C 2822 (Pages : 14) Name..................................

Reg. No..............................

U.G./P.G. ENTRANCE EXAMINATION, APRIL 2021

MATHEMATICS

Time : Two Hours Maximum : 100 Marks

Instructions :

- *Question Paper* : The Question paper contains 50 Multiple Choice Questions.

  Among the four options of each question given as (a), (b), (c) and (d), only one will be the most appropriate answer. Mark the bubble containing the letter corresponding to the most appropriate answer in the OMR answer sheet using ball point pen (blue or black).

- *Negative Marking* : Total four marks will be given for each correct answer.

  One mark will be deducted for each wrong answer.

1. If $\neg$ stands for negation, then $\neg\{p \wedge [p \vee q]\}$ is equivalent to :

   (a) $p \wedge \neg q$. (b) $\neg p \wedge q$.

   (c) $p \vee \neg q$. (d) $\neg p \wedge \neg q$.

2. If F (x) : x is a friend and if P (x) : x is perfect, then what is the logical translation of :

   "None of my friends is perfect"

   (a) $\exists x\,(\mathrm{F}(x) \wedge \neg \mathrm{P}(x))$. (b) $\exists x\,(\neg \mathrm{F}(x) \wedge \neg \mathrm{P}(x))$.

   (c) $\neg \exists x\,(\mathrm{F}(x) \wedge \mathrm{P}(x))$. (d) $\neg \exists x\,(\neg \mathrm{F}(x) \wedge \mathrm{P}(x))$.

3. Let X, Y, Z be sets of sizes 2, 3 and 2 respectively. Let W = X × Y. Let E be the set of all subsets of W. The number of functions from Z to E is :

   (a) 3^4. (b) 4^6.

   (c) 3^{12}. (d) 4^3.

Turn over

4. If A = {0, 1, 2, ... 100} and f is a function from A to A such that $f(f(k)) = k$ for all k with $0 \leq k \leq 99$, then choose the correct one from the following :

   (a) $f(k) \neq k$ for all k in {0, 1, 2, ... 100}

   (b) f is onto but not one-one.

   (c) f is one-one but not onto.

   (d) f is a bijection.

5. The derivative $\frac{d}{dx}$ of real functions is a map :

   (a) From the set of all functions to the set of all functions.

   (b) From the set of all differentiable functions to the set of all differentiable functions.

   (c) From the set of all differentiable functions to the set of continuous functions.

   (d) From the set of all differentiable functions to the set of all functions.

6. The function $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ is :

   (a) Not differentiable at 0 but continuous at 0.

   (b) Differentiable everywhere and the derivative is continuous.

   (c) Differentiable everywhere and the derivative is not continuous.

   (d) The second derivative exist at all points.

7. If $a, b \in \mathbb{R}$, then choose the correct statement :

   (a) $|a| = \sqrt{a^2}$.

   (b) $|a+b| = |a| + |b|$.

   (c) If $0 < a < 1$, $a^2 > a$.

   (d) $\sqrt{ab} = \sqrt{a}\,\sqrt{b}$.

8. What is the supremum and infimum of the set $\left\{1+\frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$?

(a) Supremum = 2, infimum = 0.

(b) Supremum = $\frac{3}{2}$, infimum = 0.

(c) Supremum = 1, infimum = $\frac{1}{2}$.

(d) Supremum = infimum = 1.

9. What is $\lim_{n \to \infty}\left(\frac{n^2+1}{2n^2-32}\right)$?

(a) 0.

(b) 1.

(c) ∞.

(d) $\frac{1}{2}$.

10. If $S = \sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$, then what is the value of S ?

(a) ∞.

(b) 1.

(c) $\frac{1}{2}$.

(d) 2.

11. Which of the functions is uniformly continuous ?

(a) $f(x) = \frac{1}{x^2}$ on $(0, \infty)$.

(b) $f(x) = \frac{1}{x^2}$ on $(1, \infty)$.

(c) $f(x) = \sin\left(\frac{1}{x}\right)$ on $(0, 1)$.

(d) $f(x) = x^2, x \in \mathbb{R}$.

12. Which of the following equation(s) has/have a real solution ?

(a) $x^2 + x + 1 = 0.$

(b) $x = \cos x, x \in \left[0, \frac{\pi}{2}\right].$

(c) $x^7 + 4x^6 - 21x^8 + 44 = 0.$

(d) $x = e^x.$

(a) (B) only.

