 2007). After receiving vocabulary instruction, majority of students improved overall understanding of mathematical concepts and, they felt that understanding mathematical words is important and it increased their achievement (Georgius, 2008). Focused vocabulary instruction is beneficial for all type of learners especially for the struggling learners as at least a 33 percent increase is there in the gains on standardized tests and their perceived self-efficacy beliefs were also improved (Gifford \& Gore, 2010). Vocabulary instruction and mathematical game improved student performance in mathematics achievement. High achieving students makes twice the gains of the underachieving students in mathematics achievement. Vocabulary instruction is equally effective on achievement in mathematics for students who were high or low achievers in reading (Tarpley, 2015). Vocabulary instruction was less effective in their understanding of mathematical vocabulary with those students who had a low
verbal comprehension (Kenyon, 2016). Vocabulary tutoring intervention improved students' vocabulary whereas it did not had effect on students with the algebraic problem-solving skills (Hollingsworth, 2019).

Recent and more powerful designs applied to language of school mathematics challenge adopts a more holistic perspective of this language. Accordingly, emphasize on oral and written communication in the mathematics classroom were found very beneficial for elementary level students (Wichelt, 2009). Mathematical communication intervention on oral and written communication skills improved success with especially higher-level skills of fourth graders whereas it improved the lower level communication skills of thirdgraders (Huggins \& Maiste, 1999). Self-confidence in solving mathematics problems increases significantly due the increase in oral communication (Sample, 2009). Students writing articles in mathematics journals everyday scored significantly higher in mathematics achievement (Flanagan, 2009). A recent study (Lomibao, Luna \& Namoco, 2016) found that students perceive mathematical communication as useful to them and majority of them also agreed that difficulty in understanding mathematical concepts reduced significantly after being exposed to write and describe on how they arrived at a solution whereas 49 percent students found it interesting and thought provoking and 57 percent students agreed that writing in mathematics helped them give more attention to accuracy. Study conclude that 76 percent students found mathematical discourse enjoyable and fun. In doing this, reciprocal peer tutoring on mathematical communication was superior to those who received one to one self-paced learning and teacher led instruction in developing students' mathematical representations and solution explanations became more accurate after the learning activity (Yang, Chang,

Cheng \& Chan, 2016). Exposure to mathematical language storybook reading intervention also can positively affect student's mathematics skills (Purpura, Napoli, Wehrspann \& Gold, 2017).

Hence, this study explores does a mathematics instruction that integrates the language aspects on which students have difficulties, as informed by evidences from a test of language of mathematics instruction and student's report of their difficulties in mathematics in general and language of its content and teaching learning process at the end of primary schooling in particular, pay off in students' achievement of valid mathematics learning outcomes? In doing this, this study among other things, will reveal how does such an evidence based language integrated mathematics instruction fare against guided practice of mathematics problem solving, an often used method to remedy students' difficulty in school mathematics, among Malayalam medium upper primary students with varying levels of previous achievement, verbal comprehension and non-verbal intelligence in enhancing their test scores and self-efficacy beliefs in mathematics and its areas and attitude towards mathematics.

## Statement of the Problem

This study is entitled as 'Enhancing Mathematics Learning through Evidence Based Instruction Focusing on Language of Mathematics in Elementary Schools of Kerala'.

It identifies language related difficulties in mathematics learning at elementary level in order to develop an instructional plan focusing on language of mathematics. It further examines the effectiveness of this instruction plan 'Language Integrated Mathematics Instruction' on achievement in mathematics,
algebra, arithmetic and geometry, self-efficacies in mathematics, algebra, arithmetic and geometry, and attitude towards mathematics of elementary school students. It further verifies whether the language integrated mathematics instruction significantly enhances elementary school students' achievement in mathematics equally for high and low levels each of previous achievement in mathematics, verbal comprehension in Malayalam, and non-verbal intelligence. This study also verifies whether effect of language integrated mathematics instruction on attitude towards mathematics and self-efficacy in mathematics of elementary school students are equal for high and low levels of verbal comprehension in Malayalam and non-verbal intelligence.

## Definition of Key Terms

The key terms that appear in the title of the study stands for the following.

## Enhancing Mathematics Learning

Enhancing mathematics learning means the improvement in mathematics learning outcomes through an active intervention as evidenced through changes in scores of achievements in mathematics, algebra, arithmetic and geometry, self-efficacies in mathematics, algebra, arithmetic and geometry and attitude towards mathematics and in the dimensions of self-efficacy and attitude towards mathematics.

## Evidence based Instruction

In this study, evidence-based instruction is the instructional programme, developed by modifying existing practices of mathematics instruction, focusing on language of mathematics based on evidences from the survey phase related to the linguistic difficulties in mathematics.

Instruction focusing on language of mathematics - language integrated mathematics instruction-made use of a set of instructional techniques focusing on imparting knowledge of mathematical vocabulary including terms and symbols, morphology of terms, syntactic structure of mathematical sentences, meaning of mathematical vocabulary and pragmatic use of mathematical language with the objective of helping students to better comprehend the language of mathematics and facilitating student understanding of the subject.

In this study, evidence-based instruction focusing on language of mathematics is an intervention that integrates mathematics instruction with the language aspects of mathematics on which students have difficulties, as informed by evidences from the test of language of mathematics and report of student difficulties in mathematics.

## Language of Mathematics

language of mathematics is the system made up of components such as vocabulary (symbols or words), grammar (rules of how to put symbols into use which is peculiar to a discourse on mathematics), syntax (placing the symbols in linear structures), discourse (strings of syntactic propositions), and meanings (to be communicated with the symbols) used by mathematicians to communicate mathematical ideas among themselves (Umeodinka \& Nnubia, 2016).

In this study, language of mathematics means the language used in mathematics textbooks, classroom interaction among teachers and students and in evaluation context, to communicate mathematical concepts - including
general terms used in mathematics teaching-learning, discipline specific terms, arithmetic and geometric symbols, morphology of terms, syntactic structure of mathematical sentences, semantics and pragmatic use of mathematical language elements.

## Elementary Schools of Kerala

Elementary schools of Kerala are the schools in Kerala offering formal education from standard I to VII. In the present study, the term elementary schools of Kerala are used to denote students attending standard VII in any of the recognized schools of Kerala where the medium of instruction is Malayalam.

## Variables of the Study

This study proceeds in three phases, with Phase I Pilot Study leading to Phase III Experiment, with a phase of design and development of appropriate tools and intervention strategies in between.

## Variables in Phase I (Pilot Study)

In phase I, the variables studied were students' perception of difficulties in mathematical tasks, reasons sourcing from nature of mathematics for these perceived difficulties in mathematical tasks, and achievement in the language of mathematics and its components. Perception of difficulties in mathematical tasks among elementary school students is conceived as the dependent variable being influenced by reasons sourcing from nature of mathematics and lack of achievement in the language of mathematics and its components.

## Variables in Phase III (Experimental Phase)

The effectiveness of an evidence-based instruction focusing on language of mathematics (Language integrated mathematics instruction) in improving students' mathematics learning outcomes in terms of achievement in mathematics, self-efficacy in mathematics and attitude towards mathematics- in comparison to guided practice in mathematics problem solving is examined. Hence there are independent, dependent and control variables in the experimental phase.

## Dependent variables

The study intended to examine the effect of language integrated mathematics instruction on mathematics learning. The dependent variables are set of cognitive and affective outcomes of mathematics learning including achievement in mathematics, self-efficacy in mathematics and attitude towards mathematics.

## Achievement variables

There are four achievement related dependent variables.

1. Achievement in mathematics. Achievement in mathematics is the weighted total of achievement students gained from the five chapters, 1) Parallel Lines, 2) Unchanging Relations, 3) Repeated Multiplication, 4) Area of Triangle and 5) Square and Square root.
2. Achievement in Algebra. It is the extent to which students has attained the cognitive objectives of learning the chapter 'Unchanging Relations' of standard seven mathematics.
3. Achievement in Arithmetic. Achievement in arithmetic is the weighted total of cognitive achievement, students gained from the two chapters'Repeated multiplication' and 'Square and square root'.
4. Achievement in Geometry. Achievement in geometry is the weighted total of cognitive achievement, students gained from the two chapters- 'Parallel lines' and 'Area of Triangle'.

## Self-efficacy variables

Effectiveness of language integrated mathematics is measured also against self-efficacy in mathematics. There are six self-efficacy variables. They are
5. Self-efficacy in Mathematics. Perceived self-efficacy is the student's beliefs about his/her capability to solve mathematics problems and to perform in mathematics learning. It is the total of the scores in selfefficacy in learning mathematics and self-efficacy in solving mathematics problems.
6. Self-efficacy in Learning Mathematics. It is student's belief about his/her capability to perform in mathematics learning contexts like school mathematics learning in general, classroom teaching-learning of mathematics and assessment practices in mathematics.
7. Self-efficacy in Solving Mathematics. It is student's belief about his/her capability to solve mathematics problems in seven areas of school mathematics viz; natural numbers, fractions, decimals, geometry, percentage, average, graph and algebra up to their grade level.

## 22 <br> EVIDENCE BASED INSTRUCTION OF LANGUAGE OF MATHEMATICS

8. Self-efficacy in Algebra. It is students' beliefs about their capabilities to perform in mathematical tasks related to the chapter 'Unchanging relations'.
9. Self-efficacy in Arithmetic. It is the weighted average of students' beliefs about their capabilities to perform mathematical tasks in the two chapters 'Repeated multiplication' and 'Square and square root'.
10. Self-efficacy in Geometry. It is the weighted average of students' beliefs about their capabilities to perform mathematical tasks in the two chapters 'Parallel lines' and 'Area of Triangle'.

## Attitude towards mathematics

Effectiveness of language integrated mathematics instruction is also measured against attitude towards mathematics and its components namely- like towards mathematics, engagement with mathematics, self-belief in mathematics, active learning of mathematics and enjoyment of mathematics.
11. Attitude towards Mathematics. It is the student's positive or negative feeling towards mathematics as a school subject, its learning, classroom practices, mathematics teacher, assessment practices, homework and involvement of parents and peers. It is the total score of five dimensions of attitude towards mathematics.
12. Like towards Mathematics. It is the student's overall like towards mathematics as a school subject, its teaching-learning activities, assessment practices, homework, parents' involvement in mathematics learning, peer involvement and mathematics teacher.
13. Engagement with Mathematics. It measures the tendency of the student to engage in or avoid mathematics related activities like classroom activities, homework, mathematics teacher and parent-peer involvement in mathematics learning.
14. Self-belief in Mathematics. It measures perception of students' belief about their ability to cope with mathematics learning activities and performance.
15. Active Learning of Mathematics. It is the measure of students' active and motivated participation in mathematics learning activities both in classroom and home, assessment context, and in interaction with mathematics teachers and peers.
16. Enjoyment of Mathematics. Enjoyment of mathematics is the measure of students positive or negative feeling towards mathematics learning activities, homework, examination and teacher.

## Independent variables

The independent variable selected for the study is the instructional strategy with two levels; Language Integrated Mathematics Instruction and Guided Practice in Solving Mathematics Problems. Experimental group is provided with language integrated mathematics instruction along with content instruction by the schoolteacher. For the same duration, guided practice in solving mathematics problems is provided to the students in the control group along with content instruction by the schoolteacher.

## Control variables

Control variables of the study are Previous Achievement in Mathematics, Verbal Comprehension in Malayalam and Non-verbal Intelligence.

## Objectives of the Study

Major objective of this study was to identify the language related difficulties in mathematics learning at elementary level and to check the effectiveness of language integrated mathematics instruction developed based on the identified difficulties, in improving their mathematics learning outcomes.

Following were the specific objectives of the present study.

1) To identify language related difficulties in mathematics learning at elementary level in order to develop an instructional plan focusing on language of mathematics.
2) To examine the effectiveness of language integrated mathematics instruction, in enhancing elementary school students':
i. Achievement in mathematics
ii. Achievement in algebra
iii. Achievement in arithmetic
iv. Achievement in geometry
3) To examine the effectiveness of language integrated mathematics instruction, in enhancing elementary school students':
i. Self-efficacy in mathematics (in total) and

Dimensions of self-efficacy in mathematics viz;
a. Self-efficacy in learning mathematics
b. Self-efficacy in solving mathematics problems
ii. Self-efficacy in areas of school mathematics viz.,
a. Self-efficacy in algebra
b. Self-efficacy in arithmetic
c. Self-efficacy in geometry
4) To examine effectiveness of language integrated mathematics instruction, in enhancing elementary school students':
i. Attitude towards Mathematics (in total) and Dimensions of Attitude towards Mathematics viz.,

1. Like towards mathematics
2. Engagement with mathematics
3. Self-belief in mathematics
4. Active Learning of mathematics
5. Enjoyment of mathematics
5) To examine the effectiveness of language integrated mathematics instruction in enhancing elementary school students' achievement in mathematics by the levels (high and low) of:
i. Previous achievement in mathematics
ii. Verbal comprehension in Malayalam
iii. Non-verbal intelligence
6) To examine the effectiveness of language integrated mathematics instruction in enhancing elementary school students' self-efficacy in mathematics by the levels (high and low) of:
i. Verbal comprehension in Malayalam
ii. Non-verbal intelligence
7) To examine the effectiveness of language integrated mathematics instruction in enhancing elementary school students' attitude towards mathematics by the levels (high and low) of:
i. Verbal comprehension in Malayalam
ii. Non-verbal intelligence

## Hypotheses of the Study

This study is designed to test the following hypotheses regarding the effectiveness of language integrated mathematics instruction on three outcome variables viz., achievement in mathematics, self-efficacy in mathematics and attitude towards mathematics, each of these outcome variables had sub variables based either on three area of school mathematics and/or dimensions of the construct like self-efficacy or attitude towards mathematics. Further, hypotheses concerned with the effect of language integrated mathematics instruction on the outcome variables after controlling the variables viz., previous achievement in mathematics, verbal comprehension in Malayalam and non-verbal intelligence. Thus, there are 23 specific hypotheses presented under seven sets of statements regarding effectiveness of language integrated mathematics instruction over guided practice in solving mathematics problems on mathematics learning outcomes of elementary school students of Kerala.

1. Language Integrated mathematics instruction significantly enhances elementary school students' achievement in mathematics.
i. Language integrated mathematics instruction significantly enhances elementary school students' achievement in mathematics equally for high and low levels of previous achievement in mathematics.
ii. Language integrated mathematics instruction significantly enhances elementary school students' achievement in mathematics equally for high and low levels of verbal comprehension in Malayalam.
iii. Language integrated mathematics instruction significantly enhances elementary school students' achievement in mathematics equally for high and low levels of non-verbal intelligence in mathematics.
2. Language Integrated Mathematics Instruction significantly enhances elementary school students':
i. Achievement in algebra
ii. Achievement in arithmetic
iii. Achievement in geometry
3. Language integrated mathematics instruction significantly enhances elementary school students' self-efficacy in mathematics.
i. Language integrated mathematics instruction significantly enhances elementary school students' self-efficacy in mathematics equally for high and low levels of verbal comprehension in Malayalam
ii. Language integrated mathematics instruction significantly enhances elementary school students' self-efficacy in mathematics equally for high and low levels of non-verbal intelligence in mathematics
4. Language integrated mathematics instruction significantly enhances elementary school students' dimensions of self-efficacy in mathematics viz;
i. Self-efficacy in learning mathematics
ii. Self-efficacy in solving mathematics problems
5. Language integrated mathematics instruction significantly enhances elementary school students':
i. Self-efficacy in algebra
ii. Self-efficacy in arithmetic
iii. Self-efficacy in geometry
6. Language integrated mathematics instruction significantly enhances elementary school students' attitude towards mathematics.
i. Language integrated mathematics instruction significantly enhances elementary school students' attitude towards mathematics equally for high and low levels of verbal comprehension in Malayalam
ii. Language integrated mathematics instruction significantly enhances elementary school students' attitude towards mathematics equally for high and low levels of non-verbal intelligence in mathematics
7. Language integrated mathematics instruction significantly enhances elementary school students' dimensions of attitude towards mathematics
i. Like towards mathematics
ii. Engagement with mathematics
iii. Self-belief in mathematics
iv. Active learning of mathematics
v. Enjoyment of mathematics

## Methodology

The study used a mixed approach by beginning with a survey to identify the language related difficulties in mathematics, development of an instructional strategy based on the identified difficulties and testing the effectiveness of the developed strategy- Language Integrated Mathematics Instruction-using a quasiexperimental pretest-posttest control group design. Henceforth the study is described in three phases.

## Procedure of the Study

The study proceeds through three phases; first a pilot study with survey and content analysis, and then a developmental phase that leads to the final experimental phase.

## Phase I: Pilot study with content analysis and survey

Content analyses of mathematics textbooks from preprimary to standard seven, and that of achievement tests used in schools were done to identify linguistic components involved in mathematics. The linguistic components of
mathematics teaching-learning were reviewed and the perception of students' about difficulty due to these factors were surveyed.

Additionally students' language related difficulties in mathematics learning were tested. Results of these surveys along with the literature reviewed guided the development of evidence-based instructional strategy focusing on language of mathematics to overcome the identified linguistic difficulties.

## Phase II: Developmental phase

An evidence-based instruction focusing on the language of mathematics in elementary level was developed based on the evidence from Phase I. Strategies for instruction to the experimental and the control groups were planned and designed. Tools for measurement in experimental phase were also developed during this phase. Test of previous achievement in mathematics, test of verbal comprehension in Malayalam, scale of self-efficacy in mathematics, attitude towards mathematics, and, tests of achievement and scales of selfefficacies for the five units of standard seven mathematics- 1. Parallel lines, 2. Unchanging relations, 3. Repeated multiplication, 4. Area of triangle and 5. Square and square root were developed. These tools were tried out and their validity and reliability were ensured.

## Phase III: Experiment

Effectiveness of the evidence-based instruction focusing on the language of mathematics is examined through a quasi-experimental pretest-posttest nonequivalent group design experiment.

1. Four intact classes of standard 7 were selected and two classes each were randomly assigned to the experimental group and the control group. Then, the analysis samples in experimental and control groups were matched on their verbal comprehension in Malayalam, non-verbal intelligence and previous achievement in mathematics.
2. Experimental and control groups were pretested on self-efficacy in mathematics and attitude towards mathematics.
3. In the experimental group, language integrated mathematics instruction was provided by the experimenter along with content instruction of five units by the schoolteacher. In the control group, for these units experimenter provided practice in solving mathematics problems along with content instruction by the schoolteacher.
4. The effectiveness of the language integrated mathematics instruction is checked with respect to dependent variables.

## Design of the experiment

The pretest-posttest non-equivalent control group design used in this study is denoted as follows.
$\begin{array}{llll}\mathrm{G}_{1} & \mathrm{O}_{1} & \mathrm{X} & \mathrm{O}_{2}\end{array}$
$\begin{array}{llll}\mathrm{G}_{2} & \mathrm{O}_{3} & \mathrm{C} & \mathrm{O}_{4}\end{array}$
$G_{1} \& G_{2}$ - Intact divisions of $7^{\text {th }}$ standard students randomly assigned to experimental and control groups and matched on previous achievement in mathematics, verbal comprehension in Malayalam and non-verbal intelligence.

X - Language integrated mathematics instruction (by experimenter) along with content instruction (by schoolteacher)

C - Guided practice in solving mathematics problems (by experimenter) along with content instruction (by Schoolteacher)
$\mathrm{O}_{1} \& \mathrm{O}_{3}$. Pretests on self-efficacy in mathematics and attitude towards mathematics.
$\mathrm{O}_{2} \& \mathrm{O}_{4}$ - Posttests on achievements and self-efficacies in 1) Parallel lines, 2) Unchanging relations, 3) Repeated Multiplication, 4) Area of Triangle and 5) Square and square root; and self-efficacy in mathematics and attitude towards mathematics.

## Sample used for the Study

There are three different sets of samples. Part I survey drew 300, eighth standard students randomly for identification of perception of difficulties in mathematical tasks and reasons for difficulty thereof and Part II survey drew 1050, eighth standard students, randomly for identification of language related difficulties in learning mathematics.

The experimental phase of the study used a sample of standard VII students from a government aided school of rural background following Kerala syllabus in Malayalam medium. Experimental and control groups consist of 45 students each, and the groups were matched on the levels of

- Previous achievement in mathematics,
- Verbal comprehension in Malayalam and
- Non-verbal intelligence

Tools were standardized in a different set of samples.

## Tools used for the Study

Content analysis, questionnaire, test of achievements and attitude and self-efficacy scales were used in the study. In initial phase the following tools were used.

1. Questionnaire on students' difficulties in learning
2. Test of difficulties in language of mathematics (3 Sets)

In experimental phase the following measuring tools were used.

1. Test of previous achievement in mathematics
2. Test of verbal comprehension in Malayalam
3. Raven's standard progressive matrices (Raven, 1994)
4. Scale of attitude towards mathematics
5. Scale of self-efficacy in mathematics
6. Achievement tests \& scales of self-efficacy in -
i. Parallel lines
ii. Unchanging relations
iii. Repeated multiplication
iv. Area of triangle
v. Square and square root

In addition to the measuring tools, language integrated mathematics instruction and guided practice in solving mathematics problems were also developed. Techniques used in language integrated mathematics instruction
strategy was designed to overcome the linguistic difficulties in learning mathematics among elementary school students. The lessons were prepared by incorporating the techniques, viz., anchoring mathematics with language, vocabulary bank, labeling vocabulary, word walls, word trails, listen and write, possible sentences, guess what?, justifying their reasoning and translation game.

Practice in solving mathematics problems was provided to the control group for an equal duration of time in which students were given guided practice in mathematics problem solving for each unit.

## Statistical Techniques used in the Study

In addition to the basic descriptive statistics, the following statistical analyses were used.

1. Percentage analysis
2. Significance of difference between two correlated percentages
3. Pearson's r
4. Significance of a coefficient of correlation
5. Comparison of correlations from dependent samples
6. Shapiro- Wilk test of normality
7. Levene's test of homogeneity of variances
8. Independent samples $t$ test
9. Mann Whitney $U$ test
10. Two-way ANOVA
11. Effect Size (Cohen's $d$ )
12. Partial eta squared

## 34

## Scope and Delimitation of the Study

This study intended to enhance students' mathematics learning in terms of achievement, self-efficacy and attitude towards mathematics by overcoming their difficulties in learning mathematics due the language of mathematics. Reasons for difficulty in learning mathematics were studied at elementary level including language difficulties, perceived task difficulty and perceived difficulties sourcing from nature of mathematics. This study explored the language difficulties in transacting mathematics through Malayalam among students with Malayalam as mother tongue. Test of difficulties in language of mathematics developed for the study incorporated language components up to standard VIII in Kerala.

The effect of language integrated strategy on mathematics learning outcomes namely achievement, self-efficacy and attitude towards mathematics, and its components are examined using a pretest-posttest nonequivalent control group design. Four intact classroom were selected for experimental intervention and two groups were randomly allotted for control and experimental group each. Intervention was given by the researcher along with content instruction by the teacher for five units in standard seven mathematics. Out of the 13 units in standard seven mathematics, five were selected for experimental study. One algebra unit out of two units, two geometry units out of four and two arithmetic units out of six units were included in the intervention.

There are three different sets of samples drawn from Malayalam medium schools of Kerala. Fourteen standardized tools with reasonable reliability and
validity were developed for data collection, including five achievement tests, five scales of self-efficacies in mathematics units, scale of self-efficacy in mathematics, scale of attitude towards mathematics, test of previous achievement in mathematics and test of verbal comprehension in Malayalam. A questionnaire on students' difficulties in learning mathematics and battery of tests of difficulties in language of mathematics were also developed. The battery of tests of difficulties in language of mathematics can be used for identification and diagnosis of language difficulties in mathematics learning.

During the survey phase, in studying the language related difficulties, owing to the limitations of study design which was largely quantitative and objective, only the language used in more formal instructional contexts especially in textbooks, teachers' handbooks and question papers which are presumably more or less uniform across all Malayalam medium schools of Kerala state only were considered; though, colloquial language localized across the regions of state of Kerala too are prone to cause difficulties especially in classroom interaction contexts.

This study identified language related difficulties only through a written test. Oral communication skills, listening skills and expressive communication like speaking were not measured. The plan to incorporate classroom observation to find out the difficulties emanating from actual process of classroom communication could not be materialized due to practical difficulties in implementations and time required.

Initial surveys on perceived difficulties in mathematical tasks and language related difficulties in mathematics, were not statewide, only students
from Kozhikode and Malappuram districts of Kerala were considered. The experimental study is delimited to standard seven elementary school students. The language integrated mathematics instruction is a combination of different techniques, but this study does not explore individual contributions of these techniques.

## Chapter II

## REVIEW OF RELATED LITERATURE

- Theoretical Overview of Language of School Mathematics
- Nature, Components and Instructional Approaches of language of School Mathematics
- Summary remarks on the structure of and approaches to language of school mathematics
- Objectives, Strategies and Issues of Language of Mathematics in Schools
- Summarizing the Mathematics Language related Issues, Objectives, and strategies in schools
- Studies Related to Language of Mathematics
- Survey studies related to language of mathematics
- Studies on Students' perception on Language of mathematics and attendant difficulties in learning
- Studies on Teachers' perception on Language of mathematics and attendant difficulties in learning and instruction
- Experimental studies related to Language of Mathematics
- Vocabulary interventions on Language of Mathematics
- Communication interventions on Language of Mathematics
- Summary of studies related to language of Mathematics
- Conclusions from Review of Literature

This study explored the difficulties in learning mathematics sourcing from the language of mathematics, and then went on to examine the effect of an instructional strategy focusing on the language of mathematics on achievement and self-efficacy in mathematics and attitude towards mathematics among elementary school students. The language integrated instructional strategy was synthesized using the techniques reviewed from related literature. The literature discussed here under were obtained with the search term "mathematics" or "mathematical" in combination with language, communication, register, vocabulary, terminology, discourse, grammar, speaking, reading and writing, and also in combination with instruction, school, classroom, strategies, method, technique and the like.

This review of literature on language of mathematics, related instructional strategies, practices in schools and researches thereon is organized in two broad sections namely 1) theoretical overview of language of school mathematics and 2) studies related to language of mathematics.

## Theoretical Overview of Language of School Mathematics

This section broadly covers 1) nature and structure of and instructional approaches to language of school mathematics and 2) objectives, strategies and issues of language of mathematics in schools. This review was conducted with a clear understanding of that there are multiple uses of the term language in the context of mathematics learning. Mathematics communication in school is a hybrid of specialized language of mathematics and the adapted model of natural language for the discourses in mathematics. This review makes a distinction between specialized language of mathematicians and the
specialized language of mathematics classroom dialogue (Morgan, Craig, Schuette \& Wagner, 2014) and focus concepts, examples and issues that will support to identify and appreciate the difficulties in learning mathematics due to the language of mathematics at the school level. This especially was intended to equip the researcher and other stakeholders to understand and appreciate the difficulties emerging from deep and surface structure of mathematics language as used in Malayalam medium schools of Kerala, owing to the realization that though mathematical language is universal and shared by all those doing mathematics, the language of 'doing mathematics within the classroom' is far from being universal (Gorgorio \& Planas, 2001; Novotna \& Moraova, 2005).

## Nature, Components and Instructional Approaches of Language of

 School MathematicsThis part discusses nature, components and instructional approaches of language of mathematics under three major sections. Section 1, nature and characteristics of the language of mathematics begins with clarifying the meaning of language and discusses how much mathematics shares features of natural language, and finally identifies what features of mathematics distinguish it from other languages. Section 2, components of language of mathematics, elaborated on the characteristics of the language of mathematics, especially its surface structure features like general, technical, and sub technical vocabulary in mathematics which may or may not vary by the medium of instruction in schools, and more universal surface structures like notations and symbols before moving on to deeper structure of language of mathematics like syntax, especially in equations and expressions. Section 3, Instructional approaches to language of
mathematics discusses about various instructional approaches related to language of mathematics like vocabulary approach, beyond focus on vocabulary, mathematics register, mathematics discourse and communication approach.

## Nature and characteristics of the language of mathematics

This section discusses the meaning of language, and the relation between language of mathematics and natural language and compares the former with the latter to get clarity about the distinguishing features of the language of mathematics.

## What is language?

Language is both medium for expression or communication and a tool for understanding and knowing. As a medium of communication, language can take verbal and visual (written) forms in order to express facts, opinions, thoughts, ideas, feelings, desires or commands. The communication may be at one time or from one time to another, between different people or within one person at different times (Baber, 2011). As a tool for understanding and knowing, language helps to constitute and organize thoughts. Grammar, structure and traditions of language help in putting together and organizing thoughts in thinking, analyzing or reasoning (Akkus, 2015). The structure of language facilitates its functions. Umeodinka and Nnubia (2016) in their definition, for example, identified six components of language namely vocabulary (symbols or words), grammar (rules of how to put symbols into use), syntax (placing the symbols in linear structures), discourse or narrative (strings of syntactic propositions), community (people using and understanding the symbols) and meanings (to be communicated with the symbols).

## Relation between language of mathematics and natural language

"Mathematics is considered as a language, and often mathematics students feel their teacher is speaking a language foreign to them" (Lee, 2006). However, there are multiple uses of the term language in the context of the language of mathematics.

The relationship between natural language and mathematical language is a major theme in research related to language of mathematics. Natural language, 'informal language' or 'colloquial language' (Radford \& Barwell, 2016) is often compared with language of mathematics. Woodin (1995) refers to mathematics as a language. It is not a natural language but a formal language (Gough, 2007), constructed while using natural everyday language in teaching the mathematical concepts (Leshem \& Markovits, 2013). Mathematics is variously described not a general purpose language, but a discipline-specific language (Moursund, 2016), the authentic language and the only universal language (Changeux \& Connes, 1998), a pure language - the language of science (Adler, 1991), or a logical language (Cole, 2010). Many researchers consider it as a universal language, mathematics itself as a language, while others focus on how mathematical language is a problem (Moschkovich, 2012) including in instructional and learning contexts.

Patkin (2011) cites Usiskin as suggesting that like other languages, mathematics fosters the organization of the ideas within the learner; has its own letters (numbers, symbols and signs for example, $\perp,=, \cong$ ), verbs (eg. subtract), syntactic rules, (eg. expressions such as ' $3+4$ '), vocabulary (though with its own unique features), and also lends and borrows words with other languages (eg. use
of Latin alphabet in algebra; and the Greek alphabet in geometry like ellipse, parabola).

Following account on strengths and weaknesses of language of mathematics are largely based on Peat (1990).

## 1. Mathematics as a more restrictive limited form of language

Peat (1990) observed that mathematics is both more, and less, than a language. Mathematics is often considered as a more restrictive limited form of language as it does not fully fit the structure and functions of the natural language; and hence may not look anything like natural language. For example, compared to natural languages mathematics is less fluent, less narrative and is rarely spoken aloud. Mathematics is also less than a language and precisely is a limited, technical language. It cannot express deep human values and is weak in richness, nuance, inherent ambiguity and strategies for dealing with this ambiguity compared to natural language.

## 2. Economy of mathematics over ordinary language

Peat (1990) further observed that everything mathematics have their origin in language. Rich and abstract proofs and theorems of mathematics voiced in natural languages are long winded and cumbersome thoughts and arguments. Mathematics is more abstract than natural languages, as it uses numbers and symbols to make calculations. Highly codified forms of mathematics makes it easy to carry out calculations, to demonstrate proofs and to arrive at true assertions. Though this can be seen as a surface difference, a feature of convenience and economy of mathematics over ordinary language. The power of
language, whether mathematical or natural, is in conveying meaning through form and transformation.

## 3. Mathematics as a more formal extension of natural language

However according to Peat (1990) mathematical thought has direct access to a form of thinking that is deeper and more primitive than anything available in any natural language. Mathematics goes beyond language, in which mathematics involves a kind of visual and sensorimotor thinking. Some parts of mathematics dealing with the properties and relationships of shapes, involving direct, internal visualization and even involving an internal sense of movement indicative of "non-verbal" thinking, a form of mental activity that goes beyond the domain of a spoken or written language. This prelinguistic mental activity is taken as indication of common source from both mathematics and ordinary language. Thus, mathematics is a more formal extension of natural language (Peat, 1990).

## Comparison between natural language and the language of mathematics

Similarities and differences between the language of mathematics and natural languages like English were perceived by educators for long. Mathematics, like natural languages, requires learning many rules, with the former being more objective and the latter being more subjective and emotive. Similarities between natural language and the language of mathematics are in them being medium for expressing and communicating and for thinking, reasoning, and analyzing, using a set of defined symbols for composing sentences and expressions, both having well-defined rules of syntax (grammar) for composing meaningful, acceptable, correct sentences and expressions (Baber, 2011).

Language of mathematics differs from the language of ordinary speech in their universes of discourse, level of tolerance for ambiguity and imprecision, time-reference, and capability to express feelings (Baber, 2011). Firstly, while the universe of discourse of the language of mathematics is very much limited to values, variables, functions and expressions, the universe of discourse of natural languages encompasses concrete and abstract aspects of the unlimited human environment and experience.

Second, the structure of mathematical language is more precise and less flexible than the structure of natural language (Ilany \& Margolin, 2010) while almost every sentence other than very simple statements in natural languages are prone to ambiguity in some way, every expression in the language of mathematics is unambiguous and suitable for rational logical reasoning, Consequently, vague or imprecise statements cannot be formulated in the language of mathematics (Baber, 2011). Susceptibility for multiple interpretation is almost non-existent in mathematics (Leshem \& Markovits, 2013). This quality of mathematical language is reflected in the paucity of language that expresses itself. There is only one type of noun - numbers, functions, and only two relational signs - equality and inequality (Bloedy-Vinner as cited in Ilany \& Margolin, 2010). This precision of language of mathematics causes enormous difficulties for neophytes. Clarity, less flexibility and lack of ambiguity in mathematics makes it foreign for students. Students' experience with natural language give little practice in forming clear, precise sentences and often lack the patience to do so. Such students constantly search for the hidden assumptions in mathematical assertions where there are none, inevitably changing the stated meaning leading to misunderstanding (Jamison, 2000). Related to this characteristic of mathematics is especially mathematics facts being viewed as a simpler, more consistent and more regular language than

English. Simple sentence facts formed by numbers (nouns) and operations (verbs), put together by rules (syntax), is a much easier, simpler, and unambiguous language than natural language.

Thirdly, the concept of time. Though time can applied in the external interpretation of a mathematical model and be modeled mathematically, the concept of time itself is not within the realm of mathematics (Baber, 2011). This is because mathematics is non-temporal and hence there is no past, present, or future in mathematics; everything just "is". Fourth, mathematical language is devoid of emotional content, though this factor causes no difficulty for students (Jamison, 2000).

## Characteristics of the language of mathematics

Language of mathematics is the system made up of components such as technical terms and grammatical conventions peculiar to a discourse on mathematics and a symbolized or coded rule used by mathematicians to communicate mathematical ideas among themselves and to pass mathematics ideas across to one another (Umeodinka \& Nnubia, 2016). The language of mathematics is characterized by:

1) Precision (able to make very fine distinctions)
2) Conciseness (able to say things briefly)- the conventional mathematical style has no extraneous words. The style that is conventionally mathematical communicates only what is necessary. There should be no 'extra' or redundant words in the communication (Lee, 2006).
3) Power (able to express complex thoughts with relative ease)
4) Economy (easy to express the kinds of thoughts that mathematicians like to express)
5) Learnable
6) Requires the efforts needed to learn any foreign language (Burns, 2018)
7) Written and oral language, symbols, visual representations such as graphs (Schleppegrell, 2007)
8) Gestures as a form of communication (McNeill 1992; 2000)
9) Expressed in a "foreign language" with its own grammar (principles that govern the correct use of a language), syntax (the part of grammar that concerns rules of word order), vocabulary, word order, synonyms, negations, conventions, abbreviations, sentence structure and paragraph structure (Esty, 1992)
10) It is both a means of communication and an instrument of thought (Esty, 1992; Kaput, 1998)
11) Truth of sentences- Sentences can be true or false. The notion of truth is of fundamental importance in the mathematical language (Burns, 2018)
12) Conventions in languages- Mathematics has its conventions, which help readers distinguish between different types of mathematical expressions (Burns, 2018).

## Components of Language of Mathematics

Descriptions of mathematical language focus on vocabulary and symbolism and some limited areas of specialist grammatical structures uncommon in everyday language (Morgan, 1996). Skemp as noted in Orton (1987), identifies two levels of language: deep structures and surface structures.

The vocabulary issues are the "surface structures" used to transmit ideas that lead to the "deep structures" of mathematical concepts (Thompson \& Rubenstein, 2000). In a sense, "the deep structure represent the meaning, and the surface structure is the actual sentence we see." (Aarts, Chalker \& Weiner, 2014). And hence, elements like clauses, and phrases; nouns and pronouns; conjunctions and conventions can be seen as surface structures and grammatical rules, syntaxes can be seen as deep structures.

## Surface structures language of mathematics

Linguistic elements in the language of mathematics include verbs, clauses, and phrases; nouns and pronouns; adjectives, adverbs, and prepositional phrases; conjunctions, negation and parts of speech and naming conventions for functions and variables (Baber, 2011).

## Noun

'Nouns' of mathematics are used to name mathematical objects of interest. Many view mathematics has only one type of noun - numbers, (Ilany \& Margolin, 2010), which though are infinite in themselves. There are nouns, such as place, borrow, and product (Schleppegrell, 2007). Use of long, dense noun phrases is a characteristic of the language of mathematics. Mathematical analogue of a 'noun' is called an expression. An expression is a name given to a mathematical object of interest (Burns, 2018). Expression is a group of signs and numbers that show a particular quantity or idea.

Use of the passive voice and deletion of personal pronouns is a feature of mathematical discourse and these contribute to the 'distant authorial voice' (Morgan, 1996) which is common in mathematical texts (Lee, 2006).

## Expression

An expression in the Language of Mathematics is like a phrase, clause, or sentence in English Expressions are combinations of values, variables, and functions. Values, variables, and functions are hence the fundamental elements in expressions written in the Language of Mathematics. Values are basic constants, arbitrary components not requiring further mathematical definition. Variables are abstract representations of values, often unknown or undetermined. Functions are ways of determining a value from other values (Baber, 2011). Expressions can be combined in certain ways to form another expression.

## Verbs

A noun vs. verb classification in mathematics is suggested as tremendous benefit as without such distinction, novices fall prey to common syntax errors like "stringing things together with equal signs", as if '=' means 'I'm going on to the next step.' (Burns, 2018). Most popular verbs in mathematics include five operational signs $(+,-, x, /,=)$ (Leshem \& Markovits, 2013); "greater than or equal to" symbol (eg. $2 \mathrm{x}=6$ ) (Burns, 2018; Umeodinka \& Nnubia, 2016); and words expressing relation that also are common in mathematics (Schleppegrell, 2007).

## Conjunctions

A conjunction is a compound statement formed by joining two statements with the connector 'AND'. The conjunction "p and q" is symbolized by 'pq'.

## General, Technical and sub technical Vocabulary in Mathematics

Nouns, expressions, verbs, clauses, phrases and conjunctions vocabulary found in mathematics textbooks can be of three types; viz., general vocabulary (largely, commonly used everyday words), special vocabulary (general vocabulary words with specialized meanings in particular content areas), and technical vocabulary (words having usage and application in a particular field only) (Vacca \& Vacca as cited in Salinas \& Ortlieb, 2011) are part of.

1) General vocabulary are words typically part of common language, (eg. liter, gallon, more than and less than); (Monroe \& Panchyshyn as cited in Salinas \& Ortlieb, 2011).
2) Technical vocabulary refers to terms specific to mathematics like multiplicand, quadrilateral (Pimm, 1987); equilateral, quotient, probability (Barwell, 2008); coefficient (Thomas, Garderen, Scheuermann \& Lee, 2015); quotient, reciprocal, and square root (Monroe \& Panchyshyn as cited in Salinas \& Ortlieb, 2011). Technical vocabulary consists of words that are specific to the content area. Mathematics uses additional technical terms resulting in a difficult situation for students to learn the meaning of this type of vocabulary (Pimm, 1987).
3) Sub technical vocabulary consists of words that have multiple meanings. These words may exist within a student's vocabulary but they are adapted for specialised completely foreign mathematical meanings and needs to be explained with common or familiar language for comprehension
(Monroe \& Panchyshyn, 1997; Umeodinka \& Nnubia, 2016). These include terms such as range, degree, face, root, ring, field, category, term and factor (Umeodinka \& Nnubia, 2016); function, expression, difference, area (Barwell, 2008); prime and leg (Thomas, Garderen, Scheuermann \& Lee, 2015); line, factor, frequency (Barwell, 2008); and similar, face, volume, product (Pimm, 1987).

## Vocabulary issues unique to mathematics

There are educators (Thomas, Garderen, Scheuermann \& Lee, 2015) highlighting the difference mathematics vocabulary has from other content vocabulary beyond these three broad categories of vocabulary common to other school subjects, and seeing it as reason for why mathematic vocabulary often confounds comprehension. They for example suggest that words such as thousand and thousandth are related but have different meanings; and the words 'pi' and 'pie' are homophones and that technical words like density occurs in both mathematics and science, but with different technical meanings.

## Notations, symbols and mathematical jargons

Notations, symbols and mathematical jargons are other distinguishing elements of mathematics vocabulary. Mathematics has assimilated notations specific to mathematics from symbols in many different alphabets; including the symbolic notations used as multiple variables like x , y and z . (Umeodinka \& Nnubia, 2016). In addition, there are symbolic vocabulary, on the other hand, consisting of numerals and symbols used in mathematics. For example, $>, \pi, \neq$,
$23,3.1$, and the other symbols like abbreviations representing units such as oz, $l b$, and in (Monroe \& Panchyshyn as cited in Salinas \& Ortlieb, 2011) or symbols like, 3-D (Pimm, 1987). Mathematics has phrases (mathematical jargons) that have specific meanings. The examples are as follows: "if and only if", "necessary and sufficient", "without loss of generality", "complete the following", "simplify the following", etc. Observe that their meanings are not far from their facial meanings when analyse, eg 'if and only if' is used for phenomena that is 'doubled barred' 'A $\Leftrightarrow$, says A implies B and B implies A . (Umeodinka \& Nnubia, 2016)

## Deep structures of language of mathematics

Deep structures of language of mathematics are grammatical rules and its syntaxes. Academic language consists of several components other than vocabulary such as language function, discourse and syntax (Lim, Stallings \& Kim, 2015).

## The Grammar of Mathematics

There are two viewpoints on grammar of mathematics. One, the grammar employed for mathematical discourse is essentially the grammar of natural language but with mathematics-specific peculiarities (Umeodinka \& Nnubia, 2016). Two, the mathematical grammar, especially in formulae are unique to mathematics and are shared internationally by the global community of mathematicians, independent of natural language.

## Syntax

Mathematics is highly technical, with characteristic patterns of vocabulary and grammar that allows statements of mathematics to be quite
precise. Language of mathematics is free of many of the vagueness and ambiguities of ordinary speech. Syntax refers to the mathematics-specific rules, special forms, conventions, and/or grammar associated with writing or speaking. (Lim, Stallings \& Kim, 2015).

Mathematical discourse includes specialist syntax, particularly in relation to the expression of logical relationships (Barwell, 2008). Syntax is the set of conventions for expressing ideas including symbols, words and phrases (Kersaint, Thompson \& Petkova, 2009; Lim, Moseley, Son \& Seelke, 2014); and consists of symbols, notations, expressions and sentences (Lim, Stallings \& Kim, 2015). Little attention has been paid to the grammatical structure of mathematical texts. It may be because school students are less involved in production of their own texts (Morgan, 1996).

Grammatical patterning in mathematics includes the use of long, dense noun phrases. These long noun phrases then participate in constructing complex meaning relationships (Schleppegrell, 2007; Veel, 1999). Highly complex mathematical sentences though render such sentences unintelligible. Likewise, mathematical verbs construct different kinds of relational processes. Attributive process constructs information about membership in a class or part-whole relationship. Identifying process, constructs relationships of identity and equality. Attributive clauses are non-reversible; whereas the identifying clauses construct relationships of equality, and hence, are reversible (examples from Veel, 1999). Attributive clauses classify objects and events, identifying clauses define technical terms. Identifying clauses bridge between technical and less technical ways of presenting knowledge in mathematics (Schleppegrell, 2007). Another unique grammatical structure of language of mathematics is the extensive use of nominalization (forming a
noun from a verb and hence an object from a process, as in permutation, relation, rotation) (Morgan, 1996).

## Equation

An equation can be given the status of a sentence or sentential phrase (Umeodinka \& Nnubia, 2016). Formula can be a part of speech in a natural language, phrase, though (Umeodinka \& Nnubia, 2016). At times it is not easy to read aloud and understand the formulas without a written or spoken explanation; though they can be vocalized (spoken aloud) using underlying natural language. The vocalization has to be learned; for example "f(x)" is pronounced "eff of eks" (Umeodinka \& Nnubia, 2016). Other mathematical statements like axioms, conjectures, theorems, lemmas and corollaries also have complex taxonomy (Umeodinka \& Nnubia, 2016). Whether in formulas, or in their explanations, even if mathematical grammar borrows from natural language it has its own specialized syntax (eg., the use of words like and, a, or if ) and ways of talking and writing (eg., word problems, writing a solution, giving an explanation) (Pimm, 1987).

## Instructional approaches to language of mathematics

Instructional approaches to language of mathematics are the ways mathematics educators try to incorporate language explicitly into classroom instruction. As the assumption that students learning mathematics will automatically pick up on and "absorb" the discourse and be able to communicate mathematical ideas (Tharpe, 2017) is increasingly under challenge and instruction focusing on the language of mathematics is gaining in acceptance. Explicit incorporation of the language of mathematics to instruction with the
objective of helping students to better comprehend the language of mathematics and facilitating student understanding of the subject may be at any of the four levels, namely mathematics vocabulary, mathematics register, mathematical discourse and or mathematics communication. These Instructional approaches to language of mathematics describe the language used in mathematics and its education settings from progressively increasing levels of comprehensiveness of the notion of language of mathematics.

The instructional approaches focusing on language of mathematics progressively address the issues like distinctive features and terms used in mathematics, language functions useful to better comprehend mathematics, roles language plays in the processes of doing mathematics and producing mathematical knowledge, and interaction of person, context and language in understanding mathematics (Morgan, Craig, Schuette \& Wagner, 2014). It may be conceived that mathematical vocabulary are put together into meaningful sentences and phrases according to the conventions of mathematics register which enables written or spoken discourses among experts and novices in mathematics, which in turn are only a part of mathematics communication.

## Vocabulary approach

One prominent approach to the language of mathematics instruction is learning to use the vocabulary of mathematics. Many of them reduce the meaning of the role of academic language in mathematics teaching to highly technical and abstract words and the proper use of grammar (Cavanagh, 2005). The rationale is that polysemous words aggravate vocabulary struggles for learners (Rojas, 2009) or that there are multiple meanings for the same term or
phrase (Pimm, 1987). Thus, though the language of mathematics emphasize clarity, it is a complex language that is typically not clear to students causing confusion when it is not explicitly taught. Proper teaching of the vocabulary and forms of notation can support students to pick up various meanings and slight differences in meanings on their own (Tharpe, 2017).

Even the natural language that is read and spoken in mathematics nears a technical jargon (Hersh, 1997), technical or sub technical vocabulary is but one category of the adapted form of language for mathematics communication. Hence, incorrect terminology is only one of the causes for students failing to gain full and clear understanding of mathematics concepts (Tharpe, 2017). However, to determine aspects of the language that are likely to be problematic, teachers must identify the key vocabulary and subject-specific terminology that students need to understand through a linguistic analysis and review of written texts including textbooks, worksheets and study guides (Lucas, Villegas \& Freedson-Gonzalez, 2008). Effective teachers then shape mathematical language by modelling appropriate terms and communicating vocabulary meaning endorsed by the wider mathematical community in ways that students understand vocabulary. However, drill and practice are not the most effective instructional practice for learning either vocabulary or mathematics (Blachowicz \& Fisher, 2000; Pressley, 2000). Vocabulary learning may involve making links between mathematical language, students' intuitive understandings and the home language (Anthony \& Walshaw, 2010).

## Beyond a focus on vocabulary

Learning to communicate mathematically is not a matter of learning vocabulary only. Vocabulary acquisition in a first or second language, and
learning terms and definitions although important is not sufficient to learn mathematics communication. Since language is complex and consists of more than just a string of words and sentences, mathematical communication is more than vocabulary. Students have to learn to describe patterns, make generalizations, and use representations to support their claims in mathematics through discussions in classrooms (Moschkovich, 2012). Opportunities to read the terms and definitions, ask questions about them, use them in sentences, (Cuevas, 1991) are important. It is even better if such terms are used in the same context in which students encounter them, than being taught in isolation (Cuevas, 1991), since mathematical terms are best understood when students use the language of mathematics in a meaningful setting (Cummins \& Swain 1986; Krashen 1981). Open-ended questions that require students to articulate their mathematical thinking might be useful too (Reynolds, 2010).

Learning mathematical language is analogous to learning a foreign language. In addition to vocabulary, an understanding of syntax, word order, and abbreviations unique to mathematics is also needed. It also entails knowledge of sociolinguistic aspects of language usage as well. Linguistic challenges of the mathematics register, suggests the need to expand understanding of the language issues in mathematics classrooms beyond a focus on vocabulary or specialized terminology (Simpson \& Cole, 2015). This calls for instructional contexts that are language-rich, that actively involve students in using language, that require both receptive and expressive understanding, and require students to use words in multiple ways over extended periods of time (Blachowicz \& Fisher, 2000; Moschkovich, 2012; Pressley, 2000). It needs to consider that, mathematics reasoning uses patterns of language that draw on grammatical constructions that create dense clauses linked with each other in conventionalized ways that are different from ordinary informal language use (Schleppegrel, 2007).

## The mathematical register

When communicating mathematics, mathematicians speak and write in a special register of the language. A register is not just a collection of words but is a way of using language to express concepts and even a characteristic mode of presenting and discussing an argument (Halliday 1975). The mathematics register is a way of using symbols, specialist vocabulary, precision in expression, grammatical structures, formality and impersonality that results in ways of expression that are recognisably mathematical. Mathematics register is a set of deep-seated linguistic conventions and expectations that have been developed over many centuries and that regulate the way discourse about mathematics takes place (Lee, 2006).

The mathematical register uses special technical words, as well as ordinary words, phrases and grammatical constructions with special meanings that may be different from their meaning in ordinary English, or any other natural language. It is typically mixed with expressions from the symbolic language (Wells, 2003). Schleppegrell (2007) identified two aspects of mathematics register namely multiple semiotic systems and grammatical patterns. Semiotic systems in mathematics register include mathematics symbolic notation, oral language, written language and graphs and visual displays. Grammatical patterns characteristic of mathematics register includes technical vocabulary, dense noun phrases, being and having verbs, conjunctions with technical meanings and implicit logical relationships. Pupils need to learn to use the register in order to have control over the concepts of mathematics (Lee, 2006).

The symbolic language of mathematics is an independent special-purpose language consisting of the symbolic expressions and statements (Wells, 2003)
and mathematicians' informal jargon consists of expressions such as "conceptual proof" and "intuitive". These communicate something about the process of doing mathematics but do not themselves communicate mathematics (Wells, 2003).

It is, however, important to remember that, while linguistics can provide means of describing mathematical texts, their interpretation is highly dependent on knowledge of the discourses in which a text is produced and consumed and on the analyst's theoretical perspective on the activities and social relationships within those discourses (Morgan, 1996).

## Mathematical discourse

Discourse is the mathematical communication that occurs in a classroom. Mathematical discourse is the written and spoken language, mathematicians and students use for communicating. In a broad sense, this is "specialised ways of talking" (Barwell, 2008) "communication" (Wells, 2003), "classroom discussion" within norms (Moschkovich, 2007), communication of definitions and proofs and about approaches to problem solving, typical errors, and attitudes and behaviors (Wells, 2003). Mathematical discourse involves mathematical symbols, ranging from numerals to more specialized notation. Written and spoken Mathematical discourse in classrooms on explanation, proof or definition, or text type (Barwell, 2008) has three components - mathematical register, symbolic language and informal jargon. Mathematical discourse approach is going beyond the traditional focus on vocabulary and symbolism to interrogate written and oral texts produced within mathematical contexts (Morgan, 1996).

Effective discourse happens when students articulate their own ideas and seriously consider their peers' mathematical perspectives as a way to construct mathematical understandings. Encouraging students to construct their own
mathematical understanding through discourse is an effective way to teach mathematics (NCTM, 2010). This takes creativity on the part of the teacher to design environments where students are comfortable and engaged in using the language of mathematics through different activities. Teachers striving to obtain effective communication in their classrooms must start somewhere, and that somewhere is the language of mathematics (Tharpe, 2017).

Discourse in the mathematics classroom may take three forms- traditional discourses, probing discourses and rich discourses (Kenney, 2005). Traditional discourses occurring in larger classrooms takes place between a student and teacher to which other students listen to. This form mostly takes question directed at specific students which are responded by the students as pre planned by the teacher through appropriate prompts. Probing discourse are still between teacher and individual student but uses more open questions that encourage students thinking and generate ideas that pick the interest of the whole class. Discourse rich classrooms have a culture of understanding that encourage sharing of ideas each other among students and the teacher (Kenney, 2005) which encourage questioning and probing one another's thinking in order to clarify underdeveloped ideas (Stigler \& Hiebert, 2009).

NCTM (1991) identifies communication, with discourse as a key component for teaching mathematics. Discourse encompasses the ways of representing, thinking, talking, and agreeing and disagreeing that teachers and students use to engage in those tasks. In this, teachers have to pose questions and tasks, listen to, ask for clarification and justification of student ideas orally or in writing, pursue them in depth, attach mathematical notation and language to such ideas; monitor and encourage student participation; elicit, engage, and
challenge each student's thinking. Teachers provide further information, clarification and also model the discourse when the students struggle. Students have to listen to, respond to and question the teacher, initiate problems and questions, make conjectures and present solutions, explore examples and counterexamples and use mathematical evidence and argument. This involves reasoning, making connections, solving problems and communicating. Discourse can be facilitated using computers, calculators, concrete models, graphic aids and tables, terms and symbols, metaphors, analogies, and stories, written hypotheses, explanations, and arguments as well as oral presentations and dramatization (NCTM, 1991).

## Communication approach

Communication is an essential part of mathematics. According to Novotna and Moraova (2005), mathematical communication is broader notion than mathematical discourse, and hence surely than mathematics register. Communication in mathematics involves making use of the processes of reading, writing, speaking, listening and thinking as one communicates with one's self and with other people (Moursund, 2016). The language of mathematics can also refer to language used in aid of an individual doing mathematics alone and hence "inner speech" as well as language employed with the intent of communicating with others (Novotna \& Moraova, 2005)

According to Pirie (1998), mathematical communication in the classroom can be of six categories;

1. Ordinary language: the language current in the everyday vocabulary of any particular child varying according to students of different ages and stages of understanding.
2. Mathematical verbal language: "using words" either spoken or written.
3. Symbolic language: mathematical symbols.
4. Visual representation: not strictly a "language", but a powerful means of mathematical communication.
5. Unspoken but shared assumptions: not strictly a "language"; means by which mathematical understanding is communicated and on which new understanding is created.
6. Quasi-mathematical language: this language, usually, but not exclusively, that of the pupils, has, for them, a mathematical significance not always evident to an outsider (even the teacher).

Communication approach in mathematics education involves sharing ideas, discussions, justifying solutions, clarify student understanding (Hatano \& Inagaki, 1991). Students who have opportunities, encouragement, and support for speaking, writing, reading, and listening in mathematics classes communicate to learn mathematics; they learn to communicate mathematically (NCTM, 2010). Through communication, ideas become objects of reflection, refinement, discussion and amendment. By thinking and reasoning about mathematics and communicating the results of their thinking orally or in writing, students learn to be clear and convincing. Listening to others' explanations develop student understanding. Conversations from multiple perspectives help the participants sharpen their thinking and make connection (NCTM, 2010). According to NCTM (2010), there are four ways in which communication supports the learning of mathematics. 1) It helps students to organize and consolidate their mathematical thinking which require that students need to try to clarify their own thinking before they make their ideas public, 2) allows students to develop mathematical terminology, 3) help students become more critical thinkers of
mathematics and 4) to appreciate the "power and precision of mathematical language".

New Jersey Mathematics Curriculum Framework (1996) has advocated many advantages of communication of mathematics in the classroom in a variety of forms including via orally, in writing, and using symbols and visual representations for learning and using mathematics. One, communication help students to make critical connections among physical, pictorial, graphic, symbolic, verbal and mental representations of mathematical ideas. Two, it is important in making mathematics meaningful and enabling students to construct links between their informal, intuitive notions and the abstract language and symbolism of mathematics. Three, they learn appropriate use of mathematical language and symbols. Four, by realizing that some ways of representing a problem are more helpful than others, students understand the flexibility and usefulness of mathematics. Five, such communication of mathematical ideas in written and oral form with their classmates, teachers and parents, students clarify and solidify understanding of mathematics and develop confidence in themselves as mathematics learners. Six, students learn to use and interrelate the use of tables, charts, graphs, manipulatives, equations, computers and calculators (New Jersey Mathematics Curriculum Framework, 1996).

## Categories of mathematical communication

Communication in mathematics classrooms is usually categorized based on expressive (speaking, writing) and receptive (listening, reading) functions of language (Thomas, Garderen, Scheuermann \& Lee, 2015) though, it can also be based on expressions and organisation of ideas and thinking (eg., clarity of expression, logical organization), oral, visual, and written forms (eg., pictorial, graphic, dynamic, numeric, algebraic forms), audiences (eg., peers,
teachers) and purposes (eg., to present data, justify a solution, express a mathematical argument in oral, visual, and written forms) and use of conventions, vocabulary and terminology of the discipline (eg., terms, symbols) (Ontario Ministry of Education, 2005). Among the major expressive and receptive forms of communication speaking and listening are developmental skills and reading and writing are learned skills. Understanding these components of communication helps in better appreciating the role of communication in mathematics teaching and learning. Such understanding benefits students who struggle to learn mathematics more than others (Thomas, Garderen, Scheuermann \& Lee, 2015).

## 1. Listening

Thomas, Garderen, Scheuermann and Lee (2015) observes that research on mathematics and listening is almost nonexistent, apart from those attempting testing of cognitive capacity including oral recall and listening comprehension, despite the fact that listening is a critical foundational skill as a major part of teaching and learning in mathematics is based on listening to teachers. Listening skill is required to follow demonstrations of procedures and orally presented directions for tasks and assignments. Listening skills is required also to follow and evaluate peers during whole-class and small-group discussions.

## 2. Reading

Learning to read mathematics takes work (Esty, 1992). The National Reading Panel (2000) identified (a) decoding, (b) vocabulary and c) comprehension - as important aspects of learning to read. Reading mathematics text requires decoding and comprehending words, signs and symbols including pictures which may refer to mathematical operations and
expressions. Since pictorial language uses visual models to communicate (Kenney, 2005), they need to be learned much like the 'sight' words in other natural languages (Barton \& Heidema, 2000). Sight vocabulary is not limited to individual letters, letter combinations and word parts but extents to numbers, operations, variables, concepts and equalities/inequalities (Thomas, Garderen, Scheuermann \& Lee, 2015).

## 3. Comprehension

Comprehension requires conceptual understanding in mathematics. Mathematics text is the most difficult content material to read in comparison to text in other content areas, as the former is conceptually dense, with more ideas, and more complex ideas, embedded in minimal amounts of text (Barton \& Heidema, 2000). Achieving good comprehension of mathematics text requires students to draw on prior knowledge (Graham \& Perin, 2007) of mathematic vocabulary and symbols (McNeil \& Alibali, 2005; Thomas, Garderen, Scheuermann \& Lee, 2015; Thompson \& Rubenstein, 2000).

To facilitate reading in the mathematics classroom, mathematics teachers need not be reading specialists (Kenney, 2005). However, they do need to recognize that students need their help in reading mathematics texts in contexts and make the strategic processes necessary for understanding mathematics explicit to students, help students to acquire vocabulary and to read word problems for meaning. Students are helped when they don't understand the text by asking them questions so that they internalize such questions to use them on their own (Metsisto, 2005). Teachers usually don't see literacy as part of their skill set; nor do they appreciate that reading a mathematics text or problem is really very different and require specific strategies unique to mathematics. On
the other hand, most reading teachers do not teach the skills necessary to successfully read in mathematics class. Teachers can model their thinking out loud as they read and figure out what the problem is asking them to do; they can do dialoguing with students about any difficulties students may have in understanding a problem (Metsisto, 2005).

Metsisto (2005) have elaborated on the special reading requirements for mathematics text. Mathematics texts contain more concepts per sentence and paragraph and are written in a very compact style; with a lot of information and with little redundancy. They contain words as well as numeric and nonnumeric symbols to be decoded. Graphics which at times even may be distracting also must be understood for the text to make sense. Page layout of mathematics texts at times require eye to travel in a different pattern than the traditional left-to-right pattern. Students often fail to differentiate among problem statements, explanatory information and supportive prose. Asking students questions about the text structure can help them to focus on the idea that texts have an underlying organization, that different texts may have different structures and that it is important to analyze the structure of the text being used (Metsisto, 2005).

## 4. Speaking

Coming to Expressive aspect of mathematics language, especially speaking, there is a need for students to be provided with opportunities to communicate about mathematics through various partner, small-group and large-group discussions (Draper, 2002). While written or oral communications are important, class discussions are especially important "for developing the ability to formulate problems, to explore, conjecture, reason
logically and to evaluate whether something makes sense" (NCTM, 1991). Discussions help students to make sense of mathematics and to develop mathematical proficiency (Baxter, Woodward \& Olson, 2005; Thomas, Garderen, Scheuermann \& Lee, 2015). Developing mathematical ways of talking requires rich opportunities for students to explain their thinking as in structured pair or group work (Barwell, 2008).

## 5. Writing

Writing in mathematics is a large part of its learning. Writing about mathematics positively impact cognition (Pugalee, 2004). Writing in mathematics demonstrate and create comprehension (Barlow \& McCrory, 2011), clarify and analyze own thinking and hence help important metacognitive processes (Pugalee, 2004), reveal student misconceptions and levels of selfefficacy and hence deepen understanding and improve engagement (Barlow \& McCrory, 2011; Baxter, Woodword \& Olson, 2005; Thomas, Garderen, Scheuermann \& Lee, 2015). Students keep records, solve problems, explain ideas and describe their learning process in writing (Urquhart, 2009).

Mathematics writing requires hard work and planning on the part of the teacher. It takes time and practice to develop. If teachers facilitate this type of learning, they come to know their students well. New variations emerge as students reflect on the solutions of their classmates. For mathematics writing to be effective, problem must be appropriate for the students. Students need to develop confidence in their ability to respond to the problem and must feel comfortable sharing their answers in discussion with the whole class (Tuttle, 2005).

Ontario Ministry of Education (2006) observes the following on the importance of oral and written communication in the mathematics classroom. If students participate in talking, listening, questioning, explaining, defining, discussing, describing, justifying, and defending in an active, focused, and purposeful way, they better understand mathematics. Written communication enables students to think about and articulate what they know. Mathematical writing also provides evidence of students' mathematical understanding. However, before any writing task, students need experiences in listening to others' ideas and expressing orally as quality of writing is significantly improved by participation in a class dialogue before writing (Ontario Ministry of Education, 2006).

## Summary Remarks on the Structure of and Approaches to Language of School Mathematics

Mathematics and the language of mathematics are not the same. When it comes to language of mathematics, there are multiple uses of the term language. It denotes 1) language used in classrooms, in the home and community, 2) language used by mathematicians, 3) language used in textbooks, and 4) even that in test items or 5) language as a socio-cultural-historical activity. Moschkovich (2012) has observed that it is crucial to clarify how we use the term, what set of phenomena one refer to, and have a focus on. The mathematics register draws on a range of modalities, constructing meaning by deploying multisemiotic resources that interact with each other (Schleppegrel, 2007). In classrooms, mathematical discourse has a social dimension, the particular ways that students and teachers talk in mathematics classes. Though these ways are not specifically mathematical, they are still associated with mathematics (Barwell, 2008).

Essentially, there are three ways in which language influences mathematics education. 1. nature of natural language applied in mathematics classrooms 2. special language and grammar of mathematics itself 3 . language in multi linguistic contexts as in non-English classrooms where English words, letters, abbreviations, are used in addition to natural language. Thus three areas of concern for more substantial and coordinated research effort are 1) identifying linguistic competences and knowledge required for participation in mathematical practices, 2) the processes and mechanisms by which students develop linguistic competence and knowledge in mathematics and 3) knowledge and skills teachers need and apply in order to support the development of students' linguistic mathematical competence (Morgan, Craig, Schuette \& Wagner, 2014). Though these issues can be universal to school level mathematics education, the answers to them need not be as universal like the issues themselves. This lack of universality arises because mathematical communication is not culture free (Novotna \& Moraova, 2005), and is affected by linguistic peculiarities and pedagogic conventions and practices of different cultures. This study is especially to equip the stakeholders to understand and appreciate the difficulties emerging from deep and surface structure of mathematics language as used in Malayalam medium schools of Kerala where this study is situated.

## Objectives, Strategies and Issues of Language of Mathematics in Schools

This part discusses learning objectives and strategies in relation to language of mathematics in schools and sums up by identifying persisting issues connected with language of mathematics in three sections. Section 1, learning objectives in relation to language of mathematics, section 2, instructional strategies for language of mathematics and, section 3 , issues arising from the language of mathematics in
elementary schools. For the sake of supporting the emphasis on mathematics communication and acquisition of related language skills in school, the review of literature available globally, attempts to bring together language related objectives of teaching mathematics, and steps that teachers can take to create and adopt appropriate environment and strategies to implement mathematics language skills in schools and instructional goals and suggested instructional strategies /techniques from existing literature in relation to LSRW (listening, speaking, reading \& writing) skills in mathematics at school level.

## Learning objectives in relation to Language of Mathematics

Hill-Bonnet and Lippincott (2010) uses the term language function to refer to the measurable verbs embedded in objectives and ways (eg., classifying, describing, explaining, interpreting, and comparing) to engage students in both receptive (eg., listening, reading) and productive language skills (eg., speaking, writing) to increase mathematics understanding. Language function is defined as "basically the purpose or reason for using language in a learning task." and expects teachers to specify the language function in a written objective or learning outcome (Lim, Stallings \& Kim, 2015).

Objectives of learning mathematics involve, in part, acquiring precise use of mathematical language (Moschkovich, 2012). The NCTM (2010) refers to mathematics as a language of communication. NCTM (2010) stipulates that instructional programmes from pre-kindergarten through grade 12 should enable all students to communicate their mathematical thinking coherently and clearly to peers, and others and use the language of mathematics to express mathematical ideas (Leshem \& Markovits, 2013). Niss (2003) identified that to master language of mathematics means to understand and interpret oral and written expressions by others and to speak and express oneself orally and in writing.

Acquiring mathematical competence also include competencies in handling mathematical symbols and formalisms and communicating in, with, and about mathematics. Handling mathematical symbols and formalisms involves decoding and interpreting symbolic and formal mathematical language, and understanding the formal mathematical systems, in turn include both syntax and semantics and its relations to natural language; and ability to translate from natural language to formal/symbolic language and ability to handle and manipulate statements and expressions with symbols and formulae. Communicating in, with, and about mathematics requires understanding written, visual or oral 'texts', in a variety of linguistic registers, and expressing oneself, in oral, visual or written form, about such matters (Niss, 2003). Language skill is one type of mathematics skills along with number fact skill, arithmetic skill, information skill and visual spatial skill (Tambychik \& Meerah, 2010). One focus in mathematical discourse is students' ability to communicate by clarifying and justifying their ideas and procedures (NCTM, 1991).

## Objectives of mathematics language related instruction

Without basic understanding and fluency with vocabulary words, the purposeful and effective use of the language of mathematics will likely not occur (Riccomini, Smith, Hughes \& Fries, 2015) and hence objectives of teaching vocabulary in school level are;

1. To help students become fluent and maintain the word meaning over time (Riccomini, Smith, Hughes \& Fries, 2015).
2. To easily and accurately use the language of mathematics to explain and justify mathematical concepts and relationships (Riccomini, Smith, Hughes \& Fries, 2015).

Beyond, fluency in using vocabulary, instructional programmes focusing on mathematics language related instruction from prekindergarten through grade 12 should enable all students to listen, think and reflect, respond, communicate and argue, analyze, and evaluate on grade appropriate mathematics content. Organize and consolidate mathematical thinking through communication. Accordingly, Mathematics language related instruction is to enable learners to:

1. Become better at listening, paraphrasing, questioning, and interpreting others' ideas (NCTM, 2000).
2. Be aware of, and responsive to, their audience and to be aware of whether they are convincing and whether others can understand them.
3. Examine the methods and ideas of others in order to determine their strengths and limitations.
4. Understand the role of mathematical definitions and should use them in mathematical work.

In relation to thinking, mathematics language instruction is to enable learners to:
5. Gain proficiency in organizing and recording their thinking (NCTM, 2000).
6. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
7. Analyze and evaluate the mathematical thinking and strategies.
8. Consider, evaluate, and build on the thinking of others.
9. reflect an increasing array of ways to justify their procedures and results
10. Question and probe one another's thinking in order to clarify underdeveloped ideas.

In relation to communication, mathematics language instruction is to enable learners to:
11. Learn to explain their answers and describe their strategies.
12. Learn to communicate in more-formal mathematical ways, using conventional mathematical terminology, through the middle grades and into high school.
13. Practice communication and express themselves increasingly clearly and coherently.
14. Develop an appreciation of the need for precise definitions and for the communicative power of conventional mathematical terms by first communicating in their own words (NCTM, 2010).
15. Use the language of mathematics to express mathematical ideas precisely.

In relation to mathematical arguments, mathematics language instruction is to enable learners to:
16. Acquire and recognize conventional mathematical styles of dialogue and argument.
17. Internalize standards of dialogue and argument so that they always aim to present clear and complete arguments and work to clarify and complete them when they fall short by the time students graduate from high school (NCTM, 2010).
18. Become more mathematically rigorous and increasingly state in their supporting arguments the mathematical properties they used.
19. Become proficient in constructing and articulating viable arguments and conjectures using informal and formal language of mathematics, asking

## 72 <br> EVIDENCE BASED INSTRUCTION OF LANGUAGE OF MATHEMATICS

students to perform linguistically complex tasks that involve more than learning new vocabulary (Simpson \& Cole, 2015)
20. Write well-constructed mathematical arguments using formal vocabulary Essentially, students need to learn the language used in mathematical and mathematics education settings (Morgan, Craig, Schuette \& Wagner, 2014), though the goal need not be limited to these.

## Instructional strategies for language of mathematics

Instructional strategies reviewed from literature in connection with teaching learning of language of mathematics in schools are classified broadly into vocabulary strategies used in mathematics teaching learning and strategies to enhance mathematics communication in classrooms and presented accordingly hereunder.

## Vocabulary strategies used in mathematics teaching learning

Strategies or techniques obtained from the literature in connection with teaching - learning of mathematics, especially its vocabulary, could be categorized as strategies that make meanings of exclusive technical terms, selection of a class of terms, building up vocabulary, and hands-on vocabulary strategies. Each technique is described briefly focusing specifically on the focus, procedure of the strategy and the roles of teacher and learner while they applied for vocabulary learning. Most of the strategies here under more or less reflects vocabulary instruction (Marzano, Pickering \& Pollack, 2001; Salinas \& Ortlieb, 2011), where students encounter words in context more than once, learning those words in context, and associate an image with it, especially using direct instruction on words that are critical to new content.

## Single or few words strategy

These strategies have in common the selection of a vocabulary word from a set of words in the lesson, students analysing the meaning of words by its parts or examples and depicting it with select graphic summaries, with the help of teacher.

## 1. Concept of definition map (Mink, 2016)

Concept of definition map graphic organizer (Schwartz \& Raphael 1985) helps students make connections with words by teaching definitions of vocabulary words used in mathematics. This is done by outlining a variety of ways and settings for students to learn the meaning of a word, making use of students' senses and their prior knowledge to learn new word meanings. The characteristics of the new term are analyzed, including through simple definition (what is it?), comparative descriptions (what is it like?), and examples of the new term (what are some examples and illustrations?) to promote long-term memory and to personally connect with the word. The steps followed are 1) determine the words that students will not understand, 2) Select one of these words and write it on the board, 3) make an overhead transparency of the concept of definition map and write the word at the center of the map, 4) work as a class to complete the map and encourage students to use all their senses to understand the new word, 5) ask the questions- what is it?, what are some things you know about it?, what is it like?, what are some examples of it?, 6) assign students a passage of mathematics text that incorporates the new word, encouraging them to add any new information to their maps. Teachers have to take care to allow time for students to share their maps and write examples on the board of good definitions and analogies that students have generated.

## 2. Alike and different

Alike and different strategy best utilized after students have had some exposure to a chosen list of vocabulary words, makes students examine either orally or in writing the ways in which selected vocabulary words are both alike and different by analyzing the key aspects of the words and make connections that deepen their understanding. Teachers have to provide students time to discuss the words, the connections that exist among the words, and the reasons why students identified those connections. The procedure include 1 . choosing a list of vocabulary words associated with the content of lesson being taught, 2) pairing the words in a way that makes sense by seeing how the words are both alike and different, 3) writing the word pairs on the board or overhead, 4) students record each word pair on the activity sheet, 5) teacher reading the first pair of words aloud and students repeat the words, 6) asking students to tell what they already know about each word pair, 7) students thinking about and discuss how the word pairs are alike- independently, in pairs, in small groups, or as a class and keeping a record their ideas in the correct place, 8) students thinking about how the words are different and record their ideas as earlier, 9) repeating steps 4-7 with the remaining pairs of words and 10) reviewing the word pairs as a class and talk about how the words are both alike and different, when the activity sheet is completed (Mink, 2010).

## 3. Total physical response

This technique using physical movements as a way to acquire language skills in which students apply actions with oral language to concepts and procedures by performing the action while chorally saying the select word. This is done by 1) choosing a set of vocabulary words related to the mathematical
concept being covered in class, 2) writing the vocabulary words on the board or overhead, 3) discussing each word and choosing a physical action to represent itlike for 'circle' students joining fingers above their heads to create the shape of a circle with their arms, 4) demonstrating the correct action for students to selfcheck students, 5) practicing the actions either in pairs or in small groups, 6) calling out each vocabulary word, saying, "Show me __" for which students chorally repeating the word and demonstrating the correct physical action for the called word by standing around the room, and if needed and 7) displaying the vocabulary word on a word card while being called out to associate the written word with its verbal equivalent. If necessary, call out each word more than once. (Mink, 2010)

## 4. Root word tree

It is a graphic organizer that makes students to examine and decipher the meaning of the different parts of a single vocabulary word placed at the base of the tree by breaking the word apart into recognizable chunks and then writing words that are associated with the word parts to help students remember the definition. This Procedure include 1) Choosing a list of vocabulary words that are associated with the content lesson and writing them on the board or overhead, 2) providing students copies of the graphic organizer, Root Word Tree for each of the words, 3) students writing select vocabulary word (eg., pictograph) at the base of the tree, 4) the class breaking down the word into known word parts and write those parts (eg., pict and graph) on the large limbs of the tree, 5) students writing down other words with those same word parts (eg., picture and pictorial; autograph and photograph) on the branches, 6) students sharing and discussing their new words and ideas about the meaning
of the word and finally and 7) the class deciding on a definition that students write down beneath the tree.

## 5. Vocabulary flip book

It is a foldable flap that can be made using regular paper for organizing and defining key vocabulary terms where students write a word on the top of each flap and on flipped pages students draw pictures, write definitions, or create symbols to remind them of the meanings of the words. The main steps are 1) choosing a list of vocabulary words that are associated with the lesson being taught, 2) writing those words on the board or overhead, 3) distributing copies of vocabulary flip book, 4) on the top of each flap students writing each vocabulary word being studied, 5) the class, discussing the meaning of each word and symbols and pictures associated with it, 6) students drawing pictures, symbols, associated words, and definitions in the necessary spaces and 7) finally, reviewing their words with partners using the information they included in their vocabulary flip books, while teacher encourages them to discuss if and why the content under their flaps is different than their partners' (Mink, 2004).

## 6. Flashcards

These are index cards having a vocabulary term on one side and definition, and a visual on the other which permit fluency building through multiple exposures, with frequent but brief 5 to 10 -minutes activities. There are modifications like division of cards into quadrants; with the new vocabulary word is listed in the top right quadrant, definition in the bottom right quadrant and picture supporting the definition of the word in the left two quadrants and the students description of the relationship between the picture and the new term on the back of the card.

## 7. Mnemonic and keyword strategies, vocabulary diagram and mystery word

Mnemonic strategies provide students with a tool to anchor a new term with an already known similar-sounding word. Keyword strategies use picture representations that highlights the critical attributes of the new term. Vocabulary diagrams helps to clarify word's meaning by summarizing related words, drawings and examples, synonyms and antonyms, parts, sentence from text using the word, and original sentences into one place. In mystery word, a vocabulary word is selected from a list, on which the leader provides clues until the class is able to surmise what the word is.

## 8. Collection of words

These strategies are meant to stimulate vocabulary growth as the more exposure students have to the written word, the more their vocabulary increases. By generating lists of words, students become more sensitive to and aware of words and their meanings (Brummer \& Clark, 2013).

## 9. Mathematics word wall

It is a display of key vocabulary or concept words created on a bulletin board or on a large piece of paper taped to the wall. It is an effective way to keep track of new words students are learning and an easy reference for students, even better if students are involved with the creation and upkeep of the wall. There are many activities that can be done in a mathematics class using math word wall. 1) make a list - have students classify the mathematics word wall words by part of speech, roots, prefixes, suffixes, etc., 2) defining sentence - students create a
sentence for the assigned word that defines it, 3) what's at the end? - Identify and discuss words with similar endings, 4) be a mind reader - class members try to guess the word from given clues about a word like the beginning or ending letter, rhyme, the definition of its roots, prefixes, or suffixes, number of letters in the word, etc., 5) guess the covered word - students to guess which word belongs in a sentence, 6) find it first! - see which student can find it first and use it in a sentence, 7) seek and find - students to search newspapers, brochures, letters, business cards, etc., to highlight mathematics word wall words, and 8) crossword puzzles - students use the words on the mathematics word wall to make crossword puzzles, exchange the crossword puzzles, and solve them (Brummer \& Clark, 2013).

## 10. Vocabulary self-collection

Vocabulary Self-Collection strategy helps students to create a list of vocabulary words they would be interested in learning and researching. For this, 1) students create a list of words from their mathematics reading materials that are of interest to them, 2) students nominate one of the words to be studied by the class, 3) write these words on the board or on an overhead, 4) students define them and justify the selection, 5) meaning of each word and clear up any misunderstandings, using a dictionary if needed, 6) delete words that most students already know, duplicates of words, and words of little interest to the students, 7) students write down the selected words and their meanings in their vocabulary journal and post them on the mathematics word wall and 8) incorporate these words into lessons and writing activities. And, Encourage students to use these words as often as possible in their own writing to move the new vocabulary words (Brummer \& Clark, 2013).

## 11. Math hunt

Students are gathering objects and pictures related to specific mathematical concepts that deepen their understanding of mathematical vocabulary and gain direct and indirect experiences with the words. 1) create a list of approximately 10 vocabulary words both familiar and unfamiliar concepts related to the mathematics concept being taught, 2) groups of three to four students work on a math hunt list ideally one week before the concept is actually taught in class on which the pictures and objects are collected, shared and discussed, 3) provide a point system to use for objects they would likely to collect- 4 points for building a model or collage, 3 points for bringing in a picture, 2 points for finding a book or magazine, and 1 point for drawing a picture or writing a sentence, 4) students plan their "itemcollection strategy" and brainstorm ideas for each word, 5) teams organize their items by the word they represent and 6) share one item for each word on the list and total the points for each team with winning team creating a classroom display of vocabulary words and the items (Mink, 2010).

## 12. Picture and mathematics dictionaries

Pictures are connected with written descriptions in students' own words and the teams try to guess mathematics terms or concepts from pictures drawn by their teammates.

Providing access to mathematical dictionaries in the classroom and encouraging children to make use of them is helpful.

## Set of related words graphic organizers

A strong relationship exists between word knowledge and reading comprehension. Without word knowledge, readers read less and are more apt to
be poor readers (Anderson \& Freebody, 1982). Seldom do words stand isolated from and unrelated to other words. Students need to have a repertoire of strategies to use when they face unknown words in their reading. The strategies involving set of related words and graphic organizers reduce the language load by displaying information with pictures, phrases and labels and connecting related terms and concepts. These strategies use a visual picture of the many interrelationships, have students brainstorm aspects of the topic, and use a summary tool in which students take all the related concepts from a broader area like a unit. These are especially useful for students who struggle with written or verbal strategies.

## 13. Frayer model

Frayer model (Frayer, Frederick \& Klausmeier, 1969) is a graphic organizer often used when teaching vocabulary designed to help students understand relationships and similarities between concepts. The framework of the Frayer model consists of the concept word, the definition, characteristics of the concept word, with the key element of this model being an example of what the concept is and what it is not. It can be a sheet of paper divided into four quadrants with first quadrant defining a given term, the second listing any facts that students know about the word, the third listing examples of the given term, and the fourth listing non-examples (Metsisto, 2005). This requires 1) copies of the Frayer Model graphic organizer be distributed, 2) students have to write the concept of the lesson at the center oval as a concept phrase or a single word, 3) students to determine the clear, concise and easy to understand definition of this concept with their textbooks or a variety of resources, 4) students to determine the characteristics or attributes of this concept, 5) finally
the class determine what the concept is and what it is not and 6) assigning the students to write a paragraph about the concept using the Frayer Model graphic organizer. As a guide teachers have to encourage students to generate their own examples and non-examples and allow time for students to discuss their findings with the class (Brummer \& Clark, 2013).

## 14. List-group-label

List-group-label (Taba, 1967), a classification strategy that can be done at the beginning of a lesson to introduce students to new words and concepts or following a lesson as a review of concepts, encourages brainstorming to categorize and organize mathematics vocabulary. It helps in seeing hierarchical relationships between words, as well as word parts and word associations since most words are associated with other words, and grouping these words in meaningful ways clarifies understanding of words and their meanings by bridging between students' background knowledge and the new mathematics vocabulary. The procedure is 1) selecting a word or phrase that describes the topic of the lesson which is written on the center of the board or a transparency, 2) students thinking and listing words that are associated with this word specifying the connection to the focus word, 3 ) after a list of 20-30 words is generated, students individually, with partners, or in small groups cluster the words into categories based on attributes, characteristics, or features that the words have in common, and assign each category in the list-group-label activity sheet and label the cluster accordingly and 4) groups sharing its version of categorizing and organizing the vocabulary words and discussing until an agreement can be reached on the categories, labels, and the respective words included in each category. Keep the students focused on words and categories that are directly related to the lesson objectives. Teachers may bear in
mind that students may need to generate further categories to group all of the words that some words may need to be eliminated if they do not fit into the categories, students be encouraged to explain and justify their decisions for the selected categories and labels and placement of words in the categories (Brummer \& Clark, 2013).

## 15. Content links

Content Links is a strategy best utilized after students have had some exposure to the chosen vocabulary words that helps students to see how vocabulary words are connected to each other through meaningful conversations and discussions about the vocabulary words and relationships between the words. In this strategy, 1) choose a list of vocabulary words to use of which students are already familiar in meanings, 2 ) write one each vocabulary word on enough number of note cards for each student to receive a different word, 3) hold up each word card and read the word on the card and give clarification on meaning if necessary, 4) each student receives a card with a vocabulary word written on it, 5) class mingles and each student finds someone with whom they can link based on the vocabulary word on his or her card, for which there is not just one correct answer, and students can make their own decisions about why the words are linked and 6) at the end of each round of mingling, students are invited to share their thinking with the class (Mink, 2010).

## 16. Word trails

Word trails strategy build connections or "trails" from newly introduced words to familiar ones. The bridges or "trails" can be the following. 1) root words knowing which can help students determine meaning, 2) prefixes and suffixes, 3) synonyms or similar words and 4) antonyms (Brummer \& Clark, 2013).

## 17. Vocabulary bingo (Frei, 2007)

In this activity, the students create their own boards where in 1) teacher writes the vocabulary words specific to the day's lesson on the board and gives each student a blank three-by-three grid, 2) students write one word in each square (but, there can be empty squares, may write previous mathematical vocabulary words, or write the words twice), 3) students draw small pictures or examples next to the location where they placed the word on their bingo charts, 4) the bingo learning activity where the teacher reads a description of the vocabulary word or shows an example or representative picture, 5) students locate the word and cover it with a marker, 6) teacher monitor that students are covering the correct words, 7) "winner" can review the words and definitions out loud for the benefit of the whole class, the winner being one who mark off all their words.

## Instruction or activities

In these strategies, students read write, edit and share sentences and reinforce proper vocabulary use each time they say the words in sentences. Sharing Mathematics helps students to develop reasoning skills as they listen for correct vocabulary usage in sentences.

## 18. Possible sentences

Possible sentences strategy is a way to teach vocabulary words introduced in a text that help students to make predictions about new words while reading, providing a purpose for reading, and encouraging interest in text and use the text to rewrite and refine their predictions (Brummer \& Clark, 2013). For this, make a list of important vocabulary words from the mathematics text and write them on the board or transparency. Model correct
pronunciation aloud. Students select two words from the list to use in one sentence that might appear in a mathematics text. Record sentences on the board and underline each vocabulary word. Encourage students to generate sentences until all the vocabulary words have been used in at least one sentence. Remind students to edit and revise their work. Next, students read the selected text and compare the class sentences with the actual sentences in the text. Students examine the sentences to see if they have written accurately. Students edit and revise sentences as needed and then write revised sentences independently using their new knowledge and understanding of vocabulary words (Brummer \& Clark, 2013).

## 19. Chart and Match

Write the vocabulary words specific to the day's lesson on the board. Students have a three-column grid labeled with word, illustration or example and definition or description. Words are written down the left side of the grid row by row. Teacher then introduces the words and leads a whole-class discussion with examples and draws pictures and do the same in the middle column of the grid, against the vocabulary word. In the final column, the class decides on a description or definition of the word but not a dictionary definition. After reviewing the finished grid, the students cut up the squares on which the word, the picture or example, and the definition or description are written or drawn. After each student receive one piece from one cut-up completed grid, they walk around, read their pieces to other students, and trade cards. Each group of three (one student with the word card, one student with the illustration or example card, and one student with the description or definition card) will stand together and present the vocabulary word to the class (Frei, 2007).

## 20. Which statement is inaccurate? (Frei, 2007)

A vocabulary word specific to the day's lesson is used in four written sentences, numbered one to four, of which three accurate plus one inaccurate sentences are displayed on an overhead projector. 1) students work in groups of four to read the sentences aloud, deciding which sentence is inaccurate, 2) showing answer number with their fingers hiding it from their group, 3) individuals show his or her conclusion to the team, 4) the team discuss the answer in order to reach a consensus and write down the answer on a small whiteboard or piece of paper, 5) when all teams have reached consensuses, each team display its answer to the rest of the class and the class discuss each team's results and finally 6) teams convert the inaccurate sentence to an accurate sentence.
21. Sharing markers (Frei, 2007)

This is a sharing activity where everyone shares vocabulary sentences equally by giving three to four markers to each student, who then form small groups of three to five students and every time a student says a sentence with a vocabulary word in it, he or she gives away a marker until all students have used all of their markers, and once a student's markers are "spent" he or she is not allowed to say anything more until all students have "spent" their markers.

## 22. Sentence frames for vocabulary (Frei, 2007)

Procedure for sentence frame for vocabulary is, 1) first, the teacher shares the vocabulary words specific to the day's lesson, 2) then teacher shares a simple sentence that frames the vocabulary in a proper mathematical context with blanks in which the students will substitute information, 3) the teacher models complete
sentences using the vocabulary, for example: The equation "_" is equivalent to the equation "_" and writes sample answers to put in the blanks, like ( $3+5=8 ; 6-2$ $=4 ; 4+0=4 ; 9-1=8) ; 4$ ) students work in pairs to practice orally rehearsing the vocabulary with the right substitutions like in example "The equation $3+5=8$ is equivalent to the equation $9-1=8$ " and "The equation $6-2=4$ is equivalent to the equation $4+0=4$ " and 5 ) students share the answers they came up with.

## 23. Holistic strategies

These use writing, speaking and interaction more than focus on select vocabulary and has more in common with strategies that focus on communication of language than vocabulary strategies.

## 24. Writing strategies

These include 1) student journals, 2) take-home problem-solving assignments where students fold their paper vertically down the middle and on the left, they record their problem-solving work and on the right, they write explanations of their thinking and 3) blend written descriptions with visual images where students write definitions and draw or identify examples and nonexamples.

## 25. "Say something"

Here, partners reading mathematics exposition stop intermittently to share aloud emerging understandings, comments, and questions.

## 26. Use computer technology

ICT enables visual and auditory stimuli and interactive simulations. These include apps, streaming audio and video, software programs, computer
simulations, video and audio demonstrations, and graphics programs (eg., graphing calculators).

## Strategies to enhance mathematics communication in classrooms

A variety of explicit and implicit, immediate as well as long term outcomes of instructional practices that purposefully provide students opportunities to listen, speak, read and write and discourse in mathematics are recognized in literature. Obvious direct effects of including mathematics language skills in instruction is that students acquire the ability to listen, speak, read and write mathematics and acquire the ability to engage in mathematics discourses (NALDIC,2002). Beyond those immediate outcomes, exercising and developing the skills in listening is proposed to help learners to become critical thinkers about mathematics (NCTM, 2010) and to negotiate the symbols, diagrams, and technical language (Schleppegrell, 2007).

## Developing listening skills in mathematics

## 1. Listening to teachers and peers

Listening can be encouraged by teacher explanations that use mathematical language and vocabulary as well as non-mathematical explanations of mathematical ideas; and teacher and pupils exploring mathematical processes, reasoning and proving the solutions to a problem (NALDIC, 2002). Students need to listen closely to the thinking of others, take their ideas seriously." One aspect of taking students' ideas seriously is ensuring that their classmates attend to the ideas and work to understand them. This requires also that classroom activities should be structured to ensure that students have ample time and encouragement to process others' ideas (Clark, Jacobs, Pittman \& Borko, 2005).

## Developing speaking skills in mathematics

Speaking mathematics gives students opportunity to explain, develop and name mathematical theories, and promote greater clarity in their thinking and verbalization (Lee, 1997). Speaking with current understanding of mathematical ideas enables learners to become aware of, develop and reorganize their knowledge; to remember what they have worked with, and makes the knowledge available for them to use and control. Essentially, by speaking mathematics and involving in mathematical discourse, students learn to assign meanings to words and phrases which are shared within a community, learn mathematical concepts, and turn out to be self-confident of solving mathematical problems (Lee, 2006).

## 1. Inquiry environments and questioning

Skills in speaking mathematics require an inquiry environment in the mathematics classroom. This involves inviting students to share their strategies, pose questions, and "think out loud" (Clark, Jacobs, Pittman \& Borko, 2005; Cobb, Wood, Yackel \& McNeal, 1992). Effective questioning of pupils stimulates an inquiry environment. Give students time to think; and expect them to demonstrate and explain their reasoning. Another way is to explore reasons for wrong answers if any. Students can also be required to explore mathematical concepts; describe shapes, movements and constructions; explain calculation strategies and methods for solving problems; reason solution plans and justify results; compare different efficiency and effectiveness of different mathematical procedures; discuss which mathematical equipment and materials to use; and present their findings to an audience (NALDIC, 2002). Students need be involved orally and in writing in explaining solution processes, describing conjectures, proving conclusions and presenting arguments (Schleppegrell, 2007). Teachers have to model how to talk formally, for instance, by using numbers or symbols
instead of the pronouns and demonstratives (Pimm, 1987). NCTM (2010) also suggests formulating questions that puzzles students, then students presenting their methods for solving problems and justifying their reasoning to their peers and teachers in a coherent and clear manner as facilitating speaking in mathematics. Student-led discussions regarding the problems of the day and allowing for agreement and disagreement from others will encourage thoughts and interactions (Sample, 2009).

## 2. Think aloud

An important means to develop speaking skills is to create rich opportunities for students to explain their thinking (Barwell, 2008). Especially, children in the early grades may be made to "think out loud," to learn to explain their answers and describe their strategies through thoughtful questions that provoke reasoning. Such questions provoking them to reexamine their reasoning can be posed by a teacher or classmate, and as they gain experience of think aloud procedures, student proficiency in organizing and recording their thinking is enhanced (NCTM, 2010). By presenting their methods for solving problems, and giving verbal accounts and explanations, students gain insights into own thinking (Silver, Kilpatrick \& Schlesinger as cited in NCTM, 2010). "Think aloud" is possible even while explaining their solution strategies for multiplechoice and short answer items (Capraro \& Joffrion, 2006). Working out arithmetic aloud has other advantages too. This procedure allows the group to know what other members are doing and to check their own answer (Lee, 2006). Other methods to enhance thinking and speaking in mathematics advocated by Lee (2006) include stating and restating the problem, vocalizing the arithmetical workings, and challenging others' observations and providing answers when challenged and revealing when they feel uncertain about the solution to a
problem. Interacting in such a way that give access to reasoning of others scaffold mathematical thinking-speaking.

## 3. Code switching

Code switching is the most frequently used technique to connect everyday language and mathematics language. This involves alternation in use of more than one language in a single speech act. Code-switching can be between languages or between the mathematical and everyday registers. Teachers can switch codes in order to translate or clarify instructions and to reformulate and model appropriate mathematical language use. Students can be allowed to switch codes to seek clarification and to express their ideas or arguments (Zazkis, 2000).

## Developing reading skills in mathematics

## 1. Nine-stage instruction model

Through reading mathematics, students learn to recognize and comprehend terminology, numbers, mathematical symbols and expressions, formulae, charts, diagrams, tables and graphs. This in turn facilitates understanding the technical language of mathematics, patterns and relationships in mathematical problems. Reading and comprehending these various forms of mathematics representations essentially helps them to consolidate learning in mathematics (NALDIC, 2002).

Reading skills can be developed in mathematics problem solving situations. Nine-stage instruction model developed by Ilany and Margolin (2010) utilizes reading as a means for problem solving. This involves, among others, reading the problem repeatedly. (1) Initial reading is from words to the whole text as a way of
collecting details and understanding meaning. (2) Reading is for understanding the keywords, sentences and describing the problem in own words. (3) Understanding the mathematical situation through explicit or implicit data in the problem. Processing the literal information and changing it into a mathematical exercise or an algebraic equation is done using the literal clues like words that support (helpful clues) or the words that deceive (misleading clues) as clues for choosing the arithmetic operations needed to solve the problem.

## 2. Reading word problems

Another way to encourage reading using mathematics problems is to read word problems out loud, elaborating and commenting on what it says that encourage students to talk about the meanings (Adams, 2003; Lemke as cited in Schleppegrell, 2007). Meaningful reading can be facilitated via equipping learners to be strategic readers. Teachers can model the strategic reading process by reading the problem out loud (Metsisto, 2005). Strategic readers preview the text, look for title, the pictures, and thereby activate appropriate prior knowledge, vocabulary, and clarify their purposes for reading by asking questions on what to learn. While reading, strategic learners paraphrase the author's words, monitor comprehension, use context clues to figure out unknown words, and imagine, infer, and predict in and out of text to integrate new concepts with existing knowledge. Teachers can model raising questions relevant to mathematics problem (Metsisto, 2005).

## 3. Reading books in mathematics

Reading books in mathematics can emphasize quantitative language (eg., more, fewer, a lot, a little) and spatial language (eg., higher, lower, above, below, before, after) wherever possible by incorporating questions that asks for
responses in such language. Such prompting questions in reading books need be increasingly more complex to make children familiar with mathematical content and language before advancing to more in-depth prompts and discussions including distancing strategies. In using such reading books teachers can also ask questions that call for spontaneous responses and feedback in the classroom to reinforce children's understanding of the mathematical language used. Teachers may even explicitly define or explain mathematical language terms if their class demonstrates a lack of understanding of the terms involved in the questions or text. However, teachers need not explicitly define each word for the whole class. This will be achieved by learners through the context of the pictures, text, and questions. Such dialogic reading frameworks are known to use strategies like PEER (prompt, evaluate, expand, and repeat) strategy and CROWD (completion, recall, open-ended, wh- [what, where, why), and distancing during readings. Note cards placed at specific points in text book can also prompt dialogic reading (Purpura, Napoli, Wehrspann \& Gold, 2017)

## Developing writing skills in mathematics

## 1. Exposure to writing authentic genres in mathematics

Though students write a lot while learning mathematics, student exposure to writing different genres of mathematics is often limited. Redrafting the presented information or learning to prepare a mathematical argument or justification or formulate conclusions (NALDIC, 2002) are valuable mathematics learning outcomes in themselves. Writing helps learners to be metacognitive by reflecting on their work, clarifying their thoughts about the ideas, and re-reading the record of their own thoughts (NCTM, 2000).

Mathematics communication requires giving practice in writing authentic genres of mathematics like presentation of procedures, descriptions and classes
of things, explanations of judgments or findings, and arguments about theorems and other mathematical tasks (Schleppegrell, 2007) and correcting someone else's mathematical writing (Lee, 2006). Students need to be provided with a wide variety of writing samples relating to mathematics (Ryder \& Graves as cited in Brummer \& Clark, 2013). In this context, a dynamic and live bulletin board in the classroom that displays newer writing samples adds to curiosity about the new additions (Brummer \& Clark, 2013).

## 2. Common approaches

A variety of common sense approaches like providing plenty of time for students to experience the writing process, Offering daily writing opportunities, Allowing time for students to evaluate others' writing and receive teacher feedback, Encouraging learning new mathematics words, Focusing on students’ reading and writing on big ideas, providing opportunities to read, understand, and write about increasingly complex text (Corona, Spangenberger \& Venet as cited in Brummer \& Clark, 2013). Sample (2009) for example, provided approximately 10 minutes each day to give each students time to write out their solutions to the problem of the day.

## 3. Structured strategies

There are many structured strategies that support, and direct students writing on specific mathematics learning related tasks. In composing with key word's, students compose mathematically related sentences and paragraphs with words from Taxonomies (Rothstein, Rothstein \& Lauber, 2006). Another specific strategy is 'defining format' where a three-column format defines a mathematical term (eg., number), using a question, the category, and the characteristics. Yet another structured strategy is using profiles and frames.

Profiles are templates into which students plug appropriate information to solve a problem or explain a mathematical concept. In Frames, however, the students are given the syntactic structure, which includes stem or partial sentences. Completing these partial sentences helps students focus on the content of their writing without concern for grammatical or structural aspects of the text. (Rothstein, Rothstein \& Lauber, 2006). Worksheets with title, the learning target, concept notes with illustrative examples and vocabulary of key terms will also be useful (Rothstein, Rothstein \& Lauber, 2006).

## 4. Reflection writing

Students can also be asked to write their reflection on what they learned, quantity of work done during the lesson and to identify topics they did not understand and have found difficult (Lomibao, Luna \& Namoco, 2016). Such metacognitive frames enhances self-awareness of mathematical knowledge by making students write statements such starter phrases that you must complete, I know that I know something about..., First I know; In addition, I know; 'Finally, I know'; 'Now you know something that I know'.

## Developing discourse skills in mathematics

Discourses in mathematics classrooms apart from being helpful for teachers to evaluate students' ability to use technical language appropriately, develops in students a register of technical language of mathematics that in turn enables them to develop connections between the everyday meanings of words and their mathematical meanings (Schleppegrell, 2007). Posing rich open-ended and challenging tasks that promote discussions in classrooms encourage students to think collaboratively and build upon one another's ideas (Stein, Smith, Henningsen \& Silver as cited in Clark, Jacobs, Pittman \& Borko, 2005) which further enhances such mathematical discourse (Lee, 2006).

## 1. Think, talk, write, read and re-draft

Think, talk, write, read and re-draft strategy by Lee (2006) incorporates all four language skills into a single strategy. Strategy elements are as follows. 1) give time to think quietly by themselves for a few seconds and ask them to write five words that they associate with a concept in a whole-class questioning session. 2) discuss their ideas with one or more partners ('response partners' or 'study buddies') and make some decisions for between 30 seconds and a few minutes for an answer with a minimum length, depending on the question; rehearsing the answer helps pupils feel confident to add their contributions to a whole-class mind-map or spider diagram. 3) read to themselves and to other people and Re-draft based on feedback received, to improve their work and to produce high-quality communications and 4) write only when they have thought and talked about their ideas. Display pupils writings for everyone and ask them to consider which wording or phrasing expresses the ideas most clearly to help pupils towards fluency with mathematical language.