(b) (A) and (C).

(c) (C) only.

(d) (B) and (C).

13. Find the integral $\int_1^4 \frac{\sqrt{1+\sqrt{t}}}{\sqrt{t}}\,dt.$

(a) 1.

(b) 0.

(c) $\frac{4}{3}\left(3^{3/2} - 2^{3/2}\right).$

(d) $\frac{2}{3}\left(3^{3/2} - 2^{3/2}\right).$

14. Which of the following functions is not Riemann integrable on [0, 1] ?

(a) $f(x) = \begin{cases} x+1, & \text{if } x \text{ is irrational} \\ 0, & \text{if } x \text{ is rational} \end{cases}$

(b) $f(x) = \begin{cases} 1, & \text{if } 0 \le x \le \frac{1}{2} \\ 0, & \text{if } \frac{1}{2} < x \le 1 \end{cases}.$

(c) $f(x) = \sin x.$

(d) $f(x) = \begin{cases} x^3, & \text{if } 0 \le x \le \frac{1}{2} \\ x - \frac{1}{2}, & \text{if } \frac{1}{2} < x \le 1 \end{cases}.$

15. Which of the following sequence of functions is uniformly convergent ?

(a) $f_n(x) = x^n, x \in [0, 1]$.

(b) $f_n(x) = \dfrac{\sin(n^7 x^{11})}{\sqrt{n}}, x \in \mathbb{R}$.

(c) $f_n(x) = \dfrac{x^2 + nx}{n}, x \in \mathbb{R}$.

(d) $f_n(x) = \dfrac{nx}{1 + n^2 x^2}, x \in [0, \infty)$.

16. Pick out the correct statement.

(a) $\lim_{n \to \infty} \int_0^1 f(x)\, dx = \int_0^1 f_n(x)\, dx$ implies f_n converges to f uniformly.

(b) f_n converges to f uniformly implies f'_n converges to f' uniformly.

(c) $M_n = \sup\{|f_n(x) - f(x)| : x \in E\}$, then f_n converges to f uniformly on E if and only if $\lim_{n \to \infty} M_n = 0$.

(d) f_n converges to f pointwise on [0, 1] implies $\lim_{n \to \infty} \int_0^1 f_n(x)\, dx = \int_0^1 f(x)\, dx$.

17. Number of real roots for $f(x) = x^5 + 23x + 2$ is :

(a) 1.

(b) 5.

(c) 3.

(d) 4.

18. Compute the integral $\int_{|z|=1} z^2 \sin\left(\frac{1}{z}\right) dz$:

(a) $\frac{i\pi}{3}$.

(b) $\frac{-i\pi}{3}$.

(c) $\frac{2i\pi}{3}$.

(d) $\frac{-2i\pi}{3}$.

19. Pick out the correct statement.

(a) The map $f(z) = \frac{z+i}{z-i}$ maps unit circle onto itself.

(b) The map $f(z) = \bar{z}$ is analytic everywhere in $\mathbb{C}$.

(c) Let $f : \mathbb{C} \to \mathbb{C}$ be analytic everywhere and $\lim_{z \to \infty} f(z) = c$, for some $c \in \mathbb{C}$. Then f is a constant function.

(d) The function $f(z) = \sin z$ is bounded.

20. Find $b \in \mathbb{C}$ with $|b| < 1$ such that $\int_{|z|=1} \frac{e^z}{z-b} dz = \frac{2\pi i}{3}$.

(a) $\log \frac{2\pi}{3} + \frac{i\pi}{2}$.

(b) $-\log \frac{2\pi}{3} + \frac{i\pi}{2}$.

(c) $\log \frac{2\pi}{3} - \frac{i\pi}{2}$.

(d) $-\log \frac{2\pi}{3} - \frac{i\pi}{2}$.

21. Consider the following power series ; $f(z) = \sum_{n=1}^{\infty} n \log n z^n$, $g(z) = \sum_{n=1}^{\infty} \frac{e^{n^2}}{n} z^n$. If r and R are the radii of convergence of f and g respectively, then :

(a) $r = 0, \text{R} = 1$.

(b) $r = 1, \text{R} = 0$.

(c) $r = 1, \text{-R} = \infty$.

(d) $r = \infty, \text{R} = 1$.