## 2. RPTMC activity flow

Yang, Chang, Cheng and Chan (2016) developed reciprocal peer tutoringenhanced mathematical communication activity flow (RPTMC Activity flow) where every two students were paired as a mathematical communication group. Four sub activities are creating, reciprocal peer tutoring, revising and staging.
i. Creating: required students to prepare tutoring materials involved four steps. 1) understanding the problem where Students read the word problem on their own tablet PCs and discuss the solution with their peers to understand the conditions given and the problem asked. 2) drawing a representation where Students use words, symbols, models, and
manipulative materials as their mathematical representations to devise a plan as well as to convey their ideas and communicate information. 3) writing a solution where Students write their mathematical equations for solving the problem and 4) explaining the solution were Students reflected on how and why they had solved the problem and explained their solution in writing. Because students may need guidance in learning how to express their mathematical concepts before they could write a complete sentence explaining their solutions, a text-based scaffold was provided.
ii. Reciprocal peer tutoring: Paired students sat together to reciprocally teach their mathematics creations. One student, who played the role of a tutor, taught his/her peer why and how to solve word problems by displaying mathematics creation in the 'sharing zone' (designed for easy display of mathematics creations). While the other student who played a tutee, received instruction with the tutor's mathematics creation in his/ her own tablet PC. Subsequently the tutee has to ask questions about solution strategy. The paired students then switch their roles.
iii. Revising: students had to revise their mathematics creation based on the peer feedback in the previous activity for improving the clarity of their own mathematics creations. Revising also served as a time for selfreflection and preparation for the next activity staging.
iv. Staging: Teacher encouraged the students in each group to display their mathematics creation of the whole class. They had to explain their solution with their representations to the audience. Then they had to answer questions asked by the audience. In the end, the teacher used students work to demonstrate how to explain the mathematics concepts and to clarify mistakes made by students for preventing similar ones in
future. Moreover, teachers may ask some relevant questions to promote students' thinking for communicating their own mathematical concepts and thinking with others.

## 3. Other discourse strategies

Other methods found in literature for encouraging discourse in mathematics classrooms include open-ended problem/questions (Lomibao, Luna \& Namoco, 2016; Wichelt, 2009), Revoicing (technique for interaction) (Chapin, O’Connor, O'Connor \& Anderson, 2009; Anthony \& Walshaw, 2010; Moschkovich as cited in Schleppegrell, 2007), leading mathematical conversations (Patkin, 2011), activity sheets, and classroom discussions (Schleppegrell, 2007).


Figure 1. A strategy for instruction that focus on the language of mathematics

This review helped to finalize a strategy for instruction focusing on the language of mathematics as in Figure 1, by realizing that teachers and educators increasingly recognize mathematics as a language with oral and written communication with its own vocabulary, symbols, and concepts, rules and conventions. Listening, comprehending and speaking mathematics also require special skills. This requires that facilitating precise communication in and about mathematics needs to be an important part of learning mathematics in schools. Despite this, other than vocabulary instruction, globally, developing language skills in mathematics like listening, speaking, writing and speaking, though not to the same extent, are largely neglected in teaching and learning of mathematics in schools, but especially in multilingual classrooms in countries including in India.

## Issues arising from the language of mathematics in elementary schools

Most of the issues mentioned in literature as arising in relation to language of mathematics in school are at word level, and practically every example cited here under is from English language context.

## Mathematical meaning is more precise than ordinary language

Many terms shared with English have comparable meanings, but the mathematical meaning is more precise though they sound like everyday English words (Thompson \& Rubenstein, 2000) or everyday English terms have different meanings in mathematics classrooms (Council of Australian Governments, 2008; Meiers \& Trevit, 2010). Even familiar words, and 'sub technical vocabulary words' that have a common meaning, but also have a mathematical denotation that must be specifically taught because mathematics has many everyday words that have specific mathematical meanings (Irujo, 2007). These are terms used in everyday English, which have different or much more specific meanings in mathematics. Meanings differ in mathematical language and natural language. Terms mean
different things in mathematics and non-mathematics contexts, not only because two different words sound the same, but also because more than one word is used to describe the same concept (Kenney, 2005) as in 15 minutes past vs. quarter after (Rubenstein \& Thompson as cited in Riccomini, Smith, Hughes \& Fries, 2015). Such multiple meanings can create obstacles in mathematical conversations because students often use the colloquial meanings of terms, while teachers and other students may use the mathematical meaning of terms (Moschkovich, 2012).

## Even familiar words have special meanings

Many familiar words are assigned special definitions in mathematics as in "similar" and "prime" (Heuer, 2005), face, take away, match, odd, lots of, product, difference, mean, volume, value, integrate (Lee, 2006), cone as in the shape vs. cone as in what one eats (Thompson \& Rubenstein, 2000). For example "similar" means "alike" in everyday usage, whereas in mathematics it means that the ratios of the corresponding sides of two shapes are equivalent and corresponding angles are equal. Thus in everyday English, all rectangles are "similar" because they are alike whereas in mathematics they are "similar" only if the ratio of the short sides equals the ratio of the long sides. Prime, median, mean, mode, product, combine, dividend, height, difference, example, and operation all have distinct meanings in mathematics (Metsisto, 2005).

## Sub technical vocabulary

Another example is the sub technical vocabulary. "true" that in everyday language means accurate, the opposite of false, but has a technical definition in mathematics problems of a number sentence where the value to the left of the equal sign is the same as the value to the right (Meiers \& Trevit, 2010). Clearly, primary school teachers have to anticipate possible confusions when using such words as
these (Haylock, 2007). For example, in relation to subtraction the 'difference between 8 and $13^{\prime}$ is not that one has one digit and the other has two digits. Other familiar examples would include: 'volume' (in everyday English used mainly for levels of sound); and 'right' as used in 'right angle' (not the opposite of a left angle!). Mathematics uses 'odd' to refer to every other counting number, which is hardly consistent with the everyday use of the word (Pimm, 1987).

## Everyday language vs. language of mathematics

In ordinary everyday English, many mathematical words are misused or used with a degree of sloppiness. For example 'Sugar cubes' are usually cuboids, but not all of them are actually cubes. Adults do not mean a time interval of one second when they say, 'Just a second!' The phrase 'a fraction of the cost' uses the word 'fraction' imprecisely to mean 'a small part of'. The word 'half' is often used to mean one of two parts are not necessarily equal. Moreover, many teachers themselves use mathematical language carelessly, such as confusing 'amount' with 'number', or using 'sum' to refer to a calculation other than an addition (Haylock, 2007). Or, students and teachers may adopt informal terms instead of mathematical terms for example diamond vs. rhombus, the house vs. in the division bracket. Mathematical meanings are more precise, for example, "product" as the solution to a multiplication problem vs. the product of a company. specific challenges with translated words like mesa vs. table are also mentioned or related but different words circumference vs. perimeter are also notable (Rubenstein \& Thompson as cited in Riccomini, Smith, Hughes \& Fries, 2015).

## Special meanings of pronouns, prepositions, and conjunctions

Related to this issue is the use of pronouns, prepositions and conjunctions. In English there are many small words, such as pronouns,
prepositions, and conjunctions that make a big difference in student understanding of mathematics problems. The words 'of' and 'off' cause a lot of confusion in solving percentage problems, as the percent 'of' something is quite distinct from the percent 'off' something. The word can mean "any" in mathematics. When asking students to "show that a number divisible by 6 is even," teacher is not asking for a specific example, but for the students to show that all numbers divisible by 6 have to be even. When we take the area "of" a triangle, we mean what the students think of as "inside" the triangle. The square (second power) "of" the hypotenuse gives the same numerical value as the area of the square that can be constructed "on" the hypotenuse (Metsisto, 2005).

## Technical jargon.

Language one read and speak in mathematics class is actually a technical jargon (Hersh, 1997). For example, zero is not really a number in everyday language - "a number of books" in English never mean zero (or one, for that matter). But in mathematics, 0 and 1 are both acceptable answers denoting the concept of "a number." "Add" in English invariably mean that we are increasing something, In mathematics, however, addition can result in an increase, a decrease, or no change at all depending on what number is being added (Metsisto, 2005). May mathematical phrases must be learned and understood in their entirety (Thompson \& Rubenstein, 2000).

## Technical vocabulary are rarely used in everyday conversation

Technical vocabulary are rarely used in everyday conversation, specific to the subject area, found only in a mathematical context (Thompson \& Rubenstein, 2000) for example rhombus, hypotenuse and integer (Heuer, 2005); multiple, factor, trapezium, denominator, polygon, parallelogram, imaginary
number (Thompson \& Rubenstein, 2000). These also include Terms that have a meaning only in mathematical language like hypotenuse, isosceles, coefficient, graph, (Lee, 2006). Some words in mathematics are shared with science. Some have different technical meanings in the two disciplines (Thompson \& Rubenstein, 2000). These are technical words that are not usually met or used by primary school pupils outside mathematics lessons, 'Parallelogram' and 'multiplication'. Such words are not being reinforced in everyday usage. These mathematical terminology makes pupils to perceive mathematics as being something that happens in school that is unrelated to their everyday lives outside school (Haylock, 2007).

## Graphic representations

Graphic representations may be confusing because of formatting variations or because the graphics are not consistently read in the same direction, for example bar graphs vs line graphs (Kenney, 2005). Also, Shorthand or abbreviations are often used in place of the complete word or phrase, even if students must pronounce the entire word when verbalizing the shorthand (Thompson \& Rubenstein, 2000).

## Specialized symbols and expressions of mathematical language

Many Symbols may be confusing either because they look alike (eg., the division and square root symbols) (Meiers \& Trevit, 2010) or because different representations may be used to describe the same process as in $\bullet, *$, and $\times$ for multiplication (Kenney, 2005). The confounding potential of symbolic representation cannot be overstated. Younger students can be quite mystified by the fact that changing the orientation of a symbol-for example, from horizontal $(=)$ to vertical (\|)—can completely change its meaning (Kenney, 2005). With
symbols students face a multilevel decoding process as they must recognize and separate out the confusing mathematical symbols (eg.,,$+<$ ) without any phonic cues; then they must translate each symbol into English; and finally they must connect the symbol to the concept and then carry out the operations indicated (Metsisto, 2005). Also, technical symbols (logograms) signs standing for whole words have no sound-symbol relationship for students to decode, $\Sigma, \Delta$ (Heuer, 2005).

## Imprecise and ambiguous descriptions

Beyond terms, imprecise use and ambiguous descriptions of mathematical vocabulary causes misconception. For example description of rectangle as 'a shape with four right angles and two pairs of equal sides', could lead to children not recognizing that a square is also a rectangle, or not understanding that a rectangle is also a type of parallelogram and quadrilateral. A good definition should be complete and concise, for example 'a rectangle is a four-sided shape, all four of whose angles are right-angles' (Clissold, 2014).

## Semantax and syntax

There are issues emerging in language of mathematics beyond word level, though less studied. Semantax (a term that is used more and more frequently in linguistics to refer to the interrelationship of semantics and syntax) is based partly on word meanings (vocabulary/lexicon/semantics) and partly on "grammar", morphology, and partly on sentence structure (syntax) (Irujo, 2007).

Syntax of the conventional style causes problems for pupils engaging with mathematics. The conventional presentations of mathematics - deleting
personal reference and the consequent use of the passive voice - sometimes make complex syntax inevitable if the writer feels they must use the conventional, impersonal, passive voice then complex syntax is sometimes inevitable (Lee, 2006).

Statements and questions are understood differently when made in a nonmathematical context. Right angles are often drawn with one vertical line and with one perpendicular line extending from it to the right. When shown a right angle with the perpendicular line extending to the left, students doubt "Is that a left angle?" (Metsisto, 2005).

## Cognitive dissonance based on cultural dependence

Cognitive dissonance based on cultural dependence (Bagchi \& Wells, 1998) creates an expectation in the student that the object has properties different from the ones it actually has as the connotations of a word or phrase used to name a type of mathematical object sometimes create an expectation to the contrary. The suggestiveness of such names are inevitably culturedependent. For example, one wonders whether a word such as "clockwise" will convey anything to students twenty years from now. Mathematicians and other scientists are used to inventing their own terminology. The definition of a word such as "group" in a mathematics text may require students to abandon most of their previous understanding of the word and start afresh, the exact opposite of what is expected in a literature course; and students are seldom, if ever, told this. Mathematical authors are of course free to change the language (Bagchi \& Wells, 1998).

Lee (2006) observe that the use of metaphor to convey meaning is involved at every level of mathematical discourse like functions obey rules, and an equation is a balance. The idea that an equation is a balance works very well until negative numbers are included in the problems to be solved. If the pupil is relying on that metaphor, and not the underlying mathematical concepts about equations, then they will have no idea how to proceed when the metaphor breaks down. The persistent use of metaphor rather than simile (for example, a function is a machine rather than is like a machine) has great potential for confusion and misconception as pupils make their way into the mathematics register.

## Summarizing the Mathematics Language Related Issues, Objectives, and

## Strategies in Schools

The review of the available literature shed light on some relevant, immediate as well as long term goals of mathematics teaching when one considers mathematics as a language. Yet, these goals are generally neglected in schools. An emphasis on the language of mathematics and communicating through mathematics language in teaching engenders learning that is more meaningful, and conceptually integrated. More reading brings in better consolidated learning of mathematics. Discourses in mathematics help to develop a register of technical language of mathematics, making connections between everyday meanings of words and their mathematical meanings possible for students. Discourse driven mathematics classrooms are helpful for teachers as well, as they reveal students' ability in the area being discussed.

This review has brought together an array of strategies suitable for developing listening, speaking, reading and writing skills in mathematics among
school students. Attributes of classroom strategies that enhances student acquisition of mathematics language skills include teacher explanations, inquiry environment, oral and writing participation, modeling and scaffolding by teachers on the intended skills, formulating questions/puzzles that engenders discussion, opportunities for students to explain their thinking, code switching, strategic reading of problems, opportunity in writing authentic genres, reflective writing on mathematics learning along with common sense approaches and specifically structured strategies.

Obviously, integration of language into mathematics classroom will develop student skills in using technical language appropriately. Paying attention to the language of mathematics in classrooms, apart from the acquisition of listening speaking reading and writing skills, help learners; to become aware of, recognize, develop and reorganize their knowledge, to negotiate the language, to articulate their understanding, to consolidate their learning, to develop critical thinking about mathematics, to develop connections between mathematics and life, to think collaboratively and to build upon one another's ideas and to increasingly engage in mathematical discourses. Apart from structured and specific instructional procedures, instruction focusing on the language of mathematics frequently make use of exploring mathematical processes, talking, questioning, stating and restating problems and uncertainties, reasoning, thinking aloud, challenging others' observations and providing answers, building explanations, and justifying and the like in whole class and varied group environments. This in turn will help them to get greater clarity in their thinking and verbalization, and hence in mathematics communication. Improved communication among peers and with
teachers turn learners to meet cognitively and critically reflect on mathematics. Hence, an increased emphasis on communication through language of mathematics in schools will bring in for their students deeper engagement and understanding, greater independence and self-regulation and stronger competence with mathematical processes.

## Studies Related to Language of Mathematics

This part consists of studies related to language of mathematics - both survey studies and experimental.

## Survey Studies Related to Language of Mathematics

This section contains 29 survey studies related to the language difficulties in mathematics learning. They were broadly classified as 1 ) students' perception on language of mathematics and attendant difficulties in learning and 2) teachers' perceptions on language of mathematics and attendant difficulties in learning and instruction.

## Studies on students' perception on language of mathematics and attendant difficulties in learning

Nineteen studies here under reveals that research attention during the last two decades after the onset of new millennium on language factor in mathematics learning among school students largely focussed on either problemsolving skills in mathematics, especially in word problems, and factors influencing it, especially complexity of words or symbols involved. Apart from the relation of mathematics problem solving to linguistic complexity of items and influence of linguistic factors in testing, the research attention during this
period in the area of language of mathematics in schools were on students' ability to switch between symbolic and verbal expressions, lack of ability in understanding the content as well as instructional language, students' proficiency in their native language and English, difficulty with reading comprehension, the impact of reduced linguistic complexity, inadequacy of mere listing keywords to help learners understand the mathematics problem better and need for code switching during instruction.

Chow and Ekholm (2019) explored the language domain predictors of mathematics performance among 365 students in first and second grade by assessing their language ability and mathematics performance. Structural equation model revealed that syntax as the strongest predictor of mathematics performance among young children and vocabulary did not significantly predict mathematics performance. Vocabulary cannot be considered as an index of language ability among primary school students in the context of mathematics learning.

Rao, Ramaa and Gowramma (2017) studied the relationship between reading mathematics terminology, mathematical language and performance in mathematics on a purposive sample of 50 fourth grade students who learn mathematics in Kannada medium, using test of mathematics vocabulary reading (MVR) in Kannada and test of mathematical language (TML) in Kannada. The 60 -item TML group test was developed multiple choice, fill in the blanks and matching type items based on review of vocabulary in textbooks from standard one to four. The 60 -item MVR individual diagnostic test on number concepts, addition, subtraction, multiplication and division
consists of terms that explains a concept or give instructions to solve problems. There was a high correlation between language ability in mathematics, reading ability in mathematics and performance in arithmetic skills. Qualitative analysis of data revealed common errors in spatial terms namely, short vs. long, up vs. down, right vs. left, more vs. less, horizontal vs. vertical etc. Percentage of error is more in common words used both in general language and mathematics than in spatial terms.

Powell, Driver, Roberts and Fall (2017) investigated the mathematical vocabulary knowledge of elementary grade students to explore how general vocabulary knowledge and mathematics computation are related to mathematics vocabulary performance, particularly for students with varying stages of mathematics vocabulary knowledge. Sample of 193 students included 65 students from five 3rd-grade classes from one elementary school and 128 students from seven 5th-grade classes drawn from an intermediate school. Extend of attainment in general vocabulary, mathematics computation skills and mathematical vocabulary of students were measured. General vocabulary was measured using the subtest of Gates-MacGinitie reading tests, level 3 to third grade students and Level 5 to fifth grade students. Mathematics computation skills of students were measured using the math computation subtest of the wide range achievement test (WRAT) with 40 computation problems of increasing difficulty. For assessment of mathematical vocabulary tool was developed for grade 3 and 5. Vocabulary was identified a thorough analysis of textbooks and textbook glossaries of standard three and five rather than vocabulary used in assessment practices. More extensive set of terms that students see and use daily were included in the test. Through textbook analysis

133 novel mathematics-vocabulary terms were identified and 129 questions for measurement were developed. Terms were included in the vocabulary list based on five methods; 1) term appeared in the glossaries of both third and fifth grade mathematics curricula ( $\mathrm{n}=77$ ), 2) terms appeared in a glossary at third or fifth grade in addition to being explicitly named within the common core state standards ( $\mathrm{n}=22$ ), 3) terms featured in a glossary at third or fifth grade but not explicitly named in the $\operatorname{CCSS}(\mathrm{n}=6), 4)$ terms explicitly named within the CCSS but not named in a third- or fifth-grade textbook glossary (n $=16)$ and 5) terms not in a textbook glossary or explicitly named within the $\operatorname{CCSS}(\mathrm{n}=12)$. Investigators underlined the mathematics-vocabulary term of each question or prompt to emphasize that vocabulary. Three levels of questions for each vocabulary term viz., recall, comprehension, and use in complex tasks were considered for development of questions and prompts. Thus, vocabulary test comprised of items in which $42 \%$ of items featured recall questions, $27 \%$ featured comprehension questions, and $31 \%$ featured use-in-task questions. Each right answer received one score which makes a maximum score of 129 . Cronbach's $\alpha$ for the vocabulary test was 0.92 at third grade and 0.96 at fifth grade indicated internal consistency of the measurement tool. Performance in mathematical vocabulary was found comparatively low among third grade students than fifth grade students. Approximately $37 \%$ of the Variance in mathematics vocabulary performance of third grade students is accounted from general vocabulary and $45 \%$ of the variance from computation. Whereas at fifth grade, $43 \%$ of mathematics vocabulary variance was contributed by general vocabulary, and $39 \%$ of variance was contributed by computation skills. In both grades, general vocabulary and mathematics computation had statistically significant effects
on mathematics vocabulary. The relationships among general vocabulary, mathematics computation, and mathematics vocabulary were significantly stronger in the third-grade students than in the fifth-grade students. The study concludes that general vocabulary was a stronger predictor for third-grade students with lower mathematics vocabulary scores. Mathematics computation was a better predictor for third-grade students with higher mathematicsvocabulary performance. At fifth grade, mathematics computation was a stronger predictor for students with lower mathematics-vocabulary performance.

Cruz and Lapinid (2014) identified the difficulties encountered by students in translating word problems into mathematical equations among 204 grade five students. Errors in translating mathematical problem was identified by a 20 -item problem solving test involving the four fundamental operations with every operation consisting of five items. Students were asked to translate the given word problems into mathematical symbols. Findings of the study revealed that 42 percent of students are under the satisfactory level of performance in translating word problems to mathematics symbols, 20 percent belongs to very poor performance and 22 percent can be labelled as poor in translating word problems. Students' difficulties in translating mathematics word problems are classified under six categories viz., misinterpretation of the problem, lack of comprehension of the problem posed, incorrect use of operation, carelessness, interchanging values and unfamiliar words.

Vukovic and Lesaux (2013 ${ }_{\mathrm{a}}$ ) conducted a longitudinal study on ways in which language impacts students' mathematics performance; whether children's early language ability can predict their later performance in arithmetic, algebra,

## 112 EVIDENCE BASED INSTRUCTION OF LANGUAGE OF MATHEMATICS

geometry and probability, and whether the relation between these variables differ between native English speakers and language minority students. Language ability was measured in terms of general vocabulary and listening comprehension and the study hypothesized that children' capability to comprehend oral communication reflect their ability to comprehend mathematical content. Gain in arithmetic was measured using Computation subtest of the Stanford Diagnostic Math Test-Fourth Edition in terms of comprehending basic number system and their ability in basic arithmetic operations. Achievement in algebra is measured in terms of their ability to work with number sentences and representing mathematical relations. Achievement in geometry is measured in terms of their ability to analyze and compare shapes, problem solving and use of visualization to solve problems. Achievement in probability is measured in terms of ability in interpreting tables and estimating probability. General achievement and visual-spatial working memory were controlled. Children's language ability can predict performance in geometry and probability; but not in arithmetic and algebra after controlling their reading ability and visual-spatial working memory and this effect did not differ between native English speakers and language minority students. Children's mathematics learning will be influenced by their language ability, but it does not influence arithmetic procedures. As the early language experiences influences their later development in mathematics, intensive and targeted language instruction for students is inevitabile.

Vukovic and Lesaux (2013 ${ }_{\mathrm{b}}$ ) examined the relationship between general verbal ability and phonological skills with arithmetic knowledge among 287 third grade students. Students were individually assessed on verbal analogies, phonological decoding, symbolic number skills, and arithmetic word problems;
and group tested on procedural arithmetic. Analogy subtest of the WJ-III Reading Vocabulary test was used to measure general verbal ability in which students solve analogy read aloud by the examiner. Verbal analogies can be considered as a representation of overall verbal ability as it mirrors acquired knowledge and skill related to language. Word attack test of the WJ-III was used to assess phonological decoding in which students had to pronounce pseudo words that conform to English spelling rules. Phonological decoding was measured based on the assumption that encoding and manipulating numerical symbols and lexical symbols have similar processes. Number series subtest of the WJ-III Quantitative Concepts test measured symbolic number skills in which students have to use inductive and deductive reasoning to determine the next number in a sequence. Procedural arithmetic knowledge was measured using calculation test of the WJ-III: research edition which measures the extent to which students have learned addition and subtraction facts and are able to use procedural skills to solve whole number arithmetic problems. Skills in arithmetic word problems was measured with the Applied Problems test of the WJ-III: Research Edition in which students have to solve practical problems in mathematics. General verbal ability influences students' symbolic number skills, whereas phonological skills influence their arithmetic performance. It is concluded that even though mathematical thinking can be considered as language free, students need language to understand, express and learn mathematics, and that knowledge about student's ability to express, understand and learn mathematics is necessary to design interventions that addresses the needs of less performed students in mathematics.

Domingo, Molina, Canadas and Castro (2012) studied the errors in translation of algebraic statements among 26 secondary level students.

Objectives of the study was to develop an instrument that can be used to explore the translation process between the symbolic and verbal representation systems, to analyze and classify students' errors on such translations, and to explore the relation between verbal and symbolic representations of algebraic statements that students create. Instrument based on the game domino was developed in which students built the dominoes pieces individually by translating some statements from verbal to symbolic representations and vice versa. The game involves six statements which are presented verbally and the other six symbolically. Among the six statements of each type additive, multiplicative, powers are included. And the remaining three had the combinations of these relations. Higher frequency of errors (75\%) made in translation from verbal to symbolic representation were mostly due to the special features of algebraic language. The most frequent errors while translating from symbolic to verbal expression were due to misperception of the power and product operations (41\%).

Jegede (2011) studied the use of code switching in mathematics teachinglearning process in primary schools in Kenya. Five mathematics teachers and fifty students from five primary schools were purposively sampled and data collected from them using questionnaire, structured and unstructured interviews and participant observation. Teachers interview collected data on language usually used during mathematics teaching, use of code-switching as a strategy and the reason for it, and implications of using code-switching as a communicative approach in mathematics teaching-learning process. Students were also interviewed to elicit data on their preferences for code-switching and reasons for their view. Use of code switching by teachers allowed students to use their language in a meaningful way in classroom activities and it is an efficient way in multilingual classrooms to ensure meaningful attainment of mathematical concepts.

Barbu and Beal (2010) studied the effect of linguistic complexity in questions and difficulty in solving mathematical word problems among 41 grade 6 and 7 students. Eight mathematics word problems were included in the test which consists of two-word problems having easy mathematics problem presented in easy linguistic structure, two-word problems having easy mathematics problem presented in complex linguistic structure, two-word problems having hard mathematics problem presented in easy linguistic structure, and two-word problems having hard mathematics problem presented in complex linguistic structure. Mathematics problems including single digit addition and multiplication was considered as easy word problem and those with multi digit multiplication and division was considered as difficult mathematics word problems. The linguistic complexity of word problems was changed by varying the lexicon and syntactic structure of the problem. However, the overall word count was kept constant. Individual interview was used as a method of data collection. Students less performed in mathematics problem solving when the text of the word problem had advanced vocabulary and complex syntactic structure compared to problems in easy linguistic structure while the actual mathematics operation to be done is the same. The least performance in mathematics problems was seen when both the mathematics content and language presented were complex.

Beal, Adams and Cohen (2009) studied the relationship of reading proficiency and mathematics problem solving ability among high school students. Mathematics problem solving ability was measured in terms of state mathematics test scores, problem solving in an online mathematics tutorial, and mathematics self-concept was also measured using self-reporting scale. English conversational and reading proficiency data were also collected. Mathematics performance for the English language learners increased with English-reading
proficiency and English-reading proficiency can predict student's mathematics test scores, progress in the online mathematics tutorial, and their mathematical self-concept.

Gooding (2009) conducted a case study on difficulties in solving mathematics word problems among five primary school students. Data was collected through interviews with probing questions on students' views of their difficulties in solving word problems discussion with children while they solved mathematics problems. Qualitative data were thematically analyzed based on categories of difficulties explored from existing literature. The study explored seven categories of difficulties viz., 1) reading and comprehension, 2) reading all the information, 3) distracting information, 4) imagining the context, 5) writing a number sentence, 6) carrying out the calculation, and 7) interpreting the answer in the context of the question. Three of them have subcategories of difficulties. related to reading and comprehension are decoding the words in a word-problem and understanding the meaning of the words and sentences difficulties associated with carrying out the calculations are 1) Lack of accurate methods for calculating and 2) making a mistake when calculating. Difficulties related to interpreting the answer in the context of the question are 1) giving an answer that is possible or likely and 2) transferring an answer into the required format.

Vilenius-Tuohimaa, Aunola and Nurmi (2008) investigated association between mathematical word problems skills and reading comprehension among 225 children of Grade four in Finland. Text comprehension was tested using Lindeman's (2000) ALLU primary school reading test which has two subtests technical reading and text comprehension. The test consists of two subtests
based on narrative context and two subtests based on expository context. Four subtests with narrative context and expository context were included in the test with four types of items cause-effect, concept, conclusion and main idea. Children's text comprehension and mathematical word problem-solving performance was tested. Based on technical reading skills students were categorized as good or poor readers. Subtest from the NMART Counting skills test was used to assess skills in solving mathematical word problems. Skill in mathematics word problems was strongly related to skill in reading comprehension and there was no gender difference in word problem solving skills.

Shaftel, Belton-Kocher, Glasnapp and Poggio (2006) investigated the linguistic features that affect the difficulty of mathematics test items which leads to poor performance in mathematics among students, especially English language learners and those with disabilities. Approximately 2000 students across each of three grade levels- $4^{\text {th }}, 7^{\text {th }}$, and $10^{\text {th }}$ grades were studied, including students with disabilities and English language learners. Items in the Kansas general mathematics assessments were given at grades four, seven, and ten with four parallel forms. Items from four mathematical domains viz., number and computation, algebra, geometry, and data were included in the test. The item pool includes 208 items at 4th grade, 203 items at $7^{\text {th }}$ grade, and 183 items at $10^{\text {th }}$ grade. These multiple-choice word problems have words per item ranging from two words (in six items at $4^{\text {th }}$ grade) to 177 words (in three items at $10^{\text {th }}$ grade), with a mean of 45 words. Regression analysis revealed that linguistic features had moderate effects on item difficulty at $4^{\text {th }}$ grade, dropping to small-to-medium effects at $10^{\text {th }}$ grade, and this effect is consistent for both English language learners and students with a disability. Vague or terms with multiple meanings
contributes to item difficulty at 4th grade. Study suggests that care should be taken while preparing test items that includes terms that are unclear or colloquial usage which have multiple meanings based on the context and comparative terms, as the use of such terms have statistically significant influence on student's mathematics performance.

Capraro and Joffrion (2006) examined whether middle school students were able to translate word problems to algebraic equations by adequate conceptual understanding and comprehension. Sample selected for the study includes 668 students of 25 middle-school teachers' classrooms. Additionally, 60 random incorrect responses were examined to identify patterns in student responses. As a confirmatory procedure, five students who solved certain tasks were interviewed in a cognitive lab. Data were analyzed both qualitatively and quantitatively. Students were assessed on three specific algebra tasks including two multiple choice and one short answer question. In interview, students were requested to 'think aloud' and describe their reasoning for choosing the answers for the three algebraic questions. Investigator listened and notes were made based on strategies explained by the students. These strategies were categorized as conceptual or procedural. Students were characterized as having conceptual understanding if they demonstrated comprehension and understanding. Students were characterized as having procedural understanding if they are merely reading words, looks for keywords, and follows rote procedure. Only nine percent of students answered all the three algebraic problems correctly which means that students were not procedurally or conceptually equipped to translate from the written word to mathematical equations even at the secondary level. Teachers commonly give a list of keywords that indicate different operations
while solving word problems and the results of this study indicates that this is not enough to equip students with conceptual understanding of the problem.

Latu (2005) investigated the language characteristics that influences mathematics learning of senior secondary students for whom English is an additional language. In addition to interview and classroom observations, questionnaire was also used as a tool for data collection. Questionnaire had sections on self-report on competency in English, mathematical instructions, mathematical vocabulary and mathematical language and word problems. Classroom observations show up classroom practices were discussing mathematics was not practiced as group investigation, problem solving, group discussions or hands on type activities were not used. Students had to work individually, classroom activities do not promote inter student interactions and they were not exposed to language of mathematics. However, code switching was a common practice between students with the same mother tongue. In mathematics classrooms, students used their first language in their mathematical conversations after teacher's instruction. Then teachers used to explain or translate between English and first language for students individually. Teachers should be aware that code-switching is a common practice in mathematics classrooms among bilingual students. It also shows up the importance of mastering general language and mathematical language so that students' comprehension skills will be improved that will make them successful in solving mathematics problems.

Nordin (2005) surveyed the perception among 279 lower secondary school students from three schools in rural area Malaysia on learning mathematics in English to find out the problem in mathematics teaching-learning process in English. Questionnaire of 16-items which was planned to identify
students' difficulties while teaching and learning mathematics occurs in English, was used for data collection. Students feel difficulty in learning science and mathematics through English language even though they agreed about the importance of English in everyday life as well as career. Students points out lack of ability in understanding the content as well as instructional language as the reason for this difficulty.

Howie (2003) conducted a study on language and other background factors that affects performance in mathematics among secondary students in South Africa. Study sample was 8000 students from 200 schools. Students' language proficiency was measured along with TIMSS-R mathematics and science tests to measure performance in mathematics. Background questionnaire was used to gather information about the use of English inside and outside of school by teachers and students. Relative input of these factors to students' achievement along with other background variables from the students and teachers were analyzed using partial least square analysis. The study concludes that student's proficiency in English can predict their achievement in mathematics while home language was not found to have significant effect on achievement in mathematics.

Abedi and Lord (2001) studied the language related factors in mathematical tests that affect test performance of students among 1,174 eight grade students. Test items were selected from the national assessment of educational progress mathematics assessment test. Items parallel to selected test items, which are modified to reduce linguistic complexity, were also developed and given to students. Linguistic features of test items were modified such as unfamiliar or infrequent words were changed, passive phrase were changed to
active voice, long nominals were shortened, conditionals were replaced with separate sentences, or the order of conditional and main clause was changed, and complex question phrases were changed to simple question words. Students who were proficient speakers of English have an advantage over English language learners (ELLs) on the mathematics test score. Mathematics performance is varied based on student's socio-economic status but not by gender. Linguistic modification of test items caused significant differences in mathematics performance as indicated in the slightly higher scores on the linguistically modified version. And the benefit of reduced linguistic complexity was more among students who were low or average in their mathematics performance.

Adetula (1990) studied the influence of language factor (English / native language) on children's performance in mathematics word problems among 48 students in primary level. Students were asked to solve 10 arithmetic word problems (including "more" or "less" as the cue term) which are presented in English and in their native language. To identify how students analyses each problem in order to find out the operation needed to solve the problem, retrospective clinical interviews were also conducted. Both public and private school students' performance was better in mathematical problem-solving skills and strategies to solve problems when problems were presented in their native language in comparison to problems presented in English.

## Studies on teachers' perception on language of mathematics and attendant difficulties in learning and instruction

The 10 studies reviewed below show that during the last two decades, teachers are becoming increasingly aware of the relevance of language, both medium of instruction and the technical language of mathematics, in

## 122 EVIDENCE BASED INSTRUCTION OF LANGUAGE OF MATHEMATICS

mathematics learning of their students, with earlier studies highlighting the first and latter studies recognising the second.

Turner, McDuffie, Sugimoto, Aguirre, Bartell, Drake and Witters (2019) explored the practices and understandings related to language in mathematics instruction among six early career teachers' (ECTs) of elementary and middle school level through multiple case studies. Classroom observations and interviews before and after observations of mathematics lessons used to collected data. Investigators observed 8-12 mathematics lessons per teacher, per year and recorded classroom activities in scripted format including gestures, actions, visual representations etc. and collected photos, lesson plans, and student work samples. Early career teachers purposefully provided opportunities for students to listen to and practice using mathematical language, such as teacher modeling vocabulary, partner talk, turn and talk, whole group choral talk, writing activities, small group discussions. They try to connect vocabulary to students' knowledge and experiences by eliciting students' prior knowledge and experiences related to mathematical term. Teachers promote precise and regular use of mathematics language through teacher activities like correcting, prompting, or re-voices a term. Teachers focus on the role, purpose, use of multiple languages - both English and mother tongue in mathematics teaching and learning. They use code-switching, translation, re-voicing across languages to promote the use of multiple languages. Teachers also focuses on mathematical communication (oral, written, nonverbal), participation in mathematical discussion, mathematical discourse practices like explaining and justifying ideas and attention to how students participate in mathematical discourse. There is inconsistency in early career teachers focus on how to teach mathematical vocabulary, anticipations for students' precise use of mathematical terminology, and the use of multiple languages during classroom instruction.

Livers and Elmore (2018) examined the existing vocabulary support given by middle school mathematics teachers within their mathematics classroom instruction in terms of vocabulary support, activities, and usage qualitatively by classroom observations. Three tiers of language namely common everyday vocabulary (cents, cups, discount etc.), descriptive vocabulary (adjacent, highest, lowest etc.) and domain-specific vocabulary (Pythagorean theorem, vertex, decimal etc.) put forward by Beck and McKeown (1985) were considered in classroom observations. Classroom observations revealed that all three types of vocabulary were evident in middle school mathematics classrooms in which domain specific terms are used more occasionally, second highest percentage is for everyday vocabulary and the least one is descriptive vocabulary. Even though the teachers exhibited fluency and vocabulary needed for content understanding, students do not have enough vocabulary and they were not the owners of domain-specific vocabulary and hence the study recommended, teachers to enable and encourage accurate student use of domainspecific vocabulary along with symbols, notation, and operations.

Kabael and Baran (2016) qualitatively studied teacher's awareness on developing mathematical communication skills among ten middle school mathematics teachers through clinical interviews that lasted for 15 minutes for each teacher. All the teachers were found aware that mathematics has a special language for communication, but only two of them adequately explained the properties and structure of mathematical language by focusing on its syntax and semantics. All the teachers participated in the study perceived that, in using the language of mathematics effectively, mathematics teachers must be a model for their students. However, teachers were not adequately equipped with their responsibilities in developing mathematical communication skills.

## 124 EVIDENCE BASED INSTRUCTION OF LANGUAGE OF MATHEMATICS

Sibanda (2015) investigated the nature of linguistic challenges of annual national assessments (ANA) test items for standard four students along with students' experience as they solve mathematical problems and teachers' perception regarding linguistic challenges in this test. Content analysis and language complexity analysis of annual national assessments (ANA) items using linguistic complexity checklist (Shaftel, Belton-Kocher, Glassnap \& Poggio, 2006), analysis of 106 written scripts of students, interview with students using Newman error analysis and thematic analysis of teacher questionnaires were done. Twenty-six students, and two Grade four mathematics teachers were selected for interview. Linguistic complexity checklist included four major areas viz., Basics, word level, sentence level, and paragraph level characteristics. While analyzing each items in the test, instances of given characteristics were counted like number of sentences in each item and number of words used. In word level characteristics, counted the number of different words with 7 letters or more, relative pronouns, examples of slang, homophones and homonyms, and number of abbreviations. In sentence level characteristics counted the Number of prepositional phrases, infinitives, complex verbs, complex/compound sentences, conditional constructions, and number of comparative constructions. In paragraph level characteristics counted the number of cultural/or experiencespecific references and number of American holidays. Linguistic Complexity Index (LCI) was calculated by multiplying the number of sentences with sum of characteristics in four areas viz., number of words, sum of word level characteristics, sum of sentence level characteristics and sum of paragraph level characteristics. Findings of the content analysis done on the 2013 mathematics ANA test items point out several linguistic complexities in most of the test items, predominantly frequent use of 7 or more letter words, homophones,
prepositional phrases and specific mathematics vocabulary across most questions. The language used in test items was unfamiliar for standard four students which contribute to the poor performance of students. Students experience difficulties in reading and comprehension skills also especially for those who were less proficient in English. Teachers also perceive that mathematical language used in the test is difficult for the students to comprehend and students who have poor reading skills struggle to comprehend the questions.

Naidoo (2015) explored the teaching strategies used by mathematics teachers to overcome the language related difficulties in multilingual schools. Three mathematics lessons that were taught by each of the six teachers in grade 11 was observed along with semi-structured interview of these teachers. In addition to this, focus group interviews with students of small group of six to ten learners from each participating school $(\mathrm{N}=48)$ were also conducted. Study indicates that teachers used mnemonics, manipulatives and collaborative work in mathematics instruction. The teachers also describe some mathematical terms like function, face and figure that had varied meanings outside the classroom. The teachers did not stick to the traditional method, rather they used collaborative work by interacting with the learners throughout their mathematics instruction so that students could comprehend the key concepts meaningfully in a mathematics context. Teachers also used manipulatives to overcome the linguistic challenges. Teachers were indicated as aware about the linguistic challenges in mathematics instruction and the influence of language on students' performance in mathematics.

Serio (2014) qualitatively studied the extend of teacher enrichment of both verbal and written communication in mathematics classrooms whether student-to student or teacher-to-student communication. The study also tries to
find out the factors that may hamper a teacher's ability of enriching mathematical discourse in their mathematics classrooms, and the suggestions of teachers on strategies that can be used in mathematics classrooms to facilitate mathematical discourse. Three elementary school teachers, ranging in years of teaching and training experience were the sample for the study. The teachers were interviewed through semi-structured interview mode which gathered information on background details of the teachers, understandings of importance of mathematical discourse, perceptions of the benefits of mathematical communication, challenges in infusing mathematical discourse, perceptions of his/her own capabilities to facilitate mathematical communication and Suggestion regarding strategies that can be used in mathematics classroom to improve mathematical discourse. The study indicates teachers' perceptions of the challenges in employing student discourse in mathematics classrooms such as teachers' perceptions and experiences with mathematics, students' difficulties with the subject matter, English language learners, external curriculum demands and assessments, and time. Suggestions provided by teachers to improve mathematics discourse are enhance more peer-to-peer interactions, use of strategies from language instruction, be open to incorrect responses, have the students visualize the math, assess student discourse in their group activities, include math games and give chance to students to explain their thinking.

Leshem and Markovits (2013) studied the similarities and differences between English and mathematics as perceived by small sample of five teacherstwo mathematics teachers and three English teachers- studying at a teacher education college and teachers' opinions on the possibility of collaboration between teachers of the two disciplines through in-depth interviews. All teachers, except one has the opinion that mathematics can be considered as
language also. Teachers perceive mathematics and English as a universal language of communication with rules and structures and share analytical thinking. However, mathematics descriptions of reality are more objective compared to English descriptions which are more subjective. For example, a 'sentence' in mathematics has always one reading ( $3=3$ ), while a sentences in English might have diverse interpretations. It further uses the metaphor of music to explain similarity between mathematics and English, explaining that both English and mathematics have their exclusive music. Teaching vocabulary in English was compared to teaching fractions in mathematics, as both involves illustrations and practice. The need to learn many rules, to solve mathematical problems or perform in a language other than mother tongue, are the reasons for poor performance and anxiety among students. In spite of similarities in teaching mathematics and English, these teachers do not find any reason to collaborate, as they conceive both subjects as entirely different from one another.

How the mathematical language influences achievement in mathematics among 661 students and 71 mathematics teacher at secondary level in Kenya (Mbugua, 2012) was explored through an ex post facto design. Information about presentation and level of explanation of mathematical terms, symbols and structures, which ones were left out in classroom explanation by the teacher was obtained through observation. Level of inclusion of explanation of mathematical terms, symbols and structures in lessons by teachers is solicited by using mathematics teachers' questionnaire (MTQ), and student's opinion on understanding of the same is solicited using Students' Questionnaire (SQ). Students' understanding of mathematical terms, symbols and structures in mathematical sentences from variety of topics in mathematics were tested. Term end examination marks were analyzed to identify the extent to which questions
on mathematical language are included. Students' achievement in mathematics is highly correlated with their understanding of mathematical language and to some extent mistakes committed by students in solving mathematics problems is owing to their lack of comprehension of mathematical language. Including the components of mathematical language such as terms, symbols and syntactic structure in teaching will improve student performance in mathematics.

Yamat, Maarof, Maasum, Zakaria and Zainuddin (2011) studied the use of code-switching in teaching of mathematics and science among secondary school students and teachers from eight schools of four zones in Malaysia. This mixed method design made use of tools and techniques like survey questionnaire, interview and competency test for data collection. Interview gathered data on how and why teachers use code switching as a means of scaffolding their students' learning and the themes were identified and categorized regarding use of code-switching. Majority of teachers use national language of Malaysia (Bahasa Melayu) while teaching mathematics and science (around 60 percent and 70 percent of classroom teaching) whenever they needed to help their students comprehend the concept more meaningfully. Teachers perceive that they had to use Bahasa Melayu to help backward students, clarify ideas, explain ideas, to manage classroom discipline, to give instruction and demonstrate procedures/activities. Further, the study indicates that in occasions where other means of conveying concept such demonstrating procedures/ activities, code switching is less.

Cantoni (2007) qualitatively studied role of instructional language in primary schools of Namibia in terms of relationship students have with the English language before starting school, transfer from mother tongue instruction
to English, languages teachers and students use in the classroom and during breaks and reason for choosing one language rather than the other. Sample of the study was 400 students from grades 1 to 10 and 18 teachers and the mother tongue of the majority is one of the eight dialects within the Oshiwambo language group. Interviews and classroom observations were done. Most pupils do not speak English outside school, before starting the fourth grade. Moreover, the sudden change in instruction language from mother tongue to English creates difficulties in student participation in learning activities. However, teachers have a positive opinion regarding English as a medium of instruction.

## Experimental Studies Related to Language of Mathematics

This section consists of 21 experimental studies which experimented effect of interventions related to language of mathematics and its impact on mathematics learning outcomes. They are further near equally categorised as interventions on vocabulary and interventions on more holistic elements of mathematics language.

## Vocabulary interventions on language of mathematics

Chiphambo (2019) experimented the influence of integrating mathematics dictionary and polygon pieces into the mathematics instruction. Purposeful sampling was done after diagnostic test in order to identify low performing students, average student and high performing students. Only nine students were selected for the study to elicit more in-depth data. Nine intervention tasks were developed and implemented during intervention in which, students were asked to answer each one of the questions using cut polygon pieces of the given triangle, after measuring and comparing angles and sides of the same triangles.

## 130 EVIDENCE BASED INSTRUCTION OF LANGUAGE OF MATHEMATICS

Mathematics dictionary was given to students in order to support with vocabulary, spellings and terminology. After the intervention activities, all the nine students had to write a post-test. In posttest, students performed well in test based on van Hiele theory of geometric thinking. The questions measured students' geometrical proficiency such as geometric vocabulary, terminology and conceptual understanding. The study suggests that independent learning can be promoted through teacher's integration of mathematics dictionary and polygon pieces into the teaching and learning of geometry.

Hollingsworth (2019) studied the effect of a vocabulary tutoring intervention on vocabulary development and algebraic problem-solving skills among five college students who struggle with mathematics. Participants completed two vocabulary tutoring sessions each week and completed layeredlook books which includes the vocabulary word, definition, an example, and non-example. Intervention was given in a guided practice strategy. The post test was conducted on a six items test with three vocabulary short answer questions and three multiple-choice algebraic problems. The study found out that vocabulary tutoring intervention improved students vocabulary whereas it did not affect on students with the algebraic problem-solving skills.

Kenyon (2016) studied the effect Frayer model of teaching academic vocabulary on students' overall mathematical ability among secondary school students. Participants were 15 students each in experimental and control group who were low in their verbal comprehension level. Explicit vocabulary instruction and graphic organizers were used as strategies to improve student vocabulary. Duration of intervention was for a period of six weeks. The study finds out that the class who received the vocabulary instruction showed
significant improvements over the control group. However, the strategy was less effective with those students who had a low verbal comprehension. It also indicates that students who got vocabulary instructional strategy had greater improvements in their understanding of mathematical vocabulary in the test.

Tarpley (2015) examined the effect of rich instruction of mathematical academic vocabulary on elementary students' achievement in mathematics and vocabulary through a pretest-posttest experimental design. Two mathematics units were selected for the experiment. Experimental group of 63 students in fourth grade spent 10 to 15 minutes in vocabulary instruction and 71 students played non-digital math games for a comparable amount of time. Intervention was conducted in classrooms of three mathematics teachers. First, teacher incorporated the vocabulary intervention into the daily mathematics instruction in two classes and used a comparable amount of time playing non-vocabulary, non-digital mathematics academic games in the third class. Other two teachers incorporated the vocabulary intervention into the daily instruction of one class each and used a comparable amount of time playing mathematics games with the remaining two classes. Intervention focused on vocabulary instruction to recognize and identify examples and non-examples, opportunities for students to discuss relationships between focus words and justification of their reasoning when they were asked to select the correct term for a concept. Vocabulary list was developed and reduced based on the suggestions from practicing teachers who identified important terms for the selected topics. The vocabulary included in the intervention was selected from terms highlighted within instruction and terms used in unit test instructions. Vocabulary instruction and mathematical game improved student performance in mathematics achievement, but there was no significant difference between the effect of these two interventions, and high
achieving students makes twice the gains of underachieving students in mathematics achievement; moreover vocabulary instruction is equally effective on achievement in mathematics for students who were high or low achievers in reading.

Gillmor, Poggio and Embretson (2015) experimented with the effect of reducing cognitive load of mathematics test items on performance in mathematics and anxiety among eighth grade students. The control group was given a traditional test with 15 items that were chosen to represent representative levels of cognitive load and the experimental group was given the same items but modified to reduce cognitive load using research-based strategies. Strategies used to reduce cognitive load of items were reducing word count, use of diagrams to represent spatial information, focusing attention with signals and cues, eliminating unnecessary visuals and text, asking question first to give a direction to the item, and then include supporting information, ordering the answer options logically and placing text near corresponding features on figures. Further numerical complexity was reduced by using smaller, rounded, and familiar numbers when values are construct irrelevant. Reducing the cognitive load of mathematics test items improves student performance. Among the abovementioned strategies to reduce mathematical cognitive load in test items, three strategies were identified as particularly effective for reducing item difficulty viz., focusing attention with signals and cues, ordering items logically and placing text near corresponding features on figures, and removing unnecessary content. However, reducing cognitive load in mathematical test items does not impact student anxiety. The study suggests that items that gives focus on important information, are logically organized, and are free of unnecessary information can improve student performance in mathematics.

Gifford and Gore (2010) experimented with the effect of focused academic vocabulary instruction on performance in mathematics tests among underperforming mathematics students of 6th grade. The study used the book 'Building Academic Vocabulary: Teacher’s Manual’ (Marzano \& Pickering, 2005) for designing the intervention. Long term intervention was started at the beginning of a chapter by reintroducing the students to the vocabulary in that chapter by demonstrating the terms through an example or explaining a concept using pictures or diagrams, which will lead to brainstorming session till students formulate descriptions in their own words. Well-developed descriptions with the help of teacher probing during brainstorming session were posted on the board which students copied to their mathematical vocabulary journal. Whenever a terms is introduced in the class, the teacher asks the students its meaning and encourage them to describe it in their own words rather than definitions. Explanations from the students were never considered as a wrong answer, rather they were helped to improve their descriptions through probing questions and further classroom discussions. Also, students occasionally played vocabulary games and they were given academic vocabulary tests in which students need to match terms with student-drafted definitions, and they also need to illustrate each word. The study further studied students' perception of their efficacy to succeed through a pretest-posttest survey in which the items focused on students' feeling of readiness, willingness to take the test, high score expectation, general understanding of questions, vocabulary in the test, anticipation of results, and enjoyment in taking the test. Findings of the study indicates that focused academic vocabulary instruction is beneficial for all types of learners, especially for struggling learners as at least a 33 percent increase is there in the gains on standardized tests and their perceived self-efficacy beliefs were also improved.

## 134 EVIDENCE BASED INSTRUCTION OF LANGUAGE OF MATHEMATICS

Long terms effects of academic vocabulary instruction were also analyzed as the study spanned through three academic years in which first year with no focus on vocabulary instruction, and in its second year, vocabulary instruction was an integral part.

Flanagan (2009) studied the effect of vocabulary games and mathematics writing on achievement in mathematics among secondary school students in New York. The experimental group got additional vocabulary game and writing practices, while control group was taught with traditional methods. Vocabulary games selected for the intervention were bingo, around the world and go fish. Students had a bingo board and they have to put in the vocabulary word. Teacher read aloud the vocabulary word from the index card and students find out the matching definition. While playing around the world, one student goes against the student next to them. Teacher would have the cards of vocabulary words, and if the student makes the correct definition, he can proceed to the next one. Go fish was played with six pairs of cards with vocabulary and its definition. Students have to match vocabulary and its definition. Everyday students have to write articles in mathematics journals. Results of the study indicated that experimental group scored significantly higher in mathematics achievement tests compared to students in the control group.

Georgius (2008) done an action research on improving mathematics communication through vocabulary and writing among 6th grade mathematics students. Specific objectives of the study were to find out the extent of student use of mathematical vocabulary after direct instruction, influence of vocabulary instruction on written problem explanations, mathematical understanding and achievement and to find out the correlation between vocabulary, written expression, and achievement. Vocabulary instruction mainly includes a
vocabulary book for every student in which students and teacher recorded each vocabulary word from the textbook on one page. Each page has four square, for the word, definition, examples, a memory trick, and a word problem example. At the end of each chapter teacher gives a list of vocabulary and list of their definitions. To get an idea about vocabulary that the students already knew, and which words need emphasis, at the beginning of each chapter, teacher gave students a pretest of the vocabulary words including both review words and novice terms. Two days before the test, students were required to cut out the definition and paste it to the correct vocabulary word. Additionally, students played a review game called, "I am... I have..." in which each student got a slip of paper with an answer and a question. But the answer and question did not match. One student started by reading, "I have..." and the question to which the student with the correct answer reply, "I am..." and the answer. This game continued till it came back to the first person. Weekly quizzes were conducted in which students must write how they solved problems, which is graded by the teacher based on fourpoint rubric. These rubric was given to students so that they can understand how their writings are evaluated. A poster was hanged on the classroom wall which explains what to write on what they did, why they did it, and the meaning of their solution. During the intervention and after it, students were surveyed with a Likert scale and interviewed to explore their feelings about the intervention. After receiving vocabulary instruction, majority of students improved in their overall understanding of mathematical concepts and they were more precise in their communication. It was also found that students felt that understanding mathematical words is important and it increased their achievement.

Larson (2007) done an action research to study the effect of giving a daily vocabulary word, vocabulary quizzes and vocabulary-based math games
on students' understanding of mathematics concepts and comprehension of mathematical vocabulary among sixth grade students through interview and survey. For the intervention on mathematics vocabulary, textbook analysis was done to list vocabulary in mathematics which were most frequently used and significant in mathematics learning and the list of vocabulary was given to students. Fun activities that were not regarded as typical schoolwork were also included in the intervention. Students were made to use the words from the list again and again each week to achieve comprehension. Student activities included creating word search puzzle for other students to solve and coding various definitions of terminology using a cipher wheel and then a partner have to decode the definition to discover the actual word. At the beginning of each assignment students have to list the keywords related to the particular content. Along with these activities, weekly vocabulary quiz was conducted, and grade of students were recorded so that they can track their progress in attainment of mathematical vocabulary. Effect of all these mathematical vocabulary activities were measured in terms of their test scores in the northwest evaluation association, measures of academic progress test. Pretestposttest scores were analyzed. In addition to testing, survey was done among students through interview related to their mathematics related issues concerning vocabulary. The study concludes that exposure to mathematical vocabulary through daily activities improved students test scores as well selfefficacy beliefs in learning mathematics in comparison with those who did not attend vocabulary activities.

Yushau and Bokhari (2005) experimented language barrier of preparatory year mathematics students, who are studying English as a new language of instruction at King Fahd University of Petroleum \& Minerals, Saudi Arabia.

Sample of the study involved first mathematics course students of the second semester during 2001-2002 academic session. The experiment consisted of eighteen units of students with seven teachers. Experimental instruction was given in the preliminaries, equations and inequities in the college algebra book, which cover more than $2 / 3$ rd of the mathematical terminology of the whole course. While medium of instruction was English, students were given with a handbook of Arabic translations of mathematics terminology of the entire course which was to help students to read and recall their previous knowledge of vocabulary in Arabic and make a link with the preparatory course lesson. Findings of the study shows that the Arabic translation of keywords and concepts helped majority of the students to recall the concepts they had learnt previously, and it minimized language difficulties in learning mathematics. It also helped to increase teacher-student interaction in the classroom.

## Communication interventions on language of mathematics

Moffett and Eaton (2018) studied the impact of the promoting early number talk on classroom mathematics talk through mixed-methods approach using quantitative and qualitative data. Participants were five teachers working with children in their first year of primary education. Intervention was for a period of six months. A questionnaire was designed to elicit information on the effectiveness of early number talk. Before the intervention, students were introduced to the book and the rationale for early number intervention. They were also asked to complete the questionnaire concerning mathematical beliefs and attitudes. Interviews with teachers elicited information on their views on number sense and the development of mathematical language. Questionnaire administration and interviews were done at the middle of the project and at the
end of the project. Early number talk had a positive impact on teachers' professional development in terms of appreciation of the importance of mathematical language, awareness of ideas and opportunities for number talk, knowledge for teaching mathematics, and reflection on practice. The intervention also influenced children's learning in terms of development of number sense and their attitude towards mathematics.

Purpura, Napoli, Wehrspann and Gold (2017) experimented with the impact of dialogic reading intervention on students' mathematical knowledge among 47 children in one school who were randomly assigned to either the treatment ( $\mathrm{n}=24$ ) or comparison condition $(\mathrm{n}=23)$. Student age and highest parental education were covariates. Mathematical language storybook reading intervention was given to treatment group and students participated in small groups for approximately 15 to 20 minutes per day, two to three days per week, for eight weeks. To ensure that the same students were not always together in sessions, activity was given to students in small groups for each session. The framework of the storybook reading intervention was modeled after dialogic reading and focused on terms, concepts, and pictures related to mathematical language. It included only mathematical language terms such as more, less, near etc. and not mathematical knowledge content such as number names, counting etc. The PEER (Prompt, Evaluate, Expand, and Repeat) strategy and CROWD (Completion, Recall, Open-ended, Wh- [what, where, why], and Distancing) prompts were for readings. Question prompts related to mathematics language components were asked during the reading intervention. Such questions made students more familiar with the language used in books. Moreover, investigator used dialogic reading strategies and responded in such a way that it did not
reinforce student's numeracy skills. Six books were selected for intervention in which three stressed on quantitative language and three emphasized spatial language. Interventionists integrated these two types of mathematical language into the prompting questions. If the student shows lack of understanding of the term, interventionists explicitly explained that terms. Students after mathematical language storybook reading intervention significantly outperformed the students in the comparison group on mathematical language and mathematical knowledge assessments. However, on expressive vocabulary measure, the intervention group is not significantly higher than the comparison group. The study concluded that exposure to mathematical language can positively affect student's mathematics skills.

Yang, Chang, Cheng and Chan (2016) experimented influence of computer supported reciprocal peer tutoring on mathematical communication abilities among second grade students. Before this reciprocal peer tutoringenhanced mathematical communication (RPTMC) activity, the researchers explained the purpose and process of RPTMC activity to teachers and students. Then, mathematical communication groups were created by every two students. The learning activity of RPTMC had four sub activities- creating, reciprocal peer tutoring, revising and staging. In the first stage students were required to prepare tutoring materials having four steps. 1) understanding the problem in which students read the word problem on their own tablet PCs and discusses the solution with their peers to recognize the context in the given problem, 2) drawing a representation using words, symbols, models, and manipulative materials to convey their ideas, 3) writing mathematical equations for solving the problem, and 4) students reflected on how and why they had solved the problem
and explained their solution in writing. Before students could write a complete sentence explaining their solutions, a text-based scaffold was given as they require guidance in expressing their mathematical concepts effectively. In the second stage paired students reciprocally teach their mathematics creations. By displaying mathematics creation, one student played the role of a tutor, and taught his/her peer why and how to solve word problems. Subsequently the tutee had to ask questions about solution strategy. The paired students then switch their roles. In the third stage, students had to reflect on their mathematics creation based on self-reflection and peer feedback in the previous activity for improving that creation. Teacher encouraged the students in each group to display their mathematics creation to the whole class. After presentation, students had to answer questions asked by the audience. In the final stage, teacher demonstrate how to explain the mathematics concepts using student workbook and to clarified errors made by students. Furthermore, teacher asked relevant questions to promote student thinking for communicating their own mathematical concepts. Mathematical communication ability was assessed based on three major criteria- 1) express individual mathematical concepts, 2) comprehend others mathematical equations and 3) comprehend others mathematical thought. Ability to express individual mathematical concepts was measured in terms of a) ability to understand the meaning of mathematics problem, b) ability to use of mathematical symbols and c) knowledge to apply mathematics language to describe mathematical concepts. Ability in comprehending others mathematical equations was measured in terms of a) ability to understand others' mathematical equations and evaluate it, b) ability to provide meaningful explanations for correct equations or explain reasons for
incorrect equations. Skills to comprehend others' mathematical thought was measured in terms of, a) ability to comprehend others' mathematical thought and evaluate it, and b) ability to use mathematical language to convert others' mathematical thought into mathematical equations or ask meaningful questions and explain the reasons for the incorrect equation. Findings of the study revealed that experimental group performed well in comparison with the control group in the assessment of communication ability. Mathematical communication abilities of students engaged in RPTMC activities was superior to those who received one to one self-paced learning and teacher led instruction. Results also indicate that students' mathematical representations and solution explanations became more accurate after the learning activity.

Lomibao, Luna and Namoco (2016) experimented with the effect of mathematical communication on mathematics performance and anxiety among secondary school students in Bulua National high school, Philippines. Mixed method research including both quantitative and qualitative phase was conducted. Quasi experimental phase was of a pretest-posttest control group design. Experimental sample consists a total of 188 fourth year high school students with 94 students randomly assigned as the experimental group and the other two groups with 94 students as control group. Mathematics performance was measured using a 24 -item test and additional five open ended questions was given to qualitatively measure ability to communicate the procedure used, mathematical reasoning and justification of the steps of their solution. Perception of students about the use of mathematical communication as a teaching-learning process was also assessed. Experimental intervention involved activities to develop student's communication through discourse and writing. Worksheets
were used to develop student communication through writing. Worksheets contained title of the lesson, learning outcome with the description of the skills that students should acquire, concept notes with illustrative examples. Key terms were also included in the worksheet so that students can define in their own words. Exercises were also given which required students to show algorithm and open-ended questions to which students should write their justification and explanation on how they arrived at the solution. Moreover, students were also asked to write their review of the work done in terms of quantity of work done during the lesson and topics they did not understand or found difficult to comprehend. Developing students' mathematical communication through discourse was done through open-ended problems. After presentation of openended problems, students were asked a series of questions which guided them to analyze and reflect on thoughts of strategies used to solve the given mathematical task. Throughout the intervention, teacher helped students to recall previous lessons to connect the concept behind the problem. In addition to this, students were given activity sheets with guide questions on how to solve the problem. In the activity, students were asked to form small groups of three or four members to have discourse to solve the problems. Small group activities accommodated shy students who felt uncomfortable in presenting their ideas to the whole class and encouraged slow learners to be open. The teacher monitored the discourse by asking essential questions related to the topic. This was done to direct students' line of thinking and reasoning. Group reporting encouraged each group to present their process of arriving at the answer which helped them improve their oral communication skills as well as to enhance their conceptual understanding. Findings of the study shows that the group exposed to communication process of learning had better conceptual understanding and
mathematics achievement than the control group. In addition to this mathematical communication intervention significantly reduced mathematical anxiety of the experimental group. Content analysis of the students' answers revealed that students were able to make connections and had applied previously learned concepts. Moreover, students perceive that mathematical communication was useful to them. Majority of the students agreed that difficulty in understanding mathematical concepts reduced significantly after being exposed to write and describe on how they arrived at a solution whereas 49 percent students found it interesting and thought provoking and 57 percent students agreed that writing in mathematics helped them give more attention to accuracy. Study concludes that 76 percent students found mathematical discourse enjoyable and fun.

Gillmor, Poggio and Embretson (2015) experimented with the effect of reducing cognitive load of mathematics test items on performance in mathematics and anxiety among eighth grade students. The control group was given a traditional test with 15 items that were chosen to represent representative levels of cognitive load and the experimental group was given the same items but modified to reduce cognitive load using research-based strategies. Strategies used to reduce cognitive load of items were reducing word count, use of diagrams to represent spatial information, focusing attention with signals and cues, eliminating unnecessary visuals and text, asking question first to give a direction to the item, and then include supporting information, ordering the answer options logically and placing text near corresponding features on figures. Further numerical complexity was reduced by using smaller, rounded, and familiar numbers when values are construct irrelevant. Findings of the study indicates that reducing the cognitive load of mathematics test items improves
student performance. Among the above-mentioned strategies to reduce mathematical cognitive load in test items, three strategies were identified as particularly effective for reducing item difficulty viz., focusing attention with signals and cues, ordering items logically and placing text near corresponding features on figures, and removing unnecessary content. However, reducing cognitive load in mathematical test items does not impact student anxiety. The study suggests that items that gives focus on important information, are logically organized, and are free of unnecessary information can improve student performance in mathematics.

Patkin (2011) studied the literacy activities pre-service teachers use to deal with problems related to language and mathematics. The study analyzed reflections of the pre-service teachers to get an insight into the importance of integrating literacy activities in the teaching of mathematics. The study was conducted among 22 pre-service teachers, twelve of them specializing at junior high school and the remaining in primary school mathematics, at a college of education in Israel. Qualitative analysis of diaries and activities were done. The intervention was conducted as an annual course dealing with 'mathematics teaching and assessment', with 30 weekly sessions of two hours duration, a total of 15 sessions in each semester. Recorded mathematics lessons of pre-service teachers were observed and analyzed. In order to monitor the feelings and difficulties of pre-service teachers, they were asked to write a diary describing their feelings, difficulties they encountered and reflections throughout the year. These diaries were qualitatively analyzed. The first session began with a poem. Students were divided into small working groups with a maximum of four members. From every subgroup one student had to read the poem aloud. The second step was to write the poem in their own words, following marking of
words in the poem representing mathematical concepts and explanation of the meaning of these words in the poem. Then the students had to check the meaning of unknown terms in dictionary - regular or mathematical. In the next session, pre-service teachers were asked to read articles regarding use of everyday language and mathematics. Then pre-service teachers were exposed to integration of literacy activities in teaching. They were taught to operate instructional strategy namely 'leading mathematical conversations' (verbalization of mathematical relations). At the end of each session pre-service teachers had to list all terms with double meaning. They also had to design an intervention of four or five lessons, by integrating literacy activities. Then they had to implement the developed programme in their own classes. The weekly sessions involved discussions, reflection and analysis of findings. Major output of this study was the 'bank' of words with double meanings which can result in misunderstanding and errors and a pool of ideas for literacy activities in mathematics. Analysis of the diaries exposed understanding of importance of knowledge about double meanings, positive attitude towards the continuation of literacy integrated teaching, and increased level of self-confidence.

Ilany and Margolin (2010) developed a nine-stage instructional model to solve mathematical word problem which focuses to bridge the gap between natural language and mathematical language. The study was conducted as case studies on three samples- one students from fifth grade, one from ninth grade and one in college. The instructional model presents an interactive multi-level process that helps in decoding of the mathematical text by decoding symbols and graphs. First stage involves reading the problem from bottom to top to collect the details, then reading the problem for a second time for understanding the linguistic characteristics of the problem in terms of words, sentences, keywords
and describing the problem in learners' words. Third stage involves collection of data regarding the problem in terms implicit and explicit data given in the problem. Fourth stage involves reading the problem from top down for identifying literal clues or certain words that help as a clue for choosing the operation required for solving the problem and possibility of demonstrating the problem by means of a picture, table, diagram, or graph. In the fifth stage students must construct a schema for solving the problem based on past experiences. Then sixth stage is to check those schemas in detail and retain only relevant ideas. Building a mathematical model is the seventh stage in which the student builds a mathematical model of the mathematical principles by understanding the relationships and the conditions related to the problem and use the mathematical model. Eighth stage is to think about the possible solution of the problem. Final stage is to revisit the problem again and check whether the solution make sense, whether the solution is appropriate to the linguistic situation, whether the solution is appropriate to the mathematical situation and whether the mathematical model used fit the problem. Findings of the study indicates that after being taught the model, fifth grade student, ninth grade student, and college student specializing in mathematics teaching, performed well in mathematical problem solving.

Wichelt (2009) done an action research to study the impact of communication in mathematical classrooms among seventh grade students by observing lessons and group work by students. Rubric was used to observe classes. When more open-ended questions were asked, students had a better understanding of vocabulary in mathematics. Moreover, students felt more comfortable in communicating mathematics with peers and teachers after exposure to more open-ended questions. Study conclude that emphasize on oral
and written communication in the mathematics classroom can be very beneficial for elementary level students.

Sample (2009) studied the effect of oral and written communication in mathematics classrooms on students' level of understanding and self-confidence in solving mathematics problems through a pretest-posttest experimental design research. Experimental intervention consists of daily journals noting observations, weekly journals, oral and written solutions from the students, individual interviews, and daily work. Journals noting observations were written two to three times per week. At the end of each week, researcher reflected on the activities of the week, and recorded it in a weekly journal. Students presented oral solutions to the whole class once per week and written solutions daily. Approximately 10 minutes was given to students to write out their solutions to the problem every day. Interviews were conducted with students to explore the student level of understanding and self-confidence in solving mathematics problems with special reference to attitude towards solving mathematics problems and working in groups. Student level of understanding was not necessarily increased due increase in written communications. However, student level of self-confidence in solving mathematics problems increases significantly due the increase in oral communication.

Brown and Hirst (2007) studied the mediating role of talk in mathematics classroom and the characteristics of classroom talk that encourage student engagement in mathematics activities at individual and group levels. Sample of the study consists of students of two standard seven classrooms, one class with 22 students and the other with 24 students. Two teachers of the class were also included in the sample. Video records of both the classes twice during the research and anecdotal records of teacher-learner and learner-learner interactions
were made. However, when these videotaped sessions were made, both the teachers do not have a formalized information of the discourse formats. Teachers and some selected students were asked to keep a reflective journal. In addition to this, both the teachers were interviewed towards the end of the year regarding their perception of mathematics teaching and learning. The video recordings of classroom activities were used as a catalyst for reflection and discussion during this interview. To identify the formats of classroom talk deployed, videotaped lessons were transliterated. Transcripts were then analyzed for teacher interaction with students at individual and group levels based on Renshaw and Brown discourse characterization of four types of talk that teachers and students use when interacting with each other viz., replacement, interweaving, contextual privileging, and pastiche. In replacement talk, space was given to student's language and representation which acts as a bridge to formal mathematical language. Gradually, teacher replaces students' representations with formal mathematical language. Interweaving denotes to classroom interaction where students combine their mathematical ideas with the ideas of others into a form of talk. In contextual privileging, students are urged to accept certain ways of talk because they are proper to the situation. Pastiche format of classroom talk gives opportunity for multiple representations of concepts by the students. In this study, one teacher promoted classroom talk which facilitated relation of student's personal understanding with conventional mathematical understanding. In the second class, classroom communication does not stick on correct/incorrect or true/false. This format of classroom talk promotes the view that students also occupy roles in the process of learning and communication. Findings of the study indicates that teachers must learn how to balance interaction between content knowledge, instructional techniques, and contextual understandings with
the institutional requirements for the transformation of teacher practice in the mathematics classrooms. This study also suggests that to improve the quality of the learning opportunities presented to students in mathematics classroom, teachers need to be reflective and critical users of classroom talk and understand their role in mathematical discourse.

Huggins and Maiste (1999) experimented with the effect of mathematical communication intervention on oral and written communication skills in mathematics among primary school students. Class surveys, student interviews, and teacher made tests were used for survey on experiences in oral and written communication in mathematics classrooms. Survey results shows that communication in mathematics, except for signs and symbols, has been clearly ignored. Interview with teachers revealed that there is also widespread concern on mathematical communication. Review of curricular focus in the area of mathematics revealed that communication is emphasized much less than calculation. Correspondingly, student surveys revealed experiences given to students in oral and written communication in mathematics is very less. Intervention strategies were decided by review of existing instructional strategies, analysis of problem setting and expert opinion. These strategies included the use of mathematics journals, cooperative groups, real life problem solving, and emphasis on mathematical vocabulary. Throughout the intervention, students worked in cooperative groups to solve and discuss problems, direct instruction of problem-solving strategy and writing mathematical journal. A rubric was developed to assess student's communication abilities in terms of their solution to a problem, explanation of process of obtaining the solution and justification of selecting that solution method for which students would earn points. The rubric was based on the rubric used by the 'Illinois goals assessment
program' for scoring written communication in mathematics. The maximum score was 9 , if they communicated in all areas successfully, while the minimum was zero for not communicating their results in any of the areas mentioned. It is notable that whether the answer is correct or not is not considered while scoring, only student ability to communicate is considered. Findings of the experimental phase indicates that there is an increase in the oral and written mathematical communication skills among both third and fourth grade students. The fourthgrade students improved in all areas of mathematical communication skills especially in higher-level skills like explaining how they got their answers and why they solved a problem the way they did. However, the success of the thirdgrade class was restricted to lower level communication skills, like explaining what their answer is.

## Summary of Studies Related to Language of Mathematics

The following are the guidelines that could be drawn from the surveyed studies.

Linguistic features affect the difficulty of mathematics across primary grade levels

Existing literature generally indicate that linguistic features affect the difficulty of mathematics across primary grade levels. Primary school children in grade four performed better both in skills and strategies when word problems were presented in their native language than when presented in English (Adetula, 1990). Mathematical language was difficult for the Grade four learners especially when their reading skills were poor and as they struggled in reading comprehension. Linguistic complexities in test items, particularly in relation to recurrent use of seven or more letter words, homophones, prepositional phrases
and specific mathematics vocabulary, is a key contributing factor to learners' poor performance in mathematics tests. Along with these linguistic factors, learners experienced difficulties in reading, comprehension, transformation, process and encoding with the greatest difficulties in comprehension and in reading (Sibanda, 2015).

Complexities of languages both of instruction and of mathematics impacts mathematics learning

Gooding (2009) has identified explicit language competencies namely reading and comprehension along with writing number sentences, carrying out the calculation and interpreting the answer in the context of the question as mathematical difficulties in solving word problems among year five children of English primary schools. In grade five, one fifth of learners had very poor performance in translating word problems to mathematics symbols and equations (Cruz \& Lapinid, 2014). In Grade six and seven also, learners were less successful in problem solving when the text has higher grammatical complexity and more advanced vocabulary and the poorest performance expectedly were on problems with both complex text and relatively difficult mathematics (Barbu, 2010).

Mathematics learning and teaching in primary as well as secondary levels are impacted both by language of instruction and by nature of mathematics language as such, and sometimes through an interaction of the nature of language of instruction and language of mathematics. Accordingly, performance on mathematics word problems in grade four was strongly related to performance in reading comprehension, and fluent technical reading abilities. Independent of technical reading abilities, reading comprehension in native
language adds to mathematics word problem solving as both require overall reasoning abilities. May be for these reasons, factors like parental levels of education also predicted children's mathematics word problem-solving performance (Tuohimaa, Aunola \& Nurmi, 2008). In grade eight, English language learners (ELLs) scored lower on the mathematics test than proficient speakers of English, and socioeconomic status continued its sway on mathematics learning. Consequently, linguistic modification of test items resulted in significant differences in mathematics performance, in particular, for students in low-level and average mathematics classes (Abedi \& Lord, 2001). There is a high correlation between language ability in mathematics, reading ability in mathematics and performance in arithmetic skills among fourth grade students and percentage of error is more in common words used both in general language and mathematics than in spatial terms (Rao, Ramaa \& Gowramma, 2017)

Language related difficulties in the instructional-learning contexts in mathematics can be studied through a variety of means

Interviews, observation, testing, and correlation methods are used by previous research to identify, gauge student difficulties in learning mathematics emerging from the use natural language, special language of mathematics and the interaction of these two in varied language and teaching learning contexts. One area of consistent attention was how these difficulties interact in solving word problems and many researches adopted parallel form tests based on textbook and other documents analysis. Another procedure employed was comparing the facility in translation among arithmetic, algebraic and geometric languages.

## Effect of language features, especially surface level characters, on

 mathematics performance are pronounced in earlier gradesSome language characteristics though had varied effects on student performance across school levels, there are reasons to doubt that the effect of language features on mathematics performance reduce as students move up the grades. Still, learning lower secondary school mathematics in a language other than native language is perceived very difficult and demanding (Nordin, 2005). Language features had moderate effects on item difficulty at 4th grade that dropped to small-to-medium effects by Grade 10. Difficult mathematics vocabulary had a consistent effect on performance of all students at all grades. Ambiguous or multiple-meaning words, Words that are unclear, colloquial, or slang, or that have multiple meanings depending on context had increased item difficulty at 4th grade. Comparison problems are more difficult for students. For example, the use of problems requiring comparative terms are more common at Grade seven where it significantly impact student performance. Comparisons have an almost-significant effect at Grade 10 also (Shaftel, Kocher, Glasnapp \& Poggio, 2006).

By secondary level, importance of formal language over home language on success in mathematics become clearer

By secondary level, though, home language did not significantly affect mathematics achievement, proficiency of English was a strong predictor of their success in mathematics along with several background variables on student and class-level (Howie, 2003). For high school students, mathematics performance besides test scores such as progress in the online tutorial, and mathematics selfconcept for the ELLs increased with English-reading proficiency though
nonlinearly (Beal, Adams \& Cohen, 2010). Finally, apart from linguistic features, reading skills, comprehension, and proficiency in English language, students' understanding of mathematical language also highly influences achievement, as mistakes secondary students commit with mathematics problems is partly due to student's lack of understanding of mathematical language (Mbugua, 2012).

## General verbal ability, and non-verbal ability may impact students’

## mathematics difficulties

Even as mathematical thinking can exist independent of language, children need language to express, understand, and learn mathematics. Relationships were stronger in the third-grade sample than in the fifth-grade sample for general vocabulary and mathematical vocabulary. At third grade, general vocabulary accounted for approximately $37 \%$ of variance in mathematics-vocabulary performance. At fifth grade, $43 \%$ of mathematicsvocabulary variance was explained by general vocabulary, general vocabulary was a stronger predictor for third-grade students with lower mathematicsvocabulary scores and a more robust predictor across quantiles in fifth grade (Powella, Driverb, Robertsc \& Fallc, 2017). Verbal analogies were indirectly related to arithmetic knowledge in third grade children through symbolic number skills. Phonological decoding directly contributes to their arithmetic performance. General verbal ability influences how children understand and reason with numbers, whereas phonological skills are involved in executing conventional arithmetic problems (Vukovic \& Lesaux, 2013 ${ }^{\text {b }}$ ). Early language experiences are important for later mathematical development regardless of language minority and native status. But, language influences how children
make meaning of mathematics but is not involved in complex arithmetical procedures. This is evident from language ability in initial grades predicting later gains in grade four in data analysis/probability and geometry, but not in arithmetic or algebra, after controlling for visual-spatial working memory, reading ability, and gender (Vukovic \& Lesaux, 2013 ${ }_{\mathrm{a}}$ ).

## Dimensions of languages impacts different areas of school mathematics

## differently

Chow \& Ekholm (2019) found syntax as the strongest predictor of mathematics performance among young children and vocabulary was not a significant predictor of mathematics performance. They also suggests that vocabulary cannot be considered as an index of language ability among primary school students in the context of mathematics learning. Only less than 10 percent of students at the seventh and eighth grade level were procedurally or conceptually ready even to translate written words into algebraic equations (Capraro \& Joffrion, 2006). In secondary grades, likewise higher frequency of errors originated in the translation from the verbal to the symbolic representation system, especially due to the peculiar features of the algebraic language like variables and structural compilation errors (Domingo, Molina, Canadas \& Castro, 2012).

Teachers of various levels of schooling have an array of strategies to language of mathematics at their disposal

Early language experiences are important for later mathematical development regardless of language background, denoting the need for intensive and targeted language opportunities for language minority and native English learners to develop mathematical concepts and representations (Vukovic \&

Lesaux, 2013). Middle school mathematics teachers recognize the need for them to model their students in using the language of mathematics effectively but fail to realize their responsibilities in developing students' mathematical communication skills (Kabael \& Baran, 2016). Among primary mathematics teachers, there are recent attempts to employ strategies focussing on language of mathematics like code switching (Jegede, 2011), student discourse (Serio, 2014), mnemonics, manipulatives and collaborative work (Naidoo, 2015) with a view to overcome challenges created by the language of instruction within multilingual mathematics classrooms and to help their learners to understand key concepts and recognize mathematics meaning.

However, there are inconsistencies though in early career teachers focus on how to teach mathematical vocabulary, anticipation for students' precise use of mathematical terminology, and the use of multiple languages during classroom instruction (Turner, Roth McDuffie, Sugimoto, Aguirre, Bartell, Drake \& Witters, 2019). They use code-switching, translation, re-voicing across languages to promote the use of multiple languages. Teachers also focuses on mathematical communication (oral, written, nonverbal), participation in mathematical discussion, mathematical discourse practices like explaining and justifying ideas and attention to how students participate in mathematical discourse.

## More integrated strategies for instruction of language of mathematics

 are being tried out for their effectsInterventions based on language of mathematics were conducted from preschool to college level, with many studies focusing at grade 5-8 level. Intervention duration ranged from three year long ones to those limited to a few
chapters. An intervention was limited to weekly hourly sessions for 30 weeks (Patkin, 2011). Earlier interventions used were focusing around vocabulary only. For example, an early study applied mathematics journals, cooperative groups, real life problem solving, and an increased emphasis on mathematical vocabulary among third and fourth graders (Huggins \& Maiste, 1999). Others used techniques like handout of translations of the mathematics terminology at university level (Yushau \& Bokhari, 2005), or vocabulary quizzes, vocabularybased mathematics games and activities (Larson, 2007) or applying building academic vocabulary developed by Marzano \& Pickering (2005) among 6th graders (Gifford \& Gore,2010). Independent learning can be promoted through teacher's integration of mathematics dictionary and polygon pieces into the teaching and learning of geometry (Chiphambo, 2019).

## Most intervention on language of mathematics were on vocabulary though movement away from this trend has begun especially at post-primary level

Vocabulary tutoring intervention improved students vocabulary whereas it did not have an effect on students' algebraic problem-solving skill (Hollingsworth, 2019). Irrespective of grade level, later interventions moved beyond vocabulary, to different forms of talk applied in 7th graders (Brown \& Hirst, 2007), students oral presentations of solutions once per week and daily written solutions to the problems for 6th graders (Sample, 2009), and openended questions with grade one students (Wichelt, 2009). Recent studies tend to focus on whole language interventions, mostly in primary grades or pre primary level. This is evidenced in dialogic reading intervention among pre primary students (Purpura, Napoli, Wehrspann \& Gold, 2017) and computer supported
reciprocal peer tutoring among 2nd graders (Yang, Chang, Cheng \& Chan, 2016) or communication through writing in mathematics and discourse used among fourth year high school students (Lomibao, Luna \& Namoco, 2016).

Early number talk had a positive impact on teachers' professional development in terms of appreciation of the importance of mathematical language, awareness of ideas and opportunities for number talk, knowledge for teaching mathematics, and reflection on practice. The intervention also influenced children's learning in terms of development of number sense and their attitude towards mathematics (Moffett \& Eaton, 2018).

After 'mathematical language storybook reading intervention' students significantly outperformed the students in the comparison group on mathematical language and mathematical knowledge assessments but not in expressive vocabulary measure (Purpura, Napoli, Wehrspann \& Gold, 2017). At secondary level, the class who received the vocabulary instruction showed significant improvements in mathematics vocabulary. However, the strategy was less effective with those students who had a low verbal comprehension (Kenyon, 2016). Vocabulary instruction and mathematical game improved student performance in mathematics achievement, but there was no significant difference between the effect of these two interventions (Tarpley, 2015). Also, high achieving students makes twice the gains of underachieving students in mathematics achievement. The study indicates that vocabulary instruction is equally effective on achievement in mathematics for students who were high or low achievers in reading. Vocabulary games and mathematics writing had a significant effect on achievement in mathematics among secondary school students in New York (Flanagan, 2009).

Holistic communication in mathematics is still being neglected in instructional practices

Communication in mathematics, with the exception of signs and symbols, has been clearly neglected and students have not been given many experiences in oral and written communication in mathematics (Huggins \& Maiste, 1999) at least till the turn of previous century. In the last one and a half decades, mostly in primary grades or pre primary level, studies on language of mathematics, many of them with their focus on whole language interventions, targeted to improve mathematics understanding, achievement and communication of these younger learners, with only a few having explicit intention of improving mathematics vocabulary of them. For example, mathematical language (vocabulary test, comparative language and spatial language and mathematical knowledge) (preschool early numeracy skills) of Preschoolers (Purpura, Napoli, Wehrspann \& Gold, 2017), achievement and mathematical communication abilities of second-graders (Yang, Chang, Cheng \& Chan, 2016), communication abilities of third and fourth graders along with their mathematical vocabulary (Huggins \& Maiste, 1999) were targeted through various interventions. Among sixth graders, written problem explanations and mathematical understanding (Georgius, 2008), and level of understanding of mathematics concepts and selfconfidence in solving mathematics problems (Sample, 2009) were the aim of mathematics language-based interventions.

## Integration of language into mathematics instruction may impact

 related cognitive and affective outcomes, but the issue is unsettledAmong students beyond primary grades, the interventions had their sight on affective outcomes, especially those related to problem solving in mathematics along with mathematics test scores and vocabulary. The outcomes
studied included academic vocabulary, perceptions of potential for success, test scores in mathematics, reduced cognitive load and anxiety among eighth-graders (Gillmor, Poggio \& Embretson, 2015), mathematical word problems solving skills and mathematics performance among fourth year high school students (Lomibao, Luna \& Namoco, 2016).

Beyond the mathematics learning outcomes manifested in vocabulary scores, test scores, word problem solving and an array of affective outcomes, mathematics language based interventions especially those with whole language approach enhanced a host of classroom processes including Interaction in classroom, communication, meaningful participation in mathematics problem solving in classroom across grade levels. This is the case even when they failed to achieve the direct targets of interventions like expressive vocabulary (Purpura, Napoli, Wehrspann \& Gold, 2017) or understanding mathematics (Sample, 2009).

In preschool, for example, increasing exposure to mathematical language through dialogic reading intervention did not improve expressive vocabulary measure, but it positively affected their general mathematics skills (Purpura, Napoli, Wehrspann \& Gold, 2017). Likewise, among class one students, oral and written communication proved useful in the mathematics classroom. Students were more exact in their communication after receiving vocabulary instruction. Students had a better understanding of vocabulary and felt a lot more comfortable about communicating with peers and teachers after being exposed to more open-ended questions. Vocabulary make students more aware of the words they see in their daily assignments which will then translate to higher vocabulary scores as well as test scores (Wichelt, 2009). Among second graders also, mathematical representations and solution explanations became more accurate
after computer supported reciprocal peer tutoring (Yang, Chang, Cheng \& Chan, 2016).

All types of interventions focusing on the language of mathematics do not have identical effects on student outcomes is clear from many studies conducted on sixth grade students. As in the case of early grades, initially, interventions focusing on the language of mathematics at upper primary level also had their focus on vocabulary. Among sixth graders, for example, focus on vocabulary made students more aware of the words they see in their daily assignments which then translate to higher vocabulary scores as well as test scores (Larson, 2007) and expectedly, knowing the definition of mathematical words increased student achievement (Georgius, 2008). Focused instruction on academic vocabulary made students confident, helped in understanding all of the questions and to finish the timed tests early. While, teaching academic vocabulary can benefit all types of learners, it is more beneficial to struggling learners (Gifford \& Gore, 2010). However, written communication could not effect understanding mathematics even as an increase in oral communication could raise selfconfidence (Sample, 2009). Research among grade seven students also imply that teachers need to have a variety of discourse formats at their disposal and be able to use them intentionally, to achieve specific learning goals (Brown \& Hirst, 2007).

Language oriented instruction increase teacher-student interaction in the classroom, even beyond primary grades, and enhance students learning process (Yushau \& Bokhari, 2005). Promoting communication in the class made mathematics enjoyable and fun, had contributed to the reduction of the students' mathematical anxiety, and brought in significantly higher conceptual
understanding of students in high school students (Lomibao, Luna \& Namoco, 2016). Even beyond instruction, cognitive load-reducing techniques in tests ensures that student responses reflect their understanding as observed among eighth graders (Gillmor, Poggio \& Embretson, 2015).

## Conclusions from Review of Literature

This review strengthened the recognition that language cannot be separated from what is taught and learned in school, including in mathematics, and it is the medium through which students gain access to the curriculum and through which they display-and are assessed for-what they have learned. It also shed light on the ways by which academic language of mathematics poses special challenges for learners, especially in a 'multilingual' mathematics classroom. This closing remarks to a detailed review of literature puts synoptically together what has been reiterated in previous summary sections on the structure of and approaches to language of school mathematics; the related issues, objectives, and strategies in schools; and what the previous researches, both descriptive and interventional, on teaching and learning of language of mathematics in schools have detailed.

An overview of the literature on language of school mathematics, especially its nature and characteristics, its relation, similarities and differences with language; structure, and function in particular was obtained. Presently, mathematics is considered a more formal extension of natural language, though restricted and limited in certain aspects like form of language, but its economy over ordinary language being its strength. Reviewed literature showcased valuable learning objectives in relation to language of mathematics in schools
and an array of instructional strategies attempted by teachers and by research interventions as suitable for developing listening, speaking, reading and writing skills in mathematics were identified.

It is clear that an emphasis on the language of mathematics and communicating through this language in classroom teaching engenders learning that is more meaningful, and conceptually integrated. For example, more reading brings in better consolidated learning in mathematics; and discourses in mathematics help in developing a register of its technical language enabling students to connect between everyday meanings of words and their mathematical meanings appropriately. Teachers paying attention to the language of mathematics and integrating such language into mathematics classroom help learners to become aware of, recognize, develop and reorganize their knowledge, to negotiate the language, to articulate their understanding, to consolidate their learning, to develop critical thinking about mathematics, to develop connections between mathematics and life, to think collaboratively and build upon one another's ideas and to increasingly engage in mathematical discourses. It also brings in deeper engagement and understanding of students.

Mathematics learning and teaching in primary as well as secondary levels are impacted at multiple levels of language in classrooms, 1) language of instruction , 2) by nature of mathematics language as such, and 3) through an interaction of the nature of language of instruction and language of mathematics. Regarding language of instruction, difficulty with reading skills, for example affected performance in arithmetic skills among fourth grade students. General verbal ability influences how children understand and reason with numbers, whereas phonological skills are involved in executing conventional arithmetic
problems. However, it is also suggested that language influences how children make meaning of mathematics but is not involved in complex arithmetical procedures.

Whereas many studies demonstrated the importance of various categories of vocabulary for mathematics learning in schools, some others instead suggested that more deeper structures of language like syntax had stronger impact on mathematics performance among young children. Reviewed literature also suggested that impact of multiple levels and different components of language in mathematics classroom are complex and evolves by level of schools and grades in which students learn. This is evident from language ability in initial grades predicting later gains in grade four in data analysis/probability and geometry, but not in arithmetic or algebra. The literature suggests that the effect of language features on mathematics performance reduce as students move up the grades.

Another principle that could be drawn from this review of literature was that early language experiences are important for later mathematical development regardless of language background, whether through vocabulary or through more discourse or communication strategies. It also demonstrates the need for intensive and targeted language opportunities for language minority and native English learners to develop mathematical concepts and representations. However, there are inconsistencies in teachers even on how to facilitate learning mathematics in tandem with language, on how to teach mathematical vocabulary, anticipating students' precise use of mathematical terminology, and the use of multiple languages during classroom instruction. Neither are all aspects of mathematics communication nurtured in an integrated way. For
example Thomas, Garderen, Scheuermann and Lee (2015) observes that research on mathematics and listening is almost nonexistent.

Post 2010, many studies focusing at grade 5-8 level attempted interventions of which duration ranged from three year long ones to those limited to a few chapters. Even as vocabulary instructional strategy remained popular and brought out improvements in students' understanding of mathematical vocabulary, recent interventions especially in later grades moved beyond vocabulary and tried mathematics journals, cooperative groups, real life problem solving, and irrespective of grade level, to different forms of talk. It was evidenced that such exposure to mathematical language can positively affect student's mathematics skills. However, communication in mathematics, with the exception of signs and symbols, has been clearly neglected and students have not been given many experiences in oral and written communication in mathematics.

The literature throws light also on the kinds of outcomes which the integration of language in mathematics teaching learning can bring in. Such language of mathematics if integrated in classroom teaching, results in improved academic vocabulary, perceptions of potential for success, test scores in mathematics, conceptual understanding, mathematical word problems solving and mathematics performance, and reduced cognitive load effect on student performance and anxiety. Promoting communication in the class also made mathematics enjoyable and fun.

This review revealed longstanding interest in the issues involved in teaching and learning mathematics in different languages, reflective of the special context of political struggles over choice of language of instruction. Further research will add to understanding of how characteristics of specific
languages, for example Malayalam, may affect the nature of the mathematics teaching and student learning. Such research needs to attend to the nature of natural language applied in mathematics classrooms, and improve special language and grammar of mathematics itself, as well as language in multi linguistic contexts as in non-English classrooms where English words, letters, abbreviations, are used in addition to natural language. Concern for such substantial and coordinated research efforts may take into account identification of linguistic competences and knowledge required for participation in mathematical practices, the processes and mechanisms by which students develop linguistic competence and knowledge in mathematics; and the skills teachers need and apply in order to support the development of linguistic mathematical competence in students. Hence, this study is especially to equip the stakeholders of mathematics education in schools to understand and appreciate the difficulties emerging from deep and surface structures of mathematics language as used in Malayalam medium schools of Kerala where this study is situated and it test the effect of a language integrated mathematics instruction on achievement, self-efficacy and attitudes related to elementary school mathematics among these students.

## Chapter III

## METHODOLOGY

- Design of the Study
- Variables of the Study
- Procedure of the Study
- Tools and Techniques used for the Study
- SAMPLE USED FOR THE STUDY
- Statistical Techniques Used

This study analyzes language related difficulties in learning mathematics to develop an evidence-based instruction focusing on the language of mathematics, the effect of which is then examined in enhancing mathematics learning in terms of achievement in mathematics, self-efficacy in mathematics and attitude towards mathematics. This chapter discusses in detail the design of the study, variables, tools, sample and statistical techniques used.

## Design of the Study

This study has a survey phase preceding an experimental phase with the former giving basis for latter phase by providing the details of language related difficulties in learning mathematics. Hence it can be considered as adopting a quan $\rightarrow$ QUAN, deductive-sequential design, where both components are quantitative; and hence can be considered a multimethod design (Morse \& Niehaus, 2009) with questionnaire and survey testing in an earlier phase informing the development of an instruction programme for a later experimental phase. Mixed methods study usually consists of a qualitative or quantitative core component and a supplementary component. This supplementary component consists of qualitative or quantitative research strategies that is not a complete study in itself (Morse \& Niehaus, 2009). In between the survey and experimental phase, several tools were developed in preparation to pretesting and post-testing. Hence, the whole design of this study can be outlined as in Figure 2.


Figure 2. Outline of the study

## Variables of the Study

This study can be described in three phases, with Phase I Pilot Study proceeding to Phase III Experiment, after a phase of design and development of appropriate tools and intervention strategies in between.

## Variables in Phase I (Pilot Study)

Through content analysis and Part 1 survey, components of language of mathematics was identified. Based on the results from this survey, a test to
identify linguistic difficulties was developed and part 2 survey was conducted using this test to verify the linguistic difficulties in learning mathematics when the medium of instruction is Malayalam. Hence in phase 1, the variables studied were students' perception of difficulties in mathematical tasks, reasons sourcing from nature of mathematics for these perceived difficulties in mathematical tasks, and achievement in the language of mathematics and its components. Perception of difficulties in mathematical tasks among elementary school students is conceived as the dependent variable being influenced by reasons sourcing from nature of mathematics and achievement in the language of mathematics and its components.

After survey phase, using the research evidence, instruction focusing on the language of mathematics was developed based on the identified linguistic difficulties in learning mathematics.

## Variables in Phase III (Experimental Phase)

The effectiveness of an evidence-based instruction focusing on language of mathematics (Language integrated mathematics instruction) in improving students' mathematics learning outcomes in terms of achievement in mathematics, selfefficacy in mathematics and attitude towards mathematics- in comparison to guided practice in mathematics problem solving is examined. There are independent, dependent and control variables in the experimental phase.

## Independent variables

Independent variable in this study is instructional method. For the experimental group, language integrated mathematics instruction is provided and for control group practice in solving mathematics problems is provided.

## Dependent variables

The study was intended to examine the effect of language integrated mathematics instruction on mathematics learning. The dependent variables, hence, are a set of cognitive and affective outcomes of mathematics learning including achievement in mathematics, self-efficacy in mathematics and attitude towards mathematics.

## Achievement variables

There are four achievement related dependent variables.

1. Achievement in Mathematics. It is the weighted total of achievement in the five chapters of mathematics viz., 1) Parallel Lines, 2) Unchanging Relations, 3) Repeated Multiplication, 4) Area of Triangle and 5) Square and Square root, prescribed for standard seven pupils by State Council of Educational Research and Training, Kerala.
2. Achievement in Algebra. It is the extent to which students have attained the cognitive objectives of learning the unit 'Unchanging Relations'.
3. Achievement in Arithmetic. It is the weighted total of achievement in the two units- 'Repeated multiplication' and 'Square and square root'.
4. Achievement in Geometry. Achievement in geometry is the weighted total of cognitive achievement of students in the two units- 'Parallel lines' and 'Area of Triangle’.

## Self-efficacy variables

Effectiveness of language integrated mathematics is measured also against self-efficacy in mathematics. There are six self-efficacy variables. They are:
5. Self-efficacy in Mathematics. Perceived self-efficacy is the measure of student beliefs about their capabilities to solve mathematics problems and to perform in mathematics learning, denoted by the total score in selfefficacy in learning mathematics and self-efficacy in solving mathematics problems.
6. Self-efficacy in Learning Mathematics. It is the measure of student belief about their capabilities to perform in mathematics learning contexts like school mathematics learning in general, classroom teaching-learning of mathematics and assessment practices in mathematics.
7. Self-efficacy in Solving Mathematics. It is the measure of student belief about their capabilities to solve mathematics problems in seven areas of school mathematics viz; natural numbers, fractions, decimals, geometry, percentages, averages, graphs and algebra up to standard.
8. Self-efficacy in Algebra. It is the measure of student beliefs about their capabilities to perform in mathematical tasks related to the chapter 'Unchanging relations'.
9. Self-efficacy in Arithmetic. It is the weighted average of student beliefs about their capabilities to perform mathematical tasks in the two chapters 'Repeated multiplication' and 'Square and square root'.
10. Self-efficacy in Geometry. It is the weighted average of student beliefs about their capabilities to perform mathematical tasks in the two chapters 'Parallel lines' and 'Area of Triangle'.

## Attitude towards Mathematics

Effectiveness of language integrated mathematics instruction is also measured against attitude towards mathematics and its components namely- like towards mathematics, engagement with mathematics, self-belief in mathematics, active learning of mathematics and enjoyment of mathematics.
11. Attitude towards Mathematics. It is the sum total of student's positive or negative feelings towards mathematics as a school subject, its learning, classroom practices, mathematics teacher, assessment practices, homework, and involvement of parents and peers. It is the total score of the five dimensions of attitude towards mathematics.
12. Like towards Mathematics. It is the student's overall like towards mathematics as a school subject, its teaching-learning activities, assessment practices, homework, parents' involvement in mathematics learning, peer involvement and mathematics teacher.
13. Engagement with Mathematics. It measures the tendency of the student to engage in or avoid mathematics related activities like classroom activities, homework, mathematics teacher and parent-peer involvement in mathematics learning.
14. Self-belief in Mathematics. It measures students' beliefs about their ability to cope with mathematics learning activities and performance.
15. Active Learning of Mathematics. It is a measure of students' active and motivated participation in mathematics learning activities both in the classroom and home, assessment context and in interaction with mathematics teachers and peers.
16. Enjoyment of Mathematics. Enjoyment of mathematics is the measure of students positive feelings towards mathematics learning activities, homework, exam and teacher.

## Control variables

Verbal comprehension in Malayalam, Nonverbal intelligence and Previous achievement in mathematics were controlled among the experimental and control groups; by matching mean scores of these variables among the groups.

## Procedure of the Study

The study proceeds through three phases; first a pilot study with survey and content analysis, and then a developmental phase that leads to the final experimental phase.

## Phase I: Pilot Study with content analysis and survey

In order to identify the language related difficulties in learning mathematics for elementary school students the following were done. Content analyses of mathematics textbooks from preprimary to standard seven, and that of achievement tests used in schools were done to identify linguistic components involved in it. Through survey and review of related literature, the linguistic components of mathematics teaching-learning and the perception of students about difficulty due to these factors were identified.

Part 1 survey identified eighth standard students' perceived difficulties in mathematical tasks and reasons for difficulty thereof. Then, development of a battery of tests to identify linguistic difficulties in mathematics teaching-learning process was guided by extensive review of the literature on language related difficulties in mathematics and the content analysis to identify the linguistic components in mathematics textbooks, and that of mathematics achievement tests used in school examinations. By using this battery of tests, part 2 survey identified students' language related difficulties in mathematics learning. Results of Part 1 and part 2 surveys guided the development of evidence-based instructional strategy focusing on language of mathematics to overcome the identified linguistic difficulties by choosing appropriate instructional strategies from reviewed related literature.

## Phase II: Developmental Phase

In the second phase, an evidence-based instruction focusing on the language of mathematics in elementary level was developed based on the evidence from the pilot study. Strategies for language integrated mathematics instruction to the experimental group and that for guided practice in solving mathematics problems for the control group were planned and designed. Tools for measurement in experimental phase were also developed during this phase. Test of previous achievement in mathematics, test of verbal comprehension in Malayalam, scale of self-efficacy in mathematics, attitude towards mathematics, and achievement tests and scales of self-efficacy for the five units of standard seven mathematics- 1. Parallel lines, 2. Unchanging relations, 3. Repeated multiplication, 4. Area of triangle and 5. Square and square root were
developed. These tools were tried out and their validity and reliability were ensured.

## Standardization of tools for experimental phase

Since, elementary school students of Kerala studying in Malayalam medium comprise the population of the study, for standardization of tools, try out sample was drawn from seven schools of Kozhikode, Malappuram, Palakkad and Alappuzha districts. Draft tools were tried out in different samples drawn from these schools. Test of previous achievement in mathematics, scale of attitude towards mathematics, scale of self-efficacy in mathematics and test of verbal comprehension in Malayalam were administered on different samples of 370 students each. Data from additional students were also drawn on scale of attitude towards mathematics, scale of self-efficacy in mathematics, and test of verbal comprehension in Malayalam in order to facilitate factor analysis and/or validity as required. Tests of achievement and scales of self-efficacy in five units of mathematics were administered on different samples of 200 students each. The data required for validation, viz., Self-efficacy for learning and performance subscale of Motivated Strategies for Learning Questionnaire (Pintrich, Smith, Garcia \& Mckeachie, 1991), Selfefficacy in learning Mathematics (Abidha \& Gafoor, 2018), Test of Malayalam reading comprehension for grade 7 students (Gafoor \& Aneesh, 2018) were also drawn along with the respective tools for validation. List of schools from where students were drawn into sample for standardization of tools are given in Table 1.

Table 1
List of Schools from where Students were Drawn into Sample for Standardization of Tools

| SI. No. | Name of Schools | District |
| :---: | :--- | :--- |
| 1 | Govt. U P School, Kodal Nadakkavu | Kozhikode |
| 2 | Govt. Ganapath UP School, Pokkunnu | Kozhikode |
| 3 | AMUPS, Kambliparamba | Kozhikode |
| 4 | AUPS, Puthurmadam | Kozhikode |
| 5 | GMHSS, CU Campus School | Malappuram |
| 6 | AUPS, Varode | Palakkad |
| 7 | NSSHSS, Panavally | Alappuzha |

## Phase III: Experiment

Effectiveness of the evidence-based instruction focusing on the language of mathematics is examined through a quasi-experimental pretest-posttest nonequivalent control group design experiment as given hereunder.

1. Four intact classes of standard seven were selected and two classes each were randomly assigned to the experimental group and the control group. Then, the analysis samples in experimental and control groups were matched on their verbal comprehension in Malayalam, non-verbal intelligence and previous achievement in mathematics.
2. Experimental and control groups were pretested on self-efficacy in mathematics and attitude towards mathematics.
3. In the experimental group, language integrated mathematics instruction was provided by the experimenter along with content instruction of five chapters by the schoolteacher (1. Parallel lines, 2. Unchanging relations, 3. Repeated multiplication, 4. Area of Triangle and 5. Square and square root). In the control group, experimenter provided practice in solving
mathematics problems along with content instruction by the schoolteacher for these five units.
4. Subsequently the effectiveness of the language integrated mathematics instruction is checked with respect to all dependent variables.

## Design of the experiment

The pretest-posttest control group non-equivalent group design used in this study is denoted as follows.

| $\mathrm{G}_{1}$ | $\mathrm{O}_{1}$ | X | $\mathrm{O}_{2}$ |
| :--- | :--- | :--- | :--- |

$\begin{array}{llll}\mathrm{G}_{2} & \mathrm{O}_{3} & \mathrm{C} & \mathrm{O}_{4}\end{array}$
$G_{1} \& G_{2}$ - Intact divisions of $7^{\text {th }}$ standard students randomly assigned to experimental and control groups and matched on previous achievement in mathematics, verbal comprehension in Malayalam and non-verbal intelligence.

X - Language integrated mathematics instruction (by experimenter) along with content instruction (by schoolteacher)

C - Practice in solving mathematics problems (by experimenter) along with content instruction (by Schoolteacher)
$\mathrm{O}_{1} \& \mathrm{O}_{3}$. Pretests on self-efficacy in mathematics and attitude towards mathematics.
$\mathrm{O}_{2} \& \mathrm{O}_{4}-\quad$ Posttests on achievement and self-efficacies in 1) Parallel lines, 2) Unchanging relations, 3) Repeated Multiplication, 4) Area of Triangle and 5) Square and square root; and self-efficacy in mathematics and attitude towards mathematics.

## Tools and Techniques used for the Study

Content analysis, questionnaire, achievement testing, and attitude and self-efficacy scales were used in the study. In initial phase the following tools were used.

1. Questionnaire on students' difficulties in learning mathematics
2. Test of difficulties in the language of mathematics (3 Sets)

In experimental phase the following measuring tools were used.

1. Test of previous achievement in mathematics
2. Test of verbal comprehension in Malayalam
3. Raven's standard progressive matrices (Raven, 1998)
4. Scale of attitude towards mathematics
5. Scale of self-efficacy in mathematics
6. Tests of achievement and scales of self-efficacy in -
i. Parallel lines
ii. Unchanging relations
iii. Repeated multiplication
iv. Area of triangle
v. Square and square root

In addition to the measuring tools, lesson manuals for language integrated mathematics instruction and guided practice in solving mathematics problems were also developed. Techniques used in language integrated mathematics instruction strategy were designed to overcome the linguistic difficulties in learning mathematics among elementary school students. The lessons were prepared by employing the techniques, viz., anchoring mathematics with language, vocabulary bank, labeling vocabulary, word walls, word trails, listen and write, possible sentences, guess what?, justifying their reasoning and translation game.

Practice in solving mathematics problems was provided to the control group for an equal period of time in which students were given guided practice in mathematics problem solving for each select unit.

Each tool used for data collection are described in this section.

## 1. Questionnaire on Students' Difficulties in Learning

This questionnaire was developed to identify student perception of difficulties in mathematical tasks and reasons for difficulty thereof. It measures students' perceived difficulty in 26 mathematical tasks and the reason for difficulty sourcing from nature of mathematics content and its teaching-learning process.

## Planning

The questionnaire was planned to have two sections. One section of the questionnaire was planned to be a checklist of mathematical tasks. After exploring the mathematical tasks through review of literature (Barbu \& Beal, 2010; Barton \& Heidemma, 2002; Cruz \& Lapinid, 2014; Gooding, 2009; Shaftel, Belton-Kocher, Glasnapp \& Poggio, 2006) and SCERT Kerala textbooks up to $6^{\text {th }}$ grade, it was decided to make a checklist of 26 mathematical tasks, under five heads viz., number concept, mathematical symbols and notations, mathematical operations, mathematical abstractions and problem solving.

The other section is planned to have rating scale of reasons for difficulty sourcing from nature of subject and it was not limited to mathematics in this questionnaire. All major school subjects were included in the questionnaire to reduce respondents cueing the intention of the test-administrator's interest in mathematics in particular. Difficulties in learning is sourced from structure of the subject, the process of learning that subject and irrelevance of that subject in daily life. Ramanujam, Subramanian \& Sachdev (2006) identifies cumulative nature of mathematics as one of the major reasons for fear of mathematics. Kerala

Curriculum Framework (2007) observes difficulty in mathematics learning sourcing from imbibing basic tenets and unpalatable theories of mathematics, difficulties with methods of forming ideas of mathematics, the repetitive nature of exercises to gain proficiency in mathematical calculations, mismatch between mathematics in daily life and school mathematics, introduction of symbols and figures and also from the over emphasize given to the established methods of calculation. Nature of subject which makes difficulty in learning were identified through extensive review and 13 reasons were listed.

## Item writing

Items were prepared for questionnaire on students' difficulties in learning based on the plan. One section of the questionnaire has 14 items. One item was to rate the school subjects viz., Malayalam, physics, chemistry, biology, social science and mathematics in the order of feeling of difficulty. Remaining items comprise reasons related to nature of school subjects that makes the subject difficult to learn. The reasons included are need for regularity in attending classes, prominence to problem solving, need for strenuous attention, repeated practice, number of concepts, difficulty of concepts, need for external support, understanding questions, unfamiliar terms, prevalence of symbols and notations, need for rote learning and impracticability in life. Participants have to rate their feelings of difficulty of school subjects for each of these reasons in 3-point scale. Responses were structured differently for each item to match the stem. Out of the 13 reasons, one item had only two response options. Second part of the questionnaire was in the form of checklist of 26 tasks, under 5 heads viz. number concept, mathematical symbols and notations, mathematical operations, mathematical abstractions and problem solving. At the end of the questionnaire one open ended item drawing any additional reason for difficulty in learning mathematics was given.

## Finalization of the questionnaire

Exploratory factor analysis was conducted on the nature of elementary school mathematics that makes it difficult for students to learn, to find the latent structure if any in the 13 reasons for mathematics being difficult as perceived by the students. The results are provided in the Table 2.

Table 2
Factor Loading and Communalities from Principal Component Analysis of Perceived Nature of Elementary School Mathematics that Makes it Difficult to Learn

|  | Components |  |
| :--- | :--- | :---: |
| Perceived nature of mathematics owing to <br> difficulty in learning it | Nature of <br> mathematics content | Nature of <br> mathematics <br> teaching learning |
| Difficulty of concepts | 0.438 |  |
| Need for strenuous attention | 0.427 |  |
| Prevalence of symbols and notations | 0.759 |  |
| Need to learn unfamiliar terms | 0.741 |  |
| Impracticability in Daily Life | 0.648 |  |
| Need for Precision in understanding | 0.542 | 0.511 |
| Number of concepts | 0.391 | 0.774 |
| Prominence of Problem Solving |  | 0.731 |
| Need for Regularity in Attending Classes |  | 0.671 |
| Need for Repeated Practice |  | 0.654 |
| Need for rote learning |  | 0.496 |
| Need for external support | $23.45 \%$ | $29.16 \%$ |
| Difficulty in understanding questions |  | $52.61 \%$ |
| Variance explained (Eigen value) |  |  |
| Total variance explained |  |  |

Two categories of source of difficulties in mathematics learning were arrived through exploratory factor analysis namely nature of mathematics content and nature of mathematics teaching-learning.

Further, exploratory factor analyses were conducted with principal component analysis on each of the five areas namely number concept,

## 182 EVIDENCE BASED INSTRUCTION OF LANGUAGE OF MATHEMATICS

mathematical symbols and notations, mathematical operations, mathematical abstractions and problem solving. Two areas viz., mathematical symbols and notations and mathematical abstractions derived only one factor each while the other three areas derived more than one factor. The results of the factor analyses are given in Appendix A1. Summary of the factors derived is given in Table 3.

Table 3
Summary of the Factors Derived in Exploratory Factor Analyses of the Perceived Task Difficulties in the Five Areas of Elementary School Mathematics

| Area | Factor derived | Tasks involved |
| :--- | :--- | :--- |
| Number <br> concept | Number systems <br> Comprehending <br> numbers | Using fraction and using decimals <br> place value |
| Mathematical <br> Symbols <br> \& notations | Mathematical <br> Symbols <br> \& notations | Understanding algebraic problems, analyzing <br> geometrical figures, understanding symbols and <br> notations and drawing geometrical figures |
| Mathematical <br> operations | Problem solving <br> competence | Doing calculations with speed and concentrating <br> for long time to solve problems |
| operations | Doing mental arithmetic, following rules while doing <br> calculations, remembering numbers while doing <br> operations and doing basic arithmetic operations |  |
| Mathematical | Mathematical <br> abstractions | Comprehending process unrelated to daily life and <br> comprehending concepts unrelated to daily life |
|  | Understanding <br> word problems | Identifying irrelevant information in word problems, <br> understanding word problem without external help, <br> identifying key words and identifying mathematics <br> problem in word problems |

## Administration and scoring procedure

The questionnaire on students' difficulties in learning is a 3-point Likerttype scale. One item was to rate the school subjects based on their feeling of
difficulty. Remaining statements were related to nature of subjects that makes difficulty in learning it. Students may respond to each statement in three different ways. Each item had different response options. However, every item had only 3 response statements indicating 3- high, 2- moderate and 1-low rate of difficulty. Appropriate instructions were provided in the tool. Space was provided in the questionnaire to write name and gender of the student. There is no total score for the tool. Each item was considered separately for analysis. Each item has a minimum score of 1 and a maximum score of 3 in case of a subject. Percentage of students who perceive low / moderate / high difficulty due to each of these reasons was obtained.

The checklist of mathematical tasks was scored one score for each task, if the student marked a tick mark to indicate their perceived difficulty in that task. Percentage of students who perceive difficulty in 26 mathematical tasks was calculated. In order to get difficulty level in the nine identified factors, the percentages of all the tasks involved in the factor were averaged.

## Validity

Reasons for difficulty sourcing from nature of the school mathematics were identified by reviewing Kerala Curriculum Framework (2007) and National Focus Group report by Ramanujam, Subramanian and Sachdev (2006). Two categories of source of difficulties in mathematics learning were further arrived through exploratory factor analysis. The high factor loadings of the sources of difficulty namely nature of mathematics content and nature of mathematics teaching-learning further validates them.

The 26 tasks in elementary school mathematics were identified through a detailed content analysis of SCERT mathematics textbooks up to standard seven extant during the academic year 2015-16, along with the review of literature on
the same. Exploratory factor analysis also validated the nine factors of task difficulty in elementary school mathematics namely number systems, comprehending numbers, mathematical symbols and notations, problem solving competence, arithmetic operations, mathematical abstractions, understanding word problems, equations and operations and translation of word problems.

Copies of Malayalam and English versions of questionnaire on students difficulties in learning are provided as Appendices A2 and A3 respectively.

## 2. Test of Difficulties in Language of Mathematics (3 Sets)

This test was developed to identify the language related difficulties in learning mathematics among elementary school students when the medium of instruction is Malayalam.

## Planning

Textbooks from preprimary to standard seven and teacher made achievement tests were analyzed to find out the linguistic components of mathematics teaching-learning. This culminated in a detailed and categorized glossary of terms and symbols in elementary school mathematics. This glossary of terms and symbols in elementary school mathematics where medium of instruction is Malayalam is provided as Appendix B.

The content analysis helped to identify components of language of mathematics other than words and symbols and thus to compare language used in mathematics with a natural language and a thematic structure of language of mathematics was thus developed as in Figure 3. Components of a natural language can be enumerated as its content, structure and function. Content includes lexicon and grapheme. Vocabulary or lexicon is the basic component of any natural language. Most of the natural language has its own grapheme which is the smallest
unit used in describing the writing system of a language. Structure of language is governed by rules related to phonology, morphology and syntax. Phonology deals with sound system of language while morphology deals with the rules related to formation of words. Syntax is rules related to formation of sentence that we commonly refer as grammar in language. Then, there is the functional aspect of language - Semantics and pragmatics. semantics patterns the meaning of words and sentences whereas pragmatics is a system that outlines the use of language in context.

Parallelism between natural language and language of mathematics was examined. Language of mathematics can also be made into components as content, structure and function.

Mathematics has its own language with unique content that comprises grapheme and lexicon as natural language do.


Figure 3. Components of language of mathematics

Mathematical grapheme includes something which is unique to mathematics viz., diagrams, numbers and symbolic expressions. Though mathematics is considered as a universal language, some of the mathematical graphemes are not universal. For example, numbers have different written character forms in Arabic and Malayalam languages. Mathematical symbols can be classified as

- Object/Concept Symbols (E.g. numbers)
- Operation/ Process symbols (E.g. arithmetic operations)
- Relation symbols (E.g. perpendicular, parallel)
- Auxiliary Symbols (E.g. parentheses)

Mathematical vocabulary / lexicon includes verbal expressions/ terms, which can be broadly classified as discipline specific terms, and common words that have a different meaning in mathematics. For example, sign, volume, figure, odd, face has a different meaning in mathematics. Mathematical lexicon also includes variables, numbers and symbolic expression.

Unlike natural languages, mathematics has no special phonology to deal with speech sounds but require morphology to deal with grammar in formation of words. A number of mathematical terms has prefix from Latin or Greek. Morphological study of such mathematical terms will upturn the level of comprehension of mathematical vocabulary. Structure of mathematical language also deals with syntax that constitutes rules related to sentence formation. Mathematical sentence has its own grammar. For example, an
equation is a mathematical sentence with noun as expression and verb as "=" (is equal to).

The third component of the language of mathematics is the functional aspect of language - semantics and pragmatics. Semantics concern with meaning of mathematical terms whereas pragmatics deals with the contextual usage of mathematics.

Item to diagnose difficulties in each of the components were planned for which different format of items were needed.

## Item writing

Based on the components of language of mathematics, items were prepared to diagnose difficulties in each of these components. Four types of items namely multiple choice, true/false, matching type and fill in the blanks (completion type) were included in this test.

Due to large number of items, the tool was made into a battery of three parallel tests of three (Set A, B \& C). Details of the three sets of tests of difficulties in language of mathematics are given in Table 4.

Table 4
Component and Subcategory wise Items in Three Sets of Tests of Difficulties in Language of Mathematics

| Components | Subcategory | Item Numbers |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | SET A | SET B | SET C |
| Morphology | Parts of words | 46, 47, 48 | 43, 44, 45, 60, 61 | 42, 43 |
| Terms | General terms | $\begin{gathered} 39,40,41,42,43 \\ 44,51,52,53,54 \\ 55,56 \end{gathered}$ | $\begin{gathered} 40,46,48,49,50 \\ 51,52,53,54,55, \\ 56,58,59,63 \end{gathered}$ | $\begin{gathered} 39,46,47,48,49 \\ 51,52,53,54,57 \\ 58,59,60,61 \end{gathered}$ |
|  | Mathematics specific terms | $\begin{gathered} 4,20,21,22,45 \\ 48,49,57,58,59 \\ 60,61,62 \end{gathered}$ | $\begin{gathered} 2,21,22,23,41 \\ 42,47,57,62 \end{gathered}$ | $\begin{gathered} 18,19,20,21 \\ 40,41,44,45,50 \\ 55,56,62 \end{gathered}$ |
| Symbols | Basic mathematical symbols | 1, 63, 64, 65 | 64, 65, 66 | 63, 64, 65, 66 |
|  | Symbols in geometry | 12, 13, 15 | 15 | 2, 13 |
| Semantics | Word meaning in specific context | $3,14,16,23,24$ | $\begin{gathered} 3,11,16,17,18 \\ 19,24,25 \end{gathered}$ | $\begin{gathered} 1,5,14,15,16 \\ 22,23,24 \end{gathered}$ |
| Syntax | Mathematical conventions | 2 | 1 | 4 |
|  | Natural language | $\begin{gathered} 11,25,26,27,28, \\ 29,30 \end{gathered}$ | $\begin{gathered} 13,29,30,31,32 \\ 33,34 \end{gathered}$ | $\begin{gathered} 12,27,30,31,32 \\ 33,34 \end{gathered}$ |
|  | Numeric expression | 31, 32 | 26, 27 | 25, 26 |
|  | Algebraic expression | 10, 33, 34 | 12, 28, 35 | 11, 28, 29 |
|  | Geometric figures | 35, 36, 37, 38 | 36, 37, 38, 39 | $35,36,37,38$ |
| Pragmatics | Commonly used fractions | 18, 19 | 7, 10, 14 | 3 |
|  | Real world problems | 6, 8 | 6, 8 | 7,10 |
|  | Reading graphs | 17 | 20 | 17 |
|  | Identifying operations from key terms | 5, 7, 9 | 4, 5, 9 | 6, 8, 9 |
| Total Number of items |  | 65 | 66 | 66 |

Note. Figures in italics indicate item numbers.

## Administration and scoring procedure

Test of linguistic difficulties in mathematics is an objective- test with three parallel forms. Multiple choice items have four responses out of which one
is right answer and rests are distracters. Students have to circle their answers on the test booklet itself. For true / false items students have to mark a tick or cross against the statement given. Four set of matching items were given in each test. Fill in the blank items were scored one if the student has given a correct answer. Appropriate instructions were included in the test. Students were given 40 minutes to complete the test. Space was provided in the test to write name and gender of the student.

There is no total score for this tool as it was developed to diagnose the linguistic difficulties in mathematics learning. However, each section namely, morphology, terms, symbols, semantics, syntax and pragmatics have a total score. Procedure for this was as follows.
i. Each right answer is given one score and wrong answer is given a zero.
ii. Estimating the percentage of students who answered each items correctly.

- If there are more than one items the percentage of students who answered the items in a subcategory is averaged.
iii. Calculating the weighted average of percentage of students who answered the parallel items/items in a subcategory in all the three parallel tests.
iv. Calculating the weighted average of percentage of students who answered the items in each component.


## Validity

Content validity of test of difficulties in the language of mathematics was ensured by covering major language components used in mathematics textbooks and mathematics question papers from preprimary to standard seven. Ambiguous items were modified according to suggestions of expert.

Copies of the three tests of difficulties in the language of mathematics (Set A, B \& C) are provided as Appendices C1, D1 and E1 respectively, and corresponding scoring keys are provided as Appendices C2, D2 and E2.

## 3. Raven's Standard Progressive Matrices

It is an internationally accepted valid measure of intelligence. The standard progressive matrices (SPM) was designed to cover the widest possible range of mental ability and to be equally useful with persons of all ages, whatever be their education, nationality or physical condition (Paul, 1986). Raven's standard progressive matrices was used to match the experimental groups in their non-verbal intelligence. The tool consisted of 5 sets, each containing 12 items which makes a total of 60 items. Items in this tool are arranged in the increasing order of difficulty. Items were in the form of puzzle pictures with one missing part. One score was given to each right answer, and the total raw score can be calculated by adding these scores.

The SPM was developed in the mid 1930's and it was standardized in different set of populations, but no recent percentile norms are available for SPM on Indian population. Hence only raw scores of SPM were used for grouping purposes. Students are classified as high and low on non-verbal intelligence by using median of raw scores ( $50^{\text {th }}$ percentile) as the cut point.

## 4. Scale of Attitude towards Mathematics

This scale is intended to assess the attitude of students towards mathematics as a school subject. An attitude is typically conceptualized as being a feeling toward an object, a social institution, or a group (Mehrens \& Lehmann, 1984). This scale assesses students' positive or negative feelings towards mathematics. The five dimensions included in the attitude scale are like towards mathematics, engagement with mathematics, self-belief in mathematics, active learning of mathematics and enjoyment of mathematics.

## Planning

Various theoretical views about attitudes has been emerged. The tricomponential theory propose that attitude is a single entity with three aspects namely affective, behavioral, and cognitive. An affective or emotional component is the feelings and emotions one has toward the object, behavioral component consists of one's action tendencies toward the object and cognitive component comprises the ideas and beliefs about the object (Oskamp \& Schultz, 2005). There may or may not be congruence among feelings, behavior and beliefs. Further, a number of factors are found to be affecting mathematics learning namely, nature of mathematics in general, its learning, classroom activities, teacher, assessment practices, homework, involvement of parents and peers, and content of school mathematics. Extensive review of literature on attitude towards mathematics revealed major dimensions of attitude as follows.

1. Liking-enjoyment of mathematics (Adelson \& McCoach, 2011; Alken, 1974; Chapman, 2009; Guce \& Talens, 2013; Michaels \& Forsyth, 1977; Palacios, Arias \& Arias, 2014; Sandman, 1980; Tapia \& Marsh, 2005)
2. Anxiety towards mathematics (Fennema \& Sherman, 1976; Palacios, Arias \& Arias, 2014; Sandman, 1980; Tapia \& Marsh, 2004)
3. Perception of difficulty (Palacios, Arias \& Arias, 2014)
4. Perceived utility (Fennema \& Sherman, 1976; Guce \& Talens, 2013; Michaels \& Forsyth, 1977; Palacios, Arias \& Arias, 2014; YanezMarquina \& Villardon-Gallego, 2015)
5. Value (Alken, 1974; Chapman, 2003; Sandman, 1980; Tapia \& Marsh, 2004)
6. Mathematical self-concept (Palacios, Arias \& Arias, 2014; Sandman, 1980, Yanez-Marquina \& Villardon-Gallego, 2015)
7. Perception of efficacy (Adelson \& McCoach, 2011; Chapman, 2003; Guce \& Talens, 2013; Tapia \& Marsh, 2004)
8. Confidence (Fennema \& Sherman, 1976; Tapia \& Marsh, 2004)
9. Motivation (Fennema \& Sherman, 1976; Sandman, 1980; Tapia \& Marsh, 2004)
10. Parent / teacher expectations (Tapia \& Marsh, 2004)
11. Interest for mathematics (Yanez-Marquina \& Villardon-Gallego, 2015)
12. Tendency to engage in or avoid in mathematics activities (Guce \& Talens, 2013)

These dimensions were abridged into five by merging similar ones. They are enjoyment of mathematics, utility, anxiety, self-concept and motivation. Based on the eight factors affecting mathematics learning and the identified dimensions of attitude towards mathematics, a two-dimensional 8 X 5 grid was made. It was planned to prepare items for each of the cell. The grid used to prepare items for scale of attitude towards mathematics learning is given in Table 5 .

Table 5
Dimension wise and Factor wise Distribution of Items of Scale of Attitude towards Mathematics

| Dimensions <br> Factors affecting mathematics learning |  | $\frac{\grave{y}}{ \pm}$ | $\frac{\vec{\pi}}{\frac{\pi}{x}}$ | $$ |  | $\stackrel{\square}{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Nature of mathematics in general | 1 | 4 | 1 | 1 | 1 | 8 |
| Learning | 3 | 2 | 1 | 3 | 5 | 14 |
| classroom activities | 2 | 1 | 3 | 1 | 2 | 9 |
| Teacher | 1 | 1 | 1 | 1 | 2 | 6 |
| Assessment practices | 2 | 1 | 2 | 1 | 1 | 7 |
| Homework | 1 | 1 | 1 | 1 | 2 | 6 |
| Involvement of parents / peers | 1 | 1 | 2 | 1 | 2 | 7 |
| Content of school mathematics | 1 | 2 | 1 | 1 | 1 | 6 |
| Positive items | 9 | 11 | 1 | 1 | 13 | 35 |
| Negative items | 3 | 2 | 11 | 9 | 3 | 28 |
| Total | 12 | 13 | 12 | 10 | 16 | 63 |

## Item writing

Pool of items were developed based on the grid. Thirty-five positive and 28 negative statements were prepared which were clear, unambiguous and relevant. Draft scale consisted of 63 items.

## Administration and scoring procedure

The scale of attitude towards mathematics is a 5-point Likert-type scale. Each statement can be responded on anyone of the five points: 1. True, 2. Mostly True, 3. Somewhat True, 4. Rarely True, and 5. Not True. Students were asked to read each of the statements carefully and decide on how accurate the statement is in their case. The positive items are scored " $5,4,3,2$, and 1 " whereas the negative items are scored " $1,2,3,4$ and 5 " respectively. Sum of the scores of all statements give the total score of attitude towards mathematics, and it can range between 63 and 315 .

## Item Analysis

Item analysis was done on a sample of 370 students. Data were arranged in ascending order of total score in scale of attitude towards mathematics. Upper 27 percentage (100 Numbers) and lower 27 percentage (100 numbers) were selected for analysis. Response to each of the items were analyzed. Critical ratio was calculated. Statements having $t$-value $\geq 2.58$ were selected for the final version. Three items with inadequate discrimination power and 10 items which are duplications were removed. Details of the results of item analysis of the scale of attitude towards mathematics is given in Appendix F1.

Factor analysis of the scale was done on a sample of 509 students which confirmed the five dimensions of attitude towards mathematics learning.

Dimension wise factor loading of each items of the scale of attitude towards mathematics is given in Table 6.

Table 6
Dimension wise Factor Loading of Each Items of Scale of Attitude towards Mathematics

| Like towards Mathematics |  | Engagement with Mathematics |  | Self-belief in Mathematics |  | Active learning of Mathematics |  | Enjoyment of Mathematics |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Item <br> No. | Factor Loading | Item <br> No. | Factor Loading | Item <br> No. | Factor Loading | Item <br> No. | Factor Loading | Item <br> No. | Factor Loading |
| 43 | . 708 | 25 | . 693 | 28 | . 700 | 23 | . 758 | 40 | . 784 |
| 15 | . 705 | 13 | . 675 | 7 | . 671 | 18 | . 728 | 30 | . 737 |
| 26 | . 670 | 21 | . 669 | 17 | . 652 | 34 | . 721 | 19 | . 699 |
| 2 | . 644 | 11 | . 618 | 3 | . 649 | 29 | . 661 | 1 | . 637 |
| 36 | . 632 | 5 | . 604 | 9 | . 628 | 10 | . 655 | 12 | . 628 |
| 27 | . 611 | 32 | . 594 | 44 | . 519 | 39 | . 614 | 4 | . 616 |
| 31 | . 604 | 41 | . 557 | 8 | . 480 | 38 | . 558 | 24 | . 597 |
| 16 | . 574 | 37 | . 507 | - | - | 22 | . 422 | 33 | . 517 |
| 35 | . 521 | 42 | . 406 | - | - | - | - | 14 | . 389 |
| 20 | . 440 | - | - | - | - | - | - | - | - |
| 6 | . 366 | - | - | - | - | - | - | - | - |

After factor analysis six more items without enough factor loading on any of the five factors were also excluded from the draft scale.

Thus, final tool consisted of 44 items falling under 5 dimensions namely like towards mathematics, engagement with mathematics, self-belief in mathematics, active learning of mathematics and enjoyment of mathematics. Factor analysis confirmed these 5 dimensions. Utility related items were removed as there was no factor loading. Items in anxiety dimension after exploratory factor analysis loaded with other items which could be better described as reflecting like toward mathematics. Motivation measures the students' behavioral component regarding mathematics learning and hence were classified into active learning in mathematics and engagement with mathematics.

Illustrative items with corresponding item numbers in final tool are given in Table 7.

Table 7
Illustrative Items from Five Dimensions of Scale of Attitude towards Mathematics

| Dimensions | Item No. <br> (Final Tool) | Illustrative Items |
| :---: | :---: | :---: |
| Like towards mathematics | $\begin{gathered} 2,6,15,16,20 \\ 26,27,31,35,36 \\ 40 \text { (11 items) } \end{gathered}$ | I am scared of mathematics (Item no. 2) <br> I feel sick on the day of mathematics examination (Item no. 26) |
| Engagement with mathematics | $\begin{gathered} 5,11,13,21,25 \\ 32,37,41,42(9 \\ \text { Items) } \end{gathered}$ | I will try my best to avoid mathematics textbook (Item no. 5) <br> I do not like to attend mathematics class (Item no. 13) |
| Self-belief in mathematics | $\begin{gathered} 3,7,8,9,17,28 \\ 44 \text { (7 items) } \end{gathered}$ | I do not have enough accuracy and precision to learn mathematics (Item no. 7) <br> I am adept at learning mathematics (Item no. 9) |
| Active learning of mathematics | $\begin{gathered} \text { 10, 18, 22, 23, 29, } \\ 34,38,39(8 \\ \text { items) } \end{gathered}$ | I study difficult areas of mathematics repeatedly (Item no. 10) <br> I study mathematics every day (Item no. 34) |
| Enjoyment of mathematics | $\begin{gathered} 1,4,12,14,19 \\ 24,30,33,40(9 \\ \text { items) } \end{gathered}$ | Mathematics is interesting (Item no. 1) <br> I don't know where the time goes while studying mathematics (Item no. 4) |
| Total No. of items | 44 |  |

## Reliability and validity

Reliability of the scale of attitude towards mathematics is established through split-half method, test-retest method and Cronbach's Alpha. Split half reliability was calculated by correlating scores on one half of the test with scores on the other half of the test. scales were made to two halves by sorting the items on each of the five dimensions in ascending order of discriminating power. Testretest reliability was calculated with an interval of 2 weeks. Cronbach Alpha value was also calculated. Reliability coefficients of the scale of attitude towards mathematics and its dimensions are given in Table 8.

Table 8
Coefficients of Reliability of Scale of Attitude towards Mathematics and its Dimensions

| Dimensions of scale of attitude towards <br> mathematics | Test-retest <br> $(\mathrm{N}=75)$ | Reliability <br> Split-half <br> $(\mathrm{N}=509)$ | Cronbach alpha <br> $(\mathrm{N}=509)$ |
| :--- | :---: | :---: | :---: |
| Like towards mathematics | .68 | .82 | .79 |
| Engagement with mathematics | .72 | .78 | .77 |
| Self-belief in mathematics | .72 | .71 | .73 |
| Active learning of mathematics | .73 | .81 | .79 |
| Enjoyment of mathematics | .69 | .80 | .80 |
| Scale of Attitude towards Mathematics | .86 | .95 | .94 |

Items in the scale of attitude towards mathematics were prepared as per the literature available on attitude towards mathematics incorporating the factors that are the most plausible to determine the attitudes of learners towards the subject, and hence the scale is considered theoretically valid. Construct validity of the scale is ensured by preparing items reflective of tri-componential theory of attitude and is validated by the correlation of its scores with that of scale of selfefficacy in mathematics $(\mathrm{r}=.82, \mathrm{~N}=167)$, which were significantly below the test retest and Cronbach alpha reliability coefficients obtained.

Copies of the draft and final versions of scale of attitude towards mathematics along with their English translation are provided as Appendices F2, F3, F4 and F5 respectively. Copies of final versions of response sheet (Malayalam \& English versions) are provided as Appendices F6 \& F7 respectively.

## 5. Scale of Self-efficacy in Mathematics

This scale is developed to measure self-efficacy in mathematics of elementary school students. Perceived self-efficacy is defined as people's beliefs about their capabilities to produce designated levels of performance that exercise influence over events that affect their lives (Bandura, 2006). Beliefs about one's own competence are not identical to beliefs about the likely outcome that one's actions will produce (Usher \& Pajares, 2009).

## Planning

A student might have either high or low self-efficacy across the mathematics spectrum (Bandura, 2006). Hence the scale was planned in such a way that self-efficacy in learning and in solving problems are assessed separately. Efficacy beliefs in a construct differ in level, generality and strength. Bandura (2006) alerts that, while measuring self-efficacy, sub skills must be analyzed and included. So conceptual and comprehensive analysis of mathematical tasks demanded up to standard seven was done. Learning outcomes of each unit of mathematics from standard one to seven were listed. Then these were grouped into areas of school mathematics viz; natural numbers, fractions, decimals, geometry, percentage, average, graph and algebra. Learning outcomes were sorted and merged based on difficulty level. This part constitutes the first dimension of scale of self-efficacy namely self-efficacy in solving mathematics problems. Second part, scale- self-efficacy in learning mathematics consists of 3 components namely- mathematics learning in general, classroom teachinglearning of mathematics and assessment practices in mathematics.

## Item writing

Items were developed based on learning outcomes and cognitive behavior expected in each domain of mathematics learning. Item writing was based on level, strength and generality. Level of efficacy was assessed in terms of expected cognitive behavior for each mathematical tasks or problems. Strength of efficacy was assessed through the logically ordered response patterns namely definitely, usually, sometimes, occasionally and never. Generality of selfefficacy is the disparity in people's perceived efficacy across domains of construct. Efficacy may change according to the sub tasks or context. So to assess generality of efficacy different domains of mathematics learning were included such as mathematics learning in general, classroom teaching-learning of mathematics and assessment practices in mathematics, and in different areas of
elementary school mathematics like natural numbers, fractions, decimals, geometry, percentage, average, graph and algebra. There are 32 items in the draft scale. Self-efficacy in solving mathematics problems is assessed through 17 items, whereas self-efficacy in learning mathematics is assessed through 15 items. All statements are stated positively. Illustrative statements with corresponding item numbers in draft tool are given in Table 9.

Table 9
Illustrative Items from the two Dimensions of Scale of Self-efficacy in Mathematics

| Dimensions | Item no. (draft) | Category | Illustrative example |
| :---: | :---: | :---: | :---: |
|  | 1 to 4 | Natural Numbers | I can quickly solve applied problems using the four basic arithmetic operations (addition, subtraction, multiplication, division) (Item no. 1) |
|  | 5 to 7 | Fractions | I can find simplified form of fractions (Item no. 7) |
|  | 8 | Decimals | I can solve applied problems by performing the four basic arithmetic operations of decimals (addition, subtraction, multiplication, division) (Item no. 8) |
|  | 9 to 12 | Geometry | I can draw angles, rectangles, squares, and circles with given measurements (Item no. 12) |
|  | 13 | Percentage | I can solve applied problems using percentage (Item no. 13) |
|  | 14 | Average | I can solve applied problems involving average (Item no. 14) |
|  | 15 to 16 | Graph | I can understand and classify information given in a graph (Item no. 16) |
|  | 17 | Algebra | I can understand the relation between measurements and counts when they are indicated using letters (Item no. 17) |
|  | $\begin{gathered} 18 \text { to } 23, \\ 32 \end{gathered}$ | Mathematics learning in general | I can learn mathematics like any other subject (Item no 18) |
|  | 24 to 29 | Classroom teachinglearning | I can teach mathematics to my classmates (Item no. 29) |
|  | 30,31 | Assessment practices in Mathematics | I can prepare for mathematics examination without fear (Item no. 31) |

## Administration and scoring procedure

Students were asked to respond to what extent they are confident in solving mathematics problems or perform mathematics related tasks indicated in each statement. Students have to mark their responses on the test booklet itself. Proper instruction was given at the beginning of scale administration. Space was provided for writing name and gender of the student. Five response categories were given. They were 1. Definitely, 2. Usually, 3. Sometimes, 4. Occasionally and 5. Never. The response to statements in the scale are given points " $5,4,3,2$, and $1 "$. Sum of scores of each statement is considered as the total score on the scale of self-efficacy in mathematics. The lowest score in the scale is 32 and the highest score is 160 .

## Item analysis

Item analysis was done on a sample of 370 students of standard seven. Data were arranged in ascending order of total score in scale of self-efficacy in mathematics. Upper 27 percentage ( 100 Numbers) and lower 27 percentage (100 numbers) were selected for analysis. Response to each of the items were analyzed. Critical ratio was calculated. Items that have $t$ value $\geq 2.58$ were selected for the final tool. Result of item analysis of scale of self-efficacy in mathematics is given in Appendix G1. Copies of the draft and final form of scale of self-efficacy in mathematics (Malayalam and English versions) are provided as Appendices G2, G3, G4 and G5 respectively.

## Validity and reliability

Concurrent validity of the test is established by correlating the scores on the scale of self-efficacy in mathematics with scale of self-efficacy for learning mathematics (Abidha \& Gafoor, 2018) which has 20 items and self-efficacy for learning and performance subscale of MSLQ which has 8 items. Correlation with
the total scale and two dimensions of the scale were calculated. Scale of selfefficacy in learning mathematics (Abidha \& Gafoor, 2018) measures students' perceived ability to succeed in mathematics and in related situations. MSLQ is envisioned to assess self-efficacy for learning any subject.

Reliability was established by split-half method, test-retest method with an interval of two weeks, and Cronbach Alpha.

Split-half reliability of the whole scale is calculated by grouping the items on the basis of index of discriminating power. Correlation of scores in self-efficacy in solving mathematics problems with self-efficacy in learning mathematics is found to be .71 and that with total scale was .91 . Correlation of scores in Selfefficacy in mathematics learning with the total scale is found to be .94 .

Reliability coefficients for the two dimensions of the scale of self-efficacy in learning mathematics and for the total scale were calculated. Validity and reliability coefficients indicate that the scale is reliable and valid.

Reliability and validity coefficients of scale of self-efficacy in mathematics and its dimensions is given in Table 10.

Table 10
Coefficients of Reliability and Validity of Scale of Self-efficacy in Mathematics and its Dimensions

|  | Reliability |  | Validity (Concurrent) ( $\mathrm{N}=75$ ) |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Measure | Test- <br> Retest <br> $(\mathrm{N}=75)$ | Split-half <br> $(\mathrm{N}=509)$ | Cronbach's <br> alpha <br> $(\mathrm{N}=509)$ | Scale of self- <br> efficacy <br> $(\mathrm{N}=75)$ | MSLQ <br> $(\mathrm{N}=75)$ |
| Self-efficacy in solving <br> mathematics problems | .79 | .87 | .86 | .74 | .69 |
| Self-efficacy in learning <br> mathematics | .78 | .91 | .90 | .72 | .71 |
| Self-efficacy in <br> mathematics | .85 | .94 | .93 | .78 | .75 |

[^0]Validity and reliability coefficients indicate that the scale is reliable and valid. Confirmatory factor analysis done on a sample of 509 students confirmed the two dimensions of scale of self-efficacy in mathematics viz., self-efficacy in solving mathematics problems and self-efficacy in learning mathematics. Dimension wise factor loading of each item of scale of self-efficacy in mathematics is given in Table 11.

Table 11
Dimension wise Factor Loading of Each Items of Scale of Self-efficacy in Mathematics

| Self-efficacy in learning mathematics | Self-efficacy in solving mathematics <br> problems |  |  |
| :---: | :---: | :---: | :---: |
| item no (Final) | Factor loading | item no (Final) | Factor loading |
| 24 | .723 | 8 | .678 |
| 13 | .692 | 9 | .663 |
| 21 | .687 | 3 | .636 |
| 12 | .646 | 6 | .616 |
| 18 | .642 | 1 | .590 |
| 15 | .639 | 10 | .582 |
| 20 | .639 | 2 | .561 |
| 23 | .626 | 4 | .560 |
| 16 | .618 | 5 | .553 |
| 17 | .601 | 11 | .551 |
| 14 | .561 | 7 | .482 |
| 22 | .527 | - | - |
| 19 | .524 | - | - |

## 6. Test of Previous Achievement in Mathematics

This test was prepared to match the students in experimental and control groups on their previous achievement in mathematics. It measures students' cognitive achievement in mathematics learning from standards 1 to 6 in areas namely Algebra, Average, Fractions, Decimals, Geometry, LCM-HCF, Percentage and Graph.

## Planning

Achievement test in mathematics was planned to assess the level of student achievement in mathematical concepts and learning outcomes prescribed for standards 1 to 6 , covering eight areas of school mathematics. Cognitive behaviors measured in these eight areas were identified based on Bloom's revised taxonomy of cognitive objectives. Items of easy, average and difficult level were included in the test. Number of items in each content area and time duration of the test is fixed. A two-dimensional blueprint was prepared including distribution of item and content wise objectives. For convenience, all items were decided to be multiple choice. Details are given in Table 12.

Table 12
Blueprint for Test of Previous Achievement in Mathematics

|  |  |  | - <br> $\stackrel{0}{2}$ <br> $\frac{0}{2}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Algebra |  |  | 29 |  |  | 30,31,32 | 4 |
| Average |  |  | 8, 9 | 10 |  |  | 3 |
| Fractions, Decimals |  | 1,4,6 | 2,3,5 | 7 |  |  | 7 |
| Geometry |  | 14,16,17 | 11,12,13,15 |  |  |  | 7 |
| LCM, HCF | 18 | 19,20 | 21,22 | 23,24 |  |  | 7 |
| Percentage |  |  | 26,27 | 25 | 28 |  | 4 |
| Graph |  |  |  | 34 | 33,35 |  | 3 |
| Total | 1 | 8 | 14 | 6 | 3 | 3 | 35 |

Note. Figures in italics indicate item number.

## Item Writing

Items were constructed based on the blueprint prepared. Precise, unambiguous and relevant questions were included with the help of an expert in mathematics teaching. Test of previous achievement in mathematics has 35 items.

Each question had 4 alternatives. Alternatives are carefully made and logically ordered so as to decrease guess in responding. Instructions were included. Space was given in the response sheet to provide name and gender of the student. Illustrative items for each cognitive domain objective are given in Table 13.

Table 13
Illustrative Items from Test of Previous Achievement in Mathematics by the Cognitive Objectives

| Cognitive |  | Illustrative Items |
| :---: | :---: | :---: |
| Objective | Item No. (draft) | Item |
| Remembering | 18 | Which one of the given numbers is neither composite nor prime? <br> a) 1 <br> b) 3 <br> c) 5 <br> d) 7 |
| Understanding | 6 | What is the decimal form of $\frac{3}{40}$ ? <br> a) 0.007 <br> b) 0.075 <br> c) 0.750 <br> d) 7.500 |
| Applying | 11 | What is the length of one side of the square of perimeter 200 meters? <br> a) 20-meter <br> b) 50 meter <br> c) 100 meter <br> d) None of these |
| Analyzing | 7 | A 5-meter-long ribbon is cut into 3 equally long pieces. What is the length of the quarter of a piece? <br> a) $\frac{3}{20}$ <br> b) $\frac{5}{12}$ <br> c) $\frac{12}{5}$ <br> d) $\frac{20}{3}$ |
| Evaluating | 28 | The price for a television last year was 18000 Rupees. The price has increased up to 20160 Rupees. What is the increase in percentage? <br> a) $12 \%$ <br> b) $20 \%$ <br> c) $21 \%$ <br> d) $24 \%$ |
| Creating | 32 | Let $A$ be the age of $A n u$ and $B$ be the age of Sonu. How do you indicate using letters that Anu's age is 4 years less than that of Sonu? <br> a) $A=B-4$ <br> b) $B=A-4$ <br> c) $A=B+4$ <br> d) $B=4-A$ |

## Administration and Scoring procedure

Test of previous achievement in mathematics is an objective- multiple choice test with 35 items. Each item has four responses out of which one is the right answer and rests are distracters. Each right answer is given one score and
wrong response is given a zero, with a possible total score ranging from zero to 35. Students have to mark their answers using a tick mark in a separate response sheet provided with the test booklet. Students were given 40 minutes to complete the test.

## Item analysis

Item analysis was done by administering the test on a sample of 370 students by examining student responses to each question in order to assess the quality of items and of the test. Discrimination power and item difficulty was calculated based on responses of upper- and lower-27 percent students (100 in each group). Difficulty index (DI) and discriminating power (DP) of each item were calculated using the following equations.

$$
\mathrm{DI}=\frac{U H+L H}{2 N}
$$

and

$$
\mathrm{DP}=\frac{U H-L H}{N}
$$

Where,

DI - Difficulty index

DP - Discrimination power

UH- Number of right answers among the $27 \%$ of students with the highest test scores

LH- Number of right answers among the $27 \%$ of students with the lowest test scores

N - Number of students in the lower/upper group $(\mathrm{N}=100)$

Data and results of item analysis of test of previous achievement in mathematics is given in Appendix H1. Items with discriminating power greater than 0.3 and difficulty index between .35 and .6 were selected for the final test, which made a 20 -item test. Copies of draft and final Malayalam and English versions along with scoring key are provided as Appendix H2, H3, H4, H5, H6 and H 7 respectively. Final form of response sheet is given as Appendix H8.

## Validity and reliability

Content validity of test of previous achievement in mathematics is ensured by covering major learning objectives of mathematics topics from standard 1 to 6 in consultation with an expert in mathematics teaching. Ambiguous items were modified according to suggestions of expert.

Concurrent validity was established by correlating the scores on this test with the marks obtained in mathematics for the previous term examination. The coefficient of correlation obtained $0.65(\mathrm{~N}=75)$, established that the test has concurrent validity.

Reliability is estimated by split-half method. The items were grouped based on their discrimination power. Index of reliability obtained by SpearmanBrown prophecy formula is $\mathrm{r}=.85(\mathrm{~N}=370)$.

## 7. Tests of Achievement in Mathematics Units

Tests of achievement in five units of mathematics in standard seven were prepared to measure the level of student achievement of learning objectives in the five units namely 1) parallel lines, 2) unchanging relations, 3) repeated multiplication, 4) area of triangle and 5) square and square root, in order to evaluate the influence of language integrated mathematics instruction
on achievement in mathematics, algebra, arithmetic and geometry. Test for the five units were developed based on the learning objectives and expected learning outcomes related to the unit and were independently administered as posttest in experimental phase. Achievement in arithmetic is then obtained as the weighted total of the achievement in repeated multiplication and achievement in the area of triangle. Achievement in algebra is the total score in achievement in unchanging relations. Achievement in geometry is the weighted total of the achievement in parallel lines and achievement in square and square root.

## Planning

In the planning stage, textbook and teacher's handbook of mathematics in Grade seven (SCERT, 2016) were thoroughly analyzed which helped to identify and list the concepts and learning outcomes of each unit, which broadly fits into Bloom's revised taxonomy of objectives viz., remembering, understanding, applying, analyzing, evaluating and creating.

Learning outcomes of the unit 'parallel lines' are explaining parallel lines as lines which are a constant distance apart, explaining parallel lines in terms of perpendicularity and slant, drawing parallel lines using different methods and proving that they are parallel, explaining parallel lines using models, computing the other angles and justifying the computations when given one angle made by a line cutting across a pair of parallel lines, explaining the classification of pairs of corresponding, alternate, co-interior and co-exterior angles and proving that the sum of the angles of a triangle is $180^{\circ}$ (SCERT, 2016).

Learning outcomes of the unit 'unchanging relations' are finding general principles in arithmetical operations, writing general principles in
ordinary languages, expressing relations between numbers and operations using letters and using general principles to make computations easier (SCERT, 2016).

Learning outcomes of the unit 'powers' are describing exponentiation as the operation of repeated multiplication, justifying the rules of exponentiation, using the rules of exponentiation to solve problems, describing the positional system of notation using exponentiation and logically justifying number relations associated with powers (SCERT, 2016).

Learning outcomes of the unit 'area of triangle' are explaining the methods to compute the area of right-angled triangle, explaining how the area of any triangle can be computed by splitting into right angled triangles and solving problems on computation of triangular areas (SCERT, 2016).

Learning outcomes of the unit 'square and square roots' are describing squares and perfect squares with examples, explaining the peculiarities of squares logically, describing methods to compute the square root of a perfect square, explaining the peculiarities of square roots with examples and solving practical problems using square and square root (SCERT, 2016).

Items were planned based on revised Bloom's taxonomy. Appropriate weighting was given to content of each unit so that number of items varied for each unit. All items were planned as multiple choice as they are easy to score and devoid of subjectivity. Items of easy, average and difficult level were included. Number of items and time duration of the tests in each unit area were fixed. A blueprint was prepared including objective and content wise distribution of items. Details are given in Table 14.

Table 14
Blueprint for Tests of Achievement in Mathematics Units

| Objectives <br> Content |  |  | $\frac{\stackrel{00}{\lambda}}{\frac{0}{2}}$ | $\begin{aligned} & \stackrel{\infty}{N} \\ & \stackrel{N}{N} \\ & \frac{N}{N} \\ & \frac{1}{4} \end{aligned}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Parallel Lines | 2 | 1,3,4,5,12,13 | 6,7,8,14,15,16 |  | 9,10,11 |  | 16 |
| Unchanging Relations |  | 2,3,4,5,12 |  | 6,7,8,9 | $\begin{array}{r} 10,11,13 \\ 14,15,16 \end{array}$ | 1 | 16 |
| Repeated Multiplication |  | 1,2,3,4,5,17 | $\begin{gathered} \text { 6,9,10,11,12 } \\ 13,16,18 \end{gathered}$ | 7,8,14,15 |  |  | 18 |
| Area of Triangle | 2 |  | 1,3,5,6,11,12 | 7,8,9,10 |  |  | 12 |
| Square and Square root |  | $\begin{gathered} \text { 1,2,5,9,11 } \\ 13,16 \end{gathered}$ | $\begin{gathered} 6,7,12,14,15 \\ 17,19,20 \end{gathered}$ | 3,4,8,10 | 18 |  | 20 |

Note. Figures in italics indicate item number.

## Item writing

Item were constructed based on the blueprint. Precise, unambiguous and relevant items were prepared with the help of an expert in mathematics teaching. Each question had 4 alternatives in which one right answer was given, and the rest were distracters. Alternatives are carefully made and logically ordered to decrease guessing. Space was given in the response sheet to provide name and gender of the student. Illustrative items for each cognitive domain objective are given in Table 15.

Table 15
Illustrative Items for each Cognitive Domain Objective from the Test of Achievement in Mathematics Unit tests

| Unit | Illustrative Items |  |  |
| :---: | :---: | :---: | :---: |
|  | Cognitive Objective |  | Items |
| Parallel Lines | Remembering | 2 | The sum of all angles of a parallelogram will be --- <br> a) $90^{\circ}$ <br> b) $180^{\circ}$ <br> c) $240^{\circ}$ <br> d) $360^{\circ}$ |
|  | Understanding | 3 | -C |
|  |  |  | $A-$ $\qquad$ B <br> How many parallel lines can be drawn to the line $A B$ that passes through the point C? |
|  |  |  | a) 0 <br> b) 1 <br> c) 2 <br> d) Many |
|  | Applying | 7 |  <br> In the figure given, <EBD+<MCA=--- <br> a) $90^{\circ}$ <br> b) $180^{\circ}$ <br> c) $240^{\circ}$ <br> d) $360^{\circ}$ |
|  | Evaluating | 9 |  <br> Which of the pairs given below are not equal angles <br> a) $\angle \mathrm{CBM}, \angle \mathrm{ABE}$ b) $\angle \mathrm{DEN}, \angle \mathrm{FEB}$ <br> c) $\angle \mathrm{DEN}, \angle \mathrm{FEN}$ d) $\angle \mathrm{MBA}, \angle \mathrm{CBE}$ |
| Unchanging Relations | Understanding | 12 | Anu had 63 candies. She gave one each to all 35 students in her class, and 5 to her teacher. What numerical operation to be used to figure out the number of remaining candies? |
|  |  |  | $\begin{array}{lllll}\text { a) } 63-35 & \text { b) } 63-(35+5) & \text { c) } 63-(35-5) & \text { d) } 63+(35-5)\end{array}$ |
|  | Analyzing | 6 | $(47-93 / 4)+1 / 4=$ |
|  |  |  | a) $47+(93 / 4-1 / 4) ~$ b) $47+(93 / 4+1 / 4)$ |
|  |  |  | c) $47-(93 / 4+1 / 4) ~ d) ~ 47-(93 / 4-1 / 4)$ |
|  | Evaluating | 10 | If the sum of two numbers is 30 , and their difference is 4 , which is bigger number? |
|  |  |  | $\begin{array}{llll}\text { a) } 13 & \text { b) } 16 & \text { c) } 17 & \text { d) } 18\end{array}$ |
|  | Creating | 1 | If the length, width, and perimeter of a rectangle are $a$ and $b$, respectively, how can their relationship be expressed? |
|  |  |  | a) c=2ab $\quad$ b) $\mathrm{c}=\mathrm{a}+\mathrm{b}$ |
|  |  |  | c) $\mathrm{c}=2+\mathrm{a}+\mathrm{b} \quad$ d) $\mathrm{c}=2(\mathrm{a}+\mathrm{b})$ |


| Unit | Illustrative Items |  |  |
| :---: | :---: | :---: | :---: |
|  | Cognitive Objective | Item No. (Draft) | Items |
| Repeated Multiplication | Understanding | 4 | Write one lakh as the power of ten <br> a) $10^{2}$ <br> b) $10^{5}$ <br> c) $10^{10}$ <br> d) $10^{15}$ |
|  | Applying | 16 | $3^{x}=242,3^{x+1}=\ldots$ <br> a) 243 <br> b) 245 <br> c) 484 <br> d) 726 . |
|  | Analyzing | 14 | Which of the following is an equivalent pair? <br> a) $2^{2}, 4^{2}$ <br> b) $2^{2}, 4^{4}$ <br> c) $2^{3}, 4^{3}$ <br> d) $2^{4}, 4^{2}$ |
| Area of Triangle | Remembering | 2 | Choose the characteristics of a trapezoid <br> a) Lengths of all sides would be equal <br> b) One pair of opposite sides would be parallel. <br> c) All angles would be 90 degrees <br> d) The total of all angles would be 180 degrees |
|  | Applying | 1 | Figure out the surface area of a rectangle that is length 12 cm long and 8 cm wide <br> a) $20 \mathrm{~cm}^{2}$ <br> b) $40 \mathrm{~cm}^{2}$ <br> c) $96 \mathrm{~cm}^{2}$ <br> d) $192 \mathrm{~cm}^{2}$ |
|  | Analyzing | 9 | What would be the operation to find the surface area of $\Delta X Y Z$ ? <br> a) $1 / 2 \times 12 \times 5$ <br> b) $1 / 2 \times 15 \times 5$ <br> c) $15 \times 5$ <br> d) $12 \times 5$ |
| Square and Square root | Understanding | 2 | Why 36 is a perfect square? <br> a) because 36 is the square of square of 3 <br> b) because 36 is a multiple of 6 and 3 <br> c) because 36 is completely divisible by 6 <br> d) because 36 is the square of the natural number 6 |
|  | Applying | 20 | What is the length of one side of a square of surface area $1225 \mathrm{~m}^{2}$ ? <br> a) 12 meter <br> b) 24 meter <br> c) 35 meter <br> d) 48 meter |
|  | Analyzing | 4 | Which is not equivalent to $111^{2}$ ? <br> a) $110^{2}+221$ <br> b) $12100+221$ <br> c) $110^{2}+(110+111)$ <br> d) $110^{2}+(111+111)$ |
|  | Evaluating | 18 | The operations to find the square root of 5184 is given in order. Find out the operation that is incorrect among them. <br> a) $5184=2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 9 \times 9$ <br> b) $5184=2^{6} \times 9^{2}$ <br> c) $\sqrt{5184}=2 \times 9$ <br> d) $\sqrt{5184}=18$ |

## Administration and scoring procedure

Tests achievement in mathematics units are objective- multiple choice test with varying number of items administered independently after instruction in each unit. Students have to mark their responses using a tick mark in a separate response sheet provided with the test booklet. Appropriate instructions were given in the test. Each right answer is given one score and wrong answer is given a zero.

## Item Analysis

Item analysis was done by administering the five-unit tests on a sample of 200 students each. The discriminating power (DP) and difficulty index (DI) of the items in each of the five-unit tests were calculated by using conventional method. Discrimination power and item difficulty of each item were calculated based on responses of students in the upper and lower group (50 in each group). Difficulty index (DI) and discriminating power (DP) of each item were calculated the same conventional procedure followed in the case of Test of Previous Achievement in Mathematics. Data and results of item analysis of tests of achievement in mathematics units are given in Appendix I1. Items with discriminating power greater than 0.3 and difficulty index between .35 and .6 were selected for the final tests. Draft and final Malayalam and English versions and response sheets of the tests in parallel lines and repeated multiplication and Malayalam and English versions and response sheets of unchanging relations, area of triangle and square and square roots are appended as follows.

| Test of achievement in unit | No. of items (Final) |  | Time in minutes |
| :--- | :---: | :---: | :---: |
| Parallel Lines (Appendices I 2 to I 6) | 14 | 24 |  |
| Unchanging relations (Appendices I 7 to I 9) | 16 | 26 |  |
| Repeated multiplication (Appendices I 10 to I 4) | 16 | 26 |  |
| Area of triangle (Appendices I 15 to I 7) | 12 | 22 |  |
| Square and square root (Appendices I 18 to I 20) | 20 | 30 |  |

Scoring keys of tests of achievement in mathematics units is given in Appendix I 21.

## 212 EVIDENCE BASED INSTRUCTION OF LANGUAGE OF MATHEMATICS

## Validity and reliability

Content validity of the tests were ensured by covering major concepts and learning objectives in each unit which were further verified by an expert in mathematics teaching and assessment. Accordingly, irrelevant items were removed, and ambiguous items were modified as per the suggestion of expert.

Concurrent validity was established by correlating the scores on tests of achievement in each unit with the marks obtained in mathematics for the previous term examination and is given in Table 16.

Reliability is estimated by split-half method by correlating scores in half of the test with the score on the other half. The items were grouped after ranking the items based on their discrimination power, with every alternate item being assigned to the two halves. Split half reliability, Cronbach alpha and concurrent validity indices were given in Table 16.

Table 16
Coefficients of Reliability and Validity of the Tests of Achievement in Mathematics Units

| Unit | Split half Reliability <br> $(\mathrm{N}=200)$ | Cronbach Alpha <br> $(\mathrm{N}=200)$ | Concurrent Validity ${ }^{\#}$ <br> $(\mathrm{~N}=40)$ |
| :--- | :---: | :---: | :---: |
| Parallel Lines | 0.67 | 0.66 | 0.61 |
| Unchanging Relations | 0.69 | 0.68 | 0.59 |
| Repeated Multiplication | 0.65 | 0.65 | 0.62 |
| Area of Triangle | 0.69 | 0.67 | 0.60 |
| Square and Square root | 0.86 | 0.85 | 0.76 |

\# Correlation with marks obtained in mathematics for the previous term examination

Coefficients of concurrent validity against marks obtained in mathematics for the previous term examination and coefficients of split-half reliability and Cronbach alpha indicate that the tests of achievement in mathematics units are valid and reliable.

## 8. Scales of Self-Efficacy in Units of Mathematics

Scales of self-efficacy in the five units of mathematics in standard seven were prepared to measure the level of self-efficacy of students in learning and performing tasks related to the five units. This was used to evaluate the influence of language integrated mathematics instruction on self-efficacy in algebra, arithmetic and geometry. They were used as posttest in experimental phase. These scales were developed based on the learning objectives and expected learning outcomes related to the units. Scales of self-efficacy were made for five units namely, 1) parallel lines, 2) unchanging relations, 3) repeated multiplication, 4) area of triangle and 5) square and square root. Self-efficacy in each unit measures students' appraisal regarding their capability to carry out mathematical tasks related to that unit.

## Planning

A thorough analysis of mathematical tasks and learning outcomes in each chapter lead to the writing of items. Learning outcomes of each unit is explained in the planning section of tests of achievements in mathematics. It was planned to make items for scales of self-efficacy for five units, parallel to the questions in the achievement tests of corresponding unit. Appropriate weightage was given to content of each unit so that number of items varied for each unit. A five-point Likert type scale was planned as in the case of scale of self-efficacy in mathematics.

## Item writing

Items were prepared based on the learning objectives, content and mathematical tasks to be accomplished of each unit. Precise, relevant and unambiguous statements were constructed. Illustrative statements with corresponding item numbers in the draft scales are given in Table 17.

Table 17
Illustrative Items from Scales of Self-efficacy in the Five Select Mathematics Units of Standard VII Mathematics

|  |  | Illustrative Items |
| :--- | :---: | :--- |
| Unit | Item <br> No. <br> (Draft) |  |
| Parallel <br> Lines | 1 | I can find other angles of a parallelogram if the value of one of the <br> angles is given |

Number of items and range of score of the five scales of self-efficacy in mathematics units with their corresponding appendix numbers is given in Table 18.

## Administration and Scoring Procedure

Students respond to statements about the extent to which they are confident in solving mathematics problems or perform in mathematics related tasks in the five mathematics units by marking their responses against the
statements on any one of the five response categories - definitely, usually, sometimes, occasionally and never which carries a score of " $5,4,3,2$, and 1 " respectively. Only positive statements were given. Sum of scores of all the statements in a unit provided the measure of self-efficacy in that unit. Space was provided in the tool for writing name and gender of the student.

Self-efficacy in arithmetic is obtained as the weighted total score of selfefficacy in repeated multiplication and square and square root. Self-efficacy in algebra is the total score of self-efficacy in unchanging relations. Self-efficacy in geometry is obtained as the weighted total score of self-efficacy in parallel lines and area of triangle.

## Item analysis

The five scales of self-efficacy in mathematics units were tried out on a sample of 200 students of standard seven for item analysis. Result of item analysis of scales of self-efficacy in units of mathematics is given in Appendix J1. Items that have $t$ value $\geq 2.58$ were selected for the final scale.

Details of draft and final versions of each of the five-unit tests were given in Table 18.

Table 18
Number of Items and Range of Scores in the Draft and Final Forms of Scales of Self-efficacy in the Five Select Mathematics Units of Standard VII with Corresponding Appendix Numbers of their Malayalam and English Versions

| Mathematics Units | Draft |  |  |  | Final |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | No. of items | Range of score | Appendix |  | No. of items | Range of score | Appendix |  |
|  |  |  | Mal. | Eng. |  |  | Mal. | Eng. |
| Parallel Lines | 18 | 18-90 | J2 | J3 | 14 | 14-70 | J3 | J5 |
| Unchanging relations | 19 | 19-95 | J6 | J7 | 14 | 14-70 | J8 | J9 |
| Repeated multiplication | 16 | 16-80 | J10 | J11 | 14 | 14-70 | J 12 | J13 |
| Area of triangle | 10 | 10-50 | J14 | J15 | 10 | 10-50 | -- | -- |
| Square and square root | 14 | 14-70 | J16 | J17 | 14 | 14-70 | -- | -- |

## Reliability and validity

Concurrent validity of the scale is established by correlating the scores on the scale of self-efficacy in each of the five units of mathematics with that of scale of self-efficacy in mathematics which had 24 items.

Reliability was established by split-half method, test-retest method with an interval of two weeks in between the administration, and by calculating Cronbach alpha.

Split-half reliability of the whole scale is calculated by grouping the items into two groups based on index of discrimination power. Reliability and validity coefficients of five scales of self-efficacy in units of mathematics are given in Table 19.

Table 19
Coefficients of Reliability and Validity of the Scales of Self-efficacy in Mathematics units

| Unit | Split half <br> Reliability <br> $(N=200)$ | Cronbach <br> Alpha <br> $(N=200)$ | Test-Retest <br> $(N=40)$ | Concurrent <br> Validity ${ }^{\#}$ <br> $(N=40)$ |
| :--- | :---: | :---: | :---: | :---: |
| Parallel Lines | 0.88 | 0.89 | 0.79 | 0.68 |
| Unchanging Relations | 0.78 | 0.74 | 0.70 | 0.69 |
| Repeated Multiplication | 0.80 | 0.78 | 0.73 | 0.62 |
| Area of Triangle | 0.81 | 0.80 | 0.78 | 0.64 |
| Square and Square root | 0.89 | 0.86 | 0.81 | 0.75 |

\# Correlation with scale of self-efficacy in mathematics

Coefficients of concurrent validity obtained against the scale of selfefficacy in mathematics and the coefficients of split-half reliability, Cronbach alpha and test-retest reliability indicates that the scales of self-efficacy in mathematics units are valid and reliable.

## 9. Test of Verbal Comprehension in Malayalam

This test was prepared to match the students in the experimental and control groups on their level of verbal comprehension in Malayalam. It measures students' ability to read and understand different types of written texts in terms of vocabulary knowledge, syntactic knowledge, text-structure awareness, mainideas comprehension, inferences about text information, summarization abilities and evaluation and critical reading.

Planning.

Extensive review on verbal comprehension test construction was done to find out the major component abilities in verbal comprehension and assessment format for these abilities. Items to assess students' vocabulary knowledge, syntactic knowledge, and pragmatics knowledge were planned. Seven types of items were planned namely analogies, general vocabulary, syntactic errors, sentence comprehension, altering syntax, sentence sequencing and passage comprehension. Content for these items were selected from old SCERT standard six textbooks of social science, physical science and literature with special attention to include non-technical vocabulary related to mathematics. Number of items under each comprehension skill and time duration of the test was fixed. Total marks for the test and time to finish the test were also decided. Distribution of items on each comprehension skill and type of item to be used were also planned. For convenience in scoring, all items were decided to be multiple choice. However, item tasks varied according to the skill involved. Details are given in Table 20.

## 218 EVIDENCE BASED INSTRUCTION OF LANGUAGE OF MATHEMATICS

Table 20
Distribution of Items in the Test of Verbal Comprehension in Malayalam by the Components and the Type of Items Used

| Components of verbal Comprehension | Type of Items | Descriptions | Item number in draft tool |
| :---: | :---: | :---: | :---: |
| Vocabulary Knowledge | Analogies | Two pairs of words will be given. Students have to find out how these words are related and then think of two other words that share a similar relationship. For example, Teacher works in school; Advocate works in court. Teacher: school; Advocate: Court. | 1, 2, 3 |
|  | General vocabulary | One sentence or a paragraph is given with one word underlined. Students have to find out similar word from the alternatives which can best replace the underlined word. This type of items is a measure of vocabulary knowledge and not the ability to derive meaning from context. | $4,5,6,7,8,30$ |
| Syntactic <br> Knowledge | Syntactic errors | The student identifies one syntactically correct sentences from a set of four that have the same meaning. | 9, 10, 11, 12, 13 |
|  | Sentence comprehension | One or two sentences with two or more concepts embedded is given. Students have to comprehend the meaning and find out the correct sentence which rightly convey the meaning from the four alternatives. | $\begin{gathered} 14,15,16,17 \\ 18,19,20,21 \end{gathered}$ |
|  | Altering syntax | Altering syntax - One or two statements are given. Students have to complete the sentence when the start of the sentence is altered. | 22, 23 |
| Pragmatic Knowledge | Sentence sequencing | Sentence sequencing- Four pragmatic sentences are presented without order, and the student have to rearrange them to make sense. | 26 |
|  | Passage comprehension | Passage comprehension - After reading given paragraphs / poem students have to answer content specific questions. | $\begin{gathered} 24,25,27,28 \\ 29,31,32,33,34 \end{gathered}$ |

## Item writing

Test of verbal comprehension in Malayalam has 34 items. Each item has four responses out of which one is the right answer and the rest are distracters.

Alternatives are carefully made and logically ordered to avoid guessing.

## Administration and scoring procedure

Students mark their responses on response sheet by tick marking as per instructions for the items. Each right answer is given one score and wrong response is given a zero and hence total score ranged from zero to 33 . Students were given 45 minutes to complete the test.

## Item analysis

Item analysis was done by administering the test on a sample of 370 students by examining student responses to each item in order to assess the quality of items and of the test. The discriminating power (DP) and difficulty index (DI) of each item were calculated by using conventional method as mentioned previously in the test of previous achievement in mathematics. DP and DI were calculated based on the responses of students in the upper and lower group (100 in each group). Items with discriminating power greater than 0.3 and difficulty index between .35 and .6 were selected for the final test which made a 24 items test. Data and results of item analysis of test of verbal comprehension in Malayalam is given in Appendix K1. Copies of draft and final versions of the test of verbal comprehension in Malayalam, along with scoring key and final format of response sheet are provided as Appendices K2, K3, K4, K5 and K6 respectively.

## Validity and reliability

Concurrent validity was established by correlating the scores of tests of verbal comprehension in Malayalam with that of test of Malayalam reading comprehension for grade seven students (Gafoor \& Aneesh, 2018) ( $\mathrm{r}=.76 ; \mathrm{N}=$ 45) and also against score on terminal Examination in Malayalam( $\mathrm{r}=.74 ; \mathrm{N}=45$ ) indicating concurrent validity.

Reliability of test of verbal comprehension in Malayalam is indicated in internal consistency (Cronbach alpha=.80; $\mathrm{N}=503$ ) and Spearman - Brown prophecy coefficient obtained by split-half method $(\mathrm{r}=.80 ; \mathrm{N}=503)$.

## 10. Language Integrated Mathematics Instruction

Five chapters in standard seven mathematics were taught to the experimental and control groups. The content instruction in these five units of mathematics were supplemented in experimental group with language of mathematics instruction and control group was given guided practice in solving mathematics problems. In both the groups, content instruction in mathematics was given in all the five units by the schoolteacher. Along with this, the experimenter integrated language of mathematics instruction in the experimental group and guided practice in solving mathematics problems in the control group, through one or two sessions per week. For every unit, both the experimental and the control groups received 4 lessons in as many class periods.

Language integrated mathematics instruction makes use of different types of techniques to improve the linguistic skills (Listening, Speaking, Reading and writing) of students in mathematics along with mathematics content instruction. Categories of strategies used in language integrated mathematics instruction and student and teacher activity while applying these strategies are given in Table 21.

Table 21
Techniques used in Language integrated mathematics instruction

| SI. <br> No. | Strategy | Aim | Student Activity | Teacher Activity |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Anchoring math with language | To assure student readiness to learn the language of mathematics | Participates in discussions about the linguistic features of mathematics | Teacher explanations about language related factors in mathematics- terms, symbols and conventions used in Mathematics and benefits of focusing on linguistic factors in mathematics. |
| 2 | Vocabulary <br> Bank | To develop a vocabulary bank in every unit of mathematics | Teacher asks students to note down the key words and symbols in their textbook and list them. | Teacher helps students to develop the word bank with already learned terms and symbols. |
| 3 | Labeling Vocabulary | To categorize listed vocabulary in every unit of mathematics | Identified and listed terms, are labeled under different categories. | Monitoring |
| 4 | Word Wall | To construct word walls in every unit of mathematics and learn mathematical vocabulary through visual clues | Students work in groups to prepare word walls using mathematics words and visual clues | Teacher selects vocabulary that can be made into word wall and monitors students work |
| 5 | Word Trails | To learn the etymology of terms | Students were asked to write one term called out by the teacher at the top of hierarchy diagram. Break down the word into known word parts and write those parts on the next branch of the diagram. Then students were asked to write down the meaning of known words. Groups shared their meaning with the class. These were discussed in class and etymology of the word is written down. | Teacher calls out a word and helps to break down the word and find out the meaning of word parts. |
| 6 | Listen and Write | To improve listening to communication in mathematics | Students were asked to listen and write down the statements given orally by the teacher | Teacher selects sentences to readout in class |

## 222 EVIDENCE BASED INSTRUCTION OF LANGUAGE OF MATHEMATICS

| Sl. <br> No. | Strategy | Aim |  | Student Activity |
| :--- | :--- | :--- | :--- | :--- |

Allocation of lessons and strategies in language integrated mathematics
instruction in each unit is given in Table 22.

Table 22
Allocation of Lessons and Strategies in Language Integrated Mathematics Instruction in Each of the Five Select units of Standard VII Mathematics

| $\begin{aligned} & \dot{i} \\ & \dot{n} \end{aligned}$ | Units | $\begin{aligned} & \tilde{n} \\ & 0 \\ & \tilde{u} \\ & \frac{0}{4} \\ & \dot{0} \\ & \dot{z} \end{aligned}$ | Number of times the strategies were used per unit |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\begin{aligned} & \sqrt[n]{7} \\ & 3 \\ & \frac{0}{0} \\ & 3 \\ & 3 \end{aligned}$ | n 끈 0 0 3 |  | $\begin{aligned} & \tilde{N} \\ & \frac{0}{0} \\ & \frac{\pi}{3} \\ & \tilde{\sim} \\ & \frac{1}{3} \end{aligned}$ |  |
| 1 | Parallel lines | 4 | 1 I | I | 1 | 11 | 1 | 1 |
| 2 | Unchanging relations | 4 | 1 I |  |  | 1 |  | 1 IIII |
| 3 | Repeated Multiplication | 4 | 11 |  |  | 11 | 1 | 1 |
| 4 | Area of Triangle | 4 | 1 I | I | 1 | 1 l | 1 | 1 |
| 5 | Square and Square root | 4 | 1 | 1 |  | 1 l | 1 | 1 |
|  | Total | 20 | 5 5 | 3 | 2 | 54 | 4 | $5 \quad 4$ |

To facilitate language integrated mathematics instruction, a workbook was constructed and supplied to every student. A model of the workbook is given in Appendix L.

## 11. Practice in Solving Mathematics Problems

Practice in solving mathematics problems was provided to the control group by the experimenter along with content instruction by the schoolteacher. Five chapters of mathematics in standard seven namely parallel lines, unchanging relations, repeated multiplication, area of triangle and square and square root were taught by the schoolteacher. In addition to this, guided practice in mathematics problem solving was provided for each of these units by the experimenter. The guided practice was in each unit for the same duration, number of lessons and periods, as was used for language integrated mathematics instruction in the same unit. This guided practice was focusing on the unit end exercises, and all the unit end exercises in the select units were solved individually with guidance from experimenter and wherever possible from peers. In units where the number of exercises at the end of the units were few and were not enough to match the number of lessons, additional exercises were constructed by the experimenter. A workbook was constructed and supplied to every student. A model of the workbook is given as Appendix M.

## Sample used for the Study

The study used three sets of samples, two sets of samples for survey and the third set for experimental phase, in addition to the try out samples used for item analysis of different measuring tools.

## Sample of the Survey Study

Two separate samples were used for studying perception of difficulties in mathematical tasks cum reasons for difficulty thereof and then to specifically test students' difficulties in the language of mathematics.

## Part 1-Survey

Part 1 survey was to identify perception of difficulties in mathematical tasks and reasons for difficulty thereof. Participants were 300, eighth standard students randomly selected from government and aided schools of Kozhikode district. Both boys and girls from intact classrooms in these co- educational schools were sampled. Though locality of school is not a consideration in this study, schools from both urban and rural areas were sampled. List of seven schools from where data was collected for the part 1 survey phase is given in Table 23.

Table 23
Details of Part 1 Survey Sample used to Identify Student Perception of Difficulties in Mathematical Tasks and their Reasons for Difficulty

| SI. No. | Name of Schools | Number of Students |
| :---: | :--- | :---: |
| 1 | Ganapath Girls HSS, Chalappuram | 39 |
| 2 | GHSS, Nallalam | 55 |
| 3 | VHSS, Kinassery | 44 |
| 4 | GHSS, Perumanna | 64 |
| 5 | HS, Pantherakavu | 29 |
| 6 | HSS, Azhchavattom | 34 |
| 7 | Achuthan Girls HSS, Chalappuram | 35 |

## Part 2 - Survey

Part 2 survey was to identify students' language related difficulties in mathematics learning. Random sample of 1050, eighth standard students from 14 schools of Kozhikode and Malappuram districts were selected for part 2 survey. Both boys and girls from intact classrooms in these co- educational schools were
sampled. Though locality of school is not a consideration in this study, schools from both urban and rural areas were sampled. List of 14 schools from where data was collected for the survey phase is given in Table 24.

Table 24
List of Schools from which Data was Collected for Part 2 Survey

| SI. No. | Name of Schools | District |
| :---: | :--- | :--- |
| 1 | GHSS, Poonoor | Kozhikode |
| 2 | GVHSS, Balussery | Kozhikode |
| 3 | GHSS, Nallalam | Kozhikode |
| 4 | CMRHSS, Chennamangallur | Kozhikode |
| 5 | Ganapath Boys HSS, Chalappuram | Kozhikode |
| 6 | Ganapath Girls HSS, Chalappuram | Kozhikode |
| 7 | GHSS, Perumanna | Kozhikode |
| 8 | VHSS, Kinassery | Kozhikode |
| 9 | Achuthan Girls HSS, Chalappuram | Kozhikode |
| 10 | HS, Pantherakavu | Kozhikode |
| 11 | Calicut Girls' VHSS, Kundungal | Malappuram |
| 12 | PTM HSS, Thazhekode | Malappuram |
| 13 | THSS, Thachinganadam | Malappuram |
| 14 | GVHSS, Vengara |  |

## Sample of the Experimental Phase

Four intact standard seven classrooms from Government aided upper primary school, Puthurmadam, Kozhikode were selected for experimental study. The exact procedure followed was as follows. Four intact classes of standard seven were selected and then the two classes each were randomly assigned to experimental and control groups. These two groups (experimental and control)

## 226 <br> EVIDENCE BASED INSTRUCTION OF LANGUAGE OF MATHEMATICS

were matched on verbal comprehension in Malayalam, non-verbal intelligence and previous achievement in mathematics. For this the students were classified into high and low groups based on their verbal comprehension in Malayalam, non-verbal intelligence and previous achievement in mathematics using median score as the cut point. Then 45 students each in experimental group and control group who were identical on their level of verbal comprehension in Malayalam, non-verbal intelligence and previous achievement in mathematics were identified.

The match of experimental and control groups on verbal comprehension in Malayalam, non-verbal intelligence and previous achievement in mathematics are demonstrated in Table 25.

Table 25
Distribution of Experimental Sample by the Levels of Verbal comprehension in Malayalam, Non-verbal Intelligence and Previous Achievement in Mathematics

| Sub samples based on Control Variables | Category | Number of students in the |  |
| :--- | :--- | :---: | :---: |
|  | Control Group |  |  |
| Verbal Comprehension in Malayalam | High | 23 | 23 |
|  | Low | 22 | 22 |
| Non-verbal Intelligence | High | 25 | 24 |
|  | Low | 20 | 21 |
| Previous Achievement in Mathematics | High | 26 | 24 |
|  | Low | 19 | 21 |

Further, experimental and control groups were studied for the nature of distribution of the control variables in them and then tested for their match using the independent samples $t$ tests/ Mann-Whitney $U$ test analyses as suitable for the nature of data and the results were as follows.

## Match of experimental and control groups on verbal comprehension

 in MalayalamBefore intervention, verbal comprehension in Malayalam in the control group was symmetric (Skewness $=0.24, S E=0.35$, skewness $/ S E=0.69$ ) and nearly mesokurtic (Kurtosis $=-0.89, S E=0.69$, kurtosis/ $S E=1.29$ ) indicating normality of distribution. In the experimental group also, verbal comprehension in Malayalam is symmetric (Skewness $=0.14, S E=0.35$, skewness $/ S E=0.40$ ) and nearly mesokurtic Kurtosis $=-0.81, S E=0.69$, kurtosis $/ S E=1.17$ ) indicating normality of distribution in the experimental group.

Distributions of verbal comprehension in Malayalam were normal in the control $(S-W=.96, \mathrm{df}=45, p>.05)$ and experimental $(S-W=.97, d f=45, p>.05)$ groups and their variances were homogeneous $[F(1,88)=0.13, p>.05]$. The groups had almost identical histograms as well as box plots and the linearity of the points in normal $\mathrm{Q}-\mathrm{Q}$ plots further confirmed that verbal comprehension in Malayalam before interventions in both the groups are normally distributed (Appendix N). Hence, the match between the experimental and control groups in verbal comprehension in Malayalam before intervention was tested using independent samples $t$-test. Result is given in Table 26.

Table 26
Test of Significance of Difference between the Mean Scores of Verbal Comprehension in Malayalam of the Control and Experimental Groups

| Groups | Mean | SD | Critical Ratio |
| :--- | :---: | :---: | :---: |
| Control Group | 53.33 | 19.69 |  |
| Experimental Group | 52.69 | 20.65 | 0.15 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$.

Table 26 shows that verbal comprehension in Malayalam before intervention do not differ significantly between the control $(M=53.33, S D=$ 19.69) and experimental $(M=52.69, S D=20.65)$ groups, $[t=0.15 ; p>.05]$.

## Match of experimental and control groups on non-verbal intelligence

Before intervention, non-verbal intelligence in the control group is symmetric (Skewness $=-0.18, S E=0.35$, skewness $/ S E=0.51$ ) and nearly mesokurtic (Kurtosis $=-0.74, S E=0.69$, kurtosis $/ S E=1.07$ ) indicating normality of distribution. In the experimental group also, non-verbal intelligence was symmetric (Skewness $=-0.36, S E=0.35$, skewness $/ S E$ $=1.03$ ) and nearly mesokurtic (Kurtosis $=-0.39, S E=0.69$, kurtosis $/ S E=$ 0.57 ) indicating normality of distribution.

Distribution of non-verbal intelligence before the intervention were normally distributed in the control $(S-W=.97, d f=45, p>.05)$ and experimental $(S-W=.97, d f=45, p>.05)$ groups and their variances were homogeneous $[F(1$, 88) $=0.00, p>.05]$. The groups have almost identical histograms as well as box plots and the linearity of the points in normal Q-Q plots further confirmed that non-verbal intelligence before interventions in both the groups are normally distributed (Appendix O). Hence, the match between the experimental and control groups in non-verbal intelligence before intervention was tested using independent samples $t$-test. Result is given in Table 27.

Table 27
Test of Significance of Difference between the Mean Scores of Non-Verbal Intelligence of the Control and Experimental Groups

| Groups | Mean | SD | Critical Ratio |
| :--- | :---: | :---: | :---: |
| Control Group | 36.64 | 7.60 | 0.51 |
| Experimental Group | 35.82 | 7.57 |  |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$.

Table 27 shows that non-verbal intelligence before intervention do not differ significantly between the control $(M=36.64, S D=7.60)$ and experimental $(M=35.82, S D=7.57)$ groups $[t=0.51, p>.05]$.

## Match of experimental and control groups on previous achievement in mathematics

Before intervention, previous achievement in mathematics in the control group was positively skewed (Skewness $=0.93, S E=0.35$, skewness $/ S E=2.66$ ) and nearly mesokurtic (Kurtosis $=0.21, S E=0.69$, kurtosis $/ S E=0.30$ ) indicating deviation from normality. In the experimental group also, previous achievement in mathematics, was positively skewed (Skewness $=0.57, S E=0.35$, skewness $/ S E=1.63$ ) and platykurtic (Kurtosis $=-0.57, S E=0.69$, kurtosis $/ S E=$ 0.83 ) indicating deviation from normality.

Distribution of scores of previous achievement in mathematics before intervention deviated from normality in the control $(S-W=.91, d f=45, p<.01)$ and experimental $(S-W=.93, \mathrm{df}=45, p<.01)$ groups through their variances were homogeneous $[F(1,88)=4.49, p>.05]$. The groups had almost identical histograms as well as box plots on previous achievement in mathematics before intervention. The non-linearity of the points in normal Q-Q plots further confirmed that the distribution of previous achievement in mathematics before interventions deviated from normality in both the control and the experimental
groups (Appendix P). Hence, the match between the control and experimental groups in previous achievement in mathematics before intervention was tested using Mann Whitney $U$ test. Result is given in Table 28.

Table 28
Mann-Whitney Test of Significance of Difference between Median Scores of Previous Achievement in Mathematics of the Control and Experimental Groups

| Groups | Median | Range | Mean Rank | Sum of Ranks | Mann-Whitney U |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Control Group | 35 | $80(10-90)$ | 43.54 | 1959.50 |  |
| Experimental Group | 35 | $65(15-80)$ | 47.46 | 2135.50 | 924.50 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$.
Table 28 shows that previous achievement in mathematics before intervention do not differ significantly between the control $(M d n=35)$ and experimental $(M d n=35)$ groups, $[U=924.50 ; p>.05]$.

Thus, before the intervention, the distribution of the verbal comprehension in Malayalam and non-verbal intelligence in both the control and experimental groups were symmetric and nearly mesokurtic, normal, homogenous, and did not differ significantly from one another. The distribution of previous achievement in mathematics, however, deviated from normality both in control and experimental groups, but their variances are equal and did not differ significantly from one another.

## Statistical Techniques Used

Survey data was analyzed mainly using percentage analysis in order to identify the extent of perceived difficulties in mathematical tasks and the reasons thereof and student achievement in mathematical language. To find out the relation between perceived difficulty in mathematical tasks and perceived
difficulty sourcing from nature of mathematics, Pearson's product-moment coefficient of correlation were calculated. Significance of difference of correlation of perceived difficulty in mathematical tasks with perceived difficulty sourcing from the nature of mathematics content and the nature of mathematics teaching-learning process were tested.

In the experimental phase, two groups were randomly selected for experimental treatment. Appropriate statistical tests were used to match the groups on the control variables and dependent variables and, to test the formulated hypotheses. Data analyses were done with the help of SPSS (Statistical Package for Social Sciences) software and online statistical calculators. The statistical techniques used for analysis of data in the survey and experimental phases are described briefly in the following headings.

## Survey Phase

In the survey phase, a series of tests were conducted in a planned way to find out the language related difficulties in learning mathematics among elementary school students. For that, Percentage analysis, Significance of difference between two correlated percentages, Pearson's $r$, Significance of a coefficient of correlation and Comparison of correlations from dependent samples were used. The pattern of statistical procedures applied on survey data is depicted in the scheme of analysis in Figure 4.


Figure 4. Scheme of analysis of the phase 1 survey study

## Percentage analysis

Percentage of students feeling difficulty in 26 mathematical tasks, related to nine factors, were analyzed. Subsequently, extent of perception of difficulty, sourcing from thirteen factors related to the nature of mathematics were studied. Also, extent of student achievement in six components of language of mathematics and their subcategories, were compared and relatively more difficult and less difficult components of language of mathematics were identified.

## Significance of difference between two correlated percentages

Responses recorded in percentages usually correlated when the same group gives answers to the same item (Ferguson, 1959). The obtained difference between percentages when divided by the standard error gives a critical value. If the critical value is greater than 1.96 (. 05 level) or 2.58 (. 01 level), reject the null hypothesis and conclude that the groups differed significantly in their answers to the item. The critical value was calculated using online statistics calculator (The statistical calculator, n.d.).

## Pearson's $r$

The relation between perceived difficulty in mathematical tasks and perceived difficulty sourcing from the nature of mathematics content and the nature of mathematics teaching-learning were studied using Pearson's $r$.

## Significance of coefficient of correlation

An observed correlation coefficient may result from chance or sampling error, and the test to determine its statistical significance is appropriate (Best \&

Kahn, 2006). Significance of each of the correlation coefficients obtained for the relationship of perceived difficulty in mathematical tasks with the nature of mathematics content and the nature of teaching-learning were tested using a two tailed test at the .05 level or .01 level with corresponding degrees of freedom, and then checked whether the obtained $t$ value exceeded the critical value.

## Comparison of correlations from dependent samples

Test of significance of difference between the two correlations was used to find out the significance of difference between correlation of perceived difficulty in mathematical tasks with the perceived difficulty sourcing from 1) nature of mathematics content and 2) nature of mathematics teaching-learning process. The test statistic was calculated using online statistical calculator (Psychometrica, n.d.).

## Experimental Phase

A series of tests were conducted in planned way to answer the research hypotheses and to ensure that various conditions for using the statistical procedures are satisfied. Basic descriptive statistics, Shapiro- Wilk test of normality, Levene's test of homogeneity of variances, Independent samples $t$ test, Mann-Whitney $U$ test, Two-way ANOVA, Effect Size (Cohen's $d$ ) and Partial eta squared were used for analysis of data from the experimental phase. The pattern of statistical procedures applied on experimental data is depicted in the scheme of analysis in Figure 5.


Figure 5. Scheme of analysis of the phase III experimental study

## Basic descriptive statistics

Normality of distribution is a requirement for many parametric tests including test of significance of difference between means and analysis of variance. The present study employed statistical indices, tests and graphical method for examining the distribution of data. Statistical indices from descriptive statistics and tests of normality and homogeneity include inferences.

Basic descriptive statistics such as mean, median, mode, standard deviation, skewness, kurtosis, ratio of skewness to its standard error and ratio of kurtosis to its standard error were calculated to study the distribution of control variables, and pretest and posttest measures. Histograms and normal Q-Q plots were plotted to further study these distributions. Box plots were also plotted to compare between the distribution of the variables in control and experimental group.

## Shapiro- Wilk test of normality

Shapiro - Wilk test of normality is the measure of deviation from normality. It identifies whether a sample comes from a non-normal distribution. The value of Shapiro- Wilk statistic $(S-W)$ lies between zero and one. Small values of $S-W$ indicate the rejection of normality whereas a value near one indicates normality of distribution. The test rejects the null hypothesis of normality when the $p$-value is less than or equal to .05 . If the $p$ value is greater than .05 , it indicates that the distribution is not significantly deviated from normality.

## Levene's test of homogeneity of variances

Homogeneity of variance is a requirement for parametric tests including independent sample $t$ test and analysis of variance. Levene's test is used to test the equality of variances for a variable calculated for two or more groups. Null hypothesis of this test states that the variances among all the groups are equal
whereas alternative hypothesis states that there will be a statistically significant difference in the variance of at least one group. So, if significance level of Levene's test statistic is greater than .05 , then the samples have equal variance and if it is less than .05 , then the variances are unequal among groups.

## Independent samples $\boldsymbol{t}$ test

To test the significance of differences between experimental and control groups, mean differences were tested using independent samples $t$ test, if the assumptions of independent $t$ test are met. This test was used for two purposes. To test the match of the control and experimental groups on control variables and to test the effect of language integrated instruction on achievement, self-efficacy and attitude towards mathematics.

## Mann-Whitney $\boldsymbol{U}$ test

When the assumptions of parametric tests are not met, i.e. if the distribution of data was not normal even for the control group, then an equivalent of the parametric $t$ test, Mann-Whitney $U$ test was used to compare experimental and control group. The null hypothesis proposed is that there is no significant difference between the two populations.

## Two-way ANOVA

The purpose of two-way ANOVA is to find out the interaction between two independent variables on the dependent variable. In this study 1) In order to verify whether language integrated mathematics instruction significantly enhances elementary school students' achievement in mathematics equally for high and low levels each of verbal comprehension in Malayalam, previous achievement in mathematics and non-verbal intelligence, factorial ANOVAs were used. And 2) in order to verify whether language integrated mathematics
instruction significantly enhances elementary school students' self-efficacy in mathematics equally for high and low levels each of verbal comprehension in Malayalam and non-verbal intelligence.

## Effect Size (Cohen's d)

'Effect size' is simply a way of quantifying the size of the difference between the two groups (Coe, 2002). Cohen's $d$ is the proper effect size measure if the two groups have similar standard deviations and are of the same size. To find out the extent of effect of language integrated instructional strategy, effect size was calculated using Cohens' $d$.

For the independent samples $t$ test, Cohen's $d$ is determined by calculating the mean difference between the two groups, and then dividing the result by the pooled standard deviation. Cohen's $d$ was calculated using an online statistical calculator (Social science statistics, n.d). Cohen (1965) suggested that value of $d=0.2$ as 'small' effect size, 0.5 as a 'medium' effect size and 0.8 as a 'large' effect size. Effect size is interpreted in terms of percentile ranks as per interpretation of Coe (2002).

For the Mann-Whitney $U$ test, effect size is calculated using the online statistical calculator (Psychometrica, n.d.)

## Partial eta squared

Partial eta- squared is calculated to determine the size of interaction effect of language integrated mathematics instruction after two-way ANOVA. Partial etasquared can be calculated using the following formula (Levine \& Hullett, 2010).

$$
\text { Partial } \eta 2=\frac{\text { ssbetween }}{\text { (ssbetween }+ \text { sserror) }}
$$

Partial eta-squared values are interpreted as $.09=$ small, $.14=$ medium, and $.22=$ large (Richardson, 2011; Fay \& Boyd, 2010).

## Chapter IV

## ANALYSIS

- Language Related Difficulties in Mathematics Learning
- Students' Perception of Difficulties in Mathematical Tasks
- Difficulty in Mathematics Sourcing from Nature of Mathematics
- Achievement in Components of Language of Mathematics
- Summary of Language related Difficulties in Learning Mathematics among Elementary School Students of Kerala
- Effects of Language Integrated Mathematics Instruction
- Main Effects of Language Integrated Mathematics Instruction on Mathematics Learning Outcomes
- Interaction of Language Integrated Mathematics Instruction with Control Variables on Mathematics Learning Outcomes
- Summary of Effect of Language Integrated Mathematics Instruction on Mathematics Learning among Elementary School Students in Kerala

This study examined the effectiveness of an evidence-based instruction focusing on language of mathematics in enhancing mathematics learning outcomes among elementary school students. It has three phases. First phase was a survey with two parts. Part 1 survey identified perception of difficulties in mathematical tasks and reasons for difficulty thereof. Part 2 survey identified students' language related difficulties in mathematics learning. Phase 2 was developmental phase in which experimental intervention and tools for measurement in experimental phase were developed. The survey data is analyzed mainly by percentage analysis. In phase 3 , experimental phase, effect of language integrated mathematics instruction on achievement in mathematics, self-efficacy in mathematics and attitude towards mathematics is studied using mean difference analyses of pretests and posttest data as per scheme of analysis. Before mean difference analysis, the data were analyzed to verify whether these measures are normally distributed, and the variance of comparison groups were equal. If the measures were normal in the control group and the variance of the comparison groups are equal, $t$ tests were used or else Mann-Whitney U tests were used. Then, interaction effect of treatment with verbal comprehension, non-verbal intelligence, and previous achievement in mathematics were studied as per objectives after ensuring the normality of distribution of residuals and homogeneity among comparison groups, using 2-way ANOVA. If the conditions were not met, interaction were studied through multiple comparisons using $t$ tests.

Results of analysis are presented under two sections, 1. Language related difficulties in mathematics learning and 2. Effect of language integrated mathematics instruction on achievement in mathematics, self-efficacy in mathematics and attitude towards mathematics.

## Language Related Difficulties in Mathematics Learning

Student perception of difficulties in mathematical tasks, reasons for difficulty thereof and student achievement in language of mathematics were studied. For this, percentage of students feeling difficulty in 26 mathematical tasks, related to nine factors, were analyzed and presented under five categories viz., number concept, mathematical symbols and notations, mathematical operations, mathematical abstractions and problem solving. Subsequently, extent of perception of difficulty, sourcing from thirteen factors related to nature of mathematics were studied. Then, the relation between perceived difficulty in mathematical tasks and perceived difficulty sourcing from nature of mathematics content and nature of mathematics teaching-learning were studied using Pearson's $r$. Significance of coefficient of correlation was also calculated. Finally, extent of student achievement in six components of language of mathematics viz., terms, symbols, morphology, syntax, semantics, and pragmatics, and their subcategories, were compared and relatively more difficult and less difficult components of language of mathematics were identified.

## Students' Perception of Difficulties in Mathematical Tasks

Percentage of students feeling difficulty in 26 mathematical tasks related to nine factors are analyzed and presented under five categories viz., number concept, mathematical symbols and notations, mathematical operations, mathematical abstractions and problem solving.

## Perceived difficulties in number concept

Difficulty with number concept has two factors - difficulty with number systems and understanding numbers. Percentage of students perceiving difficulty in factors related to number concept by tasks involved is given in Table 29.

Table 29
Percentage of Students Perceiving Difficulty in Tasks Involved in Factors of Number Concept

| Factor | Task | Percentage |
| :--- | :--- | :---: |
| Number systems | Using Decimals | 64.33 |
|  | Using Fractions | 56.33 |
| Understanding numbers | Understanding large numbers | 24.00 |
|  | Understanding Place value | 24.33 |

Note. $\mathrm{N}=300$

Tasks in number systems viz., using decimals (64.33\%) and fractions $(56.33 \%)$ are felt difficult for majority of students whereas only a small proportion of students feel difficulty in tasks related to understanding numbers viz., understanding large numbers (24 \%) and place value (24.33\%).

## Perceived Difficulties in Mathematical Symbols and Notations

Percentage of students perceiving difficulty in mathematical symbols and notations are given in Table 30.

Table 30
Percentage of Students Perceiving Difficulty in Tasks Involved in Mathematical Symbols and Notations

| Factor | Task | Percentage |
| :--- | :--- | :---: |
| Mathematical Symbols Understanding algebraic problems | 56.33 |  |
|  | Analyzing geometrical figures | 44.67 |
|  | Understanding symbols and notations | 33.00 |
|  | Drawing geometrical figures | 22.33 |

Note. $\mathrm{N}=300$

Tasks related to Mathematical symbols and notations viz., understanding algebraic problems $(56.33 \%)$ is felt difficult for majority of students. More than
$1 / 3^{\text {rd }}$ of the students feel difficulty in analyzing geometrical figures (44.67\%) and in understanding symbols and notations (33\%). Only a small proportion of students feel difficulty in drawing geometrical figures (22.33\%).

## Perceived Difficulties in Mathematical operations

Difficulty in mathematical operations has two factors- difficulty in problem solving competence and arithmetic operations. Percentage of students perceiving difficulty in factors related to mathematical operations by tasks involved is given in Table 31.