22. Find the harmonic conjugate of $u : \mathbb{R}^2 \to \mathbb{R}$ defined by $u(x, y) = e^x \sin y$:

(a) $-e^y \cos x + C$.

(b) $-e^y \sin x + C$.

(c) $-e^x \cos y + C$.

(d) $e^{-x} \cos y + C$.

23. The function $f(z) = \dfrac{e^z + 1}{e^z - 1}$ has :

(a) A removable singularity at $z = 0$.

(b) Residue 2 at $z = 0$.

(c) Residue 1 at $z = 0$.

(d) An essential singularity at $z = 0$.

24. Let $A = \begin{bmatrix} 2 & 5 & 6 \\ 3 & 1 & -4 \\ 1 & 4 & 8 \end{bmatrix}$. The the co-factor of a_{12} is :

(a) -26.

(b) -27.

(c) -28.

(d) -29.

25. For what values of x, is the matrix $A = \begin{bmatrix} x-1 & x^2 & x^4 \\ 0 & x+2 & x^3 \\ 0 & 0 & x-4 \end{bmatrix}$ singular ?

(a) 1, 2, 4.

(b) -1, 2, 4.

(c) 1, -2, 4.

(d) 1, 2, -4.

26. Which of the following is not a subspace of $\mathbb{R}^3 = \{(x, y, z) : x, y, z \in \mathbb{R}\}$?

I All vectors of the form $(a, 0, 0)$.

II All vectors of the form $(a, 1,1)$.

III All vectors of the form (a, b, c), such that $b = a + c$.

IV All vectors of the form $(a, b, 0)$.

(a) I. (b) II.

(c) III. (d) IV.

27. Let $f_1(x) = x, f_2(x) = -\cos(x)$. Then the Wronskian of f_1 and f_2 at $\frac{\pi}{2}$ is :

(a) 0. (b) $\frac{\pi}{4}$.

(c) $\frac{\pi}{2}$. (d) π.

28. The row-echelon form of the matrix $\begin{bmatrix} 1 & -3 & 4 \\ 2 & -6 & 9 \\ -1 & 3 & -4 \end{bmatrix}$ is :

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. (b) $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

(c) $\begin{bmatrix} 1 & 1 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. (d) $\begin{bmatrix} 1 & -3 & 4 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

29. The rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & -4 \\ 0 & 0 & 5 \end{bmatrix}$ is :

(a) 0. (b) 1.

(c) 2. (d) 3.

30. The group of units of $\mathbb{Z}_{25}$ is :

(a) Isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_5$.

(b) Isomorphic to $\mathbb{Z}_4 \times \mathbb{Z}_5$.

(c) A group of order 24.

(d) A non abelian group.

31. Which of the following is a group ?

(a) $\mathbb{Z}$ under subtraction.

(b) $GL_2(\mathbb{R})$ under matrix addition.

(c) $\mathbb{R} \setminus \mathbb{Q}$ under multiplication.

(d) $\left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} \in M_2(\mathbb{R}) \mid a \in \mathbb{R} \setminus \{0\} \right\}$ under matrix multiplication.

32. If x, y and z are elements of a group such that $xyz - 1$, then which of the following is true ?

(a) $yzx = 1$.

(b) $yxz = 1$.

(c) $zyx = 1$.

(d) None of these.

33. What is the order of $\sigma = (1\ 2\ 3)\ (234) \in S_9$.

(a) 2.

(b) 3.

(c) 6.

(d) 9.

34. Which of the following is not a normal subgroup of S_4.

(a) D_4.

(b) A_4.

(c) $\{(1)\}$.

(d) $H = \{(1), (1\ 2)\ (3\ 4), (1\ 3)\ (2\ 4)\}, (1\ 4)\ (3\ 2)\}$.

35. Let G be a simple group and $\varphi : G \to G'$ be a non trivial homomorphism on G. Then :

(a) φ is one to one.

(b) φ is on to.

(c) φ is an isomorphism.

(d) None of the above.

36. The co-efficient of $\left(x - \frac{1}{2}\right)^5$ in the expansion of e^x at $x = \frac{1}{2}$ is :

(a) $\frac{\sqrt{e}}{5!}$.

(b) $\frac{e^2}{5!}$.

(c) $\frac{e^5}{5!}$.

(d) $\frac{1}{5!}$.

37. The real root of $x^3 + x + 1 = 0$ lies between :

(a) -2 and -1.