Table 31
Percentage of Students Perceiving Difficulty in Tasks Involved in Factors of Mathematical Operations

| Factor | Task | Percentage |
| :--- | :--- | :---: |
| Problem solving <br> competence | Concentrating for long time to solve problems | 61.00 |
|  | Doing calculations with speed | 60.33 |
| Arithmetic operations | Remembering numbers while doing operations | 35.33 |
|  | Following rules while doing calculations | 31.33 |
|  | Doing basic arithmetic operations | 22.33 |
|  | Doing Mental arithmetic | 22.33 |

Note. N $=300$
Tasks related to problem solving competence is felt difficult for majority of students compared to tasks in arithmetic operations. Majority of students feel difficulty in concentrating for a long time to solve problems (61\%) and doing calculations with speed ( $60.33 \%$ ). More than $1 / 3^{\text {rd }}$ of the students feel difficulty in remembering numbers while doing operations (35.33\%) and following rules while doing calculations ( $31.33 \%$ ). Relatively small proportion of students feel difficulty in tasks viz., doing basic arithmetic operations (22.33\%) and doing mental arithmetic (22.33\%).

## Perceived Difficulties in Mathematical abstractions

Percentage of students perceiving difficulty in tasks related to Mathematical abstractions is given in Table 32.

Table 32
Percentage of Students Perceiving Difficulty in Tasks Involved in Mathematical Abstractions

| Factor | Task | Percentage |
| :---: | :--- | :---: |
| Mathematical abstractions | Comprehending Process unrelated to daily life | 41.00 |
|  | Comprehending Concepts unrelated to daily life | 40.67 |

Note. N= 300
Both comprehension of processes ( $41 \%$ ) and concepts ( $40.67 \%$ ) unrelated to daily life are felt difficult for more than $1 / 3^{\text {rd }}$ of the students.

## Perceived Difficulties in Problem solving

Difficulty in problem solving has three factors- understanding word problems, equations and operations, and translation of word problems. Percentage of students perceiving difficulty in factors related to problem solving by tasks involved is given in Table 33.

Table 33
Percentage of Students Perceiving Difficulty in Tasks Involved in Factors of Problem Solving

| Factor | Task | Percentage |
| :--- | :--- | :---: |
|  | Identifying irrelevant information in word problems | 48.67 |
| Understanding | Identifying key words in word problems | 47.33 |
| word problems | Identifying mathematics problem in story problems | 46.00 |
|  | Understanding word problem without external help | 41.00 |
|  | Identifying equations | 51.67 |
| Equations and | Analyzing lengthy word problems | 46.00 |
| operations | Doing mathematical operations in sequence | 36.33 |
|  | Selecting mathematical operations | 26.33 |
| Translation of | Translating mathematical expression to verbal expression | 43.00 |
| word problems | Translating word problems into mathematical expression | 38.00 |

Note. N= 300

Difficulty in tasks related to understanding word problems such as identifying irrelevant information in word problems (48.67\%), identifying key words in word problems (47.33\%), identifying mathematics problem in story problems (46\%) and understanding word problem without external help (41\%) are felt difficult for more than $1 / 3^{\text {rd }}$ of students. Selecting mathematical operations is felt difficult for relatively small proportion of students (26.33\%). Identifying equation for a given problem (51.67\%) is felt difficult for majority of students. Around $40 \%$ of Students feel difficulty in translating mathematical answer to verbal expression (43\%) and translating word problems into mathematical expression (38\%).

## Summary of Difficulty in Tasks of School Mathematics

Percentage of students perceiving difficulty in mathematical tasks is summarized in Figure 1. Majority of students feel difficulty in tasks involved in problem solving competence (61\%) and number system like decimals and fractions $(60 \%)$. Tasks related to understanding word problems (46\%), mathematical abstractions (41\%), translation of word problems (41\%), equations and operations (40\%) and, symbols and notations (39\%) are felt difficult for more than $1 /{ }^{\text {rd }}$ of the students. Tasks related to arithmetic operations (28\%) and understanding numbers ( $24 \%$ ) are felt difficult for about $1 / 4^{\text {th }}$ of students.


Figure 6. Percentage of standard VIII students perceiving difficulty in mathematical tasks.

## Difficulty in Mathematics Sourcing from Nature of Mathematics

Extent of difficulty in learning mathematics sourcing from thirteen factors related to nature of mathematics were studied in terms of percentage of students and is given in Table 34.

Table 34
Percentage of Students Perceiving Difficulty Sourcing from Nature of Mathematics

| Source of difficulty | High Difficulty <br> $(\%)$ | Average Difficulty <br> $(\%)$ | No Difficulty <br> $(\%)$ |
| :--- | :---: | :---: | :---: |
| Cumulative nature of Content | 53 | 38 | 9 |
| Need for strenuous attention | 52 | 34 | 15 |
| Number of concepts | 46 | 40 | 14 |
| Problem solving | 46 | 46 | 9 |
| Need for repeated practice | 45 | 43 | 12 |
| Difficulty of concepts | 45 | 42 | 13 |
| Difficulty in understanding questions | 43 | 41 | 16 |
| Need for external support | 40 | 49 | 11 |
| Need to learn unfamiliar words | 37 | 40 | 24 |
| Need for rote learning | 33 | 46 | 22 |
| Prevalence of symbols and notations | 31 | 39 | 39 |
| Impracticability in daily life | 22 | --- | 19 |
| Need for Precision in understanding | 81 |  | 27 |

Note. N= 300
Table 34 along with Figure 7 shows that thirteen factors related to nature of mathematics cause difficulty in learning mathematics. Above 60 percent of students rate all thirteen reasons as causing high or average difficulty. For around 90 percent of students, prominence of problem solving and cumulative nature of content are felt as the major reasons for difficulty in learning Mathematics. Need for external support, need for repeated practice, difficulty of concepts, number of concepts, need for strenuous attention, difficulty in
understanding questions and need for precision in understanding are felt as difficult for around 80 percent of students. Prevalence of symbols and notations, necessity to learn unfamiliar words, need for rote learning and impracticability in daily life are considered as elements creating high difficulty for around $1 /{ }^{\text {rd }}$ of students. However, in these factors, only one reason i.e. impracticability in daily life is considered as creating no difficulty by around 40 percent of students. It is evident that prominence of problem solving and cumulative nature of content are the major reasons for difficulty. Need for strenuous attention, number of concepts, need for repeated practice, toughness of concepts, need for precision in understanding, difficulty in understanding questions and need for external support are moderate reasons for difficulty. Need to learn unfamiliar terms, need for rote learning, prevalence of symbols and notations and impracticability in daily life are perceived by around $3 / 4^{\text {th }}$ of students as reason for difficulty.


Figure 7. Percentage of standard VIII students perceiving difficulty sourcing from nature of mathematics content and teaching-learning.

Factors in nature of mathematics contributing to task difficulty in school mathematics

The relation of difficulty in mathematical tasks as perceived by students with their felt difficulty sourcing from nature of mathematics and its components - difficulties sourcing from nature of mathematics content and teaching-learning process - were studied using Pearson's $r$. Correlation of nature of mathematics and its factors with task difficulties in mathematics is given in Table 35.

Table 35
Correlation of Task Difficulties in Mathematics with Nature of Mathematics and its Factors

|  | Pearson's correlation coefficient |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Factors of Task Difficulty in <br> school Mathematics | Nature of <br> Mathematics <br> content | Nature of <br> Mathematics <br> Teaching-Learning | Nature of <br> Mathematics <br> (Total) | $z^{\text {a }}$ |
| Equations and operations | $.41^{* *}$ | $.32^{* *}$ | $.39^{* *}$ | $2.38^{*}$ |
| Symbols and notations | $.38^{* *}$ | $.28^{* *}$ | $.36^{* *}$ | $2.61^{* *}$ |
| Arithmetic operations | $.37^{* *}$ | $.23^{* *}$ | $.34^{* *}$ | $3.61^{* *}$ |
| Problem solving competence | $.26^{* *}$ | $.25^{* *}$ | $.28^{* *}$ | 0.25 |
| Understanding word problems | $.26^{* *}$ | $.21^{* *}$ | $.26^{* *}$ | 1.26 |
| Translation word problems | $.24^{* *}$ | $.20^{* *}$ | $.24^{* *}$ | 1.00 |
| Understanding numbers | $.24^{* *}$ | $.14^{*}$ | $.21^{* *}$ | $2.49^{* *}$ |
| Number systems | $.21^{* *}$ | $.13^{*}$ | $.18^{* *}$ | $1.98^{*}$ |
| Mathematical abstractions | $.20^{* *}$ | .11 | $.17^{* *}$ | $2.23^{*}$ |
| Total perceived task difficulty | $.46^{* *}$ | $.34^{* *}$ | $.44^{* *}$ | $3.24^{* *}$ |

Note. $N=300$
$Z^{\text {a }}$ critical value obtained for test of significance of difference of correlation of perceived difficulty in mathematical tasks with difficulty sourcing from nature of mathematics content and nature of mathematics teaching-learning process.
*p<.05. ${ }^{* *} p<.01$.
Table 35 shows that every task difficulty factor in mathematics has significant positive correlation with nature of mathematics. Nature of mathematics content has higher correlation with relatively easier tasks involving equations and
operations, symbols and notations and, arithmetic operations. Tasks related to equations and operations have significant positive substantial correlation with nature of mathematics content $(r=.41, p<.01)$ and low correlation with nature of mathematics teaching learning $(r=.32, p<.01)(z=2.38, p<.01)$.

Tasks related to symbols and notations, arithmetic operations, problemsolving competence, understanding word problems and translation of word problems have significant positive low correlation with both nature of mathematics content ( $p<.01$ ) and nature of mathematics teaching learning ( $p<.01$ ). There is significantly higher correlation for nature of mathematics content, than for nature of teaching-learning process with perceived difficulty in tasks involving symbols and notations $(z=2.61, p<.01)$ and arithmetic operations ( $z=3.61, p<.01$ ). However, correlation for nature of mathematics content and nature of mathematics teaching-learning process with task difficulty involving problem-solving competence ( $z=0.25, p>.05$ ), understanding word problems $(z=1.26, p>.05)$ and translation of word problems $(z=1.00, p>.05)$ are not significant.

Tasks related to understanding numbers and number systems exhibits significant positive low correlation with nature of mathematics content ( $p<.01$ ) and negligible correlation with nature of mathematics teaching learning ( $p<.01$ ). There are significant positive low correlations for nature of mathematics content ( $p<.01$ ) and negligible correlation for nature of mathematics teaching-learning ( $p<.01$ ) against difficulty in tasks involving understanding numbers $(z=2.49$, $p<.01)$ and number systems ( $z=1.98, p<.05)$.

Tasks related to using mathematical abstractions have significant positive low correlation with nature of mathematics content $(r=.20, p<.01)$, whereas it
does not have significant correlation with nature of mathematics teaching learning $(r=.11, p>.05)(z=2.23, p<.05)$.

Perceived difficulty in mathematical tasks are related more with difficulty sourcing from nature of mathematics content ( $r=.46$ ) than with nature of mathematics teaching-learning process $(r=.34, z=3.24, p<.01)$.

## Achievement in Components of Language of Mathematics

Extent of student achievement in terms of percentage score in six components of language of mathematics viz., terms, symbols, morphology, syntax, semantics, and pragmatics, and their subcategories, were analyzed. Achievement in their subcategories were also studied. Percentage of achievement in components of language of mathematics of elementary school students is given in Figure 8.


Figure 8. Achievement in components of language of mathematics among standard VIII students.

Achievement in morphology (25.84\%), terms (38.99\%) and semantics $(46.03 \%)$ are less than average achievement in language of mathematics (47\%). Achievement in symbols (51.31\%) and Syntax (53.63\%) are comparatively higher. The component of language of mathematics with highest achievement is pragmatics (66.25\%). The language related difficulties in learning mathematics for students, source more from vocabulary components like morphology, terms and semantics than from sentence level components like syntax and pragmatics.

## Achievement in subcategories of components of language of mathematics

Student achievement in subcategories of six components of language of mathematics were further analyzed in term of percentage score and comparison of percentage scores of subcategories within each of the six components of language of mathematics were done. Percentage achievement in subcategories of components of language of mathematics is given in Figure 9.


Figure 9. Achievement in subcategories of components of language of mathematics among standard VIII students.

Achievement in subcategories of components of mathematical language is below average in parts of words (25.84\%), general terms (38.23\%) and mathematics specific terms (40.36\%), word meaning in specific context (46.03\%), basic mathematics symbols (41.01\%) and mathematical conventions (37.07\%). Higher achievement is observed in subcategories of components of mathematics language viz., algebraic expressions (48.57\%), natural language ( $50.41 \%$ ), geometric figures ( $62.52 \%$ ), numeric expressions ( $63 \%$ ), commonly used fractions ( $63.68 \%$ ), real life word problems (64.38\%), reading graph (67.8\%), identifying operations from key terms (68.7\%) and geometrical symbols (74.5\%).

Test of significance of difference between two correlated percentages was performed to determine whether there were significant differences between achievement in subcategories of components of language of mathematics. The percent of achievement in subcategories within each component were compared.

In case of subcategories of terms, there is no significant difference between achievement in general terms (38.23\%) and mathematics specific terms (40.36\%) ( $C R=0.45, p>.05)$.

Achievement in basic mathematics symbols (41.01\%) is significantly low compared to achievement in geometric symbols (74.5\%) ( $C R=6.14, p<.01$ ).

In case of subcategories of syntax, achievement in mathematical conventions (37.07\%) is significantly low compared to achievement in algebraic expression $(48.57 \%, C R=2.34, \mathrm{p}<.05)$, natural expression $(50.41 \%, C R=2.69$, $p<.01$ ), geometric figures ( $62.52 \%, C R=4.93, p<.01$ ) and numeric expression $(63.00 \%, C R=5.02, p<.01)$. However, when the same content is expressed in different formats, achievement in algebraic expression (48.57\%) is not significantly different from achievement in natural expression $(50.41 \%, C R=$ $0.35, p>.05$ ), but is significantly low compared to achievement in numeric expression $(63 \%, C R=2.58, p<.01)$. When the same content is expressed in natural language (50.41\%) and numeric language (63\%) achievement is significantly high in numeric expression ( $C R=2.27, p<.05$ ).

Achievement in subcategories of pragmatics doesn't show any significant difference. That is, achievement in commonly used fractions (63.68\%) is not significantly different from achievement in real life word problems ( $64.38 \%, C R$ $=0.12, p>.05)$, reading graph $(67.8 \%, C R=0.67, p>.05)$ and achievement in identifying operations from key words ( $68.7 \%, C R=0.82, p>.05$ ). Also,
achievement in real life word problems (64.38\%) is not significantly different from achievement in reading graph $(67.8 \%, C R=0.56, p>.05)$ and achievement in identifying operations from key words ( $68.7 \%, C R=0.56, p>.05$ ). Furthermore, achievement in identifying operations from key words (68.7\%) is not significantly different from achievement in reading graph (67.8\%, $C R=0.14$, $p>.05)$.

## Summary of Language related Difficulties in Learning Mathematics among Elementary School Students of Kerala

High percentage of students feels difficulty not only in problem solving competence in mathematics but also in number systems, especially in using decimals and fractions. Also, understanding algebraic problems and identifying equations to solve problems are difficult for majority of students. Word problems, mathematical abstractions, selecting mathematical operations and, symbols and notations are felt difficult for more than $1 / 3^{\text {rd }}$ students. Tasks related to equations and operations, though are less difficult, has substantial correlation with difficulty due to nature of mathematics content. Perceived difficulty in tasks related to mathematical abstractions is correlated only with nature of mathematics content and not with nature of mathematics teachinglearning.

Thus, difficulty in learning mathematics is contributed to both by nature of mathematics content and nature of mathematics teaching-learning. Perceived difficulty in mathematical tasks are related more with perceived difficulty sourcing from nature of mathematics content ( $r=.46$ ) than with nature of mathematics teaching-learning process $(r=.34, z=3.24, p<.01)$.

Among the above cited students' perceived difficulties in mathematics, tasks like understanding algebraic problems, analyzing geometrical figures, understanding and translating word problems and using number systems are evidently related to its language. Likewise, difficulties sourced from nature of mathematics such as difficulty in understanding questions, unfamiliar words and, symbols and notations also relate to language of mathematics.

The language related difficulties in learning mathematics for students are more from vocabulary components like morphology, terms and semantics than from sentence level components like syntax and pragmatics. This is evidenced from percentage achievements in mathematical terms, both general (38.23\%) and mathematics specific (40.36\%), basic mathematics symbols (41.01\%), syntactic conventions ( $37.07 \%$ ), algebraic expression (48.57\%), and natural language ( $50.41 \%$ ) being significantly less. Achievement in other components of mathematical language such as that in numeric expression (63\%), commonly used fractions ( $63.68 \%$ ), real life word problems (64.38\%), reading graph ( $67.8 \%$ ) and identifying operations from key words (68.7\%) though are relatively high.

In summary, students perceive difficulties in mathematical tasks including understanding algebraic problems, analyzing geometrical figures, understanding and translating word problems - which relate to nature of its content and its teaching-learning process, and much of these natures in turn link to its language elements such as understanding questions, unfamiliar words and, symbols and notations. Language related difficulties in school mathematics is large especially in general and mathematical terms, mathematical symbols,
syntactic conventions and, algebraic expressions. There are less but substantial difficulties emerging from numeric expressions, commonly used fractions, real life word problems, reading graph and identifying operations from key words in word problems.

## Effects of Language Integrated Mathematics Instruction

Initially, effectiveness of language integrated mathematics instruction on achievement in mathematics, self-efficacy in mathematics and attitude towards mathematics were studied using $t$-test or Mann-Whitney $U$ test as appropriate. This was followed up with analysis of interaction of language integrated mathematics instruction with verbal comprehension in Malayalam, non-verbal intelligence and previous achievement in mathematics on select mathematics learning outcomes using factorial ANOVA or multiple independent samples $t$ tests as suitable. The results are presented under two broad sections- 1) Main effects of language integrated mathematics instruction and 2) Interaction effects of language integrated mathematics instruction with verbal comprehension in Malayalam, non-verbal intelligence and previous achievement in mathematics on mathematics learning outcomes.

## Main Effects of Language Integrated Mathematics Instruction

Effectiveness of language integrated mathematics instruction among elementary school students in improving their achievement in mathematics, selfefficacy in mathematics and attitude towards mathematics were studied using $t$ test or Mann-Whitney $U$ test as applicable, after verifying the distribution of data of each dependent variable for fulfillment of the assumptions of t-test, using

Shapiro-Wilk ( $S-W$ ) test of normality and Levene's test of homogeneity. Before treatment, it was also verified that experimental and control groups did not differ significantly on dependent variables- self-efficacy in mathematics and attitude towards mathematics.

## Effects of Language Integrated Mathematics Instruction on Achievement in Mathematics

Distribution of posttest scores of achievement in mathematics of control and experimental groups were studied. Posttest score of achievement in mathematics in the control group was symmetric (Skewness $=0.22, S E=0.35$, Skewness $/ S E=0.63$ ) and nearly mesokurtic (Kurtosis $=-0.65, S E=0.69$, Kurtosis/SE $=0.94$ ) indicating normality. In the experimental group also, distribution of achievement in mathematics after intervention was symmetric (Skewness $=-0.05, S E=0.35$, Skewness $/ S E=0.14$ ) and nearly mesokurtic (Kurtosis $=-0.83, S E=0.69$, sKurtosis $/ \mathrm{SE}=1.20$ ) indicating normality. Distributions of achievement in mathematics after intervention are normal in both control $(S-W=.98, d f=45, p>.05)$ and experimental $(S-W=.98$, $d f=45, p>.05)$ groups and the variances of the two groups are homogeneous $[F(1,88)=0.20, p>.05]$.

Difference in distribution of posttest scores of achievement in mathematics of control and experimental groups is studied using histograms as well as box plots and the linearity of the points in normal Q-Q plots further confirmed that posttest scores of achievement in mathematics in both control and experimental groups are normally distributed (Appendix Q). Hence, the difference in posttest scores of achievement in mathematics between the
experimental and control groups was tested using independent samples t -test. Result is given in Table 36.

Table 36
Test of Significance of Difference between Means of Achievement in Mathematics after Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems (Control)

| Groups | $M$ | $S D$ | $t$ | Cohen's $d$ |
| :--- | :---: | :---: | :---: | :---: |
| Practice in solving mathematics problems | 46.18 | 14.89 |  |  |
| Language integrated mathematics | 56.18 | 13.42 | $3.35^{* *}$ | 0.71 |
| Instruction |  |  |  |  |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45 ;{ }^{* *} p<.01$

Table 36 shows that mean posttest scores of achievement in mathematics after language integrated mathematics instruction $(M=56.18, S D=13.42)$ is significantly higher than that after practice in solving mathematics problems ( $M$ $=46.18, S D=14.89$ ) $[t=3.35 ; p<.01]$. Effect size (Cohen's $d=0.71$ ) indicates that, achievement in mathematics after language integrated mathematics instruction is medium, over and above that after practice in solving mathematics problems. As per Coe (2002), Cohen's $d=0.70$ means that 76 percent of students in language integrated mathematics instruction group are above the average student in the group which practiced solving mathematics problems and this is further demonstrated in Figure 10 showing smoothed frequency curves of the distributions of posttest scores of achievement in mathematics of experimental and control groups.


Figure 10. Ogives of the scores of achievement in mathematics after language integrated mathematics instruction and practice in solving mathematics problems.

Figure 10 shows that after language integrated mathematics instruction, achievement in mathematics of students, especially those in between first and third quartiles, have an advantage of approximately 25 percentile ranks than that after practice of mathematics problem solving; while those in the upper and lower quartiles have an advantage of approximately 15 percentile ranks.

## Effect of language integrated mathematics instruction on achievement

## in algebra

Distribution of posttest scores of achievement in algebra of control and experimental groups were studied. Distribution of posttest scores of achievement in algebra in the control group is symmetric (Skewness $=0.00, S E=0.35$, skewness/ $S E=0)$ and nearly mesokurtic (Kurtosis $=-0.38, S E=0.69$, Kurtosis $/ \mathrm{SE}=0.55$ ) indicating normality. In the experimental group also, distribution of achievement in algebra after intervention is symmetric (Skewness $=-0.31, S E=0.35$,

Skewness $/ \mathrm{SE}=0.89$ ) and nearly mesokurtic (Kurtosis $=0.62, S E=0.69$, Kurtosis/SE $=0.89$ ) indicating normality. Likewise, distributions of achievement in algebra after intervention are normal in both control $(S-W=.97, d f=45$, $p>.05)$ and experimental $(S-W=.96, d f=45, p>.05)$ groups and the variances of the two groups are homogeneous $[F(1,88)=1.75, p>.05]$.

Difference in distribution of posttest scores of achievement in algebra of control and experimental groups is studied using histograms as well as box plots and the linearity of the points in normal Q-Q plots further confirmed that posttest scores in achievement in algebra in control group is normal (Appendix R). Hence, the difference in posttest scores of achievement in algebra between the experimental and control groups was tested using independent samples t-test. Result is given in Table 37.

Table 37
Test of Significance of Difference between Means of Achievement in Algebra after Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems (Control)

| Groups | $M$ | $S D$ | $t$ | Cohen's $d$ |
| :---: | :---: | :---: | :---: | :---: |
| Practice in solving mathematics problems | 40.42 | 13.89 |  |  |
| Language integrated mathematics Instruction | 50.56 | 12.34 | $3.66^{* *}$ | 0.77 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45 ;{ }^{* *} p<.01$
Table 37 shows that mean posttest score of achievement in algebra after language integrated mathematics instruction $(M=50.56, S D=12.34)$ is significantly higher than that after practice in solving Mathematics problems ( $M=$ 40.42, $S D=13.89$ ) $[t=3.66 ; p<.01]$. Effect size (Cohen's $d=0.77$ ) indicates that, increase in achievement in algebra after language integrated mathematics instruction is medium, compared to that after practice in solving mathematics problems. As per Coe (2002), Cohen's $d=0.7$ means that 76 percent of students in
language integrated mathematics instruction group is above the average student in the group which practiced solving mathematics problems and this is further demonstrated in Figure 11 showing smoothed frequency curves of the distributions of posttest scores of achievement in algebra of experimental and control groups.


Figure 11. Ogives of the scores of achievements in algebra after language integrated mathematics instruction and practice in solving mathematics problems.

Figure 11 shows that effect of language integrated mathematics instruction on achievement in algebra is highest in the lower quartile with an advantage of approximately 30 percentile ranks over and above that after practice in solving problems, an advantage of approximately 25 percentile ranks at the median, and an advantage of less than 15 percentile ranks at the third quartile, indicating that language integrated mathematics instruction is especially beneficial for students in the lower achievement strata though others are also benefited significantly.

Effect of language integrated mathematics instruction on achievement in arithmetic

Distribution of posttest scores of achievement in arithmetic of control and experimental groups were studied. In the control group, distribution of posttest scores of achievement in arithmetic is symmetric (Skewness $=0.23, S E=0.35$, Skewness $/ \mathrm{SE}=0.66$ ) and nearly mesokurtic (Kurtosis $=-0.59, S E=0.69$, Kurtosis/SE $=0.86$ ) indicating normality. In the experimental group also, distribution of achievement in arithmetic after intervention is symmetric (Skewness $=0.17, S E=0.35$, Skewness $/ \mathrm{SE}=0.49$ ) and nearly mesokurtic (Kurtosis $=-0.87$, $S E=0.69$, Kurtosis $/ \mathrm{SE}=1.26$ ) indicating normality. Likewise, distributions of achievement in arithmetic after intervention is normal in both control ( $S-W=.97, d f$ $=45, p>.05)$ and experimental $(S-W=.96, d f=45, p>.05)$ groups and the variances of the two groups are homogeneous $[F(1,88)=0.00, p>.05]$.

Difference in distribution of posttest scores of achievement in arithmetic of control and experimental groups is studied using histograms as well as box plots and the linearity of the points in normal Q-Q plots further confirmed that posttest scores of achievement in arithmetic in both control and experimental groups are normal (Appendix S). Hence, the difference in posttest scores of achievement in arithmetic between the experimental and control groups was tested using independent samples $t$ test. Result is given in Table 38.

Table 38
Test of Significance of Difference between Means of Achievement in Arithmetic after Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems (Control)

| Groups | $M$ | $S D$ | $t$ | Cohen's $d$ |
| :---: | :---: | :---: | :---: | :---: |
| Practice in solving mathematics problems | 45.12 | 15.26 |  |  |
| Language integrated mathematics Instruction | 52.96 | 14.30 | $2.51^{*}$ | 0.53 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45 ;{ }^{*} p<.05$

Table 38 shows that mean posttest score of achievement in arithmetic after language integrated mathematics instruction ( $M=52.96, S D=14.30$ ) is significantly higher than that after practice in solving mathematics problems ( $M=45.12, S D=15.26$ ) $[t=2.51 ; p<.05]$. Effect size (Cohen's $d=0.53$ ) indicates that, increase in posttest score of achievement in arithmetic after language integrated mathematics instruction is medium, compared to that after practice in solving mathematics problems. As per Coe (2002), Cohen's $d=0.5$ means that 69 percent of students in language integrated mathematics instruction group are above the average student in the group which practiced solving mathematics problems and this is further demonstrated in Figure 12, showing smoothed frequency curves of the distributions of posttest scores of achievement in arithmetic of experimental and control groups.


Figure 12. Ogives of the scores of achievement in arithmetic after language integrated mathematics instruction and practice in solving mathematics problems.

Figure 12 shows that after language integrated mathematics instruction, achievement in arithmetic of students, especially those at median, have an advantage of approximately 20 percentile ranks than that after practice of mathematics problem solving; while those in the upper and lower quartiles have an advantage of above 10 percentile ranks.

## Effect of language integrated mathematics instruction on achievement in geometry

Distribution of posttest scores of achievement in geometry of control and experimental groups were studied. Posttest scores of achievement in geometry in the control group is symmetric (Skewness $=0.13, S E=0.35$, skewness/ $S E=0.37$ ) and nearly mesokurtic (Kurtosis $=-1.01, S E=0.69$, Kurtosis $/ \mathrm{SE}=$ 1.46) indicating normality of distribution. In the experimental group also, distribution of posttest scores of achievement in geometry is symmetric (Skewness $=-0.34, S E=0.35$, Skewness/ $/ \mathrm{SE}=0.97$ ) and nearly mesokurtic (Kurtosis $=-0.52, S E=0.69$, Kurtosis $/ \mathrm{SE}=0.75$ ) indicating normality. Likewise, distributions of achievement in geometry after intervention are normal in both control $(S-W=.96, d f=45, p>.05)$ and experimental $(S-W=.97, d f=45$, $p>.05)$ groups and the variances of the two groups are homogeneous $[F(1,88)=$ $0.58, p>.05]$.

Difference in distribution of posttest scores of achievement in geometry of control and experimental groups is studied using histograms as well as box plots and the linearity of the points in normal Q-Q plots further confirmed that posttest scores of achievement in geometry in both control and experimental groups are normal (Appendix T). Hence, the difference in
posttest scores of achievement in geometry between the experimental and control groups was tested using independent samples $t$ test. Result is given in Table 39.

Table 39
Test of Significance of Difference between Means of Achievement in Geometry after Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems (Control)

| Groups | $M$ | $S D$ | $t$ | Cohen's $d$ |
| :--- | :---: | :---: | :---: | :---: |
| Practice in solving mathematics problems | 51.20 | 16.72 |  |  |
| Language integrated mathematics Instruction | 64.10 | 15.52 | $3.79^{* *}$ | 0.79 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45 ;{ }^{* *} p<.01$

Table 39 shows that mean posttest score of achievement in geometry after language integrated mathematics instruction $(M=64.10, S D=15.52)$ is significantly higher than that after practice in solving mathematics problems ( $M=51.20, S D=16.72$ ) $[t=3.79 ; p<.01]$. Effect size (Cohen's $d=0.79$ ) indicates that, increase in posttest scores of achievement in geometry after language integrated mathematics instruction is medium, compared to that after practice in solving mathematics problems. As per Coe (2002), Cohen's $d=0.7$ means that 76 percent of students in language integrated mathematics instruction group are above the average student in the group which practiced solving mathematics problems, and this is further demonstrated in Figure 13 showing smoothed frequency curves of the distributions of posttest scores of achievement in geometry of experimental and control groups.


Figure 13. Ogives of the scores of achievements in geometry after language integrated mathematics instruction and practice in solving mathematics problems.

Figure 13 shows that effect of language integrated mathematics instruction on achievement in geometry is the highest in the lower quartile with an advantage of approximately 30 percentile ranks over and above that after practice in solving problems, an advantage of approximately 25 percentile ranks at the median, and an advantage of approximately 15 percentile ranks at the third quartile, indicating that language integrated mathematics instruction is especially beneficial for students in the lower achievement strata though others are also benefited significantly.

## Effects of Language Integrated Mathematics Instruction on Self-efficacy in Mathematics

Before intervention, self-efficacy in mathematics was symmetric (Skewness=-0.59, $S E=0.35$, Skewness/ $\mathrm{SE}=1.68$ ) and nearly mesokurtic (Kurtosis=$0.63, S E=0.69$, Kurtosis $/ \mathrm{SE}=0.01$ ) indicating normality of distribution in the control group. In the experimental group also, self-efficacy in mathematics is symmetric
(Skewness $=-0.68, S E=0.35$, Skewness/ $\mathrm{SE}=1.94$ ) and nearly mesokurtic (Kurtosis $=-0.50, S E=0.69$, Kurtosis $/ \mathrm{SE}=0.72$ ) indicating normality of distribution.

Distributions of pretest scores of self-efficacy in mathematics is normal in both control $(S-W=.96, d f=45, p>.05)$ and experimental $(S-W=.96, d f=45$, $p>.05)$ groups and their variances are homogeneous $[F(1,88)=1.08$, $p>.05]$. The groups had almost identical histograms as well as box plots, and the linearity of the points in normal Q-Q plots further confirmed that self-efficacy in mathematics before interventions in both the groups were normally distributed (Appendix U). Hence, comparability of self-efficacy in mathematics in the experimental and control groups was tested using independent samples $t$ test. And, it was found that self-efficacy in mathematics did not differ significantly between the control ( $M=66.41, S D=10.97, N_{l}=45$ ) and experimental $\left(M=60.41, S D=12.58, N_{2}=45\right)$ groups before intervention $[t=1.21 ; p>.05]$.

Thus, before the intervention, the distribution of self-efficacy in mathematics in the control and experimental group are symmetric and nearly mesokurtic, normal, homogenous, and did not differ significantly one another.

After intervention, distribution of gain score of self-efficacy in mathematics of control and experimental groups were studied. distribution of posttest scores of Self-efficacy in Mathematics is provided in Appendix V. Gain scores of selfefficacy in mathematics in the control group is symmetric (Skewness= $0.07, S E=$ 0.35, Skewness/SE=0.20) and nearly mesokurtic (Kurtosis $=-0.42, S E=0.69$, Kurtosis/SE $=0.61$ ) indicating normality of distribution. However, in the experimental group, gains score of self-efficacy in mathematics is symmetric (Skewness $=0.33, S E=0.35$, Skewness $/ \mathrm{SE}=0.94$ ) but leptokurtic $($ Kurtosis $=1.66$, $S E=0.69$, Kurtosis $/ \mathrm{SE}=2.41$ ) indicating deviation from normality of the distribution. Likewise, distributions of gain score of self-efficacy in mathematics are normal in the control group ( $S-W=.97, d=45, p>.05$ ) but deviated from
normality in the experimental group $(S-W=.94, d f=45, p<.01)$ and variances of the two groups were not homogeneous $[F(1,88)=4.11, \mathrm{p}<.01]$.

Difference in distribution of gain scores of self-efficacy in mathematics of control and experimental groups is studied using histograms as well as box plots and the linearity of the points in normal Q-Q plots further confirmed that gain score of self-efficacy in mathematics in control group is normal; but this is not true in the case of experimental group (Appendix W). Hence, the difference in gain score of self-efficacy in mathematics between the experimental and control groups was tested using independent samples $t$-test. Result is given in Table 40.

Table 40
Test of Significance of Difference between Mean Gain Scores of Self-efficacy in Mathematics after Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems (Control)

| Groups | $M$ | $S D$ | $t$ | Cohen's d |
| :--- | :---: | :---: | :---: | :---: |
| Practice in solving mathematics problems | 9.57 | 5.11 |  |  |
| Language integrated mathematics Instruction | 16.05 | 4.05 | $6.67^{* *}$ | 1.41 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$; ${ }^{* *} p<.01$
Table 40 shows that mean gain score of self-efficacy in mathematics after language integrated mathematics instruction $(M=16.05, S D=4.05)$ is significantly higher than that after practice in solving mathematics problems ( $M=9.57, S D=$ 5.11) $[t=6.67 ; p<.01]$. Effect size (Cohen's $d=1.41$ ) indicates that, gain in selfefficacy in mathematics after language integrated mathematics instruction is large, compared to that after practice in solving mathematics problems. As per Coe (2002), Cohen's $d=1.4$ means that 92 percent of students in language integrated mathematics instruction group are above the average student in the group which practiced solving mathematics problems and this is further demonstrated in Figure 14 showing smoothed frequency curves of the distributions of gain scores of selfefficacy in mathematics of experimental and control groups.


Figure 14. Ogives of the gain scores of self-efficacy in mathematics after language integrated mathematics instruction and practice in solving mathematics problems.

Figure 14 shows that effect of language integrated mathematics instruction on self-efficacy in mathematics is highest in the lower quartile with an advantage of approximately 45 percentile ranks over and above that after practice in solving problems, an advantage of approximately 40 percentile ranks at the median, and an advantage of approximately 20 percentile ranks at the third quartile, indicating that language integrated mathematics instruction is especially beneficial for students in the lower self-efficacy strata though others are also hugely benefited.

## Effect of language integrated mathematics instruction on dimensions of self-efficacy in mathematics

The effect of language integrated mathematics instruction among elementary school students on two dimensions of their self-efficacy in
mathematics viz., self-efficacy in learning mathematics and self-efficacy in solving mathematics problems were further examined. The results are presented under distinct headings.

## Effect of Language Integrated Mathematics Instruction on Self-

## efficacy in Learning Mathematics.

Before intervention, in the control group, self-efficacy in learning mathematics was slightly negatively skewed (Skewness $=-0.72, S E=0.35$, Skewness/SE=2.05) and leptokurtic (Kurtosis=-0.88,SE=0.69, Kurtosis/SE=2.51) indicating small deviation from normality. In the experimental group, self-efficacy in learning mathematics was slightly negatively skewed (Skewness=-0.89, $S E=0.35$, Skewness $/ \mathrm{SE}=2.54$ ) and leptokurtic $($ Kurtosis $=1.36, S E=0.69$, kurtosis $/ S E=1.97$ ) indicating small deviation from normality.

Further examination revealed that, distributions of self-efficacy in learning mathematics before intervention was normal in the control $(S-W=.97, d f=45$, $p>.05$ ) and experimental groups ( $S-W=.95, d f=45, p>.05$ ) and their variances were homogeneous $[F(1,88)=0.00, p>.05]$. Both the groups have almost identical histograms as well as box plots on self-efficacy in learning mathematics before intervention and the linearity of the points in normal Q-Q plots further confirmed that self-efficacy in learning mathematics before interventions in both the groups are normally distributed (Appendix X). Hence, the match in selfefficacy in learning mathematics between the control and experimental groups before intervention was tested using independent samples $t$ test. And it was found that self-efficacy in learning mathematics did not differ significantly between the control $\left(M=67.18, S D=13.11, N_{l}=45\right)$ and experimental $(M=70.15, S D=$ 13.08, $N_{2}=45$ ) groups before intervention $[t=1.08 ; p>.05]$.

Thus, before the intervention, the distribution of self-efficacy in learning mathematics in the control and experimental group are slightly negatively skewed and leptokurtic, normal, homogenous, and did not differ significantly one another.

After intervention, distribution of gain scores of self-efficacy in learning mathematics of control and experimental groups were studied. Distribution of posttest scores of self-efficacy in learning mathematics is provided in Appendix V. Distribution of gain scores of self-efficacy in learning mathematics in the control group was positively skewed (Skewness $=0.82, S E=0.35$, Skewness $/ \mathrm{SE}=2.34$ ) and leptokurtic (Kurtosis $=1.32, S E=0.69$, Kurtosis/SE $=1.91$ ) indicating deviation from normality. In the experimental group, distribution of gain scores of self-efficacy in learning mathematics is symmetric (Skewness $=0.49, S E=0.35$, Skewness/SE = 1.4) and nearly mesokurtic (Kurtosis $=-0.37, S E=0.69$, Kurtosis/SE = 1.01) indicating normality of distribution. Distributions of gain score of self-efficacy in learning mathematics is deviated from normality in the control group ( $S-W=.94, d f=45, p<.01$ ) but normal for the experimental group $(S-W=$ $.96, d f=45, p>.05$ ) and the variances of the two groups are homogeneous $[F(1,88)=0.53, p>.05]$.

Difference in distribution of gain score of self-efficacy in learning mathematics of control and experimental groups are studied using histograms as well as box plots, and the non-linearity of the points in normal Q-Q plots further confirmed that gain scores of self-efficacy in learning mathematics in control group is deviated from normality; but this is not true in the case of experimental group (Appendix Y). Hence, the difference in gain scores of self-efficacy in learning mathematics between the experimental and control groups was tested using Mann-Whitney $U$ test. Result is given in Table 41.

Table 41
Mann-Whitney Test of Significance of Difference between Median Gain Scores of Selfefficacy in Learning Mathematics after Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems (Control)

| Groups | Median | Range | Mean <br> Rank | Sum of <br> Ranks | Mann- <br> Whitney U | Cohen's <br> $d$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Practice in solving <br> mathematics problems | 9.23 | $32(0-32)$ | 33.70 | 1516.50 |  |  |
| Language integrated <br> mathematics Instruction | 16.92 | $28(8-35)$ | 57.30 | 2578.50 |  | $481.50^{* *}$ |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45 ;{ }^{* *} p<.01$

Table 41 shows that mean gain score of self-efficacy in learning mathematics after language integrated mathematics instruction $(M d n=16.92)$ is significantly higher than that after practice in solving mathematics problems (Mdn = 9.23), [ $U=481.50 ; p<.01]$. Effect size (Cohen's $d=1.01$ ) indicates that, gain score of self-efficacy in learning mathematics after language integrated mathematics instruction is large, compared to that after practice in solving mathematics problems. As per Coe (2002), Cohen's $d=1.0$ means that 84 percent of students in language integrated mathematics instruction group are above the average student in the group which practiced solving mathematics problems and this is further demonstrated in Figure 15 showing smoothed frequency curves of the distributions of gain scores of self-efficacy in learning mathematics of experimental and control groups.


Figure 15. Ogives of the gain scores of self-efficacy in learning mathematics after language integrated mathematics instruction and practice in solving mathematics problems.

Figure 15 shows that effect of language integrated mathematics instruction on self-efficacy in learning mathematics is the highest in the lower quartile with an advantage of approximately 35 percentile ranks over and above that after practice in solving problems, while there is an advantage of approximately 30 percentile ranks at the median, and an advantage of approximately 20 percentile ranks at the third quartile, indicating that language integrated mathematics instruction is especially beneficial for students in the lower self-efficacy strata though others are also hugely benefited.

## Effect of Language Integrated Mathematics Instruction on Self-

## efficacy in Solving Mathematics Problems.

Before intervention, self-efficacy in solving mathematics problems in the control group was symmetric (Skewness $=-0.22, S E=0.35$, Skewness/SE $=$ 0.63 and nearly mesokurtic Kurtosis $=-0.69, S E=0.69$, Kurtosis/SE = 1) indicating normality of distribution; in the experimental group also, it was
symmetric (Skewness $=-0.59, S E=0.35$, Skewness/SE $=1.68$ ) and nearly mesokurtic (Kurtosis $=0.21, S E=0.69$, Kurtosis $/ \mathrm{SE}=0.30$ ) indicating normality of distribution.

Distributions of self-efficacy in solving mathematics problems before intervention were normal in control $(S-W=.97, d f=45, p>.05)$ and experimental groups ( $S-W=.97, d f=45, p>.05$ ) and the variances of the two groups were homogeneous $[F(1,88)=2.64, p>.05]$. The groups had almost identical histograms as well as box plots and the linearity of the points in normal Q-Q plots further confirmed that self-efficacy in solving mathematics problems before interventions in both the groups were normally distributed (Appendix Z). Hence, comparability of self-efficacy in solving mathematics problems in the experimental and control groups before interventions was tested using independent samples $t$ test. It was found that self-efficacy in solving mathematics problems, did not differ significantly between the control ( $M=$ 65.49, $\left.S D=11.36, N_{l}=45\right)$ and experimental $\left(M=68.53, S D=14.27, N_{2}=45\right)$ groups before intervention $[t=1.12 ; p>.05]$.

Thus, before the intervention, the distribution of self-efficacy in solving mathematics problems in the control and experimental group are symmetric and nearly mesokurtic, normal, homogenous, and did not differ significantly one another.

After intervention, distribution of gain scores of self-efficacy in solving mathematics problems of control and experimental group is studied. Distribution posttest scores of self-efficacy in solving problems of control and experimental groups is provided in Appendix V. Distribution of gain scores of self-efficacy in
solving mathematics problems in the control group is symmetric (Skewness $=$ $0.31, S E=0.35$, Skewness $/ \mathrm{SE}=0.89$ ) and nearly mesokurtic (Kurtosis $=0.31$, $S E=0.69$, Kurtosis $/ \mathrm{SE}=0.45$ ) indicating normality. In experimental group also, distribution of gain scores of self-efficacy in solving mathematics problems is symmetric (Skewness $=0.62, S E=0.35$, Skewness $/ \mathrm{SE}=1.77$ ) and nearly mesokurtic (Kurtosis $=0.69, S E=0.69$, Kurtosis/SE $=1$ ) indicating normality of distribution. Distributions of gain scores of self-efficacy in solving mathematics problems are normal in both control $(S-W=.95, d f=45, p>.05)$ and experimental groups ( $S-W=.96, d f=45, p>.05$ ) and the variances of the two groups were homogeneous $[F(1,88)=0.67, p>.05]$.

Difference in distribution of gain scores of self-efficacy in solving mathematics problems in control and experimental groups are studied using histograms as well as box plots, and the linearity of the points in normal Q-Q plots further confirmed that gain in self-efficacy in solving mathematics problems in both the control and experimental groups are normal (Appendix AA). Hence, the difference in gain in self-efficacy in solving mathematics problems between the experimental and control groups was tested using independent samples $t$ test. Result is given in Table 42.

## Table 42

Test of Significance of Difference between Mean Gain Scores of Self-efficacy in Solving Mathematics Problems after Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems (Control)

| Groups | $M$ | $S D$ | $t$ | Cohen's $d$ |
| :--- | :---: | :---: | :---: | :---: |
| Practice in solving mathematics problems | 13.01 | 5.71 |  |  |
| Language integrated mathematics Instruction | 19.64 | 6.75 | $5.03^{* *}$ | 1.06 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45 ;{ }^{* *} p<.01$

Table 42 shows that mean gain score of self-efficacy in solving mathematics problems after language integrated mathematics instruction ( $M=$ $13.01, S D=5.71$ ) is higher than that after practice in solving mathematics problems ( $M=19.64, S D=6.75$ ) $[t=5.03 ; p<.01]$. Effect size (Cohen's $d=$ 1.06) indicates that, gain in self-efficacy in solving mathematics problems after language integrated mathematics instruction is large, compared to that after practice in solving mathematics problems. As per Coe (2002), Cohen's $d=1.0$ means that 84 percent of students in language integrated mathematics instruction group are above the average student in the group which practiced solving mathematics problems and this is further demonstrated in Figure 16 showing smoothed frequency curves of the distributions of gain scores of self-efficacy in solving mathematics problems of experimental and control groups.


Figure 16. Ogives of the gain scores of self-efficacy in solving mathematics problems after language integrated mathematics instruction and practice in solving mathematics problems.

Figure 16 shows that effect of language integrated mathematics instruction on self-efficacy in solving mathematics problems is the highest in the lower quartile with an advantage of approximately 40 percentile ranks over and above that after practice in solving problems, an advantage of approximately 34 percentile ranks at the median, and an advantage of approximately 15 percentile ranks at the third quartile, indicating that language integrated mathematics instruction is especially beneficial for students in the lower self-efficacy strata though others are also highly benefited.

## Effect of language integrated mathematics instruction on area-wise self-efficacy in mathematics

The effect of language integrated mathematics instruction among elementary school students on self-efficacies in three areas of mathematics viz., self-efficacies in algebra, arithmetic and geometry were also examined. For this posttest scores of these self-efficacies in experimental and control groups were compared using independent samples $t$ tests. The results are presented under three headings.

## 1. Effect of Language Integrated Mathematics Instruction on Selfefficacy in Algebra.

Distribution of posttest scores of self-efficacy in algebra of control and experimental groups were studied. Distribution of self-efficacy in algebra after intervention in the control group is symmetric (Skewness $=-0.33, S E=0.35$, Skewness/SE=0.94) and nearly mesokurtic (Kurtosis $=0.19, S E=0.69$, kurtosis/ $S E=0.28$ ) indicating normality. In the experimental group also, distribution of self-efficacy in algebra after intervention is symmetric (Skewness= -0.27,
$S E=0.35$, Skewness $/ \mathrm{SE}=0.77$ ) and nearly mesokurtic (Kurtosis $=-0.46, S E=$ 0.69 , Kurtosis/SE $=0.67$ ) indicating normality. Hence, distributions of selfefficacy in algebra after intervention are normal in both control $(S-W=.99, d f=45$, $p>.05$ ) and experimental ( $S-W=.96, d f=45, p>.05$ ) groups and the variances of the two groups are homogeneous $[F(1,88)=0.11, p>.05]$.

Difference in distribution of posttest scores of self-efficacy in algebra of control and experimental groups is studied using histograms as well as box plots, and the linearity of the points in normal Q-Q plots further confirmed that posttest scores of self-efficacy in algebra in both the control and experimental groups are normal (Appendix AB). Hence, the difference in posttest scores of self-efficacy in algebra between the experimental and control groups was tested using independent samples $t$ test. Result is given in Table 43.

Table 43
Test of Significance of Difference between Mean Posttest Scores of Self-efficacy in Algebra after Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems (Control)

| Groups | $M$ | $S D$ | $t$ | Cohen's $d$ |
| :--- | :---: | :---: | :---: | :---: |
| Practice in solving mathematics problems | 70.51 | 11.12 |  |  |
| Language integrated mathematics Instruction | 79.30 | 10.24 | $3.90^{* *}$ | 0.82 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45 ;{ }^{* *} p<.01$
Table 43 shows that mean posttest score of self-efficacy in algebra after language integrated mathematics instruction $(M=79.30, S D=10.24)$ is significantly higher than that after practice in solving mathematics problems $(M=70.51, S D=11.12)[t=3.90 ; p<.01]$. Effect size (Cohen's $d=.82)$ indicates that, increase in posttest scores of self-efficacy in algebra after language integrated mathematics instruction is large, compared to that after practice in solving mathematics problems. As per Coe (2002), Cohen's $d=0.8$ means that

79 percent of students in language integrated mathematics instruction group are above the average student in the group which practiced solving mathematics problems and this is further demonstrated in Figure 17 showing smoothed frequency curves of the distributions of posttest scores of self-efficacy in algebra of experimental and control groups.


Figure 17. Ogives of the scores of self-efficacy in algebra after language integrated mathematics instruction and practice in solving mathematics problems.

Figure 17 shows that effect of language integrated mathematics instruction on self-efficacy in algebra is uniform across the distribution with an advantage of around 25 percentile ranks over and above that after practice in solving problems.

## 2. Effect of Language Integrated Mathematics Instruction on Selfefficacy in Arithmetic.

Distribution of posttest scores of self-efficacy in arithmetic of control and experimental groups were studied. Distribution of posttest scores of self-efficacy in arithmetic in the control group is symmetric (Skewness $=-0.09, S E=0.35$,

Skewness/SE $=0.26$ ) and nearly mesokurtic (Kurtosis $=-1.07, S E=0.69$, kurtosis $/ S E=1.55$ ) $\quad$ indicating normality of distribution. In the experimental group also, distribution of self-efficacy in arithmetic after intervention is symmetric (Skewness $=-0.38, S E=0.35$, Skewness/SE $=1.09$ ) and nearly mesokurtic (Kurtosis $=-1.08, S E=0.69$, Kurtosis/SE $=1.57$ ) indicating normality of distribution. However, distributions of self-efficacy in arithmetic after intervention deviated from normality in both control $(S-W=.95, d f=45$, $p<.01$ ) and experimental groups $(S-W=.92, d f=45, p<.01)$ and the variances of two groups are homogeneous $[F(1,88)=1.38, p>.05]$.

Difference in distribution of posttest scores of self-efficacy in arithmetic of control and experimental groups are studied using histograms as well as box plots and the non-linearity of the points in normal Q-Q plots further confirmed that posttest scores of self-efficacy in arithmetic both in control and experimental groups deviate from normality (Appendix AC). Hence, the difference in posttest scores of self-efficacy in arithmetic between the experimental and control groups was tested using Mann-Whitney $U$ test. Result is given in Table 44.

Table 44
Mann-Whitney Test of Significance of Difference between Medians of Posttest Scores of Self-efficacy in Arithmetic after Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems (Control)

| Groups | Median | Range | Mean <br> Rank | Sum of <br> Ranks | Mann- <br> Whitney U | Cohen's <br> $d$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Practice in solving <br> mathematics problems | 74.67 | $46(47-93)$ | 39.30 | 1768.50 |  |  |
| Language integrated <br> mathematics Instruction | 80.00 | $38(56-94)$ | 51.70 | 2326.50 |  | 0.49 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45 ;{ }^{*} p<.05$
Table 44 shows that median of posttest scores of self-efficacy in arithmetic after language integrated mathematics instruction ( $M d n=80$ ) is significantly higher
than that after practice in solving mathematics problems ( $M d n=74.67$ ), $[U=$ 733.50; $p<.01$ ]. Effect size (Cohen's $d=0.49$ ) indicates that, increase in posttest scores of self-efficacy in arithmetic after language integrated mathematics instruction is small, compared to that after practice in solving mathematics problems. As per Coe (2002), Cohen's $d=0.4$ means that 66 percent of students in language integrated mathematics instruction group are above the average student in the group which practiced solving mathematics problems and this is further demonstrated in Figure 18 showing smoothed frequency curves of the distributions of posttest scores of self-efficacy in arithmetic of experimental and control groups.


Figure 18. Ogives of the scores of self-efficacy in arithmetic after language integrated mathematics instruction and practice in solving mathematics problems.

Figure 18 shows that effect of language integrated mathematics instruction on self-efficacy in arithmetic is uniform below the upper quartile with an advantage of approximately 16 percentile ranks over and above that after practice in solving problems, with only negligible advantage for students above the upper quartile.

## 3. Effect of Language Integrated Mathematics Instruction on Selfefficacy in Geometry.

Distribution of posttest scores of self-efficacy in geometry of control and experimental groups were studied. Distribution of posttest scores of self-efficacy in geometry in the control group is symmetric (Skewness $=-0.20, S E=0.35$, Skewness/SE $=0.57$ ) and nearly mesokurtic (Kurtosis $=-1.12, S E=0.69$, Kurtosis/SE $=1.62$ ) indicating normality of distribution. In the experimental group, distribution of self-efficacy in geometry after intervention is negatively skewed (Skewness $=-0.87, S E=0.35$, Skewness/SE $=2.49$ ) and nearly mesokurtic (Kurtosis $=-0.06, S E=0.69$, Kurtosis $/ \mathrm{SE}=0.09$ ) indicating normality of distribution. However, distributions of self-efficacy in geometry after intervention deviate from normality in both control $(S-W=.95, d f=45$, $p<.01$ ) and experimental ( $S-W=.90, d f=45, p<.01$ ) groups and the variances of two groups are homogeneous $[F(1,88)=2.03, p>.05]$.

Difference in distribution of posttest scores of self-efficacy in geometry of control and experimental groups are studied using histograms as well as box plots and the non-linearity of the points in normal Q-Q plots further confirmed that posttest scores of self-efficacy in geometry in both control and experimental groups deviated from normality (Appendix AD). Hence, the difference in posttest scores of self-efficacy in algebra between the experimental and control groups were tested using Mann-Whitney $U$ test. Result is given in Table 45.

Table 45

Mann-Whitney Test of Significance of Difference between Medians of Posttest Scores of Self-efficacy in Geometry after Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems (Control)

| Groups | Median | Range | Mean <br> Rank | Sum of <br> Ranks | Mann- <br> Whitney U | Cohen's <br> $d$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Practice in solving <br> mathematics <br> problems | 71.54 | $44(47-91)$ | 34.33 | 1545 |  |  |
| Language integrated <br> mathematics <br> Instruction | 86.15 | $38(58-95)$ | 56.67 | 2550 |  | 0.95 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45 ;{ }^{* *} p<.01$

Table 45 shows that median of posttest score of self-efficacy in geometry after language integrated mathematics instruction $(M d n=86.15)$ is significantly higher than that after practice in solving Mathematics problems $(M d n=71.54),[U=510 ; p<.01]$. Effect size (Cohen's $d=0.95$ ) indicates that, posttest scores of self-efficacy in geometry after language integrated mathematics instruction is large, compared to that after practice in solving mathematics problems. As per Coe (2002), Cohen's $d=0.9$ means that 82 percent of students in language integrated mathematics instruction group are above the average student in the group which practiced solving mathematics problems and this is further demonstrated in Figure 19 showing smoothed frequency curves of the distributions of posttest scores of self-efficacy in geometry of experimental and control groups.


Figure 19. Ogives of the scores of self-efficacy in geometry after language integrated mathematics instruction and practice in solving mathematics problems.

Figure 19 shows that effect of language integrated mathematics instruction on self-efficacy in geometry is uniform up to the upper quartile with an advantage of approximately 32 percentile ranks over and above that after practice in solving problems; and is still high but only approximately an advantage of 23 percentile ranks for students above the upper quartile.

## Effects of Language Integrated Mathematics Instruction on Attitude towards Mathematics

Before intervention, attitude towards mathematics in the control group was symmetric (Skewness $=-0.53, S E=0.35$, Skewness/SE $=1.51$ ) and nearly mesokurtic (Kurtosis $=-0.40, S E=0.69$, Kurtosis $/ \mathrm{SE}=0.58$ ) indicating normality of distribution. In the experimental group also, attitude towards mathematics, was symmetric (Skewness $=-0.24, S E=0.35$, Skewness $/ \mathrm{SE}=$ 0.68 ) and nearly mesokurtic (Kurtosis $=-0.93, S E=0.69$, Kurtosis/SE $=1.35$ ) indicating normality of distribution.

Distributions of attitude towards mathematics before intervention were normal in the control $(S-W=.96, d f=45, p>.05)$ and experimental groups $(S-W=$ $.95, d f=45, p>.05)$ and their variances were homogeneous $[F(1,88)=0.00$, $p>.05]$. The groups had almost identical histograms as well as box plots and the linearity of the points in normal Q-Q plots further confirmed that attitude towards mathematics before interventions in both the groups were normally distributed (Appendix AE). Hence, comparability of attitude towards mathematics in the experimental and control groups was tested using independent samples $t$ test. It was found that attitude towards mathematics did not differ significantly between the control $\left(M=67.62, S D=9.73, N_{l}=45\right)$ and experimental $(M=65.90, S D=$ 9.37, $N_{2}=45$ ) groups before intervention [ $\left.t=0.85 ; p>.05\right]$.

Thus, before the intervention, the distribution of attitude towards mathematics in the control and experimental group were symmetric and mesokurtic, normal, homogenous, and did not differ significantly one another.

After the intervention, distribution of gain scores of attitude towards mathematics of control and experimental groups were studied. Distribution of posttest scores of attitude towards mathematics of control and experimental groups is provided in Appendix V. Gain scores of attitude towards mathematics in the control group is symmetric (Skewness $=0.42, S E=0.35$, Skewness $/ \mathrm{SE}=$ 1.2 ) and nearly mesokurtic (Kurtosis $=-0.20, S E=0.69$, Kurtosis $/ \mathrm{SE}=0.29$ ) indicating normality of distribution. However, in the experimental group, distribution of gain scores of attitude towards mathematics is positively skewed $($ Skewness $=0.96, S E=0.35$, Skewness $/ S E=2.74)$ and leptokurtic (Kurtosis $=-$ 1.23, $S E=0.69$, Kurtosis/SE = 1.78). Accordingly, distributions of gain scores of attitude towards mathematics are normal in the control group $(S-W=.97, d f=$ $45, p>.05$ ) but deviated from normality in the experimental group $(S-W=.93, d f$
$=45, p<.01)$ and the variances of the two groups are not homogeneous, after intervention $[F(1,88)=13.84, p<.01]$.

Difference in distribution of gain scores of attitude towards mathematics of control and experimental groups are studied using histograms as well as box plots and the linearity of the points in normal Q-Q plots further confirmed that gain scores of attitude towards mathematics in control group is normal; but this is not true in the case of experimental group (Appendix AF). Hence, the difference in gain scores of attitude towards mathematics between the experimental and control groups was tested using independent samples $t$ test. Result is given in Table 46.

Table 46
Test of Significance of Difference between Mean Gain Scores of Attitude towards Mathematics after Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems (Control)

| Groups | $M$ | $S D$ | $t$ | Cohen's $d$ |
| :--- | :---: | :---: | :---: | :---: |
| Practice in solving mathematics problems | 2.94 | 1.56 |  |  |
| Language integrated mathematics Instruction | 12.34 | 3.38 |  |  |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45 ;{ }^{* *} p<.01$
Table 46 shows that mean gain score of attitude towards mathematics after language integrated mathematics instruction $(M=12.34, S D=3.38)$ is significantly higher than that after practice in solving mathematics problems ( $M=2.94, S D=$ 1.56) $[t=16.96 ; p<.01]$. Effect size (Cohen's $d=3.57$ ) indicates that, gain in attitude towards Mathematics after language integrated mathematics instruction is large, compared to that after practice in solving mathematics problems. As per Coe (2002), Cohen's $d>3$ means that 99.9 percent of students in language integrated mathematics instruction group are above the average student in the group which practiced solving mathematics problems and this is further demonstrated in Figure 20 showing smoothed frequency curves of the distributions of gain scores of attitude towards mathematics of experimental and control groups.


Figure 20. Ogives of the gain scores of attitude towards mathematics after language integrated mathematics instruction and practice in solving mathematics problems.

Figure 20 shows that only 2 percent of the control and experimental groups have similar gain in attitude towards mathematics; i.e., 98 percent of students who received language integrated mathematics instruction had higher gain in attitude towards mathematics than those who had practiced mathematics problem solving in meantime.

## Effect of language integrated mathematics instruction on dimensions of attitude towards mathematics

The effect of language integrated mathematics instruction among elementary school students on five dimensions of their attitude towards mathematics viz., like towards mathematics, engagement with mathematics, self-belief in mathematics, active learning of mathematics and enjoyment of mathematics were further examined. The results are presented under distinct headings.

## 1. Effect of Language Integrated Mathematics Instruction on Like

 towards Mathematics.Before intervention, like towards mathematics in the control group was symmetric (Skewness $=-0.08, S E=0.35$, Skewness $/ \mathrm{SE}=0.23$ ) and nearly mesokurtic (Kurtosis $=-0.60, S E=0.69$, Kurtosis/SE $=0.87$ ) indicating normality of distribution. In the experimental group also, like towards mathematics was symmetric (Skewness $=-0.24, S E=0.35$, Skewness $/ \mathrm{SE}=$ 0.69 ) and nearly mesokurtic (Kurtosis $=-0.83, S E=0.69$, Kurtosis $/ \mathrm{SE}=0.97$ ) indicating normality of distribution.

Distributions of like towards mathematics before intervention is normal in the control $(S-W=.97, \mathrm{df}=45, \mathrm{p}>.05)$ and experimental $(S-W=.95, \mathrm{df}=45, \mathrm{p}>.05)$ groups and their variances were homogeneous $[F(1,88)=0.46, p>.05]$. The groups had almost identical histograms as well as box plots and the linearity of the points in normal $\mathrm{Q}-\mathrm{Q}$ plots further confirmed that like towards mathematics before interventions in both the groups were normally distributed (Appendix AG). Hence, comparability of like towards mathematics of control and experimental groups before intervention was tested using independent samples $t$ test. It was found that like towards mathematics did not differ significantly between the control ( $M=$ 62.51, $\left.S D=14.26, N_{l}=45\right)$ and experimental $\left(M=61.86, S D=14.79, N_{2}=45\right)$ groups before intervention $[t=0.21 ; p>.05]$.

Thus, before the intervention, the distributions of like towards mathematics in the control and experimental group are symmetric and nearly mesokurtic, normal, homogenous, and did not differ significantly one another.

After the intervention, distribution of gain scores of like towards mathematics of control and experimental groups were studied. Distribution of
posttest scores of like towards mathematics of control and experimental group is provided in Appendix V. Distribution of gain scores of like towards mathematics in the control group is negatively skewed (Skewness $=-0.67, S E$ $=0.35$, Skewness $/ \mathrm{SE}=1.91$ ) and nearly mesokurtic (Kurtosis $=-0.63, S E=$ 0.69, Kurtosis $/ \mathrm{SE}=0.91$ ) indicating normality of distribution. However, in the experimental group, distribution of gain scores of like towards Mathematics is positively skewed (Skewness $=0.89, S E=0.35$, Skewness $/ \mathrm{SE}=2.54$ ) and nearly mesokurtic (Kurtosis $=-0.66, S E=0.69$, Kurtosis $/ \mathrm{SE}=0.96$ ) indicating normality of distribution. However, distributions of gain scores of like towards mathematics deviate from normality for both control $(S-W=.92$, $d f=45, p<.01)$ and experimental $(S-W=.93, d f=45, p<.01)$ groups and the variances of the two groups are not homogeneous $[F(1,88)=33.46, p<.01]$.

Difference in distribution of gain scores of like towards mathematics of control and experimental groups are studied using histograms as well as box plots, and the non-linearity of the points in normal Q-Q plots further confirmed that gain score of attitude towards mathematics in both control and experimental groups deviate from normality (Appendix AH). Hence, the difference in gain scores of like towards mathematics between the experimental and control groups was tested using Mann-Whitney $U$ test. Result is given in Table 47.

Table 47
Mann-Whitney Test of Significance of Difference between Medians of Gain Scores of Like towards Mathematics after Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems (Control)

| Groups | Median | Range | Mean <br> Rank | Sum of <br> Ranks | Mann- <br> Whitney U | Cohen's <br> $d$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Practice in solving <br> mathematics problems | 9.09 | $16(0-16)$ | 29.16 | 1312.00 |  |  |
| Language integrated <br> mathematics Instruction | 18.18 | $47(4-51)$ | 61.84 | 2783.00 | $277.00^{* *}$ | 1.60 |

Table 47 shows that, gain scores of like towards Mathematics after language integrated mathematics instruction $(M d n=18.18)$ is significantly higher than that after practice in solving mathematics problems ( $M d n=9.09$ ), [ $U=277 ; \mathrm{p}<.01$ ]. Effect size (Cohen's $d=1.60$ ) indicates that, gain in like towards mathematics after language integrated mathematics instruction is large, compared to that after practice in solving mathematics problems. As per Coe (2002), Cohen's $d=1.6$ means that 95 percent of students in language integrated mathematics instruction group are above the average student in the group which practiced solving mathematics problems and this is further demonstrated in Figure 21 showing smoothed frequency curves of the distributions of gain scores of like towards mathematics of experimental and control groups.


Figure 21. Ogives of the gain scores of like towards mathematics after language integrated mathematics instruction and practice in solving mathematics problems.

Figure 21 shows that after language integrated mathematics instruction, students have an advantage of approximately 50 percentile ranks in like towards mathematics than that after practice in solving mathematics problems.

## 2. Effect of Language Integrated Mathematics Instruction on <br> Engagement with Mathematics.

Before intervention, engagement with mathematics in the control group is symmetric (Skewness $=-0.26, S E=0.35$, Skewness $/ \mathrm{SE}=0.74$ ) and nearly mesokurtic (Kurtosis $=-0.39, S E=0.69$, Kurtosis $/ \mathrm{SE}=0.57$ ) indicating normality of distribution. In the experimental group also, engagement with mathematics is symmetric (Skewness $=-0.53, S E=0.35$, Skewness $/ \mathrm{SE}=1.51$ ) and nearly mesokurtic (Kurtosis $=-0.19, S E=0.69$, Kurtosis $/ \mathrm{SE}=0.28$ ) indicating normality of distribution.

Distributions of pretest scores of engagement with mathematics are normal in the control $(S-W=.98, d f=45, p>.05)$ and experimental $(S-W=.96$, $d f=45, p>.05)$ groups and their variances are homogeneous $[F(1,88)=2.82$, $p>.05]$. The groups have almost identical histograms as well as box plots on engagement with mathematics and the linearity of the points in normal Q-Q plots further confirmed that engagement with mathematics before interventions in both the groups are normally distributed (Appendix AI). Hence, the match in engagement with mathematics before intervention between the control and experimental groups was tested using independent samples $t$ test and found that engagement with mathematics did not differ significantly between the control $\left(M=65.58, S D=16.61, N_{l}=45\right)$ and experimental $\left(M=64.89, S D=13.93, N_{2}=\right.$ 45) groups before intervention $[t=0.21 ; p>.05]$.

Thus, before the intervention, the distribution of engagement with mathematics in the control and experimental groups were symmetric and nearly mesokurtic, normal, homogenous, and did not differ significantly one another.

After the intervention, distribution of gain scores of engagement with mathematics of control and experimental groups were studied. Distribution of posttest scores of engagement with mathematics of control and experimental groups is provided in Appendix V. Distribution of gain scores of engagement with mathematics in the control group is positively skewed (Skewness $=0.96, S E=0.35$, Skewness/ $\mathrm{SE}=2.74$ ) and leptokurtic (Kurtosis $=1.26, \mathrm{SE}=0.69, \mathrm{Kurtosis} / \mathrm{SE}=$ 1.83) indicating deviation from normality. However, in the experimental group, engagement with mathematics is positively skewed (Skewness $=0.79, S E=0.35$, Skewness/SE $=2.26$ ) and nearly mesokurtic (Kurtosis $=0.11, S E=0.69$, Kurtosis/SE $=0.16$ ) indicating deviation from normality. Consequently, distributions of gain scores of engagement with mathematics deviate from normality for both control $(S-W=.89, d f=45, p<.01)$ and experimental $(S-W=.93, d f=45$, $p<.01)$ groups and their variances are not homogeneous $[F(1,88)=20.95, p<.01]$.

Difference in distribution of gain scores of engagement with mathematics of control and experimental groups are studied using histograms as well as box plots and the non-linearity of the points in normal Q-Q plots further confirmed that gain score of engagement with mathematics in control and experimental groups deviate from normality (Appendix AJ). Hence, the difference in gain score of engagement with mathematics between the experimental and control groups was tested using Mann-Whitney $U$ test. Result is given in Table 48.

Table 48
Mann-Whitney Test of Significance of Difference between Medians of Gain Scores of Engagement with Mathematics after Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems (Control)

| Groups | Median | Range | Mean <br> Rank | Sum of <br> Ranks | Mann- <br> Whitney U | Cohen's <br> $d$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Practice in solving <br> mathematics problems | 8.89 | $22(0-22)$ | 33.11 | 1490.00 |  |  |
| Language integrated <br> mathematics Instruction | 15.56 | $40(4-44)$ | 57.89 | 2605.00 | $455^{* *}$ | 1.08 |

[^1]Table 48 shows that, median of gain scores of engagement with mathematics after language integrated instruction $(M d n=15.56)$ is significantly higher than that after practicing solving mathematics problems ( $M d n=8.89$ ), [ $U=455 ; p<.01$ ]. Effect size (Cohen's $d=1.08$ ) indicates that gain in engagement with mathematics after language integrated mathematics instruction is large, compared to that after practice in solving mathematics problems. As per Coe (2002), Cohen's $d=1.0$ means that 84 percent of students in language integrated mathematics instruction group are above the average student in the group which practiced solving mathematics problems and this is further demonstrated in Figure 22 showing smoothed frequency curves of the distributions of gain scores of engagement with mathematics of experimental and control groups.


Figure 22. Ogives of the gain scores of engagement with mathematics after language integrated mathematics instruction and practice in solving mathematics problems.

Figure 22 shows that effect of language integrated mathematics instruction on engagement with mathematics is uniform up to the upper quartile with an advantage of approximately 40 percentile ranks over and above that after
practice in solving problems; and is high yet only approximately an advantage of 21 percentile ranks for students above the upper quartile.

## 3. Effect of Language Integrated Mathematics Instruction on Selfbelief in Mathematics.

Before intervention, self-belief in mathematics in the control group was symmetric (Skewness $=-0.34, S E=0.35$, Skewness $/ S E=0.97$ ) and nearly mesokurtic (Kurtosis $=-0.62, S E=0.69$, Kurtosis/SE $=0.89$ ) indicating normality of distribution. In the experimental group also, self-belief in mathematics is symmetric (Skewness $=-0.24, S E=0.35$, Skewness $/ \mathrm{SE}=0.69$ ) and nearly mesokurtic (Kurtosis $=-0.45, S E=0.69$, Kurtosis $/ \mathrm{SE}=0.65$ ) indicating normality of distribution.

Distributions of self-belief in mathematics before intervention were normal in the control $(S-W=.97, d f=45, p>.05)$ and experimental $(S-W=.97, d f=45, p>.05)$ groups and their variances were homogeneous $[F(1,88)=1.58, p>.05]$. The groups have almost identical histograms as well as box plots on self-belief in mathematics and the linearity of the points in normal Q-Q plots further confirmed that self-belief in mathematics before interventions in both the groups were normally distributed (Appendix AK). Hence, the match in self-belief in mathematics before intervention between the control and experimental groups was tested using independent samples $t$ test. It was found that self-belief in mathematics did not differ significantly between the control ( $M=63.62, S D=15.42, N_{l}=45$ ) and experimental $(M=59.62$, $S D=13.30, N_{2}=45$ ) groups before intervention $[t=1.32 ; p>.05]$.

Thus, before the interventions, the distribution of self-belief in mathematics in the control and experimental groups are symmetric and nearly mesokurtic, normal, homogenous, and did not differ significantly one another.

After the intervention, distributions of gain scores of self-belief in mathematics of control and experimental groups were studied. Distribution of posttest scores of self-belief in mathematics of control and experimental groups is given in Appendix V. Distribution of gain scores of self-belief in mathematics in the control group is symmetric $($ Skewness $=-0.34, S E=-0.34$, Skewness $/$ SE $=0.97$ ) and nearly mesokurtic (Kurtosis $=-0.23, S E=0.69$, Kurtosis $/ \mathrm{SE}=0.33$ ) indicating normality of distribution. In the experimental group, distribution of gain score of self-belief in mathematics is symmetric (Skewness $=0.03$, $S E=-0.34$, Skewness $/ \mathrm{SE}=0.09$ ) and nearly mesokurtic (Kurtosis $=-0.92$, $S E=0.69$, Kurtosis/SE $=1.33$ ) indicating normality of distribution. Distributions of gain scores of self-belief in mathematics are normal in both control ( $S-W=$ $.95, d f=45, p>.05$ ) and experimental ( $S-W=.96, d f=45, p>.05$ ) groups and their variances are not homogeneous $[F(1,88)=46.34, p<.01]$.

Difference in distribution of gain score of self-belief in mathematics of control and experimental groups are studied using histograms as well as boxplots and the linearity of the points in normal Q-Q plots further confirmed that selfbelief in mathematics before interventions in both the groups were normally distributed (Appendix AL). Hence, the difference in gain scores of self-belief in mathematics between experimental and control groups was tested using independent samples $t$ test. Result is given in Table 49.

Table 49
Test of Significance of Difference between Mean Gain Scores of Self-belief in Mathematics after Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems (Control)

| Groups | $M$ | $S D$ | $t$ | Cohen's $d$ |
| :--- | :---: | :---: | :---: | :---: |
| Practice in solving mathematics problems | 10.67 | 4.49 |  | $6.09^{* *}$ |
| Language integrated mathematics Instruction | 22.41 | 12.14 |  |  |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45 ;{ }^{* *} p<.01$

Table 49 shows that mean gain score of self-belief in mathematics after language integrated mathematics instruction $(M=22.41, S D=12.14)$ is significantly higher than that after practice in solving mathematics problems ( $M=$ 10.67, $S D=4.49$ ) $[t=6.09 ; p<.01]$. Effect size (Cohen's $d=1.28$ ) indicates that, gain in self-belief in mathematics after language integrated mathematics instruction is large, compared to that after practice in solving mathematics problems. As per Coe (2002), Cohen's $d=1.2$ means that 88 percent of students in language integrated mathematics instruction group are above the average student in the group which practiced solving mathematics problems and this is further demonstrated in Figure 23 showing smoothed frequency curves of the distributions of gain scores of self-belief in mathematics of experimental and control groups.


Figure 23. Ogives of the gain scores of self-belief in mathematics after language integrated mathematics instruction and practice in solving mathematics problems.

Figure 23 shows that after language integrated mathematics instruction, self-belief in mathematics of students beyond first quartile have an advantage of almost 50 percentile ranks than that after practice of mathematics problem solving; but a little less, approximately 37 percentile ranks advantage to students in the lower quartile.

## 4. Effect of Language Integrated Mathematics Instruction on Active

 Learning of Mathematics.Before intervention, active learning of mathematics in the control group was negatively skewed (Skewness $=-1.09, S E=0.35$, Skewness/SE $=3.11$ ) and leptokurtic (Kurtosis $=2.33, S E=0.69$, Kurtosis/SE $=3.38$ ) indicating deviation from normality of distribution. However, in the experimental group, active learning of mathematics was symmetric (Skewness $=-0.07, S E=0.35$, Skewness/SE $=0.20$ ) and nearly mesokurtic (Kurtosis $=-0.89, S E=0.69$, Kurtosis $/ \mathrm{SE}=1.29$ ) indicating normality of distribution.

Distribution of pretest scores of active learning of mathematics deviated from normality in the control group ( $S-W=.94, d f=45, p<.01$ ) but was normal in the experimental group $(S-W=.97, d f=45, p>.05)$ and their variances were not homogeneous $[F(1,88)=4.49, p<.01]$. Distribution of active learning of mathematics is densely clustered towards center in control group; but in experimental group the tails are much thinner and longer than that in control group as in histograms and boxplots in Appendix AM. The linearity of the points in normal Q-Q plots further confirmed that distribution of scores of active learning of mathematics before intervention in experimental group is normally distributed; but this is not true in the case of control group (Appendix W). Hence, the match in active learning of mathematics between the control and experimental groups before intervention was tested using Mann-Whitney $U$ test. It was found that, before intervention, active learning of mathematics did not differ significantly between the control (Mdn $=77.50$, Mean rank $=48.71$, Sum of ranks $\left.=2192, \mathrm{~N}_{1}=45\right)$ and experimental $(M d n$ $=75$, Mean rank $=42.29$, Sum of ranks $=1903, \mathrm{~N}_{2}=45$ ) groups $[U=868 ; p>.05]$.

Thus, before the intervention, the distribution of active learning of mathematics in the control group was negatively skewed and leptokurtic and hence not normal whereas that in experimental group it was symmetric and
mesokurtic and normal. The groups were not homogeneous, but they didn't differ significantly one another in the median score.

After the intervention, distribution of gain scores of active learning of mathematics of the control and experimental groups were studied. Distribution of posttest scores of active learning of mathematics of the control and experimental groups is given in Appendix V. Gain scores of active learning of mathematics in the control group is symmetric (Skewness $=0.12, S E=0.35$, Skewness $/ \mathrm{SE}=0.34$ ) and nearly mesokurtic (Kurtosis $=1.08, S E=0.69$, Kurtosis/SE $=1.57$ ) indicating normality of distribution. In the experimental group, distribution of gain scores of active learning of mathematics is positively skewed (Skewness $=0.87, S E=-0.34$, Skewness $/ \mathrm{SE}=2.49$ ) and nearly mesokurtic (Kurtosis $=-1.04, S E=0.69$, Kurtosis/SE = 1.51). Distributions of gain scores of active learning of mathematics are normal in both control ( $S-W=$ $.96, d f=45, p>.05)$ and experimental $(S-W=.95, d f=45, p>.05)$ groups and their variances are not homogeneous $[F(1,88)=18.46, p<.01]$.

Difference in distribution of gain scores of active learning of mathematics of control and experimental groups are studied using histograms as well as box plots and the linearity of the points in normal Q-Q plots further confirmed that gain score of active learning of mathematics in control and experimental groups is normal (Appendix AN). Hence, the difference in gain scores of active learning of mathematics between the experimental and control groups was tested using independent samples $t$ test. Result is given in Table 50.

Table 50
Test of Significance of Difference between Mean Gain Scores of Active Learning of Mathematics after Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems (Control)

| Groups | $M$ | $S D$ | $t$ | Cohen's $d$ |
| :--- | :---: | :---: | :---: | :---: |
| Practice in solving mathematics problems | 10.44 | 4.10 |  | $0.10^{* *}$ |
| Language integrated mathematics Instruction | 16.94 | 9.81 |  |  |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45 ;{ }^{* *} p<.01$

Table 50 shows that mean gain score of active learning of mathematics after language integrated mathematics instruction $(M=16.94, S D=9.81)$ is significantly higher than that after practice in solving Mathematics problems ( $M=10.44, S D=4.10$ ) $[t=4.10 ; p<.01]$. Effect size (Cohen's $d=0.86$ ) indicates that, gain in active learning of mathematics after language integrated mathematics instruction is large, compared to that after practice in solving mathematics problems. As per Coe (2002), Cohen's $d=0.86$ means that 79 percent of students in language integrated mathematics instruction group are above the average student in the group which practiced solving mathematics problems and this is further demonstrated in Figure 24 showing smoothed frequency curves of the distributions of gain scores of active learning of mathematics of experimental and control groups.


Figure 24. Ogives of the gain scores of active learning of mathematics after language integrated mathematics instruction and practice in solving mathematics problems.

Figure 24 shows that after language integrated mathematics instruction, active learning of mathematics of students beyond the first quartile have an advantage of almost 50 percentile ranks than that after practice of mathematics problem solving; but a reduced, approximately 20 percentile ranks advantage to students in the lower quartile.

## 5. Effect of Language Integrated Mathematics Instruction on

 Enjoyment of Mathematics.Before intervention, enjoyment of mathematics in the control group was symmetric (Skewness=-0.39, SE=0.35, Skewness/SE = 1.11) and nearly mesokurtic (Kurtosis $=-0.38, \quad S E=0.69, \quad$ Kurtosis $/ \mathrm{SE}=0.55$ ) indicating normality of distribution. In the experimental group also, enjoyment of mathematics is symmetric (Skewness $=-0.47, S E=0.35$, Skewness $/ \mathrm{SE}=0.68$ ) and nearly mesokurtic (Kurtosis $=-0.08, S E=0.69$, Kurtosis $/ \mathrm{SE}=0.00$ ) indicating normality of distribution.

Distributions of enjoyment of mathematics before intervention was normal in the control $(S-W=.97, d f=45, p>.05)$ and experimental groups ( $S-W$ $=.97, d f=45, p>.05)$ and their variances were homogeneous $[F(1,88)=3.86$, $p>.05]$. The groups have almost identical histograms as well as box plots on enjoyment of mathematics and the linearity of the points in normal Q-Q plots further confirmed that enjoyment of mathematics before interventions in both the groups are normally distributed (Appendix AO). Hence, the match in enjoyment of mathematics between the control and experimental groups before intervention was tested using independent samples $t$ test. It was found that, enjoyment of mathematics before intervention did not differ significantly between the control $\left(M=72.05, S D=14.23, N_{l}=45\right)$ and experimental $\left(M=70.17, S D=10.86, N_{2}=\right.$ 45) groups before intervention $[\mathrm{t}=0.70 ; \mathrm{p}>.05]$.

Thus, before the intervention, the distribution of enjoyment of mathematics in the control and experimental groups were symmetric and nearly mesokurtic, normal, homogenous, and did not differ significantly one another.

After the intervention, distribution of gain scores of enjoyment of mathematics of control and experimental groups were studied. Distribution of posttest scores of enjoyment of mathematics of control and experimental groups is given in Appendix V. Distribution of gain scores of enjoyment of mathematics in the control group is symmetric (Skewness $=-0.08, S E=0.35$, Skewness/SE $=$ 0.23 ) and nearly mesokurtic (Kurtosis $=0.29, S E=0.69$, Kurtosis/ $\mathrm{SE}=0.42$ ) indicating normality of distribution. In the experimental group, gain scores of enjoyment of mathematics is positively skewed (Skewness $=1.10, S E=0.35$, Skewness $/ \mathrm{SE}=3.14$ ) and leptokurtic (Kurtosis $=2.46, S E=0.69$, Kurtosis $/ \mathrm{SE}=$ 3.56) indicating deviation from normality of the distribution. However, distributions of gain scores of enjoyment of mathematics deviate from normality for both control $(S-W=.94, d f=45, p<.01)$ and experimental $(S-W=.93, d f=45$, $p<.01)$ groups and their variances are not homogeneous $[F(1,88)=19.31, p<.01]$.

Difference in distribution of gain scores of enjoyment of mathematics of control and experimental groups is studied using histograms as well as boxplots, and the non-linearity of the points in normal Q-Q plots further confirmed that gain score in enjoyment of mathematics in both control and experimental groups deviate from normality (Appendix AP). Hence, the difference in gain score of enjoyment of mathematics between the experimental and control groups was tested using Mann-Whitney $U$ test. Result is given in Table 51.

Table 51
Mann-Whitney Test of Significance of Difference between Medians of Gain Scores of Enjoyment of Mathematics after Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems (Control)

| Groups | Median | Range | Mean <br> Rank | Sum of <br> Ranks | Mann- <br> Whitney U | Cohen's <br> $d$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Practice in solving <br> mathematics problems | 8.89 | $16(0-16)$ | 29.70 | 1336.50 |  |  |
| Language integrated <br> mathematics Instruction | 15.56 | $47(2-49)$ | 61.30 | 2758.50 |  | 1.52 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45 ;{ }^{* *} p<.01$

Table 51 shows that gain scores of enjoyment of mathematics after language integrated mathematics instruction $(M d n=15.56)$ is significantly higher than that after practice in solving mathematics problems $(M d n=8.89)$, $[U=$ 301.50; $p<.01]$. Effect size (Cohen's $d=1.52$ ) indicates that, gain in enjoyment of mathematics after language integrated mathematics instruction is large, compared to that after practice in solving mathematics problems. As per Coe (2002), Cohen's $d=1.52$ means that 93.3 percent of students in language integrated mathematics instruction group are above the average student in the group who practiced solving mathematics problems and this is further demonstrated in Figure 25 showing smoothed frequency curves of the distributions of gain scores of enjoyment of mathematics of experimental and control groups.


Figure 25. Ogives of the gain scores of enjoyment of mathematics after language integrated mathematics instruction and practice in solving mathematics problems.

Figure 25 shows that after language integrated mathematics instruction, students have an advantage of approximately 50 percentile ranks in enjoyment of mathematics than that after practice in solving mathematics problems.

## Interaction of Language Integrated Mathematics Instruction with Control Variables on Mathematics Learning Outcomes

In order to verify whether language integrated mathematics instruction significantly enhances elementary school students' achievement in mathematics equally for high and low levels each of verbal comprehension in Malayalam, previous achievement in mathematics and non-verbal intelligence, factorial ANOVAs were used. Then, effectiveness of language integrated mathematics instruction on attitude towards mathematics and self-efficacy in mathematics of elementary school students who were high or low each on verbal comprehension in Malayalam and non-verbal intelligence were also studied either by factorial ANOVAs or independent samples $t$ tests. The results of analysis pertaining to the three dependent variables are provided under separate headings.

## Effects of language integrated mathematics instruction on achievement in mathematics by the levels of verbal comprehension, previous achievement and non-verbal intelligence

Effectiveness of language integrated mathematics instruction on achievement in mathematics of elementary school students who were high or low each on verbal comprehension in Malayalam, previous achievement in mathematics and non-verbal intelligence were studied using three $2 \times 2$ ANOVAs. Results are given in three sections.

## Effect of Language Integrated Mathematics Instruction on

## Achievement in Mathematics by Verbal Comprehension in Malayalam.

Effectiveness of language integrated mathematics instruction on achievement in mathematics of elementary students who were either high or low on verbal comprehension in Malayalam were examined with $2 \times 2$ ANOVA. Residuals of achievement in mathematics by the levels of verbal comprehension in Malayalam were normal both for control $(S-W=.96, d f=45, p>.05)$ and experimental $(S-W=.98, d f=45, p>.05)$ groups, and the error variances of the dependent variable are equal across these groups $[F(3,86)=2.12, p>.05]$. Result of $2 \times 2$ ANOVA is given in Table 52 .

Table 52
Result of $2 \times 2$ ANOVA of Achievement in Mathematics by Treatment and Verbal Comprehension in Malayalam

| Source | Sum of <br> Squares | $d f$ | Mean <br> Square | $F$ |
| :--- | :---: | :---: | :---: | :---: |
| Model 246123.53 4 61530.88 $552.08^{* *}$ <br> Treatment (Language integrated <br> mathematics instruction vs. Practice in <br> solving mathematics problems) 2238.87 1 2238.87 $20.08^{* *}$ <br> Verbal comprehension in Malayalam 8084.05 1 8084.05 $72.53^{* *}$ <br> Treatment * Verbal comprehension in <br> Malayalam 22.64 1 22.64 0.20 <br> Error 9584.88 86 111.45 $\quad 255708.41$ | 90 |  |  |  |

** $p<.01$

Table 52 shows that there is no significant interaction between language integrated mathematics instruction and verbal comprehension in Malayalam in effecting achievement in mathematics of elementary level students $[F(1,86)=$
$0.20, p>.05]$. Student achievement in mathematics enhanced after language integrated mathematics instruction, than after practice in solving mathematics problems (control), among students with high verbal comprehension ( $M=65.94$, $S D=9.09, N=23$; control: $M=54.96, S D=13.63, N=23, t=3.21, p<.01, d=$ 0.95 ) as well as those with low verbal comprehension ( $M=45.98, S D=8.79, N$ $=22$; control: $M=37, S D=9.89, N=22, t=3.18, p<.01, d=0.96)$. In other words, language integrated mathematics instruction enhanced achievement in mathematics of students irrespective of their level (low or high) of verbal comprehension in Malayalam. This effect is demonstrated in the line graphs of mean scores with error bars for experimental and control groups in Figure 26.


Figure 26. Line graph with error bars of achievement in mathematics of students with high and low verbal comprehension in the control (practice in solving mathematics problems) and experimental (language integrated mathematics instruction) groups.

Figure 26 illustrates that achievement in mathematics after language integrated mathematics instruction is higher than that after practice in solving mathematics problems (control) in students whether they are low or high on verbal comprehension in Malayalam. However, achievement in mathematics was
higher in students with high verbal comprehension than those with low verbal comprehension whether they were taught with language integrated mathematics instruction or with practice in solving mathematics problems.

## Effect of Language Integrated Mathematics Instruction on

## Achievement in Mathematics by Previous Achievement in Mathematics.

Effectiveness of language integrated mathematics instruction on achievement in mathematics of elementary students who were either high or low on previous achievement in mathematics were examined with $2 \times 2$ ANOVA. Residuals of achievement in mathematics by the levels of previous achievement in mathematics were normal both for control $(S-W=.98, d f=45, p>.05)$ and experimental $(S-W=.97, d f=45, p>.05)$ groups, and the error variances of the dependent variable are equal across these groups $[F(3,86)=1.79, p>.05]$. Result of $2 \times 2$ ANOVA is given in Table 53 .

Table 53
Result of $2 \times 2$ ANOVA of Achievement in Mathematics by Treatment and Previous Achievement in Mathematics

| Source | Sum of <br> Squares | df | Mean <br> Square | $F$ |
| :--- | :---: | :---: | :---: | :---: |
| Model | 249371.21 | 4 | 62342.80 | $846.03^{* *}$ |
| Treatment (Language integrated <br> mathematics instruction vs. Practice in <br> solving mathematics problems) | 1835.51 | 1 | 1835.51 | $24.90^{* *}$ |
| Previous achievement in mathematics <br> Treatment * Previous achievement in <br> mathematics | 11325.19 | 1 | 11325.19 | $153.69^{* *}$ |
| Error | 6337.19 | 86 | 73.68 | 0.25 |

[^2]Table 53 shows that there is no significant interaction between language integrated mathematics instruction and previous achievement in mathematics in effecting achievement in mathematics of elementary level students $[F(1,86)=$ $0.25, p>.05]$. Student achievement in mathematics enhanced after language integrated mathematics instruction, than after practice in solving mathematics problems (control), among students with high previous achievement ( $M=65.34$, $S D=8.61, N=26$; control: $M=57.16, S D=10.12, N_{2}=24, t=3.09, p<.01, d=$ 0.87 ) as well as those with low previous achievement ( $M=43.66, S D=7.13, N$ = 19; control: $M=33.64, S D=7.79, N=21, t=4.23, p<.01, d=1.34$ ). Language integrated mathematics instruction enhanced achievement in mathematics of students irrespective of their level (low or high) of previous achievement in mathematics. This effect is demonstrated as line graphs of mean scores with error bars for experimental and control groups in Figure 27.


Figure 27. Line graph with error bars of achievement in mathematics of students with high and low previous achievement in mathematics in the control (practice in solving mathematics problems) and experimental (language integrated mathematics instruction) groups.

Figure 27 illustrates that achievement in mathematics after language integrated mathematics instruction is higher in students whether they are low or high on previous achievement in mathematics than after practice in solving mathematics problems (control). However, achievement in mathematics was higher in students with high previous achievement than those with low previous achievement whether they were taught with language integrated mathematics instruction or with practice in solving mathematics problems.

## Effect of Language Integrated Mathematics Instruction on

## Achievement in Mathematics by Non-verbal Intelligence.

Effectiveness of language integrated mathematics instruction on achievement in mathematics of elementary students who were either high or low on non-verbal intelligence were examined with $2 \times 2$ ANOVA. Residuals of achievement in mathematics by the levels of non-verbal intelligence (NVI) were normal both for control $(S-W=.97, d f=45, p>.05)$ and experimental $(S-W=.97$, $d f=45, p>.05)$ groups, and the error variances of the dependent variable are equal across these groups $[F(3,86)=1.57, p>.05]$. Result of $2 \times 2$ ANOVA is given in Table 54.

Table 54
Result of $2 \times 2$ ANOVA of Achievement in Mathematics by Treatment and Non-verbal Intelligence

| Source | Sum of <br> Squares | df | Mean <br> Square | $F$ |
| :--- | :---: | :---: | :---: | :---: |
| Model | 244915.92 | 4 | 61228.98 | $487.90^{* *}$ |
| Treatment (Language integrated    <br> mathematics instruction vs. Practice in 2042.24 1 2042.24 <br> solving mathematics problems)  $16.27^{* *}$  <br> Non-verbal intelligence 6895.50 1 6895.50 <br> Treatment * Non-verbal intelligence (NVI) 4.94 1 4.94 <br> Error 10792.48 86 125.49 <br>  255708.41 90 $\quad$Total |  |  |  |  |

[^3]Table 54 shows that there is no significant interaction between language integrated mathematics instruction and non-verbal intelligence in effecting achievement in mathematics of elementary level students $[F(1,86)=0.03$, $p>.05]$. Student achievement in mathematics enhanced after language integrated mathematics instruction, than after practice in solving mathematics problems (control), among students with high non-verbal intelligence ( $M=64.21, S D=$ 9.33, $N=25$; control: $M=54.17, S D=10.09, N=24, t=3.62, p<.01, d=1.03$ ) as well as those with low non-verbal intelligence $(M=46.15, S D=10.79, N=$ 20; control: $M=37.06, S D=9.81, N=21, t=2.83, p<.01, d=0.88$ ). Language integrated mathematics instruction enhanced achievement in mathematics of students irrespective of their level (low or high) of non-verbal intelligence. This effect is demonstrated as line graphs of mean scores with error bars for experimental and control groups in Figure 28.


Figure 28. Line graph with error bars of achievement in mathematics of students with high and low non-verbal intelligence in the control (practice in solving mathematics problems) and experimental (language integrated mathematics instruction) groups.

Figure 28 illustrates that achievement in mathematics after language integrated mathematics instruction is higher in students whether they are low or
high on non-verbal intelligence than after practice in solving mathematics problems (control). However, achievement in mathematics was higher in students with high non-verbal intelligence than those with low non-verbal intelligence whether they were taught with language integrated mathematics instruction or with practice in solving mathematics problems.

Effects of language integrated mathematics instruction on selfefficacy in mathematics by the levels of verbal comprehension and nonverbal intelligence

Effectiveness of language integrated mathematics instruction on selfefficacy in mathematics of elementary school students who were high or low each on verbal comprehension in Malayalam and non-verbal intelligence were studied using two, $2 \times 2$ ANOVAs. Results are given in two sections.

## Effect of language integrated mathematics instruction on self-efficacy

 in mathematics by verbal comprehension in Malayalam.Effectiveness of language integrated mathematics instruction on selfefficacy in mathematics of elementary students who were either high or low on verbal comprehension in Malayalam were examined with $2 \times 2$ ANOVA. Residuals of self-efficacy in mathematics by the levels of verbal comprehension in Malayalam were normal both for control $(S-W=.98, d f=45, p>.05)$ and experimental $(S-W$ $=.93, d f=45, p<.05)$ groups, and the error variances of the dependent variable are equal across these groups $[F(3,86)=1.24, p>.05]$. Result of $2 \times 2$ ANOVA is given in Table 55.

Table 55
Result of $2 \times 2$ ANOVA of Gain in Self-efficacy in Mathematics by Treatment and Verbal Comprehension in Malayalam

| Source | Sum of Squares | df | Mean Square | $F$ |
| :---: | :---: | :---: | :---: | :---: |
| Model | 16039.09 | 4 | 4009.77 | 222.61** |
| Treatment (Language integrated mathematics instruction vs. Practice in solving mathematics problems) | 960.14 | 1 | 960.14 | 53.30** |
| Verbal comprehension in Malayalam | 195.76 | 1 | 195.76 | 10.86** |
| Treatment * Verbal comprehension in Malayalam | 126.04 | 1 | 126.04 | 6.99* |
| Error | 1549.05 | 86 | 18.01 |  |
| Total | 17588.14 | 90 |  |  |

${ }^{*} p<.05,{ }^{* *} p<.01$
Table 55 shows that there is significant interaction between language integrated mathematics instruction and verbal comprehension in Malayalam in effecting self-efficacy in mathematics of elementary level students $[F(1,86)=$ 6.99, $\left.p<.05, \eta_{p}{ }^{2}=0.09\right]$. Post hoc comparisons were done using test of significance of difference between means for independent samples. The results are given in Table 56.

Table 56
Comparison of Mean Gain Scores of Self-efficacy in Mathematics among students with Low or High Verbal Comprehension in Malayalam after Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems (Control)

| Level of Verbal <br> Comprehension | Treatment | $M$ | $S D$ | $N$ | $t$ | $d$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Practice in solving mathematics <br> problems | 6.85 | 4.09 | 22 |  |  |
| Language integrated mathematics <br> instruction | 15.75 | 3.09 | 22 | $8.14^{* *}$ | 2.46 |  |
| Practice in solving mathematics <br> problems | 12.17 | 4.66 | 23 | $2.96^{* *}$ | 0.88 |  |
| Language integrated mathematics <br> instruction | 16.34 | 4.85 | 23 |  |  |  |

[^4]Table 56 shows that self-efficacy in mathematics enhanced more after language integrated mathematics instruction than after practice in solving mathematics problems (control), among students with low verbal comprehension ( $t=8.14, p<.01, d=2.46$ ) as also among those with high verbal comprehension ( $t=2.96, p<.01, d=0.88$ ). This means that, on self-efficacy in mathematics, approximately 99 percent of students with low verbal comprehension $(d=2.46)$ and 80 percent of students with high verbal comprehension $(d=0.88)$ who received language integrated mathematics instruction are above the average student who received practice in solving mathematics problems.

There was large effect of verbal comprehension on self-efficacy in mathematics after practice in solving mathematics problems with advantage for high verbal comprehension group ( $M=12.17, S D=4.67, N=23$ ) over that of low verbal comprehension group $(M=6.85, S D=4.09, N=22)(t=4.06, p<.01$, $d=1.21$ ). This means that after practice in solving mathematics problems, around 88 percent students in low verbal comprehension group have lower selfefficacy in mathematics, than a student who is average on self-efficacy in mathematics in high verbal comprehension group. However, after language integrated mathematics instruction, the disadvantage for low verbal comprehension group ( $M=15.75, S D_{l}=3.09, N=22$ ) in comparison to high verbal comprehension group ( $M=16.34, S D=4.85, N=23$ ) $(t=0.48, p>.05)$ disappeared. This means that after language integrated instruction, self-efficacy of at least 38 percent more students with low verbal comprehension enhanced at par with that of those who had high verbal comprehension than if they had practice in solving mathematics problems. This effect is demonstrated as line graphs of mean scores with error bars for experimental and control groups in Figure 29.


Figure 29. Line graph with error bars of self-efficacy in mathematics of students with high and low verbal comprehension in Malayalam in the control (practice in solving mathematics problems) and experimental (language integrated mathematics instruction) groups.

Figure 29 illustrates that self-efficacy in mathematics after language integrated mathematics instruction, compared to that after practice in solving mathematics problems (control), enhanced more in students with low verbal comprehension than among students with high verbal comprehension in Malayalam.

Effect of language integrated mathematics instruction on self-efficacy in mathematics by non-verbal intelligence

Effectiveness of language integrated mathematics instruction on selfefficacy in mathematics of elementary students who were either high or low on nonverbal intelligence were examined with $2 \times 2$ ANOVA. Residuals of self-efficacy in mathematics by the levels of non-verbal intelligence were normal both for control $(S-W=.98, d f=45, p>.05)$ and experimental $(S-W=.93, d f=45, p<.01)$ groups, and the error variances of the dependent variable are equal across these groups $[F(3,86)=1.13, p>.05]$. Result of $2 \times 2$ ANOVA is given in Table 57.

Table 57
Result of $2 \times 2$ ANOVA of Gain in Self-efficacy in Mathematics by Treatment and Non-verbal Intelligence

| Source | Sum of <br> Squares | df | Mean <br> Square | $F$ |
| :--- | :---: | :---: | :---: | :---: |
| Model | 15928.27 | 4 | 3982.06 | $206.31^{* *}$ |
| Treatment (Language integrated    <br> mathematics instruction vs. Practice in 959.27 1 959.27 <br> solving mathematics problems)   $49.70^{* *}$ <br> Non-verbal intelligence 158.84 1 158.84 <br> Treatment * Non-verbal intelligence 51.42 1 51.42 | $8.23^{* *}$ |  |  |  |
| Error | 1659.86 | 86 | 19.30 | 2.66 |
|  | 17588.14 | 90 |  |  |

** $p<.01$

Table 57 shows that there is no significant interaction between language integrated mathematics instruction and non-verbal intelligence in effecting selfefficacy in mathematics of elementary level students $[F(1,86)=0.03, p>.05]$. Student self-efficacy in mathematics enhanced after language integrated mathematics instruction, than after practice in solving mathematics problems (control), among students with high non-verbal intelligence ( $M=16.56, S D=$ 4.11, $N_{l}=25$; control: $\left.M=11.52, S D=4.94, N_{2}=24, t=3.89, p<.01, d=1.11\right)$ as well as those with low non-verbal intelligence $\left(M=15.41, S D=3.99, N_{l}=\right.$ 20; control: $\left.M=7.34, S D=4.41, N_{2}=21, t=6.13, p<.01, d=1.92\right)$. Language integrated mathematics instruction enhanced self-efficacy in mathematics of students irrespective of their level (low or high) of non-verbal intelligence. This effect is demonstrated as line graphs of mean scores with error bars for experimental and control groups in Figure 30.


Figure 30. Line graph with error bars of self-efficacy in mathematics of students with high and low non-verbal intelligence in the control (practice in solving mathematics problems) and experimental (language integrated mathematics instruction) groups.

Figure 30 illustrates that self-efficacy in mathematics after language integrated mathematics instruction is higher in students whether they are low or high on non-verbal intelligence than after practice in solving mathematics problems (control). However, Figure 25 shows that the disadvantage in selfefficacy in mathematics after practice in solving mathematics problems of low non-verbal intelligence group, in comparison to their high non-verbal intelligence counter parts, disappeared after language integrated mathematics instruction.

## Effects of language integrated mathematics instruction on attitude towards mathematics by the levels of verbal comprehension in Malayalam and non-verbal intelligence

Before moving on to testing the effect of language integrated mathematics instruction on attitude towards mathematics by the levels of
verbal comprehension in Malayalam and non-verbal Intelligence, the fit of the distribution for factorial ANOVA were verified. Residuals of attitude towards mathematics by the levels of verbal comprehension in Malayalam deviated from normality both for control $(S-W=.93, d f=45, p<.01)$ and experimental $(S-W=.94, d f=45, p<.01)$ groups, and the error variances of the dependent variable were not equal across these groups $[F(3,86)=6.21, p<.01]$. Also, residuals of attitude towards mathematics by the levels of non-verbal intelligence were normal for control group $(S-W=.95, d f=45, p>.05)$ and deviated from normality for experimental group ( $S-W=.93, d f=45, p<.01$ ), and the error variances of the dependent variable are unequal across these groups $[F(3,86)=5.44, p<.01]$. Hence, effectiveness of language integrated mathematics instruction on attitude towards mathematics of elementary school students who were either high or low each on 1) verbal comprehension in Malayalam and 2) non-verbal intelligence were studied using independent samples $t$ tests. Results are given in two sections.

## Effect of language integrated mathematics instruction on attitude

 towards mathematics by verbal comprehension in Malayalam.Interaction of verbal comprehension in Malayalam with effect of language integrated mathematics instruction on attitude towards mathematics among elementary school students was studied using independent samples $t$ tests and mean plots. Results of comparison of mean scores of attitude towards mathematics among students with low or high verbal comprehension in Malayalam after language integrated mathematics instruction and practice in solving mathematics problems (Control) are given in Table 58.

Table 58

Comparison of Mean Gain Scores of Attitude towards Mathematics among Students with Low or High Verbal Comprehension in Malayalam after the Language Integrated Mathematics Instruction and the Practice in Solving Mathematics Problems (Control)

| Level of verbal comprehension | Treatment | M | SD | $N$ | $t$ | $d$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Low | Practice in solving mathematics problems | 2.31 | 1.72 | 22 | 12.86** | 3.88 |
|  | Language integrated mathematics instruction | 13.00 | 3.49 | 23 |  |  |
| High | Practice in solving mathematics problems | 3.54 | 1.14 | 23 | 11.53** | 3.39 |
|  | Language integrated mathematics instruction | 11.72 | 3.21 | 23 |  |  |

** $p<.01$
Table 58 shows that attitude towards mathematics enhanced more after language integrated mathematics instruction than after practice in solving mathematics problems (control), among students with low verbal comprehension ( $t=12.86, p<.01, d=3.88$ ) as also among those with high verbal comprehension ( $t=11.53, p<.01, d=3.39$ ). As per Coe (2002), these obtained Cohen's $d>3$ means that, on attitude towards mathematics, 99.9 percent of students who received language integrated mathematics instruction are above the average student who received practice in solving mathematics problems irrespective of their level (high or low) of verbal comprehension in Malayalam.

If students practice solving mathematics problems, there was large advantage on attitude towards mathematics for high verbal comprehension group ( $M=3.54, S D=1.14, N=23$ ) over low verbal comprehension group ( $M=2.31$, $S D=1.72, N=22)(t=2.83, p<.01, d=0.84)$. In other words, after practice in solving mathematics problems, around 79 percent students in low verbal
comprehension group have lower attitude towards mathematics than a student who is average on attitude towards mathematics in high verbal comprehension group. However, after language integrated mathematics instruction, the disadvantage for low verbal comprehension group ( $M=13, S D=3.49, N=22$ ) in comparison to high verbal comprehension group ( $M=11.72, S D=3.21, N=$ 23) $(t=1.28, p>.05)$ disappeared.

This means that language integrated mathematics instruction enhanced attitude of at least 29 percent more students in low verbal comprehension group at par with their counter parts in high verbal comprehension group.

The influence of verbal comprehension on attitude towards mathematics of students who received language integrated mathematics instruction or practice in solving mathematics problems is demonstrated as line graphs of mean scores with error bars for experimental and control groups in Figure 31.


Figure 31. Line graph with error bars of attitude towards mathematics of students with high and low verbal comprehension in Malayalam in the control (practice in solving mathematics problems) and experimental (language integrated mathematics instruction) groups.

Figure 31 illustrates that attitude towards mathematics after language integrated mathematics instruction, compared to that after practice in solving mathematics problems (control), enhanced more in students with low verbal comprehension than among students with high verbal comprehension in Malayalam. Language integrated mathematics instruction could compensate the tendency for lower attitude towards mathematics among students with low verbal comprehension.

## Effect of language integrated mathematics instruction on attitude

 towards mathematics by non-verbal intelligenceInteraction of non-verbal intelligence with effect of language integrated mathematics instruction on attitude towards mathematics among elementary school students was studied using independent samples $t$ tests and mean plots. Results of comparison of mean scores of attitude towards mathematics among students with low or high non-verbal intelligence after language integrated mathematics instruction and practice in solving mathematics problems (Control) are given in Table 59.

Table 59
Comparison of Mean Gain Scores of Attitude towards Mathematics among Students with Low and High Non-verbal Intelligence after the Language Integrated Mathematics Instruction and the Practice in Solving Mathematics Problems (Control)

| Level of non- <br> verbal intelligence | Treatment | $M$ | $S D$ | $N$ | $t$ | $d$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Practice in solving <br> mathematics problems | 2.43 | 1.79 | 21 | $11.90^{* *}$ | 3.69 |
|  | Language integrated <br> mathematics instruction | 12.18 | 3.28 | 20 |  |  |
|  | Practice in solving <br> mathematics problems | 3.39 | 1.19 | 24 | $12.01^{* *}$ | 3.46 |
| Language integrated <br> mathematics instruction | 12.47 | 3.51 | 25 |  |  |  |

[^5]Table 59 shows that attitude towards mathematics enhanced more after language integrated mathematics instruction than after practice in solving mathematics problems (control), among students with low non-verbal intelligence $(t=11.90, p<.01, d=3.69)$ as also among those with high nonverbal intelligence ( $t=12.01, p<.01, d=3.46$ ). As per Coe (2002), these obtained Cohen's $d>3$ means that, on attitude towards mathematics, 99.9 percent of students who received language integrated mathematics instruction are above the average student who received practice in solving mathematics problems irrespective of their level (high or low) of non-verbal intelligence.

If students practice mathematics problem solving, there was medium advantage on attitude towards mathematics for high non-verbal intelligence group ( $M=3.39, S D=1.19, N=24$ ) over low non-verbal intelligence group ( $M$ $=2.43, S D=1.79, N=21)(t=2.15, p<.05, d=0.63)$. In other words, after practice in solving mathematics problems, around 73 percent students in low non-verbal intelligence group have lower attitude towards mathematics than a student who is average on attitude towards mathematics in high non-verbal intelligence group. However, after language integrated mathematics instruction, the disadvantage for low verbal comprehension group ( $M=12.18, S D=3.28, N$ $=20$ ) in comparison to high non-verbal intelligence group ( $M=12.47, S D=$ $3.51, N=25)(t=0.28, p>.05)$ disappeared.

This means that language integrated mathematics instruction enhanced attitude of at least 23 percent more students in low non-verbal intelligence group at par with their counter parts in high non-verbal intelligence group.

The influence of non-verbal intelligence on attitude towards mathematics of students who received language integrated mathematics instruction or practice
in solving mathematics problems is demonstrated as line graphs of mean scores with error bars for experimental and control groups in Figure 32.


Figure 32. Line graph with error bars of attitude towards mathematics of students with high and low non-verbal intelligence in the control (practice in solving mathematics problems) and experimental (language integrated mathematics instruction) groups.

Figure 32 illustrates that attitude towards mathematics after language integrated mathematics instruction, compared to that after practice in solving mathematics problems (control), enhanced more in students with low non-verbal intelligence than among students with high non-verbal intelligence. Language integrated mathematics instruction could compensate the tendency for lower attitude towards mathematics among students with low non-verbal intelligence.

## Summary of Effect of Language Integrated Mathematics Instruction on Mathematics Learning among Elementary School Students in Kerala

Language integrated mathematics instruction has significant effect of medium size on achievement in mathematics of elementary school students over and above the effect of practice in solving mathematics problems irrespective of
them being high or low on verbal comprehension in Malayalam, non-verbal intelligence and previous achievement in mathematics. The effect of language integrated mathematics instruction on achievement, over and above the effect of practice in solving mathematics problems is true for the areas of school mathematics viz., algebra, arithmetic and geometry.

Language integrated mathematics instruction has significant and large effect on self-efficacy in mathematics of elementary school students over and above the effect of practice in solving mathematics problems. This effect is true for efficacy in both learning mathematics and solving mathematics problems. Whereas this effect is large for self-efficacies for algebra and geometry in school mathematics, it is only small on self-efficacy in arithmetic. While the large effect of language integrated mathematics instruction on self-efficacy is true for students high or low on verbal comprehension as well, such instruction compensates for adverse effect of low verbal comprehension on self-efficacy and hence is of additional value for them as self-efficacy of students with low verbal comprehension enhanced at par with that of those who had high verbal comprehension. Likewise, large effect of language integrated mathematics instruction on self-efficacy is true for students high or low on non-verbal intelligence as well, such instruction compensates for adverse effect of low nonverbal intelligence on self-efficacy and hence is of additional value for them as self-efficacy of students with low non-verbal intelligence enhanced at par with that of those who had high non-verbal intelligence.

Language integrated mathematics instruction has significant and large effect on attitude towards mathematics of elementary school students over and above the effect of practice in solving mathematics. This effect is true for dimensions of attitude viz., like towards mathematics, engagement with
mathematics, self-belief in mathematics, active learning of mathematics and enjoyment of mathematics. While the large effect of language integrated mathematics instruction on attitude is true for students high or low on verbal comprehension as well, such instruction compensates for adverse effect of low verbal comprehension on attitude and hence is of additional value for them as attitude of students with low verbal comprehension enhanced at par with that of those who had high verbal comprehension. Likewise, large effect of language integrated mathematics instruction on attitude is true for students high or low on non-verbal intelligence as well, such instruction compensates for adverse effect of low non-verbal intelligence on attitude and hence is of additional value for them as attitude of students with low non-verbal intelligence enhanced at par with that of those who had high non-verbal intelligence.

Essentially, language integrated mathematics instruction has large effects on affective outcomes like attitude towards mathematics and self-efficacy in mathematics and has medium effect on achievement in mathematics.

## Chapter V

## SUMMARY, MAJOR FINDINGS AND SUGGESTIONS

- Restatement of the Problem
- Variables of the Study
- Hypotheses of the Study
- Methodology in Brief
- Major Findings of the Study
- Tenability of the Hypotheses
- Discussion of Findings
- Limitations of the Study
- Conclusion
- Educational Implications
- Suggestions for Further Research

This chapter presents outline of the important aspects of the study such as procedure of the study, summary of findings, relevance of practice in elementary school mathematics instruction and suggestions for further research.

## Restatement of the Problem

This study is entitled as 'Enhancing Mathematics Learning through Evidence Based Instruction Focusing on Language of Mathematics in Elementary Schools of Kerala’.

It identified language related difficulties in mathematics learning at elementary level in order to develop an instructional plan focusing on language of mathematics. Effectiveness of this instruction plan - 'Language Integrated Mathematics Instruction' on achievement in mathematics, algebra, arithmetic and geometry, self-efficacies in mathematics, algebra, arithmetic and geometry, and attitude towards mathematics of elementary school students were verified. It further verified whether the language integrated mathematics instruction significantly enhances elementary school students' achievement in mathematics equally for high and low levels each of previous achievement in mathematics, verbal comprehension in Malayalam and, non-verbal intelligence. This study also verified whether effect of language integrated mathematics instruction on attitude towards mathematics and self-efficacy in mathematics of elementary school students is equal for high and low levels each of verbal comprehension in Malayalam and non-verbal intelligence.

## Variables of the Study

This study can be described in three phases, with Phase I Pilot Study proceeding to Phase III Experiment, after a phase of design and development of appropriate tools and intervention strategies in between.

## Variables in Phase I (Pilot Study)

In phase 1 the variables studied were students' perception of difficulties in mathematical tasks, reasons sourcing from nature of mathematics for these perceived difficulties in mathematical tasks, and achievement in the language of mathematics and its components. Perception of difficulties in mathematical tasks among elementary school students is conceived as the dependent variable being influenced by reasons sourcing from nature of mathematics and achievement in the language of mathematics and its components.

## Variables in Phase III (Experimental Phase)

The effectiveness of an evidence-based instruction focusing on language of mathematics (Language integrated mathematics instruction) in improving students' mathematics learning outcomes in terms of achievement in mathematics, self-efficacy in mathematics and attitude towards mathematics- in comparison to practice in mathematics problem solving is examined. There are independent, dependent and control variables in the experimental phase.

1. The dependent variables of the study were mathematics learning outcomes in terms of
i. Achievement in Mathematics
ii. Achievement in Algebra
iii. Achievement in Arithmetic
iv. Achievement in Geometry
v. Self-efficacy in Mathematics
vi. Self-efficacy in Learning Mathematics
vii. Self-efficacy in Solving Mathematics Problems
viii. Self-efficacy in Algebra
ix. Self-efficacy in Arithmetic
x. Self-efficacy in Geometry
xi. Attitude towards Mathematics
xii. Like towards Mathematics
xiii. Engagement with Mathematics
xiv. Self-belief in Mathematics
xv. Active Learning of Mathematics
xvi. Enjoyment of Mathematics
2. The independent variable selected for the study is the instructional strategy with two levels; Language Integrated Mathematics Instruction and Practice in Solving Mathematics Problems.
3. Control variables of the study are Previous Achievement in Mathematics, Verbal Comprehension in Malayalam and Non-verbal Intelligence.

## Hypotheses of the Study

Hypotheses of this study were the following:

1. Language Integrated mathematics instruction significantly enhances elementary school students' achievement in mathematics.
i. Language integrated mathematics instruction significantly enhances elementary school students' achievement in mathematics equally for high and low levels of previous achievement in mathematics.
ii. Language integrated mathematics instruction significantly enhances elementary school students' achievement in mathematics equally for high and low levels of verbal comprehension in Malayalam.
iii. Language integrated mathematics instruction significantly enhances elementary school students' achievement in mathematics equally for high and low levels of non-verbal intelligence in mathematics.
2. Language Integrated Mathematics Instruction significantly enhances elementary school students':
i. Achievement in algebra
ii. Achievement in arithmetic
iii. Achievement in geometry
3. Language integrated mathematics instruction significantly enhances elementary school students' self-efficacy in mathematics.
i. Language integrated mathematics instruction significantly enhances elementary school students' self-efficacy in mathematics equally for high and low levels of verbal comprehension in Malayalam
ii. Language integrated mathematics instruction significantly enhances elementary school students' self-efficacy in mathematics equally for high and low levels of non-verbal intelligence in mathematics
4. Language integrated mathematics instruction significantly enhances elementary school students' dimensions of self-efficacy in mathematics viz;
i. Self-efficacy in learning mathematics
ii. Self-efficacy in solving mathematics problems
5. Language integrated mathematics instruction significantly enhances elementary school students':
i. Self-efficacy in algebra
ii. Self-efficacy in arithmetic
iii. Self-efficacy in geometry
6. Language integrated mathematics instruction significantly enhances elementary school students' attitude towards mathematics.
i. Language integrated mathematics instruction significantly enhances elementary school students' attitude towards mathematics equally for high and low levels of verbal comprehension in Malayalam
ii. Language integrated mathematics instruction significantly enhances elementary school students' attitude towards mathematics equally for high and low levels of non-verbal intelligence in mathematics
7. Language integrated mathematics instruction significantly enhances elementary school students' dimensions of attitude towards mathematics
i. Like towards mathematics
ii. Engagement with mathematics
iii. Self-belief in mathematics
iv. Active learning of mathematics
v. Enjoyment of mathematics

## Methodology in Brief

The study proceeded through three phases, first pilot study involving content analysis and surveys, developmental phase and then an experimental phase.

## Phase I: Pilot Study

Pilot study includes two surveys. Part 1 survey identified perception of difficulties in mathematical tasks and reasons for difficulty thereof. Content analysis to identify the linguistic components in mathematics teaching-learning process and extensive review of literature on language related difficulties in mathematics, lead to development of a set of tests to identify linguistic
difficulties in mathematics teaching-learning. Part 2 survey was to test the linguistic difficulties using this battery of tests. This guided to the development of evidence-based instructional strategy focusing on language of mathematics to overcome the identified linguistic difficulties by choosing appropriate instructional strategies from review of related literature.

## Phase II: Developmental Phase

In the second phase, an evidence-based instruction focusing on language of mathematics in elementary level was developed based on the evidence from the pilot study. An intervention for the control group (practice in solving mathematics problems) was also developed. Strategies for language integrated mathematics instruction were planned. Tools for measurement in experimental phase were developed.

## Phase III: Experiment

Effectiveness of the evidence-based instruction focusing on language of mathematics is examined with the help of a quasi-experimental pretest-posttest nonequivalent group design as following.

1. Four intact classes of standard seven were selected for the experiment.
2. The classes were randomly assigned to experimental and control groups.
3. These two groups (experimental \& control) were matched on verbal comprehension in Malayalam, non-verbal intelligence and previous achievement in mathematics.
4. Experimental and control groups were pretested on self-efficacy in mathematics and attitude towards mathematics.
5. Language integrated mathematics instruction is done by the experimenter along with content instruction of five units by the schoolteacher
(1. Parallel lines, 2. Unchanging relations, 3. Repeated multiplication, 4. Area of Triangle and 5. Square and square root) to the experimental group. For the control group, for an equal duration, guided practice in solving mathematics problems were given along with content instruction by the schoolteacher for the five units.
6. Effectiveness of the language integrated mathematics instruction is checked with respect to all dependent variables.

## Design of the quasi experimental phase of the study

The pretest-posttest non-equivalent control group design used in this study is as follows.
$\mathrm{G}_{1} \mathrm{O} 1 \mathrm{X} \mathrm{O} 2$
$\mathrm{G}_{2} \mathrm{O} 3 \mathrm{CO}$
$\mathrm{G}_{1} \& \mathrm{G}_{2}-$ Intact divisions of $7^{\text {th }}$ standard students randomly assigned to experimental and control groups and matched on previous achievement in mathematics, verbal comprehension in Malayalam and non-verbal intelligence.

X - $\quad$ Language integrated mathematics instruction along with content instruction (by schoolteacher)

C - Practice in solving mathematics problems along with content instruction (by schoolteacher)
$\mathrm{O}_{1} \& \mathrm{O}_{3}$. Pretests on self-efficacy in mathematics and attitude towards mathematics.
$\mathrm{O}_{2} \& \mathrm{O}_{4}$ - Posttests on achievements and self-efficacies in 1) Parallel lines, 2) Unchanging relations, 3) Repeated Multiplication, 4) Area of Triangle and 5) Square and square root; and self-efficacy in mathematics and attitude towards mathematics.

## Sample

## Phase I: Pilot Study

There were two different sets of random samples. Part 1 survey used 300, eighth standard students to identify perception of difficulties in mathematical tasks and reasons for difficulty thereof. Part 2 survey used 1050, eighth standard students for testing of linguistic difficulties in learning mathematics.

## Phase III: Experimental phase

The experimental phase of the study was conducted on a sample of standard VII students from a government aided school of rural background following Kerala syllabus in Malayalam medium. Experimental and control groups consisted of 45 students each, and the groups were matched on the levels of

- Previous achievement in mathematics,
- Verbal comprehension in Malayalam and
- Non-verbal intelligence

Also, this sample is not significantly different from the population in terms of self-efficacy in mathematics and attitude towards mathematics.

## Tools used for the Study

Content analysis, questionnaire, achievement tests, attitude and selfefficacy scales were used in the study. In phase 1 the following tools were used.

1. Questionnaire on students' difficulties in learning
2. Test of difficulties in language of mathematics (3 Sets)

In experimental phase the following measuring tools were used.

1. Test of previous achievement in mathematics
2. Test of verbal comprehension in Malayalam
3. Raven's standard progressive matrices (Raven, 1994)
4. Scale of attitude towards mathematics
5. Scale of self-efficacy in mathematics
6. Achievement Tests \& Scales of self-efficacies in -
i. Parallel Lines
ii. Unchanging Relations
iii. Repeated Multiplication
iv. Area of Triangle
v. Square and Square root

In addition to the measuring tools, language integrated mathematics instruction and practice in solving mathematics problems were also developed. Techniques used in language integrated mathematics instruction strategy was designed to overcome the linguistic difficulties in learning mathematics among elementary school students. The lessons were prepared by employing the techniques, viz., anchoring mathematics with language, vocabulary bank, labeling vocabulary, word walls, word trails, listen and write, possible sentences, guess what?, justifying their reasoning and translation game.

Practice in solving mathematics problems was provided to the control group for an equal period of time in which students were given practice in mathematics problem solving for each unit.

## Statistical Techniques used in the Study

In addition to the basic descriptive statistics, the following statistical procedures were used for analysis of data.

1. Percentage analysis
2. Significance of difference between two correlated percentages
3. Pearson's r
4. Significance of a coefficient of correlation
5. Comparison of correlations from dependent samples
6. Shapiro- Wilk test of normality
7. Levene's test of homogeneity of variances
8. Independent samples $t$ test
9. Mann Whitney $U$ test
10. Two-way ANOVA
11. Effect Size (Cohen's $d$ )
12. Partial eta squared

## Major Findings of the Study

AI. Understanding and solving especially algebraic problems, and using decimals and fractions cause difficulty for majority of elementary students, while understanding and translating word problems, equations, operations and, symbols and notations are difficult for more than $1 / 3^{\text {rd }}$ of them, whereas arithmetic and its operations are easier.

1) Majority of students feels difficulty in tasks viz., 1. using decimals (64.33\%), 2. concentrating for a long time to solve problems (61\%), 3. doing calculations with speed ( $60.33 \%$ ), 4. fractions (56.33\%), 5. understanding algebraic problems ( $56.33 \%$ ), 6. Identifying equation for a given problem (51.67\%)
2) More than $1 / 3^{\text {rd }}$ of students feel difficulty in tasks viz., 1. identifying irrelevant information in word problems (48.67\%), 2. identifying key words in word problems (47.33\%), 3. identifying mathematics problem in word problems (46\%), 4. analyzing geometrical figures (44.67\%), 5. translating
mathematical answer to verbal expression (43\%), 6. understanding word problems without external help (41\%), 7. comprehension of processes ( $41 \%$ ), 8. and concepts ( $40.67 \%$ ), 9. translating word problems into mathematical expressions (38\%), 10. remembering numbers while doing operations (35.33\%), 11. understanding symbols and notations (33\%), 12. following rules while doing calculations (31.33\%)
3) Only around $1 / 4^{\text {th }}$ of students have difficulty in selecting mathematical operations ( $26.33 \%$ ), 2. understanding large numbers ( $24 \%$ ) and place value ( $24.33 \%$ ), 3 . drawing geometrical figures (22.33\%), 4. doing basic arithmetic operations (22.33\%), 5. doing mental arithmetic (22.33\%).

## A II. Along with problem solving and attendant learning requirements related to the nature of mathematics, students' difficulties in mathematics source from density of difficult concepts, prevalence of symbols, notations, and unfamiliar words that make their learning be strenuous, rote and uninteresting.

4) Around 90 percent of students, prominence of problem solving and need for regularity in attending classes are felt as the major reasons for difficulty in learning mathematics.
5) For around 80 percent of students, need for external support, need for repeated practice, difficulty of concepts, number of concepts, need for strenuous attention, difficulty in understanding questions and need for precision in understanding are felt as difficult.
6) For around $1 / 3^{\text {rd }}$ of students, prevalence of symbols and notations, necessity to learn unfamiliar words, need for rote learning and impracticability in daily life are considered as elements creating high difficulty.

## A III. Task difficulty in mathematics significantly correlate with nature of content of mathematics, more than with its teaching learning, but for word problem related tasks both the nature of content and teaching learning are equally important.

7) Perceived difficulty in mathematical tasks are related more with difficulty sourcing from nature of mathematics content $(r=.46)$ than from nature of mathematics teaching-learning process $(r=34, z=3.24, p<.01)$.
i. Tasks related to equations and operations have significant positive and substantial correlation with nature of mathematics content ( $r$ $=.41, p<.01)$ and low correlation with nature of mathematics teaching and learning $(r=.32, p<.01)(z=2.38, p<.01)$.
ii. There is significant positive low correlation for both nature of mathematics content ( $p<.01$ ) and nature of mathematics teaching learning ( $p<.01$ ) with tasks related to symbols and notations ( $r_{l}=$ $.38, p<.01 ; r_{2}=.28, p<.01$ ), arithmetic operations ( $r_{1}=.37, p<.01$; $r_{2}=.23, p<.01$ ), problem-solving competence ( $r_{1}=.26, p<.01 ; r_{2}=$ $.25, p<.01$ ), understanding word problems ( $r_{1}=.26, p<.01 ; r_{2}=.21$, $p<.01$ ) and translation of word problems ( $r_{1}=.24, p<.01 ; r_{2}=.20$, $p<.01$ ). There is significantly higher correlation for nature of mathematics content, than for nature of teaching-learning process with perceived difficulty in tasks involving symbols and notations ( $z$ $=2.61, \mathrm{p}<.01)$ and arithmetic operations $(z=3.61, p<.01)$. However, difference in correlation for nature of mathematics content and nature of mathematics teaching-learning process with task difficulty involving problem-solving competence ( $z=0.25, p>.05$ ), understanding
word problems $(z=1.26, \mathrm{p}>.05)$ and translation of word problems ( $z$ $=1.00, \mathrm{p}>.05$ ) are not significant.
iii. Tasks related to understanding numbers and number systems exhibits significant positive low correlation with nature of mathematics content ( $r_{1}=.24, P<.01 ; r_{2}=.21, p<.01$ ) and negligible but significant correlation with nature of mathematics teachinglearning ( $r_{1}=.14, P<.05 ; r_{2}=.13, p<.05$ ). There are significant positive low correlations for nature of mathematics content ( $p<.01$ ) and negligible correlation for nature of mathematics teachinglearning ( $p<.01$ ) against difficulty in tasks involving understanding numbers $(z=2.49, p<.01)$ and number systems $(z=1.98, p<.05)$.
iv. Tasks related to using mathematical abstractions have significant positive low correlation with nature of mathematics content ( $r=.20$, $\mathrm{p}<.01$ ), whereas it does not have significant correlation with nature of mathematics teaching learning $(r=.11, \mathrm{p}>.05)(z=2.23, p<.05)$.

A IV. Language related difficulties in learning mathematics for students, source more from vocabulary components such as morphology, general and specific terms, symbols and conventions and semantics than from sentence level components such as syntax of algebraic or numeric expressions and pragmatics.
8) Achievement in morphology ( $25.84 \%$ ), terms ( $38.99 \%$ ) and semantics ( $46.03 \%$ ) are less than average of achievement in the language of mathematics (47\%). Achievement in symbols (51.31\%) and syntax (53.63\%) are comparatively higher. The component of language of mathematics with highest achievement is pragmatics (66.25\%).

A V Achievement in mathematical language is the lowest in parts of words, general and mathematics specific terms, word meaning in specific context, basic mathematics symbols and mathematical conventions, and comparatively lower in expressions in algebraic and natural language.
9) Achievement in mathematical language is below average in parts of words (25.84\%), general terms (38.23\%) and mathematics specific terms (40.36\%), word meaning in specific context (46.03\%), basic mathematics symbols (41.01\%) and mathematical conventions (37.07\%). Higher achievement is observed in subcategories of components of mathematics language viz., algebraic expressions (48.57\%), natural language (50.41\%), geometric figures ( $62.52 \%$ ), numeric expressions ( $63 \%$ ), commonly used fractions ( $63.68 \%$ ), real life word problems ( $64.38 \%$ ), reading graph (67.8\%), identifying operations from key terms (68.7\%) and geometrical symbols (74.5\%).
i. There is no significant difference between achievement in general terms (38.23\%) and mathematics specific terms (40.36\%) ( $C R=0.45$, $p>.05$ ).
ii. Achievement in basic mathematics symbols (41.01\%) is significantly low compared to achievement in geometric symbols (74.5\%) ( $C R=6.14, p<.01$ ).
iii. Achievement in mathematical conventions is significantly low compared to achievement in algebraic expression ( $p<.05$ ), natural expression $(p<.01)$, geometric figures $(p<.01)$ and numeric expression ( $p<.01$ ).
iv. When the same content is expressed in different formats, achievement in algebraic expression is not significantly different from achievement
in natural expression ( $p>.05$ ) but is significantly lower than achievement in numeric expression $(p<.01)$. When the same content is expressed in natural language and numeric language, achievement is significantly high in numeric expression ( $p<.05$ ).
v. Achievement in subcategories of pragmatics do not show any significant difference ( $p>.05$ ).

## B I Language integrated mathematics instruction has significant effect of

 medium size on achievement in mathematics of elementary school students over and above the effect of practice in solving mathematics problems irrespective of them being high or low on verbal comprehension in Malayalam, non-verbal intelligence and previous achievement in mathematics.10) After language integrated mathematics instruction, mean posttest scores of achievement in mathematics $(M=56.18, S D=13.42)$ is significantly higher than that after practice in solving mathematics problems ( $M=46.18, S D=14.89$ ) $[t=3.35 ; p<.01]$ with a medium effect at the mean level (Cohen's $d=0.71$ ), equal to an advantage of approximately 25 percentile ranks for those in between first and third quartiles, and of approximately 15 percentile ranks for those in the upper and lower quartiles.
i. There is no significant interaction between language integrated mathematics instruction and previous achievement in mathematics in effecting achievement in mathematics of elementary level students [ $F$ $(1,86)=0.25, p>.05]$.

Student achievement in mathematics enhanced after language integrated mathematics instruction, than after practice in solving mathematics problems (control), among students with high previous achievement ( $M=65.34, S D=8.61, N_{I}=26$; control: $M=57.16, S D$ $\left.=10.12, N_{2}=24, t=3.09, p<.01, d=0.87\right)$ as well as those with low previous achievement ( $M=43.66, S D=7.13, N_{l}=19$; control: $M=$ 33.64, $\left.S D=7.79, N_{2}=21, t=4.23, p<.01, d=1.34\right)$.
ii. There is no significant interaction between language integrated mathematics instruction and verbal comprehension in Malayalam in effecting achievement in mathematics of elementary school students $[F(1,86)=0.20, p>.05]$.

Student achievement in mathematics enhanced after language integrated mathematics instruction, than after practice in solving mathematics problems, among students with high verbal comprehension $\left(M=65.94, S D=9.09, N_{I}=23\right.$; control: $M=54.96$, $\left.S D=13.63, N_{2}=23, t=3.21, p<.01, d=0.95\right)$ as well as those with low verbal comprehension ( $M=45.98, S D=8.79, N_{l}=22$; control: $\left.M=37, S D=9.89, N_{2}=22, t=3.18, p<.01, d=0.96\right)$.
iii. There is no significant interaction between language integrated mathematics instruction and non-verbal intelligence in effecting achievement in mathematics of elementary level students $[F(1,86)=$ $0.03, p>.05]$.

Student achievement in mathematics enhanced after language integrated mathematics instruction, than after practice in solving
mathematics problems (control), among students with high nonverbal intelligence ( $M=64.21, S D=9.33, N_{l}=25$; control: $M=54.17$, $\left.S D=10.09, N_{2}=24, t=3.62, p<.01, d=1.03\right)$ as well as those with low non-verbal intelligence $\left(M=46.15, S D=10.79, N_{l}=20\right.$; control: $\left.M=37.06, S D=9.81, N_{2}=21, t=2.83, p<.01, d=0.88\right)$.

## B II The effect of language integrated mathematics instruction on

 achievement, over and above the effect of practice in solving mathematics problems, is true for the areas of school mathematics viz, algebra, arithmetic and geometry.11) After language integrated mathematics instruction, mean posttest scores of achievement in algebra ( $M=50.56, S D=12.34$ ) is significantly higher than that after practice in solving mathematics problems ( $M=$ 40.42, $S D=13.89$ ) $[t=3.66 ; p<.01]$; the effect of language integrated mathematics instruction being medium at the mean level (Cohen's $d=$ 0.77 ), with the highest advantage of approximately 30 percentile ranks in the lower quartile, of approximately 25 percentile ranks at the median, and an advantage of less than 15 percentile ranks at the third quartile, indicating that language integrated mathematics instruction is especially beneficial for students in the lower achievement strata though others are also benefited significantly.
12) After language integrated mathematics instruction, mean posttest scores of achievement in arithmetic $(M=52.96, S D=14.30)$ is significantly higher than that after practice in solving mathematics problems $(M=45.12, S D=15.26)[t=2.51 ; \mathrm{p}<.05]$ with a medium effect at the mean level (Cohen's $d=0.53$ ), equal to an advantage of
approximately 20 percentile ranks for those at the median and of 10 percentile ranks for those in the upper and lower quartiles.
13) After language integrated mathematics instruction, mean posttest scores of achievement in geometry $(M=64.10, S D=15.52)$ is significantly higher than that after practice in solving mathematics problems $(M=51.20, S D=16.72)[t=3.79 ; p<.01]$ with a medium effect at the mean level (Cohen's $d=0.79$ ), which is highest in the lower quartile with an advantage of approximately 30 percentile ranks, of approximately 25 percentile ranks at the median, and an advantage of approximately 15 percentile ranks at the third quartile, indicating that language integrated mathematics instruction is especially beneficial for students in the lower achievement strata though others are also benefited significantly.

## B III Language integrated mathematics instruction, in comparison to practice

 in solving mathematics problems, has significant and large effect on selfefficacy in mathematics of elementary school students and it compensates for adverse effect of low verbal comprehension as well as low non-verbal intelligence on self-efficacy.14) After language integrated mathematics instruction, mean gain scores of self-efficacy in mathematics ( $M=16.05, S D=4.05$ ) is significantly higher than that after practice in solving mathematics problems ( $M=$ 9.57, $S D=5.11$ ) $[t=6.67 ; p<.01]$; the effect of language integrated mathematics instruction being large at the mean level (Cohen's $d=1.41$ ), with the highest advantage of approximately 45 percentile ranks in the lower quartile, of approximately 40 percentile ranks at the median, and of approximately 20 percentile ranks at the third quartile.
i. There is significant interaction between language integrated mathematics instruction and verbal comprehension in Malayalam in effecting self-efficacy in mathematics of elementary level students [ $F$ $\left.(1,86)=6.99, p<.05, \eta_{p}{ }^{2}=0.08\right]$.
a. After language integrated mathematics instruction, the disadvantage in self-efficacy for low verbal comprehension group ( $M=15.75, S D$ $=3.09, N_{l}=22$ ) in comparison to high verbal comprehension group $\left(M=16.34, S D=4.85, N_{l}=23\right)(t=0.48, p>.05)$ was compensated such that, self-efficacy of at least 38 percent more students with low verbal comprehension enhanced at par with that of those having high verbal comprehension.
b. Among students with high verbal comprehension, practice in solving mathematics problems has large effect on their self-efficacy in mathematics ( $M=12.17, S D=4.67, N_{I}=23$ ) compared to those with low verbal comprehension $\left(M=6.85, S D=4.09, N_{2}=22\right)(t=$ 4.06, $p<.01, d=1.21$ ) such that around 88 percent students in low verbal comprehension group having lower self-efficacy in mathematics than an average student on self-efficacy in mathematics in high verbal comprehension group.
c. Language integrated mathematics instruction, compared to practice in solving mathematics problems, enhanced self-efficacy in mathematics among students with low verbal comprehension ( $M=$ 15.75, $S D=3.09, N_{l}=22$; control: $M=6.85, S D=4.09, N_{2}=22, t=$ 8.14, $p<.01, d=2.46$ ) and also among those with high verbal comprehension ( $M=16.34, S D=4.85, N_{I}=23$; control: $M=12.17$, $S D=4.66, N_{2}=23, t=2.96, p<.01, d=0.88$ ) such that
approximately 99 percent of those with low verbal comprehension and 80 percent of those with high verbal comprehension having received language integrated mathematics instruction had selfefficacy above the average student who received practice in solving mathematics problems.
ii. There is no significant interaction between language integrated mathematics instruction and non-verbal intelligence in effecting selfefficacy in mathematics of elementary level students $[F(1,86)=$ 0.03, $\mathrm{p}>.05]$.
a. Student self-efficacy in mathematics enhanced after language integrated mathematics instruction, than after practice in solving mathematics problems, among students with high non-verbal intelligence $\left(M=16.56, S D=4.11, N_{l}=25\right.$; control: $M=11.52, S D$ $\left.=4.94, N_{2}=24, t=3.89, p<.01, d=1.11\right)$ as well as those with low non-verbal intelligence ( $M=15.41, S D=3.99, N_{l}=20$; control: $M=$ 7.34, $S D=4.41, N_{2}=21, t=6.13, p<.01, d=1.92$ ).

## B4 Effect of language integrated mathematics instruction on self-efficacy,

 over and above the effect of practice in solving mathematics problems, is true for Self-efficacies in learning mathematics and in solving mathematics problems.15) After language integrated mathematics instruction, median of gain scores of self-efficacy in mathematics $(\operatorname{Mdn}=16.92)$ is significantly higher than that after practice in solving mathematics problems (Mdn $=$ 9.23), $[\mathrm{U}=481.50 ; \mathrm{p}<.01]$; the effect of language integrated mathematics instruction being large at the median level (Cohen's $\mathrm{d}=1.01$ ), with an
advantage of approximately 30 percentile ranks, of approximately 35 percentile ranks in the lower quartile, and of approximately 20 percentile ranks at the third quartile.
16) After language integrated mathematics instruction, mean gain scores of self-efficacy in solving mathematics problems $(M=13.01, S D=5.71)$ is significantly higher than that after practice in solving mathematics problems ( $M=19.64, S D=6.75$ ) $[t=5.03 ; p<.01]$; the effect of language integrated mathematics instruction being large at the mean level (Cohen's $d=1.06$ ), with the highest advantage of approximately 40 percentile ranks in the lower quartile, of approximately 34 percentile ranks at the median, and of approximately 15 percentile ranks at the third quartile.

## B 5 Effect of language integrated mathematics instruction is large, over and

 above the effect of practice in solving mathematics problems, for selfefficacies in algebra and geometry in school mathematics; it is only small on self-efficacy in arithmetic.17) After language integrated mathematics instruction, mean posttest scores of self-efficacy in algebra ( $M=79.30, S D=10.24$ ) is significantly higher than that after practice in solving mathematics problems ( $M=$ 70.51, $S D=11.12$ ) $[t=3.90 ; p<.01]$ _with a large effect (Cohen's $d=$ 0.82 ) which is uniform across the distribution with an advantage of around 25 percentile ranks.
18) After language integrated mathematics instruction, median of posttest scores of self-efficacy in arithmetic $(M d n=80)$ is significantly higher than that after practice in solving mathematics problems ( $M d n=74.67$ ), $[U=$ 733.50; $p<.01]$; the effect of language integrated mathematics instruction
being small at the median level (Cohen's $d=0.49$ ), with uniform advantage of approximately 16 percentile ranks upto the upper quartile, and only negligible advantage for students above the upper quartile.
19) After language integrated mathematics instruction, median of posttest scores of self-efficacy in geometry $(\operatorname{Mdn}=86.15)$ is significantly higher than that after practice in solving mathematics problems $(\mathrm{Mdn}=86.15)$, $[\mathrm{U}=510 ; \mathrm{p}<.01]$; the effect of language integrated mathematics instruction being large at the median level (Cohen's $\mathrm{d}=0.95$ ) with uniform advantage of approximately 32 percentile ranks up to the upper quartile; but only an advantage of approximately 23 percentile ranks for students above the upper quartile.

## $B 6$ Language integrated mathematics instruction has significant and large

 effect on attitude towards mathematics of elementary school students over and above the effect of practice in solving mathematics and it compensates for adverse effect of low verbal comprehension as well as low non-verbal intelligence on attitude towards mathematics.20) After language integrated mathematics instruction, mean gain scores of attitude towards mathematics $(M=12.34, S D=3.38)$ is significantly higher than that after practice in solving mathematics problems ( $M=$ 2.94, $S D=1.56$ ) $[t=16.96 ; p<.01]$ with large effect at the mean level (Cohen's $d=3.57$ ). Only 2 percent of the control and experimental groups have similar gain in attitude towards mathematics; i.e., 98 percent of students who received language integrated mathematics instruction had higher gain in attitude towards mathematics than those who had practiced mathematics problem solving in meantime.
i. After language integrated mathematics instruction, the disadvantage for low verbal comprehension group ( $M=13, S D=3.49, N_{l}=22$ ) in comparison to high verbal comprehension group ( $M=11.72, S D=3.21$, $\left.N_{I}=23\right)(t=1.28, p>.05)$ was compensated such that, attitude of at least 29 percent more students with low verbal comprehension enhanced at par with that of those having high verbal comprehension group.
a) After practice in solving mathematics problems, there was large effect for high verbal comprehension on attitude towards mathematics $\left(M=3.54, S D=1.14, N_{l}=23\right)$ over that of low verbal comprehension group $\left(M=2.31, S D=1.72, N_{2}=22\right)(t=$ 2.83, $p<.01, d=0.84$ ) with around 79 percent students in low verbal comprehension group having lower attitude towards mathematics than an average student on attitude towards mathematics in high verbal comprehension group.
b) Language integrated mathematics instruction, compared to practice in solving mathematics problems, enhanced attitude towards mathematics among students with low verbal comprehension ( $M=$ 13, $S D=3.49, N_{l}=23$; Control: $M=2.31, S D=1.72, N_{2}=22, t=$ $12.86, p<.01, d=3.88)$ and also among those with high verbal comprehension ( $M=11.72, S D=3.21, N_{l}=23$; Control: $M=3.54$, $\left.S D=1.14, N_{2}=23, t=11.53, p<.01, d=3.39\right)$.
ii. After language integrated mathematics instruction, the disadvantage for low non-verbal intelligence group ( $M=12.18, S D=3.28, N_{l}=20$ ) in comparison to high non-verbal intelligence group ( $M=12.47, S D=$ $\left.3.51, N_{l}=25\right)(t=0.28, p>.05)$ was compensated such that, attitude of at least 23 percent more students with low non-verbal intelligence enhanced at par with that of those having high non-verbal intelligence.
a) After practice in solving mathematics problems, there was medium effect for high non-verbal intelligence on attitude towards mathematics $\left(M=3.39, S D=1.19, N_{l}=24\right)$ over that of low non-verbal intelligence $\left(M=2.43, S D=1.79, N_{2}=21\right)(t=$ 2.15, $p<.05, d=0.63$ ) with around 73 percent students in low non-verbal intelligence group having lower attitude towards mathematics than an average student on attitude towards mathematics in high non-verbal intelligence group.
b) Language integrated mathematics instruction, compared to practice in solving mathematics problems, enhanced attitude towards mathematics among students with low non-verbal intelligence ( $M=12.18, S D=3.28, N_{1}=20$; control: $M=2.43, S D=1.79, N_{2}=$ $21, t=11.90, p<.01, d=3.69)$ and also among those with high nonverbal intelligence $\left(M=12.47, S D=3.51, N_{l}=25\right.$; control: $\left.M=3.39, S D=1.19, N_{2}=24, t=12.01, p<.01, d=3.46\right)$.

B 7 Effect of language integrated mathematics instruction is significant and large, over and above the effect of practice in solving mathematics, on dimensions of attitude towards Mathematics viz., like towards mathematics, engagement with mathematics, self-belief in mathematics, active learning of mathematics and enjoyment of mathematics.
21) After language integrated mathematics instruction, median of gain scores of like towards mathematics $(M d n=18.18)$ is significantly higher than that after practice in solving mathematics problems $(M d n=9.09)$, [ $U=277$; $\mathrm{p}<.01$ ] with large effect at the median level (Cohen's $\mathrm{d}=1.6$ ) equal to an advantage of approximately 50 percentile ranks in like towards mathematics.
22) After language integrated mathematics instruction, median of gain scores of engagement with mathematics $(M d n=15.56)$ is significantly higher than that after practice in solving mathematics problems ( $M d n=8.89$ ), $[U$ $=455 ; p<.01]$; the effect of language integrated mathematics instruction being large at the median level (Cohen's $\mathrm{d}=1.08$ ), with an advantage of approximately 40 percentile ranks up to the upper quartile and an advantage of approximately 21 percentile ranks above the upper quartile.
23) After language integrated mathematics instruction, mean gain scores of self-belief in mathematics $(M=22.41, S D=12.14)$ is significantly higher than that after practice in solving mathematics problems ( $M=$ 10.67, $S D=4.49$ ) $[t=6.09 ; p<.01]$ with large effect at the mean level (Cohen's d = 1.28); self-belief in mathematics of students beyond first quartile have an advantage of almost 50 percentile ranks than that after practice of mathematics problem solving; but a little less, approximately 37 percentile ranks advantage to students in the lower quartile.
24) After language integrated mathematics instruction, mean gain scores of active learning of mathematics $(M=16.94, S D=9.81)$ is significantly higher than that after practice in solving mathematics problems ( $M=$ 10.44, $S D=4.10$ ) $[t=4.10 ; p<.01]$ with large effect at the mean level (Cohen's $d=0.86$ ) equal to an advantage of approximately 50 percentile ranks for students beyond the first quartile; but a reduced, approximately 20 percentile ranks advantage to students in the lower quartile.
25) After language integrated mathematics instruction, median gain scores of enjoyment of mathematics $(M d n=15.56)$ is significantly higher than that after practice in solving mathematics problems ( $M d n=8.89$ ), $[U=$ 301.50; $p<.01$ ] with large effect at the median level (Cohen's $\mathrm{d}=1.52$ ) equal to an advantage of approximately 50 percentile ranks in enjoyment of mathematics.

## Summary of the Effect of Evidence based Instruction Focusing on Language of Mathematics on Self-efficacy and Attitude Measures

The findings from the experimental phase regarding the effect of language integrated mathematics instruction on the achievement, self-efficacy and attitude can be put in context by comparing their mean scores obtained 1 ) during the test development phase in a sample representative of the population (norm group), 2) during pretesting of control group 3) during pretesting of experimental group 4) during post testing of control group and 5) during post-testing of experimental group as shown in Figure 33.


Figure 33. Multiple bar diagram of self-efficacy in mathematics and attitude towards mathematics and their components with error bars showing 95\% confidence intervals giving a visual comparison of their mean scores obtained from 1) survey of a larger sample representative of the population 2) \& 3) pretest scores in control and experimental groups and 4) \& 5) post-test scores in control and experimental groups respectively

As per Figure 33, the study sample was not significantly different from population in case of attitude towards mathematics and self-efficacy in mathematics and hence the findings from the experiment are generalizable to the population of standard seven students in elementary school of Kerala.

Figure 33 reveals that self-efficacy in mathematics enhances even after practice in mathematics problem solving, compared to standard seven students (in the norm group) who merely received mathematics instruction usually provided in the school; i.e., guided practice in problem solving enhances selfefficacy in solving mathematics problems. However, language integrated mathematics instruction strengthens self-efficacy beliefs both in learning mathematics and in solving mathematics problems much more than that after practice in solving mathematics problems.

Figure 33 further reveals that attitude towards mathematics or its dimensions do not enhance even after guided practice in mathematics problem solving beyond regular instruction on the topics. However, language integrated mathematics instruction, if used instead of guided practice in mathematics problem solving, enhances attitude towards mathematics and its dimensions viz., like towards mathematics, engagement with mathematics, self-belief in mathematics, active learning in mathematics and enjoyment of mathematics not only significantly but to a large extent.

The effect of language integrated mathematics instruction, in terms of gain in percentile rank points, on mathematics learning outcomes viz; achievement in mathematics, algebra, geometry, and arithmetic; self-efficacy in mathematics, its dimensions, attitude towards mathematics and its dimensions are summarized in Figure 34.


Figure 34. Gain in percentile rank points after language integrated mathematics instruction over and above practice in solving mathematics problems

Figure 34 shows that language integrated mathematics instruction has medium effect on achievement in arithmetic and small effect on self-efficacy in arithmetic. But on all other variables viz., achievements in mathematics in total or in algebra or geometry, there is a gain of 25 percentile rank points after language integrated mathematics instruction, indicating large effects.

Figure 34 further shows that self-efficacy in mathematics or in algebra or in geometry whether to learn mathematics or to solve problems in mathematics is further strengthened through language integrated mathematics instruction over and above practice in solving mathematics problems equivalent to a gain of 30 or more percentile ranks.

Figure 34 further shows that the outcome most influenced by language integrated mathematics instruction was attitude towards mathematics. Emotional or cognitive dimensions of attitude such as like towards mathematics, enjoyment of mathematics and self-belief in mathematics are influenced by language
integrated mathematics instruction more than psychomotor aspects of attitude like active learning in mathematics or enjoyment in mathematics.

The effect of language integrated mathematics instruction on mathematics learning outcomes in terms of gain in percentile rank points of achievement in mathematics, self-efficacy in mathematics and attitude towards mathematics on students in the three quartiles of distribution of the respective variables are summed up in Figure 35.


Figure 35. Gain in percentile rank points after language integrated mathematics instruction over and above practice in solving mathematics problems in three quartiles.

Figure 35 shows that generally, effect of language integrated mathematics instruction is more among the students in the lower quartile, and comparatively less among the students in the upper quartile, of achievement and self-efficacy in mathematics; especially so for achievements in algebra and geometry and self-
efficacies in mathematics, both for learning and for solving problems. However, attitude towards mathematics, specifically like towards and enjoyment in mathematics benefitted, from language integrated mathematics instruction, equally among students irrespective of their previous levels on these variables. The only outcomes that had lesser benefit among lower quartile students, than upper quartile students, were active learning in mathematics which enhanced significantly less among pupil in the lower quartile than the others.

## Tenability of the Hypotheses

Tenability of the hypotheses formulated for the study are verified based on the findings and is stated in Table 60.

Table 60
Tenability of the Hypotheses

| $i$ $i$ $i$ |  | Hypothesis | $$ |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | Language integrated mathematics instruction significantly enhances elementary school students' achievement in mathematics | Accepted | 10 |
| 2 | 1 i | Language integrated mathematics instruction significantly enhances elementary school students' achievement in mathematics equally for high and low levels of previous achievement in mathematics | Accepted | 10 i |
| 3 | 1 ii | Language integrated mathematics instruction significantly enhances elementary school students' achievement in mathematics equally for high and low levels of verbal comprehension in Malayalam | Accepted | 10 ii |
| 4 | 1 iii | Language integrated mathematics instruction significantly enhances elementary school students' achievement in mathematics equally for high and low levels of non-verbal intelligence in mathematics | Accepted | 10 iii |


| i |  | Hypothesis |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 2 i | Language integrated mathematics instruction significantly enhances elementary school students' achievement in algebra | Accepted | 11 |
| 6 | 2 ii | Language integrated mathematics instruction significantly enhances elementary school students' achievement in arithmetic | Accepted | 12 |
| 7 | 2 iii | Language integrated mathematics instruction significantly enhances elementary school students' achievement in geometry | Accepted | 13 |
| 8 | 3 | Language integrated mathematics instruction significantly enhances elementary school students' self-efficacy in mathematics | Accepted | 14 |
| 9 | 3 i | Language integrated mathematics instruction significantly enhances elementary school students' self-efficacy in mathematics equally for high and low levels of verbal comprehension in Malayalam | Not accepted | 14 i |
| 10 | 3 ii | Language integrated mathematics instruction significantly enhances elementary school students' self-efficacy in mathematics equally for high and low levels of non-verbal intelligence in mathematics | Accepted | 14 ii |
| 11 | 4 i | Language integrated mathematics instruction significantly enhances elementary school students' self-efficacy in learning mathematics | Accepted | 15 |
| 12 | 4 ii | Language integrated mathematics instruction significantly enhances elementary school students' self-efficacy in solving mathematics problems | Accepted | 16 |
| 13 | $5 i$ | Language integrated mathematics instruction significantly enhances elementary school students' self-efficacy in algebra | Accepted | 17 |
| 14 | 5 ii | Language integrated mathematics instruction significantly enhances elementary school students' self-efficacy in arithmetic | Accepted | 18 |


| i | $\begin{aligned} & \stackrel{n}{0} \\ & \stackrel{y}{ \pm} \\ & \stackrel{0}{2} \\ & \underset{x}{2} \dot{2} \end{aligned}$ | Hypothesis | n <br> $\substack{0 \\ \sim \\ \sim \\ \hline}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| 15 | 5 iii | Language integrated mathematics instruction significantly enhances elementary school students' self-efficacy in geometry | Accepted | 19 |
| 16 | 6 | Language integrated mathematics instruction significantly enhances elementary school students' attitude towards mathematics | Accepted | 20 |
| 17 | $6 i$ | Language integrated mathematics instruction significantly enhances elementary school students' attitude towards mathematics equally for high and low levels of verbal comprehension in Malayalam | Not accepted | 20 i |
| 18 | 6 ii | Language integrated mathematics instruction significantly enhances elementary school students' attitude towards mathematics equally for high and low levels of non-verbal intelligence in mathematics | Not accepted | 20 ii |
| 19 | 7 i | Language integrated mathematics instruction significantly enhances elementary school students' like towards mathematics | Accepted | 21 |
| 20 | 7 ii | Language integrated mathematics instruction significantly enhances elementary school students' engagement with mathematics | Accepted | 22 |
| 21 | 7 iii | Language integrated mathematics instruction significantly enhances elementary school students' self-belief in mathematics | Accepted | 23 |
| 22 | 7 iv | Language integrated mathematics instruction significantly enhances elementary school students' active learning of mathematics | Accepted | 24 |
| 23 | 7 v | Language integrated mathematics instruction significantly enhances elementary school students' enjoyment of mathematics | Accepted | 25 |

## Discussion of Findings

## Content and Language of Mathematics are Perceived as Much Difficult as

## Problem Solving in Mathematics

High percentage of students feel difficulty in number systems, especially in using decimals and fractions; as much as specific tasks in problem-solving like understanding algebraic problems and identifying equations to solve problems, are difficult for majority of students. Ramanujam, Subramanian and Sachdev (2007) has observed that the introduction of operations on fractions coincides with the beginnings of fear of mathematics. Students of 12-13 age group have difficulty in solving equations with letter symbols, than those with numbers; this may be because development of algebraic thinking unfolds over a long period of time (Susac, Bubic, Vrbanc \& Planinic, 2014). Others have also found that only less than 10 percent of students at the seventh and eighth grade level were procedurally or conceptually ready even to translate written words into algebraic equations (Capraro \& Joffrion, 2006) and even in secondary grades, higher frequency of errors originated in the translation from the verbal to the symbolic representation system due to the peculiar features of the algebraic language like variables and structural compilation errors (Domingo, Molina, Canadas \& Castro, 2012).

One in three students feels difficulty in word problems, mathematical abstractions, selecting mathematical operations, and symbols and notations; may be because students of this age group mostly use concrete strategies and have limitations with abstract reasoning. Ramanujam, Subramanian and Sachdev (2007) has observed that operations on natural numbers usually form a major part of primary mathematics syllabi, at the expense of development of number sense and related skills, and suggested that operations should be introduced contextually, followed by the development of language and symbolic notation, and that algebraic notation be best seen as a means by which students gain fluency in using the new language.

Tasks like understanding algebraic problems, analyzing geometrical figures, understanding and translating word problems and using number systems are evidently related to language of mathematics. Other studies have also found similar results. For example, $1 / 5^{\text {th }}$ of grade five learners had very poor performance in translating word problems to mathematics symbols and equations (Cruz \& Lapinid, 2014). In Grades six and seven also, learners were less successful in problem solving when the text had higher grammatical complexity and more advanced vocabulary and the poorest performance expectedly were on problems with both complex text and relatively difficult mathematics (Barbu, 2010).

Generally, difficulty in learning mathematics is contributed to both by nature of mathematics content and nature of mathematics teaching-learning. However, this study reveals specifically that difficulties in mathematical tasks are perceived as related more with nature of its content than with nature of its teachinglearning processes. Difficulties with equations, operations and mathematical abstractions have substantial relation with perceived difficulties sourcing from nature of mathematics content but not with difficulties from nature of mathematics teaching-learning. This may be because the development of algebraic thinking is a process which unfolds over a long period of time as it requires more abstract, rulebased strategies (Susac, Bubic, Vrbanc \& Planinic, 2014).

## For elementary school students, difficulties in learning mathematics source

 equally or even more from surface structures than deep structures of its language.Language related difficulties in learning mathematics for elementary school students, source more from vocabulary components such as morphology, general and specific terms, symbols, conventions and semantics than from
sentence level components such as syntax of algebraic or numeric expressions and pragmatics.

Analysis of achievements in various aspects of mathematical language revealed that it is the lowest in parts of words, general and mathematics specific terms, word meaning in specific context, basic mathematics symbols and mathematical conventions and comparatively lower in expressions in algebraic and natural language. Percentage achievements in mathematical terms, both general (38.23\%) and mathematics specific (40.36\%), basic mathematics symbols (41.01\%), syntactic conventions (37.07\%), algebraic expression (48.57\%), and natural expression (50.41\%) are significantly less. Achievement in syntactic and pragmatic elements of mathematical language such as that in numeric expression (63\%), commonly used fractions (63.68\%), real life word problems (64.38\%), reading graph (67.8\%) and identifying operations from key words ( $68.7 \%$ ) though are relatively high. Achievement in subcategories of pragmatics do not show any significant difference. Another recent research has also highlighted the role of "a robust vocabulary knowledge base, flexibility, fluency and proficiency with numbers, symbols, words, and diagrams; and comprehension skills" in students learning mathematics (Riccomini, Smith, Hughes \& Fries, 2015). But there were contrary observations that syntax though a strong predictor of mathematics performance among young children, vocabulary is not a significant predictor of mathematics performance among primary school students (Chow \& Ekholm, 2019). When the same content is expressed in natural language and numeric language, achievement is significantly higher in numeric expression, is echoed by other researches (Geary, 1996; Koedinger \& Nathan, 2004) reporting that children make more errors in solving word problems than in solving number problems.


#### Abstract

Along with problem solving and attendant learning requirements related to the nature of mathematics, elementary school students' difficulties in mathematics source from density of difficult concepts, prevalence of symbols, notations, and unfamiliar words that make their learning strenuous, rote and uninteresting


Vast majority of upper primary students attribute prominence of problem solving and need for regularity in attending classes as reasons for difficulty in learning mathematics, and to near equal percentage of students, need for external support, need for repeated practice, difficulty of concepts, large number of concepts, need for strenuous attention, difficulty in understanding questions and need for precision in understanding also cause difficulty. Prevalence of symbols and notations, necessity to learn unfamiliar words, need for rote learning and impracticability in daily life are considered as elements creating high difficulty for $1 / 3^{\text {rd }}$ of students. Previous research also identified word problems as belonging to the most difficult and complex problem types that pupils encounter during their elementary-level mathematical development. Such studies (Gooding, 2009; Daroczy, Wolska, Meurers \& Nuerk, 2015) also identified a number of linguistic verbal components not directly related to arithmetic, like the linguistic complexity of the problem text, the numerical complexity of the problem, and the relation between the linguistic and numerical complexity of a problem as contributing greatly to their difficulty. Writing number sentences, carrying out the calculation and interpreting the answer in the context of the question also were mathematical difficulties in solving word problems among year five children of English primary schools (Gooding, 2009).

## Task difficulty in mathematics correlate more with nature of content of mathematics, than with its teaching learning; though for word problem related tasks, both the nature of content and teaching-learning are equally important.

Except for problem solving related tasks, difficulties in tasks in school mathematics correlate more with nature of mathematics content than with its teaching learning. Task difficulty in equations and operations is substantially correlated with nature of mathematics content, whereas it has low correlation with nature of mathematics teaching-learning. Task difficulty in symbols and notations, and arithmetic operations have significant though low correlation with both nature of mathematics content as well as nature of mathematics teachinglearning, but these difficulties correlate significantly more with the nature of content than with nature of teaching learning. Difficulty in problem solving related tasks viz., problem solving competence, understanding word problems, and translation word problems have comparable, low yet significant, correlation with nature of content and nature of teaching learning of mathematics. Task difficulties in understanding numbers, number systems and mathematical abstractions have significant but low correlation with nature of mathematics content; but have only negligible correlation with nature of mathematics teaching-learning. Relation between difficulties from nature of content of mathematics and task difficulties in mathematics tends to increase as the tasks becomes complex and abstract. These observations reflect previous research observation that correlations between measures of language and cognitive development and mathematics achievement tended to increase from grade one to six (Souviney, 1983).

## Language integrated mathematics instruction has significant effect of medium size over the effect of practice in solving mathematics problems, on achievement in mathematics-including algebra, arithmetic and geometryirrespective of students' level of verbal comprehension, non-verbal intelligence and previous achievement in mathematics

After language integrated mathematics instruction, achievement in mathematics gains approximately 25 percentile ranks for those in between the first and third quartiles and approximately 15 percentile ranks for those in the upper and lower quartiles, such that 76 percent of students who had language integrated mathematics instruction being above the average student who instead practiced solving mathematics problems. Still, achievement in mathematics was higher in students with high previous achievement, high verbal comprehension or high non-verbal intelligence than those low on these abilities whether they were taught with language integrated mathematics instruction or with practice in solving mathematics problems. That reading skills and comprehension as well as students' understanding of mathematical language (Mbugua, 2012), general verbal ability (Vukovic \& Lesaux, 2012) highly influence achievement does echo previous research findings.

Benefits from language integrated mathematics instruction for achievement in algebra and geometry are higher for students in the lower achievement strata with a gain of approximately 30 percentile ranks in the lower quartile, of approximately 25 percentile ranks at the median, and of less than 15 percentile ranks at the third quartile. However, its benefit on achievement in arithmetic is lesser, of medium level, with gain of approximately 20 percentile ranks for those at the median and of 10 percentile ranks for those in the upper and lower quartiles such that 69 percent of students in language integrated
mathematics instruction group are above the average student who practiced solving mathematics problems in the meantime.

Most of the previous researches also have reported beneficial effects of language of mathematics instruction, but they largely focused on vocabulary than other aspects of language, but a few studies fail to observe such benefits on achievement or problem solving in mathematics too. Georgius (2008) for example found that for the majority of students understanding mathematical words is important and it increased their achievement and that after receiving vocabulary instruction, majority of students improved in their overall understanding of mathematical concepts that in turn increased their achievement. And, exposure to mathematical vocabulary through daily activities improve students test scores as well self-efficacy beliefs in learning mathematics in comparison with those who did not attend vocabulary activities (Larson, 2007) is also previously observed.

Gifford and Gore (2010) also found out that focused academic vocabulary instruction is beneficial for all types of learners, especially for struggling learners as at least a 33 percent increase is there in the gains on standardized tests and their perceived self-efficacy beliefs were also improved. Long terms effects of academic vocabulary instruction were also analyzed as the study spanned through three academic years in which first year with no focus on vocabulary instruction, and in its second year, vocabulary instruction was an integral part. There is contradicting findings. For example, fourth-grade students improved in all areas of mathematical communication skills especially in higher-level skills like explaining how they got their answers and why they solved a problem the way they did. However, the success of the third-grade class was restricted to lower level communication skills, like explaining what their answer is (Huggins
\& Maiste, 1999). vocabulary instruction strategy was less effective with those students who had a low verbal comprehension. It also indicates that students who got vocabulary instructional strategy had greater improvements in their understanding of mathematical vocabulary in the test (Kenyon, 2016). Exposure to mathematical language can positively affect students' mathematics skills (Purpura, Napoli, Wehrspann \& Gold, 2017). But there are studies against this trend of positive outcomes of vocabulary instruction, for example, vocabulary tutoring intervention improved students' vocabulary whereas it did not have an effect on students with the algebraic problem-solving skills (Hollingsworth, 2019). However, the present study, is different from the above cited ones in that the language integrated mathematics instruction employed had focused on deeper structures of the language of mathematics as well, though one important focus in it was vocabulary.

Effect of Language integrated mathematics instruction on self-efficacy in mathematics is large, specifically in algebra, and geometry- and also on selfefficacy in learning mathematics and in solving mathematics problems but it is significant yet small on self-efficacy in arithmetic.

The large advantage from language integrated mathematics instruction on self-efficacy in mathematics is the highest in the lower quartile of students who gain approximately 45 percentile ranks, gain is near 40 percentile ranks at the median, and is of approximately 20 percentile ranks at the third quartile, such that 92 percent of students in language integrated mathematics instruction group are above the average student in the group which practiced solving mathematics problems.

Self-efficacy in solving mathematics problems seems to gain more, after language integrated instruction, among the students in lower quartile where gain
is the highest with approximately 40 percentile ranks advantage, of approximately 34 percentile ranks at the median, and of approximately 15 percentile ranks at the third quartile. Self-efficacy in learning mathematics seems to gain lesser, after language integrated instruction, with a gain of approximately 30 percentile ranks at the median, of approximately 35 percentile ranks in the lower quartile, and of approximately 20 percentile ranks at the third quartile. However, the average gain is equal in both self-efficacies- in learning and in solving problems, with 84 percent of students who had language integrated mathematics instruction are above the average student who practiced solving mathematics problems.

The gain in self-efficacy in geometry after language integrated mathematics instruction is the most pronounced up to the upper quartile -with gain of approximately 32 percentile ranks- and an advantage of only approximately 23 percentile ranks for students in the upper quartile such that 82 percent of students who had language integrated mathematics instruction being above the average student who instead practiced solving mathematics problems. The large gain in self-efficacy in algebra after language integrated mathematics instruction is uniform across the distribution with a gain of around 25 percentile ranks. The gain from language integrated mathematics instruction on self- efficacy in arithmetic is small, with approximately 16 percentile ranks gain up to the upper quartile, and only negligible advantage for students in upper quartile.

Advantage in self-efficacy in mathematics learning and solving problems after language integrated instruction were reported by earlier studies as well. For instance, Larson, (2007) also found that exposure to mathematical vocabulary through daily activities improve students' self-efficacy beliefs in learning mathematics, and Sample (2009) observed that student level of self-confidence in solving mathematics problems increases significantly due to the increase in oral
communication. However, this study could refine these findings, by specifying the effects of language integrated mathematics instruction on the three areas of school mathematics at the end of elementary stage of schooling and in gauging the size of such effects. It specified that effect of language integrated mathematics instruction on self-efficacy in mathematics is large, specifically in algebra, and geometry, and also on self-efficacy in learning mathematics and in solving mathematics problems but it is significant yet small on self-efficacy in arithmetic. Observation that language ability has more effect on predicting later gains in primary grades geometry, than of arithmetic or algebra (Vukovic \& Lesaux, 2013) partly matches with that of with this study.

## Language integrated mathematics instruction compensates for adverse effect

 of low verbal comprehension as well as low non-verbal intelligence on selfefficacy in mathematicsAfter practice in solving mathematics problems, 88 percent of students with low verbal comprehension had lower self-efficacy in mathematics than a student average on self-efficacy in high verbal comprehension group. But, after language integrated mathematics instruction, 99 percent of students with low verbal comprehension and 80 percent of those with high verbal comprehension had self-efficacy in mathematics above an average student in respective groups but had received practice in solving mathematics problems only. After language integrated instruction, self-efficacy of at least 88 percent more students with low verbal comprehension enhanced at par or beyond that of those average on selfefficacy in students with high verbal comprehension but had practice in solving mathematics problems only.

Student self-efficacy in mathematics enhanced after language integrated mathematics instruction, than after practice in solving mathematics problems,
among students with high non-verbal intelligence. Disadvantage in self-efficacy in mathematics of low non-verbal intelligence group after practice in solving mathematics problems in comparison to their high non-verbal intelligence counter parts also disappeared if they are provided language integrated mathematics instruction. Non-verbal intelligence is found to be a significant predictor of measures of mathematics performance at secondary level.

## Language integrated mathematics instruction has significant large effect over

 the effect of practice in solving mathematics problems, on attitude towards mathematics and its components such as like towards mathematics, engagement with mathematics, self-belief in mathematics, active learning of mathematics and enjoyment of mathematicsNinety eight percent of elementary school students who received language integrated mathematics instruction had higher gain in attitude towards mathematics than those who practiced problem solving in mathematics. Students who received language integrated mathematics gained approximately 50 percentile ranks in like towards mathematics, and enjoyment of mathematics, than if they had instead practiced solving mathematical problems.

In self-belief in mathematics, language integrated mathematics instruction yielded a gain of almost 50 percentile ranks in students beyond the first quartile and of approximately 37 percentile ranks in the lower quartile than that after practice of mathematics problem solving such that 88 percent of students in language integrated mathematics instruction group are above the average student in the group which practiced solving mathematics problems.

In engagement with mathematics, language integrated mathematics instruction, however yielded a lesser gain only of approximately 40 percentile
ranks up to the upper quartile and of approximately 21 percentile ranks in the upper quartile, such that approximately 84 percent of students after language integrated mathematics instruction were above the students average on engagement with mathematics after practice in solving mathematics problems.

In active learning of mathematics, language integrated mathematics instruction, yielded a still reduced gain only of approximately 50 percentile ranks but remarkably for already better off students beyond the first quartile and of approximately 20 percentile ranks only to students in the lower quartile, such that approximately 79 percent of students after language integrated mathematics instruction were above the students average on active learning in mathematics after practice in solving mathematics problems.

Effect of language integrated mathematics instruction on psychomotor aspects of attitude towards mathematics viz., engagement and active learning in mathematics is large but is smaller in size than affective and cognitive aspects of attitude.

## Language integrated mathematics instruction compensates for adverse effect

 of low verbal comprehension as well as low non-verbal intelligence on attitude towards mathematicsAfter practice in solving mathematics problems, among students low on verbal comprehension, 79 percent of students had lower attitude towards mathematics compared to an average student on attitude towards mathematics in high verbal comprehension group. However, after language integrated mathematics instruction, attitude towards mathematics of 29 percent more students with low verbal comprehension enhanced at par with that of those having high verbal comprehension group.

Student attitude towards mathematics enhanced after language integrated mathematics instruction, than after practice in solving mathematics problems, among students with high non-verbal intelligence.

Disadvantage in attitude towards mathematics of low non-verbal intelligence group after practice in solving mathematics problems in comparison to their high non-verbal intelligence counter parts also disappeared if they are provided language integrated mathematics instruction.

Language integrated mathematics instruction has large effects on affective outcomes namely attitude towards mathematics and self-efficacy in mathematics and has medium effect on achievement in mathematics

After language integrated mathematics instruction, achievement in mathematics has gain of 25 percentile rank points and self-efficacy in mathematics has gain of 30 percentile rank points. Whereas after language integrated mathematics instruction, attitude towards mathematics has gain of 50 percentile ranks. Effect of language integrated mathematics instruction on achievement and self-efficacy in mathematics is more at the lower quartile, than elsewhere, but that on attitude towards mathematics is high all along the distribution.

Practice in mathematics problem solving does enhance Self-efficacy in mathematics, but not Attitude towards mathematics or its dimensions

Practicing problem solving enhances self-efficacy in solving mathematics problems. However, language integrated mathematics instruction strengthens self-efficacy beliefs both in learning mathematics and in solving mathematics problems much more than that after practice in solving mathematics problems.

Attitude towards mathematics or its dimensions does not enhance even after continued practice in mathematics problem solving for a significant period.

However, language integrated mathematics instruction for the same duration significantly and to large extend enhances attitude towards mathematics and its dimensions. Other researchers (Larson, 2007; Mbugua, 2012) have also found that as the students understood the language of mathematics, their confidence, attitudes, and scores all began to improve.

Attitude towards mathematics, especially the like towards mathematics and enjoyment of mathematics gained very high all across the distribution, whereas self-belief and active learning in mathematics improved less in the lower quartile, than in the upper or middle levels of distribution. Though engagement with mathematics enhanced, it was to a less extent than like or enjoyment of mathematics.

The above inferences from this study reinforces observation by national focus group (2007) that language used in our textbooks must be sensitive to language uses of all children, equally applies more to teachers and their classroom language as it is they who translated the textbook and its language to local context and the life experience and language background of individual learner. The findings reiterate that students learning of mathematics require "a robust vocabulary knowledge base, flexibility, fluency and proficiency with numbers, symbols, words, and diagrams; and comprehension skills." Precise and unambiguous use of language and rigor in formulation are important characteristics of mathematical treatment, and these constitute values to be imparted by way of mathematics education in deliberate, conscious and stylized way of notation, conventions and their use (Ramanujam, Subramanian \& Sachdev, 2007) be seen as endpoints than means of mathematics education at upper primary stage and hence be explicitly taught through more learner friendly means.

## Limitations of the Study

This study found out the language related difficulties in learning mathematics and effectiveness of language integrated mathematics instruction in enhancing mathematics learning outcomes. Even though the language integrated mathematics instruction is found to be effective in enhancing cognitive and affective learning outcomes in mathematics at the elementary school level in Malayalam medium schools of Kerala, this study has some limitations also, as listed hereunder.

In this study, the language of mathematics instruction was given after or along the actual classroom instruction. Language integrated mathematics instruction was given in addition to the classroom instruction and after the normal classroom hours by the schoolteacher for logistical reasons. Actual integration of language strategies throughout the lesson was not done as that could have replaced the schoolteacher with the experimenter as the mathematics instructor. Hence to balance the additional instructional time, the instruction most often used by mathematics teachers to enhance their students' mathematics achievement, i.e., guided problem solving was added to the control group. The real magnitude of the influence of language integrated mathematics instruction could have been better gauged if there were one more no treatment control group or if the language integrated mathematics instruction was imparted at the outset and as part of normal instruction of every unit. Influence of language integrated mathematics instruction when done with the classroom instruction could have added to the contextual validity of the findings of this research. However, in either case of language integrated mathematics instruction there is no reason to believe that the impact could have been lesser.

The small sample size in the experimental phase and nature of tools for measurement might have limited, though in a meagre way, the interpretability and generalizability of the findings in some instances as in: 1) some measures did not have distributions that permit independent samples $t$ test and factorial ANOVA, thus imposing the use of non-parametric tests and repeated use of $t$ test for comparison of means. However, despite this, the obtained results were put together, through the use of effect size interpretations irrespective of the exact statistical tests used; 2) for similar reasons, language integrated mathematics instruction strategy was found not statistically effective in overcoming the influence of verbal comprehension in Malayalam and non-verbal intelligence on achievement in mathematics among elementary level students, despite the scores having improved visibly. Low sample size in subsamples based on levels of verbal comprehension in Malayalam and non-verbal intelligence may be one of the reasons for this observation.

On the experimental design, the following were identified limitation that might have restricted the interpretability and comparability of the findings. 1) Pretests was conducted in self-efficacy in mathematics and attitude towards mathematics only. Other measures namely, achievement and self-efficacy in the five select mathematics units were only post tested, in order to prevent the testing effects; as these tests were evidently content specific. This has resulted in two methods of studying effects - comparison of gain scores in the former set of variables, and comparison of posttests scores in the latter sets of variables; 2) Control and experimental intervention were given to students in the same school in order to control school contextual factors, at the expense of adding the risk of control- experiment diffusion across the peers, which though is considered minimal, and such effect if any could have been random and neutralized;
3) a retention study if added to the design could have made the results even more compelling though language integrated instruction was given for relatively longer duration of half an academic year and for five units.

## Conclusion

This study explored the language related difficulties in learning mathematics and examined the effectiveness of an evidence-based instruction focusing on the language of mathematics in enhancing mathematics learning outcomes in terms of achievement and self-efficacy in mathematics and its major areas namely arithmetic, algebra and geometry, and also in terms of attitude towards mathematics among elementary school students in Kerala. Students perceive difficulties in mathematical tasks - including understanding algebraic problems, analyzing geometrical figures, understanding and translating word problems - that relate to the nature of its content, equally or even more than to its teaching-learning process. Moreover, much of these nature of mathematics content in turn link to its language elements such as understanding questions, unfamiliar words and symbols and notations. The need and importance of language integrated mathematics instruction is evidenced from the findings that language related difficulties in school mathematics is large, especially in general and mathematical terms, mathematical symbols, syntactic conventions and algebraic expressions.

The evidence-based intervention through language integrated mathematics instruction that focus on the language of mathematics enhanced achievement in mathematics-including algebra, arithmetic and geometry- irrespective of students' level of verbal comprehension, non-verbal intelligence and previous achievement in Mathematics, than guided practice in solving mathematics problems. Such language integrated mathematics instruction enhanced also self-efficacy in

## 372

 EVIDENCE BASED INSTRUCTION OF LANGUAGE OF MATHEMATICSmathematics, specifically in algebra, and geometry- and self-efficacies in learning mathematics and in solving mathematics problems to a large extent, but in arithmetic the increase in self-efficacy is to a lesser extent. Language integrated mathematics instruction enhances attitude towards mathematics and its components such as like towards mathematics, engagement with mathematics, self-belief in mathematics, active learning of mathematics and enjoyment of mathematics. Moreover, this strategy compensates for adverse effects of low verbal comprehension and low non-verbal intelligence on self-efficacy in mathematics as well as on attitude towards mathematics.

Language integrated mathematics instruction has medium effect on achievement in arithmetic and small effect on self-efficacy in arithmetic. But on all other variables viz., achievements in mathematics in total or in algebra or in geometry, the gain is of 25 percentile rank points or more after language integrated mathematics instruction.

Self-efficacy in mathematics or in algebra or in geometry whether to learn mathematics or to solve problems in mathematics is further strengthened equivalent to a gain of 30 or more percentile ranks through language integrated mathematics instruction over and above practice in solving mathematics problems.

The outcome that was most influenced by language integrated mathematics instruction was attitude towards mathematics. Emotional or cognitive dimensions of attitude such as like towards mathematics, enjoyment of mathematics and selfbelief in mathematics are influenced by language integrated mathematics instruction more than psychomotor domains of attitude like active learning in mathematics or enjoyment in mathematics.

Generally, the effect of language integrated mathematics instruction is more at the lower quartile in case of achievement and self-efficacy in
mathematics and it is more at median level in case of attitude towards mathematics. After language integrated mathematics instruction, increase in achievement especially in algebra and geometry were the highest in the lower quartile, and the least in the upper quartile of the distribution. But the gain in achievement in arithmetic was highest in the median and less at the two ends of the distribution. Gain in self-efficacy in geometry likewise was more in the lower end and middle of the distribution than in the upper half of the distribution. Selfefficacies in learning mathematics and solving mathematics problems likewise gained more in the lower quartile and less in the upper quartile.

In summary, language integrated mathematics instruction enhances affective outcomes of mathematics namely attitude towards mathematics and self-efficacy in mathematics to a large extent and enhances achievement in mathematics to a lesser extent. Furthermore, practice in mathematics problem solving does enhance self-efficacy in mathematics, but not attitude towards mathematics or its dimensions.

The above inferences from this study reinforces observation by National focus group (2007) that language used in our textbooks must be sensitive to language uses of all children, equally applies more to teachers and their classroom language as it is they who translated the textbook and its language to local context and the life experience and language background of individual learner. The findings reiterate that students learning of mathematics require "a robust vocabulary knowledge base; flexibility; fluency and proficiency with numbers, symbols, words, and diagrams; and comprehension skills."

Truly, the challenges in learning mathematics go beyond the language issues. But, for students to be successful in attaining the aims of mathematics
learning, the linguistic challenges, which is often neglected, need also to be addressed. Mathematics teaching-learning in schools among other things has to value precise and unambiguous use of language and rigor in formulation as endpoints and hence do integrate these in a deliberate, conscious and yet holistic approach. If textbooks, teachers, classroom environments and teaching-learning processes give required focus on the language of mathematics in a learner appropriate way, it will considerably reduce the feeling of difficulty in mathematics and further enhance cognitive and affective outcomes of mathematics learning.

## Educational Implications

Implications of this study spread over different aspects of school mathematics from curricular objectives, through curriculum materials and resources, transaction strategies in out of classrooms, testing and assessment practices in school mathematics, teachers and their education and further research and refinements of mathematics education through local/regional languages.

## 1. Language of mathematics has to receive due attention in elementary school

- Mathematics instruction should not be limited to practice of problem solving. It should pay adequate attention to the nature of language used as a medium of instruction in mathematics classrooms.
- Due attention should be given to the special language and grammar of mathematics - like technical and semi technical words and their morphology, symbols, notations, syntax, expressions, equations and the like and to language in multi linguistic non-English classrooms contexts where English, Greek or/and Latin letters, abbreviations, nouns and verbs are involved.

2. Pay attention to linguistic features specific to different areas of school mathematics -arithmetic, algebra and geometry.

- Mathematics education in schools should realize that the majority of students feel difficulty in number systems, especially in using decimals and fractions; as much as specific tasks in problem-solving, like understanding algebraic problems and identifying equations to solve problems, are difficult for the majority of students.
- One in three students feels difficulty in word problems, mathematical abstractions, selecting mathematical operations and, symbols and notations.

3. Surface structures of language of mathematics like morphology, general and specific terms, symbols and conventions and semantics be given due attention

- In elementary mathematics teaching give attention to the development of a robust vocabulary knowledge base; flexibility; fluency and proficiency with numbers, symbols, words, and diagrams; and comprehension skills for word problems.
- Mathematics education objectives has to consider pupils' need to learn to use the mathematical register in order to have control over the concepts of mathematics.

4. Focus on language elements of elementary mathematics should be responsive to learner difficulties (where exactly the learners have difficulties)

- Emphasis should be given on the linguistic aspect of mathematicsespecially terms- both general and discipline specific and morphology of terms in Malayalam. Attention should be paid to other components of
language of mathematics namely basic mathematical symbols, mathematical conventions and algebraic expressions. Closer attention should be given to precise mathematics vocabulary using a variety of pedagogical techniques and tools.
- Promoting communication in the class made mathematics enjoyable and fun, had contributed to the reduction of the students' mathematical anxiety, and brought in significantly higher conceptual understanding of students in high school students. Even beyond instruction, cognitive loadreducing techniques in tests ensures that student responses reflect their understanding as observed among eighth graders.


## 5. Employ language of mathematics strategies to make school mathematics better inclusive

- Language integrated mathematics instruction is especially helpful for students at lower achievement strata, with lower verbal and non-verbal ability as it not only enhances their achievement in mathematics but also adds to their self-efficacies in learning and solving problems in mathematics and attitudinal indicators such as like towards mathematics, enjoyment of mathematics and self-belief in mathematics, active learning in mathematics or enjoyment in mathematics.

6. Language integration in school mathematics instruction has to consider the difficulty of language elements involved

- Units and areas of mathematics which are especially dense in difficult concepts, symbols, notations, and unfamiliar words that make student learning strenuous, rote and uninteresting be identified and compensated through instruction

$$
\begin{equation*}
\text { Summary, Major Findings and Suggestions } 3 \tag{377}
\end{equation*}
$$

7. Develop learning resources- textbooks, workbooks, worksheets, and design enriching environment that facilitate language of mathematics learning

- Textbooks should give space for linguistic aspects of mathematics. They need to highlight new terms, their morphology, symbols and notations with their origins, in ways that make their meaning clearer.
- Glossary of words should be provided in the textbook.
- Present textbooks are giving only cursory attention to the development of language of mathematics by highlighting a few terms. This should be extended to all new terms, definitions, and problem-solving instructions.
- Other resources that were tried out include math word walls, and word trails may be utilized.

8. Communication of mathematics, its listening, speaking, reading and writing be an integral part of classroom procedures as much as problem solving

- Mathematics instruction should focus on making the students able to understand and interpret oral expressions, decode written and graphical representations and express quantitative ideas and statements.
- An emphasis on the language of mathematics and communicating through mathematics language in teaching engenders learning that is more meaningful, and conceptually integrated. More reading brings in better consolidated learning in mathematics. Discourses in mathematics help to develop a register of technical language of mathematics, making connections between everyday meanings of words and their mathematical
meanings possible for students. Discourse driven mathematics classrooms are helpful for teachers as well, as they reveal students' ability in the area being discussed.
- Use of code switching by teachers allowed students to use their language in a meaningful way in classroom activities and it is an efficient way in multilingual classrooms to confirm meaningful attainment of mathematical concepts.

9. Multiple strategies that enhance acquisition of language of mathematics be employed in elementary classrooms

- Anchoring mathematics with language, vocabulary bank, labeling vocabulary, word walls, word trails, listen and write, possible sentences, guess what?, justifying their reasoning and translation game be employed to build up fluency with language of mathematics are some of the strategies used in this study that brought in desirable results.
- Apart from structured and specific instructional procedures, instruction focusing on the language of mathematics frequently make use of exploring mathematical processes, talking, questioning, stating and restating problems and uncertainties, reasoning, thinking aloud, challenging others' observations and providing answers, building explanations and justifying and the like in whole class and varied group environments.


## 10. Proper Testing and Identification of linguistic difficulties at early stages

- Proper identification of difficulties sourcing from language of mathematics is needed at early stages of Mathematics. Hence, age and grade appropriate tests of student's proficiency in the language of
mathematics to be developed and used to diagnose and remedy student difficulties.
- Testing and assessment practices should support teaching-learning the elements of language of mathematics. Word problems especially complex problem types require attention not only in classroom strategies but also from test developers and designers. And be aware that linguistic complexity of the problem text, the numerical complexity of the problem, and the relation between the linguistic and numerical complexity of a problem as contributing greatly to their difficulty.
- Linguistic complexities in test items, particularly in relation to recurrent use of seven or more letter words, homophones, prepositional phrases and specific mathematics vocabulary, is a key contributing factor to learners' poor performance in mathematics tests. Hence, linguistic modification of test items resulted in significant differences in mathematics performance, in particular, for students in low-level and average mathematics classes.

11. Teacher education should focus on issues of teaching language of mathematics

- Importance of integrating literacy activities in the teaching of mathematics needs to be taught to preservice teachers which increases level of self-confidence.
- Teachers must identify the key vocabulary and subject-specific terminology that students need to understand. They must also review written texts (textbooks, worksheets, study guides) associated with the various learning tasks to determine aspects of the language that are likely to be problematic.
- Teachers need to have a variety of discourse formats at their disposal and be able to use them intentionally, to achieve specific learning goals
- Middle school mathematics teachers recognize the need for them to model their students in using the language of mathematics effectively but generally fail to realize their responsibilities in developing students' mathematical communication skills. Among primary mathematics teachers, there are recent attempts to employ the language of mathematics focusing strategies like code switching, student discourse, mnemonics, manipulatives and collaborative work. These should be strengthened.
- They need to use code-switching, translation, re-voicing across languages to promote the use of multiple languages. Teachers need to be reflective and critical users of classroom talk and understand their role in mathematical discourse.
- Teacher educators can model for future teachers how to analyze the language demands of a lesson. Then, teacher candidates can practice conducting similar analyses, using instructional plans designed by others as well as themselves.
- Teacher education should focus on issues of teaching language of mathematics and different elements of it in school context and in the context of specific medium of instruction. Pedagogic analysis of content should go beyond key terms and definitions to problem words, terms and phrases which are taken for granted.


## 12. The language related instructional and learning difficulties in mathematics in the contexts of languages like English are of value for

## local languages too

- The language related instructional and learning difficulties in mathematics in the contexts of languages like Malayalam are akin to those in the contexts of languages like English. However, research on language of mathematics in the context of specific instructional languages like Malayalam to be strengthened.
- Coordinated research efforts are needed in 1) identifying linguistic competences and knowledge required for participation in mathematical practices, 2) understanding the processes and mechanisms by which students develop linguistic competence and knowledge in mathematics and 3) developing knowledge and skills teachers need and apply in order to support the development of students' linguistic mathematical competence.

Essentially, all stakeholders of elementary mathematics education have to realize and amend their practice in tune with such understanding that paying attention to the language of mathematics in classrooms, apart from the acquisition of listening, speaking, reading and writing skills, help learners; to become aware of, recognize, develop and reorganize their knowledge, to negotiate the language, to articulate their understanding, to consolidate their learning, to develop critical thinking about mathematics, to develop connections between mathematics and life, to think collaboratively and build upon one another's ideas and to increasingly engage in mathematical discourses. An
increased emphasis on communication through language of mathematics in schools will bring in for their students deeper engagement and understanding, greater independence and self-regulation, and stronger competence with mathematical processes.

## Suggestions for Further Research

1. Earlier identification of linguistic difficulties in mathematics and intense instruction focusing on language will have an impact on student's mathematical performance. There is need for developing age and grade appropriate tests of student's proficiency in the language of mathematics in local languages, including in Malayalam.
2. Language related difficulties identified in this study resulted from its focus more on expressive (speaking, writing) aspects and less on receptive (listening, reading) functions of language, and hence importance to oral communication, speaking and listening skills may also be considered by future research.
3. As the more or less quantitative and objective paradigm of research adopted in this study do not permit probing into the difficulties especially in classroom interaction contexts emerging from colloquial language influences localized across the regions of the state of Kerala, more qualitative approach to studying mathematical communication in the classroom across dialectical regions of larger linguistically diverse states, including Kerala may be taken up.
4. Gender is not included as a variable in this study, neither in identifying linguistic difficulties and nor in examining the effectiveness of language
integrated mathematics instruction. However, verbal comprehension and previous achievement in mathematics which were found to interact with effectiveness of language integrated mathematics instruction could be different by gender and hence further studies can be conducted on the gender difference in linguistic difficulties as well as the effect of language integrated instructional strategy.
5. Language integrated mathematics instruction has significant effect on mathematics learning outcomes like achievement, self-efficacy and attitude towards mathematics. Effect of language integrated mathematics instruction on other important outcomes like interest in mathematics and problem-solving ability in mathematics may be Further studied.
6. This study examined the effectiveness of language integrated mathematics instruction using ten select strategies owing to time and resource constraints, and the effect is more on affective outcomes and comparatively less on cognitive outcomes. Future research can investigate other vocabulary / language instructional strategies for even larger effect on cognitive outcomes.
7. This study examined the effect of language integrated mathematics instruction among elementary level students. A longitudinal study may be conducted, to find out whether an early intervention in language of mathematics among primary school students have an impact on their later performance in mathematics.
8. This study can be replicated in the contexts that make use of other Indian regional languages as medium of instruction for mathematics instruction.
9. It is evident from the literature review that medium of instruction has a significant influence on student's mathematical ability. Influence of level of proficiency in medium of instruction on mathematics learning outcomes, in multilingual instructional contexts especially in Indian states including in Malayalam needs further studies.
10. Areas of concern for more substantial and coordinated research include identifying linguistic competences and knowledge required for participation in mathematical practices, and knowledge and skills teachers need and apply in order to support the development of students' linguistic mathematical competence in local languages like Malayalam.

## REFERENCES

Aarts, B., Chalker, S., \& Weiner, E. (2014). The Oxford Dictionary of English Grammar. United Kingdom: Oxford University Press..

Abedi, J., \& Lord, C. (2001). The language factor in mathematics tests. Applied Measurement in Education, 14(3), 219-234.

Abidha, K. \& Gafoor, K., A. (2018). Scale of self-efficacy in learning mathematics. Doctoral Thesis, Department of Education, University of Calicut.

Adams, R. (2003). Reading and adult English language learners: A review of the research. CAL Center for Applied Linguistics.

Adelson, J. L., \& McCoach, D. B. (2011). Development and psychometric properties of the math and me survey: Measuring third through sixth graders' attitudes toward mathematics. Measurement and Evaluation in Counseling and Development, 44(4), 225-247.

Adetula, L. O. (1990). Language factor: Does it affect children's performance on word problems?. Educational Studies in Mathematics, 21(4), 351-365.

Adler. (1991). Mathematics and creativity. In T. Ferris (Ed.), The World Treasury of Physics, Astronomy and Mathematics. Brown and Co.

Akkus, R. (2015). Language and discourse in mathematics. Elementary Education Online, 14(1), 230-242.

Alken, L. R. (1974). Two scales of attitude toward mathematics. Journal for Research in Mathematics Education, 67-71.

Anderson, R.C., \& Freebody, P. (1982). Reading comprehension and the assessment and acquisition of word knowledge. Center for the Study of Reading Technical Report; No. 249.

Anthony, G., \& Walshaw, M. (2010). Effective pedagogy in mathematics. Geneva: International Bureau of Education.

Baber, R. L. (2011). The language of mathematics: Utilizing math in practice. John Wiley \& Sons.

Bagchi, A., \& Wells, C. (1998). On the communication of mathematical reasoning. problems, resources, and issues. Mathematics Undergraduate Studies, 8(1), 15-27.

Bandura, A. (2006). Guide for constructing self-efficacy scales. Self-efficacy Beliefs of Adolescents, 5(1), 307-337.

Barbu, O. C., \& Beal, C.R. (2010). Effects of linguistic complexity and math difficulty on word problem solving by English learners. International Journal of Education, 2(2), 1-19.

Barlow, A. T., \& McCrory, M. R. (2011). 3 Strategies for Promoting Math Disagreements. Teaching Children Mathematics, 17(9), 530-539.

Barton, M. L., Heidema, C., \& Jordan, D. (2002). Teaching reading in mathematics and science. Educational Leadership, 60(3), 24-29.

Barwell, R. (2008). ESL in the mathematics classroom. Retrieved from http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/ESL_m ath.pdf

Baxter, J. A., Woodward, J., \& Olson, D. (2005). Writing in mathematics: An alternative form of communication for academically low-achieving students. Learning Disabilities- Research \& Practice, 20(2), 119-135.

Beal, C. R., Adams, N. M., \& Cohen, P. R. (2010). Reading proficiency and mathematics problem solving by high school English language learners. Urban Education, 45(1), 58-74.

Beck, I., McKeown, M. G., \& Kucan, L. (2002). Bringing words to life: Robust vocabulary development. New York: Guilford.

Best, J. W., \& Kahn, J. V. (2016). Research in education. New Delhi: Pearson Education.

Blachowicz, C. L., \& Fisher, P. (2000). Vocabulary instruction. Handbook of Reading Research, 3, 503-523.

Brown, R., \& Hirst, E. (2007). Developing an understanding of the mediating role of talk in the elementary mathematics classroom. The Journal of Classroom Interaction, 18-28.

Brummer, T., \& Clark, S. K. (2013). Writing strategies for mathematics. Huntington Beach: Shell Education.

Bruun, F., Diaz, J.M., \& Dykes, V.J. (2015). The language of mathematics. Teaching Children Mathematics, 21(9), 530-536.

Burns, C., JVF. (2018). One mathematical cat, please! A first course in algebra (A truth and language book in mathematics). Retrieved from http://www.onemathematicalcat.org/cat_book.htm.

Cantoni, M. (2007). What role does the language of instruction play for a successful education? A case study of the impact of language choice in a Namibian school.

Capraro, M. M., \& Joffrion, H. (2006). Algebraic equations: Can middle-school students meaningfully translate from words to mathematical symbols? Reading Psychology, 27(2-3), 147-164.

Cavanagh, S. (2005). Math: The not-so-universal language. Education Week. Retrieved from http://www.barrow.k12.ga.us/esol/Math_The_Not_ So_Universal_Language.pdf.

Changeux, J. P., \& Connes, A. (1998). Conversations on mind, matter, and mathematics. United States: Princeton University Press.

Chapin, S. H., O'Connor, C., O'Connor, M. C., \& Anderson, N. C. (2009). Classroom discussions: Using math talk to help students learn, Grades K-6. Retrieved from https://mathsolutions.com/wp-content/uploads/978-1-935099-01-7_L1.pdf.

Chapman, E. (2009). Development and validation of a brief mathematics attitude scale for primary-aged students. The Journal of Educational Enquiry, 4(2).

Chard, D. (2003). Vocabulary strategies for the mathematics classroom. Boston, MA: Houghton Mifflin.

Chiphambo, S. M. (2019). Mathematics dictionary: Enhancing students Geometrical vocabulary and terminology. Retrieved from https://www.intechopen.com/books/metacognition-in-learning/mathem atics-dictionary-enhancing-students-geometrical-vocabulary-and-term inology.

Chow, J. C., \& Ekholm, E. (2019). Language domains differentially predict mathematics performance in young children. Early Childhood Research Quarterly, 46, 179-186.

Clark, K. K., Jacobs, J., Pittman, M. E., \& Borko, H. (2005). Strategies for building mathematical communication in the middle school classroom: Modeled in professional development, implemented in the classroom. Current Issues in Middle Level Education, 11(2), 1-12.

Clissold, C. (2014). Mathematical vocabulary. London: Rising Stars UK Ltd. Retrieved from http://www.lindfieldprimaryacademy.org.uk/docs/ Mathematical\%20Vocabulary\%20ePDF.pdf

Cobb, P., Wood, T., Yackel, E., \& McNeal, B. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. American Educational Research Journal, 29(3), 573-604.

Coe, R. (2002). It's the effect size, stupid: What effect size is and why it is important. Retrieved from https://www.leeds.ac.uk/educol/documents/ 00002182.htm

Cohen, J. (1965). Some statistical issues in psychological research. In B.B. Wolman, (Ed.), Handbook of Clinical Psychology, pp.95-121. New York: McGraw Hill.

Cole, G. H. (2010). The inverted bowl: Introductory accounts of the universe and its life. Singapore: World Scientific.

Council of Australian Governments. (2008). National numeracy review report. Canberra: Commonwealth of Australia. Retrieved from http://www.coag. gov.au/reports/docs/national_numeracy_review.pdf

Craig, T., \& Morgan, C. (2015). Language and communication in mathematics education. In The Proceedings of the $12^{\text {th }}$ International Congress on Mathematical Education (pp. 529-533), Springer, Cham.

Cruz, J.K.B.D., \& Lapinid, M.R.C. (2014). Students' difficulties in translating worded problems into mathematical symbols. In The DLSU Research Congress Proceeding.

Cuevas, G. J. (1991). Developing communication skills in mathematics for students with limited English proficiency. Mathematics Teacher, 84(3), 186-89.

Cummins, J., \& Swain, M. (1986). Linguistic interdependence: A central principle of bilingual education. Bilingualism in Education, 80-95.

Daroczy, G., Wolska, M., Meurers, W. D., \& Nuerk, H. C. (2015). Word problems: A review of linguistic and numerical factors contributing to their difficulty. Frontiers in Psychology, 6, 348.

DeKeyser, R. (Ed.). (2007). Practice in a second language: Perspectives from applied linguistics and cognitive psychology. Cambridge, United Kingdom: Cambridge University Press.