(b) 1 and 2.

(c) -1 and 0.

(d) 2 and 3.

38. Let $f : [0, 1] \to \mathbb{R}$ be differentiable such that $|f'(x)| < 1$ for all $x \in [0, 1]$. Then :

(a) There exists at most one $c \in [0, 1]$ such that $f(c) = c$.

(b) There exist more than one $c \in [0, 1]$ such that $f(c) = c$.

(c) There exist infinitely many $c \in [0, 1]$ such that $f(c) = c$.

(d) None of the above.

39. The curvature of the space curve, $\bar{\gamma}(t) = e^t \cos t \, \hat{i} + e^t \sin t \, \hat{j} + 2\hat{k}$ at $t = 1$ is :

(a) $\frac{1}{\sqrt{2}} e$.

(b) $\frac{1}{\sqrt{2}\, e}$.

(c) $\sqrt{2}\, e$.

(d) $\frac{\sqrt{2}}{e}$.

40. The work done by the force field, $\vec{F} = z\hat{i} + x\hat{j} + y\hat{k}$ over the curve $\bar{\gamma}(t) = \sin t\hat{i} + \cos t\hat{j} + t\hat{k}, \ 0 \le t \le 2\pi$ is :

(a) 2π.

(b) π.

(c) $-\pi$.

(d) 0.

41. The value of $\oint_C (6y + x)\,dx + (y + 2x)\,dy$ where C is the circle :

$(x-2)^2 + (y-3)^2 = 4$ is :

(a) -16π.

(b) 16π.

(c) 8π.

(d) -8π.

42. Let $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and S be the parallelepiped bounded by

$x = 0, y = 0, z = 0; x = 2, y = 1, z = 3$. If $\vec{n}$ is surface's outward unit normal field, then the value of

$\iint_S \vec{F}\cdot\vec{n}\,d\sigma$ is

(a) 15.

(b) 25.

(c) 26.

(d) 30.

43. The vector field, $\vec{F} = \left(6xy + z^3\right)\hat{i} + \left(3x^2 - z\right)\hat{j} + \left(3xz^2 - y\right)\hat{k}$ is :

(a) Solenoidal.

(b) Irrotational.

(c) Rotational.

(d) Non-Conservative.

44. Suppose a radio-active material has a half-life of T hours. What percentage of the original sample is left after 5T hours ?

(a) 10×2^{-5}.

(b) 100×2^{-4}.

(c) 100×2^{-5}.

(d) 10×2^{-4}.

45. Which one among the following is the Laplace transform of $\frac{1}{\sqrt{t}}$?

(a) $\sqrt{\frac{\pi}{S}}$.

(b) $\frac{\sqrt{\pi}.}{S}$.

(c) $\frac{\sqrt[3]{\pi}}{S}$.

(d) $\frac{\pi}{S}$.

46. Which one among the following differential equations represents the family of parabolas with foci at the origin and axis along the x – axis ?

(a) $y(y')^2 + 2xy' - y^2 = 0$.

(b) $y(y')^2 + 2xy' - y = 0$.

(c) $y^2(y')^2 + 2xy' - y^2 = 0$.

(d) $y(y')^2 + 2xy' + y = 0$.

47. What will be the next level approximation of a root of $f(x) = x^2 - 4$ with Newton-Raphson method correct upto two decimal places, if the initial guess is 6 ?

(a) 6.33. (b) 2.33.

(c) 5.33. (d) 3.33.

48. What will be the approximate value of y (1. 6) for the initial value problem $y' = \frac{2x - y}{3}$, $y(1) = 0$ using Euler's method with a step size 0.3 ?

(a) 0.22. (b) 0.33.

(c) 0.44. (d) 0.55.

49. Consider the LP problem: Maximize $Z = -x_1 + 2x_2$ subject to the constraints $x_1 - x_2 \le -1$ and $-0.5x_1 + x_2 \le 2$ and $x_1, x_2 \ge 0$. Choose the optimal solution from the following :

(a) 3.
(b) 4.
(c) 5.
(d) 6.

50. The LP problem Maximize $Z = 3x_1 + 2x_2$ subject to the constraints

$x_1 - x_2 \ge 1, x_1 + x_2 \ge 3$ and $x_1, x_2 \ge 0$ has :

(a) No solution.
(b) Two solutions.
(c) Unbounded solution.
(d) Multiple optimal solution.