Draper, R. J. (2002). School mathematics reform, constructivism, and literacy: A case for literacy instruction in the reform-oriented math classroom. Journal of Adolescent \& Adult Literacy, 45, 520-529.

Esty, W. W. (1992). Language concepts of mathematics. Focus on Learning Problems in Mathematics, 14(4), 31-54.

Fay, K., \& Boyd, M. J. (2010). Eta-squared. Encyclopedia of Research Design, 1, 422-425.

Fennema, E., \& Sherman, J. A. (1976). Fennema-Sherman mathematics attitudes scales: Instruments designed to measure attitudes toward the learning of mathematics by females and males. Journal for Research in Mathematics Education, 7(5), 324-326.

Ferguson, G. A. (1976). Statistical analyses in psychology and education (4 $4^{\text {th }}$ Ed.). New York: McGraw-Hill

Ferris, T., \& Fadiman, C. (1991). The world treasury of physics, astronomy, and mathematics. Boston, Massachusetts, United States: Little, Brown and Company.

Flanagan, M. (2009). Critical play: Radical game design. Cambridege, MA: MIT Press.

Frayer, D. A., Frederick, W. C., \& Klausmeier, H. J. (1969). A science for testing the level of concept mastery (Working paper No. 16). Madison: University of Wisconsin Research and Development Center for Cognitive Learning.

Frei, S. (2007). Teaching Mathematics Today (Practical Strategies for successful Classrooms). United States: Schell Education.

Gafoor, K.A., \& Aneesh, N.V. (2018). Development of tests for scaling malayalam reading comprehension of school students. Guru Journal of Behavioral Sciences, 6 (2), 832-842.

Geary D. C. (1996). Children's mathematical development: Research and practical applications. Washington, DC: American Psychological Association.

Georgius, K. (2008). Improving communication about mathematics through vocabulary and writing. Retrieved from http://digitalcommons.unl.edu/ mathmidsummative/13/

Gifford, M., \& Gore, S. (2010). The effects of focused academic vocabulary instruction on underperforming math students. ASCD Report. Alexandria, VA: Association for Supervision and Curriculum Development.

Gillmor, S. C., Poggio, J., \& Embretson, S. (2015). Effects of reducing the cognitive load of mathematics test items on student performance. Numeracy, 8(1), 4.

Gooding, S. (2009). Children's difficulties with mathematical word problems. Proceedings of the British Society for Research into Learning Mathematics, 29(3), 31-36.

Gorgorio, N., \& Planas, N. (2001). Teaching mathematics in multilingual classrooms. Educational Studies in Mathematics, 47(1), 7-33.

Gottlieb, M., \& Ernst-Slavit, G. (2014). Academic language in diverse classrooms: Definitions and contexts. Corwin Press.

Gough, J. (2007). Conceptual complexity and apparent contradictions in mathematics language. Australian Mathematics Teacher, 63(2), 8-15.

Graham, S., \& Perin, D. (2007). A meta-analysis of writing instruction for adolescent students. Journal of Educational Psychology, 99(3), 445.

Guce, I., \& Talens, J. (2013). Scale on Attitude Toward Mathematics (SATM). Educational Measurement and Evaluation, 4, 100-107.

Haggard, M. R. (1986). The vocabulary self-collection strategy: Using student interest and world knowledge to enhance vocabulary growth. Journal of Reading, 29(7), 634-642.

Halliday, M.A.K. (1975). Learning how to mean: Explorations in the development of language. London: Edward Arnold.

Hatano, G., \& Inagaki, K. (1991). Sharing cognition through collective comprehension activity. In L. B. Resnick, J. M. Levine, \& S. D. Teasley (Eds.), Perspectives on Socially Shared Cognition (p. 331-348). American Psychological Association. Retrieved from https://doi.org/ 10.1037/10096-014.

Haylock, D. (2007). Key concepts in teaching primary mathematics. Thousand Oaks, California, United States: Sage Publications.

Hersh, R. (1997). Math lingo vs. plain English: double entendre. The American Mathematical Monthly, 104(1), 48-51

Heuer, L. (2005). Graphic representation in the mathematics classroom. In J.M. Kenney. (Ed.), Literacy strategies for improving mathematics instruction. ASCD.

Hiebert, J., Stigler, J.W., \& Manaster, A.B. (1999). Mathematical features of lessons in the TIMSS Video Study. ZDM, 31(6), 196-201.

Hill-Bonnet, L., \& Lippincott, A. (2010). Academic language. Birmingham, United Kingdom: Packt Publishing.

Hollingsworth, L.N. (2019). Effects of a mathematics vocabulary tutoring intervention. Retrieved from https://diginole.lib.fsu.edu/islandora/ object/fsu\%3A709769/

Howie, S. J. (2003). Language and other background factors affecting secondary pupils' performance in mathematics in South Africa. African Journal of Research in Mathematics, Science and Technology Education, 7(1), 1-20.

Huggins, B., \& Maiste, T. (1999). Communication in mathematics. Master"s Action Research Project, St. Xavier University \& IRI/Skylight.

Ilany, B.S., \& Margolin, B. (2010). Language and mathematics: Bridging between natural language and mathematical language in solving problems in mathematics. Creative Education, 1(03), 138.

Irujo, S. (2007). What does research tell us about teaching reading to English language learners. Retrieved from https://www.readingrockets.org/ article/what-does-research-tell-us-about-teaching-reading-english-lan guage-learners

Jamison, R. E. (2000). Learning the language of mathematics. Language and Learning across the Disciplines, 4(1), 45-54.

Jegede, O. (2011). Code switching and its implications for teaching Mathematics in primary schools in Ile-Ife, Nigeria. Journal of Education and Practice, 2(10), 41-54.

Jurecska, D. E., Lee, C. E., Chang, K. B., \& Sequeira, E. (2011). I am smart, therefore I can: Examining the relationship between IQ and self-efficacy across cultures. International Journal of Adolescent Medicine and Health, 23(3), 209-216.

Kabael, T., \& Baran, A.A. (2016). Investigation of mathematics teachers' awareness of developing mathematical communication skills. Retrieved from https://www.researchgate.net/publication/315950339_One_Middle _School_Mathematics_Teacher's_and_One_Senior_Prospective_Teacher' s_Mathematical_Discourse

Kaput, J. J. (1998). Representations, inscriptions, descriptions and learning: A kaleidoscope of windows. The Journal of Mathematical Behavior, 17(2), 265-281.

Kenney, J. M. (2005). Literacy strategies for improving mathematics instruction. Alexandria, VA: Association for Supervision and Curriculum Development.

Kenyon, V. (2016). How can we improve mathematical vocabulary comprehension that will allow students to develop higher-order levels of learning?. The STeP Journal: Student Teacher Perspectives, 3(2), 47-61.

Kerala Curriculum Framework. (2007). State Council of Educational Research and Training (SCERT), Kerala.

Kersaint, G., Thompson, D., \& Petkova, M. (2009). The nature of mathematics language. Teaching Mathematics to English Language Learners, 46-52.

Kim, T.K. (2015). Barnes' type multiple degenerate Bernoulli and Euler polynomials. Applied Mathematics and Computation, 258, 556-564.

Koedinger, K. R., \& Nathan M. J. (2004). The real story behind story problems: effects of representations on quantitative reasoning. J. Learn. Sci., 13, 129-164 10.1207/s15327809jls1302_1).

Krashen, S. D. (1981). The "fundamental pedagogical principle" in second language teaching. Studia Linguistica, 35(1-2), 50-70.

Larson, C. (2007). The importance of vocabulary instruction in everyday mathematics. (Mémoire de maîtrise, Université de Nebraska-Lincoln, NE ). Retrieved from https://digitalcommons.unl.edu/cgi/viewcontent.cgi? article $=1033 \&$ context $=$ mathmidactionresearch

Latu, V.F. (2005). Language factors that affect mathematics teaching and learning of Pasifika students. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, \& A. Roche (Eds.), Building Connections: Theory, Research and Practice. Proceedings of the $28^{\text {th }}$ annual conference of the Mathematics Education Research Group of Australasia, Melbourne, pp. 483-490). Sydney: MERGA.

Lee, C. (2006). Language for learning mathematics: Assessment for learning in practice. UK: McGraw-Hill Education.

Lee, C. (1997). Effective communication in mathematics. Research project commissioned by the Teacher Training Agency.

Lemke, J. L. (1989). Using language in the classroom. USA: Oxford University Press.

Lenhard, W., \& Lenhard, A. (2014). Hypothesis tests for comparing correlations. Retrieved from https://www.psychometrica.de/correlation.html. Bibergau (Germany): Psychometrica. doi: 10.13140/RG.2.1.2954.1367.

Lerman, S. (2000). The social turn in mathematics education research. Multiple Perspectives on Mathematics Teaching and Learning, 19-44.

Leshem, S., \& Markovits, Z. (2013). Mathematics and English, two languages: Teachers' views. Journal of Education and Learning, 2(1), 211.

Levine, T. R., \& Hullett, C. R. (2010). Partial eta-squared. Encyclopedia of Research Design, 1007-1009.

Lim, W., Moseley, L. J., Son, J. W., \& Seelke, J. (2014). A snapshot of teacher candidates' readiness for incorporating academic language in lesson plans. Current Issues in Middle Level Education, 19(2), 1-8.

Lim, W., Stallings, L., \& Kim, D. J. (2015). A proposed pedagogical approach for preparing teacher candidates to incorporate academic language in mathematics classrooms. International Education Studies, 8(7), 1-10.

Livers, S. D., \& Elmore, P. (2018). Attending to precision: Vocabulary support in middle school mathematics classrooms. Reading \& Writing Quarterly, 34(2), 160-173.

Lomibao, L. S., Luna, C. A., \& Namoco, R. A. (2016). The influence of mathematical communication on students' mathematics performance and anxiety. American Journal of Educational Research, 4(5), 378-382.

Lucas, T., Villegas, A. M., \& Freedson-Gonzalez, M. (2008). Linguistically responsive teacher education: Preparing classroom teachers to teach English language learners. Journal of Teacher Education, 59(4), 361-373.

Marzano, R. J., \& Pickering, D. J. (2005). Building academic vocabulary: Teacher's Manual. Alexandria: Association for Supervision and Curriculum Development.

Marzano, R. J., Pickering, D., \& Pollock, J. E. (2001). Classroom instruction that works: Research-based strategies for increasing student achievement. Ascd. Alexandria, Virginia, United States

Mbugua, Z. K. (2012). Influence of mathematical language on achievement in mathematics by secondary school students in Kenya. International Journal of Education and Information Studies. 2 (1), 1-7. Retrieved from http://karuspace.karu.ac.ke/handle/20.500.12092/1695

McKeown, M. G., Beck, I. L., Omanson, R. C., \& Pople, M. T. (1985). Some effects of the nature and frequency of vocabulary instruction on the knowledge and use of words. Reading Research Quarterly, 522-535.

McNeil, N. M., \& Alibali, M. W. (2005). Why won't you change your mind? Knowledge of operational patterns hinders learning and performance on equations. Child Development, 76(4), 883-899.

McNeill, D. (1992). Hand and mind: What gestures reveal about thought. University of Chicago Press. Chicago, Illinois, United States

McNeill, D. (Ed.). (2000). Language and gesture (Vol. 2). Cambridge, United Kingdom: Cambridge University Press.

Mehrens, W. A., \& Lehmann, I. J. (1984). Measurement and evaluation. Education and Psychology, 1991-145.

Meiers, M., \& Trevitt, J. (2010). The digest edition 2010/2: Language in the mathematics classroom. Australian Council for Educational Research. Retrieved from https://research.acer.edu.au/cgi/viewcontent.cgi?article $=1006 \&$ context $=$ digest

Metsisto, D. (2005). Reading in the mathematics classroom. In J.M. Kenney, (Ed.), Literacy Strategies for Improving Mathematics Instruction. Alexandria, Virginia, United States: ASCD.

Michaels, L. A., \& Forsyth, R. A. (1977). Construction and validation of an instrument measuring certain attitudes toward mathematics. Educational and Psychological Measurement, 37(4), 1043-1049.

Mink, D., V. (2010). Strategies for Teaching Mathematics. Schell Education. United States.

Moffett, P., \& Eaton, P. (2018). The impact of the promoting early number talk project on the development of abstract representation in mathematics. European Early Childhood Education Research Journal, 26(4), 547-561.

Moffett, P., \& Eaton, P. (2018). The impact of the promoting early number talk project on the development of abstract representation in mathematics. European Early Childhood Education Research Journal, 26(4), 547-561.

Monroe, E., \& Panchyshyn, R. (1997).Vocabulary considerations for teaching mathematics. Childhood Education, 72, 80-83. Retrieved from http://www.pittsfordschools.org

Morgan, C. (1996). The language of mathematics": towards a critical analysis of mathematics texts. For the Learning of Mathematics, 16(3), 2-10.

Morgan, C., Craig, T., Schuette, M., \& Wagner, D. (2014). Language and communication in mathematics education: an overview of research in the field. $Z D M, 46(6), 843-853$.

Morse J., M., Niehaus L. (2009). Mixed method design: Principles and procedures. Walnut Creek: Left Coast Press.

Moschkovich, J. (1999). Supporting the participation of English language learners in mathematical discussions. For the Learning of Mathematics, 19(1), 11-19.

Moschkovich, J. (2007). Using two languages when learning mathematics. Educational Studies in Mathematics, 64(2), 121-144.

Moschkovich, J. (2012). Mathematics, the common core, and language: Recommendations for mathematics instruction for ELs aligned with the common core. Commissioned papers on language and literacy issues in the common core state standards and next generation science standards, 94, 17.

Moursund, D. (2016). Learning problem solving strategies by using games: A guide for educators and parents. Eugene, Oregon: Information Age Educations.

Naidoo, J. (2015). Exploring some teaching strategies that overcome challenges created by the language of instruction within multilingual mathematics classrooms. International Journal of Educational Sciences, 10(2), 182191.

NALDIC (2002). Access and engagement in mathematics. Retrieved from https://www.naldic.org.uk/Resources/NALDIC/Teaching\ and\ Lea rning/ma_eal.pdf

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: Author.

National Reading Panel (US), National Institute of Child Health, Human Development (US), National Reading Excellence Initiative, National Institute for Literacy (US), \& United States Department of Health. (2000). Report of the National Reading Panel: Teaching children to read: An evidence-based assessment of the scientific research literature on reading and its implications for reading instruction: Reports of the subgroups. National Institute of Child Health and Human Development, National Institutes of Health.

NCERT. (2005). National Curriculum Framework 2005, New Delhi: Author.
NCERT. (2017). Learning outcomes at the elementary stage. Retrieved from ncert.nic.in/publication/Miscellaneous/pdf_files/tilops101.pdf

NCTM. (1991). Professional standards for teaching mathematics. National Council of Teachers of Mathematics (NCTM). Reston

NCTM. (2000). Principles and standards for school mathematics. National Council of Teachers of Mathematics (NCTM). Reston

NCTM. (2010). Mathematics teaching in the middle school. The National Council of Teachers of Mathematics. Retrieved from www.nctm.org

New Jersey Mathematics Curriculum Framework. (1996). New Jersey Mathematics Coalition. Retrieved from http://archive.dimacs.rutgers. edu/nj_math_coalition/framework.html

Niss, M. (2003). Mathematical competencies and the learning of mathematics: The Danish KOM project. In $3^{\text {rd }}$ Mediterranean Conference on mathematical education, Athens, Hellas 3-4-5 (pp. 116-124). Retrieved from http://www.math.chalmers.se/Math/Grundutb/CTH/mve375/1213/ docs/KOMkompete nser.pdf

Nordin, B.A. (2005). Students' perception on teaching and learning mathematics in English. University Teknologi Malaysia, Skudai.

Novotna, J., \& Moraova, H. (2005). Cultural and linguistic problems in the use of authentic textbooks when teaching mathematics in a foreign language. ZDM, 37(2), 109-115.

Ontario Ministry of Education. (2005). The Ontario Curriculum, Grades 1 to 8: Mathematics. Toronto, ON: Queen's Printer for Ontario.

Ontario Ministry of Education. (2006). A guide to effective instruction in mathematics. Kindergarten to grade 6 (Vol.2) - Problem solving and communication. Toronto, ON: Queen's Printer for Ontario.

Ontario Ministry of Education. (2009). Equity and inclusive education in Ontario schools. Guidelines for policy development and implementation. Toronto: Queen's Printer for Ontario.

Ontario Ministry of Education. (2010). Capacity building series; Communication in the mathematics classroom. Retrieved from http://www.edu.gov.on. ca/eng/literacynumeracy/inspire/research/CBS Communication_Mathematics.pdf

Orton, R. E. (1987). The foundations of construct validity: Towards an update. Journal of Research \& Development in Education, 21(1), 23-35.

Oskamp, S., \& Schultz, P. W. (2005). Attitudes and opinions. Psychology Press. London

Palacios, A., Arias, V., \& Arias, B. (2014). Attitudes towards mathematics: Construction and validation of a measurement instrument. Revista de Psicodidáctica, 19(1), 67-91.

Patkin, D. (2011). The interplay of language and mathematics. Pythagoras, 32(2), 1-7.

Paul, S. M. (1986). The advanced Raven's Progressive Matrices: Normative data for an American University population and an examination of the relationship with Spearman'sg. The Journal of Experimental Education, 54(2), 95-100.

Peat, F. D. (1990). Mathematics and the language. Retrieved from http://www.fdavidpeat.com/bibliography/essays/maths.htm.

Pimm, D. (1987). Speaking mathematically: Communication in the mathematics classroom. London: Routledge.

Pintrich, P.R., Smith, D., Garcia, T., \& McKeachie, W. (1991). A manual for the use of the motivated strategies for learning questionnaire (MSLQ). The University of Michigan, Ann Arbor, MI.

Pirie, S. E. (1998). Crossing the gulf between thought and symbol: Language as (slippery) stepping-stones. Language and communication in the mathematics classroom, 34(2), 7-29.

Powell, S. R., Driver, M. K., Roberts, G., \& Fall, A. M. (2017). An analysis of the mathematics vocabulary knowledge of third-and fifth-grade students: Connections to general vocabulary and mathematics computation. Learning and Individual Differences, 57, 22-32.

Pressley, M. (2000). What should comprehension instruction be the instruction of? In M. Kamil, P. Mosenthal, P. D. Pearson, and R. Barr (Eds.), Handbook of Reading Research, (Vol. III) (pp. 545-562). Mahwah, NJ: Erlbaum.

Psychometrica. (n.d.). Retrieved from https://www.psychometrica.de/effect_size .html

Pugalee, D. K. (2004). A comparison of verbal and written descriptions of students' word problem solving processes. Educational Studies in Mathematics, 55, 27-47. doi: 10.1023/B:EDUC.0000017666.11367.c7

Purpura, D. J., Napoli, A. R., Wehrspann, E. A., \& Gold, Z. S. (2017). Causal connections between mathematical language and mathematical knowledge: A dialogic reading intervention. Journal of Research on Educational Effectiveness, 10(1), 116-137.

Radford, L., \& Barwell, R. (2016). Language in mathematics education research. In The second handbook of research on the psychology of mathematics education (pp. 275-313). Brill Sense.

Ramanujam, R., Subramanian, R., \& Sachdev, P. L. (2006). Position paper on national focus group on teaching of mathematics. New Delhi: NCERT.

Rao, P. K. S., Ramaa, S., \& Gowramma, I. P. (2017). Does knowledge of mathematical language play a role in mathematical ability? A preliminary study. Journal of All India Association for Educational Research, 29(2), 83-100.

Raven, J. C. (1998). Raven's progressive matrices and vocabulary scales. Oxford Psychologists Press.

Reynolds, S. L. (2010). Examining the communication in developmental mathematics classes. Research and Teaching in Developmental Education, 21-29.

Riccomini, P. J., Smith, G. W., Hughes, E. M., \& Fries, K. M. (2015). The language of mathematics: The importance of teaching and learning mathematical vocabulary. Reading \& Writing Quarterly, 31(3), 235-252.

Richardson, J. T. (2011). Eta squared and partial eta squared as measures of effect size in educational research. Educational Research Review, 6(2), 135-147.

Rodriguez-Domingo, S., Molina, M., Canadas, M. C., \& Castro, E. (2012). Errors in algebraic statements translation during the creation of an algebraic domino. Retrieved from http://funes.uniandes.edu.co/1929/

Rojas, V. (2009). Strategies for success with ELL: A toolkit for teachers. EARCOS Institute, Inc. Retrieved, from http://www.earcos.org/etc2009/ download/ELL precon.pdf

Rothstein, A., Rothstein, E., \& Lauber, G. (2006). Writing as learning: A content-based approach. New York: Corwin Press.

Salinas, S., \& Ortlieb, E. (2011). Best vocabulary practices to support mathematics in the age of common core standards. Journal of Studies in Education, 1(1).

Sammons, L. (2018). Teaching students to communicate mathematically. Alexandria, Virginia, United States: ASCD.

Sample, L. (2009). Oral and written communication in classroom mathematics. Action Research Projects, Paper 41, http://digitalcommons.unl.edu/ mathmidactionresearch/41

Sandman, R. S. (1980). The Mathematics Attitude Inventory: Instrument and User's Manual. Journal for Research in Mathematics Education, 11(2), 148-49.

SCERT. (2016). Mathematics textbook. Thiruvananthapuram: The State Council Educational Research and Training (SCERT).

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. Reading \& Writing Quarterly, 23(2), 139-159.

Schleppegrell, M. J. (2012). Academic language in teaching and learning: Introduction to the special issue. The Elementary School Journal, 112(3), 409-418.

Schoonenboom, J., \& Johnson, R. B. (2017). How to construct a mixed methods research design. KZfSS Kölner Zeitschrift für Soziologie und Sozialpsychologie, 69(2), 107-131. Retrieved from https://www.ncbi.nlm. nih.gov/pmc/articles/PMC5602001/

Schwartz, R. M., \& Raphael, T. E. (1985). Concept of definition: A key to improving students' vocabulary. The Reading Teacher, 198-205.

Serio, M. (2015). Engaging students in mathematical communication: Teaching for understanding. Retrieved from https://tspace.library.utoronto.ca/ handle/1807/67054

Shaftel, J., Belton-Kocher, E., Glasnapp, D., \& Poggio, J. (2006). The impact of language characteristics in mathematics test items on the performance of English language learners and students with disabilities. Educational Assessment, $11(2)$, 105-126.

Sibanda, L. (2015). Investigating the nature of the linguistic challenges of the Department of Basic Education (DBE) 2013 Grade 4 Mathematics ANAs and learners' and teachers' experience of them (Doctoral dissertation, Rhodes University).

Simpson, A., \& Cole, M. W. (2015). More than words: A literature review of language of mathematics research. Educational Review, 67(3), 369-384.

Social Science Statistics. (n.d.). Retrieved from https://www.socscistatistics.com/ effectsize/default3.aspx

Souviney, R. J. (1983). Mathematics achievement, language and cognitive development: Classroom practices in Papua New Guinea. Educational Studies in Mathematics, 14(2), 183-212.)

Sparrow, L., \& Hurst, C. (2010). Effecting affect: Developing a positive attitude to primary mathematics learning. Australian Primary Mathematics Classroom, 15(1), 18-24.

Stigler, J. W., \& Hiebert, J. (2009). The teaching gap: Best ideas from the world's teachers for improving education in the classroom. Retrieved from https://eric.ed.gov/?id=ED434102.

Susac, A., Bubic, A., Vrbanc, A., \& Planinic, M. (2014). Development of abstract mathematical reasoning: the case of algebra. Frontiers in Human Neuroscience, 8, 679.

Sweller, J. (1994). Cognitive load theory, learning difficulty, and instructional design. Learning and Instruction, 4(4), 295-312.

Taba, H. (1967). Teachers' handbook for elementary social studies. Retrieved from https://eric.ed.gov/?id=ED039206

Tambychik, T., \& Meerah, T. S. M. (2010). Students' difficulties in mathematics problem-solving: what do they say?. Procedia-Social and Behavioral Sciences, 8, 142-151.

Tapia, M., \& Marsh, G. E. (2005). Attitudes toward mathematics inventory redux. Academic Exchange Quarterly, 9(3), 272-277.

Tarpley, C. (2015). Rich instruction of mathematical academic vocabulary to enhance mathematics achievement of elementary school students. Retrieved from http://hdl.handle.net/10415/4595

Tharpe, K., B. (2017). Talk your math off: Communicating in the mathematics classroom. Retrieved from http://docplayer.net/28792927-Talk-your-math-off-communication-in-the-mathematics-classroom-1.html

The Statistical Calculator (n.d.). Retrieved from https:// www.statpac.com/ statistics-calculator.

Thomas, C. N., Van Garderen, D., Scheuermann, A., \& Lee, E. J. (2015). Applying a universal design for learning framework to mediate the language demands of mathematics. Reading \& Writing Quarterly, 31(3), 207-234.

Thompson, D. R., \& Rubenstein, R. N. (2000). Learning mathematics vocabulary: Potential pitfalls and instructional strategies. The Mathematics Teacher, 93(7), 568-574.

Tikhomirova, T., Voronina, I., Marakshina, J., Nikulchev, E., Ayrapetyan, I., \& Malykh, T. (2016). The relationship between non-verbal intelligence and mathematical achievement in high school students. In SHS Web of Conferences (Vol. 29, p. 02039). EDP Sciences.

Turner, E., Roth McDuffie, A., Sugimoto, A., Aguirre, J., Bartell, T. G., Drake, C., ... \& Witters, A. (2019). A study of early career teachers' practices related to language and language diversity during mathematics instruction. Mathematical Thinking and Learning, 21(1), 1-27.

Tuttle., C., L. (2005). Writing in the mathematics classroom. In J. M. Kenney, (Ed.), Literacy Strategies for Improving Mathematics Instruction. Alexandria, Virginia, United States: ASCD.

Umeodinka, A.U., \& Nnubia, C. A. (2016). The mathematics-language symbiosis: The learners' benefits. Mgbakoigba: Journal of African Studies, $6(1)$.

Urquhart, V. (2009). Using writing in mathematics to deepen student learning. Alexandira: Denver, Co: Mid-Continent Research for Education and Learning.

Usher, E. L., \& Pajares, F. (2009). Sources of self-efficacy in mathematics: A validation study. Contemporary Educational Psychology, 34(1), 89-101.

Vale, E. (2013). Vale's technique of screen and television writing. London: Routledge.

Veel, R. (1999). Language, knowledge and authority in school mathematics. In F. Christie (Ed.), Pedagogy and the Shaping of Consciousness: Linguistic and Social Processes (pp. 185-216). London: Continuum.

Vilenius-Tuohimaa, P. M., Aunola, K., \& Nurmi, J. E. (2008). The association between mathematical word problems and reading comprehension. Educational Psychology, 28(4), 409-426.

Vukovic, R. K., \& Lesaux, N. K. (2013 ${ }_{\mathrm{a}}$ ). The language of mathematics: Investigating the ways language counts for children's mathematical development. Journal of Experimental Child Psychology, 115(2), 227244.

Vukovic, R. K., \& Lesaux, N. K. (2013 ${ }_{\mathrm{b}}$ ). The relationship between linguistic skills and arithmetic knowledge. Learning and Individual Differences, 23, 87-91.

Wells, C. (2003). A handbook of mathematical discourse. PA: Infinity.
Wichelt, L. (2009). Communication: A vital skill of mathematics. Retrieved from http://digitalcommons.unl.edu/mathmidactionresearch/18

Wilkinson, G. S., \& Robertson, G. J. (2006). Wide range achievement test (WRAT4). Lutz, FL: Psychological Assessment Resources.

Woodcock, R. W., Mather, N., McGrew, K. S., \& Wendling, B. J. (2001). Woodcock-Johnson III tests of cognitive abilities (pp. 371-401). Itasca, IL: Riverside Publishing Company.

Woodin, C. (1995). The landmark method for teaching arithmetic. Landmark School, Inc. and Christopher Woodin. Retrieved from http://www.landmarkoutreach.org/MathasaLanguage.htm

Yamat, H., Maarof, N., Maasum, T. N. T. M., Zakaria, E., \& Zainuddin, E. (2011). Teacher's code-switching as scaffolding in teaching content area subjects. World Applied Sciences Journal, 15(1), 18-22.

Yanez-Marquina, L., \& Villardon-Gallego, L. (2016). Attitudes towards mathematics at secondary level: development and structural validation of the scale for assessing attitudes towards mathematics in secondary education (SATMAS). Electronic Journal of Research in Educational Psychology, 14(3), 557-581.

Yang, E. F., Chang, B., Cheng, H. N., \& Chan, T. W. (2016). Improving pupils' mathematical communication abilities through computer-supported reciprocal peer tutoring. Journal of Educational Technology \& Society, 19(3), 157.

Yopp, H. K., Yopp, R. H., \& Bishop, A. (2008). Vocabulary instruction for academic success. Teacher Created Materials.

Yushau, B., \& Bokhari, M. (2005). Language and mathematics: A meditational approach to bilingual Arabs. International Journal of Mathematics Teaching and Learning, 1-18.

Zazkis, R. (2000). Using code-switching as a tool for learning mathematical language. For the Learning of Mathematics, 20(3), 38-43.

## APPENDICES

## Appendix 11

## Factor Loadings and Communalities Based on Principal Component Analysis of Difficulties in Tasks Related to 1) Number Concept, 2) Mathematical Operations and 3) Problem Solving for elementary school students

Table A1-1. Factor loadings and communalities based on a principal component analysis of tasks related to number concept

| Number concept | Factor loading |  |
| :--- | :---: | :---: |
|  | Number systems | Comprehending numbers |
| using fractions | 0.791 |  |
| using decimals | 0.763 | 0.756 |
| understanding large numbers |  | 0.721 |
| understanding Place value | $32.29 \%$ | $30.21 \%$ |
| Variance explained (Eigen value) |  | $62.51 \%$ |
| Total variance explained |  |  |
| Table A1-2. Factor loadings and communalities | based on a principal component analysis of |  |
| tasks related to mathematical operations |  |  |


| Mathematical operations | Factor loading |  |
| :---: | :---: | :---: |
|  | Problem solving competence | Arithmetic operations |
| Doing calculations with speed | 0.823 |  |
| Concentrating for long time to solve problems | 0.733 |  |
| Doing Mental arithmetic |  | 0.734 |
| Following rules while doing calculations |  | 0.711 |
| Remembering numbers while doing operations |  | 0.591 |
| Doing basic arithmetic operations |  | 0.586 |
| Variance explained(Eigen value) | 29.16 \% | 20.53\% |
| Total variance explained | 49.68\% |  |
| Table A1-3. Factor loadings and communalities based on a principal component analysis of tasks related to Problem solving |  |  |
|  | Factor loading |  |
| Problem solving | Understanding <br> word problems  <br> operations | Translation of word problems |
| Identifying irrelevant information in word problems | 0.726 |  |
| Understanding word problem without external help | 0.641 |  |
| Identifying key words | 0.512 |  |
| Identifying mathematics problem in word problems | 0.517 |  |
| Analyzing lengthy word problems | 0.738 |  |
| Identifying equations | 0.714 |  |
| Selecting Mathematical operations | 0.382 |  |
| Doing mathematical operations in sequence | 0.371 |  |
| Translating mathematical answer to word form |  | 0.775 |
| Translating word problem into mathematical sentence |  | 0.821 |
| Variance explained (Eigen value) | $21.51 \% \quad 14.88 \%$ | 14.61 \% |
| Total variance explained | 51\% |  |

# Appendix A 2 <br> Questionnaire on Students＇Difficulties in Learning 

Dr．K．Abdul Gafoor<br>Professor<br>Sarabi．M．K<br>Research Scholar

வேด̆： $\qquad$






















 ®ดls，




|  | 0า46 <br>  Mun）0＠o <br>  <br>  | s．le <br>  MunのOCDO <br>  <br>  （खाक 6 ก̆ | （บ） <br>  <br>  <br>  <br>  |
| :---: | :---: | :---: | :---: |
| ชelmos\％ | $)$ | $)$ | $)$ |
| กฺาแท⿺𠃊⿳亠丷厂犬 | ） | ） |  |
| ๒ைைาแษา | ） | ） | ， |
| வை¢以บถ్ర | $)$ | 2 | ） |
| Mvongn | $)$ | ） | ） |
| கைைめが | $\square$ | $\bigcirc$ | $\bigcirc$ |




|  | வஇை๐ <br>  ๒๓รั |  <br>  உ๓今̆ |  |
| :---: | :---: | :---: | :---: |
| ロอツ08\％ | ） | ） | ） |
| กาญท1\％ | ） | （ | （ |
| கெைனา（ツை） | ） | ） | ） |
| வฺセン®®ฑ | ） | ） | ） |
| muvarn | $)$ | （－） | ） |
| களைமு | ） | （ | （ |



|  | ```303ロ0S゙```   ```\varrho๓``` | మ2గ리 <br>  Ø〇（ா） ロ（m） <br>  <br>  கைกவンตั |  <br>  <br>  <br>  |
| :---: | :---: | :---: | :---: |
| －อ\％0\％ |  |  | ） |
| กาญण） | ） |  | ） |
|  | ） |  | ） |
| ఎఱ010゙\％ |  |  | ） |
| Mondin jubumo | ） | ， | ） |
| களாめか | ） | ） | ） |

 ๓ృஙుาゅதรั உஸேァ๐？

|  aroluolemo <br>  <br>  <br>  |
| :---: |
|  |  |
|  |  |

』ฉายจาอแ๐
ํㅣㄹำた
か్యกロコヴ
ๆㄷ๙




| （ป） <br>  อறா円ா <br>  வெயைனைத் อளร゙ |
| :---: |
|  |  |
|  |  |
|  |  |

（ل）




|  |  <br>  <br>  |  ```001(%) ஹก】 2%%(%)<m0%0```  |  |
| :---: | :---: | :---: | :---: |
| 凹erouobio | （ | （ | ） |
| வியை無 | $\square$ | （） |  |
| ¢ைฺา（mリ｜ | ） | （ ） |  |
| ๓ை¢0リถู\％ | ） | （ |  |
|  |  |  |  |
| கை毋ถ゙ |  | （ |  |




|  |  <br>  <br>  |  <br>  ๓ைைา๙ைร้ ŋอ |
| :---: | :---: | :---: |
| ๑อツリマ๐ |  | （） |
| กிற\％1\％ | ） | ） |
| வைனி（ツை｜ |  | $\square$ |
| வ】œ00¢\％1 | ） | － |
|  | ） | （） |
| கியைல் | ） | （） |




 ఎ Шбъиด
 வைコバ






 வイஞாயன்（ைைை

 ఇ잉

| QPO00610 | $(1)$ | $(1)$ |
| :---: | :---: | :---: |
|  |  |  |
| 6） 000$][\mathrm{O}$ |  |  |
| 62， |  |  |
|  |  |  |
| $\infty 6 \Pi \infty 6$ | $3$ | $3$ |




|  |  <br>  கத5ுணை வேளைา <br>  <br>  |  <br>  વாை ஃாேைைைா <br>  கூกவンஸ |  <br>  வேளைンாைைைைா ஜாைை ஹுஙைனுక゙ ๒！ㅣㅀ |
| :---: | :---: | :---: | :---: |
| ロอ\％）8\％ | ） | （） | （） |
| กीற0\％ | ） | ） |  |
| கொிறツை | ） | O |  |
|  | $)$ | $)$ |  |
|  | $)$ | $)$ |  |
| களை冂ை | ） | $\bigcirc$ | ） |




|  |  <br>  <br>  |  <br>  <br>  |  |
| :---: | :---: | :---: | :---: |
| 2010030 | （ ） | $)$ |  |
|  |  |  |  |
| ） ABD （ CH |  |  |  |
| ๓นセリ88\％ |  | ） |  |
|  |  |  |  |
| A6¢ Ab） | ， | （ | ， |





2 －றைロறேைா கூூヘั
1 －๓lை๐ ற리

|  |  क5잉 | （ 60$)(00) 6$ G13 <br> வி） |  <br>  |
| :---: | :---: | :---: | :---: |
| 2elobebo | ） | ） | ） |
|  | ） | ） | （） |
| வெவி｜（ザ） | ） | ） | （） |
| 6ป¢以リถ\％ | ） | ） | （） |
|  | ） | ） | ） |
| ¢ | ） | （） | $\square$ |








$\square$（ราm Moฟd





(ே15)






















## Appendix A 3

## Questionnaire on Students' Difficulties in Learning

| Dr. K. Abdul Gafoor <br> Professor | Sarabi. M.K <br> Research Scholar |
| :--- | ---: |
| Name:........................................................................................ Boy/Girl/Others |  |

## Instructions

This questionnaire is meant for gathering information on students' difficulties upon learning various subjects. There would be various reasons for every person to find learning various subjects easy or difficult. Please indicate reasons for your difficulties in learning different subjects. Consider every reason you would have in the current and previous grade levels while responding. Your responses will be kept in safe custody and used for research purposes only; please cooperate with this research process by providing the most accurate information.
I. Indicate your difficulties in learning each subject using a scale of 1-3

3 -unable to learn; do not understand anything
2-difficulty at normal level like any other subject
1 - usually no difficulties
II. Ten different reasons causing difficulties in learning various subjects are given below. Indicate the appropriate level of difficulty towards each subject using a tick $(\checkmark)$ mark.

1. Is it difficult because you need help from others to understand the concepts?(requires help from teachers, parents, or friends)

| It is very difficult because |
| :---: |
| of the requirement of |
| getting help from others on |
| almost all concepts | | It is a little difficult |
| :---: |
| because of the |
| requirement of getting |
| help from others on some |
| concepts |$\quad$| It is not at all difficult |
| :---: |
| because the |
| concepts can be |
| learned by self- |
| reading |

2. Is it difficult to understand the subsequent topics if a class is missed?

| Have great |
| :---: |
| difficulty | | Have a little difficulty in understanding |
| :---: |
| some topics | | Have no difficulty of |
| :---: |
| this type |

3. Is it difficult because it requires memorization of concepts?

> It is difficult because of the requirement of memorizing a lot of concopts.

It is not much difficult because of the requirement of memorizing some concepts.

It is not at all difficult because it requires understanding the moaning of concopts only
Malayalam
4. Is it difficult because it requires repeated learning for understanding and apply?

It is difficult because of It is not much difficult the requirement of because it can be learned increased practicing without much practice

It is not at all difficult because it does not require repeated practice

| Malayalam |  |
| :--- | :--- |
| Physics |  |
| Chemistry |  |
| Biology |  |
| Social Science |  |
| Mathematics |  |

5. Is it difficult because it requires in-depth learning to have problem-solving ability?

| It is ditticult because of |
| :---: |
| the requirement of |
| emphasizing the |
| problem-solving aspect |


| It is not much ditticult |
| :---: |
| because the emphasis |
| on problem-solving is |
| not significant |


| It is not at all ditticult |
| :--- |
| because it does not have |
| much emphasis on |
| problem-solving |

Physics
Chemistry
Sology
Mathematios
6. Is there any difficulty due to the usage of signs and symbols?

| It is difficult because most of |
| :---: |
| the learning process |
| requires them |

It is not much difficult because
they are required only in some is not at all
difficult to use
themics
7. Is it difficult because of the requirement to understand the concepts with accuracy and precision?
It is difficult bocauso of kooping

accuracy and precision | It is loss difficult bocauso it only roquiros |
| :---: |
| an overall idea on concepts |

8. Is it difficult because of the use of several terms and concepts that are not usually used in real life?

| It is difficult because of |
| :---: |
| the requirement of |
| learning several such |
| terms |


| It is not much difficult |
| :--- |
| because there are only |
| a few such terms to |
| learn |

Malayalam
Physics
Chemistry
learning are used all all difficult
biology
Social science
9. 1s it difficult because a great amount of attention and concentration is required for Learning?

| It is difficult because a <br> great deal of attention <br> and concentration is <br> required | It is not much difficult as <br> it does not require much <br> attention and <br> concentration |
| :---: | :---: |
| Malayalam |  |
| Physics |  |
| Chemistry |  |
| require special attent all difficult |  |
| and concentration |  |

10. Is it difficult because learning material are not useful in daily life?

| It is difficult because |
| :---: |
| most of the materials |
| are of this type |

Malayalam
because only a few
materials are useless
III. Given below are other 3 reasons which may make difficulty in learning each subject; indicate your level of difficulty in a scale of one to three.
3 - Very much
2 - Comparatively less
1 - None
Difficulty of the
concepts
IV. A few difficulties related to mathematics learning are given below; indicate your difficulty using a check mark $(\checkmark)$

Related to numbers
$\square$ Understanding larger numbers
$\square$ understanding the place value of numbers
$\square$ Use of decimals
$\square$ Use of fractions
Related to mathematical signs and symbolsUnderstanding what is indicated by signs and symbols
$\square$
Understanding mathematical problems involving variablesDrawing geometrical figures based on given indicatorsUnderstanding geometrical figures

## Related to mathematical operations

Doing basic operations (addition, subtraction, multiplication and division)Following the orders and laws while doing mathematical operationsDoing simple operations mentallyRemembering numbers while doing operationsMaking mistakes when performing operations quicklyPaying more attention for a long time while performing larger operationsRelated to mathematics concepts and processesLearning concepts that are not related to daily life
Learning processes that are not useful in daily life

Related to problem solving abilityChoosing operations appropriatelyPerforming more than one operations in order
Understanding the key terms that helps solving problemsFinding the right formula to be usedUnderstanding the mathematical problem in story-type questionsUnderstanding long sentence type questionsUnderstanding indications that are not necessary for problem solvingConverting story problems into mathematical form
Requiring explanations on story problems from someoneConverting answers obtained in mathematical form into sentence
V. Write reasons that makes mathematics more difficult to learn other than the above listed reasons

## Appendix B

Glossary of Terms and Symbols in Elementary School Mathematics
（Pre－primary to standard 7）
PRE－PRIMARY

| Measurement | Number system | General | Questions |
| :---: | :---: | :---: | :---: |
| வలృర゙ | 1，2，3，4，5，6，7，8， 9 | ஆก®ృం | 円）（๑）？ |
| ๑ைดర̆ | พ๐வை | 6®ロ | 囚（ூ）？ |
| கதியరั | ๑ஸ゙ை |  |  |
| கூைைை゙ | สைロ๐๐ |  |  |
| อம®ం |  |  |  |
| வஸைo |  |  |  |

YEAR 1

| Position and Direction |  | Instruction | Questions |  |
| :---: | :---: | :---: | :---: | :---: |
| வுகலிண | றऽツூ\％）̆ | வロヅゃの | ஆ（）？ | 凸வ？ |
| லை๑¢ | வவஜியிண8 |  | （）（\％）？ | ஆ冂ைைงผ？ |
| வอయ゙ |  |  | ๙ை刀口： |  |
| றऽద゙ |  | ஈாய｜న్రிゅைை | 凹） | லூனவைهெி？ |
|  |  | கலைபாைைo | இிரை๑カி？ |  |
| வృmி®8 |  | வృ冈1న్రிカைை |  | ๑லை๑ை？ |
| விmிø8 |  |  | 凹） |  |
| （ロேவリ |  | ரை๐ாி®ிカூை | 毋ிரைைை？ |  |
| ปلฺ |  |  |  |  |
| வ๑1 |  |  |  |  |


| Measurement | Shapes | General |  |
| :---: | :---: | :---: | :---: |
| கூऽிめா゙ | வડం | வல๐ | ஸ⿴囗 |
| கூ毋ைைை | உ®ூலனூ | உல8னூ | உைைை |
| வలృூワ） | வைmன゙ | கூதชைஸ8 | Sel |
| வைกூ゙ | ธைிన్రయ゙ | இ๑ே | セ๐ணிフ्र |
| உ®๐๐ | ๓า¢ைன̆ | கூమmser | ற®ே |
| கதவுமை | ®ிவృன1 | றற๐，ைை๐ | －¢ேைை๐くன |
| ๓ใช๐ | Oృவ） | ๑ว๑๐ง | ๙冂セை |
| வอปన్న | ๑（1） | றறி | ¢ை『ைo |
|  |  | வைமிி | レロW） |
|  |  | ঢைゃめை | ¢00jo |
|  |  | $\omega_{8}$ Sी | கமளைபன |
|  |  | லூவวกช | 凸Оவవం |
|  |  | விைo | mาm゙ |
|  |  | （மவவைツl | விకృவேナツ |
|  |  | ๑ழி\％ | めலலாலช |


| Number System | Symbols |
| :---: | :---: |
|  | $+,-,=$ |
| 凹冂ை๐ |  |
| พ๐வை |  |
| றய®ర |  |
| ஹலூவ（ర゙．．．هைைฺฺ，20－90 |  |
| Addition \＆Substraction | Money |
| லூவை | வைை（1） |
| வேరิกบフロ\％ | றงฺ¢๐ |
|  | $\mathrm{C}_{\mathrm{g}} \mathrm{C}$ |
| 円）（6）कூி？ | セேงš |
| ゆら§3 0 |  |
| Time |  |
| ๑๐เดา | ஹறM |
| வகணర | ツ0ヤல |
| வைவிคை |  |
| உஉ्னவூ゙ | ற（9ைை |
| ๑ைவை | ®omo |
| ஸロツ๐ | สைவกை๐ |
|  |  |
| ঞাநßృం |  |
| விmใร์ |  |
| Bிவmo |  |
| ธைவ山า |  |

YEAR 2

| Position and Direction | Instruction | Questions |
| :---: | :---: | :---: |
| moomo | களைைலை | 毋（）（6） |
| றऽணை |  | இ๑ைைவெ？ |
| வอை） | ๔ே๐ஜிన్రிカலృ | ஷ（\％）？ |
|  |  | ๑）ல்๑ை？ |
| வ๑1 | ரை๐ハி®ிカைை | ஷ冂1ヵ？ |
| கூறைைை | ๔冂ல்மை๐ை | உஸேso？ |
| ปృவดs | வாமைைெロ̆ |  |
| （10sృmssm） | உறைிカாைை |  |
| ஹேலセேงş¢ே0ஸ8 |  |  |



YEAR 3

| Position and Direction | Instruction | Questions |
| :---: | :---: | :---: |
| ป！வ囚 |  | 凹）（ロ） |
| moomo |  |  |
| セேナセேงşセேナஸ8 | ரை๐ ハைைிைூு |  |
| வจา | கலறு゙の |  |
| ๓า® |  |  |
| msைவி\％8 |  |  |
|  |  |  |
|  | உறைிカல๐ |  |



| Addition，Substraction，Multiplication，Division |  | Money | Time |
| :---: | :---: | :---: | :---: |
| ढை® | กృ円m（ヵி¢） | ๓ง¢＠๐ | ธைைมัß |
| வృŋைomo | （6） | セேงร̆ | هைறับ |
| வேరిmૅ | ヘృమிカ๑๐ை | らைセ | ¢ேロッ๐ |
| 凹）（ேைது | のృmmono | $\mathrm{O}_{3} \mathrm{C}$ | จ1ก็ร |
| வைコカைை | ชைวาヵ๐ | விలவிவ（ேதிக | ๑ฺைமைกั๋ |
| வОก్欠） | வலிமிளை |  | வฺிカ๑ைช |
| வேனెmன゙ |  |  | ற๑ఱ๐ |
|  |  |  |  |
|  |  |  | ற๐๙ูกชัロวก๐ |

## YEAR 4

| Position and Direction |  | Instruction | Questions |  |
| :---: | :---: | :---: | :---: | :---: |
| றऽணை） |  |  | 毋） |  |
| வอை゙ | ஸவவலிカூு |  | 毋லாலை？ |  |
| สฺoగం |  |  | ஆிலைカカி？ |  |
| moomo | உறைிカூை |  |  |  |
| றS＠ி®8 |  |  |  |  |
| 凹ைை๐ | வாகலைெロด̆ |  |  |  |
| விவல阝லை | セேงஜி |  |  |  |
| கூவைைை |  |  |  |  |
| கேலஸேフSூகேல |  |  |  |  |
| ป！வ囚 |  |  |  |  |
| ms్வவிண் | ழைフロナカூா |  |  |  |
| ®0ృsono | வி®1ه2्रழ్మరை |  |  |  |
| வ®1 |  |  |  |  |
| Measurement | Shapes |  | General |  |
| றலకை | வร̧ | வృจவృО毋๐ | வใธை | （8ி＠）0¢3வ |
| カிறே๐ைフ○ | வைை0 |  | จใตา |  |
| Broo |  | ரைロைை๐ |  | றறி |
|  | （దி¢ேை○○ | 凸லவృం | றரைவ® | வைமிி |
| ஸை．®า． |  | விவிய | 凹ดவృ | ه120 |
| ๑ைைถูู จใกర | வอlన్నం | வழிめひర | வวセூ¢\％์ | （ூனூைகா |
| هใกช | வ๘ঞைலช | வకীக | ๗ைડరై్రఱગఱา | வகயிலூாைை¢ |
|  | வృృめひర | めら̧லアぴ | வใmใร้ | கృபை○ |
| ๓ใช๐ | றூைைைo | ๑๐¢๐๐ | வ๐ைை | ¢ை®ృo |
| வใตา | வృைை | வృைைறั゙ | ๑๑๐ | （－n）0 |
| SOOO | ปกกชอน | めமைனิறை |  | ลேวセากช์ |


| Measurement | Shapes | General |  |  |
| :---: | :---: | :---: | :---: | :---: |
| ఎகூணை <br>  <br>  ડ๓8 <br> จาํำใดใกช <br> จใกฮ์ <br>  <br>  <br> வவณิ |  |  <br> ஹற ๐ใดา <br> மைகळூ๐ | （）잉○○ <br> にはWOMo <br> வ®ிロフรา <br> 巳カைை <br> вวセே๐мั＂ <br> வロロ๐வயา <br> வృைைற゙ゥை <br> உாைை <br> （ロ0ロणృృ | வைய <br> （カிめコロூ』 <br> ற（๐。 <br> ป్రఠூ円ி <br> ஜஜวకி円ひス <br> உவிைை |
| Number System |  |  |  | Money |
| ๙ロロー |  |  | هృa |  |
| ๔๐ெ๑றை๐வை |  |  | セேงรั |  |
|  |  |  | வMo |  |
|  |  |  | ঢைக |  |
| ற®รฺ๐வృ |  |  | mிகேษロ |  |
| 3กั๐๐வ |  |  | வबு凹OMo |  |
| ๑ఘைண8ก๐வை |  |  | －دصه |  |
| мறธช |  |  | விถิญอிโ్రయ゙ |  |
| 凹ஸை๐ |  |  | ¢ใヵ๐าఱி®1กั้ |  |
|  |  |  |  |  |



YEAR 5

| Position and Direction | Instruction | Questions |
| :---: | :---: | :---: |
| றS＠ி®ర |  |  |
| வோைவேงรั |  | 毋）（カை⿰七ைப̆ |
| கூ毋ைை | வாைிறைூூ | 凹（6）？ |
| ๑ரைక̧ృృm |  |  |
| வ๑1 | விறிெカை |  |
| ¢ி® |  |  |
| ๑ைதுாை |  |  |
| வூவுカை |  |  |
|  |  |  |


| Measurement | Shapes |  | General |
| :---: | :---: | :---: | :---: |
| coo |  | （ே）¢） | م－lo |
| வகூమ1 | வி®1வั | ปハைกั（め）ఱ |  |
| هใกర | விஷ3．（ぁ⿱宀㠯犬） | வకி毋 | வையo |
| อிกగర | ธைロிக゙（வ๙๐） | விவிய |  |
| கலヘช์ | வ®ிவั | வel | ற®ே |
| 凸めம๐ | கூஜo（๑ைை，ハ0வ） | ชาตை | ஜேงธிめひర |
| வுயிலாర | வ凹ைுஜo | றைo | வைரைைைைைை |
| ๙ைロேிலா | งฯนัక్รஜ๐ | 凹லగவృ |  （in lowest term） |
| வใดา | พกัธைృృロ๐ | ¢ைऽరెm | æ๐ธา |
| （6）ర |  | （croo | வృைைஸั゙ |
| ๑๐க゙5రิ |  | மைைை | கூ๐ |
|  | வ®ி円ைை | ¢ைㅣo |  |
|  | ฺியృலை | ه๑๐ูフ¢ | வకிゃ |
| வऽహై | கேலஸ゙／angle | விைை | விШo |
| வย1న్ํ | ¢ே（毋๐）（Centre） |  | 『ハSめ口 |
| ๓ロロ®ono | வృఅ，வமด̆ | வ๐ðิ๐ |  |
| （6ヵmelo | هŞ®ృ®，هŞ | هी2 | வைフ็ூவコツ（Common） |
|  | ¢ণைன்வ๙๐ | வకીゃ | （வளைைை |
| B100 | 毋ூைঢைo | mி¢ู® | ஸுコிவூூிைைை |
| வவญช |  | ถை®ிவ्र | ๙ฺセை |
|  | O－」o | つைவைை○ |  |
| しんOO |  （Area） | mியிற |  |
| கிவேナ（6） | வ®నูชญ้ | ๙ைण¢๐ |  |
| Sஸ® | （దிகேலற） |  | வேరిカளை |
|  | வృomo（diameter） |  |  |
| வอி¢ | ชைDoo（Radius） |  |  |
| ธใช๐ | வร์ |  |  |
| カา．จา | வృmை（circle） |  |  |
|  |  |  |  |



## YEAR 6

| Position and Direction | Instruction | Questions |
| :---: | :---: | :---: |
| ป］வهs | விறிெカை | 凹）（か）？ |
| கூரைை |  | 凹ஆலดை？ |
| வேセேงร̆ |  |  |
| هujo | வாெிறைன̆ |  |
|  |  |  |
| （roగ毋mைsగo | ハை๐ாி®ிカலுカ |  |
| ๑றதூカெ |  |  |
| கூへூ毋 |  |  |
| விలஞ冂๑ை |  |  |
| moomo |  |  |
| ตி® |  |  |
| （10şmssm） |  |  |


| Measurement | Shapes | General |  |  |
| :---: | :---: | :---: | :---: | :---: |
| ๙๐லவృめひ8 | வృ円ை |  | － | 凹Sめ๐ |
| ๓ใช๐ | ロకேேナள88 <br> （Right Angle） | வைரைைளைைை | மை｜\％ | ๑กั |
| வใต1 | CேடイO | ®Oூ38வ | ๑๑๐ | ロ๐ைை |
| พી（๗）（1 ${ }^{0}$ | வி®ிவั | สைsาmصomo |  | கூ毋⿴囗 |
| จา込®ใกช | வ0న్రญั | ஜேงธา | விை | （ロハே）ைகை |
|  |  | （C8ロO | ¢ைృo | ๙ூவరెmைை |
| จใดช |  | றறி | ゅృறை○ | （6x） |
| வSШß゙ | வ๘๐ | வைゅ๐ி | mowృ | 30ృロ1） |
| （0）O | ツロコறைo |  | வృைைஸั゙ | கூSชைくன |
| カிறேロパ○ | دળை®o |  | றற○ | வைய |
| อிกగช | வృวกั¢ை（volume） |  | （ロハைヵช | ๑வฺே๐ |
| mu®somo | OூவO |  | கலツツ\％ | ๙ைறைツன゙ |
|  | อชชชவน（capacity） |  | ه120 | றை⿺𠃊丿 |
| ற0కை | வృコロヘర |  | வி¢ை | пையை๑ை |
| க๐ாి，வऽహై |  |  | வைコைைவை | 囚○வృ |
| வுமைコロ8 | －১ை๐ |  | （ヵ）¢ | வฺைவั |
| ふ0円o | ஜேிఱஜேフรา（linear pair） |  | ه๐าะ्यીડృృカ （』0றレัゥ®o ロОก็๐） |  |
| வอlన్న0 | ஷை｜రిகேナஸ์ |  |  |  |
| ปరกูชவ้ |  |  |  |  |
| ๓றைกைชิาจใกฉิ （ณฺ．๑றை．円ி．） |  |  |  |  |
| கேフஸன்ロロกிறา （Protractor） |  |  |  |  |


| Number System |  |  | $\frac{\text { Symbols }}{\angle}$ |
| :---: | :---: | :---: | :---: |
| （6ヵカி๐ | soæృพ๐வృ | ๙வைコロ○ |  |
|  | ๙iosoæృmoவృ |  |  |
| ธை๐ธ01（average） | вぃフovomoomo | ๗๓ை |  |
| พ๐வை | ¢аво | ®0m○ |  |
|  | 300000 |  |  |
| डीmo | Eிmmoவy |  |  |
|  | Elmo |  |  |
| Bu0） |  |  |  |
| moomハle |  |  |  |
| อа®๐ |  |  |  |


| Addition，Substraction，Multiplication \＆Division |  | Money | Graph |
| :---: | :---: | :---: | :---: |
| （ே冋） | ๙ிกั้ร๐ | ¢ைக |  |
| வைைカை | வృளைomo | $\mathrm{O}_{3}$ | －ைை゙ฺ |
| ヘృறைハカา® |  | هை | வใดை |
| ®๑¢๐ |  | வอบ้ | ๘ーめอ |
| めృS¢0 | ¢ேరెmบフr | ه120 |  |
|  |  | வכ¢゙న |  |
|  |  | விలாலூவั |  |
| ヘృమmo |  | கிழาவั |  |
| ベ円lo |  | விอ |  |

YEAR 7

| Position and Direction | Instruction | Questions |
| :---: | :---: | :---: |
| ๑ฺைరెவ๙๐ |  | ๑）（๑）？ |
| ป®1வั |  |  |
| د®ி¢m |  |  |
| விராைலை |  |  |
| கூ毋ைை | விه1ه2్ర¢5\％ |  |
| வలைைவை๐ |  |  |
| ๑ைSృa |  |  |
|  | （カவி） |  |
| ms్வவி\％8 | வாெைைெலூ |  |
|  |  |  |
| ชnsimssm | พவன்மை毋லை |  |
| றऽயிண | வி๙阝1ヵロிヵாை |  |
| குమ3lo353m | ரை๐ாைைிெூை |  |
| ஃய冂1sவி§ | ரைローை○ |  |
|  |  |  |


| ｀General |  |  |  |
| :---: | :---: | :---: | :---: |
| விணீவை | வைフைைவை | （ヵ） 0 | سூவிన్నிهை冂ை |
| ஜ๐งา | வை｜\％ | ه12 | ๔ூய¢๐ |
| வコセிヵカை | ๙ூவరెmைைo | （A）DO | ๔ை山つ®o |
| விவ®ใ¢ை | مصอ | ๑ெய゙ロ | வకી¢ |
| ¢ை巳ృo | வைフைைவை | வவேேை | வுంヅめด̆ |
| விШo |  | வృரைஸ゙®o | றற○ |
| வฺ๐๐ | พவியேกษ่ | ¢ைธ๑๐ | めృゝை○ |
| வડી¢ | வ๑ை |  | ๑ه2 |
| வி¢ை | வுணை | விவ®லைலช | －10 |
|  | ¢ியிற | 凸カ®ロル๐ | ロ๐ைை |
| ตาゅ๓ைை | வெரைைரைைை | （ロロリホ） |  |
| В๑ูฮl） | ヵி¢̆வழカー | ก๐வృOロ®0 |  |


| Measurement | Shapes |  |  |
| :---: | :---: | :---: | :---: |
| கிழேナパフ○ |  |  | உையவை๐ |
|  |  |  |  |
|  |  | ngled triangle） | د®ிவั |
| ஹறૅ | கலర¢ை（hypotenuse） |  | बூロo |
|  | พงவ๐กைிக๐（parall | logram） | வ๑ |
|  | வ冂ுమேலஸுめくる（alter | ate angles） | கே（ß） |
| Sஸ8゙（tonne） |  | responding angle | கேலஸ์ |
| カி¢กถิธ |  |  |  |
| هவ（5ிக |  |  |  |
| வーかついつつ。 |  |  |  |
| வృலก๊¢ | விகல்ஸை |  |  |
| வ®ัํอบ | －Şo |  |  |
| ปృกชญั |  |  |  |
| อชฺชவ้ | నたை⿺夂力。 |  |  |
| SOCOO | ツロவమృo |  |  |
| ๑ธゐß゙ |  |  |  |
| வகூ゙セை | พロวฺை๐ |  |  |
| ஹ®รை | விஷை |  |  |
| 凸めカ๐ | வひ๐ |  |  |
| றி๑ே๐ั | Ф๐ஸ๐ |  |  |
| พી（๗） |  |  |  |
|  | Number System |  | Symbols |
|  |  |  | $\angle$（ळேலஸ） |
| डीmmoவy | ชฺே๐๐ |  | ${ }^{0} \text { (พา(๗ી) }$ |
| கృறை <br> （powers of，raised to） |  <br> （algebraic expression）（positive numbers） |  | （ ） |
| கృைைセ๐（exponent） | Bu00000 |  | ปృกูชญั（P） |
| வరెળం（square） | 3000\％00000mo | ช10๐¢0 | กใช๐（1） |
| 凹๓円○（cube） |  | ற®รู๐வை | வใตை（b） |
| கృாிகலஸ๐ （exponentiation） | ¢ேロBO |  | $>,<$ |
|  | 80గ\％ |  | $\neq, \sqrt{ }$ |
| 3000000 | லவைைO○ |  | $5{ }^{2}$ |
|  |  |  |  |


| Addition，Substraction，Multiplication \＆Division | Money |  |
| :---: | :---: | :---: |
| வృળைomo | هைอவั | வelu（ interest） |
| जృmmo（into） | ঢைos | வmo |
| $\mathrm{O}_{\mathrm{j}} \mathrm{m}$ | வीอ | விలவிவ®வకிக |
| ヘృమிகை๐๐ | －10so | สூアコツめロ○ |
| ๑ூ円ைை | ตกช้ร | வுSめியாூ |
| வைวカைை | கிழาญั | வுSカவுவுவைை |
| пゅைツ๐ | றிகேษை | றலவั |
| ๑๐ฺ๐ | விలாலிழிவั | வைmைவle |
| ๑๐ฺை |  |  |
| ธிกั้ร |  |  |


| Time | Graph |
| :---: | :---: |
| บ๐๐ை๐1 | வைைை｜ைைアヒ8（Pie diagram） |
| จ1m |  |
| ๑ைைฺฺกั์ |  |
| வ冂ிவカைன |  |
| จใกัర |  |
|  |  |
| B60 |  |
|  |  |
| வேை๐ |  |

## Appendix C1

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

# Test of Difficulties in Language of Mathematics－Set A 

## （For Malayalam Medium Primary School Students）

 VIIIDr．K．Abdul Gafoor Professor<br>Sarabi．M．K<br>Research Scholar<br>வேดั：<br>$\qquad$ 










1） $4^{2}=$ $\qquad$
a） $4+4$
b） $4 \times 4$
c） $4 \times 4 \times 4$
d） $4 \times 4 \times 4 \times 4$

a） $3^{\mathrm{m}}$
b） $\mathrm{m}^{3}$
c） mx 3
d）$m \div 3$

a） 6
b） 8
c） 48
d） $48 \div 6$

a） $4+4$
b） 4 x 4
c） $4 \times 4 x 4$
d） 4 x 4 x 4 x 4


a） $25+40$
b） $25-40$
c） $40-25$
d） $40 \div 25$
 คlかぁృ。？
a） 1 カிழேナ（ゥ๐๐
b） 250 （000
c） 500 （ $ภ \circ \circ$
d） 1000 （0つO


a） $35-25$
b） $35+25$
c） $25-35$
d） $2 \times(35+25)$


a) 4 ฮักด๐
b) $4 \frac{1}{2}$ จใดถ
c) $5 \frac{1}{2}$ จาดฉ ถ
d) 6 จใดฉ

a) $45-15$ b) $45+15$
c) $45 \div 15$
d) $45 \times 15$
 ஸைைுமிதி.

a) $(\mathrm{X}+\mathrm{X}+1)-1$
b) $\mathrm{X}+\mathrm{Y}-1$
c) $\mathrm{X}+2 \mathrm{X}-1$
d) $\mathrm{X}+2 \mathrm{Y}-1$
 றிறుృ






a) $\qquad$
b) $\perp$
c) 1
d)

13)



a) AC
b) AB
c) AD
d) BD

a)

b)
$\square$
c)

d)

15)


a) $\angle \mathrm{EDA}$
b) $\angle \mathrm{ADB}$
c) $\angle \mathrm{BDE}$
d) $\angle \mathrm{DBC}$










a）ロอツைช๐
b）ஹ๐ஸูากั
c）ถி円ณ
d）ทఱை๐ักั
 18）வுமிைா
a）$\frac{3}{4}$
b）$\frac{4}{3}$
c） $3 \frac{1}{4}$
d）$\frac{1}{3}$

19）ส円ேை
a） $5 \times \frac{1}{2}$
b）$\frac{5}{\frac{1}{2}}$
c） $5 \frac{1}{2}$
d）$\frac{\frac{1}{2}}{5}$




20）กใช๐（ $l$
a）lend
b）left
c）length
d）least

21）விறை（b）
a）brief
b）bright
c）box
d）breadth

22）ปไกญฺั（ $p$ ）
a） per
b）perimeter
c）percent
d）periodical



| ட๑（1） |  |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  พ๐வృఱ๐ஸ゙． |  |
|  |  |
|  |  |
|  <br>  <br>  |  |
|  ๓ใธロ๐ஸ゙． |  |
| 31） $5^{2}=25, \sqrt{5}=25$ |  |
| 32）$(150-50)-40=150-(50+40)$ |  |
| 33）$(\mathrm{X}-\mathrm{Y})-\mathrm{Z}=\mathrm{X}-(\mathrm{Y}+\mathrm{Z})$ |  |
|  <br>  |  |
| 35） |  |
| 36） <br> $\triangle \mathrm{ABC}$ ఱிண8 $\angle \mathrm{ACB}+\angle \mathrm{ABC}+\angle \mathrm{BAC}=360^{\circ}$ ๙ூஸ゙． |  |
| 37） <br>  |  |





## Set－I



## Set－II

A
B

45）๑ளைว० カృைை
a）$\oiint \wp^{\varphi}$
46） 3000
b）ஞூก̆
47）๑ைฺฑை
c）emo
48）พూૅต
d）ఎணั
49）ளைக カ๐றை
e）வశి๓ం
50）Фுమை० ゅృைை
f）$m_{1} ก$ ॅ

| 45$)$ |  |
| :---: | :--- |
| 46$)$ |  |
| 47$)$ |  |
| 48$)$ |  |
| 49$)$ |  |
| 50$)$ |  |

g）にロேзио
h）கృதळ

Set－III


Set－IV
A

## B

57）

a）土ளூゃం
58）


59）

c）$ா$ 毋ூ』ைைo
60）

d）พงロナாரைிカ๐
61）

e）Мัธัகృฉฉ
62）



| 57$)$ |  |
| :---: | :--- |
| 58$)$ |  |
| 59$)$ |  |
| 60$)$ |  |
| 61$)$ |  |
| 62$)$ |  |

g）வஹகృణం



| ＠（3）： | 4 cobscmo 1 | $4+1$ |
| :---: | :---: | :---: |
| 63） | 4 カృハ1 10 |  |
| 64） | 4 வชิへูం |  |
| 65） |  |  |

## Appendix C2

UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

# Scoring Key for Test of Difficulties in Language of Mathematics-Set A <br> (For Malayalam Medium Primary School Students) 

VIII

| Item No. | Answer | Item No. | Answer | Item No. | Answer |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | B | 23. | * | 45. | E |
| 2. | D | 24. | $\checkmark$ | 46. | D |
| 3. | B | 25. | $\checkmark$ | 47. | F |
| 4. | B | 26. | $\times$ | 48. | A |
| 5. | C | 27. | $\times$ | 49. | H |
| 6. | C | 28. | $\checkmark$ | 50. | C |
| 7. | B | 29. | $\checkmark$ | 51. | C |
| 8. | D | 30. | $\checkmark$ | 52. | E |
| 9. | A | 31. | $\times$ | 53. | H |
| 10. | A | 32. | $\checkmark$ | 54. | G |
| 11. | D | 33. | $\checkmark$ | 55. | F |
| 12. | A | 34. | * | 56. | A |
| 13. | A | 35. | $\checkmark$ | 57. | D |
| 14. | B | 36. | $\checkmark$ | 58. | F |
| 15. | B | 37. | $\times$ | 59. | H |
| 16. | D | 38. | $\checkmark$ | 60. | C |
| 17. | B | 39. | H | 61. | A |
| 18. | A | 40. | C | 62. | G |
| 19. | C | 41. | A | 63. | $4^{10}$ |
| 20. | C | 42. | G | 64. | $4^{2}$ |
| 21. | D | 43. | F | 65. | $10 \neq 12$ |
| 22. | B | 44. | E |  |  |

## Appendix D1

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

# Test of Difficulties in Language of Mathematics－Set B 

（For Malayalam Medium Primary School Students） VIII

Dr．K．Abdul Gafoor Professor<br>Sarabi．M．K<br>Research Scholar

■ேดั： $\qquad$


## 







##  



a）$l \div \mathrm{b}$
b）$l \mathrm{xb}$
c）$l+\mathrm{b}$
d）$l-\mathrm{b}$

2） $4^{5}$ ๑ย＇ 5 ＇๑ை வกळுய ேே๑๐ாั？
a）வช்ぃ๐
b）கృறைゅ๐
c）凹ா๐
d）கэæృ๐

a） 48
b） 8
c） 6
d） $48 \div 6$

a） $45-15$
b） $45+15$
c） $45 \div 15$
d） $45 \times 15$


a） $15+10$
b） $15-10$
c） $15 \times 10$
d） $2 \times(15+10)$


a） 500
b） 1000
c） 1500
d） 2000

a）$\frac{2}{1}$
b）$\frac{1}{2}$
c）$\frac{2}{2}$
d） $1 \frac{1}{2}$


a） 3.30 ฮัดฉ
b） 3.75 จใกฉ
c） 37.5 จใกฉ
d） 37.5 ๑กอ๑า ．


a） $6 \times 42$
b） $42-6$
c） $42 \div 6$
d） $42+6$

a）$\frac{1}{2}$
b）$\frac{2}{4}$
c）$\frac{3}{6}$
d）$\frac{4}{6}$
 $\qquad$ ஹாி๑ைஜீ

a） 12
b） 24
c） 48
d） 72
 ஸைைூோృக．


a）$X+Y$
b）$X+(Y+Z)$
c）$(X+Y)+Z$
d） $\mathrm{X}+(\mathrm{X}+\mathrm{Y})$


 ๗ூゅூகா．
 ம゙ヵாகா．
 ヅゅゥぁ．

 ஆண゙？
a）

b）

c）

d）


a）$<$
b）$\Delta$
c）$\nabla$
d）$\wedge$

a）

b） $\square$
c）

d）

|  |  |
| :--- | :--- |


a）

b） $\qquad$
c）

d） $\square$


a） AC
b） AB
c） BC
d） BD

19）


a） $\mathrm{AB}, \mathrm{CD}$
b） $\mathrm{AB}, \mathrm{MN}$
c） $\mathrm{CD}, \mathrm{MN}$





a）ฉอ๗งช๐
b）உ๐๗ูากั
c）ถीณ
d）ทยை๐ฉกั




21）สฺロロ๐（ $r$ ）
a）rank
b）radius
c）rate
d）ratio

22）வנコగ००（d）
a）diameter
b）device
c）difference
d）dimension
23) கே(ா๐ (c)
a) cent
b) cell
c) centi
d) center



|  |  |
| :---: | :---: |
|  |  |
| 25) 32 ๑๑ู ๑กั พ๐வృఱวஸ゙. |  |
| 26) $23+23+23+23+23=5 \times 23$ |  |
| 27) $(110+50)+(110-50)=3 \times 100$ |  |
| 28) $X+X+X+X+X=5 X$ |  |
|  |  |
|  <br>  |  |
|  <br>  |  |
|  <br>  |  |
|  <br>  |  |
|  <br>  |  |
|  <br>  |  |
| 36) <br>  |  |
| 37) <br>  களைว๘ வாை. |  |


|  |  |
| :---: | :---: |
| 38） <br>  カlヵ』ృం． |  |
| 39） <br>  <br>  |  |




## Set－I

A
40）Bu0
41）விறั๓าชిறัณ
42）வృกัศை
43）๑ก๓ึ
வ凹
45）๙กบัร

B
a）๑๑ร̆
b）உஓฺฺธั
c）கையிா๐
d）๙๓ண
e）வாைறை
f）வロన్ํญบั

| 40$)$ |  |
| :--- | :--- |
| 41$)$ |  |
| 42$)$ |  |
| 43$)$ |  |
| 44$)$ |  |
| 45$)$ |  |

g）வாூ゙
h）mă
Set－II



Set－III

A
52）ேேேிிஃைக
53）கృரைロ
54）ஈอฉ๑ృ๐
55）வுறுృகலிமலுக
56）ऊூவனிカைைை
57）வృனळి（கロ๐
a）வைைாைைல
b）ঞーゥாைைல
c）வூへிஃாைக

e）விவ®ใளைைைก
f）வใளకృం வใளకృం

| 52$)$ |  |
| :--- | :--- |
| 53$)$ |  |
| 54$)$ |  |
| 55$)$ |  |
| 56$)$ |  |
| 57$)$ |  |




Set－IV
A

## B

58）พ๑ロ๓๐

59）هబృ๐
b）கூ๑ธயృ3®
60）ஸைை
c） ms
61）
d）றை巳ృ○
62）สைறுூூ 0 ロー
63）வவலிఱிஜீ
e）ஃாிமயிக๐
f）พృேஷฯัロ๐

| 58$)$ |  |
| :---: | :--- |
| 59$)$ |  |
| 60$)$ |  |
| 61$)$ |  |
| 62$)$ |  |
| 63$)$ |  |

g）๔ワด๐
h）ปృกாั゙


| （30： | 4 ¢bşmo 1 | $4+1$ |
| :---: | :---: | :---: |
| 64） | 4 ЗலОООСО० 5 |  |
| 65） | 4 ヱ巳mo |  |
| 66） | 3 ベmo 4 |  |

## Appendix $\mathbf{D} 2$

## UNIVERSITY OF CALICUT

 DEPARTMENT OF EDUCATION
## Scoring Key for Test of Difficulties in Language of Mathematics-Set B (For Malayalam Medium Primary School Students) <br> VIII

| Item No. | Answer | Item No. | Answer | Item No. | Answer |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | B | 23. | D | 45. | A |
| 2. | B | 24. | x | 46. | G |
| 3. | A | 25. | * | 47. | C |
| 4. | A | 26. | $\checkmark$ | 48. | F |
| 5. | A | 27. | $\times$ | 49. | A |
| 6. | B | 28. | $\checkmark$ | 50. | E |
| 7. | B | 29. | $\checkmark$ | 51. | H |
| 8. | B | 30. | $\times$ | 52. | C |
| 9. | C | 31. | $\checkmark$ | 53. | G |
| 10. | D | 32. | * | 54. | A |
| 11. | B | 33. | $\times$ | 55. | H |
| 12. | D | 34. | $\checkmark$ | 56. | F |
| 13. | B | 35. | $\times$ | 57. | E |
| 14. | D | 36. | $\checkmark$ | 58. | D |
| 15. | B | 37. | $x$ | 59. | C |
| 16. | D | 38. | $\times$ | 60. | E |
| 17. | C | 39. | $\checkmark$ | 61. | B |
| 18. | A | 40. | G | 62. | A |
| 19. | A | 41. | F | 63. | H |
| 20. | D | 42. | B | 64. | 4.5 |
| 21. | B | 43. | H | 65. | $4^{3}$ |
| 22. | A | 44. | D | 66. | $3 \times 4$ |

## Appendix E1

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

# Test of Difficulties in Language of Mathematics-Set C 

(For Malayalam Medium Primary School Students) VIII

| Dr. K. Abdul Gafoor | Sarabi. M.K <br> Professor |
| :--- | ---: |

■ேดั: $\qquad$


## 










a) 6
b) 8
c) 48
d) $48 \div 6$

a) $2: 1$
b) $1: 2$
c) $1: 1$
d) $2: 2$

a) $\frac{4}{1}$
b) $\frac{1}{2}$
c) $\frac{1}{4}$
d) $\frac{2}{4}$

a) $4 \times \mathrm{s}$
b) $4+\mathrm{s}$
c) $\mathrm{s}^{4}$
d) $4^{\text {s }}$


a) 1
b) 2
c) 4
d) 8

a) $45-15$
b) $45+15$
c) $45 \times 15$
d) $45 \div 15$



c) வృயா゙ อிంగன
b) வுmை ยlก̊
d) ๔ேேை セிกூ



a) $30+10$
b) $30-10$
c) $2+30+10$
d) $2 \times 10+2 \times 30$


a) $24 \div 8$
b) $24+8$
c) $24 \times 8$
d) $24-8$


a) $1: 2$
b) $2: 1$
c) $1: 5$
d) $5: 1$
 ஸைைுகிலுக.

a) $2 \mathrm{X} \times 3 \mathrm{X}$
b) $2 \mathrm{X}+3$
c) $2 X+3 X$
d) $2 \mathrm{X}+3 \mathrm{Y}$







a) $<$
b) $\wedge$
c) $\Delta$
d) $\vee$
14)

| $*$ | $*$ | $*$ | $*$ | $*$ |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{*}$ | $*$ | $*$ | $*$ | ${ }^{*}$ |
| ${ }^{*}$ | ${ }^{*}$ | ${ }^{*}$ | ${ }^{*}$ | ${ }^{*}$ |
| ${ }^{*}$ | ${ }^{*}$ | ${ }^{*}$ | ${ }^{*}$ | ${ }^{*}$ |


a) 5 வఠl, 4 mி®
b) 4 வ๑า, 5 mใ๑
c) 5 வ®ी, 5 円ி®
d) 4 வ๐า, 4 mา®
15)


a) $\mathrm{AB}, \mathrm{BC}$
b) $\mathrm{AB}, \mathrm{AC}$
c) $\mathrm{BC}, \mathrm{AC}$











b）கேசள๐
c）வறัடை๐
d）$\infty(\infty$

18）๑๐．๑า．





19）กอา．๓า．





20）

c）دளృఠ هிగగర



21）カึ．๓า．

c）カிேேナカォ（5ிக
b）கிேே๐ จใกั




|  |  |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
| 25) $2^{2} \times 3^{2}=(2 \times 3)^{2}$ |  |
| 26) $2^{4} \times 2^{3}=2^{4 \times 3}$ |  |
|  |  |
| 28) $X^{2} Y^{2}=(X Y)^{2}$ |  |
|  $x^{\mathrm{m}_{\mathrm{x}}^{\mathrm{n}}}$ (ேறஸ. |  |
|  <br>  |  |
|  <br>  |  |
|  <br>  |  |
|  <br>  |  |
|  <br>  |  |
| 35) <br>  (ேேமி円ிணதృం. |  |
| 36) <br>  |  |





## Set－I

## A

39）றையை๐
a）mృก̆
40）வృவகைா๐

41）巳๐ゅ๐
c）बேே̆
42）งฺฺั
d）凹ร̆
43）จาอิำ
e）கூのカிண

f）$\because \bowtie \circ$
g）बேயிஷ๐
h）கேロறリறั

| 39$)$ |  |
| :--- | :--- |
| 40$)$ |  |
| 41$)$ |  |
| 42$)$ |  |
| 43$)$ |  |
| 44$)$ |  |

Set－II

A
45）ஸிกYัง
46）๔ாறைை
47）றலவั
48）๙ைしつ๐
49）றற๐


B
a）மைை
b）வைிி
c）๙ฺロவั
d）வృ๓ృom๐
e）கூก2
f）ऊroo

| 45$)$ |  |
| :--- | :--- |
| 46$)$ |  |
| 47$)$ |  |
| 48$)$ |  |
| 49$)$ |  |
| 50$)$ |  |

g）『ş๐
h）ஸைமைி

Set－III

A

| 51） | ๑๐毋๐ | a） | エேరூカカカ | 51） |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 52） |  | b） | ¢ேலறல | 52） |  |
| 53） | கி¢าவั | c） | ๑๐ை | 53） |  |
| 54） | வ๓ைฺ | d） |  | 54） |  |
| 55） | Ȩ®®o | e） | றறி | 55） |  |
|  |  | f） | றலவั ๑ெய゙¢ ைைை | 56） |  |
|  |  | g） <br> h） | ๙ฺญவั <br> றிேைை |  |  |

## Set－IV

## A

B
57）ஞூßコळ๐
58）உロை
59）ปુญค
b）eooso

60）Bo®o
d）ウைロฯ
61）నி円ிகலுக
e）๘ேฒぃஸிைை
 காேேூ̆

| 57$)$ |  |
| :--- | :--- |
| 58$)$ |  |
| 59$)$ |  |
| 60$)$ |  |
| 61$)$ |  |
| 62$)$ |  |

a） s b

62）
f）อிดัฉ
g）ゅใஜのஸிゥை



| ๑（30： | 4 \＆ృJ̧mo 1 | $4+1$ |
| :---: | :---: | :---: |
| 63） | 4 வரைைロ○ |  |
| 64） | 3 வدกூைஸ゙ 5 |  |
| 65） | 4 พูしめ） |  |
| 66） | 15 வைைெカை๐ 5 |  |

## Appendix $\mathbf{E} 2$

UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Scoring Key for Test of Difficulties in Language of Mathematics-Set C <br> (For Malayalam Medium Primary School Students)

VIII

| Item No. | Answer | Item No. | Answer | Item No. | Answer |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | A | 23. | * | 45. | H |
| 2. | B | 24. | $\checkmark$ | 46. | D |
| 3. | C | 25. | $\checkmark$ | 47. | E |
| 4. | A | 26. | $\times$ | 48. | F |
| 5. | C | 27. | $\times$ | 49. | A |
| 6. | A | 28. | $\checkmark$ | 50. | G |
| 7. | B | 29. | * | 51. | G |
| 8. | A | 30. | * | 52. | A |
| 9. | A | 31. | $\checkmark$ | 53. | F |
| 10. | B | 32. | $\checkmark$ | 54. | H |
| 11. | C | 33. | $\checkmark$ | 55. | C |
| 12. | A | 34. | * | 56. | D |
| 13. | A | 35. | * | 57. | B |
| 14. | B | 36. | $\checkmark$ | 58. | C |
| 15. | A | 37. | $\checkmark$ | 59. | D |
| 16. | C | 38. | $\times$ | 60. | A |
| 17. | C | 39. | F | 61. | H |
| 18. | D | 40. | E | 62. | G |
| 19. | A | 41. | B | 63. | 4\% |
| 20. | D | 42. | C | 64. | $3<5$ |
| 21. | B | 43. | G | 65. | $4^{0}$ |
| 22. | $\checkmark$ | 44. | H | 66. | 15*5 or 15/5 |

## Appendix F1

## Data and Results of Item Analysis and the Items selected to the final version of Scale of Attitude towards Mathematics

Table F1-1. Data and Results of Item Analysis and the Items selected to the final version of Scale of Attitude towards Mathematics

| Item <br> no. <br> (draft <br> tool) | $\mathrm{M}_{1}$ | $\mathrm{M}_{2}$ | $\mathrm{SD}_{1}$ | $\mathrm{SD}_{2}$ | t | Item no. (final tool) | Item <br> no. <br> (draft <br> tool) | $\mathrm{M}_{1}$ |  | $\mathrm{SD}_{1}$ | $\mathrm{SD}_{2}$ | t | Item no. (final tool) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 4.89 | 3.62 | 0.37 | 1.25 | 9.77 | 1 | $33^{\text {c }}$ | 4.83 | 3.89 | 0.55 | 1.37 | 6.37 | - |
| $2^{\text {b }}$ | 4.94 | 4.63 | 0.28 | 0.88 | 3.41 | - | 34* | 4.30 | 2.72 | 1.42 | 1.62 | 7.34 | 20 |
| $3^{\text {a }}$ | 3.97 | 3.69 | 1.44 | 1.35 | 1.43 | - | 35* | 4.57 | 2.08 | 1.08 | 1.35 | 14.36 | 21 |
| $4^{\text {a }}$ | 3.88 | 3.77 | 1.58 | 1.44 | 0.52 | - | 36 | 4.38 | 3.26 | 1.11 | 1.55 | 5.88 | 22 |
| $5^{\text {b }}$ | 4.85 | 4.44 | 0.56 | 1.07 | 3.36 | - | 37 | 4.64 | 2.66 | 0.82 | 1.51 | 11.50 | 23 |
| 6* | 4.58 | 2.34 | 0.97 | 1.46 | 12.75 | 2 | 38 | 4.17 | 2.37 | 1.38 | 1.44 | 9.03 | 24 |
| 7* | 4.23 | 2.11 | 1.21 | 1.32 | 11.84 | 3 | 39* | 4.89 | 2.51 | 0.53 | 1.57 | 14.34 | 25 |
| $8^{\text {b }}$ | 4.82 | 4.39 | 0.54 | 1.15 | 3.40 | - | $40^{\text {b }}$ | 3.92 | 3.00 | 1.49 | 1.61 | 4.19 | - |
| $9^{\text {b }}$ | 4.66 | 3.80 | 0.84 | 1.44 | 5.18 | - | 41* | 4.77 | 2.37 | 0.76 | 1.38 | 15.26 | 26 |
| 10 | 4.71 | 2.88 | 0.71 | 1.66 | 10.16 | 4 | 42* | 3.87 | 1.76 | 1.47 | 1.28 | 10.80 | 27 |
| 11* | 4.54 | 2.52 | 1.03 | 1.40 | 11.61 | 5 | 43* | 4.31 | 1.71 | 1.34 | 0.99 | 15.63 | 28 |
| $12^{\text {c }}$ | 4.42 | 3.35 | 1.08 | 1.49 | 5.82 | - | 44 | 4.80 | 2.33 | 0.55 | 1.43 | 16.13 | 29 |
| $13^{\text {c }}$ | 4.71 | 3.98 | 0.71 | 1.26 | 5.04 | - | 45 | 4.84 | 2.39 | 0.56 | 1.49 | 15.38 | 30 |
| 14* | 2.96 | 2.03 | 1.69 | 1.39 | 4.26 | 6 | $46^{\text {b }}$ | 4.78 | 4.36 | 0.61 | 1.19 | 3.15 | - |
| 15* | 4.36 | 2.46 | 1.15 | 1.46 | 10.23 | 7 | 47* | 4.53 | 2.26 | 1.00 | 1.42 | 13.07 | 31 |
| 16* | 3.90 | 2.74 | 1.57 | 1.56 | 5.23 | 8 | 48* | 4.21 | 2.34 | 1.38 | 1.38 | 9.58 | 32 |
| 17 | 4.33 | 2.17 | 0.94 | 1.28 | 13.64 | 9 | 49 | 3.85 | 2.05 | 1.56 | 1.51 | 8.28 | 33 |
| $18^{\text {b }}$ | 4.62 | 3.16 | 0.90 | 1.52 | 8.23 | - | 50 | 4.34 | 1.67 | 1.08 | 1.19 | 16.59 | 34 |
| 19* ${ }^{\text {c }}$ | 4.17 | 2.67 | 1.46 | 1.60 | 6.95 | - | $51^{\text {b }}$ | 4.80 | 3.88 | 0.72 | 1.42 | 5.76 | - |
| 20 | 4.64 | 2.83 | 0.92 | 1.49 | 10.34 | 10 | $52^{\text {a }}$ | 4.83 | 4.61 | 0.67 | 0.96 | 1.88 | - |
| 21* | 4.16 | 1.93 | 1.41 | 1.47 | 10.93 | 11 | 53* | 4.09 | 2.85 | 1.54 | 1.70 | 5.42 | 35 |
| 22 | 3.76 | 1.80 | 1.51 | 1.29 | 9.87 | 12 | 54* | 4.54 | 2.48 | 1.17 | 1.66 | 10.12 | 36 |
| 23* | 4.73 | 2.42 | 0.96 | 1.42 | 13.50 | 13 | 55* | 4.56 | 2.78 | 1.09 | 1.76 | 8.62 | 37 |
| 24 | 4.65 | 3.70 | 0.90 | 1.45 | 5.55 | 14 | 56 | 4.85 | 3.68 | 0.63 | 1.62 | 6.74 | 38 |
| $25^{\text {c }}$ | 4.18 | 3.46 | 1.28 | 1.37 | 3.82 | - | 57 | 4.38 | 2.41 | 1.13 | 1.48 | 10.57 | 39 |
| $26^{*}$ b | 4.34 | 2.16 | 1.24 | 1.46 | 11.37 | - | 58 | 4.81 | 1.79 | 0.53 | 1.17 | 23.53 | 40 |
| 27* | 4.86 | 2.12 | 0.62 | 1.44 | 17.43 | 15 | 59* | 4.00 | 1.78 | 1.50 | 1.25 | 11.34 | 41 |
| 28 | 4.52 | 2.39 | 0.98 | 1.45 | 12.18 | 16 | 60* | 4.60 | 3.28 | 1.13 | 1.58 | 6.81 | 42 |
| 29* | 4.63 | 2.48 | 0.96 | 1.42 | 12.51 | 17 | 61* | 4.35 | 1.78 | 1.10 | 1.28 | 15.25 | 43 |
| 30* ${ }^{\text {c }}$ | 3.49 | 2.68 | 1.70 | 1.52 | 3.55 | - | 62* | 3.65 | 1.94 | 1.50 | 1.22 | 8.85 | 44 |
| 31 | 4.71 | 2.66 | 0.76 | 1.56 | 11.77 | 18 | $63^{\text {b }}$ | 4.87 | 4.47 | 0.53 | 1.09 | 3.26 | - |
| 32 | 4.86 | 2.77 | 0.53 | 1.52 | 13.00 | 19 |  |  |  |  |  |  |  |
| Note. * | negat <br> ${ }^{a}$ Item <br> ${ }^{\mathrm{b}}$ Item <br> ${ }^{\text {c }}$ Item |  | ems, noved moved noved | $\mathrm{N}=370$ due to due to due to | $, \mathrm{N}_{1}=$ <br> o insign <br> o dupli <br> no fa | $\mathrm{N}_{2}=100$ <br> ificant or cation tor loadin | w t val |  |  |  |  |  |  |

## Appendix F2

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Scale of Attitude towards Mathematics

（Draft）

Dr．K．Abdul Gafoor<br>Professor<br>Sarabi．M．K

## ๓าฮิ๒๕๐๘๐๘ิ












 மிகனృ









 ロロハృ○．












 ఱிகா๐





















 வ्నకృం．





 ๑ルロงஸ゙．


 กృ円ร゙.









Appendix $\mathbf{F 3}$<br>UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Scale of Attitude towards Mathematics

(Draft)

Dr. K. Abdul Gafoor<br>Professor<br>Research Scholar

## Instructions

Please see below a set of statements related to learning Mathematics. You may respond to each statement in five different ways: 1. True, 2. Mostly True, 3. Somewhat True, 4. Rarely True, and 5. Not True. Read each of the statements carefully and make a decision on how accurate they are in your case. Provide a tick mark $(\checkmark)$ in the given box against the numbers provided for each statement. Your responses will be kept in safe custody and will only be used for research purpose.

1. Mathematics is interesting.
2. Mathematics is a very useful subject.
3. Numbers and symbols used in mathematics would help to learn other subjects.
4. Mathematics is required to learn other science subjects.
5. Mathematics is important in our daily lives.
6. I am scared of mathematics.
7. I am not good at mathematics.
8. I want to be someone who knows mathematics.
9. I like to play with numbers.
10. I don't know where the time goes while studying mathematics.
11. I will try my best to avoid mathematics textbook.
12. One can achieve accuracy and precision through learning mathematics.
13. Problem-solving skills gained by learning mathematics would help in real life.
14. I am worried about learning mathematics very well.
15. I do not have enough accuracy and precision to learn mathematics.
16. I am not aware of any techniques of learning formulas.
17. I am adept at learning mathematics.
18. I have kept my mathematics notebook clean and complete.
19. I learn mathematics barely enough to get a passing grade in examinations.
20. I study difficult areas of mathematics repeatedly.
21. I get tired or sleepy whenever I try to learn mathematics.
22. I study mathematics before I begin studying other subjects.
23. I do not like to attend mathematics class.
24. I like group activities in my mathematics class.
25. Learning activities in the mathematics class would help learn other subjects.
26. I am nervous as long as my mathematics teacher is in the classroom.
27. I do not feel comfortable in my mathematics class.
28. I am very active in my mathematics class.
29. I do not understand mathematics.
30. Mathematics classes are rigorous.
31. I participate actively in my mathematics class.
32. I like my mathematics teachers.
33. Mathematics teachers can help develop the mental capacity of their students.
34. I get anxious in the presence of mathematics teachers.
35. My mathematics teacher doesn't say anything good about me.
36. My teacher thinks that I can learn mathematics very well.
37. I used to clear my doubts with my mathematics teacher.
38. It is interesting to solve complex problems in mathematics.
39. I am too lazy to go to school on the day of mathematics exam.
40. Tests in mathematics would help me perform very well in other tests.
41. I feel sick on the day of mathematics examination.
42. I feel total emptiness while taking mathematics test.
43. I do not secure high scores even if I spend time learning mathematics.
44. Mathematics examination is a fun challenge.
45. It is fun to solve mathematics homework problems.
46. Learning tasks/activities in mathematics would be helpful in the future.
47. Mathematics homework is one of the scariest things to me.
48. I am behind in completing mathematics homework.
49. I used to solve problems outside my textbook.
50. I study mathematics every day.
51. I like friends who enjoy learning mathematics.
52. My parents are very happy when I study mathematics really well.
53. Friends add to my fear of mathematics.
54. Interference of my parents make my fear toward mathematics worse.
55. I feel embarrassed among my peers when it comes to mathematics.
56. I used to clarify my doubts with my classmates or elders.
57. I compete with my friends while studying mathematics.
58. Mathematics is my favorite subject.
59. I wonder if it is necessary to teach this much mathematics in schools.
60. Things taught in mathematics are not relevant everyday life.
61. Mathematics is a tough subject.
62. I perform well in all subjects except mathematics.
63. I wish to excel in mathematics.

## Appendix F4

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Scale of Attitude towards Mathematics

## （Final）

Dr．K．Abdul Gafoor Professor

Sarabi．M．K<br>Research Scholar

## ๓าฮె๒๕๐๘๐๘่












3．களேைிடி ஸைை வேナலロロஸ゙．







 கயேு வைழுOுளక̆．

























 กృృளั.








## Appendix 55 <br> UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Scale of Attitude towards Mathematics

(Final)

Dr. K. Abdul Gafoor<br>Professor<br>Research Scholar

## Instructions

Please see below a set of statements related to learning Mathematics. You may respond to each statement in five different ways: 1. True, 2. Mostly True, 3. Somewhat True, 4. Rarely True, and 5. Not True. Read each of the statements carefully and make a decision on how accurate they are in your case. Provide a tick mark $(\checkmark)$ in the given box against the numbers provided for each statement. Your responses will be kept in safe custody and will only be used for research purpose.

1. Mathematics is interesting.
2. I am scared of mathematics.
3. I am not good at mathematics.
4. I don't know where the time goes while studying mathematics.
5. I will try my best to avoid mathematics textbook.
6. I am worried about learning mathematics very well.
7. I do not have enough accuracy and precision to learn mathematics.
8. I am not aware of any techniques of learning formulas.
9. I am adept at learning mathematics.
10. I study difficult areas of mathematics repeatedly.
11. I get tired or sleepy whenever I try to learn mathematics.
12. I study mathematics before I begin studying other subjects.
13. I do not like to attend mathematics class.
14. I like group activities in my mathematics class.
15. I do not feel comfortable in my mathematics class.
16. I am very active in my mathematics class.
17. I do not understand mathematics.
18. I participate actively in my mathematics class.
19. I like my mathematics teachers.
20. I get anxious in the presence of mathematics teachers.
21. My mathematics teacher doesn't say anything good about me.
22. My teacher thinks that I can learn mathematics very well.
23. I used to clear my doubts with my mathematics teacher.
24. It is interesting to solve complex problems in mathematics.
25. I am too lazy to go to school on the day of mathematics exam.
26. I feel sick on the day of mathematics examination.
27. I feel total emptiness while taking mathematics test.
28. I do not secure high scores even if I spend time learning mathematics.
29. Mathematics examination is a fun challenge.
30. It is fun to solve mathematics homework problems.
31. Mathematics homework is one of the scariest things to me.
32. I am behind in completing mathematics homework.
33. I used to solve problems outside my textbook.
34. I study mathematics every day.
35. Friends add to my fear of mathematics.
36. Interference of my parents make my fear toward mathematics worse.
37. I feel embarrassed among my peers when it comes to mathematics.
38. I used to clarify my doubts with my classmates or elders.
39. I compete with my friends while studying mathematics.
40. Mathematics is my favorite subject.
41. I wonder if it is necessary to teach this much mathematics in schools.
42. Things taught in mathematics are not relevant everyday life.
43. Mathematics is a tough subject.
44. I perform well in all subjects except mathematics.

## Appendix $\mathbf{F 6}$

UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Scale of Attitude towards Mathematics

(Final)

## Response Sheet

ேேดั:

| $\begin{aligned} & \dot{\dot{Z}} \\ & \dot{\theta} \end{aligned}$ |  |  |  |  | $\begin{aligned} & \text { ab } \\ & \frac{\theta}{e} \\ & \frac{9}{8} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. |  |  |  |  |  |
| 2. |  |  |  |  |  |
| 3. |  |  |  |  |  |
| 4. |  |  |  |  |  |
| 5. |  |  |  |  |  |
| 6. |  |  |  |  |  |
| 7. |  |  |  |  |  |
| 8. |  |  |  |  |  |
| 9. |  |  |  |  |  |
| 10. |  |  |  |  |  |
| 11. |  |  |  |  |  |
| 12. |  |  |  |  |  |
| 13. |  |  |  |  |  |
| 14. |  |  |  |  |  |
| 15. |  |  |  |  |  |
| 16. |  |  |  |  |  |
| 17. |  |  |  |  |  |
| 18. |  |  |  |  |  |
| 19. |  |  |  |  |  |
| 20. |  |  |  |  |  |
| 21. |  |  |  |  |  |
| 22. |  |  |  |  |  |


| $\begin{aligned} & \dot{\theta} \\ & \dot{\mathbf{Z}} \end{aligned}$ |  |  |  |  | $\begin{aligned} & \frac{\square b}{\partial} \\ & \frac{\varrho}{\varrho} \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 23. |  |  |  |  |  |
| 24. |  |  |  |  |  |
| 25. |  |  |  |  |  |
| 26. |  |  |  |  |  |
| 27. |  |  |  |  |  |
| 28. |  |  |  |  |  |
| 29. |  |  |  |  |  |
| 30. |  |  |  |  |  |
| 31. |  |  |  |  |  |
| 32. |  |  |  |  |  |
| 33. |  |  |  |  |  |
| 34. |  |  |  |  |  |
| 35. |  |  |  |  |  |
| 36. |  |  |  |  |  |
| 37. |  |  |  |  |  |
| 38. |  |  |  |  |  |
| 39. |  |  |  |  |  |
| 40. |  |  |  |  |  |
| 41. |  |  |  |  |  |
| 42. |  |  |  |  |  |
| 43. |  |  |  |  |  |
| 44. |  |  |  |  |  |

## Appendix $\mathbf{F 7}$

UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

Scale of Attitude towards Mathematics
(Final)

## Response Sheet

Name:
Boy/Girl/Others

| $\begin{aligned} & \dot{\dot{Z}} \\ & \dot{\tilde{D}} \end{aligned}$ | E. | 跑 |  |  |  | $\begin{aligned} & \dot{\dot{Z}} \\ & \dot{\vec{n}} \end{aligned}$ | 式 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. |  |  |  |  |  | 23. |  |  |  |  |  |
| 2. |  |  |  |  |  | 24. |  |  |  |  |  |
| 3. |  |  |  |  |  | 25. |  |  |  |  |  |
| 4. |  |  |  |  |  | 26. |  |  |  |  |  |
| 5. |  |  |  |  |  | 27. |  |  |  |  |  |
| 6. |  |  |  |  |  | 28. |  |  |  |  |  |
| 7. |  |  |  |  |  | 29. |  |  |  |  |  |
| 8. |  |  |  |  |  | 30. |  |  |  |  |  |
| 9. |  |  |  |  |  | 31. |  |  |  |  |  |
| 10. |  |  |  |  |  | 32. |  |  |  |  |  |
| 11. |  |  |  |  |  | 33. |  |  |  |  |  |
| 12. |  |  |  |  |  | 34. |  |  |  |  |  |
| 13. |  |  |  |  |  | 35. |  |  |  |  |  |
| 14. |  |  |  |  |  | 36. |  |  |  |  |  |
| 15. |  |  |  |  |  | 37. |  |  |  |  |  |
| 16. |  |  |  |  |  | 38. |  |  |  |  |  |
| 17. |  |  |  |  |  | 39. |  |  |  |  |  |
| 18. |  |  |  |  |  | 40. |  |  |  |  |  |
| 19. |  |  |  |  |  | 41. |  |  |  |  |  |
| 20. |  |  |  |  |  | 42. |  |  |  |  |  |
| 21. |  |  |  |  |  | 43. |  |  |  |  |  |
| 22. |  |  |  |  |  | 44. |  |  |  |  |  |

## Appendix G1

## Data and Results of Item Analysis on Scale of Self-efficacy in Mathematics

Table G1-1 . Data and Results of Item Analysis on Scale of Self efficacy in learning Mathematics

| Item no. (draft) | M | $\mathrm{M}_{2}$ | $\mathrm{SD}_{1}$ | $\mathrm{SD}_{2}$ | t | Item no. <br> (final) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 4.82 | 3.51 | 0.44 | 1.28 | 9.65* | --- |
| 2 | 4.02 | 2.02 | 1.10 | 1.21 | 12.25 | 1 |
| 3 | 4.43 | 2.76 | 0.77 | 1.31 | 11.01 | 2 |
| 4 | 4.85 | 3.54 | 0.41 | 1.27 | 9.84* | --- |
| 5 | 4.37 | 2.68 | 0.69 | 1.29 | 11.52 | 3 |
| 6 | 3.89 | 1.65 | 0.96 | 0.99 | 16.23 | 4 |
| 7 | 4.40 | 2.22 | 0.80 | 1.23 | 14.79 | 5 |
| 8 | 4.53 | 2.72 | 0.69 | 1.32 | 12.17 | 6 |
| 9 | 4.09 | 2.35 | 0.92 | 1.34 | 10.72 | 7 |
| 10 | 4.93 | 4.17 | 0.29 | 1.21 | 6.12* | --- |
| 11 | 4.53 | 3.00 | 0.67 | 1.31 | 10.38 | 8 |
| 12 | 4.96 | 4.50 | 0.32 | 1.13 | 3.91* | --- |
| 13 | 4.42 | 2.61 | 0.77 | 1.23 | 12.48 | 9 |
| 14 | 4.46 | 2.51 | 0.72 | 1.18 | 14.15 | 10 |
| 15 | 4.78 | 2.93 | 0.62 | 1.63 | 10.61 | 11 |
| 16 | 4.64 | 3.33 | 0.58 | 1.41 | 8.61* | --- |
| 17 | 4.54 | 3.01 | 0.80 | 1.42 | 9.41* | --- |
| 18 | 4.58 | 2.65 | 0.84 | 1.49 | 11.27 | 12 |
| 19 | 4.27 | 2.35 | 0.63 | 1.18 | 14.30 | 13 |
| 20 | 4.37 | 2.21 | 0.82 | 1.25 | 14.43 | 14 |
| 21 | 4.27 | 2.27 | 0.93 | 1.30 | 12.50 | 15 |
| 22 | 4.14 | 2.22 | 0.79 | 1.12 | 14.04 | 16 |
| 23 | 4.44 | 2.28 | 0.70 | 1.33 | 14.42 | 17 |
| 24 | 4.33 | 2.31 | 0.84 | 1.34 | 12.76 | 18 |
| 25 | 4.56 | 2.41 | 0.77 | 1.49 | 12.79 | 19 |
| 26 | 4.44 | 2.53 | 0.64 | 1.21 | 13.95 | 20 |
| 27 | 4.50 | 2.34 | 0.69 | 1.14 | 16.23 | 21 |
| 28 | 4.55 | 2.93 | 0.69 | 1.54 | 9.61* | --- |
| 29 | 4.57 | 2.35 | 0.64 | 1.36 | 14.77 | 22 |
| 30 | 4.10 | 1.87 | 1.02 | 1.19 | 14.22 | 23 |
| 31 | 4.52 | 2.19 | 0.98 | 1.29 | 14.44 | 24 |
| 32 | 4.81 | 3.62 | 0.53 | 1.45 | 7.72* | --- |

Note. $\mathrm{N}=370, \mathrm{~N}_{1}=\mathrm{N}_{2}=100$

## Appendix G2

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Scale of Self-efficacy in Mathematics

## (Draft)

Dr. K. Abdul Gafoor

Professor
Sarabi. M.K
Research Scholar

ேேดั: $\qquad$


## 












|  |  | ๑) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
| 1. |  <br>  ค๐๐๐ด |  |  |  |  |  |
| 2. |  கவுం களைைாைைா |  |  |  |  |  |
| 3. |  <br>  |  |  |  |  |  |
| 4. |  <br>  |  |  |  |  |  |


| $\begin{aligned} & \text { e } \\ & \varepsilon \\ & \varepsilon \\ & \text { \& } \\ & \text { \& } \end{aligned}$ |  | ๑） ¢ிळ®̆ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\frac{1}{8}$ $\frac{8}{9}$ 8 8 |
| 5. |  <br>  <br>  |  |  |  |  |  |
| 6. |  <br>  |  |  |  |  |  |
| 7. |  |  |  |  |  |  |
| 8. |  <br>  <br>  |  |  |  |  |  |
| 9. |  <br>  |  |  |  |  |  |
| 10. |  <br>  |  |  |  |  |  |
| 11. |  <br>  |  |  |  |  |  |
| 12. |  <br>  |  |  |  |  |  |
| 13. |  カிヵ๐๐กฉ |  |  |  |  |  |
| 14. |  வமிกカிகெ๐ான் |  |  |  |  |  |
| 15. |  พとんの๐）வமேி๐ா |  |  |  |  |  |
| 16. |  <br>  |  |  |  |  |  |
| 17. |  <br>  <br>  |  |  |  |  |  |
| 18. |  |  |  |  |  |  |
| 19. |  |  |  |  |  |  |


| $\begin{aligned} & \text { e } \\ & \text { ¿ } \\ & \text { A } \\ & \text { \& } \end{aligned}$ |  | ๑）mிヵが |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
| 20. |  |  |  |  |  |  |
| 21. |  வஉயால் |  |  |  |  |  |
| 22. |  உாைை களைைாைைீ |  |  |  |  |  |
| 23. |  |  |  |  |  |  |
| 24. |  <br>  |  |  |  |  |  |
| 25. |  <br>  |  |  |  |  |  |
| 26. |  களைெறைைா |  |  |  |  |  |
| 27. |  めวกฉ |  |  |  |  |  |
| 28. |  <br>  |  |  |  |  |  |
| 29. |  カー๐ก๐ |  |  |  |  |  |
| 30. |  |  |  |  |  |  |
| 31. |  |  |  |  |  |  |
| 32. |  <br>  |  |  |  |  |  |

## Appendix G3

## UNIVERSITY OF CALICUT <br> DEPARTMENT OF EDUCATION

## Scale of Self-efficacy in Mathematics

## (Draft)

Dr. K. Abdul Gafoor<br>Professor<br>Sarabi. M.K<br>Research Scholar

Name: $\qquad$ Boy/Girl/Others

## Directions:

Please see below a variety of statements related to learning mathematics. You may react to each statement in five different ways. 1. Definitely, 2. Usually, 3. Sometimes, 4. Occassionally, and 5. Never. Read each of the statements carefully and make a conclusion on how accurate they are in your case. Provide a tick $(\checkmark)$ mark in the given box against the numbers provided for each statement. Your responses will be kept in safe custody and will only be used for research purpose.

| $\dot{8}$ $\dot{\text { z }}$ $\dot{\sim}$ | Statements | $\begin{aligned} & \frac{\lambda}{0} \\ & \stackrel{0}{0} \\ & 0.0 \\ & 0 \end{aligned}$ | 交 | : |  | $\dot{D}$ $\stackrel{\rightharpoonup}{0}$ z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | I can quickly solve applied problems using the four basic arithmetic operations (Addition, subtraction, multiplication, division) |  |  |  |  |  |
| 2. | I can find the least common multiple and the greatest common factor of numbers |  |  |  |  |  |
| 3. | I can easily identify prime and composite numbers |  |  |  |  |  |
| 4. | I can check the accuracy of the solution of the problem after solving it |  |  |  |  |  |
| 5. | I can solve applied problems using the four basic arithmetic operations (Addition, subtraction, multiplication, division) of fractions |  |  |  |  |  |
| 6. | I can solve applied problems by finding the common denominator of fractions with different denominators |  |  |  |  |  |
| 7. | I can find simplified form of fractions |  |  |  |  |  |

$\left.\begin{array}{|l|l|l|l|l|l|}\hline & & & & & \\ \hline \text { 8. Statements }\end{array}\right)$

| $\dot{\dot{B}}$ z i | Statements | $\begin{aligned} & \frac{\lambda}{0} \\ & \stackrel{0}{0} \\ & \stackrel{0}{0} \\ & \hline 0 \end{aligned}$ |  |  |  | $\dot{0}$ $\vdots$ $\vdots$ Z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 25. | I get involved in classroom discussions related to mathematics learning |  |  |  |  |  |
| 26. | I can find answers to mathematics questions quickly |  |  |  |  |  |
| 27. | I can answer almost every question from my mathematics teacher |  |  |  |  |  |
| 28. | I can understand mathematics questions without much explanation from my teacher. |  |  |  |  |  |
| 29. | I can teach mathematics to my classmates |  |  |  |  |  |
| 30. | I can score A+ on mathematics examinations |  |  |  |  |  |
| 31. | I can prepare for mathematics examination without fear |  |  |  |  |  |
| 32. | I can find solutions to mathematical problems in daily lives |  |  |  |  |  |

## Appendix G4

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Scale of Self－efficacy in Mathematics

（Final）

Dr．K．Abdul Gafoor<br>Professor<br>Sarabi．M．K<br>Research Scholar

ேேดั： $\qquad$


## 







 கேலலாைைை



|  | （ロพั¢ | ๑）（1ヵヵ） |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{\|l\|} \hline \varrho \\ \varepsilon \\ \varepsilon \\ \text { A } \\ \text { © } \end{array}$ |  |  |  |  |  | a <br> 8 <br> $\frac{8}{8}$ <br> 8 |
| 1. |  கவூం களளைாைைா |  |  |  |  |  |
| 2. |  <br>  |  |  |  |  |  |
| 3. |  <br>  <br>  |  |  |  |  |  |
| 4. |  <br>  |  |  |  |  |  |
| 5. |  |  |  |  |  |  |


| $\begin{aligned} & \text { C } \\ & \varepsilon \\ & \varepsilon \\ & \text { A } \\ & \text { © } \end{aligned}$ |  | ๑） |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
| 6. |  <br>  <br>  |  |  |  |  |  |
| 7. |  <br>  |  |  |  |  |  |
| 8. |  <br>  |  |  |  |  |  |
| 9. |  カிø๐วกฉ |  |  |  |  |  |
| 10. |  வமிกカிゅெ๐ா் |  |  |  |  |  |
| 11. |  <br>  |  |  |  |  |  |
| 12. |  |  |  |  |  |  |
| 13. |  |  |  |  |  |  |
| 14. |  |  |  |  |  |  |
| 15. |  |  |  |  |  |  |
| 16. |  உாைை களைைாைைை |  |  |  |  |  |
| 17. |  |  |  |  |  |  |
| 18. |  <br>  |  |  |  |  |  |
| 19. |  <br>  |  |  |  |  |  |
| 20. |  களளइறைைா |  |  |  |  |  |
| 21. |  ＠on |  |  |  |  |  |
| 22. |  ค๐๐ก |  |  |  |  |  |
| 23. |  |  |  |  |  |  |
| 24. |  |  |  |  |  |  |

## Appendix G5

## UNIVERSITY OF CALICUT <br> DEPARTMENT OF EDUCATION

## Scale of Self-efficacy in Mathematics

(Final)

Dr. K. Abdul Gafoor<br>Professor<br>Sarabi. M.K<br>Research Scholar

Name $\qquad$ Boy/Girl/Others

## Directions:

Please see below a variety of statements related to learning mathematics.
You may react to each statement in five different ways. 1. Definitely, 2. Usually, 3. Sometimes, 4. Occasionally, and 5. Never. Read each of the statements carefully and make a conclusion on how accurate they are in your case. Provide a tick $(\checkmark)$ mark in the given box against the numbers provided for each statement. Your responses will be kept in safe custody and will only be used for research purpose.

| $\dot{z}$ $\dot{\sim}$ $\dot{s}$ | Statements |  | $\begin{aligned} & \lambda \\ & \stackrel{\lambda}{\tilde{v}} \\ & \stackrel{\rightharpoonup}{5} \end{aligned}$ | 准 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | I can find the least common multiple and the greatest common factor of numbers |  |  |  |  |  |
| 2. | I can easily identify prime and composite numbers |  |  |  |  |  |
| 3. | I can solve applied problems using the four basic arithmetic operations (Addition, subtraction, multiplication, division) of fractions |  |  |  |  |  |
| 4. | I can solve applied problems by finding the common denominator of fractions with different denominators |  |  |  |  |  |
| 5. | I can find simplified form of fractions |  |  |  |  |  |
| 6. | I can solve applied problems by performing the four basic arithmetic operations (Addition, subtraction, multiplication, division) of decimals |  |  |  |  |  |
| 7. | I can explain the relation between smaller and larger units of weight, length, and volume |  |  |  |  |  |

$\begin{array}{|c|l|l|l|l|l|}\hline \text {. } & & & & & \\ \hline \text { 8. }\end{array}$ Itatements $\left.\begin{array}{l}\text { I can solve applied problems involving volume and } \\ \text { area }\end{array}\right)$

## Appendix H 1

Data and Results of Item Analysis of Test of Previous Achievement in Mathematics
Table H1-1. Data and Results of Item Analysis of Test of Previous Achievement in Mathematics

| Item No. <br> (Draft) | L | H | DP | DI | Item No. <br> (Final) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 21 | 53 | 0.32 | 0.37 | 1 |
| 2 | 30 | 76 | 0.46 | 0.53 | 2 |
| 3 | 16 | 47 | 0.31 | 0.35 | 3 |
| 4 | 18 | 51 | 0.33 | 0.35 | 4 |
| 5 | 12 | 44 | 0.32 | 0.28* | --- |
| 6 | 6 | 34 | 0.28* | 0.2* | --- |
| 7 | 7 | 30 | 0.23* | 0.19* | --- |
| 8 | 12 | 34 | 0.22* | 0.23* | --- |
| 9 | 15 | 60 | 0.45 | 0.38 | 5 |
| 10 | 6 | 29 | 0.23* | 0.18* | --- |
| 11 | 22 | 59 | 0.37 | 0.41 | 6 |
| 12 | 13 | 42 | 0.29* | 0.28* | --- |
| 13 | 12 | 28 | 0.16* | 0.2* | --- |
| 14 | 21 | 56 | 0.35 | 0.39 | 7 |
| 15 | 20 | 76 | 0.56 | 0.48 | 8 |
| 16 | 19 | 50 | 0.31 | 0.35 | 9 |
| 17 | 18 | 37 | 0.19* | 0.28* | --- |
| 18 | 18 | 52 | 0.34 | 0.35 | 10 |
| 19 | 12 | 24 | 0.12* | 0.18* | --- |
| 20 | 27 | 64 | 0.37 | 0.46 | 11 |
| 21 | 15 | 65 | 0.5 | 0.4 | 12 |
| 22 | 30 | 79 | 0.49 | 0.55 | 13 |
| 23 | 8 | 26 | 0.18* | 0.17* | --- |
| 24 | 13 | 31 | 0.18* | 0.22* | --- |
| 25 | 12 | 24 | 0.12* | 0.18* | --- |
| 26 | 24 | 56 | 0.32 | 0.4 | 14 |
| 27 | 12 | 29 | 0.17* | 0.21* | --- |
| 28 | 14 | 69 | 0.55 | 0.42 | 15 |
| 29 | 22 | 37 | 0.15* | 0.3* | --- |
| 30 | 16 | 54 | 0.38 | 0.35 | 16 |
| 31 | 17 | 53 | 0.36 | 0.35 | 17 |
| 32 | 19 | 50 | 0.31 | 0.35 | 18 |
| 33 | 16 | 76 | 0.6 | 0.46 | 19 |
| 34 | 29 | 47 | 0.18* | 0.38 | --- |
| 35 | 22 | 80 | 0.58 | 0.51 | 20 |

Note: * indicates value outside the limits of DP or DI
$\mathrm{N}=370$, Number of students in upper group= Number of students in lower group $=100$

## Appendix H2

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Previous Achievement in Mathematics

## (Draft)

Dr. K. Abdul Gafoor

Professor

Sarabi. M.K Research Scholar
\&லญ్ల : VII
 พロఱం: 45 ฝlmృร̆

## 










1. $\frac{15}{4}=$ $\qquad$
a) $3 \frac{3}{4}$
b) $\frac{7}{15}$
c) $4 \frac{3}{4}$
c) $\frac{15}{7}$


a) $\frac{7}{15}$
b) $\frac{8}{15}$
c) $\frac{15}{7}$
d) $\frac{15}{8}$


a) 24
b) 96
c) 120
d) 144

a) $15.018,15.081,15.181,15.810$
b) $15.081,15.018,15.181,15.810$
c) $15.181,15.081,15.018,15.810$
d) $15.018,15.081,15.810,15.181$
2. $\frac{0.001 \times 0.01}{0.01 \times 0.1}=$
a) 0.01
b) 0.001
c) 0.0001
d) 0.00001

a) 0.007
b) 0.075
c) 0.750
d) 7.500


a) $\frac{3}{20}$
b) $\frac{5}{12}$
c) $\frac{12}{5}$
d) $\frac{20}{3}$


a) 35
b) 36
c) 56
d) 66


a) 600
b) 700
c) 800

3. 10 พ०வృகனூூ


a) 90
b) 100
c) 101
d) 110
毋(ே)
a) 20 ฮา.
b) 50 ฮา.
c) 100 ๑า.



a) 12 ยா ๑ก.๑า.
b) 20 ๓๓ ๑ก.๑า.
c) 24 ฺா ๑ก.๑า.
d) 48 ๓ยா ๑ก.จา.
 ปไกญัน $=\ldots$ ?
a) 6 ๑ก.จา.
b) 12 ๑กา.จา.
c) 24 ๑ก๐.จา .
d) 60 ๑ก๐.จา.


a) 2 ๑กาจา.
b) 8 ๑ก.ฉา.
c) 12 ๑ก๐.ฉา.
d) 16 ๑ก.ฉา.


a) $55^{0}$
b) $70^{0}$
c) $90^{\circ}$
d) $180^{\circ}$



a) 0.008 ๑ก.ฉา.
b) 0.08 ๑ก.ฉา.
c) 80 ๑๐๐ฉา.
d) 800 ๑๓.ฉา.
4. 



a） $75^{0}$
b） $85^{\circ}$
c） $105^{\circ}$
d） $115^{0}$
 ๓ைை゙ ஷண゙？
a） 1
b） 3
c） 5
d） 7
 ஷ®ணૅ 凸円ிை๐ஸ゙？
a） 1
b） 2
c） 3
d） 5

a） 2
b） 3
c） 4
d） 8

a） 6
b） 12
c） 18
d） 36

a） 2
b） 18
c） 24
d） 72
 ๓๐ஸ゙？
a） 29
b） 67
c） 77
d） 89






a） 10.12 AM
b） 10.14 AM
c） 10.16 AM
d） 10.20 AM


a） 60
b） 62
c） 72
d） 120


a） $5 \%$
b） $15 \%$
c） $20 \%$
d） $25 \%$


a） 1600
b） 3200
c） 6400
d） 16000

 ๑O円० (ேறஸ゙?
a) $12 \%$
b) $20 \%$
c) $21 \%$
d) $24 \%$


a) $P=2 a b$
b) $\mathrm{P}=2 \mathrm{a} \times 2 \mathrm{~b}$
c) $P=2(a+b)$
d) $P=2(a \times b)$



a) $\mathrm{C}=2 \mathrm{AB}$
b) $\mathrm{C}=4 \mathrm{AB}$
c) $\mathrm{C}=2(\mathrm{~A}+\mathrm{B})$
d) $\mathrm{C}=4(\mathrm{~A}+\mathrm{B})$



a) $C=A+\frac{B}{2}$
b) $C=\frac{A}{2}+B$
c) $C=\frac{A+B}{2}$
d) $\mathrm{C}=2(\mathrm{~A}+\mathrm{B})$



a) $A=B-4$
b) $\mathrm{B}=\mathrm{A}-4$
c) $A=B+4$
d) $\mathrm{B}=4-\mathrm{A}$






a) 1
b) 2
c) 3
d) 4

a) 2
b) 3
c) 4
d) 5

a) 1
b) 2
c) 4
d) 5

## Appendix H 3

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Previous Achievement in Mathematics (Draft)

| Dr. K. Abdul Gafoor <br> Professor | Sarabi. M.K <br> Research Scholar |
| :--- | ---: |
| Class: VII | Marks: 35 Marks |
|  | Time: 45 mts |

## Directions:

Shown below are 35 questions based on your mathematics topics up to $6^{\text {th }}$ grade. Four answer choices are given for each question. Read each question carefully and choose the right answer. Please do not write anything on your question paper. You are given an answer sheet separately. Please put a tick mark $(\checkmark)$ toward each correct answer given in the appropriate columns. Your answer sheets will be kept in safe custody and used for research purposes only.

1. $\frac{15}{4}=\ldots . . . .$. ?
a) $3 \frac{3}{4}$
b) $\frac{7}{15}$
c) $4 \frac{3}{4}$
d) $\frac{15}{7}$
2. $\frac{7}{15}$ of a farmland is completed with sowing. How much of the farmland is left?
a) $\frac{7}{15}$
b) $\frac{8}{15}$
c) $\frac{15}{7}$
d) $\frac{15}{8}$
3. Ammu has 48 candies with her. Achu has 2.5 times candies of what Ammu has. How many candies does Achu have?
a) 24
b) 96
c) 120
d) 144
4. Choose the given sets of numbers from smallest to largest?
a) $15.018,15.081,15.181,15.810$
b) $15.081,15.018,15.181,15.810$
c) $15.181,15.081,15.018,15.810$
d) $15.018,15.081,15.810,15.181$
5. $\frac{0.001 \times 0.01}{0.01 \times 0.1}=$
a) 0.01
b) 0.001
c) 0.0001
d) 0.00001
6. What is the decimal form of $\frac{3}{40}$ ?
a) 0.007
b) 0.075
c) 0.750
d) 7.500
7. A 5-meter-long ribbon is cut into 3 equally long pieces. What is the length of the quarter of a piece?
a) $\frac{3}{20}$
b) $\frac{5}{12}$
c) $\frac{12}{5}$
d) $\frac{20}{3}$
8. The average mass of 30 students in a classroom is 35 kg . The average mass becomes 36 kg when included teacher's mass. How much is the teacher's mass?
a) 35
b) 36
c) 56
d) 66
9. Manoj's income for 7 days is 5600 Rupees. How much is his average income?
a) 600
c) 800
b) 700
d) None of these
10. The average of 10 numbers is 100 . While calculating the average, 20 is added instead of 30 . What would be the average if the calculation was accurate?
a) 90
b) 100
c) 101
d) 110
11. What is the length of one side of the square of perimeter 200 meters?
a) 20 meter
b) 50 meter
c) 100 meter
d) None of these
12. A rectangular prism has a length of 6 cm , width of 4 cm , and height of 2 cm . What would be the volume of the prism?
a) $12 \mathrm{~cm}^{3}$
b) $20 \mathrm{~cm}^{3}$
c) $24 \mathrm{~cm}^{3}$
d) $48 \mathrm{~cm}^{3 \mathrm{H}}$
13. In $\triangle \mathrm{ABC}, \mathrm{AB}=4 \mathrm{~cm}, \mathrm{BC}=5 \mathrm{~cm}$, and $\mathrm{AC}=3 \mathrm{~cm}$. The perimeter of the triangle $=\ldots$ ?
a) 6 cm
b) 12 cm
c) 24 cm
d) 60 cm
14. 



The midpoint of $A B$ in the given diagram is $C$. If $A C=4 \mathrm{~cm}$,
$\mathrm{AB}=\ldots . \mathrm{cm}$.
a) 2 cm
b) 8 cm
c) 12 cm
d) 16 cm
15.


In the given diagram, if $<\mathrm{ACE}=110^{\circ},<\mathrm{BCE}=\ldots .$.
a) $55^{\circ}$
b) $70^{\circ}$
c) $90^{\circ}$
d) $180^{\circ}$
16. Anu measured the length of a rectangle in meters. What would be the length if she measured it in centimeter?
a) 0.008 cm
b) 0.08 cm
c) 80 cm
d) 800 cm
17.
 What is the angle measure shown by the protractor?
a) $75^{0}$
b) $85^{\circ}$
c) $105^{\circ}$
d) $115^{0}$
18. Which one of the given numbers is neither prime nor composite?
a) 1
b) 3
c) 5
d) 7
19. Which one of the numbers given below has an odd number of factors?
a) 1
b) 2
c) 3
d) 5
20. Which one is not a factor of 32 ?
a) 2
b) 3
c) 4
d) 8
21. What is the largest common factor of 12 and 18 ?
a) 6
b) 12
c) 18
d) 36
22. What is the least common multiple of 18 and 24 ?
a) 2
b) 18
c) 24
d) 72
23. Which of the given numbers has more than two factors?
a) 29
b) 67
c) 77
d) 89
24. Anu and Abhi are waiting for the bus at Calicut. Anu has to leave for Ramanattukara and Abhi leaves for Farooke. The frequencies of buses are 4 minutes and 10 minutes toward Ramanattukara and Farooke toward Calicut, respectively. Buses toward both places left at 10 O'clock. At what time, they will have their next buses again at the same time?
a) 10.12 AM
b) 10.14 AM
c) 10.16 AM
d) 10.20 AM
25. There are 48 female employees working in a textiles store, which is $40 \%$ of the total employees. What would be the number of male employees in that store?
a) 60
b) 62
c) 72
d) 120
26. Sara's monthly income is 25000 Rupees out of which 5000 Rupees is used for travelling. What percent of the total income is her commuting expense?
a) $5 \%$
b) $15 \%$
c) $20 \%$
d) $25 \%$
27. Appu deposits $20 \%$ of his income in a bank. How much could he deposit in the month of December if his income for that month is 32000 Rupees?
a) 1600
b) 3200
c) 6400
d) 16000
28. The price for a television last year was 18000 Rupees. The price has increased up to 20160 Rupees. What is the increase in percentage?
a) $12 \%$
b) $20 \%$
c) $21 \%$
d) $24 \%$
29. If the length of a rectangle is a cm, width bcm , and perimeter P cm , how are a , b , and P related?
a) $P=2 a b$
b) $P=2 \mathrm{a} \times 2 \mathrm{~b}$
c) $P=2(a+b)$
d) $P=2(a x b)$
30. When the side of a square that is originally is $A \mathrm{~cm}$ is increased by $B \mathrm{~cm}$, and the perimeter of the larger square is Ccm , how can the relationship between A , B , and C be expressed?
a) $\mathrm{C}=2 \mathrm{AB}$
b) $\mathrm{C}=4 \mathrm{AB}$
c) $C=2(A+B)$
d) $\mathrm{C}=4(\mathrm{~A}+\mathrm{B})$
31. The midpoint of a line by adding B cm to a line of original length Acm is Ccm from one end of the newly formed line. How do you show the relationship between A, B, and C?
a) $C=A+\frac{B}{2}$
b) $C=\frac{A}{2}+B$
c) $C=\frac{A+B}{2}$
d) $\mathrm{C}=2(\mathrm{~A}+\mathrm{B})$
32. Let $A$ be the age of Anu and B be the age of Sonu. How do you indicate using letters that Anu's age is 4 years less than that of Sonu?
a) $A=B-4$
b) $B=A-4$
c) $A=B+4$
d) $\mathrm{B}=4-\mathrm{A}$

Answer question numbers 33 to 35 based on the given figure. The number of students admitted into Grade one for six years is shown in the figure given below

33. The year with maximum difference between male and female students?
a) 1
b) 2
c) 3
d) 4
34. The year in which less than 50 boys got admitted?
a) 2
b) 3
c) 4
d) 5
35. The year with maximum number of students enrolled?
a) 1
b) 2
c) 4
d) 5

## Appendix H4

UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

Scoring Key for Test of Previous Achievement in Mathematics
(Draft)

| Item No. | Answer | Item No. | Answer | Item No. | Answer |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | A | 13. | B | 25. | C |
| 2. | B | 14. | B | 26. | C |
| 3. | C | 15. | B | 27. | C |
| 4. | A | 16. | D | 28. | A |
| 5. | A | 17. | A | 29. | C |
| 6. | B | 18. | A | 30. | D |
| 7. | D | 19. | A | 31. | C |
| 8. | D | 20. | B | 32. | A |
| 9. | C | 21. | A | 33. | D |
| 10. | C | 22. | D | 34. | B |
| 11. | B | 23. | C | 35. | B |
| 12. | D | 24. | D |  |  |

## Appendix H 5

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Previous Achievement in Mathematics

(Final)

Dr. K. Abdul Gafoor

Professor
Sarabi. M.K
Research Scholar
\&OM్N゙: VII



## 










1. $\frac{15}{4}=\ldots . . .$. ?
a) $3 \frac{3}{4}$
b) $\frac{7}{15}$
c) $4 \frac{3}{4}$
c) $\frac{15}{7}$


a) $\frac{7}{15}$
b) $\frac{8}{15}$
c) $\frac{15}{7}$
d) $\frac{15}{8}$


a) 24
b) 96
c) 120
d) 144

a) $15.018,15.081,15.181,15.810$
b) $15.081,15.018,15.181,15.810$
c) $15.181,15.081,15.018,15.810$
d) $15.018,15.081,15.810,15.181$


a) 600
b) 700
c) 800


a) 20 ๑า.
b) 50 ๑า.
c) 100 ๑ใ.



a） 2 ๑กา．จา．
b） 8 ๑ก．๓า．
c） 12 ๑กา．ฉา．
d） 16 ๑ก๐จา．

 $\qquad$
a） $55^{0}$
b） $70^{0}$
c） $90^{0}$
d） $180^{\circ}$



a） 0.008 ๑ก๐．จา．
b） 0.08 ๑ก．จา．
c） 80 ๑๓．ฉา．
d） 800 ๑ก．ฉา．
 ๙ைை̆ ஷணั？
a） 1
b） 3
c） 5
d） 7

a） 2
b） 3
c） 4
d） 8

a） 6
b） 12
c） 18
d） 36

a） 2
b） 18
c） 24
d） 72


a） $5 \%$
b） $15 \%$
c） $20 \%$
d） $25 \%$

 ロ0円ం ஞூறஸ゙？
a） $12 \%$
b） $20 \%$
c） $21 \%$
d） $24 \%$



a） $\mathrm{C}=2 \mathrm{AB}$
b） $\mathrm{C}=4 \mathrm{AB}$
c） $\mathrm{C}=2(\mathrm{~A}+\mathrm{B})$
d） $\mathrm{C}=4(\mathrm{~A}+\mathrm{B})$



a) $C=A+\frac{B}{2}$
b) $C=\frac{A}{2}+B$
c) $C=\frac{A+B}{2}$
d) $\mathrm{C}=2(\mathrm{~A}+\mathrm{B})$



a) $\mathrm{A}=\mathrm{B}-4$
b) $\mathrm{B}=\mathrm{A}-4$
c) $A=B+4$
d) $\mathrm{B}=4-\mathrm{A}$






a) 1
b) 2
c) 3
d) 4

a) 1
b) 2
c) 4
d) 5

## Appendix H 6

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

# Test of Previous Achievement in Mathematics 

(Final)

| Dr. K. Abdul Gafoor <br> Professor | Sarabi. M.K <br> Research Scholar |
| :--- | ---: |
| Class: VII | Marks : 20 Marks |
|  | Time : 30 mts |

## Directions:

Shown below are 20 questions based on your mathematics topics up to $6^{\text {th }}$ grade. Four answer choices are given for each question. Read each question carefully and choose the right answer. Please do not write anything on your question paper. You are given an answer sheet separately. Please put a tick mark $(\checkmark)$ toward each correct answer given in the appropriate columns. Your answer sheets will be kept in safe custody and used for research purposes only.

1. $\frac{15}{4}=\ldots . . . .$. ?
a) $3 \frac{3}{4}$
b) $\frac{7}{15}$
c) $4 \frac{3}{4}$
d) $\frac{15}{7}$
2. $\frac{7}{15}$ of a farmland is completed with sowing. How much of the farmland is left?
a) $\frac{7}{15}$
b) $\frac{8}{15}$
c) $\frac{15}{7}$
d) $\frac{15}{8}$
3. Ammu has 48 candies with her. Achu has 2.5 times candies of what Ammu has. How many candies does Achu have?
a) 24
b) 96
c) 120
d) 144
4. Choose the given sets of numbers from smallest to largest?
a) $15.018,15.081,15.181,15.810$
b) $15.081,15.018,15.181,15.810$
c) $15.181,15.081,15.018,15.810$
d) $15.018,15.081,15.810,15.181$
5. Manoj's income for 7 days is 5600 Rupees. How much is his average income?
a) 600
c) 800
b) 700
d) None of these
6. What is the length of one side of the square of perimeter 200 meters?
a) 20 meter
b) 50 meter
c) 100 meter
d) None of these
In the given diagram, if $\angle \mathrm{ACE}=110^{\circ}, \angle \mathrm{BCE}=\ldots .$.
a) $55^{0}$
b) $70^{\circ}$
c) $90^{0}$
d) $180^{\circ}$
7. A
C
C
8. 


a) 2 cm
b) 8 cm
c) 12 cm
c) 12 cm
d) 16 cm

The midpoint of $A B$ in the given diagram is $C$. If $A C=4 \mathrm{~cm}$, $\mathrm{AB}=\ldots . \mathrm{cm}$.
9. Anu measured the length of a rectangle in meters. What would be the length if she measured it in centimeter?
a) 0.008 cm
b) 0.08 cm
c) 80 cm
d) 800 cm
10. Which one of the given numbers is neither prime nor composite?
a) 1
b) 3
c) 5
d) 7
11. Which one is not a factor of 32 ?
a) 2
b) 3
c) 4
d) 8
12. What is the largest common factor of 12 and 18 ?
a) 6
b) 12
c) 18
d) 36
13. What is the least common multiple of 18 and 24 ?
a) 2
b) 18
c) 24
d) 72
14. Sara's monthly income is 25000 Rupees out of which 5000 Rupees is used for travelling. What percent of the total income is her commuting expense?
a) $5 \%$
b) $15 \%$
c) $20 \%$
d) $25 \%$
15. The price for a television last year was 18000 Rupees. The price has increased up to 20160 Rupees. What is the increase in percentage?
a) $12 \%$
b) $20 \%$
c) $21 \%$
d) $24 \%$
16. When the side of a square that is originally is A cm is increased by B cm , and the perimeter of the larger square is Ccm , how can the relationship between A , B , and C be expressed?
a) $\mathrm{C}=2 \mathrm{AB}$
b) $\mathrm{C}=4 \mathrm{AB}$
c) $\mathrm{C}=2(\mathrm{~A}+\mathrm{B})$
d) $\mathrm{C}=4(\mathrm{~A}+\mathrm{B})$
17. The midpoint of a line by adding B cm to a line of original length Acm is Ccm from one end of the newly formed line. How do you show the relationship between $\mathrm{A}, \mathrm{B}$, and C ?
a) $C=A+\frac{B}{2}$
b) $C=\frac{A}{2}+B$
c) $C=\frac{A+B}{2}$
d) $\mathrm{C}=2(\mathrm{~A}+\mathrm{B})$
18. Let $A$ be the age of Anu and $B$ be the age of Sonu. How do you indicate using letters that Anu's age is 4 years less than that of Sonu?
a) $\mathrm{A}=\mathrm{B}-4$
b) $\mathrm{B}=\mathrm{A}-4$
c) $A=B+4$
d) $\mathrm{B}=4-\mathrm{A}$

Answer question numbers 19 and 20 based on the given figure. The number of students admitted into Grade one for six years is shown in the figure given below

19. The year with maximum difference between male and female students?
a) 1
b) 2
c) 3
d) 4
20. The year with maximum number of students enrolled?
a) 1
b) 2
c) 4
d) 5

## Appendix H7

UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

Scoring Key for Test of Previous Achievement in Mathematics

| Item No. | Answer |
| :---: | :---: |
| 1. | $\mathbf{A}$ |
| 2. | $\mathbf{B}$ |
| 3. | $\mathbf{C}$ |
| 4. | $\mathbf{A}$ |
| 5. | $\mathbf{C}$ |
| 6. | $\mathbf{B}$ |
| 7. | $\mathbf{B}$ |
| 8. | $\mathbf{B}$ |
| 9. | $\mathbf{D}$ |
| 10. | $\mathbf{A}$ |


| Item No. | Answer |
| :---: | :---: |
| 11. | $\mathbf{B}$ |
| 12. | $\mathbf{A}$ |
| 13. | $\mathbf{D}$ |
| 14. | $\mathbf{C}$ |
| 15. | $\mathbf{A}$ |
| 16. | $\mathbf{D}$ |
| 17. | $\mathbf{C}$ |
| 18. | $\mathbf{A}$ |
| 19. | $\mathbf{D}$ |
| 20. | $\mathbf{B}$ |

## Appendix H 8

UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

Test of Previous Achievement in Mathematics
(Final)

## Response Sheet

ேேดั:


| Sl. No. | $\mathbf{a}$ | $\mathbf{b}$ | $\mathbf{c}$ | $\mathbf{d}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1. |  |  |  |  |
| 2. |  |  |  |  |
| 3. |  |  |  |  |
| 4. |  |  |  |  |
| 5. |  |  |  |  |
| 6. |  |  |  |  |
| 7. |  |  |  |  |
| 8. |  |  |  |  |
| 9. |  |  |  |  |
| 10. |  |  |  |  |


| Sl. No. | a | $\mathbf{b}$ | $\mathbf{c}$ | $\mathbf{d}$ |
| :---: | :---: | :---: | :---: | :---: |
| 11. |  |  |  |  |
| 12. |  |  |  |  |
| 13. |  |  |  |  |
| 14. |  |  |  |  |
| 15. |  |  |  |  |
| 16. |  |  |  |  |
| 17. |  |  |  |  |
| 18. |  |  |  |  |
| 19. |  |  |  |  |
| 20. |  |  |  |  |

## Appendix 11

Data and Results of Item Analysis of Tests of Achievement in Mathematics Units
Table I 1-1. Data and Results of Item Analysis of Tests of Achievement in Mathematics Units

| Item No. (draft) | $\begin{gathered} \mathrm{L} \\ (\mathrm{~N}=50) \end{gathered}$ | $\begin{gathered} \mathrm{H} \\ (\mathrm{~N}=50) \end{gathered}$ | DP | DI | Item No. (final) | Item No. | $\begin{gathered} \mathrm{L} \\ (\mathrm{~N}=50) \end{gathered}$ | $\begin{gathered} \mathrm{H} \\ (\mathrm{~N}=50) \end{gathered}$ | DP | DI |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Parallel Lines |  |  |  |  |  | Unchanging Relations |  |  |  |  |
| 1 | 20 | 35 | 0.3 | 0.55 | 1 | 1 | 4 | 36 | 0.64 | 0.4 |
| 2 | 15 | 34 | 0.34 | 0.51 | 2 | 2 | 16 | 46 | 0.6 | 0.62 |
| 3 | 17 | 33 | 0.36 | 0.48 | 3 | 3 | 9 | 29 | 0.4 | 0.38 |
| 4 | 11 | 31 | 0.4 | 0.42 | 4 | 4 | 9 | 32 | 0.46 | 0.41 |
| 5 | 15 | 33 | 0.36 | 0.48 | 5 | 5 | 8 | 32 | 0.48 | 0.4 |
| 6 | 14 | 34 | 0.4 | 0.48 | 6 | 6 | 7 | 32 | 0.5 | 0.39 |
| 7 | 9 | 37 | 0.56 | 0.46 | 7 | 7 | 10 | 33 | 0.46 | 0.43 |
| 8 | 10 | 30 | 0.4 | 0.4 | 8 | 8 | 15 | 45 | 0.6 | 0.6 |
| 9 | 13 | 19 | 0.12* | 0.32* | --- | 9 | 8 | 27 | 0.38 | 0.35 |
| 10 | 14 | 33 | 0.38 | 0.47 | 9 | 10 | 8 | 30 | 0.44 | 0.38 |
| 11 | 15 | 36 | 0.42 | 0.51 | 10 | 11 | 10 | 41 | 0.62 | 0.51 |
| 12 | 17 | 38 | 0.42 | 0.55 | 11 | 12 | 13 | 38 | 0.5 | 0.51 |
| 13 | 13 | 41 | 0.56 | 0.54 | 12 | 13 | 19 | 44 | 0.5 | 0.63 |
| 14 | 10 | 36 | 0.52 | 0.46 | 13 | 14 | 14 | 32 | 0.36 | 0.46 |
| 15 | 15 | 29 | 0.28* | 0.44 | --- | 15 | 7 | 40 | 0.66 | 0.47 |
| 16 | 10 | 38 | 0.56 | 0.48 | 14 | 16 | 5 | 29 | 0.48 | 0.34 |
| Repeated Multiplication |  |  |  |  |  | Square and Square root |  |  |  |  |
| 1 | 9 | 27 | 0.36 | 0.36 | 1 | 1 | 9 | 35 | 0.52 | 0.44 |
| 2 | 14 | 40 | 0.52 | 0.54 | 2 | 2 | 7 | 26 | 0.38 | 0.33 |
| 3 | 8 | 27 | 0.38 | 0.35 | 3 | 3 | 6 | 46 | 0.8 | 0.52 |
| 4 | 5 | 32 | 0.54 | 0.37 | 4 | 4 | 8 | 45 | 0.74 | 0.53 |
| 5 | 10 | 30 | 0.4 | 0.4 | 5 | 5 | 7 | 36 | 0.58 | 0.43 |
| 6 | 16 | 46 | 0.6 | 0.62 | 6 | 6 | 12 | 48 | 0.72 | 0.6 |
| 7 | 13 | 28 | 0.3 | 0.41 | 7 | 7 | 7 | 35 | 0.56 | 0.42 |
| 8 | 11 | 27 | 0.32 | 0.38 | 8 | 8 | 11 | 30 | 0.38 | 0.41 |
| 9 | 8 | 27 | 0.38 | 0.35 | 9 | 9 | 3 | 41 | 0.76 | 0.44 |
| 10 | 5 | 10 | 0.1* | 0.15* | --- | 10 | 5 | 45 | 0.8 | 0.5 |
| 11 | 8 | 30 | 0.44 | 0.38 | 10 | 11 | 14 | 29 | 0.3 | 0.43 |
| 12 | 5 | 34 | 0.58 | 0.39 | 11 | 12 | 1 | 49 | 0.96 | 0.5 |
| 13 | 10 | 25 | 0.3 | 0.35 | 12 | 13 | 11 | 49 | 0.76 | 0.6 |
| 14 | 8 | 38 | 0.6 | 0.46 | 13 | 14 | 9 | 45 | 0.72 | 0.54 |
| 15 | 9 | 26 | 0.34 | 0.35 | 14 | 15 | 3 | 48 | 0.9 | 0.51 |
| 16 | 13 | 17 | 0.08* | 0.3* | --- | 16 | 7 | 46 | 0.78 | 0.53 |
| 17 | 12 | 29 | 0.34 | 0.41 | 15 | 17 | 5 | 47 | 0.84 | 0.52 |
| 18 | 10 | 26 | 0.32 | 0.36 | 16 | 18 | 5 | 30 | 0.5 | 0.35 |
| - | - | - | - | - | - | 19 | 3 | 33 | 0.6 | 0.36 |
| - | - | - | - | - | - | 20 | 10 | 22 | 0.24 | 0.32 |
| Area of Triangle |  |  |  |  |  |  |  |  |  |  |
| 1 | 9 | 30 | 0.42 | 0.39 |  | 7 | 6 | 46 | 0.8 | 0.52 |
| 2 | 11 | 45 | 0.68 | 0.56 |  | 8 | 6 | 29 | 0.46 | 0.35 |
| 3 | 6 | 40 | 0.68 | 0.46 |  | 9 | 8 | 33 | 0.5 | 0.41 |
| 4 | 5 | 35 | 0.6 | 0.4 |  | 10 | 11 | 41 | 0.6 | 0.52 |
| 5 | 8 | 40 | 0.64 | 0.48 |  | 11 | 8 | 38 | 0.6 | 0.46 |
| 6 | 3 | 37 | 0.68 | 0.4 |  | 12 | 16 | 43 | 0.54 | 0.59 |

Note: * indicates value outside the limits of DP or DI

## Appendix 12

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Achievement in Parallel Lines

## (Draft)

| Dr. K. Abdul Gafoor Professor | Sarabi. M.K Research Scholar |
| :---: | :---: |
|  |  |
|  |  |

## 










a) $\mathrm{AB}, \mathrm{BC}$
b) $\mathrm{AB}, \mathrm{DC}$
c) $\mathrm{AB}, \mathrm{AC}$



a) $90^{\circ}$
b) $180^{\circ}$
c) $240^{\circ}$
d) $360^{\circ}$
 வగ్రృం?
a) 0
b) 1
c) 2
d) ாைாேே๐







## 

 களைபளைைை.
 $\qquad$ ๑ก.จา.
a) 1 ๑๓.๑า.
b) 5 ๑ก.ฉา.
c) 4 ๑ก.ฉา.
d) 9 ๑ก๐.๓า.
 $\qquad$ ?
a) $30^{\circ}$
b) $60^{\circ}$
c) $120^{\circ}$

7.

 $\qquad$
a) $90^{\circ}$
b) $180^{\circ}$
c) $240^{\circ}$
d) $360^{\circ}$
8.

 $\qquad$ $\angle \mathrm{DAB}=$ $\qquad$
a) $\angle \mathrm{ABD}=60^{\circ}, \angle \mathrm{DAB}=60^{\circ}$
b) $\angle \mathrm{ABD}=60^{\circ}, \angle \mathrm{DAB}=80^{\circ}$
c) $\angle \mathrm{ABD}=70^{\circ}, \angle \mathrm{DAB}=50^{\circ}$
d) $\angle \mathrm{ABD}=70^{\circ}, \angle \mathrm{DAB}=60^{\circ}$




a) $\angle \mathrm{CBM}, \angle \mathrm{ABE}$
b) $\angle \mathrm{DEN}, \angle \mathrm{FEB}$
c) $\angle \mathrm{DEN}, \angle \mathrm{FEN}$
d) $\angle \mathrm{MBA}, \angle \mathrm{CBE}$

a）$\angle \mathrm{CBM}, \angle \mathrm{FEB}$
b）$\angle \mathrm{CBE}, \angle \mathrm{FEN}$
c）$\angle \mathrm{MBA}, \angle \mathrm{MBC}$
d）$\angle \mathrm{MBA}, \angle \mathrm{BED}$

a）$\angle \mathrm{ABE}, \angle \mathrm{CBE}$
b）$\angle \mathrm{CBE}, \angle \mathrm{DEB}$
c）$\angle \mathrm{MBC}, \angle \mathrm{DEN}$
d）$\angle \mathrm{MBA}, \angle \mathrm{FEN}$

a）$\angle \mathrm{ABE}, \angle \mathrm{DEB}$
b）$\angle \mathrm{CBM}, \angle \mathrm{FEB}$
c）$\angle \mathrm{CBE}, \angle \mathrm{FEN}$
d）$\angle \mathrm{MBA}, \angle \mathrm{BED}$

a）$\angle \mathrm{ABE}, \angle \mathrm{CBE}$
b）$\angle \mathrm{MBA}, \angle \mathrm{FEN}$
c）$\angle \mathrm{MBC}, \angle \mathrm{FEN}$
d）$\angle \mathrm{MBC}, \angle \mathrm{DEN}$
 カリカி உாைை ก円ழூாைக




14．$\angle \mathrm{BED}=$ $\qquad$
a） $40^{\circ}$
b） $100^{\circ}$
c） $140^{\circ}$


15．$\angle \mathrm{FEH}=$ $\qquad$
a） $20^{\circ}$
b） $40^{\circ}$
c） $80^{\circ}$
d） $140^{\circ}$

16．$\angle \mathrm{IHE}=$ $\qquad$
a） $20^{\circ}$
b） $40^{\circ}$
c） $80^{\circ}$
d） $100^{\circ}$

## Appendix 13

# UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION Test of Achievement in Parallel Lines <br> (Draft) 

| Dr. K. Abdul Gafoor <br> Professor | Sarabi. M.K <br> Research Scholar |
| :--- | ---: |
| Class: VII | Marks: 16 marks |
|  | Time: 26 mts |

## Directions

Shown below are 16 questions based on your mathematics topic of Parallel Lines. Four answer choices are given for each question. Read each question carefully and choose the right answer. Please do not write anything on your question paper. You are given an answer sheet separately. Please put a tick mark $(\checkmark)$ in the appropriate column for answering. Your answer sheets will be kept in safe custody and used for research purposes only.

1. Which are the parallel sides of the given rectangle?
a) $\mathrm{AB}, \mathrm{BC}$
b) $\mathrm{AB}, \mathrm{DC}$
c) $\mathrm{AB}, \mathrm{AC}$
d) None of the above

2. The sum of all angles of a parallelogram will be ----.
a) $90^{\circ}$
b) $180^{\circ}$
c) $240^{\circ}$
d) $360^{\circ}$
$3 . \quad \bullet$


How many parallel lines can be drawn to the line AB that passes through the point C?
a) 0
b) 1
c) 2
d) Many
4. What do you mean by two given angles are supplementary?
a) the sum of the angles would be $180^{\circ}$
b) the sum of the angles would be $360^{\circ}$
c) both the angles would be opposite angles
d) both the angles would be corresponding angles

## Answer question numbers 5 and 6 based on the given parallelogram


5. In the given parallelogram, $\mathrm{AB}=$ $\qquad$ cm.
a) 1 cm
b) 5 cm
c) 4 cm
d) 9 cm
6. In the given parallelogram, $\angle \mathrm{BCD}=$ $\qquad$
a) $30^{\circ}$
b) $60^{\circ}$
c) $120^{\circ}$
d) None of these
7.


In the figure given, $\angle \mathrm{EBD}+\angle \mathrm{MCA}=----$
a) $90^{\circ}$
b) $180^{\circ}$
c) $240^{\circ}$
d) $360^{\circ}$
8.


In the figure given
a) $\angle \mathrm{ABD}=60^{\circ}, \angle \mathrm{DAB}=60^{\circ}$
b) $\angle \mathrm{ABD}=60^{\circ}, \angle \mathrm{DAB}=80^{\circ}$
c) $\angle \mathrm{ABD}=70^{\circ}, \angle \mathrm{DAB}=50^{\circ}$
d) $\angle \mathrm{ABD}=70^{\circ}, \angle \mathrm{DAB}=60^{\circ}$

## Answer question numbers 9 to 13 based on the given figure


9. Which of the pairs given below are not equal angles?
a) $\angle \mathrm{CBM}, \angle \mathrm{ABE}$
b) $\angle \mathrm{DEN}, \angle \mathrm{FEB}$
c) $\angle \mathrm{DEN}, \angle \mathrm{FEN}$
d) $\angle \mathrm{MBA}, \angle \mathrm{CBE}$
10. Which of the pairs given below are not corresponding angles?
a) $\angle \mathrm{CBM}, \angle \mathrm{FEB}$
b) $\angle \mathrm{CBE}, \angle \mathrm{FEN}$
c) $\angle \mathrm{MBA}, \angle \mathrm{MBC}$
d) $\angle \mathrm{MBA}, \angle \mathrm{BED}$
11. Which of the following pairs are not alternate angles?
a) $\angle \mathrm{ABE}, \angle \mathrm{CBE}$
b) $\angle \mathrm{CBE}, \angle \mathrm{DEB}$
c) $\angle \mathrm{MBC}, \angle \mathrm{DEN}$
d) $\angle \mathrm{MBA}, \angle \mathrm{FEN}$
12. Choose the pair of co-interior angles.
a) $\angle \mathrm{ABE}, \angle \mathrm{DEB}$
b) $\angle \mathrm{CBM}, \angle \mathrm{FEB}$
c) $\angle \mathrm{CBE}, \angle \mathrm{FEN}$
d) $\angle \mathrm{MBA}, \angle \mathrm{BED}$
13. Choose the pair of co-exterior angles
a) $\angle \mathrm{ABE}, \angle \mathrm{CBE}$
b) $\angle \mathrm{MBA}, \angle \mathrm{FEN}$
c) $\angle \mathrm{MBC}, \angle \mathrm{FEN}$
d) $\angle \mathrm{MBC}, \angle \mathrm{DEN}$

## Answer questions 14 to 16 based on the figure given



A line intersects three parallel lines. One of the angles is $40^{\circ}$. Figure out other angles without measuring them?
14. $\angle \mathrm{BED}=$ $\qquad$
a) $40^{\circ}$
b) $100^{\circ}$
c) $140^{\circ}$
d) None of these
15. $\angle \mathrm{FEH}=$ $\qquad$
a) $20^{\circ}$
b) $40^{\circ}$
c) $80^{\circ}$
d) $140^{\circ}$
16. $\angle \mathrm{IHE}=$ $\qquad$
a) $20^{\circ}$
b) $40^{\circ}$
c) $80^{\circ}$
d) $100^{\circ}$

## Appendix 4

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Achievement in Parallel Lines

（Final）

| Dr．K．Abdul Gafoor Professor | Sarabi．M．K <br> Research Scholar |
| :---: | :---: |
| ¢⿴囗⿰丨丨刃心 |  |
|  | พ๑¢）： 24 ๑าm！ร้ |

## 










a） $\mathrm{AB}, \mathrm{BC}$
b） $\mathrm{AB}, \mathrm{DC}$
c） $\mathrm{AB}, \mathrm{AC}$



a） $90^{\circ}$
b） $180^{\circ}$
c） $240^{\circ}$
d） $360^{\circ}$
 வดృం？
a） 0
b） 1
c） 2
d）ஞாேேே๐
$A$ ——B






## 

 களைபாைைை.
 $\qquad$ ๑ก.จา.
a) 1 ๑๓.๑า.
b) 5 ๑ก.ฉา.
c) 4 ๑ก.ฉา.
d) 9 ๑ก๐.๓า.
 $\qquad$ .?
a) $30^{\circ}$
b) $60^{\circ}$
c) $120^{\circ}$

7.

 $\qquad$
a) $90^{\circ}$
b) $180^{\circ}$
c) $240^{\circ}$
d) $360^{\circ}$
8.

 $\qquad$ $\angle \mathrm{DAB}=$ $\qquad$
a) $\angle \mathrm{ABD}=60^{\circ}, \angle \mathrm{DAB}=60^{\circ}$
b) $\angle \mathrm{ABD}=60^{\circ}, \angle \mathrm{DAB}=80^{\circ}$
c) $\angle \mathrm{ABD}=70^{\circ}, \angle \mathrm{DAB}=50^{\circ}$
d) $\angle \mathrm{ABD}=70^{\circ}, \angle \mathrm{DAB}=60^{\circ}$




a) $\angle \mathrm{CBM}, \angle \mathrm{FEB}$
b) $\angle \mathrm{CBE}, \angle \mathrm{FEN}$
c) $\angle \mathrm{MBA}, \angle \mathrm{MBC}$
d) $\angle \mathrm{MBA}, \angle \mathrm{BED}$

a）$\angle \mathrm{ABE}, \angle \mathrm{CBE}$
b）$\angle \mathrm{CBE}, \angle \mathrm{DEB}$
c）$\angle \mathrm{MBC}, \angle \mathrm{DEN}$
d）$\angle \mathrm{MBA}, \angle \mathrm{FEN}$

a）$\angle \mathrm{ABE}, \angle \mathrm{DEB}$
b）$\angle \mathrm{CBM}, \angle \mathrm{FEB}$
c）$\angle \mathrm{CBE}, \angle \mathrm{FEN}$
d）$\angle \mathrm{MBA}, \angle \mathrm{BED}$

a）$\angle \mathrm{ABE}, \angle \mathrm{CBE}$
b）$\angle \mathrm{MBA}, \angle \mathrm{FEN}$
c）$\angle \mathrm{MBC}, \angle \mathrm{FEN}$
d）$\angle \mathrm{MBC}, \angle \mathrm{DEN}$
 かけ」なった




13．$\angle \mathrm{BED}=$ $\qquad$
a） $40^{\circ}$
b） $100^{\circ}$
c） $140^{\circ}$


14．$\angle \mathrm{IHE}=$ ．．．．．．．
a） $20^{\circ}$
b） $40^{\circ}$
c） $80^{\circ}$
d） $100^{\circ}$

## Appendix I 5

UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Achievement Test in Parallel Lines

(Final)

| Dr. K. Abdul Gafoor | Sarabi. M.K <br> Professor |
| :--- | ---: |
| Research Scholar |  |

Shown below are 14 questions based on your mathematics topic of Parallel Lines. Four answer choices are given for each question. Read each question carefully and choose the right answer. Please do not write anything on your question paper. You are given an answer sheet separately. Please put a tick mark $(\checkmark)$ in the appropriate column for answering. Your answer sheets will be kept in safe custody and used for research purposes only.

1. Which are the parallel sides of the given rectangle?
a) $\mathrm{AB}, \mathrm{BC}$
b) $\mathrm{AB}, \mathrm{DC}$
c) $\mathrm{AB}, \mathrm{AC}$
d) None of the above
2. The sum of all angles of a parallelogram will be -----
a) $90^{\circ}$
b) $180^{\circ}$
c) $240^{\circ}$
d) $360^{\circ}$
3. $\bullet C$

## $A \longrightarrow B$

How many parallel lines can be drawn to the line AB that passes through the point C?
a) 0
b) 1
c) 2
d) Many
4. What do you mean by two given angles are supplementary?
a) the sum of the angles would be $180^{\circ}$
b) the sum of the angles would be $360^{\circ}$
c) both the angles would be opposite angles
d) both the angles would be corresponding angles

## Answer question numbers 5 and 6 based on the given parallelogram


5. In the given parallelogram, $\mathrm{AB}=$ $\qquad$ cm.
a) 1 cm
b) 5 cm
c) 4 cm
d) 9 cm
6. In the given parallelogram, $\angle \mathrm{BCD}=-----$
a) $30^{\circ}$
b) $60^{\circ}$
c) $120^{\circ}$
d) None of these
7.


In the figure given, $\angle \mathrm{EBD}+\angle \mathrm{MCA}=----$
a) $90^{\circ}$
b) $180^{\circ}$
c) $240^{\circ}$
d) $360^{\circ}$
8.

a) $\angle \mathrm{ABD}=60^{\circ}, \angle \mathrm{DAB}=60^{\circ}$
b) $\angle \mathrm{ABD}=60^{\circ}, \angle \mathrm{DAB}=80^{\circ}$
c) $\angle \mathrm{ABD}=70^{\circ}, \angle \mathrm{DAB}=50^{\circ}$
d) $\angle \mathrm{ABD}=70^{\circ}, \angle \mathrm{DAB}=60^{\circ}$

## Answer question numbers 9 to 12 based on the given figure


9. Which of the pairs given below are not corresponding angles?
a) $\angle \mathrm{CBM}, \angle \mathrm{FEB}$
b) $\angle \mathrm{CBE}, \angle \mathrm{FEN}$
c) $\angle \mathrm{MBA}, \angle \mathrm{MBC}$
d) $\angle \mathrm{MBA}, \angle \mathrm{BED}$
10. Which of the following pairs are not alternate angles?
a) $\angle \mathrm{ABE}, \angle \mathrm{CBE}$
b) $\angle \mathrm{CBE}, \angle \mathrm{DEB}$
c) $\angle \mathrm{MBC}, \angle \mathrm{DEN}$
d) $\angle \mathrm{MBA}, \angle \mathrm{FEN}$
11. Choose the pair of co-interior angles.
a) $\angle \mathrm{ABE}, \angle \mathrm{DEB}$
b) $\angle \mathrm{CBM}, \angle \mathrm{FEB}$
c) $\angle \mathrm{CBE}, \angle \mathrm{FEN}$
d) $\angle \mathrm{MBA}, \angle \mathrm{BED}$
12. Choose the pair of co-exterior angles
a) $\angle \mathrm{ABE}, \angle \mathrm{CBE}$
b) $\angle \mathrm{MBA}, \angle \mathrm{FEN}$
c) $\angle \mathrm{MBC}, \angle \mathrm{FEN}$
d) $\angle \mathrm{MBC}, \angle \mathrm{DEN}$

## Answer questions 13 and 14 based on the figure given



A line intersects three parallel lines. One of the angles is $40^{\circ}$. Figure out other angles without measuring them?
13. $\angle \mathrm{BED}=\ldots \ldots$.
a) $40^{\circ}$
b) $100^{\circ}$
c) $140^{\circ}$
d) None of these
14. $\angle \mathrm{IHE}=$ $\qquad$
a) $20^{\circ}$
b) $40^{\circ}$
c) $80^{\circ}$
d) $100^{\circ}$

## Appendix 16

UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Achievement in Parallel Lines

(Final)

## Response Sheet

ேேดั: $\qquad$


| S. No. | a | b | $\mathbf{c}$ | d |
| :---: | :---: | :---: | :---: | :---: |
| 1. |  |  |  |  |
| 2. |  |  |  |  |
| 3. |  |  |  |  |
| 4. |  |  |  |  |
| 5. |  |  |  |  |
| 6. |  |  |  |  |
| 7. |  |  |  |  |
| 8. |  |  |  |  |
| 9. |  |  |  |  |
| 10. |  |  |  |  |
| 11. |  |  |  |  |
| 12. |  |  |  |  |
| 13. |  |  |  |  |
| 14. |  |  |  |  |

## Appendix 17

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Achievement in Unchanging Relations

Dr. K. Abdul Gafoor<br>Professor<br>Sarabi. M.K<br>かOMN్N: VII<br>Research Scholar<br> พロய๐: 26 ฝาmృร̆

## 











a) $c=2 a b$
b) $c=a+b$
c) $c=2+a+b$
d) $c=2(a+b)$



a) $2 x-x$
b) $x-2 y$
c) $x-2 x$
d) $2 y-x$

a) $x+(x+5)$
b) $x+(y+5)$
c) $5+(\mathrm{x}+5)$
d) $y+(x+5)$

a) $3 x-8 y$
b) $3 x-8 x$
c) $8 x-3 x$
d) $8 x-3 y$

a) $x+(x+1)-2$
b) $(x+x)-2$
c) $x+(x+2)-2$
d) $x+x+2$
6. $(47-93 / 4)+1 / 4=$
a) $47+(93 / 4-1 / 4)$
b) $47+(93 / 4+1 / 4)$
c) $47-(93 / 4+1 / 4)$
d) $47-(93 / 4-1 / 4)$
7. $(234+8.5)-3.5=$
a) $234+(8.5-3.5)$
b) $234+(8.5+3.5)$
c) $234-(8.5-3.5)$
d) $234-(8.5+3.5)$
8. $(5 \times 13)+(25 \times 13)=$
a) $(5 \times 25)+13$
b) $(5 \times 25) \times 13$
c) $(5+25) \times 13$
d) $(5+13) \times 25$
9. $(121 / 2 \times 15)-\left(10^{1 / 2} \times 15\right)=$
a) $\left(12 \frac{1}{2}-10 \frac{1}{2}\right) \times 15$
b) $\left(12 \frac{1}{2} \times 10 \frac{1}{2}\right) \times 15$
c) $\left(12 \frac{1}{2} \times 101 / 2\right)+15$
d) $\left(12 \frac{1}{2}-10 \frac{1}{2}\right)+15$

a) 13
b) 16
c) 17
d) 18

a) 2
b) 4
c) 5
d) 10



a) $63-35$
b) $63-(35+5)$
c) $63-(35-5)$
d) $63+(35-5)$
 ஸைைுமிதி.
A. $5+7+3=5+(7+3)$
B. $37+24 \frac{1}{2}+751 / 2=37+(241 / 2+751 / 2)$
C. $44+16.5+13.5=44+(16.5+13.5)$











a) $(6.5 \times 4)-(4.5 \times 4)=(6.5-4.5) \times 4$
b) $(35 \times 12)+(28 \times 12)=(35-28) \times 12$
c) $\left(12^{1 / 2} \times 2^{3} / 4\right)-\left(6^{1 / 2} \times 2^{3} / 4\right)=\left(12^{1 / 2}+6^{1 / 2}\right) \times 2^{3 / 4}$
d) $(35-12) \times(28-12)=(35+28)-12$




a) $(x-y)-z=x-(y+z)$
b) $(x-y)-z=x-(y-z)$
c) $(x+y)-z=x+(y-z)$
d) $(x-y)-z=x+(y+z)$




a) $(x+y)+z=x+(y+z)$
b) $(\mathrm{x}+\mathrm{y})-\mathrm{z}=\mathrm{x}+(\mathrm{y}-\mathrm{z})$
c) $(x+y)+z=x+(y-z)$
d) $x+(y-z)=(x+y)+z$

## Appendix 18

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Achievement in Unchanging Relations

| Dr. K. Abdul Gafoor <br> Professor | Sarabi. M.K <br> Research Scholar |
| :--- | ---: |
| Class: VII | Marks: 16 marks |
|  | Time: 26 mts |

## Directions

Shown below are 16 questions based on your mathematics topic of 'Unchanging Relations'. Four answer choices are given for each question. Read each question carefully and choose the right answer. Please do not write anything on your question paper. You are given an answer sheet separately. Please put a tick mark $(\checkmark)$ in the appropriate column for answering. Your answer sheets will be kept in safe custody and used for research purposes only.

1. If the length, width, and perimeter of a rectangle are $a$ and $b$, respectively, how can their relationship be expressed?
a) $\mathrm{c}=2 \mathrm{ab}$
b) $\mathrm{c}=\mathrm{a}+\mathrm{b}$
c) $c=2+a+b$
d) $\mathrm{c}=2(\mathrm{a}+\mathrm{b})$

## For questions 2 to 5, choose the correct expression of statements provided using variables

2. Subtract two times of a number from the number itself
a) $2 x-x$
b) $x-2 y$
c) $x-2 x$
d) $2 y-x$
3. Add a number and 5 added to it.
a) $x+(x+5)$
b) $x+(y+5)$
c) $5+(\mathrm{x}+5)$
d) $y+(x+5)$
4. Subtract 3 times of a number from 5 times of the same number.
a) $3 x-8 y$
b) $3 x-8 x$
c) $8 x-3 x$
d) $8 x-3 y$
5. Subtract 2 from two consecutive natural numbers.
a) $x+(x+1)-2$
b) $(x+x)-2$
c) $x+(x+2)-2$
d) $x+x+2$
6. $(47-93 / 4)+1 / 4=$
a) $47+(93 / 4-1 / 4)$
b) $47+(93 / 4+1 / 4)$
c) $47-\left(9^{3} / 4+1 / 4\right)$
d) $47-(93 / 4-1 / 4)$
7. $(234+8.5)-3.5=$
a) $234+(8.5-3.5)$
b) $234+(8.5+3.5)$
c) $234-(8.5-3.5)$
d) $234-(8.5+3.5)$
8. $(5 \times 13)+(25 \times 13)=$
a) $(5 \times 25)+13$
b) $(5 \times 25) \times 13$
c) $(5+25) \times 13$
d) $(5+13) \times 25$
9. $(121 / 2 \times 15)-\left(10 \frac{1}{2} \times 15\right)=$
a) $\left(12 \frac{1}{2}-10^{1 / 2}\right) \times 15$
b) $\left(12 \frac{1}{2} \times 10 \frac{1}{2}\right) \times 15$
c) $\left(121 / 2 \times 10^{1 / 2}\right)+15$
d) $\left(12 \frac{1}{2}-10 \frac{1}{2}\right)+15$
10. If the sum of two numbers is 30 , and their difference is 4 , which is bigger number?
a) 13
b) 16
c) 17
d) 18
11. If the sum of two numbers is 12 , and their difference is 8 , which number is smaller?
a) 2
b) 4
c) 5
d) 10
12. Anu had 63 candies. She gave one each to all 35 students in her class, and 5 to her teacher. What numerical operation to be used to figure out the number of remaining candies?
a) 63-35
b) $63-(35+5)$
c) $63-(35-5)$
d) $63+(35-5)$
13. Choose the general principle that can be reached from the given examples.
A. $5+7+3=5+(7+3)$
B. $37+24 \frac{1}{2}+75 \frac{1}{2}=37+\left(241 / 2+75 \frac{1}{2}\right)$
C. $44+16.5+13.5=44+(16.5+13.5)$
a) Instead of adding to one number, two numbers one after another, add their sum.
b) Instead of adding two numbers separately to another number, the difference of the two numbers can be added to that number.
c) In order to add two numbers to another number, their difference can be added to that number.
d) Instead of adding two numbers separately to another number, half of their sum can be added to it.
14. Choose the appropriate example that matches with the given principle.
'Multiplying two numbers by a number separately and subtracting give the same result as multiplying their difference by the number'.
a) $(6.5 \times 4)-(4.5 \times 4)=(6.5-4.5) \times 4$
b) $(35 \times 12)+(28 \times 12)=(35-28) \times 12$
c) $\left(12^{1 / 2} \times 2^{3 / 4}\right)-\left(6^{1 / 2} \times 2^{3 / 4}\right)=\left(12^{1 / 2}+61 / 2\right) \times 2^{3 / 4}$
d) $35-12) \times(28-12)=(35+28)-12$
15. Choose the correct algebraic expression of the given principle.
'Instead of subtracting two numbers one by one from a given number, the sum of the two numbers can be subtracted from the original number.'
a) $(x-y)-z=x-(y+z)$
b) $(x-y)-z=x-(y-z)$
c) $(x+y)-z=x+(y-z)$
d) $(x-y)-z=x+(y+z)$
16. Choose the correct algebraic expression of the given principle. 'Instead of adding two numbers one by one to a given number, sum of these two numbers can be added to that number'
a) $(x+y)+z=x+(y+z)$
b) $(x+y)-z=x+(y-z)$
c) $(x+y)+z=x+(y-z)$
d) $x+(y-z)=(x+y)+z$

## Appendix I 9

UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Achievement in Unchanging Relations

## Response Sheet

ேேฮั:


| S. No. | a | b | c | d |
| :---: | :--- | :--- | :--- | :--- |
| 1. |  |  |  |  |
| 2. |  |  |  |  |
| 3. |  |  |  |  |
| 4. |  |  |  |  |
| 5. |  |  |  |  |
| 6. |  |  |  |  |
| 7. |  |  |  |  |
| 8. |  |  |  |  |
| 9. |  |  |  |  |
| 10. |  |  |  |  |
| 11. |  |  |  |  |
| 12. |  |  |  |  |
| 13. |  |  |  |  |
| 14. |  |  |  |  |
| 15. |  |  |  |  |
| 16. |  |  |  |  |

## Appendix 110

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Achievement in Repeated Multiplications

（Draft）

Dr．K．Abdul Gafoor

Professor
Sarabi．M．K Research Scholar
\＆્ભૅ゙：VII
 พ๑ய๐： 28 ๑า๓ృร̆

## 










a） 4
b） 5
c） 9
d） 20

2． 3 ๑กூ ாセா๐ விடே？
a） 3
b） 6
c） 9
d） 27

3． $4^{3}=$
a） 7
b） 12
c） 16
d） 64

a） $10^{2}$
b） $10^{5}$
c） $10^{10}$
d） $10^{15}$

5．$(1.3)^{3}=$
a） 0.2197
b） 2.197
c） 21.97
d） 219.7

6． $3^{5} \times 3^{4}=$
a） $3^{5+4}$
b） $3^{5-4}$
c） $3^{5 \times 4}$
d） $3^{5 \div 4}$

a）4－0ロ๐ைை கைைை
b）12－0ロ๐ைை கைறை
c）14－ગロดணைைைை
d）40－0வ๐ாை கృாை

a) 4-0ø๐ாைைைை
b) 9-0®๓ைை கைாை
c) 10-0ø๐ைை கృாை
d) 11-ગロ๐ைை கృை

a) 7
b) 11
c) 18
d) 25
10. $3^{8} \div 3^{5}=$
a) $3^{1}$
b) $3^{3}$
c) $3^{13}$
d) $3^{40}$

a) $(3 / 4)^{2}$
b) $(3 / 4)^{8}$
c) $\frac{16}{9}$
d) $\frac{9}{16}$
12. $\left[(3 / 4)^{3}\right]^{4}=$
a) $(3 / 4)^{1}$
b) $(3 / 4)^{3} \quad$ c) $(3 / 4)^{7}$
d) $(3 / 4)^{12}$
13. $\left[(1 / 2)^{3}\right]^{4}=$
a) $(1 / 2)^{3} \mathrm{x}(1 / 2)^{3} \times(1 / 2)^{3}$
b) $(1 / 2)^{4} \mathrm{x}(1 / 2)^{4} \mathrm{x}(1 / 2)^{4}$
c) $(1 / 2)^{4} \times(1 / 2)^{4} \times(1 / 2)^{4} \times(1 / 2)^{4}$
d) $(1 / 2)^{3} \times(1 / 2)^{3} \times(1 / 2)^{3} \times(1 / 2)^{3}$

a) $2^{2}, 4^{2}$
b) $2^{2}, 4^{4}$
c) $2^{3}, 4^{3}$
d) $2^{4}, 4^{2}$
15. $144=$
a) $2^{2} \times 3^{2}$
b) $2^{2} \times 3^{4}$
c) $2^{4} \times 3^{2}$
d) $2^{3} \times 3^{3}$
16. $3^{x}=242,3^{x+1}=\ldots \ldots$.
a) 243
b) 245
c) 484
d) 726






a) 4
b) 12
c) 16
d) 20

## Appendix 111

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Achievement in Repeated Multiplication

(Draft)

Dr. K. Abdul Gafoor<br>Professor<br>Sarabi. M.K<br>Class: VII

## Directions

Shown below are 18 questions based on your mathematics topic of 'Repeated Multiplication'. Four answer choices are given for each question. Read each question carefully and choose the right answer. Please do not write anything on your question paper. You are given an answer sheet separately. Please put a tick mark $(\checkmark)$ in the appropriate column for answering. Your answer sheets will be kept in safe custody and used for research purposes only.

1. What is the power of $5^{4}$ ?
a) 4
b) 5
c) 9
d) 20
2. What is cube of 3 ?
a) 3
b) 6
c) 9
d) 27
3. $4^{3}=$
a) 7
b) 12
c) 16
d) 64
4. Write one lakh as the power of ten
a) $10^{2}$
b) $10^{5}$
c) $10^{10}$
d) $10^{15}$
5. $(1.3)^{3}=$
a) 0.2197
b) 2.197
c) 21.97
d) 219.7
6. $3^{5} \times 3^{4}=$
a) $3^{5+4}$
b) $3^{5-4}$
c) $3^{5 \times 4}$
d) $3^{5 \div 4}$
7. What power of 2 is 16 times of $2^{10}$ ?
a) $4^{\text {th }}$ power
b) $12^{\text {th }}$ power
c) $14^{\text {th }}$ power
d) $40^{\text {th }}$ power
8. What power of 4 is 4 times of $4^{10}$ ?
a) $4^{\text {th }}$ power
b) $9^{\text {th }}$ power
c) $10^{\text {th }}$ power
d) $11^{\text {th }}$ power
9. What power of $\frac{1}{10}$ is obtained when $10^{7}$ is divided by $10^{18}$ ?
a) 7
b) 11
c) 18
d) 25
10. $3^{8} \div 3^{5}=$
a) $3^{1}$
b) $3^{3}$
c) $3^{13}$
d) $3^{40}$
11. When $(3 / 4)^{3}$ is divided by $(3 / 4)^{5}$
a) $(3 / 4)^{2}$
b) $(3 / 4)^{8}$
c) $\frac{16}{9}$
d) $\frac{9}{16}$
12. $\left[(3 / 4)^{3}\right]^{4}=$
a) $(3 / 4)^{1}$
b) $(3 / 4)^{3}$
c) $(3 / 4)^{7}$
d) $(3 / 4)^{12}$
13. $\left[(1 / 2)^{3}\right]^{4}=$
a) $(1 / 2)^{3} x(1 / 2)^{3} x(1 / 2)^{3}$
b) $(1 / 2)^{4} \mathrm{x}(1 / 2)^{4} \mathrm{x}(1 / 2)^{4}$
c) $(1 / 2)^{4} \times(1 / 2)^{4} \times(1 / 2)^{4} \times(1 / 2)^{4}$
d) $(1 / 2)^{3} \times(1 / 2)^{3} \times(1 / 2)^{3} \times(1 / 2)^{3}$
14. Which of the following is an equivalent pair of 16 ?
a) $2^{2}, 4^{2}$
b) $2^{2}, 4^{4}$
c) $2^{3}, 4^{3}$
d) $2^{4}, 4^{2}$
15. $144=$
a) $2^{2} \times 3^{2}$
b) $2^{2} \times 3^{4}$
c) $2^{4} \times 3^{2}$
d) $2^{3} \times 3^{3}$
16. $3^{x}=242,3^{x+1}=$ $\qquad$
a) 243
b) 245
c) 484
d) 726
17. 8128 is a perfect number. Why?
a) because all factors of 8128 are even numbers
b) because all factors of 8128 are odd numbers
c) because the sum of all factors of 8128 is a multiple of 3
d) because the sum of all factors of 8128 except 8128 is 8128 itself
18. If $\frac{10^{16}}{10^{x}}=10^{4}$, what is the value of $x$ ?
a) 4
b) 12
c) 16
d) 20

## Appendix 112

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Achievement Test in Repeated Multiplication

## （Final）

Dr．K．Abdul Gafoor

Professor
Sarabi．M．K


Research Scholar

 พロய๐： 26 ๑lmృร̆

## 










1． $5^{4}$ 亿
a） 4
b） 5
c） 9
d） 20

a） 3
b） 6
c） 9
d） 27

3． $4^{3}=$
a） 7
b） 12
c） 16
d） 64

a） $10^{2}$
b） $10^{5}$
c） $10^{10}$
d） $10^{15}$

5．$(1.3)^{3}=$
a） 0.2197
b） 2.197
c） 21.97
d） 219.7

6． $3^{5} \times 3^{4}=$
a） $3^{5+4}$
b） $3^{5-4}$
c） $3^{5 \times 4}$
d） $3^{5 \div 4}$

a）4－ナவொை கృைை
b）12－0ロカைை கృாை
c）14－0ロ๐ாைைைை
d）40－0ロ๐ாைைைை

a）4－ரロ๐ாை கృை
b）9－गロலாை கృை
c）10－ナவோை கృாை
d）11－ગロ๐ைை கృாை

a) 7
b) 11
c) 18
d) 25

a) $(3 / 4)^{2}$
b) $(3 / 4)^{8}$
c) $\frac{16}{9}$
d) $\frac{9}{16}$
11. $\left[(3 / 4)^{3}\right]^{4}=$
a) $(3 / 4)^{1}$
b) $(3 / 4)^{3}$
c) $(3 / 4)^{7}$
d) $(3 / 4)^{12}$
12. $\left[(1 / 2)^{3}\right]^{4}=$
a) $(1 / 2)^{3} \mathrm{x}(1 / 2)^{3} \times(1 / 2)^{3}$
b) $(1 / 2)^{4} \times(1 / 2)^{4} x(1 / 2)^{4}$
c) $(1 / 2)^{4} \times(1 / 2)^{4} \times(1 / 2)^{4} \times(1 / 2)^{4}$
d) $(1 / 2)^{3} \times(1 / 2)^{3} \times(1 / 2)^{3} \times(1 / 2)^{3}$

a) $2^{2}, 4^{2}$
b) $2^{2}, 4^{4}$
c) $2^{3}, 4^{3}$
d) $2^{4}, 4^{2}$
14. $144=$
a) $2^{2} \times 3^{2}$
b) $2^{2} \times 3^{4}$
c) $2^{4} \times 3^{2}$
d) $2^{3} \times 3^{3}$






a) 4
b) 12
c) 16
d) 20

## Appendix I 13

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

# Test of Achievement in Repeated Multiplication (Final) 

Dr. K. Abdul Gafoor<br>Professor Research Scholar<br>Class: VII<br>Marks: 16 marks<br>Time: 26 mts

## Directions

Shown below are 16 questions based on your mathematics topic of 'Repeated Multiplication'. Four answer choices are given for each question. Read each question carefully and choose the right answer. Please do not write anything on your question paper. You are given an answer sheet separately. Please put a tick mark $(\checkmark)$ in the appropriate column for answering. Your answer sheets will be kept in safe custody and used for research purposes only.

1. What is the power of $5^{4}$ ?
a) 4
b) 5
c) 9
d) 20
2. What is cube of 3 ?
a) 3
b) 6
c) 9
d) 27
3. $4^{3}=$
a) 7
b) 12
c) 16
d) 64
4. Write one lakh as the power of ten
a) $10^{2}$
b) $10^{5}$
c) $10^{10}$
d) $10^{15}$
5. $(1.3)^{3}=$
a) 0.2197
b) 2.197
c) 21.97
d) 219.7
6. $3^{5} \times 3^{4}=$
a) $3^{5+4}$
b) $3^{5-4}$
c) $3^{5 \times 4}$
d) $3^{5 \div 4}$
7. What power of 2 is 16 times of $2^{10}$ ?
a) $4^{\text {th }}$ power
b) $12^{\text {th }}$ power
c) $14^{\text {th }}$ power
d) $40^{\text {th }}$ power
8. What power of 4 is 4 times of $4^{10}$ ?
a) $4^{\text {th }}$ power
b) $9^{\text {th }}$ power
c) $10^{\text {th }}$ power
d) $11^{\text {th }}$ power
9. What power of $\frac{1}{10}$ is obtained when $10^{7}$ is divided by $10^{18}$ ?
a) 7
b) 11
c) 18
d) 25
10. When $(3 / 4)^{3}$ is divided by $(3 / 4)^{5}$
a) $(3 / 4)^{2}$
b) $(3 / 4)^{8}$
c) $\frac{16}{9}$
d) $\frac{9}{16}$
11. $\left[(3 / 4)^{3}\right]^{4}=$
a) $(3 / 4)^{1}$
b) $(3 / 4)^{3}$
c) $(3 / 4)^{7}$
d) $(3 / 4)^{12}$
12. $\left[(1 / 2)^{3}\right]^{4}=$
a) $(1 / 2)^{3} \times(1 / 2)^{3} \times(1 / 2)^{3}$
b) $(1 / 2)^{4} \mathrm{x}(1 / 2)^{4} \mathrm{x}(1 / 2)^{4}$
c) $(1 / 2)^{4} \times(1 / 2)^{4} \times(1 / 2)^{4} x(1 / 2)^{4}$
d) $(1 / 2)^{3} \times(1 / 2)^{3} \times(1 / 2)^{3} \times(1 / 2)^{3}$
13. Which of the following is an equivalent pair of 16 ?
a) $2^{2}, 4^{2}$
b) $2^{2}, 4^{4}$
c) $2^{3}, 4^{3}$
d) $2^{4}, 4^{2}$
14. $144=$
a) $2^{2} \times 3^{2}$
b) $2^{2} \times 3^{4}$
c) $2^{4} \times 3^{2}$
d) $2^{3} \times 3^{3}$
15. 8128 is a perfect number. Why?
a) because all factors of 8128 are even numbers
b) because all factors of 8128 are odd numbers
c) because the sum of all factors of 8128 is a multiple of 3
d) because the sum of all factors of 8128 except 8128 is 8128 itself
16. If $\frac{10^{16}}{10^{x}}=10^{4}$, what is the value of $x$ ?
a) 4
b) 12
c) 16
d) 20

## Appendix I 14

UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Achievement in Repeated Multiplications

(Final)

## Response Sheet



| Sl. No. | a | b | c | d |
| :---: | :---: | :---: | :---: | :---: |
| 1. |  |  |  |  |
| 2. |  |  |  |  |
| 3. |  |  |  |  |
| 4. |  |  |  |  |
| 5. |  |  |  |  |
| 6. |  |  |  |  |
| 7. |  |  |  |  |
| 8. |  |  |  |  |
| 9. |  |  |  |  |
| 10. |  |  |  |  |
| 11. |  |  |  |  |
| 12. |  |  |  |  |
| 13. |  |  |  |  |
| 14. |  |  |  |  |
| 15. |  |  |  |  |
| 16. |  |  |  |  |

## Appendix 115

## UNIVERSITY OF CALICUT <br> DEPARTMENT OF EDUCATION

## Test of Achievement in Area of Triangle

Dr．K．Abdul Gafoor

Professor
\＆ơ్ల゙：VII

Sarabi．M．K
Research Scholar

## 










a） 20 د．ఎกา®า．

c） 96 ص．هก．øา．
d） 192 ป．๑ก．ฉา．
 ளைsுமから




3．＂ B ．

a） 24 ఎ．๑ก．øา．

c） 60 上．๑กา．๑า．

4.







а） 6 ๑ை．®า．
b） 8 ๑กา．จา．
c） 16 ๑ก．๓า．
d） 24 ๓ก๐．ฉา．
6.


a） 30 د．๑กั．øา．
b） 42 ค．๓าก．ฉา．
c） 60 ค．๑ก．ฉา．

7.


a） 33 ป．๑กั．øา．
b） 141 لـ．๑ก．จา．
c） 74 ค．வก．ฉา．

8.


a） 200 د．هก．จใ．
b） 300 上．๓กาจาใ．
c） 400 คก๓．จา．
d） 600 د．๑ก．จา．
9.


a） $1 / 2 \times 12 \times 5$
b） $1 / 2 \times 15 \times 5$
c） $15 \times 5$
d） $12 \times 5$
10.

11.

12.


a） 20 上．๑กา．ฉา．
b） 40 ค．๑ก๐．จา．
c） 80 อก．๑าจา．
d） 120 ป．๑ก．ฉา．

a） 21 د．๑ก．ฉา．
b） 45 د．๑กา．๑า．

d） 180 －．๑กั．จา．

a） 60 上．๑ก．๓า．
b） 100 ป．๑า．ฉา．
c） 150 ป．๑กை．จา．
d） 175 لـ．๓กา．จา．

## Appendix I 16

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Achievement in Area of Triangle

Dr. K. Abdul Gafoor
Professor

Sarabi. M.K
Research Scholar
Marks: 12 marks
Time: 22 mts

## Directions

Shown below are 12 questions based on your mathematics topic of 'Area of Triangle'. Four answer choices are given for each question. Read each question carefully and choose the right answer. Please do not write anything on your question paper. You are given an answer sheet separately. Please put a tick mark $(\checkmark)$ in the appropriate column for answering. Your answer sheets will be kept in safe custody and used for research purposes only.

1. Figure out the surface area of a rectangle that is length 12 cm long and 8 cm wide
a) $20 \mathrm{~cm}^{2}$
b) $40 \mathrm{~cm}^{2}$
c) $96 \mathrm{~cm}^{2}$
d) $192 \mathrm{~cm}^{2}$
2. Choose the special character of a trapezoid
a) Lengths of all sides would be equal
b) One pair of opposite sides would be parallel.
c) All angles would be 90 degrees
d) The total of all angles would be 180 degrees
3. " In the given right triangle, sides $\mathrm{AB}=6 \mathrm{~cm}, \mathrm{BC}=8 \mathrm{~cm}$, and $\mathrm{AC}=10 \mathrm{~cm}$. What is its surface area? $\{\mathrm{X}$ ?
a) $24 \mathrm{~cm}^{2}$
b) $40 \mathrm{~cm}^{2}$
c) $60 \mathrm{~cm}^{2}$
d) $80 \mathrm{~cm}^{2}$
4. 



In the given trapezoid, what would be the first step to calculate its surface area?
a) Add the lengths of its sides
b) Multiply the lengths of the opposite sides
c) Draw a perpendicular to $C D$ from $B$
d) Draw a perpendicular to BD from AC
5. Area of a right angled triangle is $96 \mathrm{~cm}^{2}$. If length of one perpendicular side is 12 cm , find out the length of the other perpendicular side.
a) 6 cm
b) 8 cm
c) 16 cm
d) 24 cm
6.


Find out the surface area of the given figure
a) $30 \mathrm{~cm}^{2}$
b) $42 \mathrm{~cm}^{2}$
c) $60 \mathrm{~cm}^{2}$
d) $84 \mathrm{~cm}^{2}$
7.


Find out the surface area of the given figure.
a) $33 \mathrm{~cm}^{2}$
b) $141 \mathrm{~cm}^{2}$
c) $74 \mathrm{~cm}^{2}$
d) $148 \mathrm{~cm}^{2}$
8.


What would be the surface area of $\triangle \mathrm{DBE}$ in the given figure?
a) $200 \mathrm{~cm}^{2}$
b) $300 \mathrm{~cm}^{2}$
c) $400 \mathrm{~cm}^{2}$
d) $600 \mathrm{~cm}^{2}$
9.


What would be the operation to find the surface area of $\triangle \mathrm{XYZ}$ ?
a) $1 / 2 \times 12 \times 5$
b) $1 / 2 \times 15 \times 5$
c) $15 \times 5$
d) $12 \times 5$
10.


How much is the surface area of $\triangle \mathrm{ABD}$ ?
a) $20 \mathrm{~cm}^{2}$
b) $40 \mathrm{~cm}^{2}$
c) $80 \mathrm{~cm}^{2}$
d) $120 \mathrm{~cm}^{2}$
11.

12.


Find out the surface area of the trapezoid?
a) $60 \mathrm{~cm}^{2}$
b) $100 \mathrm{~cm}^{2}$
c) $150 \mathrm{~cm}^{2}$
d) $175 \mathrm{~cm}^{2}$

Appendix I 17
UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Achievement in Area of Triangle

## Response Sheet

ேேดั:


| Sl. No. | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| 1. |  |  |  |  |
| 2. |  |  |  |  |
| 3. |  |  |  |  |
| 4. |  |  |  |  |
| 5. |  |  |  |  |
| 6. |  |  |  |  |
| 7. |  |  |  |  |
| 9. |  |  |  |  |
| 10. |  |  |  |  |
| 11. |  |  |  |  |
| 12. |  |  |  |  |

## Appendix 118

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Achievement in Square and Square Root

Dr. K. Abdul Gafoor<br>Professor<br>Sarabi. M.K<br>థ్రొయMั: VII

## 










a) 10
b) 25
c) 50
d) றவம๗ைறைவை





 ๑๑円๐ก?






a) $110^{2}+221$
b) $12100+221$
c) $110^{2}+(110+111)$
d) $110^{2}+(111+111)$

a) $\frac{7}{8} \times \frac{7}{8}$
b) $\frac{7}{8} \times \frac{8}{7}$
c) $\frac{7}{8} \times \frac{1}{8}$
d) $1 \frac{7}{8}$
6. $\left(\frac{9}{4}\right)^{2}=$
a) $\frac{27}{8}$
b) $\frac{18}{8}$
c) $\frac{27}{16}$
d) $\frac{81}{16}$
7. $(5.5)^{2} \times 2^{2}$
a) 110
b) 121
c) 220
d) 605





9. $(0.4)^{2}=$
a) 0.16
b) 0.0016
c) 0.016
d) 16
10. 3.6 ๑ா ஃळூ $\boldsymbol{1}$






a) $6 \times 6 \times 5 \times 5$
b) $(6 \times 5) \times(6 \times 5)$
c) $(6 \times 5)^{2}$
d) $(6 \times 6) \times(5 \times 5 \times 5)$
12. $\left[7 \times 10^{30}\right]^{2}$
a) $14 \times 10^{15}$
b) $14 \times 10^{60}$
c) $49 \times 10^{15}$
d) $49 \times 10^{60}$


a) 2
b) 4
c) 5
d) 8

a) $\frac{5}{7}$
b) $\frac{5}{9}$
c) $\frac{7}{5}$
d) $\frac{9}{5}$
15. $25^{2}-23^{2}=$
a) $2(25+23)$
b) $2(25 \times 27)$
c) $2+(25+23)$
d) $2+(25+27)$

a) 2
b) 3
c) 4
d) 8

a) 11
b) 12
c) 24
d) 44


a) $5184=2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 9 \times 9$
b) $5184=2^{6} \times 9^{2}$
c) $\sqrt{5184}=2 \times 9$
d) $\sqrt{5184}=18$
19. 2500 ๑ึ్రి வனిตब్మృ๐?
a) 15
b) 50
c) 150
d) 500

a) 12 จากดธ
b) 24 จาใกฮ
c) 35 จากกฉ
d) 48 จาใกฮิ

## Appendix 119

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Achievement in Square and Square Root

Dr. K. Abdul Gafoor
Professor Research Scholar

Class: VII
Marks: 20 marks
Time: 30 mts

## Directions

Shown below are 20 questions based on your mathematics topic of 'Square and Square root '. Four answer choices are given for each question. Read each question carefully and choose the right answer. Please do not write anything on your question paper. You are given an answer sheet separately. Please put a tick mark $(\checkmark)$ in the appropriate column for answering. Your answer sheets will be kept in safe custody and used for research purposes only.

1. What is square of 5
a) 10
b) 25
c) 50
d) None of these
2. Why 36 is a perfect square?
e) because 36 is the square of square of 3
f) because 36 is a multiple of 6 and 3
g) because 36 is completely divisible by 6
h) because 36 is the square of the natural number 6
3. How do you figure out without any calculations that 810000 is complete square?
e) because 81 is completely divisible by 9
f) because it is a number ending in zero
g) because 81 is a perfect square and the number of zero's is even
h) because 81 is a multiple of 3 and the number of zero's is even
4. Which is not equivalent to $111^{2}$ ?
a) $110^{2}+221$
b) $12100+221$
c) $110^{2}+(110+111)$
d) $110^{2}+(111+111)$
5. The calculation for finding the square of $\frac{7}{8}$
a) $\frac{7}{8} \times \frac{7}{8}$
b) $\frac{7}{8} \times \frac{8}{7}$
c) $\frac{7}{8} \times \frac{1}{8}$
d) $1 \frac{7}{8}$
6. $\left(\frac{9}{4}\right)^{2}=$
a) $\frac{27}{8}$
b) $\frac{18}{8}$
c) $\frac{27}{16}$
d) $\frac{81}{16}$
7. $(5.5)^{2} \times 2^{2}$
a) 110
b) 121
c) 220
d) 605
8. Why is $\frac{3}{16}$ is not a perfect square of any fraction?
a) because 3 is not a perfect square
b) because 16 is not a perfect square
c) because 3 is not completely divisible by 16
d) because 16 is not completely divisible by 3
9. $(0.4)^{2}=$
a) 0.16
b) 0.0016
c) 0.016
d) 16
10. 3.6 is not a perfect square of any number. How do you know this without doing any calculations?
a) because the decimal points are not even
b) because 6 is the decimal number
c) because 6 is the ending number
d) because it is number beginning with 3
11. Which is not equivalent to $6^{2} \times 5^{2}$ ?
a) $6 \times 6 \times 5 \times 5$
b) $(6 \times 5) \times(6 \times 5)$
c) $(6 \times 5)^{2}$
d) $(6 \times 6) \times(5 \times 5 \times 5)$
12. $\left[7 \times 10^{30}\right]^{2}$
a) $14 \times 10^{15}$
b) $14 \times 10^{60}$
c) $49 \times 10^{15}$
d) $49 \times 10^{60}$
13. What would be number of decimals in the square of 0.0312 ?
a) 2
b) 4
c) 5
d) 8
14. The square root of $\frac{25}{49}$ ?
a) $\frac{5}{7}$
b) $\frac{5}{9}$
c) $\frac{7}{5}$
d) $\frac{9}{5}$
15. $25^{2}-23^{2}=$
a) $2(25+23)$
b) $2(25 \times 27)$
c) $2+(25+23)$
d) $2+(25+27)$
16. Which of these numbers is not a factor of 32 ?
a) 2
b) 3
c) 4
d) 8
17. The square root of 144 is?
a) 11
b) 12
c) 24
d) 44
18. The operations to find the square root of 5184 is given in order. Find out the operation that is incorrect among them.
a) $5184=2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 9 \times 9$
b) $5184=2^{6} \times 9^{2}$
c) $\sqrt{5184}=2 \times 9$
d) $\sqrt{5184}=18$
19. Square root of 2500 ?
a) 15
b) 50
c) 150
d) 500
20. What is the length of one side of a square of surface area $1225 \mathrm{~m}^{2}$ ?
a) 12 meter
b) 24 meter
c) 35 meter
d) 48 meter

## Appendix 120

UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Achievement in Square and Square Root

## Response Sheet

ேேด̆: $\qquad$


| Sl. No. | $\mathbf{a}$ | $\mathbf{b}$ | $\mathbf{c}$ | $\mathbf{d}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1. |  |  |  |  |
| 2. |  |  |  |  |
| 3. |  |  |  |  |
| 4. |  |  |  |  |
| 5. |  |  |  |  |
| 6. |  |  |  |  |
| 7. |  |  |  |  |
| 8. |  |  |  |  |
| 9. |  |  |  |  |
| 10. |  |  |  |  |
|  |  |  |  |  |


| Sl. No. | $\mathbf{a}$ | $\mathbf{b}$ | $\mathbf{c}$ | $\mathbf{d}$ |
| :---: | :---: | :---: | :---: | :---: |
| 11. |  |  |  |  |
| 12. |  |  |  |  |
| 13. |  |  |  |  |
| 14. |  |  |  |  |
| 15. |  |  |  |  |
| 16. |  |  |  |  |
| 17. |  |  |  |  |
| 18. |  |  |  |  |
| 19. |  |  |  |  |
| 20. |  |  |  |  |

## Appendix $I 21$

UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Scoring Key for Test of Achievement in Mathematics Units

| Parallel Lines |  |  |  | Unchanging Relations |  |  |  | Repeated Multiplication |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \dot{8} \\ & \dot{~} \\ & 0 \end{aligned}$ | $\begin{aligned} & \dot{0} \\ & \frac{3}{3} \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{aligned} & \dot{8} \\ & \dot{y} \\ & \tilde{y} \end{aligned}$ | $\begin{aligned} & \hline \ddot{0} \\ & \text { 3 } \\ & 0 \\ & \vdots \end{aligned}$ | $\begin{aligned} & \dot{8} \\ & \dot{y} \\ & 0 \end{aligned}$ |  |  |  | $\begin{aligned} & \dot{Z} \\ & \dot{Z} \\ & 0 \end{aligned}$ |  | $\begin{aligned} & \dot{8} \\ & \dot{Z} \\ & \dot{y} \end{aligned}$ | $\begin{aligned} & \dot{0} \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \dot{8} \\ & \dot{~} \\ & 0 \end{aligned}$ | 苋 |
| 1. | B | 9.* | C | 1. | D | 9. | A | 1. | A | 9. | B | 17. | D |
| 2. | D | 10. | C | 2. | C | 10. | C | 2. | D | 10.* | B | 18. | D |
| 3. | B | 11. | A | 3. | A | 11. | A | 3. | D | 11. | C |  |  |
| 4. | A | 12. | A | 4. | C | 12. | B | 4. | B | 12. | D |  |  |
| 5. | B | 13. | C | 5. | A | 13. | A | 5. | B | 13. | B |  |  |
| 6. | C | 14. | C | 6. | C | 14. | A | 6. | A | 14. | D |  |  |
| 7. | B | 15.* | D | 7. | A | 15. | A | 7. | C | 15. | C |  |  |
| 8. | C | 16. | B | 8. | C | 16. | A | 8. | D | 16.* | C |  |  |


| Area of Triangle |  |  |  |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \dot{Z} \\ & \dot{Z} \\ & \ddot{y} \end{aligned}$ | $\begin{aligned} & \stackrel{\circ}{0} \\ & 3 \\ & 0 \\ & \vdots \end{aligned}$ |  | $\begin{aligned} & \ddot{0} \\ & \dot{3} \\ & \vdots \\ & \hline \end{aligned}$ |
| 1. | C | 11. | B |
| 2. | B | 12. | D |
| 3. | A |  |  |
| 4. | C |  |  |
| 5. | C |  |  |
| 6. | A |  |  |
| 7. | C |  |  |
| 8. | A |  |  |
| 9. | A |  |  |
| 10. | A |  |  |


| Square and Square Root |  |  |  |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \dot{8} \\ & \dot{Z} \\ & \ddot{y y} \end{aligned}$ | $\begin{aligned} & \dot{0} \\ & 3 \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & \dot{Z} \\ & \dot{Z} \\ & \ddot{y y} \end{aligned}$ | $\begin{aligned} & \ddot{0} \\ & \frac{3}{0} \\ & 0 \\ & \hline \end{aligned}$ |
| 1. | B | 11. | D |
| 2. | D | 12. | D |
| 3. | C | 13. | D |
| 4. | D | 14. | A |
| 5. | A | 15. | A |
| 6. | D | 16. | B |
| 7. | B | 17. | B |
| 8. | A | 18. | C |
| 9. | A | 19. | B |
| 10. | A | 20. | C |

## Appendix J1

## Data and Results of Item Analysis on Scale of Self-efficacy in units of Mathematics

Table J1. Data and Results of Item Analysis on Scales of Self-Efficacy in Units of Mathematics

| Item no. (draft) | $\mathrm{M}_{1}$ | $\mathrm{M}_{2}$ | $\mathrm{SD}_{1}$ | $\mathrm{SD}_{2}$ | t | Item no. (final) | Item no. <br> (draft) | $\mathrm{M}_{1}$ | $\mathrm{M}_{2}$ | $\mathrm{SD}_{1}$ | $\mathrm{SD}_{2}$ | t | Item no. <br> (final) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Parallel Lines |  |  |  |  |  |  | Unchanging Relations |  |  |  |  |  |  |
| 1 | 4.86 | 3.08 | 0.35 | 1.31 | 9.30 | 1 | 1 | 4.82 | 4.64 | 0.39 | 0.78 | 1.47* | --- |
| 2 | 4.48 | 2.00 | 0.79 | 1.21 | 12.13 | 2 | 2 | 3.80 | 2.46 | 1.14 | 1.37 | 5.30 | 1 |
| 3 | 4.84 | 2.73 | 0.37 | 1.38 | 10.41 | 3 | 3 | 4.92 | 4.58 | 0.27 | 1.07 | 2.18* | --- |
| 4 | 4.78 | 3.22 | 0.42 | 1.40 | 7.53 | 4 | 4 | 4.82 | 3.84 | 0.44 | 1.23 | 5.29 | 2 |
| 5 | 4.51 | 1.98 | 0.54 | 1.25 | 13.09 | 5 | 5 | 4.02 | 3.24 | 0.87 | 1.10 | 3.94 | 3 |
| 6 | 4.34 | 1.96 | 0.80 | 1.24 | 11.38 | 6 | 6 | 3.62 | 2.42 | 1.10 | 1.43 | 4.70 | 4 |
| 7 | 4.24 | 3.84 | 1.29 | 1.36 | 1.51* | --- | 7 | 3.94 | 3.40 | 1.17 | 1.43 | 2.07* | --- |
| 8 | 3.98 | 3.71 | 1.60 | 1.51 | 0.85* | --- | 8 | 3.66 | 3.30 | 1.24 | 1.36 | 1.38* | --- |
| 9 | 3.84 | 3.54 | 1.08 | 1.33 | 1.24* | --- | 9 | 4.00 | 2.72 | 1.01 | 1.34 | 5.39 | 5 |
| 10 | 3.84 | 3.34 | 1.22 | 1.61 | 1.75* | --- | 10 | 4.52 | 3.72 | 0.81 | 1.18 | 3.95 | 6 |
| 11 | 4.72 | 2.49 | 0.50 | 1.42 | 10.51 | 7 | 11 | 4.36 | 3.14 | 0.69 | 1.23 | 6.11 | 7 |
| 12 | 4.98 | 4.06 | 0.14 | 1.45 | 4.47 | 8 | 12 | 4.14 | 3.10 | 0.97 | 1.37 | 4.37 | 8 |
| 13 | 4.72 | 2.19 | 0.50 | 1.08 | 15.09 | 9 | 13 | 3.70 | 3.16 | 0.93 | 1.23 | 2.47* | --- |
| 14 | 4.68 | 2.18 | 0.55 | 1.25 | 12.90 | 10 | 14 | 4.12 | 2.98 | 0.75 | 1.24 | 5.58 | 9 |
| 15 | 4.78 | 2.98 | 0.46 | 1.61 | 7.60 | 11 | 15 | 4.60 | 2.88 | 0.86 | 1.55 | 6.88 | 10 |
| 16 | 4.78 | 2.80 | 0.46 | 1.56 | 8.58 | 12 | 16 | 4.06 | 2.30 | 1.00 | 1.23 | 7.84 | 11 |
| 17 | 4.6 | 2.14 | 0.61 | 1.20 | 12.98 | 13 | 17 | 4.28 | 3.18 | 0.90 | 1.42 | 4.61 | 12 |
| 18 | 4.68 | 2.81 | 0.71 | 1.54 | 7.79 | 14 | 18 | 4.32 | 3.08 | 0.77 | 1.24 | 6.00 | 13 |
|  |  |  |  |  |  |  | 19 | 4.30 | 2.92 | 0.95 | 1.41 | 5.73 | 14 |
| Repeated Multiplication |  |  |  |  |  |  | Square and square root |  |  |  |  |  |  |
| 1 | 4.92 | 3.70 | 0.34 | 0.93 | 8.70 | 1 | 1 | 4.74 | 3.40 | 0.78 | 1.21 | 6.58 |  |
| 2 | 4.10 | 1.52 | 0.84 | 0.86 | 15.16 | 2 | 2 | 4.10 | 1.80 | 0.86 | 1.14 | 11.36 |  |
| 3 | 4.42 | 2.82 | 0.57 | 1.35 | 7.71 | 3 | 3 | 4.42 | 2.74 | 0.61 | 1.38 | 7.86 |  |
| 4 | 4.90 | 3.54 | 0.30 | 1.13 | 8.23 | 4 | 4 | 4.90 | 3.46 | 0.30 | 1.20 | 8.23 |  |
| 5 | 4.52 | 2.58 | 0.50 | 1.09 | 11.42 | 5 | 5 | 4.46 | 2.58 | 0.58 | 1.07 | 10.92 |  |
| 6 | 4.50 | 4.28 | 0.89 | 1.01 | 1.16* | --- | 6 | 4.38 | 1.98 | 0.70 | 1.12 | 12.87 |  |
| 7 | 4.36 | 2.10 | 0.72 | 1.10 | 12.15 | 6 | 7 | 4.32 | 2.68 | 0.87 | 1.15 | 8.05 |  |
| 8 | 4.98 | 4.74 | 0.14 | 0.72 | 2.30* | --- | 8 | 4.20 | 2.04 | 0.99 | 1.09 | 10.39 |  |
| 9 | 4.18 | 1.96 | 0.98 | 1.01 | 11.14 | 7 | 9 | 4.92 | 3.98 | 0.27 | 1.27 | 5.12 |  |
| 10 | 3.98 | 1.48 | 0.87 | 0.76 | 15.29 | 8 | 10 | 4.60 | 2.90 | 0.53 | 1.22 | 9.05 |  |
| 11 | 4.56 | 2.78 | 0.54 | 1.09 | 10.32 | 9 | 11 | 4.98 | 4.62 | 0.14 | 0.83 | 3.02 |  |
| 12 | 4.40 | 2.70 | 0.83 | 1.20 | 8.23 | 10 | 12 | 4.44 | 2.40 | 0.88 | 1.31 | 9.13 |  |
| 13 | 4.56 | 2.30 | 0.64 | 1.18 | 11.87 | 11 | 13 | 4.72 | 2.50 | 0.67 | 1.56 | 9.27 |  |
| 14 | 4.36 | 2.43 | 0.63 | 1.04 | 11.22 | 12 | 14 | 4.60 | 3.16 | 0.64 | 1.52 | 6.19 |  |
| 15 | 4.78 | 2.50 | 0.65 | 1.59 | 9.37 | 13 |  |  |  |  |  |  |  |
| 16 | 4.62 | 3.30 | 0.60 | 1.45 | 5.96 | 14 |  |  |  |  |  |  |  |
| Area of Triangle |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 4.94 | 4.10 | 0.24 | 1.23 | 4.73 |  | 6 | 4.48 | 3.04 | 0.81 | 1.41 | 6.24 |  |
| 2 | 4.58 | 2.82 | 0.61 | 1.30 | 8.64 |  | 7 | 4.74 | 2.80 | 0.53 | 1.43 | 9.01 |  |
| 3 | 4.98 | 4.00 | 0.14 | 1.48 | 4.65 |  | 8 | 4.60 | 2.76 | 0.76 | 1.51 | 7.72 |  |
| 4 | 4.40 | 2.56 | 0.73 | 1.11 | 9.80 |  | 9 | 4.48 | 2.52 | 0.68 | 1.33 | 9.29 |  |
| 5 | 4.48 | 2.29 | 0.71 | 1.14 | 11.59 |  | 10 | 4.52 | 2.64 | 0.74 | 1.50 | 7.98 |  |

[^6]
## Appendix J2

# UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION Scale of Self-efficacy in Parallel Lines 

 (Draft)Dr. K. Abdul Gafoor<br>Professor

Sarabi. M.K
Research Scholar
வேன̆: $\qquad$


## ตาฮิตฺ๙สหศั












|  |  | (4) $\mathrm{ml} \mathrm{\infty}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\begin{aligned} & \frac{a}{8} \\ & \frac{8}{9} \\ & \frac{9}{8} \end{aligned}$ |
| 1. |  களஸ்றைைா |  |  |  |  |  |
| 2. |  <br>  |  |  |  |  |  |
| 3. |  <br>  ๑ஜைியிகை๐ால் |  |  |  |  |  |
| 4. |  <br>  றைைை |  |  |  |  |  |


|  |  | ๑) ¢าヵ๐ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\frac{1}{2}$ $\overline{8}$ $\frac{9}{9}$ 8 |
| 5. |  <br>  |  |  |  |  |  |
| 6. |  <br>  |  |  |  |  |  |
| 7. |  カி๐ாర |  |  |  |  |  |
| 8. |  <br>  |  |  |  |  |  |
| 9. |  <br>  |  |  |  |  |  |
| 10. |  <br>  |  |  |  |  |  |
| 11. |  <br>  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 12. |  |  |  |  |  |  |
| 13. |  |  |  |  |  |  |
| 14. |  |  |  |  |  |  |
| 15. |  |  |  |  |  |  |
| 16. |  |  |  |  |  |  |
| 17. |  |  |  |  |  |  |
| 18. |  |  |  |  |  |  |

## Appendix J3

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Scale of Self-efficacy in Parallel Lines

(Draft)

Dr. K. Abdul Gafoor<br>Professor<br>Sarabi. M.K<br>Research Scholar<br>Name:<br>Boy/Girl/Others

## Directions

Various statements regarding your self confidence related to the activities in the chapter 'parallel lines' are given below. You may respond to each statement in five different ways. 1. Definitely, 2. Usually, 3. Sometimes, 4. Occasionally, and 5. Never. Please read each statement carefully and decide how much of those statements are applicable to you. Then, place a tick mark $(\checkmark)$ against each suitable statement. Your responses will be kept in safe custody and will only be used for research purpose.

|  |  | Statements |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1. | I can find other angles of a parallelogram if the value of <br> one of the angles is given |  |  |  |  |
| 2. | I can find the opposite side of the parallelogram if one side <br> is given |  |  |  |  |
| 3. | I can prove that the sum of all angles in a triangle would be <br> $180^{\circ}$, based on the principle of alternate angles |  |  |  |  |
| 4. | I can find other angles when one of the angles made by a <br> line crossing two parallel lines is given |  |  |  |  |
| 5. | I can create geometrical figures using parallel lines based <br> on given indications |  |  |  |  |
| 6. | I can explore and understand the geometrical figures con- <br> taining parallel lines |  |  |  |  |
| 7. | I can learn the specific terms related to parallel lines |  |  |  |  |


|  |  | Statements |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8. | I can make use of various terms related to parallel lines <br> wherever necessary |  |  |  |  |  |
| 9. | I can understand the signs and symbols related to parallel <br> lines |  |  |  |  |  |
| 10. | I can use the signs and symbols related to parallel lines |  |  |  |  |  |
| 11. | I can explain the mathematical concepts related to parallel |  |  |  |  |  |
| lines to peers |  |  |  |  |  |  |

## Appendix $\mathbf{J 4}$

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

 Scale of Self-efficacy in Parallel Lines(Final)

Dr. K. Abdul Gafoor

Professor

Sarabi. M.K
Research Scholar

வேฮั: $\qquad$


## 












|  |  | ๑)(m)øが |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\begin{aligned} & \frac{\mathrm{a}}{6} \\ & \frac{\mathrm{~b}}{9} \\ & \stackrel{y}{8} \end{aligned}$ |
| 1. |  கிளைாைைைฉ |  |  |  |  |  |
| 2. |  <br>  |  |  |  |  |  |
| 3. |  <br>  ๑ஜைியிேேைால |  |  |  |  |  |
| 4. |  <br>  ๓ைைก |  |  |  |  |  |



## Appendix $\mathbf{J 5}$

# UNIVERSITY OF CALICUT <br> DEPARTMENT OF EDUCATION <br> Scale of Self-efficacy in Parallel Lines 

(Final)

Dr. K. Abdul Gafoor<br>Professor

Sarabi. M.K
Research Scholar

Name: $\qquad$ Boy/Girl/Others

## Directions

Various statements regarding your self confidence related to the activities in the chapter 'parallel lines' are given below. You may respond to each statement in five different ways. 1. Definitely, 2. Usually, 3. Sometimes, 4. Occasionally, and 5. Never. Please read each statement carefully and decide how much of those statements is applicable to you. Then, place a tick mark $(\checkmark)$ against each suitable statement. Your responses will be kept in safe custody and will only be used for research purpose.

| $\dot{z}$ $\dot{z}$ $\dot{n}$ | Statements | $\begin{aligned} & \stackrel{i}{0} \\ & .0 \\ & 0.0 \\ & 0.0 \\ & 0 . \end{aligned}$ |  | \% |  | 亡 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | I can find other angles of a parallelogram if the value of one of the angles is given |  |  |  |  |  |
| 2. | I can find the opposite side of the parallelogram if one side is given |  |  |  |  |  |
| 3. | I can prove that the sum of all angles in a triangle would be $180^{\circ}$, based on the principle of alternate angles |  |  |  |  |  |
| 4. | I can find other angles when one of the angles made by a line crossing two parallel lines is given |  |  |  |  |  |
| 5. | I can create geometrical figures using parallel lines based on given indications |  |  |  |  |  |
| 6. | I can explore and understand the geometrical figures containing parallel lines |  |  |  |  |  |


| $\dot{z}$ $\dot{n}$ $\dot{n}$ | Statements | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & \stackrel{0}{0} \\ & \stackrel{y}{0} \\ & 0 \end{aligned}$ | $\begin{aligned} & \lambda \\ & \stackrel{\rightharpoonup}{\tilde{u}} \\ & \stackrel{\rightharpoonup}{0} \end{aligned}$ | 兑 |  | \% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7. | I can explain the mathematical concepts related to parallel lines to peers |  |  |  |  |  |
| When a line crosses parallel lines |  |  |  |  |  |  |
| 8. | I can recognize equal angles |  |  |  |  |  |
| 9. | I can recognize corresponding angles |  |  |  |  |  |
| 10. | I can recognize alternate angles |  |  |  |  |  |
| 11. | I can recognize the co-interior angles |  |  |  |  |  |
| 12. | I can find the sum of the co-interior angles |  |  |  |  |  |
| 13. | I can recognize the co-exterior angles |  |  |  |  |  |
| 14. | I can find the sum of co-exterior angles |  |  |  |  |  |

# Appendix $J 6$ <br> UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION 

## Scale of Self-efficacy in Unchanging Relations

(Draft)

## Dr. K. Abdul Gafoor

Professor
Sarabi. M.K
Research Scholar
■ேฉ̆: $\qquad$


## 












|  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| e e ¢ d ¢ |  |  |  |  |  | - ${ }_{8}^{8}$ |
|  <br>  <br>  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 2) |  <br>  |  |  |  |  |  |
| 3) |  <br>  |  |  |  |  |  |
| 4) |  <br>  |  |  |  |  |  |


| $\begin{aligned} & \text { e } \\ & \text { e } \\ & \varepsilon \\ & \text { d } \\ & \text { © } \end{aligned}$ | เロบับைவMめひర | ๑）（1ヵ๑） |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | － |
|  <br>  |  |  |  |  |  |  |
| 5） | $(47-93 / 4)+1 / 4=$ |  |  |  |  |  |
| 6） | $(234+8.5)-3.5=$ |  |  |  |  |  |
| 7） | $(5 \times 13)+(25 \times 13)=$ |  |  |  |  |  |
| 8） | $(121 / 2 \times 15)-\left(10 \frac{1}{2} \times 15\right)=$ |  |  |  |  |  |
|  <br>  |  |  |  |  |  |  |
| 9） |  <br>  <br>  |  |  |  |  |  |
| 10） |  <br>  <br>  |  |  |  |  |  |
|  <br>  |  |  |  |  |  |  |
| 11） | $5+7+3=5+(7+3)$ |  |  |  |  |  |
| 12） | $44+16.5+13.5=44+(16.5+13.5)$ |  |  |  |  |  |
| 13） | $(121 / 2 \times 23 / 4)-\left(6^{1 / 2} \times 23 / 4\right)=(121 / 2-61 / 2) \times 23 / 4$ |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 14） |  கலல கலளுறைைกช． |  |  |  |  |  |
| 15） |  |  |  |  |  |  |
| 16） |  |  |  |  |  |  |
| 17） |  カー๐ா |  |  |  |  |  |
| 18） |  <br>  |  |  |  |  |  |
| 19） |  <br>  |  |  |  |  |  |

## Appendix $\mathbf{J 7}$

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

# Scale of Self－efficacy in Unchanging Relations （Draft） 

Dr．K．Abdul Gafoor

Professor

Sarabi．M．K
Research Scholar

Name： $\qquad$ Boy／Girl／Others

## Directions

Various statements regarding your self confidence related to the activities in the chapter＇Unchanging Relations＇are given below．You may respond to each statement in five different ways．1．Definitely，2．Usually，3．Sometimes， 4．Occasionally，and 5．Never．Please read each statement carefully and decide how much of those statements is applicable to you．Then，place a tick mark $(\checkmark)$ against each suitable statement．Your responses will be kept in safe custody and will only be used for research purpose．

| $\dot{z}$ $\dot{\text { in }}$ $\dot{\text { in }}$ | Statements | $\begin{gathered} \stackrel{\rightharpoonup}{0} \\ \stackrel{y}{0} \\ \stackrel{y}{0} \\ 0 \end{gathered}$ | 交 | W | 気 | 㐫 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I．I can express the given numerical relationships using variables |  |  |  |  |  |  |
| 1） | Subtract two times of a number from itself |  |  |  |  |  |
| 2） | Add a number and five added to it |  |  |  |  |  |
| 3） | Subtract three times of a number from its five times |  |  |  |  |  |
| 4） | Subtract two from the sum of two consecutive natural numbers |  |  |  |  |  |
| II．I can solve the following operations mentally using general principles with ease |  |  |  |  |  |  |
| 5） | $(47-93 / 4)+1 / 4=$ |  |  |  |  |  |
| 6） | $(234+8.5)-3.5=$ |  |  |  |  |  |
| 7） | $(5 \times 13)+(25 \times 13)=$ |  |  |  |  |  |
| 8） | $(121 / 2 \times 15)-\left(10 \frac{1}{2} \times 15\right)=$ |  |  |  |  |  |


| $\dot{z}$ $\dot{z}$ $\dot{n}$ | Statements | 交 | 㐫 | 荅 | N | 岗 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| III．I can express the following operational principles using variables |  |  |  |  |  |  |
| 9） | Instead of adding two numbers separately to another number，the sum of the two numbers can be added to that number |  |  |  |  |  |
| 10） | Instead of subtracting two numbers separately from another number，the sum of the two numbers can be subtracted from that number |  |  |  |  |  |
| IV．I can express the general principle of the following operations in letters |  |  |  |  |  |  |
| 11） | $5+7+3=5+(7+3)$ |  |  |  |  |  |
| 12） | $44+16.5+13.5=44+(16.5+13.5)$ |  |  |  |  |  |
| 13） | $(121 / 2 \times 23 / 4)-\left(6^{1 / 2} \times 23 / 4\right)=\left(12^{1 / 2}-6^{1 / 2}\right) \times 23 / 4$ |  |  |  |  |  |
| V．I can do the following activities |  |  |  |  |  |  |
| 14） | I can figure out numbers if their sum and difference are given |  |  |  |  |  |
| 15） | I can solve algebraic problems |  |  |  |  |  |
| 16） | I can understand algebraic concepts |  |  |  |  |  |
| 17） | I can explain algebraic topics to other students |  |  |  |  |  |
| 18） | I can find answers to algebraic problems outside textbook |  |  |  |  |  |
| 19） | I can learn algebra faster than other topics in mathematics |  |  |  |  |  |

## Appendix $\mathbf{J 8}$

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

# Scale of Self-efficacy in Unchanging Relations 

## (Final)

Dr. K. Abdul Gafoor

Professor
Sarabi. M.K
Research Scholar

ロேฺ̆: $\qquad$


## 













|  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { e } \\ & \text { \& } \\ & \text { \& } \\ & \text { \& } \end{aligned}$ |  |  |  |  |  | $\frac{\square}{8}$ $\frac{8}{9}$ $\frac{8}{8}$ |
|  जીโ్షૅ พ్ము |  |  |  |  |  |  |
| 1) |  <br>  |  |  |  |  |  |
| 2) |  <br>  |  |  |  |  |  |


|  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ¢ |  |  |  |  |  | （1）${ }_{\text {a }}$ |
|  <br>  |  |  |  |  |  |  |
| 3） | $(47-93 / 4)+1 / 4=$ |  |  |  |  |  |
| 4） | $(234+8.5)-3.5=$ |  |  |  |  |  |
|  <br>  |  |  |  |  |  |  |
| 5） |  <br>  <br>  |  |  |  |  |  |
| 6） |  <br>  <br>  |  |  |  |  |  |
| 毋ிழる๓ைா |  |  |  |  |  |  |
| 7） | $5+7+3=5+(7+3)$ |  |  |  |  |  |
| 8） | $44+16.5+13.5=44+(16.5+13.5)$ |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  <br>  |  |  |  |  |  |  |
| 10） |  カー๐ால |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 12） |  カ๓๐ாช |  |  |  |  |  |
| 13） |  <br>  |  |  |  |  |  |
| 14） |  <br>  |  |  |  |  |  |

## Appendix $\mathbf{J 9}$

UNIVERSITY OF CALICUT
DEPARTMENT OF EDUCATION

# Scale of Self－efficacy in Unchanging Relations 

## （Final）

Dr．K．Abdul Gafoor<br>Professor<br>Sarabi．M．K<br>Research Scholar<br>Name：．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．Boy／Girl／Others

## Directions

Various statements regarding your self confidence related to the activities in the chapter＇Unchanging Relations＇are given below．You may respond to each statement in five different ways．1．Definitely，2．Usually，3．Sometimes，4．Occa－ sionally，and 5 ．Never．Please read each statement carefully and decide how much of those statements is applicable to you．Then，place a tick mark $(\checkmark)$ against each suitable statement．Your responses will be kept in safe custody and will only be used for research purpose．

| $\dot{z}$ $\dot{\sim}$ $i$ | Statements |  | $\begin{aligned} & \stackrel{\rightharpoonup}{\vec{N}} \\ & \stackrel{\rightharpoonup}{6} \\ & \stackrel{\rightharpoonup}{D} \end{aligned}$ | $\begin{aligned} & 0 \\ & \stackrel{0}{B} \\ & \dot{0} \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { 侖 } \\ & \text { 䔍 } \\ & \text { だ } \\ & 0 \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |

I．I can express the given numerical relationships using variables

| 1） | Add a number and five added to it |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 2） | Subtract two from the sum of two consecutive natural <br> numbers |  |  |  |  |

II．I can solve the following operations mentally using general principles with ease

| 3$)$ | $(47-93 / 4)+1 / 4=$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4$)$ | $(234+8.5)-3.5=$ |  |  |  |  |  |

III．I can express the following operational principles using variables
5）Instead of adding two numbers separately to another number，the sum of the two numbers can be added to that number


| $\dot{z}$ $\dot{z}$ $\dot{n}$ | Statements | 穴 | 㐫 | \% |  | $\dot{\circ}$ ¢ Z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6) | Instead of subtracting two numbers separately from another number, the sum of the two numbers can be subtracted from that number |  |  |  |  |  |
| IV. I can express the general principle of the following operations in letters |  |  |  |  |  |  |
| 7) | $5+7+3=5+(7+3)$ |  |  |  |  |  |
| 8) | $44+16.5+13.5=44+(16.5+13.5)$ |  |  |  |  |  |
| V. I can do the following activities |  |  |  |  |  |  |
| 9) | I can figure out numbers if their sum and difference are given |  |  |  |  |  |
| 10) | I can solve algebraic problems |  |  |  |  |  |
| 11) | I can understand algebraic concepts |  |  |  |  |  |
| 12) | I can explain algebraic topics to other students |  |  |  |  |  |
| 13) | I can find answers to algebraic problems outside textbook |  |  |  |  |  |
| 14) | I can learn algebra faster than other topics in mathematics |  |  |  |  |  |

## Appendix $\mathbf{J 1 0}$

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Scale of Self-efficacy in Repeated Multiplications

## (Draft)

Dr. K. Abdul Gafoor<br>Professor<br>Sarabi. M.K<br>Research Scholar

ேேดั: $\qquad$


## 













|  |  | ๑） m ¢ヵロ |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { e } \\ & \text { ¿ } \\ & \text { d } \\ & \text { § } \end{aligned}$ |  |  |  |  |
| 6. |  <br>  カைைாถ์ |  |  |  |
| 7. |  றாைை゙ |  |  |  |
| 8. | கிm <br>  |  |  |  |
| 9. |  றைาை゙ |  |  |  |
| 10. |  <br>  |  |  |  |
| 11. |  <br>  |  |  |  |
| 12. |  <br>  |  |  |  |
| 13. |  <br>  |  |  |  |
| 14. |  |  |  |  |
| 15. |  <br>  |  |  |  |
| 16. |  |  |  |  |

## Appendix J11

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Scale of Self-efficacy in Repeated Multiplication

## (Draft)

Dr. K. Abdul Gafoor

Sarabi. M.K
Professor

Name:
Boy/Girl/Others

## Directions

Various statements regarding your self confidence related to the activities in the chapter 'Repeated Multiplication' are given below. You may respond to each statement in five different ways. 1. Definitely, 2. Usually, 3. Sometimes, 4. Occasionally, and 5. Never. Please read each statement carefully and decide how much of those statements is applicable to you. Then, place a tick mark $(\checkmark)$ against each suitable statement. Your responses will be kept in safe custody and will only be used for research purpose.

| $\dot{z}$ $\dot{z}$ $\dot{s}$ | Statements | $\begin{aligned} & \stackrel{\lambda}{0} \\ & \stackrel{0}{0} \\ & \stackrel{y}{0} \\ & 0 \\ & \hline 0 \end{aligned}$ |  | \% |  | 㐫 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | I can find the power of natural numbers |  |  |  |  |  |
| 2. | I can find the power of decimal numbers |  |  |  |  |  |
| 3. | I can find the power of fractions |  |  |  |  |  |
| 4. | I can rewrite natural numbers as the powers of ten based on their place values |  |  |  |  |  |
| 5. | I can rewrite decimal numbers as the powers of ten based on their place values |  |  |  |  |  |
| 6. | I can find answers for questions given in the textbook on the power of a natural number |  |  |  |  |  |
| 7. | I can raise a power to a power of any natural number |  |  |  |  |  |
| 8. | I can find answers for questions given in the textbook on rasing a power to a power of fractional numbers |  |  |  |  |  |


|  |  | Statements |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| 9. | I can raise a power to a power of any fractional number |  |  |  |  |  |
| 10. | I can find the product of two different powers of the same <br> natural number |  |  |  |  |  |
| 11. | I can find the quotient of two different powers of the same <br> natural number |  |  |  |  |  |
| 12. | I can find the quotient of two different powers of the same <br> fraction |  |  |  |  |  |
| 13. | I can rewrite numbers like one hundred, ten thousand, one <br> lakh, ten lakh, one crore as powers of ten |  |  |  |  |  |
| 14. | I can explain the special properties of perfect numbers |  |  |  |  |  |
| 15. | I can solve problems involving more than one power rules |  |  |  |  |  |
| 16. | I can solve any problems related to powers |  |  |  |  |  |

## Appendix $\mathbf{J 1 2}$

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Scale of Self-Efficacy in Repeated Multiplication

 (Final)Dr. K. Abdul Gafoor

Professor
Sarabi. M.K
Research Scholar
ேேฺั: $\qquad$


## 












|  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| e ¢ ¢ ब ¢ |  |  |  |  |  | $\frac{1}{8}$ $\frac{8}{9}$ 8 |
| 1. |  |  |  |  |  |  |
| 2. |  |  |  |  |  |  |
| 3. |  |  |  |  |  |  |
| 4. |  <br>  |  |  |  |  |  |
| 5. |  <br>  |  |  |  |  |  |


|  |  | 凹） ¢าぁன |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & e \\ & \underset{\varepsilon}{\varepsilon} \\ & \varepsilon \\ & \text { A } \\ & \dot{E} \end{aligned}$ |  |  |  |  |  | － |
| 6. |  ゅ๐๓ைறாைை゙ |  |  |  |  |  |
| 7. |  றாை゙ฺ |  |  |  |  |  |
| 8. |  <br>  |  |  |  |  |  |
| 9. |  <br>  |  |  |  |  |  |
| 10. |  <br>  |  |  |  |  |  |
| 11. |  <br>  |  |  |  |  |  |
| 12. |  |  |  |  |  |  |
| 13. |  <br>  |  |  |  |  |  |
| 14. |  カ๐๐๐ |  |  |  |  |  |

## Appendix $\mathbf{J 1 3}$

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

# Scale of Self-efficacy in Repeated Multiplication (Final) 

Dr. K. Abdul Gafoor

Sarabi. M.K
Professor

Name: $\qquad$ Boy/Girl/Others

## Directions

Various statements regarding your self confidence related to the activities in the chapter 'Repeated Multiplication' are given below. You may respond to each statement in five different ways. 1. Definitely, 2. Usually, 3. Sometimes, 4. Occasionally, and 5. Never. Please read each statement carefully and decide how much of those statements is applicable to you. Then, place a tick mark $(\checkmark)$ against each suitable statement. Your responses will be kept in safe custody and will only be used for research purpose.

|  |  | Statements |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| B. |  |  |  |  |  |
| 1. | I can find the power of natural numbers |  |  |  |  |
| 2. | I can find the power of decimal numbers |  |  |  |  |
| 3. | I can find the power of fractions |  |  |  |  |
| 4. | I can rewrite natural numbers as the powers of ten based <br> on their place values |  |  |  |  |
| 5. | I can rewrite decimal numbers as the powers of ten based <br> on their place values |  |  |  |  |
| 6. | I can raise a power to a power of any natural number |  |  |  |  |
| 7. | I can raise a power to a power of any fractional number |  |  |  |  |
| 8. | I can find the product of two different powers of the same <br> natural number |  |  |  |  |
| 9. | I can find the quotient of two different powers of the same <br> natural number |  |  |  |  |


| $\dot{\dot{z}}$ $\dot{z}$ $i$ | Statements | $\begin{aligned} & \stackrel{\lambda}{0} \\ & \stackrel{\rightharpoonup}{0} \\ & \stackrel{\rightharpoonup}{0} \\ & 0 . \end{aligned}$ | 交 | 気 |  | 边 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10. | I can find the quotient of two different powers of the same fraction |  |  |  |  |  |
| 11. | I can rewrite numbers like one hundred，ten thousand，one lakh，ten lakh，one crore as powers of ten |  |  |  |  |  |
| 12. | I can explain the special properties of perfect numbers |  |  |  |  |  |
| 13. | I can solve problems involving more than one power rules |  |  |  |  |  |
| 14. | I can solve any problems related to powers |  |  |  |  |  |

## Appendix $\mathbf{J 1 4}$

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Scale of Self-efficacy in Area of Triangle

Dr. K. Abdul Gafoor<br>Professor<br>Sarabi. M.K<br>Research Scholar

ேேดั: $\qquad$


## 












|  |  | ๑)(1ヵヵ) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\begin{aligned} & \frac{\mathrm{a}}{\mathrm{~b}} \\ & \frac{\mathrm{\theta}}{9} \\ & \stackrel{1}{2} \end{aligned}$ |
| 1. |  <br>  |  |  |  |  |  |
| 2. |  |  |  |  |  |  |
| 3. |  <br>  |  |  |  |  |  |
| 4. |  <br>  <br>  |  |  |  |  |  |


|  |  | ฯ)(1ヵロั |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { é } \\ & \text { ¿ } \\ & \text { \& } \\ & \text { \& } \end{aligned}$ |  |  |  |  |  | $\square$ <br> $\frac{8}{8}$ <br> $\frac{9}{8}$ |
| 5. |  <br>  <br>  |  |  |  |  |  |
| 6. |  |  |  |  |  |  |
| 7. |  <br>  |  |  |  |  |  |
| 8. |  <br>  |  |  |  |  |  |
| 9. |  <br>  <br>  |  |  |  |  |  |
| 10. |  <br>  |  |  |  |  |  |

## Appendix $\mathbf{J 1 5}$

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Scale of Self－efficacy in Area of Triangle

Dr．K．Abdul Gafoor
Professor

Sarabi．M．K
Research Scholar
Name： $\qquad$ Boy／Girl／Others

## Directions

Various statements regarding your self confidence related to the activities in the chapter＇Area of Triangle＇are given below．You may respond to each statement in five different ways．1．Definitely，2．Usually，3．Sometimes，4．Occasionally，5．Never． Please read each statement carefully and decide how much of those statements are applicable to you．Then，place a tick mark $(\checkmark)$ against each suitable statement．Your responses will be kept in safe custody and will only be used for research purpose．

| $\begin{aligned} & \dot{\Delta} \\ & \dot{\sim} \\ & \dot{n} \end{aligned}$ | Statements | 穴 | 交 | en d b 0 0 0 0 |  | 㐫 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | I can calculate the surface area of a rectangle if its length and width are given |  |  |  |  |  |
| 2. | I can explain the specific characters of a right triangle |  |  |  |  |  |
| 3. | I can calculate the surface area of a right triangle if meas－ ures of its perpendicular sides are given |  |  |  |  |  |
| 4. | I can calculate the length of one of the perpendicular sides of a right triangle if the length of the side perpendicular to it and the surface area are given |  |  |  |  |  |
| 5. | I can calculate the surface area of a triangle if its length of one side and the length of a perpendicular from its opposite corner are given |  |  |  |  |  |
| 6. | I can find the surface area of any triangle |  |  |  |  |  |
| 7. | I can find the surface area of a figure developed by joining right angled triangles |  |  |  |  |  |
| 8. | I can find the surface area of a figure of a triangle com－ bined with a right triangle |  |  |  |  |  |
| 9. | I can find the surface area of any figures given in the textbook under the section of the surface area of a triangle |  |  |  |  |  |
| 10. | I can find the surface area of any figures comprising trian－ gles and rectangles |  |  |  |  |  |

## Appendix $\mathbf{J 1 6}$

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Scale of Self－efficacy in Square and Square Root

Dr．K．Abdul Gafoor Professor


## 






毋டைைைடை




|  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
| 1. |  |  |  |  |
| 2. |  |  |  |  |
| 3. |  |  |  |  |
| 4. |  |  |  |  |
| 5. |  カー๐ก |  |  |  |
| 6. |  ゅ๐ஸைான |  |  |  |
| 7. |  |  |  |  |


|  |  | 凹(m) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { e } \\ & \text { e } \\ & \text { 臬 } \end{aligned}$ |  |  |  |  | [36 |
| 8. |  <br>  |  |  |  |  |
| 9. |  <br>  |  |  |  |  |
| 10. |  |  |  |  |  |
| 11. |  |  |  |  |  |
| 12. |  <br>  |  |  |  |  |
| 13. |  களஸ்றைைกฉ |  |  |  |  |
| 14. |  <br>  |  |  |  |  |

## Appendix $\mathbf{J 1 7}$

## UNIVERSITY OF CALICUT <br> DEPARTMENT OF EDUCATION <br> Scale of Self-efficacy in Square and Square Root

Dr. K. Abdul Gafoor
Professor
Name:

## Directions

Various statements regarding your self confidence related to the activities in the chapter 'Square and Square root' are given below. You may respond to each statement in five different ways. 1. Definitely, 2. Usually, 3. Sometimes, 4. Occasionally, and 5 . Never. Please read each statement carefully and decide how much of those statements are applicable to you. Then, place a tick mark $(\checkmark)$ against each suitable statement. Your responses will be kept in safe custody and will only be used for research purpose.

| $\begin{aligned} & \dot{\dot{n}} \\ & \dot{\omega} \\ & \dot{n} \end{aligned}$ | Statements | $\left\|\begin{array}{l} \stackrel{\rightharpoonup}{0} \\ \stackrel{\rightharpoonup}{0} \\ \stackrel{y}{0} \\ 0.0 \end{array}\right\|$ | $\begin{aligned} & \stackrel{\lambda}{\bar{u}} \\ & \overrightarrow{\tilde{\tilde{u}}} \\ & \vec{\sigma} \end{aligned}$ | \% |  | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | I can find powers of natural numbers up to 99 |  |  |  |  |  |
| 2. | I can find powers of decimal numbers up to two digits |  |  |  |  |  |
| 3. | I can find square of fractional numbers |  |  |  |  |  |
| 4. | I can find the square of any number |  |  |  |  |  |
| 5. | I can find the square root of two-digit perfect squares |  |  |  |  |  |
| 6. | I can find the square root of fraction that is a perfect square |  |  |  |  |  |
| 7. | I can find the square root of any perfect square |  |  |  |  |  |
| 8. | I can figure out if numbers like $10,100,1000,10000$ are perfect squares or not |  |  |  |  |  |
| 9. | I can figure out if numbers like $411,61,101,1001$ are perfect squares or not |  |  |  |  |  |
| 10. | I can rewrite a decimal as a fraction |  |  |  |  |  |
| 11. | I can find the factors of composite numbers |  |  |  |  |  |
| 12. | I can utilize the specific properties of perfect squares in solving problems |  |  |  |  |  |
| 13. | I can find the length of one side of a square if its surface area is given |  |  |  |  |  |
| 14. | I can estimate the number of decimal places in the square of decimal numbers |  |  |  |  |  |

## Appendix K1

Data and Results of Item Analysis of Test of Verbal Comprehension in Malayalam
Table K1-1. Data and Results of Item Analysis of Test of Verbal Comprehension in Malayalam

| Item No. <br> (Draft) | L | H | DP | DI | Item No. <br> (Final) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 27 | 77 | 0.5 | 0.52 | 1 |
| 2 | 16 | 53 | 0.37 | 0.35 | 2 |
| 3 | 27 | 94 | 0.67 | 0.61 | 3 |
| 4 | 23 | 94 | 0.71 | 0.59 | 4 |
| 5 | 21 | 72 | 0.51 | 0.47 | 5 |
| 6 | 28 | 56 | 0.28* | 0.42 | --- |
| 7 | 23 | 55 | 0.32 | 0.39 | 6 |
| 8 | 19 | 60 | 0.41 | 0.40 | 7 |
| 9 | 14 | 26 | 0.12* | 0.20* | --- |
| 10 | 17 | 87 | 0.7 | 0.52 | 8 |
| 11 | 18 | 21 | 0.03* | 0.20* | --- |
| 12 | 14 | 71 | 0.57 | 0.43 | 9 |
| 13 | 18 | 46 | 0.28* | 0.32 | --- |
| 14 | 30 | 64 | 0.34 | 0.47 | 10 |
| 15 | 17 | 80 | 0.63 | 0.49 | 11 |
| 16 | 16 | 85 | 0.69 | 0.51 | 12 |
| 17 | 29 | 60 | 0.31 | 0.45 | 13 |
| 18 | 21 | 94 | 0.73 | 0.58 | 14 |
| 19 | 29 | 94 | 0.65 | 0.62 | 15 |
| 20 | 27 | 91 | 0.64 | 0.59 | 16 |
| 21 | 15 | 30 | 0.15* | 0.23* | --- |
| 22 | 20 | 96 | 0.76 | 0.58 | 17 |
| 23 | 26 | 95 | 0.69 | 0.61 | 18 |
| 24 | 22 | 94 | 0.72 | 0.58 | 19 |
| 25 | 18 | 56 | 0.38 | 0.37 | 20 |
| 26 | 25 | 41 | 0.16* | 0.33* | --- |
| 27 | 21 | 40 | 0.19* | 0.31* | --- |
| 28 | 18 | 31 | 0.13* | 0.25* | --- |
| 29 | 19 | 21 | 0.02* | 0.20* | --- |
| 30 | 19 | 37 | 0.18* | 0.28* | --- |
| 31 | 18 | 59 | 0.41 | 0.39 | 21 |
| 32 | 27 | 71 | 0.44 | 0.49 | 22 |
| 33 | 21 | 51 | 0.3 | 0.36 | 23 |
| 34 | 13 | 72 | 0.59 | 0.43 | 24 |

[^7]
## Appendix K2

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

# Test of Verbal Comprehension in Malayalam （Draft） 

| Dr．K．Abdul Gafoor | Sarabi．M．K |
| :---: | :---: |
| Professor | Research Scholar |
|  |  |
|  | พ๑ய๐： 45 ๑lmృร̆ |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  <br>  |  |






உகコロのカை๐：

வமிใஜ8 ：．．．．．．．．．．．．．．．．．．．．．．．．．．．
a）கேலைดை
b）ตை๐๓ง๐า』กช
c）ேேナேาறั゙
d）เロดา



1．ஹऽద̆ ：வอன̆
கூூைைை $\qquad$
a）毋ைதூ毋ெ
b）สேธรிఱிธ8
c） ）
d）வி巳லாைை

 $\qquad$
a）พ๐வியుఱகா்
b）$m s ๙$
c）டேோயோன
d）றาชฺロงตைกั
3．ปృบั ：๑வ๐รั
هـรา： $\qquad$
a）الدإه
b）விணூ
c）ปృบั
d）கฺั̆


 க（னிறைை வேறை．
a）๙ைறృఱఱノஜృロナఱ
b）கృறைロナツ
c）（レாேைாவைツ

 ఎऽ๘ъэஸ゙．
a）ஷூணைிอృం
b）றธவிதృலஓ
c）ஃmைைธவிรృญ®




a）氏ூலவுகேナロ
b）๑๐வ

d）に

a）
b） b
c） $\mathrm{O}^{2} \mathrm{D} \circ$
d）๑๓๐


a）ேேைாั
b）ஃ๓ั

d）வைกிஜ கกソั円๐


a）வவல் றிก๐
b）ఐணை றาก๐
c） d $_{2}$ mी॰
d）พుூவశண ๓าก๐


















 ळி円ிळぁை

 कึృ．










 กั้ชைாஷஸஸ゙．
 พロナாロコஸ゙．
 ఱl링．





















巳கிカூం
 อ๐ஸ゙．



 อ๐ஸ゙．






 อહીશ્રヴ．
















 （ロகைणロ๐．．．．








 $\mathrm{O}_{3} \mathrm{~m}_{3}$.













வணைைாேேைைை வュ्य0-

 வగె வனைைロாேைைை வவேஸை๐.


 மிஜி வைமலைை














## வறwles I






















a） றвிகくன

c）๑๐ைைை mв


## வmules II


 セேைைフ




 வா 凸ன゙？






## வறwles III




 ゅ๐லிளைை＂＇．

a）๘ைவாゥం
b）கைロロறロ
c）$ぃ \varnothing \varnothing \circ$
d）هழゅ๐๐ம๐






## வmuls IV













a）レ』ßフா๐
b） $2 \boldsymbol{1}$
c）๑வவகృாோ๐


a）ตlゅo வ®๐๐ ாிறை๓ைஸ゙


d）வுணை

## Appendix K3

UNIVERSITY OF CALICUT
DEPARTMENT OF EDUCATION

## Scoring Key for Test of Verbal Comprehension in Malayalam <br> (DRAFT)

| Item No. | Answer |
| :---: | :---: |
| 1. | $\mathbf{D}$ |
| 2. | $\mathbf{C}$ |
| 3. | $\mathbf{B}$ |
| 4. | $\mathbf{B}$ |
| 5. | $\mathbf{D}$ |
| 6. | $\mathbf{C}$ |
| 7. | $\mathbf{D}$ |
| 8. | $\mathbf{A}$ |
| 9. | $\mathbf{C}$ |
| 10. | $\mathbf{A}$ |
| 11. | $\mathbf{B}$ |
| 12. | $\mathbf{D}$ |
|  |  |


| Item No. | Answer | Item No. | Answer |
| :---: | :---: | :---: | :---: |
| 13. | D | 25. | B |
| 14. | A | 26. | C |
| 15. | D | 27. | C |
| 16. | D | 28. | B |
| 17. | A | 29. | C |
| 18. | D | 30. | A |
| 19. | C | 31. | B |
| 20. | B | 32. | A |
| 21. | B | 33. | C |
| 22. | C | 34. | D |
| 23. | A |  |  |
| 24. | C |  |  |

## Appendix K4

## UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

# Test of Verbal Comprehension in Malayalam 

（Final）






உ®๐』๐ற๐：

வமிใ円 ： $\qquad$
a）கேைைை
b）๓ை๐๓งผาเฉฉ
c）เேวอย1กั
d）（ఎாை



1．ஜऽळૅ ：வமळ̆
கூ円ைை $\qquad$
a）றைதூゥக
b）๙冂ை
c）ாక్మவிஐ
d）விอஞฺ๓
」リゴ（ロ）： $\qquad$

b）$m s ๙$
c）டேவியாால

3．ปృบั ：هอ๐รั
๑นรา： $\qquad$
a）
b）விணூ
c）Цைவั
d）காப̆


 カ（னிறைை வேறை．
a）๔ைறృఱఱノஜృロナఱ
b）கృறைロナツ


 ๑ร๘๐ฺั．
a）ாூணைிறృ๐
b）றธவிதృఠஒ



a）
b）ه๐ழั
c）णృూం
d）Ф円๐


a）ேேைாั
b）๑๓ั̆

d）வைกிळ கกソัต๐










 ஜி円ிகாூ

 దヵృ。










 ஸ゙๓ைฺல்ஸ゙．
 พ๑๐๐๓๐ஸ゙．
 ఱle．





















d）丹） －®ிカா๐
 อ๐ஸ゙．



 อ๐ஸ゙．












 $\qquad$







 $\qquad$








## வmulas I











## வறwlas II




 ゅ๐லிளைை＂．

a）๙ைவయゥ๐
b）கைロハவா
c）$ぃ ๑ \infty \circ$
d）هழø๐๐ம。






## வறwl\＆III













a）レßアロー
b） $2 \underset{2}{ }$
c）๑வவகృறோ๐


a）ตløo வఠ๑๐ றிறァ๓ைஸ゙




## Appendix K5

UNIVERSITY OF CALICUT
DEPARTMENT OF EDUCATION

## Scoring Key for Test of Verbal Comprehension in Malayalam

(Final)

| Item No. | Answer |
| :---: | :---: |
| 1. | $\mathbf{D}$ |
| 2. | $\mathbf{C}$ |
| 3. | $\mathbf{B}$ |
| 4. | $\mathbf{B}$ |
| 5. | $\mathbf{D}$ |
| 6. | $\mathbf{D}$ |
| 7. | $\mathbf{A}$ |
| 8. | $\mathbf{A}$ |
| 9. | $\mathbf{D}$ |
| 10. | $\mathbf{A}$ |
| 11. | $\mathbf{D}$ |
| 12. | $\mathbf{D}$ |


| Item No. | Answer |
| :---: | :---: |
| 13. | $\mathbf{A}$ |
| 14. | D |
| 15. | $\mathbf{C}$ |
| 16. | $\mathbf{B}$ |
| 17. | $\mathbf{C}$ |
| 18. | $\mathbf{A}$ |
| 19. | $\mathbf{C}$ |
| 20. | $\mathbf{B}$ |
| 21. | $\mathbf{B}$ |
| 22. | A |
| 23. | $\mathbf{C}$ |
| 24. | D |

## Appendix K6

UNIVERSITY OF CALICUT DEPARTMENT OF EDUCATION

## Test of Verbal Comprehension in Malayalam

(Final)

## Response Sheet

ேேฮั:


| Sl. No. | a | b | $\mathbf{c}$ | $\mathbf{d}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1. |  |  |  |  |
| 2. |  |  |  |  |
| 3. |  |  |  |  |
| 4. |  |  |  |  |
| 5. |  |  |  |  |
| 6. |  |  |  |  |
| 7. |  |  |  |  |
| 8. |  |  |  |  |
| 9. |  |  |  |  |
| 10. |  |  |  |  |
| 11. |  |  |  |  |
| 12. |  |  |  |  |


| SI. No. | a | b | c | d |
| :---: | :---: | :---: | :---: | :---: |
| 13. |  |  |  |  |
| 14. |  |  |  |  |
| 15. |  |  |  |  |
| 16. |  |  |  |  |
| 17. |  |  |  |  |
| 18. |  |  |  |  |
| 19. |  |  |  |  |
| 20. |  |  |  |  |
| 21. |  |  |  |  |
| 22. |  |  |  |  |
| 23. |  |  |  |  |
| 24. |  |  |  |  |

Appendix L
Model Workbook for
Language Integrated Mathematics Instruction


## Workbook



## 










 -



## 






| （ロை๐（ถ）ใดใக๐コ๐ ！！！（Labelling Vocabulary） |  |  |
| :---: | :---: | :---: |
|  | O2， | Noவை |
|  |  |  |
|  | ลـココ， 6 OBCO | ๑ெコ（6）【コ以வ |
|  |  |  |




(@)


| 1 |  |
| :--- | :--- |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |



| SI．NO． | வコロめைめくర | வコدめ |
| :---: | :---: | :---: |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |



| SI．No． | に－イ゙mை | $\checkmark / x$ |
| :---: | :---: | :---: |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |

Appendix L-8
 reasoning)

| SI. No. |  |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

Word Walls



Prepared by:
(C) Sarabi M K \& Dr. K Abdul Gafoor

Department of Education, University of Calicut, Kerala
Email: sarapadne@gmail.com

## Appendix M

## Model Workbook for Practice in Solving Mathematics Problems



## Workbook

## 








 คிอவケி毋๐ว๐．




ぶロペーmめび．．．

## 








 களஸொூூக

|  |  | ป（2） <br> （ウา $\operatorname{cas})$ |  | वூూ | ปになっo／ <br> （ウา边うか。 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | （ウ）¢ேフ円 | 7 | $\square$ |  |
| 2 | 7 |  | 8 |  |  |
| 3 |  |  | 9 |  |  |
| 4 |  |  | 10 |  |  |
| 5 |  |  | 11 |  |  |
| 6 | $>$ |  | 12 |  |  |

 $\qquad$ ธேロமากาカஃ๐

அளை


 றேフめாコロ．









$\square$
 ØSூக

|  |  | ธ๐ி/ ๑ாட̆ | (A)0 mmi |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 |  | 4 | $\square$ |  |
| 2 |  |  | 5 |  |  |
| 3 | $X$ |  | 6 |  |  |






| (めி) мறைல | $\mathrm{O}_{\text {¢ }}$ |  |
| :---: | :---: | :---: |
| 1 |  | 1) $\angle \mathrm{ABC}=$ <br> 2) $\qquad$ <br> 3) $\qquad$ |
| 2 |  | 1) $\angle \mathrm{ABC}=$ <br> 2) $\ldots \ldots . .=$ <br> 3) $\ldots \ldots . .=$ <br> 4) $\ldots . . . .=$ |
| 3 |  | 1) $\qquad$ <br> 2) $\qquad$ |
| 4 |  | 1) <br> 2) |
| 5 |  | 1) $\qquad$ <br> 2) $\qquad$ <br> 3) $\qquad$ |





| (C)D мறைன | Oூூ $\ddagger 6308$ |  |
| :---: | :---: | :---: |
| 1 | $\underbrace{A}_{D}$ |  |
| 2 | CR |  |
| 3 |  |  |





|  |  $\angle \mathrm{AOD}=\angle \mathrm{COB}$ $\angle \mathrm{AOC}=\angle \mathrm{BOD}$ <br>  <br>  $\begin{aligned} & \angle \mathrm{AOC}+\angle \mathrm{BOC}=180^{\circ} \\ & \angle \mathrm{DOA}+\angle \mathrm{DOB}=180^{\circ} \end{aligned}$ |
| :---: | :---: |



| (A). m. |  | ¢．Jobjo |  | อ⿴囗冂力八夊力。 |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  |  <br> คी（ேைைาை $\angle \mathrm{ACE}=110^{\circ}$ <br>  | a） $55^{\circ}$ <br> b） $70^{\circ}$ <br> c） $90^{\circ}$ <br> d） $180^{\circ}$ |  |
| 2 |  |  <br> คी（ேைைைาか＜ABE，＜CBE <br>  <br>  <br>  | a）$\angle \mathrm{ABE}=60^{\circ}, \angle \mathrm{CBE}=120^{\circ}$ <br> b）$\angle \mathrm{ABE}=45^{\circ}, \angle \mathrm{CBE}=90^{\circ}$ <br> c）$\angle \mathrm{ABE}=120^{\circ}$ ，$\angle \mathrm{CBE}=60^{\circ}$ <br> d）$\angle \mathrm{ABE}=90^{\circ}, \angle \mathrm{CBE}=45^{\circ}$ |  |
| 3 |  | mmา®าヵாym <br>  <br> ๔ேறேேาை＜COB＝．．． | a） $30^{\circ}$ <br> b） $45^{\circ}$ <br> c） $\mathbf{6 0}^{\circ}$ <br> d） $120^{\circ}$ |  |
| 4 |  | mைา®าொ3m คி（ேைைาை＜AOD © Ø๐కியうఱ゙＜COA． <br>  <br>  <br>  | a）$\angle \mathrm{AOD}=120^{\circ}, \angle \mathrm{DOB}=60^{\circ}$ ， $\angle \mathrm{BOC}=120^{\circ}$ ，$\angle \mathrm{COA}=60^{\circ}$ <br> b）$\angle \mathrm{AOD}=120^{\circ}, \angle \mathrm{DOB}=60^{\circ}$ ， $\angle \mathrm{BOC}=60^{\circ}, \angle \mathrm{COA}=120^{\circ}$ <br> c）$\angle \mathrm{AOD}=60^{\circ}, \angle \mathrm{DOB}=120^{\circ}$ ， $\angle \mathrm{BOC}=120^{\circ}, \angle \mathrm{COA}=60^{\circ}$ <br> d）$\angle A O D=60^{\circ}$ ，$\angle D O B=120^{\circ}$ ， $\angle \mathrm{BOC}=60^{\circ}, \angle \mathrm{COA}=120^{\circ}$ |  |
| 5 |  |  <br>  <br>  <br>  |  <br>  <br>  |  |




Prepared by:
(C) Sarabi M K \& Dr. K Abdul Gafoor

Department of Education, University of Calicut, Kerala
Email: sarapadne@gmail.com

## Appendix $\mathbf{N}$

Distribution of Verbal Comprehension in Malayalam
Table N1. Statistical Constants of the Distribution of Scores of Verbal Comprehension in Malayalam of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\text {a }}$ | Kurtosis $^{\text {b }}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 53.33 | 54.17 | 54 | 19.69 | 0.24 | -0.89 |
| Experimental | 52.69 | 54.17 | 50 | 20.65 | 0.14 | -0.81 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}} \mathrm{SE}$ of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, mean, median and mode, are almost equal for verbal comprehension in Malayalam. Ratios of skewness and kurtosis to their respective standard errors (skewness/SE $=$ 0.69 ; kurtosis/ $\mathrm{SE}=1.29$ ) are less than 1.96; In the experimental group, mean, median and mode, are almost equal for verbal comprehension in Malayalam. Ratios of skewness and kurtosis to their respective standard errors (skewness/SE $=0.40$; kurtosis $/ \mathrm{SE}=1.17$ ) are less than 1.96.

Table N2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of scores of Verbal Comprehension in Malayalam of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  | Levene's test of homogeneity |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.96 | 45 | $\mathrm{p}>.05 ;$ Normal | 0.13 | 1 | 88 | $\mathrm{p}>.05 ;$ |
| Experimental | 0.97 | 45 | $\mathrm{p}>.05 ;$ Normal |  |  |  | Equal variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of verbal comprehension in Malayalam before intervention of the Control and Experimental groups


## Appendix 0

Distribution of Non-verbal Intelligence
Table O1. Statistical Constants of the Distribution of Scores of Non-Verbal Intelligence of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\text {a }}$ | Kurtosis $^{\text {b }}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control Group | 36.64 | 37 | 34 | 7.60 | -0.18 | -0.74 |
| Experimental Group | 35.82 | 37 | 38 | 7.56 | -0.36 | -0.39 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, mean, median and mode are almost equal for non-verbal intelligence. Ratios of skewness and kurtosis to their respective standard errors (skewness/ $\mathrm{SE}=0.51$; kurtosis/ $\mathrm{SE}=1.07$ ) are less than 1.96 . In the experimental group, mean, median and mode are almost equal for non-Verbal Intelligence. Ratios of skewness and kurtosis to their respective standard errors (skewness/ $\mathrm{SE}=1.03$; kurtosis/ $\mathrm{SE}=0.57$ ) are less than 1.96 . Hence, before intervention, non-verbal intelligence is symmetric and nearly mesokurtic indicating normality of distribution in the experimental group.

Table O2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of scores of Non-Verbal Intelligence of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  | Levene's test of homogeneity |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.97 | 45 | $\mathrm{p}>.05$; Normal |  |  |  |  |
| Experimental | 0.97 | 45 | $\mathrm{p}>.05$; Normal | 0.00 | 1 | 88 | $\mathrm{p}>.05$; Equal |
| variances |  |  |  |  |  |  |  |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of non-verbal intelligence before intervention of the Control and Experimental groups.


## Appendix $P$

## Distribution of Previous Achievement in Mathematics

Table P1. Statistical Constants of the Distribution of Scores of Previous Achievement in Mathematics of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\text {a }}$ | Kurtosis $^{\text {b }}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 38.11 | 35 | 35 | 19.92 | 0.93 | 0.21 |
| Experimental | 40.33 | 35 | 45 | 18.10 | 0.57 | -0.57 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, mean is higher than median and mode, suggesting positively skewed distribution for previous achievement in mathematics. Also, Ratios of skewness to its standard errors (skewness/SE $=2.66$ ) is greater than 1.96. However, ratio of kurtosis to its standard error (kurtosis/ $\mathrm{SE}=0.30$ ) is less than 1.96. In the experimental group, median is less than mean and mode, for previous achievement in mathematics. However, ratios of skewness and kurtosis to their respective standard errors (skewness $/ \mathrm{SE}=1.63$; kurtosis $/ \mathrm{SE}=0.83$ ) are less than 1.96.

Table P2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of scores of Previous Achievement in Mathematics of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  |  | Levene's test of homogeneity |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.91 | 45 | $\mathrm{p}>.05 ;$ Not normal | 0.11 | 1 | 88 | $\mathrm{p}>.05 ;$ |
| Experimental | 0.94 | 45 | $\mathrm{p}<.01 ;$ Not normal |  |  |  | Equal variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of previous achievement in mathematics before intervention of the Control and Experimental groups


## Appendix Q

## Distribution of Achievement in Mathematics after Intervention

Table Q1. Statistical Constants of the Distribution of Achievement in Mathematics after intervention of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\text {a }}$ | Kurtosis $^{\text {b }}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 46.18 | 44.87 | 41 | 14.89 | 0.22 | -0.65 |
| Experimental | 56.18 | 58.97 | 60 | 13.42 | -0.05 | -0.83 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35,{ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, achievement in mathematics after intervention shows descending tendency from mean, median and mode which indicates positively skewed distribution. Also, ratios of skewness and kurtosis to their respective standard errors (skewness/SE $=0.63$; kurtosis/SE $=$ 0.94 ) is less than 1.96; In experimental group, achievement in mathematics after intervention shows ascending tendency from mean, median and mode which indicates negatively skewed distribution. Also, ratios of skewness and kurtosis to their respective standard errors (skewness $/ \mathrm{SE}=0.14$; kurtosis $/ \mathrm{SE}=1.20$ ) is less than 1.96.

Table Q2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Achievement in Mathematics after intervention of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  | Levene's test of homogeneity |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.98 | 45 | $\mathrm{p}>.05 ;$ Normal | 0.20 | 1 | 88 | $\mathrm{p}>.05 ;$ <br> Equal variances |
| Experimental | 0.98 | 45 | $\mathrm{p}>.05 ;$ Normal |  |  |  |  |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of achievement in mathematics after intervention of the Control and Experimental groups


## Appendix $R$

 Distribution of Achievement in Algebra after InterventionTable R1. Statistical Constants of the Distribution of Achievement in Algebra after intervention of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\mathrm{a}}$ | Kurtosis $^{\mathrm{b}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 40.42 | 43.75 | 31 | 13.886 | 0.00 | -0.38 |
| Experimental | 50.56 | 50.00 | 50 | 12.344 | -0.31 | 0.62 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, for achievement in algebra after intervention median is the highest, mode is least and mean in between them. However, ratios of skewness and kurtosis to their respective standard errors (skewness/SE $=0$; kurtosis/ $\mathrm{SE}=0.55$ ) is less than 1.96; In experimental group, mean, median and mode are almost equal for achievement in algebra after intervention. Also, ratios of skewness and kurtosis to their respective standard errors (skewness/ $\mathrm{SE}=0.89$; kurtosis/ $\mathrm{SE}=0.89$ ) is less than 1.96.
Table R2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Achievement in Algebra after intervention of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  | Levene's test of homogeneity |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.97 | 45 | $\mathrm{p}>.05 ;$ Normal | 1.75 | 1 | 88 | $\mathrm{p}>.05 ;$ |
| Experimental | 0.96 | 45 | $\mathrm{p}>.05 ;$ Normal |  |  |  | Equal variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of achievement in algebra after intervention of the Control and Experimental groups


## Appendix S

## Distribution of Achievement in Arithmetic after Intervention

Table S1. Statistical Constants of the Distribution of Achievement in Arithmetic after intervention of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\mathrm{a}}$ | Kurtosis $^{\mathrm{b}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 45.12 | 44.44 | 42 | 15.26 | 0.23 | -0.59 |
| Experimental | 52.96 | 55.56 | 36 | 14.30 | 0.17 | -0.87 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}} \mathrm{SE}$ of Skewness $=.35$, ${ }^{\mathrm{b}} \mathrm{SE}$ of Kurtosis $=.69$.
In the control group, achievement in arithmetic after intervention shows descending tendency from mean, median and mode which indicates positively skewed distribution. However, ratios of skewness and kurtosis to their respective standard errors (skewness/SE $=0.66$; kurtosis/SE $=0.86$ ) is less than 1.96; In experimental group, for achievement in arithmetic after intervention, median is highest, mode is least, and mean is in between them. However, ratios of skewness and kurtosis to their respective standard errors (skewness/SE $=0.49$; kurtosis/ $\mathrm{SE}=1.26$ ) is less than 1.96.

Table S2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Achievement in Arithmetic after intervention of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  |  | Levene's test of homogeneity |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.97 | 45 | $\mathrm{p}>.05 ;$ Normal | 0.00 | 1 | 88 | $\mathrm{p}>.05 ;$ |
| Experimental | 0.96 | 45 | $\mathrm{p}>.05$; Normal |  | Equal variances |  |  |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of achievement in arithmetic after intervention of the Control and Experimental groups


## Appendix T

## Distribution of Achievement in Geometry after Intervention

Table T1. Statistical Constants of the Distribution of Achievement in Geometry after intervention of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\mathrm{a}}$ | Kurtosis $^{\mathrm{b}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 51.20 | 50.00 | 50 | 16.72 | 0.13 | -1.01 |
| Experimental | 64.10 | 65.38 | 65 | 15.52 | -0.34 | -0.52 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}} \mathrm{SE}$ of Skewness $=.35$, ${ }^{\mathrm{b}} \mathrm{SE}$ of Kurtosis $=.69$.
In control group, mean, median and mode are almost equal for achievement in geometry after intervention. Also, ratios of skewness and kurtosis to their respective standard errors (skewness $/ \mathrm{SE}=0.37$; kurtosis $/ \mathrm{SE}=1.46$ ) are less than 1.96; In experimental group, mean, median and mode are almost equal for achievement in geometry after intervention. Also, ratios of skewness and kurtosis to their respective standard errors (skewness/SE $=0.97$; kurtosis $/ \mathrm{SE}=0.75$ ) is less than 1.96.
Table T2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Achievement in Geometry after intervention of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  |  | Levene's test of homogeneity |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.96 | 45 | $\mathrm{p}>.05 ;$ Normal | 0.58 | 1 | 88 | $\mathrm{p}>.05 ;$ Equal |
| variances |  |  |  |  |  |  |  |
| Experimental | 0.97 | 45 | $\mathrm{p}>.05$; Normal |  |  |  |  |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of achievement in geometry after intervention of the Control and Experimental groups


## Appendix U

## Distribution of Pretest Scores of Self-efficacy in Mathematics

Table U1. Statistical Constants of the Distribution of Pretest Scores of Self-efficacy in Mathematics of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\text {a }}$ | Kurtosis $^{\text {b }}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 66.41 | 67.50 | 68 | 10.97 | -0.59 | 0.63 |
| Experimental | 69.41 | 70.00 | 65 | 12.58 | -0.68 | 0.50 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, mean, median and mode are almost equal for pretest score of self-efficacy in mathematics. Ratios of skewness and kurtosis to their respective standard errors (skewness/SE $=$ 1.68 ; kurtosis/ $\mathrm{SE}=0.01$ ) are less than 1.96 ; In the experimental group, pretest score of self-efficacy in mathematics shows ascending tendency from mean through median, to mode suggesting a negatively skewed distribution. However, ratios of skewness and kurtosis to their respective standard errors (skewness/SE $=1.94$; kurtosis/ $\mathrm{SE}=0.72$ ) are less than 1.96.

Table U2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Pretest scores of Self-efficacy in Mathematics of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  | Levene's test of homogeneity |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.96 | 45 | $\mathrm{p}>.05 ;$ Normal |  |  |  | $\mathrm{p}>.05 ;$ |
| Experimental | 0.96 | 45 | $\mathrm{p}>.05 ;$ Normal |  | 1 | 88 | Equal variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of self-efficacy in mathematics before intervention of the Control and Experimental groups.


## Appendix V

## Statistical Constants of the Distribution of

 Posttest Scores of Attitude towards Mathematics, Self-efficacy in Mathematics and Dimensions of attitude and self-efficacyTable V1. Statistical Constants of the Distribution of Posttest Scores of Attitude towards Mathematics, Self-efficacy in Mathematics and Dimensions of attitude and self-efficacy of the Control (Practice in Solving Mathematics Problems) and Experimental (Language Integrated Mathematics Instruction) groups

| Variable | Groups | Mean | Median | Mode | SD | Skewness $^{\mathrm{a}}$ | Kurtosis $^{\mathrm{b}}$ |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: |
| Attitude towards | Control | 69.65 | 70.00 | 60 | 9.60 | -0.58 | 0.05 |
| mathematics | Experimental | 77.33 | 78.18 | 85 | 9.19 | -0.32 | -0.85 |
| Like towards | Control | 63.92 | 61.82 | 53 | 14.69 | -0.03 | -0.61 |
| mathematics | Experimental | 75.03 | 76.36 | 93 | 15.09 | -0.61 | -0.21 |
| Engagement with | Control | 68.00 | 66.67 | 64 | 16.48 | -0.48 | -0.22 |
| mathematics | Experimental | 75.75 | 77.78 | 80 | 14.91 | -0.56 | 0.09 |
| Self-belief in <br> mathematics | Control | 65.71 | 65.71 | 77 | 15.52 | -0.42 | -0.47 |
| Experimental | 73.46 | 74.29 | 86 | 16.43 | -0.36 | -0.35 |  |
| Active learning <br> of mathematics | Control | 78.39 | 80.00 | 80 | 12.66 | -0.84 | 1.26 |
| Enjoyment of <br> mathematics | Control | 73.58 | 75.56 | 62 | 15.14 | -0.51 | -0.15 |
| Experimental | 80.89 | 80.00 | 76 | 10.43 | 0.04 | -0.84 |  |
| Self-efficacy in <br> mathematics | Control | 70.15 | 70.83 | 68 | 12.70 | -0.65 | 0.88 |
| Self-efficacy in | Control | 70.77 | 72.31 | 71 | 15.23 | -0.91 | 1.22 |
| learning <br> mathematics | Experimental | 80.10 | 84.62 | 85 | 14.79 | -1.47 | 1.82 |
| Self-efficacy in <br> solving <br> mathematics <br> problems | Control | 69.41 | 69.09 | 65 | 12.75 | -0.12 | 0.04 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35,{ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.

## Appendix W

## Distribution of Gain Scores of Self-efficacy in Mathematics

Table W1. Statistical Constants of the Distribution of gain Scores of Self-efficacy in Mathematics of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\text {a }}$ | Kurtosis $^{\mathrm{b}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 9.57 | 9.16 | 6.66 | 5.11 | 0.07 | -0.42 |
| Experimental | 16.05 | 15.83 | 15.00 | 4.05 | 0.33 | 1.66 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35,{ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
Table ... shows that, in the control group, mean and median are almost equal, and mode is the least for gain score of self-efficacy in mathematics, indicating positively skewed distribution. Also, ratios of skewness and kurtosis to their respective standard errors (skewness/ $\mathrm{SE}=0.20$; kurtosis $/ \mathrm{SE}=0.61$ ) are less than 1.96 ; In the experimental group, mean, median and mode are almost equal for gain score of self-efficacy in mathematics.Also, ratio of skewness to its respective standard error (skewness/ $\mathrm{SE}=0.94$ ) is less than 1.96 , but ratio of kurtosis to its standard error is (kurtosis/ $\mathrm{SE}=2.41$ ) is greater than 1.96 .

Table W2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of gain scores of Self-efficacy in Mathematics of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  | Levene's test of homogeneity |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.97 | 45 | $\mathrm{p}>.05 ;$ Normal | 4.11 | 1 | 88 | $\mathrm{p}<.01 ;$ |
| Experimental | 0.94 | 45 | $\mathrm{p}<.01 ;$ Not normal |  |  |  | Unequal variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of gain scores in self-efficacy in mathematics of the Control and Experimental groups


## Appendix X

## Distribution of Pretest Scores of Self-efficacy in Learning Mathematics

Table X1. Statistical Constants of the Distribution of Pretest Scores of Self-efficacy in learning Mathematics of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\text {a }}$ | Kurtosis $^{\text {b }}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 67.18 | 69.23 | 69 | 13.11 | -0.72 | 0.88 |
| Experimental | 70.15 | 70.77 | 68 | 13.09 | -0.89 | 1.36 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, mean, median and mode, are almost equal for pretest scores of self-efficacy in learning mathematics. But, ratios of skewness and kurtosis to their respective standard errors (skewness/SE $=2.05$; kurtosis/ $\mathrm{SE}=2.51$ ) are greater than 1.96 ; In the experimental group, mean, median and mode, are almost equal for pretest scores of self-efficacy in learning mathematics. But, ratios of skewness and kurtosis to their respective standard errors (skewness $/ \mathrm{SE}=2.54$; kurtosis/ $\mathrm{SE}=1.97$ ) are greater than 1.96 .

Table X2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Pretest scores of Self-efficacy in learning Mathematics of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  | Levene's test of homogeneity |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.97 | 45 | $\mathrm{p}>.05$; Normal | 0.00 | 1 | 88 | $\mathrm{p}>.05 ;$ |
| Experimental | 0.95 | 45 | $\mathrm{p}>.05$; Normal |  |  |  | Equal variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of self-efficacy in learning mathematics before intervention of the Control and Experimental groups.


## Appendix Y

# Distribution of Gain Scores of Self-efficacy in Learning Mathematics 

Table Y1. Statistical Constants of the Distribution of gain Scores of Self-efficacy in learning Mathematics of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\mathrm{a}}$ | Kurtosis $^{\mathrm{b}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 11.28 | 9.23 | 7.69 | 6.54 | 0.82 | 1.32 |
| Experimental | 17.64 | 16.92 | 10.77 | 6.81 | 0.49 | -0.37 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, gain scores of self-efficacy in learning mathematics shows descending tendency from mean, through median, to mode, suggesting a positively skewed distribution. Also, ratios of skewness to its standard error (skewness/ $\mathrm{SE}=2.34$ ) is greater than 1.96). But ratio of kurtosis to its standard error (kurtosis/ $\mathrm{SE}=1.91$ ) is less than 1.96 ; in the experimental group also, gain scores of self-efficacy in learning mathematics shows descending tendency from mean, through median to mode suggesting a positively skewed distribution. However, ratios of skewness and kurtosis to their respective standard errors (skewness/ $\mathrm{SE}=1.4$; kurtosis $/ \mathrm{SE}=1.01$ ) are less than 1.96 .

Table Y2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Gain scores of Self-efficacy in learning Mathematics of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  |  | Levene's test of homogeneity |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.94 | 45 | $\mathrm{p}<.01$; Not normal | 0.53 | 1 | 88 | $\mathrm{p}>.05 ;$ |
| Experimental | 0.96 | 45 | $\mathrm{p}>.05 ;$ Normal |  |  |  | Equal variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of gain scores in self-efficacy in learning mathematics of the Control and Experimental groups.


## Appendix Z

## Distribution of Pretest Scores of Self-efficacy in Solving Mathematics Problems

Table Z1. Statistical Constants of the Distribution of Pretest Scores of Self-efficacy in solving Mathematics problems of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\text {a }}$ | Kurtosis $^{\text {b }}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 65.49 | 63.64 | 60 | 11.36 | -0.22 | 0.69 |
| Experimental | 68.53 | 70.91 | 71 | 14.27 | -0.59 | 0.21 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, pretest scores of self-efficacy in solving mathematics problems shows ascending tendency from mean, through median, to mode, suggesting a negatively skewed distribution. However, ratios of skewness and kurtosis to their respective standard errors (skewness/SE $=0.63$; kurtosis/ $\mathrm{SE}=1$ ) are less than 1.96 ; In the experimental group, mean, median and mode, are almost equal for pretest scores of self-efficacy in solving mathematics problems. But, ratios of skewness and kurtosis to their respective standard errors (Skewness/SE $=1.68$; kurtosis $/ \mathrm{SE}=0.30$ ) are less than 1.96.

Table Z2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Pretest scores of Self-efficacy in solving Mathematics problems of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  | Levene's test of homogeneity |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.97 | 45 | $\mathrm{p}>.05 ;$ Normal | 2.64 | 1 | 88 | $\mathrm{p}>.05 ;$ |
| Experimental | 0.97 | 45 | $\mathrm{p}>.05 ;$ Normal |  |  |  | Equal variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of self-efficacy in solving mathematics problems before intervention of the Control and Experimental groups


## Appendix AA

## Distribution of Gain Scores of Self-efficacy in Solving Mathematics Problems

Table AA1. Statistical Constants of the Distribution of gain Scores of Self-efficacy in solving Mathematics problems of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\mathrm{a}}$ | Kurtosis $^{\mathrm{b}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 13.01 | 10.91 | 9.09 | 5.71 | 0.31 | 0.31 |
| Experimental | 19.64 | 20.00 | 16.36 | 6.75 | 0.62 | 0.69 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45 .{ }^{\mathrm{a}} \mathrm{SE}$ of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, gain scores of Self-efficacy in solving mathematics problems shows descending tendency from mean, through median, to mode, suggesting a positively skewed distribution. However, ratios of skewness and kurtosis to their respective standard errors (skewness $/ \mathrm{SE}=0.89$; kurtosis $/ \mathrm{SE}=0.45$ ) are less than 1.96 ; In the experimental group, mean and median are almost equal, and mode is the least for gain scores of self-efficacy in solving mathematics problems suggesting a positively skewed distribution. However, ratios of skewness and kurtosis to their respective standard errors (skewness/ $\mathrm{SE}=1.77$; kurtosis $/ \mathrm{SE}=1$ ) are less than 1.96.

Table AA2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Gain scores of Self-efficacy in solving Mathematics problems of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  |  | Levene's test of homogeneity |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.95 | 45 | $\mathrm{p}>.05 ;$ Normal | 0.67 | 1 | 88 | $\mathrm{p}>.05 ;$ |
| Experimental | 0.96 | 45 | $\mathrm{p}>.05$; Normal |  |  |  | Equal variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of gain scores in self-efficacy in solving mathematics problems of the Control and Experimental groups


## Appendix AB

## Distribution of Self-efficacy in Algebra after Intervention

Table AB1. Statistical Constants of the Distribution of Self-efficacy in Algebra after intervention of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\mathrm{a}}$ | Kurtosis $^{\mathrm{b}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 70.51 | 71.43 | 73 | 11.12 | -0.33 | 0.19 |
| Experimental | 79.30 | 78.57 | 71 | 10.24 | -0.27 | -0.46 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35,{ }^{\mathrm{b}} \mathrm{SE}$ of Kurtosis $=.69$.
In control group, self-efficacy in algebra after intervention shows ascending tendency from mean, median and mode which indicates negatively skewed distribution. Also, ratios of skewness and kurtosis to their respective standard errors (skewness/ $\mathrm{SE}=0.94$; kurtosis $/ \mathrm{SE}=0.28$ ) is less than 1.96; In experimental group, for self-efficacy in algebra after intervention, mean and median are almost equal, and mode is the least. However, ratios of skewness and kurtosis to their respective standard errors (skewness $/ \mathrm{SE}=0.77$; kurtosis $/ \mathrm{SE}=0.67$ ) is less than 1.96.

Table AB2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Selfefficacy in Algebra after intervention of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  | Levene's test of homogeneity |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.99 | 45 | $\mathrm{p}>.05 ;$ normal |  |  |  |  |
| Experimental | 0.96 | 45 | $\mathrm{p}>.05 ;$ normal | 0.11 | 1 | 88 | $\mathrm{p}>.05$; Equal |
| variances |  |  |  |  |  |  |  |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of self-efficacy in algebra after intervention of the Control and Experimental groups.


## Appendix AC

## Distribution of Self-efficacy in Arithmetic after Intervention

Table AC1. Statistical Constants of the Distribution of Self-efficacy in Arithmetic after intervention of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\text {a }}$ | Kurtosis $^{\text {b }}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 73.47 | 74.67 | 92 | 13.19 | -0.09 | -1.07 |
| Experimental | 79.72 | 80.00 | 73 | 11.40 | -0.38 | -1.08 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In control group, mean and median are almost equal, and mode is the greatest for self-efficacy in arithmetic after intervention. However, ratios of skewness and kurtosis to their respective standard errors (skewness $/ \mathrm{SE}=0.26$; kurtosis/ $\mathrm{SE}=1.55$ ) is less than 1.96; In experimental group, for self-efficacy in arithmetic after intervention, mean and median are almost equal, and mode is the least. However, ratios of skewness and kurtosis to their respective standard errors (skewness/SE = 1.09; kurtosis/SE = 1.57) is less than 1.96.

Table AC2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Selfefficacy in Arithmetic after intervention of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  |  | Levene's test of homogeneity |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.95 | 45 | $\mathrm{p}<.01 ;$ Not normal | 1.38 | 1 | 88 | $\mathrm{p}>.05 ;$ |
| Experimental | 0.92 | 45 | $\mathrm{p}<.01 ;$ Not normal |  |  |  | Equal variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution self-efficacy in arithmetic after intervention of the Control and Experimental groups


## Appendix AD

## Distribution of Self-efficacy in Geometry after Intervention

Table AD1. Statistical Constants of the Distribution of Self-efficacy in Geometry after intervention of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\text {a }}$ | Kurtosis $^{\text {b }}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 71.90 | 71.54 | 55 | 12.38 | -0.20 | -1.12 |
| Experimental | 82.05 | 86.15 | 86 | 10.48 | -0.87 | -0.06 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In control group, mean and median are almost equal, and mode is the least for Self-efficacy in Geometry after intervention. Also, ratios of skewness and kurtosis to their respective standard errors (skewness $/ \mathrm{SE}=0.57$; kurtosis $/ \mathrm{SE}=1.62$ ) is less than 1.96 ; In experimental group, for Self-efficacy in Geometry after intervention, median and mode are almost equal, and mean is the least. However, ratios of skewness to its standard error (skewness/SE $=2.49$ ) is greater than 1.96. Ratio of kurtosis to its standard error (kurtosis/ $\mathrm{SE}=0.09$ ) is less than 1.96.

Table AD2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Self-efficacy in Geometry after intervention of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  |  | Levene's test of homogeneity |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.95 | 45 | $\mathrm{p}<.01$; Not normal | 2.03 | 1 | 88 | $\mathrm{p}>.05 ;$ |
| Experimental | 0.90 | 45 | $\mathrm{p}<.01 ;$ Not normal |  |  |  | Equal variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution self-efficacy in geometry after intervention of the Control and Experimental groups


Control Group



Experimental Group


## Appendix AE

## Distribution of Pretest Scores of Attitude towards Mathematics

Table AE1. Statistical Constants of the Distribution of Pretest Scores of Attitude towards Mathematics of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\text {a }}$ | Kurtosis $^{\text {b }}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 67.62 | 68.18 | 75 | 9.73 | -0.53 | -0.40 |
| Experimental | 65.90 | 68.18 | 69 | 9.37 | -0.24 | -0.93 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, pretest scores of attitude towards mathematics shows ascending tendency from mean, through median, to mode, suggesting a negatively skewed distribution. ratios of skewness and kurtosis to their respective standard errors; (skewness/ $\mathrm{SE}=1.51$; kurtosis/ $\mathrm{SE}=$ 0.58 ) are less than 1.96 ; In the experimental group, pretest scores of attitude towards mathematics shows ascending tendency from mean, through median, to mode suggesting a negatively skewed distribution. Ratios of skewness and kurtosis to their respective standard errors (skewness $/ \mathrm{SE}=0.68$; kurtosis $/ \mathrm{SE}=1.35$ ) are less than 1.96.

Table AE2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Pretest scores of Attitude towards Mathematics of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  | Levene's test of homogeneity |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | .96 | 45 | $\mathrm{p}>.05$; Normal | 0.00 | 1 | 88 | $\mathrm{p}>.05 ;$ |
| Experimental | .95 | 45 | $\mathrm{p}>.05$; Normal |  |  |  | Equal variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of pretest scores of attitude towards mathematics of the Control and Experimental groups


## Appendix AF

## Distribution of Gain Scores of Attitude towards Mathematics

Table AF1. Statistical Constants of the Distribution of gain Scores of Attitude towards Mathematics of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\mathrm{a}}$ | Kurtosis $^{\mathrm{b}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 2.94 | 2.73 | 2.73 | 1.56 | 0.42 | 0.20 |
| Experimental | 12.34 | 11.82 | 10.46 | 3.38 | 0.96 | 1.23 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, mean, median and mode are almost equal for gain score of attitude towards mathematics. Also, ratios of skewness and kurtosis to their respective standard errors (skewness $/ \mathrm{SE}=1.2$; kurtosis $/ \mathrm{SE}=0.29$ ) are less than 1.96 ; in the experimental group, gain score of attitude towards mathematics shows ascending tendency from mean, through median, to mode. Also, ratio of skewness to its respective standard error (skewness/ $\mathrm{SE}=2.74$ ) is greater than 1.96 suggesting deviation from normality. However, ratio of kurtosis to its standard error is (kurtosis/SE = 1.78) is less than 1.96.
Table AF2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of gain scores of Attitude towards Mathematics of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  |  | Levene's test of homogeneity |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.97 | 45 | $\mathrm{p}>.05 ;$ Normal | 13.84 | 1 | 88 | $\mathrm{p}<.01 ;$ |
| Experimental | 0.93 | 45 | $\mathrm{p}<.01 ;$ Not normal |  |  |  | Unequal variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of gain scores in attitude towards mathematics of the Control and Experimental groups


## Appendix AG

Distribution of Pretest Scores of Like towards Mathematics
Table AG1. Statistical Constants of the Distribution of Pretest Scores of Like towards Mathematics of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\text {a }}$ | Kurtosis $^{\mathrm{b}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 62.51 | 60 | 60 | 14.26 | -0.08 | -0.60 |
| Experimental | 61.86 | 60 | 51 | 14.79 | -0.24 | -0.83 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35,{ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, mean, median and mode, are almost equal for pretest scores of like towards mathematics. Ratios of skewness and kurtosis to their respective standard errors (skewness/ $\mathrm{SE}=$ 0.23 ; kurtosis/SE $=0.87$ ) are less than 1.96 ; In the experimental group, pretest scores of like towards mathematics shows ascending tendency from mean, through median, to mode suggesting a negatively skewed distribution. However, ratios of skewness and kurtosis to their respective standard errors (skewness $/ \mathrm{SE}=0.69$; kurtosis $/ \mathrm{SE}=0.97$ ) are less than 1.96.

Table AG2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Pretest scores of Like towards Mathematics of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  | Levene's test of homogeneity |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.97 | 45 | $\mathrm{p}>.05 ;$ Normal | 0.46 | 1 | 88 | $\mathrm{p}>.05 ;$ |
| Experimental | 0.95 | 45 | $\mathrm{p}>.05$; Normal |  |  |  | Equal variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of pretest scores of like towards mathematics of the Control and Experimental groups


## Appendix AH

## Distribution of Gain Scores of Like towards Mathematics

Table AH1. Statistical Constants of the Distribution of gain Scores of Like towards Mathematics of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\mathrm{a}}$ | Kurtosis $^{\mathrm{b}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 8.68 | 9.09 | 7.27 | 3.57 | -0.67 | 0.63 |
| Experimental | 20.44 | 18.18 | 10.91 | 10.67 | 0.89 | 0.66 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, median is highest, and mean is greater than mode which indicates skewed distribution of gain score of Like towards mathematics. However, ratios of skewness and kurtosis to their respective standard errors (skewness/SE $=1.91$; kurtosis/ $\mathrm{SE}=0.91$ ) are less than 1.96; In the experimental group, gain score of like towards mathematics shows descending tendency from mean, through median, to mode which indicates a positively skewed distribution. Also, ratio of skewness to its respective standard error (Skewness/ $\mathrm{SE}=2.54$ ) is greater than 1.96, suggesting deviation from normality. However, ratio of kurtosis to its standard error is (kurtosis/SE $=0.96$ ) is less than 1.96.

Table AH2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of gain scores of Like towards Mathematics of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  | Levene's test of homogeneity |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.92 | 45 | $\mathrm{p}<.01 ;$ Not normal | 33.46 | 1 | 88 | $\mathrm{p}<.01 ;$ |
| Experimental | 0.93 | 45 | $\mathrm{p}<.01 ;$ Not normal |  |  |  | Unequal variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of gain scores in like towards mathematics of the Control and Experimental groups.


## Appendix AI

## Distribution of Pretest Scores of Engagement with Mathematics

Table AI 1. Statistical Constants of the Distribution of Pretest Scores of Engagement with Mathematics of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\text {a }}$ | Kurtosis $^{\text {b }}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 65.58 | 64.44 | 58 | 16.61 | -0.26 | -0.39 |
| Experimental | 64.89 | 64.42 | 64 | 13.93 | -0.53 | -0.19 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, pretest scores of engagement with mathematics shows ascending tendency from mean, through median, to mode suggesting a negatively skewed distribution. However, ratios of skewness and kurtosis to their respective standard errors (skewness/SE $=0.74$; kurtosis $/ \mathrm{SE}=0.57$ ) are less than 1.96 ; In the experimental group, mean, median and mode, are almost equal for pretest scores of engagement with mathematics. Ratios of skewness and kurtosis to their respective standard errors (skewness/ $\mathrm{SE}=1.51$; kurtosis/ $\mathrm{SE}=0.28$ ) are less than 1.96.

Table AI 2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Pretest scores of Engagement with Mathematics of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  | Levene's test of homogeneity |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.98 | 45 | $\mathrm{p}>.05$; Normal | 2.82 | 1 | 88 | $\mathrm{p}>.05 ;$ Equal variances |
| Experimental | 0.96 | 45 | $\mathrm{p}>.05 ;$ Normal |  |  |  |  |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of engagement with mathematics before intervention of the Control and Experimental groups.


## Appendix AJ

## Distribution of Gain Scores of Engagement with Mathematics

Table AJ1. Statistical Constants of the Distribution of gain Scores of Engagement with Mathematics of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\mathrm{a}}$ | Kurtosis $^{\mathrm{b}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 9.09 | 8.89 | 6.67 | 4.69 | 0.96 | 1.26 |
| Experimental | 17.53 | 15.56 | 6.67 | 9.68 | 0.79 | 0.11 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, gain scores in engagement with mathematics shows descending tendency from mean, median and mode which indicates positively skewed distribution. Also, ratios of skewness to its standard errors (skewness/SE $=2.74$ ) is greater than 1.96 indicates deviation from normality. However, ratio of kurtosis to its standard error (kurtosis/SE = 1.83) is less than 1.96.; In the experimental group also, gain score of engagement with mathematics shows descending tendency from mean, through median, to mode. Also, ratio of skewness to its respective standard error (skewness/SE $=2.26$ ) is greater than 1.96, indicates deviation from normality. However, ratio of kurtosis to its standard error is (kurtosis/ $\mathrm{SE}=0.16$ ) is less than 1.96.

Table AJ2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of gain scores of Engagement with Mathematics of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  |  | Levene's test of homogeneity |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.89 | 45 | $\mathrm{p}<.01$; Not normal | 20.95 | 1 | 88 | $\mathrm{p}<.01 ;$ |
| Experimental | 0.93 | 45 | $\mathrm{p}<.01$; Not normal |  |  |  | Unequal variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of gain scores in engagement with mathematics of the Control and Experimental groups


## Appendix AK

## Distribution of Pretest Scores of Self-belief in Mathematics

Table AK1. Statistical Constants of the Distribution of Pretest Scores of Self-belief in Mathematics of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\mathrm{a}}$ | Kurtosis $^{\mathrm{b}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 63.62 | 62.86 | 63 | 15.42 | -0.34 | -0.62 |
| Experimental | 59.62 | 60.00 | 60 | 13.29 | -0.24 | -0.45 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}} \mathrm{SE}$ of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, mean, median and mode are almost equal for pretest scores of self-belief in mathematics. Ratios of skewness and kurtosis to their respective standard errors (skewness/SE $=0.97$; kurtosis $/ \mathrm{SE}=0.89$ ) are less than 1.96 ; In the experimental group, mean, median and mode, are almost equal for pretest scores of self-belief in mathematics. Ratios of skewness and kurtosis to their respective standard errors (skewness $/ \mathrm{SE}=0.69$; kurtosis $/ \mathrm{SE}=0.65$ ) are less than 1.96 .

Table AK2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Pretest scores of Self-belief in Mathematics of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  | Levene's test of homogeneity |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.97 | 45 | $\mathrm{p}>.05 ;$ Normal | 1.58 | 1 | 88 | $\mathrm{p}>.05 ;$ <br> Equal variances |
| Experimental | 0.97 | 45 | $\mathrm{p}>.05 ;$ Normal |  |  |  |  |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of self-belief in mathematics before intervention of the Control and Experimental groups


## Appendix AL

## Distribution of Gain Scores of Self-belief in Mathematics

Table AL1. Statistical Constants of the Distribution of Gain Scores of Self-belief in Mathematics of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\mathrm{a}}$ | Kurtosis $^{\mathrm{b}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 10.67 | 11.43 | 8.57 | 4.49 | -0.34 | -0.23 |
| Experimental | 22.41 | 22.86 | 8.57 | 12.14 | 0.03 | -0.92 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}} \mathrm{SE}$ of Skewness $=.35,{ }^{\mathrm{b}} \mathrm{SE}$ of Kurtosis $=.69$.
In the control group, mean and median are almost equal, and mode is the least for distribution of gain score of self-belief in mathematics. However, ratios of skewness and kurtosis to their respective standard errors (skewness/SE $=0.97$; kurtosis/ $\mathrm{SE}=0.33$ ) are less than 1.96 ; In the experimental group also, mean and median are almost equal, and mode is the least for distribution of gain score of self-belief in mathematics. However, ratios of skewness and kurtosis to their respective standard errors (skewness/ $\mathrm{SE}=0.09$; kurtosis/ $\mathrm{SE}=1.33$ ) are less than 1.96.

Table AL2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of gain scores of Self-belief in Mathematics of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  | Levene's test of homogeneity |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.95 | 45 | $\mathrm{p}>.05$; Normal | 46.34 | 1 | 88 | $\mathrm{p}<.01 ;$ |
| Experimental | 0.96 | 45 | $\mathrm{p}>.05$; Normal |  |  |  | Unequal variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of gain scores in self-belief in mathematics of the Control and Experimental groups.


## Appendix AM

## Distribution of Pretest Scores of Active Learning of Mathematics

Table AM1. Statistical Constants of the Distribution of Pretest Scores of Active learning of Mathematics of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\mathrm{a}}$ | Kurtosis $^{\mathrm{b}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 75.44 | 77.50 | 80 | 11.39 | -1.09 | 2.33 |
| Experimental | 72.22 | 75.00 | 80 | 14.07 | -0.07 | -0.89 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, pretest scores of active learning of mathematics shows ascending tendency from mean, through median, to mode, suggesting a negatively skewed distribution. Moreover, ratios of skewness and kurtosis to their respective standard errors (skewness/ $\mathrm{SE}=3.11$; kurtosis/SE $=3.38$ ) are greater than 1.96; In the experimental group also, pretest scores of active learning of mathematics shows ascending tendency from mean, through median, to mode suggesting a negatively skewed distribution. However, ratios of skewness and kurtosis to their respective standard errors (skewness/ $\mathrm{SE}=0.20$; kurtosis $/ \mathrm{SE}=1.29$ ) are less than 1.96.
Table AM2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Pretest scores of Active learning of Mathematics of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  |  | Levene's test of homogeneity |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.94 | 45 | $\mathrm{p}<.01$; Not normal |  |  |  | $\mathrm{p}<.01 ;$ |
| Experimental | 0.97 | 45 | $\mathrm{p}>.05$; Normal | 4.49 | 1 | 88 | Unequal <br> variances |

Histograms with normal curve which best fit on them, Box plots, and normal Q-Q plots of the distribution of active learning of mathematics before intervention of the Control and Experimental groups.


## Appendix AN

## Distribution of Gain Scores of Active Learning of Mathematics

Table AN1. Statistical Constants of the Distribution of gain Scores of Active learning of Mathematics of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\text {a }}$ | Kurtosis $^{\text {b }}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 10.44 | 10.00 | 10.00 | 4.10 | 0.12 | 1.08 |
| Experimental | 16.94 | 17.50 | 7.50 | 9.81 | 0.87 | -1.04 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35$, ${ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, mean, median and mode are almost equal for gain score of active learning of mathematics. Also, ratios of skewness and kurtosis to their respective standard errors (skewness $/ \mathrm{SE}=0.34$; kurtosis $/ \mathrm{SE}=1.57$ ) are less than 1.96 ; In the experimental group, mean and median are almost equal, and mode is the least for distribution of gain score of active learning of mathematics. However, ratios of skewness to its standard errors (skewness/SE = 2.49 ) is greater than 1.96 suggesting deviation from normality. However, ratio of kurtosis to its standard error (kurtosis/ $\mathrm{SE}=1.51$ ) is less than 1.96.

Table AN2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of gain scores of Active learning of Mathematics of the Control and Experimental groups

| Groups | Shapiro-Wilk test of <br> normality |  |  | Levene's test of homogeneity |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.96 | 45 | $\mathrm{p}>.05 ;$ Normal | 18.46 | 1 | 88 | $\mathrm{p}<.01 ;$ |
| Experimental | 0.95 | 45 | $\mathrm{p}>.05 ;$ Normal |  |  | Unequal variances |  |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of gain scores in active learning of mathematics of the Control and Experimental groups


Control Group



Experimental Group


## Appendix AO

## Distribution of Pretest Scores of Enjoyment of Mathematics

Table AO1. Statistical Constants of the Distribution of Pretest Scores of Enjoyment of Mathematics of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\mathrm{a}}$ | Kurtosis $^{\mathrm{b}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 72.05 | 73.33 | 84 | 14.23 | -0.39 | -0.38 |
| Experimental | 70.17 | 68.89 | 69 | 10.86 | -0.47 | -0.08 |

Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45$. ${ }^{\mathrm{a}}$ SE of Skewness $=.35,{ }^{\mathrm{b}}$ SE of Kurtosis $=.69$.
In the control group, pretest scores of enjoyment of mathematics shows ascending tendency from mean, through median, to mode, suggesting a negatively skewed distribution. However, ratios of skewness and kurtosis to their respective standard errors (skewness/ $\mathrm{SE}=1.11$; kurtosis $/ \mathrm{SE}=0.55$ ) are less than 1.96 ; In the experimental group, mean, median and mode, are almost equal for pretest scores of enjoyment of mathematics. Ratios of skewness and kurtosis to their respective standard errors (skewness $/ \mathrm{SE}=0.68$; kurtosis $/ \mathrm{SE}=0.00$ ) are less than 1.96.

Table AO2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of Pretest scores of Enjoyment of Mathematics of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  |  | Levene's test of homogeneity |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.97 | 45 | p>.05; Normal | 3.86 | 1 | 88 | $\mathrm{p}>.05$ <br> Equal variances |
| Experimental | 0.97 | 45 | p>.05; Normal |  |  |  |  |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of enjoyment of mathematics before intervention of the Control and Experimental groups


## Appendix AP

## Distribution of Gain Scores of Enjoyment of Mathematics

Table AP1. Statistical Constants of the Distribution of gain Scores of Enjoyment of Mathematics of the Control and Experimental groups

| Groups | Mean | Median | Mode | SD | Skewness $^{\mathrm{a}}$ | Kurtosis $^{\mathrm{b}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 8.20 | 8.89 | 6.67 | 3.57 | -0.08 | 0.29 |
| Experimental | 17.39 | 15.56 | 15.56 | 8.77 | 1.10 | 2.46 |
| Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45 .{ }^{\text {a }}$ SE of Skewness $=.35,{ }^{\mathrm{b}}$ SE of Kurtosis $=.69$ |  |  |  |  |  |  |

In the control group, mean and median are almost equal, and mode is the least for gain score of enjoyment of mathematics. However, ratios of skewness and kurtosis to their respective standard errors (skewness $/ \mathrm{SE}=0.23$; kurtosis $/ \mathrm{SE}=0.42$ ) are less than 1.96 ; In the experimental group, median and mode are equal, and mean is the highest for distribution of gain score of enjoyment of mathematics, indicating positively skewed distribution. Also, ratio of skewness and kurtosis to their respective standard errors (skewness/ $\mathrm{SE}=3.14$; kurtosis/ $\mathrm{SE}=3.56$ ) is greater than 1.96 suggesting deviation from normality.
Table AP2. Result of Shapiro- Wilk Test of Normality and Levene's Test of Homogeneity of gain scores of Enjoyment of Mathematics of the Control and Experimental groups

| Groups | Shapiro-Wilk test of normality |  | Levene's test of homogeneity |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Interpretation | Statistic | $\mathrm{df}_{1}$ | $\mathrm{df}_{2}$ | Interpretation |
| Control | 0.94 | 45 | $\mathrm{p}<.01$; Not normal | 19.31 | 1 | 88 | $\mathrm{p}<.01$; Unequal |
| Experimental | 0.93 | 45 | $\mathrm{p}<.01$; Not normal |  |  |  | variances |

Histograms with normal curve which best fit on them, box plots, and normal Q-Q plots of the distribution of gain scores in enjoyment of mathematics of the Control and Experimental groups.



[^0]:    \# Correlation with scale of self-efficacy for learning mathematics (Abidha \& Gafoor, 2018)

[^1]:    Note. $\mathrm{N}_{1}=\mathrm{N}_{2}=45 ;{ }^{* *} p<.01$

[^2]:    ${ }^{* *} p<.01$

[^3]:    **p<. 01

[^4]:    ${ }^{* *} p<.01$

[^5]:    **p<. 01

[^6]:    Note. $\mathrm{N}=200, \mathrm{~N}_{1}=\mathrm{N}_{2}=50$

[^7]:    Note: * indicates value outside the limits of DP or DI $\mathrm{N}=370$, Number of students in upper group= Number of students in lower group $=100$

